A strong field dynamo in the solar tachocline

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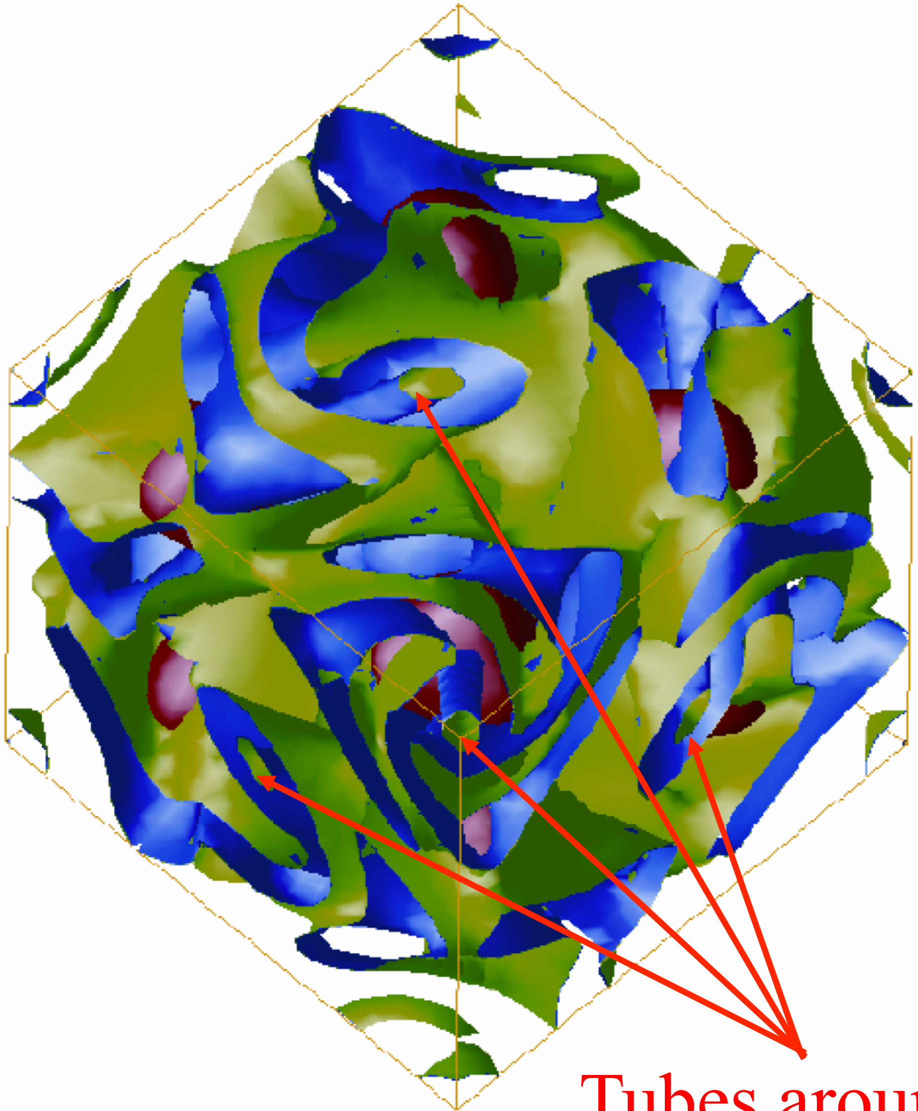
Strong field dynamos

Dynamos with U and B comparable (in Alfvenic scaled units) over a large fraction of the flow domain.

Example: Archontis dynamo has **U = +B** everywhere, or **U = -B** everywhere, with error of order the diffusivities

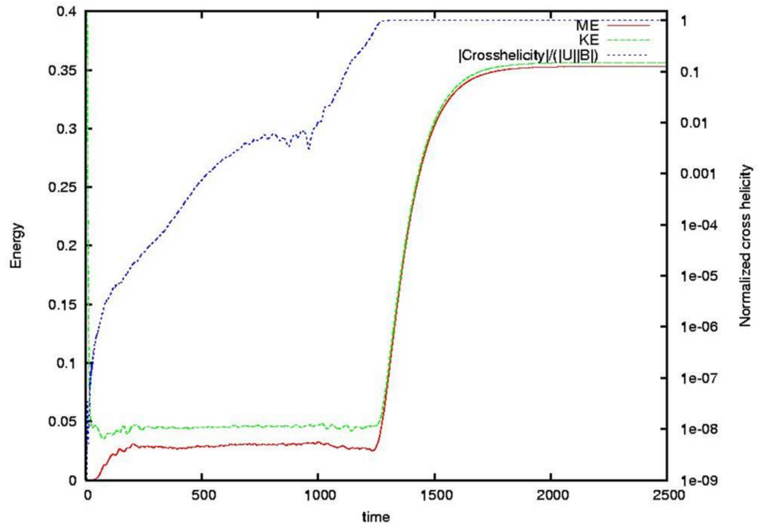
F=ν(sin z, sin x, sin y); we took ν=η (but see later)

→ **U/B** =0.5(sin z, sin x, sin y) + few % terms



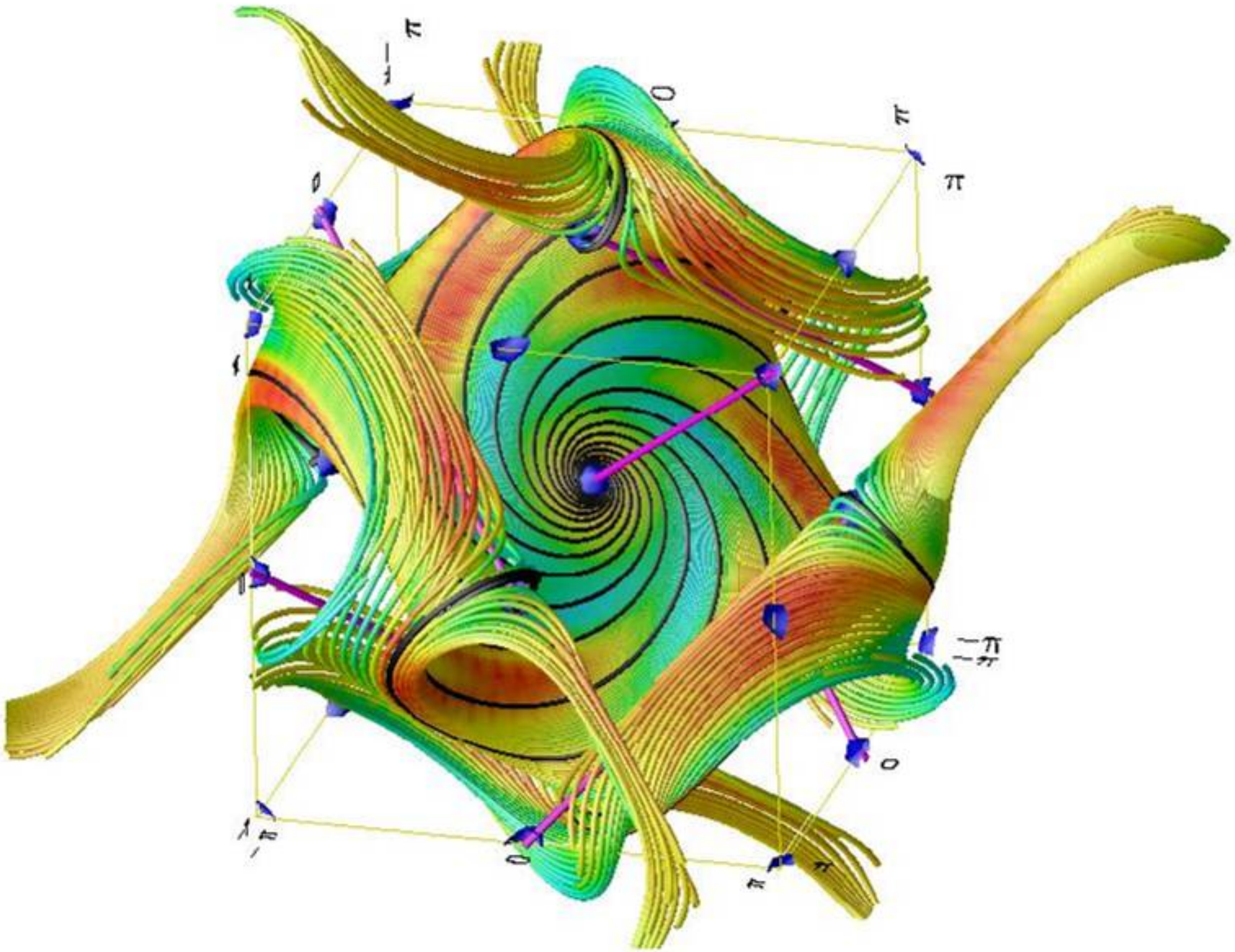
Tubes around heteroclinic orbits

Isosurface of |**u-B**| **(0.75 of max)**



Evolution of KE and ME starting from small seed field. The upper curve shows the evolution of the cross-helicity, which is the integral of **U.B** over the box.

Results for Re=Rm=200



π
0
π
π
0
−π
0
π

A scaling argument: dynamos to order

Suppose we have any steady solution $\mathbf{B_0}$ to the induction equation when solved with a velocity field $\mathbf{U_0}$ and a magnetic diffusivity η_0. We can now generate an equilibrium solution to the whole dynamo problem (including the momentum equation) for $\eta=\varepsilon\eta_0$. This is $\mathbf{U_1}=\varepsilon\mathbf{U_0}+\mathbf{B_0}$, $\mathbf{B_1}=\mathbf{B_0}$, $\mathbf{F}=\varepsilon^2\mathbf{U_0}\cdot\nabla\mathbf{U_0}-\varepsilon\nu\nabla^2\mathbf{U_0}+2\varepsilon\mathbf{B_0}\cdot\nabla\mathbf{U_0}$. This dynamo has the property that **U** tends to **B** as the diffusivity tends to zero.

Note the stability of the resulting object is uncertain.

However, experimentation with the Archontis dynamo shows that this idea works and that the results are often stable, with heuristic arguments to support this. Friedlander and Vishik have shown that the ideal MHD case is neutrally stable.

Some new work: the issue of low magnetic Prandtl number

The magnetic Prandtl number $p_m = \nu/\eta$, the ratio of viscous to magnetic diffusivity.

Various authors (Boldyrev, Cattaneo, and others at KITP Santa Barbara 2008 “Dynamo Theory”) have claimed that dynamos cannot function when this ratio is low as inside the Sun and other stars. These worries arise from the use of turbulent models, usually involving the mean field approximation.

The Archontis dynamo works with

$u = B \approx (\sin z, \sin x, \sin y) /2$

at $\nu = 1/100$, $\eta = 1/400$ ($p_m = 4$)

$\nu = 1/100$, $\eta = 1/100$ ($p_m = 1$)

$\nu = 1/400$, $\eta = 1/100$ ($p_m = 1/4$).

Why? An interesting new argument gives some insight.

Re-derivation of $\underline{u} = \underline{B}$ for general ν/η —

$$\frac{\partial \underline{u}}{\partial t} = \underline{u} \times (\nabla \times \underline{u}) - \nabla\left(\frac{1}{2}u^2 + \frac{p}{\rho}\right) + \frac{(\nabla \times \underline{B}) \times \underline{B}}{\mu_0} + \underline{F} + \nu \nabla^2 \underline{u}$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} .$$

Energy equation (after de-divving → surface integrals etc)

$$\frac{d}{dt} \int_V (u^2 + B^2)\, dV = \int_V \underline{F} \cdot \underline{u}\, dV + \nu \int_V \underline{u} \cdot \nabla^2 \underline{u}\, dV + \eta \int_V \underline{B} \cdot \nabla^2 \underline{B}\, dV$$

Cross-helicity equation:

$$\frac{d}{dt} \int_V \underline{u} \cdot \underline{B}\, dV = \int_V \underline{F} \cdot \underline{B}\, dV + \nu \int_V \underline{B} \cdot \nabla^2 \underline{u}\, dV + \eta \int_V \underline{u} \cdot \nabla^2 \underline{B}\, dV$$

Now suppose $\underline{B}$, $\underline{u}$ and $\underline{F}$ all have the same form, which is an eigenvalue of the Laplacian $\nabla^2 \underline{F} = -k^2 \underline{F}$ (for Archontis, $k=1$).

Then in the steady state, putting $\underline{u} = \lambda \underline{F}$, $\underline{B} = \mu \underline{F}$:

$$\lambda \int_V F^2 dV - k^2 \nu \lambda^2 \int_V F^2 dV - k^2 \eta \lambda^2 \int_V F^2 dV = 0$$

$$\mu \int_V F^2 dV - k^2 \nu \mu \lambda \int_V F^2 dV - k^2 \nu \mu \lambda \int_V F^2 dV = 0 .$$

The integrals cancel; the second equation has a root $\mu = 0$ corresponding to no magnetic field. If we exclude that, then $\lambda = k^2(\nu\lambda^2 + \eta\mu^2)$ and $\lambda k^2(\nu+\eta) = 1$.

Hence $\lambda = \dfrac{1}{k^2(\nu+\eta)}$, $\mu^2 = \dfrac{\lambda - k^2\nu\lambda^2}{\eta k^2}$.

With $k=1$ (Archontis), $\lambda = \dfrac{1}{\nu+\eta}$ and then

$$\mu^2 = \left[\frac{1}{\nu+\eta} - \nu\left(\frac{1}{\nu+\eta}\right)^2\right] \Big/ \eta = \frac{\nu+\eta-\nu}{(\nu+\eta)^2\eta} = \frac{1}{(\nu+\eta)^2} .$$

Hence $\boxed{\underline{u} = \underline{B} = \dfrac{1}{\nu+\eta}\underline{F}}$.

(joint work with Chris Jones)

Dynamos to order

Take any known pet kinematic dynamo and concentrate on the case where it is steady, at marginal magnetic Reynolds number. Common non-numerical dynamos include Herzenberg, Gibson, Ponomarenko,…

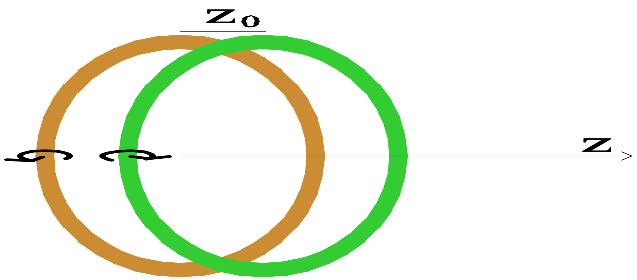
Scale up according to the recipe, and let $\varepsilon \to 0$ so that the diffusivities are small and **U** and **B** are nearly aligned.

Gibson 3-sphere dynamo works and is qualitatively similar to the Archontis dynamo (Cameron and Galloway 2006b).

Ponomarenko has so far not proved useful.

Strong-field Gailitis dynamo

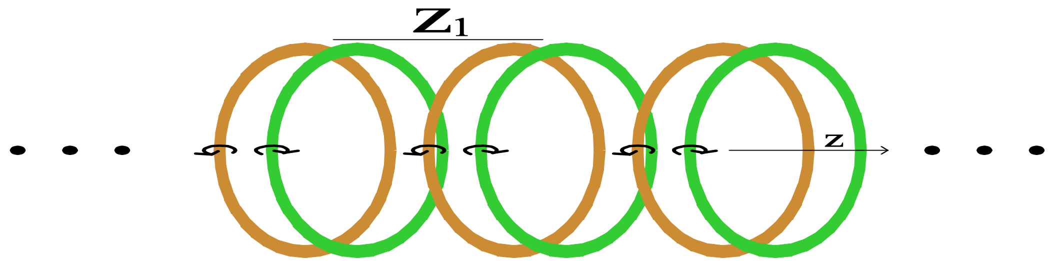
Gailitis's 1970 kinematic dynamo consists of two axisymmetric rings rotating in opposite directions in their meridional plane. Cowling's theorem tells us no axisymmetric dynamo is possible, but a nonaxisymmetric field where the field from one ring acts as a seed field for the other can be shown to work.



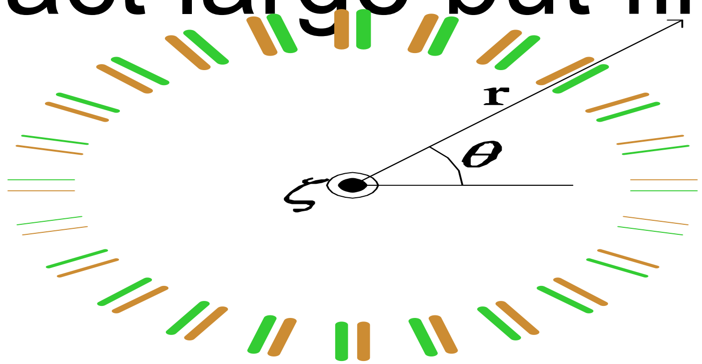
Let c be the distance out from the Z-axis to the centre of the cross-section of each ring, a be the radius of the cross-section, and Z_0 be the separation as shown. Then Gailitis's theory gives the critical magnetic Reynolds number for kinematic dynamo action as $(c^2/a^2F(Z_0/c))$, where F is an integral.

Here we are assuming an $e^{i\varphi}$ dependence for the magnetic field, where φ is the angle around the z-axis. The field is predominantly from L to R at the back of the rings and from R to L at the front (say).

This can be generalised to a long line of such pairs; the critical magnetic Reynolds number is now $(c^2/(a^2H))$, where $H=F(z_0/c)+\Sigma(F((nz_1+z_0)/c) - (F((nz_1-z_0)/c))$, and the sum runs from n=1 from to ∞. The field components can be calculated by evaluating the integrals numerically.)



Now identify the +∞ and -∞ ends of the row, supposing the number of ring pairs is in fact large but finite.



….then scaled up into a U=B dynamo as specified earlier

The set of rings can then be parked at the tachocline (a strong shear layer at the base of the Sun's convection zone).

Superimposing a meridional flow (which is thought for other reasons to be a feature of the tachocline) and letting it have a circulation time of 22 years, the $e^{i\varphi}$ is wafted around to give fields of different polarities to be picked up by the convection zone and carried quickly to the surface every 11 years - a new theory for the solar cycle! Inferred velocity of around 1m/s is reasonable.

The details need filling in!

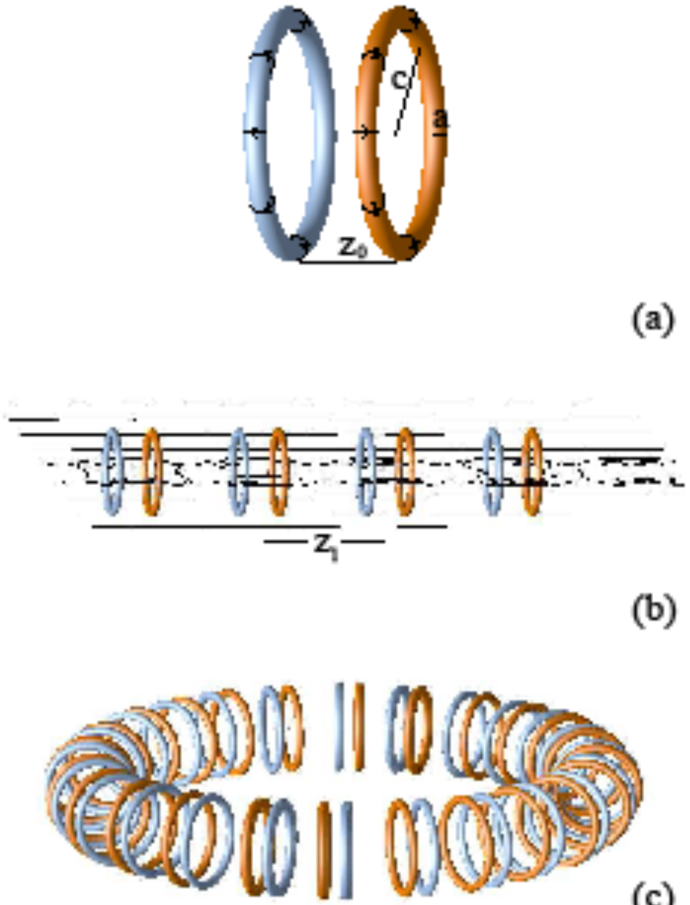


Figure 1. The geometries of the three dynamos considered in this paper: (a) a single pair of Gailitis rings; (b) a small segment of a line of such pairs and some of the magnetic field lines; and (c) an indicative plot of the geometry when the line is bent to form a circle. It is the third of these which, when embedded in a circulating tachocline, can reproduce the observed properties of the solar cycle.

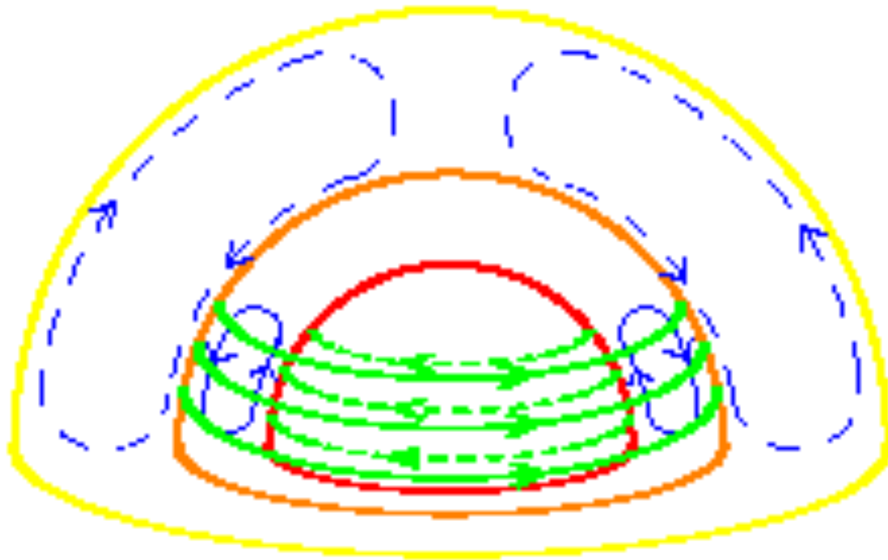


Figure 2. The engine room of the solar dynamo? A schematic illustration of the large-scale features of the proposed dynamo mechanism. The Sun is here divided into a radiative interior (below the red curve), a tachocline (which lies between the red and orange curves), and a convection zone (which lies between the tachocline and the solar surface which is shown in yellow. For illustrative purpose the thickness of the tachocline has been vastly exagerated. The assumed meridional velocity field is shown using the blue curves and the large scale component of the magnetic field is shown in green. Note that the field has opposite directions at the top and bottom of the tachocline. This field is advected by the meridional circulation and thus presents oppositely directed poloidal field at the base of the convection zone over the course of a 22-year magnetic cycle.

- Surface reacts almost instantaneously to BC presented by tachocline to lower boundary of convection zone (timescale is around 1 month)
- Field strength is limited by the balance with differential rotation - estimates give predicted field strengths of around 1T in tachocline. This agrees well with estimates based on how field evolves up to surface via magnetic buoyancy, if the buoyant flux evacuates soon after setting off.
- Explains Hale polarity laws, equatorwards progression of butterfly diagram, and most or all other aspects of the solar cycle
- No attempt so far to couple hemispheres via interactions near the equator: slight asymmetries could explain Gnedyshev-Ohl rule on odd/even cycles
- In this model fields in the photosphere/corona/solar wind are lost as waste products from what is happening deep down

Uncertainties (many!):

- Depends on interactions with differential rotation within the tachocline: latter is not at all understood
- Needs u=B in top/bottom of tachocline, u=-B in bottom/top (because differential rotation apparently does not change sign with solar cycle)
- **U=λB** dynamos so far only produced in periodic geometries (or infinite for Gailitis); effects of boundary conditions must modify things at least locally
- Convection zone aspects: picture as proclaimed so far (rising twists to give tilts, magnetic buoyancy, etc. etc) to be largely taken over lock stock and barrel---perhaps peaceful coexistence is possible!

Conclusion

The specific model presented here is presented as a thought-experiment and cannot literally be what is occurring on the Sun.

But…the idea that the tachocline somehow generates a permanent magnetic structure which moves round to present an alternating magnetic boundary condition to the base of the convection zone seems an interesting alternative to other models suggested till now. It avoids the Herculean problem of rebuilding the flux system every 11 years, and helps explain the amazing regularity of the Hale polarity laws.