Inflation within MSSM

Kari Enqvist
University of Helsinki
and
Helsinki Institute of Physics
Inflaton is a great idea …but many open questions

particle theory origins?
- usually assumed to be gauge singlet
  (radiative corrections can spoil flatness)
- Not motivated by theory: no known gauge singlets

coupling to baryons?
- reheating/origin of matter

If singlet, no testable consequences in the lab
- how can ever make sure inflation is correct?
Inflation from MSSM

- MSSM has scalars: sleptons, squarks, higgses
- low scale $\rightarrow$ low $H$; field values $<< M_p$
- perturbations $\sim H/\epsilon$ $\rightarrow$ need very flat potential
- but then: radiative corrections can spoil everything
  
  *need extra protection*

susy, gauge symmetry $\rightarrow$ directions in MSSM scalar field space with $V=0$ in the limit of exact susy
Superpotential: $F = \lambda LHe + \lambda' QHu + \ldots$

unbroken susy:

$$V = \sum |F_i|^2 + \frac{1}{2}D^2$$

$$F_i = \frac{\partial F}{\partial \phi_i} = 0$$ \hspace{1cm} F-flat

$$D^a = \phi_i^* T^a \phi_i = 0$$ \hspace{1cm} D-flat

global $U(1)$: $\phi_i \rightarrow \phi_i e^{i\theta}$

flat directions typically carry $B$ and/or $L$

(curved) trajectory

d=49

$$\varphi = \sum a_i f(\varphi_i)$$

squarks, sleptons
MSSM flat directions all classified:

\[ H_u H_d \]
\[ L H_u \]
\[ u^c d^c d^c \]
\[ L L e^c \]
\[ Q Q Q L \]
\[ u^c u^c d^c e^c \]
\[ Q Q u^c u^c e^c \]
\[ u^c u^c u^c e^c e^c \]
\[ Q Q L L d^c d^c \]
\[ Q Q Q Q L L d^c \]
\[ Q L u^c Q Q d^c d^c \]

\[ B=L=0 \]
purely leptonic
purely baryonic

Dine, Randall, Thomas

example: udd

\[ u_1^\alpha = d_2^\beta = d_3^\gamma = \frac{1}{\sqrt{3}} \phi \quad (\alpha \neq \beta \neq \gamma) \]
FLAT DIRECTIONS AND GRAVITY

- Inflation means gravity
- Gravity is not renormalizable
- MSSM + gravity is a non-renormalizable effective theory valid at scales $\ll M_p$

$\rightarrow$ must include all the non-renormalizable superpotential terms

- for each flat direction, the first non-renormalizable term allowed by gauge and supersymmetry has a definite dimension $n$
- in addition, supersymmetry is broken softly in the Nature

flat directions ”lifted”: $V \neq 0$
Lifting flat directions

soft susy breaking

\[ V = \frac{1}{2} m^2 |\phi|^2 \]

non-renormalizable terms

\[ F = \frac{\phi^n}{M^{n-3}}, \psi \phi^{n-1} / M^{n-3} \]

soft breaking A-terms

\[ V = |F_k|^2 = \phi^{2(n-1)} / M^{2(n-3)} \]

\[ V = \frac{1}{2} \text{Re } AF / M^{(n-3)} \]

All flat directions lifted by operators dim \(\leq 9\)
example: udd, lifted by dim=6 operators

\[ w \supset \frac{1}{M_p^3} (uud)(uud) \]
Flat direction potential

\[ \phi = \phi e^{i\theta} \]

\[ V = \frac{1}{2} m_\phi^2 \phi^2 + A \cos(n\theta + \theta_A) \frac{\lambda_n \phi^n}{nM_P^{n-3}} + \lambda_n^2 \frac{\phi^{2(n-1)}}{M_P^{2(n-3)}} \]

A-term \( \rightarrow \) n-fold set of minima when \( \cos = -1 \) provided

\[ A^2 \geq 8(n-1)m_\phi^2 \]

(but not too large A: otherwise true minimum breaking colour)
barrier if $A^2 > 8(n-1)m^2$ inflation never ends

barrier vanishes if

then

$V'(\phi_0) = V''(\phi_0) = 0$

$V'''(\phi_0) = 2(n-2)^2 \frac{m^2}{\phi_0}$

$saddle point$

$V(\text{Im} \phi) \propto m^2 \phi^2$

very flat!
at the saddle point

\[
\phi_0 = \left( \frac{m_\phi M_P^{n-3}}{\lambda_n \sqrt{2n-2}} \right)^{1/(n-2)}
\]

\[
H = \frac{n-2}{\sqrt{6n(n-1)}} \frac{m_\phi \phi_0}{M_P}
\]

saddle point $\leftrightarrow$ eternal inflation regime

2-point correlator spreads in time

classical slow roll takes over when

\[
(\phi_0 - \phi) = \left( \frac{m_\phi \phi_0^2}{M_P^3} \right)^{1/2} \phi_0
\]

business as usual
the model:

$L_i L_j e_k$ or $u d_i d_j$ flat direction

(others: wrong amplitude)

$m_\phi \approx O(1) \, TeV; n = 6; A = \sqrt{40} m_\phi; \lambda \approx O(1)$

$H_{\text{inf}} \approx O(1) \, GeV; \phi_0 \approx O(10^{14}) \, GeV$

slow roll $\rightarrow$ saddle point condition must hold very precisely

$\left(\frac{A}{m_\phi}\right)^2 = 40(1 + O(10^{-16}))$
# of total e-folds
\[ N = \int H_{\text{inf}} \frac{d\phi}{\dot{\phi}} \approx 10^3 \]

# of observationally relevant e-folds (immediate decay)
\[ N_{\text{COBE}} \approx 47 \]

amplitude of perturbations
\[ \delta_H = \frac{1}{5\pi} \frac{H^2}{\dot{\phi}} \approx 1.9 \times 10^{-5} \]

slow roll parameters
\[ \varepsilon \approx \frac{1}{N_{\text{COBE}}^4} ; \eta \approx -\frac{2}{N_{\text{COBE}}} \]

spectral tilt
\[ n_s = 1 - \frac{4}{N_{\text{COBE}}} \approx 0.92 \]
\[ \frac{dn_s}{d \ln k} = -\frac{4}{N_{\text{COBE}}} \approx -0.002 \]

no gravitational waves
REHEATING

after inflation oscillation frequency ~ $m_\phi \sim 10^3H$

expansion negligible

udd breaks SU(3) U(1)
Lle breaks SU(2)×U(1)
susy conserving masses ~ $g\phi$

instant preheating

$\phi \rightarrow$ gauge bosons, gauginos $\rightarrow$ squarks/quarks within 1 Hubble time

details remains to be worked out
### Measuring inflaton parameters at colliders

**CMB amplitude**

\[ m_\phi (\phi_0) = 340 \text{GeV} \lambda^{-1} \]

#### run down to TeV:

For \( \text{LLe} \):\[
\begin{array}{|c|c|c|}
\hline
\xi \ (\text{gaugino/flat}) & m_\phi (\text{TeV}) \\
\hline
2 & (1.9)^{\frac{1}{2}} \ 340 \ \text{GeV} \\
1 & (1.3)^{\frac{1}{2}} \ 340 \ \text{GeV} \\
0.5 & (1.1)^{\frac{1}{2}} \ 340 \ \text{GeV} \\
\hline
\end{array}
\]

LHC slepton mass limits can rule the model out!
WHY SADDLE POINT?

What fixes A/m?

In MSSM just parameters put in by hand

must go beyond MSSM

KE, Mether, Nurmi
Nurmi
SUPERGRAVITY CHANGES THINGS

expect corrections

\[ \delta V_{\text{SUGRA}} = H^2 M^2 \left( \frac{\phi}{M} \right)^2 \]

\[ H \approx \frac{m_\phi \phi_0}{M} \implies \delta V_{\text{SUGRA}} \ll V_{\text{MSSM}} \]

small, but the saddle point is finetuned:

\[ \delta \left( \frac{m}{A} \right)^2 \approx \frac{\phi_0^2}{M^2} \approx 10^{-10} \gg 10^{-16} \]

depends on the Kähler potential
\[ V_{SUGRA} = e^G (G_i G_{\bar{j} \bar{j}} G^{\bar{j} j} - 3) \]

\[ G_{\bar{i} \bar{j}} = G_{\bar{i} \bar{j}} (h, \phi) \]

Kähler metric

hidden

observable

\[ m^2 = \frac{\partial^2}{\partial \phi \partial \bar{\phi}} V_{SUGRA} (h) \]

\[ A_6 = V^{(6)}_{SUGRA} (h) \]
Require:

\[ A = \sqrt{40} m_\phi \]

\[ \phi = \phi_0 \]

not accidental but holds for all values of hidden sector fields

Kähler potential, moduli fields
Let us assume

\[
W = \hat{W}(h_m) + \frac{\hat{\lambda}(h_m) \phi^6}{6M_P^3}
\]

Kähler potential

\[
G = K + \ln |W|^2
\]

\[
K = \hat{K}(h_m, \bar{h}_m) + \sum_{n=1}^{\infty} \hat{Z}_{2n}(h_m, \bar{h}_m) \phi^{2n}
\]

does there exist \( K \) such that \( A = \sqrt{40} m_\phi \) identically?

KE, Mether, Nurmi

YES
to lowest order

\[ V = (V_0 + V_2 \phi^2 + V_6 \phi^6 + V_{10} \phi^{10})(1 + O(\phi^2)) \]

\[ V_2 = e^{\bar{K}} \left| \hat{W} \right|^2 \hat{Z}_2 (\hat{K}_m^m \hat{K}_m + \hat{K}_m^m \hat{K}_n (\hat{Z}_2^{-2} \hat{Z}_{2m} \hat{Z}_{2n} - \hat{Z}_2^{-1} \hat{Z}_{2m\bar{n}}) - 2) \]

etc. saddle point condition

\[ \left| \hat{K}_m^m \hat{K}_m - 6\hat{Z}_2^{-1} \hat{K}_m^m \hat{Z}_{2m} + 3 \right|^2 = 20(\hat{K}_m^m \hat{K}_m + \hat{K}_m^m \hat{K}_n (\hat{Z}_2^{-2} \hat{Z}_{2m} \hat{Z}_{2n} - \hat{Z}_2^{-1} \hat{Z}_{2m\bar{n}}) - 2) \]

= partial differential equation of two unknown functions

assume: \( V_0 = 0 \)
The simplest case: only one hidden sector field

\[ \partial_h \hat{K} \partial_{\bar{h}} \hat{K} = -\beta \partial_h \partial_{\bar{h}} \hat{K} \implies \hat{K} = \beta \log(h + \bar{h}) \]

no-scale sugra

MSSM inflaton potential as the leading order

\[ \hat{Z}_2 = (h + \bar{h})^{-2/9 + \beta/6} \left[ c_1 (h + \bar{h})^\omega + c_1 (h + \bar{h})^{-\omega} \right]^{5/9} \]

\[ \omega = \frac{1}{2} \sqrt{-17 - 6\beta} \]
several hidden fields: try the Ansatz

\[ K = \sum_m \beta_m \log(h_m + \overline{h}_m) + \kappa \prod_m (h_m + \overline{h}_m)^{\alpha_m} \phi^2 + O(\phi^4) \]

"modular weights"

like abelian orbifold compactification of the heterotic string

- \( \beta = \text{number of moduli} \)

saddle point if

\[ \alpha \cdot 36\alpha + 16 - 12\beta + \beta + 7^2 = 0 \]

\[ \alpha = \sum_m \alpha_m, \quad \beta = \sum_m \beta_m \]
look for solutions that are rational numbers:

\[
\begin{array}{|c|c|}
\hline
\beta = \sum_{m} \beta_m & \alpha = \sum_{m} \alpha_m \\
-3 & -\frac{4}{9} \\
-7 & 0 \\
-7 & -\frac{25}{9} \\
-11 & -\frac{1}{9} \\
-11 & -4 \\
\hline
\end{array}
\]

saddle point if

these values not found in abelian orbifold compactification
\[ K = O(\phi^2) \quad \Rightarrow \quad V = V_{\text{MSSM}} (1 + O(\phi^2)) \]

do these spoil flatness?

**order by order**

\[
\Delta V_1 = V_4 \phi^4 + V_8 \phi^8 + V_{12} \phi^{12} \\
\Delta V_2 = V_6 \phi^6 + V_{10} \phi^{10} + V_{14} \phi^{14}
\]

**third and higher order negligible**
must consider

\[ K = \sum_{m} \beta_m \log(h_m + \bar{h}_m) + \hat{Z}_2 \phi^2 + \hat{Z}_4 \phi^4 + \hat{Z}_6 \phi^6 + \ldots \]

saddle point, flatness maintained if

\[ \Delta V'_1 = \Delta V''_1 = 0 \]
\[ \Delta V'_2 = 0 \]

with coefficients determined by \( \alpha_m, \beta_m \)

K expansion in canonically normalized field
d=6 flat MSSM inflaton potential guaranteed by

\[
K = \sum_m \beta_m \log (h_m + \bar{h}_m) + \kappa \prod_m (h_m + \bar{h}_m)^{\alpha_m} \phi^2 \\
+ \mu \left( \kappa \prod_m (h_m + \bar{h}_m)^{\alpha_m} \right)^2 \phi^4 + \nu \left( \kappa \prod_m (h_m + \bar{h}_m)^{\alpha_m} \right)^3 \phi^6
\]

\[
\beta = \sum_m \beta_m \\
\alpha = \sum_m \alpha_m \\
\gamma = \sum_m \alpha_m^2 / \beta_m \\
\mu = \frac{1507}{91854} + \delta \frac{1}{21} \\
\nu = \frac{1158182}{203391} + \delta \frac{29}{186}
\]

\[
\delta = \sum_m \alpha_m^3 / \beta_m^2 \quad \text{free parameter}
\]
in shorthand:

\[
K = \ln Z(\beta) + \sum_{m=1}^{\infty} a_m (Z(\alpha)|\phi|^2)^m
\]

known, several solutions

\[
Z(x) = \prod_{n} Z_{x_n}^x
\]

\[
Z_n = h_n + h_n^*
\]
Constraints relaxed if one does not require

\[ \phi = \phi_0 \]

saddle point shifted by sugra

constraints on the Kähler potential parameters
slightly changed

some particularly simple logarithmic solutions
\[ K = -\ln \left( \prod_m (h_m + h_m^*)^{-\beta_m} - \kappa \prod_m (h_m + h_m^*)^{\alpha_m - \beta_m} \mid \phi \mid^2 \right) \]

<table>
<thead>
<tr>
<th>( \beta = \Sigma \beta_m )</th>
<th>( \alpha = \Sigma \alpha_m )</th>
<th>( \gamma = \Sigma \alpha_m^2 / \beta_m )</th>
<th>( \delta = \Sigma \alpha_m^3 / \beta_m^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>0</td>
<td>-5/4</td>
<td>free</td>
</tr>
<tr>
<td>-7</td>
<td>-25/9</td>
<td>-145/81</td>
<td>-985/720</td>
</tr>
<tr>
<td>-11</td>
<td>-1/9</td>
<td>-89/81</td>
<td>-721/720</td>
</tr>
<tr>
<td>-11</td>
<td>-4</td>
<td>-17/8</td>
<td>-91/64</td>
</tr>
</tbody>
</table>

Example of a solution for \( \alpha = 0, \beta = -7 \):

\[ \beta_m = -1, \alpha_1 = 1, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = -\frac{1}{4}, \alpha_6 = \alpha_7 = 0 \]

proof of existence of a solution
supergravity corrections to inflation

- non-trivial Kähler potential $\rightarrow$ non-minimal kinetic terms
  $\rightarrow$ normalize to get canonical

- corrections from $e^K$

  small (to lowest order) linear term

  small, does not spoil MSSM inflation

  small change in the spectral index: $n = 0.92 \ldots 0.94$

N.B. for $K=\log(\ldots)$ corrections vanish
1-loop radiative corrections:

\[ A^2 \Rightarrow A^2 \times \left( 1 + K_1 - \frac{2}{3} K_2 + \frac{1}{5} K_3 \right) \]

from RGEs, depend on the flat direction

seemingly fine-tuning required; e.g. LLe with unification:

\[ K_1 \approx -0.017 \xi^2; \quad K_2 \approx -0.009 \xi; \quad K_3 \approx -0.029; \]

\[ \xi = \frac{M_g}{m_0} \]

A/m shifted by \( O(10^{-2}) \)
a saddle point remains

but: assumes \( A \) and \( m_0 \) are independent parameters instead of functions of the moduli fields
wave function renormalization

\[ |\phi| \rightarrow Z^{1/2} |\phi| \]

can be absorbed by scaling the moduli: e.g.

\[ K = -\ln(x^{-\beta_1} y^{-\beta_2} - k x^{\alpha_1 - \beta_1} y^{\alpha_2 - \beta_2} |\phi|^2) \]

\[ \rightarrow K (Z^{1/2} |\phi|) = K \]

in the superpotential

\[ \lambda \rightarrow Z^{-3} \lambda \]

vertex corrections?
moduli dynamics? Lalak, Turzynski

if moduli shifted during inflation \(\rightarrow\) inflation affected

what stabilizes the moduli?

superpotential for the moduli?

(cosmological constant)

open questions:

initial condition
Kähler potential fixed \( \rightarrow \) implications for sparticle phenomenology

e.g. non-flat direction \( \psi \)

\[ W = \frac{1}{3} \hat{\lambda} \psi^3 \]

assume non-flat directions have all the same "modular weights" as the inflaton direction

trilinear A-term

phases

\[ A_3 = \frac{4 \cos \xi (\alpha - \beta/3)}{\sqrt{\alpha - \beta - 2}} m_\psi \]

at scale \( \phi_0 \)

\textit{in principle testable}
Conclusions

• $n=6$ MSSM flat directions have all the ingredients for successful inflation
• inflaton is a gauge invariant combination of squarks or sleptons: couplings to matter known
• requires saddle point: but fine tuning can be a consequence of sugra (string theory?)
• proof of existence of a class of Kähler potentials that give rise to the saddle point: expansion in terms of the canonically normalized inflaton
• parameters of the inflaton potential (e.g. inflaton mass) can in principle be determined in laboratory
• many open questions