Stellar Parameter Determination

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Composition

Luminosity ($L$), Radius ($R$), Mass ($M$), Effective temperature ($T_{\text{eff}}$), Surface gravity ($\log g$)

Model Atmosphere

Spectra, Photometry
Effective Temperature

\[ \sigma T_{\text{eff}}^4 \equiv \int_{0}^{\infty} F_\nu \, d\nu = F_* = \frac{L}{4\pi R^2} \]

- Temperature of a black body that gives the same total power per unit area.
- Physically related to \( F_* \) total radiant power per unit area at stellar surface.
- \( T_{\text{eff}} \) of star is temperature of blackbody with same luminosity and radius as the star.
$T_{\text{eff}}$: Observable quantities

$F_* = \frac{\theta^2}{4} f_\oplus$

- $f_\oplus \text{ total}$ flux at earth (UV, optical, IR)
  - Corrected for interstellar reddening
- $\theta$ is angular diameter
  - Directly: interferometry, lunar occultations
    - Use limb-darkening corrected values
  - Indirectly from eclipsing binary systems with known distances (parallaxes): $\theta \propto R/d \propto R \pi$
  .. and asteroseismology!
Surface Gravity

\[ g = g_\odot \frac{M}{R^2}, \quad \rho = \rho_\odot \frac{M}{R^3} \quad g = R \rho = M^{1/3} \rho^{2/3} \]

• Directly given by stellar mass and radius.
  – An indirect measure of photospheric pressure

• Obtainable from:
  – Pairs (binary stars) R and M
  – Pulsations (astereoseismology) \( \rho \) and R
  – Planets (transits) Just \( \rho \) (but need M or R)
Fundamental Stars

- Fundamental stars can give accurate values of $T_{\text{eff}}$ and $\log g$ for selected stars only.
  - Except for the Sun, good to no better than 1~2 %
- Composition is not directly measured
  - Closest is the Sun via solar system material
    - Fe $7.50 \pm 0.04$ (photosphere) $7.45 \pm 0.01$ (meteorites)
      - Asplund et al., 2009, ARA&A, 47, 481

Everything else is model dependent!
F,G and K Reference Stars

- For example, the GAIA Benchmark Stars
  - 34 Stars with well determined parameters
    - Not all have fundamental $T_{\text{eff}}$ and $\log g$.
Indirect Methods

- Direct determination is usually impractical for most stars.
- Have to use indirect methods:
  - Photometric calibrations
  - Infrared Flux Method
  - Spectrophotometric flux fitting
  - Balmer Profiles
  - Line ratios
  - Equivalent Width Analysis
  - Spectrum Synthesis
Equivalent Width

- Measure of number of absorbers
  - Abundance
  - No information on profile shape

- Measure EWs of spectral lines
  - Manually
  - Automatically (ARES, DAOSPEC)

\[
W_\lambda = \int_0^\infty \left(1 - \frac{F_\lambda}{F_0}\right) d\lambda
\]

Small errors in continuum can lead to relatively large errors in EW

- Avoid strong lines with wings
  - Profile truncation leads to underestimated EW
Metal Line Diagnostics

- log A versus Excitation Potential ($T_{\text{eff}}$)
  - Abundances from the same element should agree for all excitation potentials, i.e. no trend
- log A versus EW (microturbulence)
  - Adjust $V_{\text{micro}}$ until no trend with EW
- Ionization Balance ($\log g$)
  - Average log A obtained from differing ionization stages of the same element must agree
    - Fe I/Fe II ratio can be used as a $T_{\text{eff}}$ – log g diagnostic
Effect of changing parameters

Simulation
Base model
$T_{\text{eff}} = 6000$ K
$\log g = 4.5$
$\log A(\text{Fe}) = 7.50$

Fe I lines (not pressure sensitive in Solar-type stars)
Wavelength range 5000-6000Å
$5 < \text{EW} < 100$ mÅ (avoid very weak or saturated lines)
Spectral Fitting

• Measuring equivalent widths might not always be practical.
  – Blending, high rotation, etc.

• Take all or selected parts of spectrum
• Vary input parameters and calculate synthetic spectrum.
• Fit best fitting solution (minimize $\chi^2$)
  – Error estimates are usually just internal precision

What about missing or incorrect line data?
χ² Correlations

- Correlation between [M/H] and $T_{\text{eff}}$
- Weak sensitivity to log $g$

Simulation
Base model
$T_{\text{eff}} = 6000$ K
log $g = 4.5$
log A(Fe) = 7.50

Generated with all lines with EW >5mÅ in wavelength range 5000-6000Å
Assumed S/N 100:1 for χ² calculation
A complex stellar recipe

- **Atomic/Molecular data**
  - Log $gf$, damping constants, missing/bad lines, hyperfine structure, isotopes

- **Atmospheric Physics**
  - NLTE, convection, turbulence, spots, abundance clouds

- **Modelling Code internals**
  - Partition functions, continuous opacities, numerical precision

- **Analysis Methods**
  - Equivalent widths, profile fitting, choice of lines and wavelength regions

- **Data Quality**
  - S/N, scattered light, continuum normalisation, telluric/interstellar lines

- **Stellar properties**
  - Binarity, variability

Plus other ingredients!
Pedagogical Heuristic Simulations
Collisional Broadening

  - Even weak lines can be affected by damping
  - Damping errors depend on excitation potential
    - errors in $v_{\text{mic}}$ and $T_{\text{eff}}$

![Graph showing the relationship between $\gamma/\gamma(U)$ and $\chi$ (eV)]
Effect of damping

- Effect of +20% error in van der Waals damping constant
  - could lead to errors in microturbulence and $T_{\text{eff}}$

$T_{\text{eff}} = 6000 \text{ K}$

Log $g = 4.5$
Astrophysical \textit{gf} values

- Line data is often inaccurate or missing
- Take spectrum of star with known properties and adjust synthesis line data until fits
  - Usually the Sun for late-type stars
  - Assume abundances are known
  - Mostly adjust just oscillator strengths (\textit{gf} values)
- Widely-used and can give good results
  - \textbf{But} values do depend on model and assumed parameters.
    - Damping constants, microturbulence, convection, ...
Astrophysical $gf$ Systematics

- Astrophysical $gf$ values created at 6000 K but with +20% error in van der Waals damping.
  - Plots show difference in at 6500 K.
Microturbulence

- A **free** parameter introduced to ensure that abundances from weak and strong lines agree
- Extra source of line broadening
  - added to thermal broadening
  - Small scale motions within the atmosphere

Microturbulence varies with $T_{\text{eff}}$
  - increases with increasing temperature
  - peaking around mid-A type


Not need in 3-d simulations
Solar Microturbulence Value

- Edvardsson et al. 1993 \((A&A, 275, 101)\) 1.15 km/s
- Bruntt et al. 2010 \((MNRAS, 405, 1907)\) 0.95 km/s
- Valenti & Fischer 2005 \((ApJS, 159, 141)\) 0.85 km/s
- Santos et al. 2004, \((A&A, 415, 1153)\) 1.00 km/s
- Magain 1984 \((A&A, 134, 189)\) 0.85 km/s (centre of solar disk)

Which to use in Astrophysical \(gf\) determination?
Astrophysical $gf$ Systematics

- Astrophysical $gf$ values created at 6000 K but with microturbulence too low by 0.1 km/s.
  - 0.9 km/s instead of “true” 1.0 km/s
  - Plots show difference at 6500 K
Valenti & Fischer 2005 found:

“strongly correlated values of $v_{\text{mic}}$ and [M/H], suggesting that $v_{\text{mic}}$ and [M/H] are partially degenerate.”

Adopted fixed value.

Gray 2001 fit by Smalley 2004, IAUS, 224, 131
Bruntt et al., 2010, MNRAS, 405, 1907
Fixing log $g$ can lead to incorrect other parameters.
Surface Gravity from Spectroscopy

Spectroscopic log $g$ can be accurately determined to ±0.1 dex by spectral analysis alone!
Starspots

• Simulate a spotted star with 5% spot coverage.
• Take two models: \( T_{\text{eff}} = 6000 \text{ K} \) and \( T_{\text{eff}} = 5000 \text{ K} \)
  – Both with \( \log g = 4.5 \)
• Generate spectra and combine 95% and 5%
• Fit with single \( T_{\text{eff}} \) model
• \( \text{H}_\alpha \) gives 5950 K. Agrees with Stefan's Law:
  \[
  (0.95 \times 6000^4 + 0.05 \times 5000^4)^{1/4} = 5953
  \]
• But, what \( \log g \) does Na D give?
Effect of “Spot” on Na D line

Spectroscopic log $g$ overestimated in spotted stars?
Spots and EWs

- Effect on determination of $T_{\text{eff}}$ and $\log g$
  - depends on choice of lines.
Macroturbulence

Doyle et al. 2014 ArXiv 1408.3988

Difficult to disentangle between $v \sin i$ and macroturbulence on line profile

Base on $v \sin i$ from Kepler astereoseismology

Doyle et al. 2014 ArXiv 1408.3988
A few Case Studies from the Literature
### WASP-13

#### Spectroscopically-determined Stellar Parameters of WASP-13

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SME</th>
<th>EW/ UCLSYN</th>
<th>ARES/ MOOG</th>
<th>ARES/ MOOG</th>
<th>Weighted Mean*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{eff}}$ (K)</td>
<td>6003 ± 65</td>
<td>5955 ± 75</td>
<td>5919 ± 30</td>
<td>6025 ± 21</td>
<td>5989 ± 16 ± 48</td>
</tr>
<tr>
<td>$\log g$</td>
<td>4.16 ± 0.08</td>
<td>4.13 ± 0.11</td>
<td>4.02 ± 0.06</td>
<td>4.19 ± 0.03</td>
<td>4.16 ± 0.03 ± 0.07</td>
</tr>
<tr>
<td>$\log A(\text{Fe})$</td>
<td>7.54 ± 0.06</td>
<td>7.60 ± 0.09</td>
<td>7.54 ± 0.05</td>
<td>7.58 ± 0.05</td>
<td>7.56 ± 0.03 ± 0.03</td>
</tr>
<tr>
<td>[Fe/H]</td>
<td>0.04 ± 0.05</td>
<td>0.10 ± 0.09</td>
<td>0.04 ± 0.02</td>
<td>0.08 ± 0.02</td>
<td>0.06 ± 0.01 ± 0.03</td>
</tr>
<tr>
<td>$v \sin i$ (km s$^{-1}$)</td>
<td>5.79 ± 0.08</td>
<td>5.26 ± 0.25</td>
<td>...</td>
<td>...</td>
<td>5.74 ± 0.08 ± 0.38</td>
</tr>
<tr>
<td>$v_t$ (km s$^{-1}$)</td>
<td>1.01 ± 0.17</td>
<td>0.95 ± 0.10</td>
<td>1.53 ± 0.09</td>
<td>1.28 ± 0.10</td>
<td>1.27 ± 0.06 ± 0.29</td>
</tr>
</tbody>
</table>

*Spread in values*

- H$_\alpha$ 5950 ± 70 K; log $g$ (Transit) 4.10 ± 0.04
- IRFM: 5935 ± 183 K

Gaia-ESO Survey

- The analysis of 1301 FGK-type stars (2014arXiv1409.0568S)
- 13 independent groups and methods
  - All using MARCS models (no Kurucz ATLAS models)
- *Method-to-method dispersion* of the atmospheric parameters
  - $T_{\text{eff}}$ 55 K, $\log g$ 0.13 dex, [Fe/H] 0.07 dex
- *Systematic biases* are estimated to be between
  - $T_{\text{eff}}$ 50-100 K, $\log g$ 0.10-0.25 dex, [Fe/H] 0.05-0.10
- The typical method-to-method dispersion of elemental abundances varies between 0.10 and 0.20 dex.

All spectral analysis methods are well developed and yield satisfactory results.
Summary

• Analyses should include sufficient reference stars
  – use exactly the same methods and quality of spectra.
• Use as many diagnostics as possible
  – Spectroscopic and photometric.
• Realistically the typical errors:
  – $T_{\text{eff}} \pm 50\text{~to~}100\text{K}$
  – $\log g \pm 0.1\text{~to~}0.2$ dex
  – Abundances $\pm 0.05\text{~to~}0.10$ dex

Errorbars in stellar analyses usually show how well the model fits to the data and not how good is the model.

High precision fitting to high S/N data is possible, but overall accuracy of parameters is less certain.