The Trouble with Cosmological Magnetic Fields

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1. Introduction: What’s the problem?

2. Effects of magnetic fields on the CMB

3. Causal generation of primordial magnetic fields
   - The spectrum
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5. Conclusions
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- **Estimates by equi-partition** (e.g. of magnetic field and thermal or turbulent energy).
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- **Galaxies:** Most galaxies host magnetic fields of the order of $B \sim 1 - 10\mu\text{Gauss}$ with coherence scales as large as $10\text{kpc}$. This is also the case for galaxies at redshift $z \sim 1 - 2$. 
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- **Filaments**: There are indications of magnetic fields in filaments with strengths up to $B \sim 10^{-8} - 10^{-7}$Gauss and coherence scales over 1Mpc (Kronberg 2010).
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- **Intergalactic space, voids**: The fact that certain blazars do emit TeV $\gamma$-radiation but not GeV, means that lower energy electrons which are produced by scattering with intergalactic background light and which then generate a cascade of GeV photons by inverse Compton scattering must be deflected out of the beam. This requires intergalactic fields of $B \gtrsim 10^{-15}$Gauss (direct) to $B \gtrsim 10^{-17}$Gauss (delayed) with coherence scales of 1Mpc (Neronov & Vovk, 2010, Taylor, Vovk & Neronov, 2011).
Observations...

from Taylor, Vovk & Neronov 2011
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In this talk I concentrate on the possibilities to generate \textit{primordial magnetic fields} and on their limitations.
**Possible Solutions**

- **Second order perturbations:** To generate magnetic fields in the cosmic fluid one needs vorticity and a charge and current density. The first can be obtained only in second order perturbation theory (first order vector perturbations decay) and the second only in second order in the tight coupling limit. Estimates have shown that typical fields on cluster scales are of the order of $10^{-28}$ Gauss (Matarrese et al. ’04, Ichiki et al. ’07, Fenu et al. ’10). This is far too small to be consistent with the Neronov-Vovk bound or with the minimal amplitude needed for dynamo amplification.
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- **Inflation:** The electromagnetic field is conformally coupled to gravity and is therefore usually not generated during inflation. However, if conformal symmetry is explicitly broken or if the electromagnetic field is coupled to the inflation, it can also be generated during inflation (Turner & Widrow ’88, Ratra ’92, Anber & Sorbo ’06, RD, Hollenstain & Jain ’10).
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In the remainder of this talk I restrict to the 2nd and 3rd possibility.
A constant magnetic field affects the geometry of the universe by introducing shear. It generates an anisotropic stress $\Pi_{ij} \propto B_i B_j \neq 0$. This leads to a well studied homogeneous Bianchi I model.
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However, if a massless free-streaming particle is present, the different expansion rate in different directions will also generate an anisotropic stress in this free-streaming component which exactly compensates the one from the magnetic field $\Rightarrow$ the Universe is isotropized (Adamek et al. ’11).
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From Adamek, RD, Fenu & Vonlanthen, ’11
A constant magnetic field still interacts with the electrons in the cosmic plasma and leads to Faraday rotation of the CMB photons. Since a constant magnetic field breaks parity, its Faraday rotation leads to parity odd correlations between B-polarization and temperature anisotropies (and E- and B-polarization) in the CMB (Scannapieco & Ferreira, ’97). This leads to limits of the order of $B < 10^{-8}$ Gauss.
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CMB limits on magnetic fields are (almost) all of this order. This is not surprising since

$$\Omega_B = 10^{-5} \Omega_\gamma \left( \frac{B}{10^{-8} \text{ Gauss}} \right)^2$$

Magnetic fields of the order $3 \times 10^{-9}$ Gauss (on CMB scales) will leave 10% effects on the CMB anisotropies while $10^{-9}$ Gauss will leave 1% effects. It is thus clear that we can never detect magnetic fields of the order of $10^{-16}$ Gauss in the CMB.
Magnetic fields effect the CMB via

- their energy-momentum tensor which leads to metric perturbations \( \Rightarrow \) perturbed photon geodesics
- magnetosonic waves affect the acoustic peaks in the CMB spectrum
- Alvèn waves (vector perturbations)
- Faraday rotation can turn E-mode polarization into B-modes

All these lead to magnetic field limits on the order of \( 10^{-9} \) Gauss on CMB scales. Depending on the spectral index this leads to different limits on galactic scales \( \lambda \sim 0.1 \text{Mpc} \).
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**Caveat:** During recombination small scale (kpc) magnetic fields may induce large fluctuations in the baryon density and modify the recombination history $\Rightarrow$ modify the CMB peaks. Planck may detect magnetic fields as small as $B \sim 10^{-11}$ Gauss by this imprint! (Abel & Jedamzek ’11)
Other effects on the CMB

(from: Paoletti, Finelli & Paci ’08)
The spectrum of causally generated magnetic fields

We assume that the process leading to a magnetic field is statistically homogeneous and isotropic. A magnetic field spectrum generated by such a process is of the form

\[
\langle B_i(k) B^*_j(\eta, q) \rangle = \frac{(2\pi)^3}{2} \delta(k - q) \left\{ (\delta_{ij} - \hat{k}_i \hat{k}_j) P_S(k) - i\epsilon_{ijn} \hat{k}_n P_A(k) \right\}
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The Dirac–\(\delta\) is due to statistical homogeneity and the requirement \(\nabla \cdot \mathbf{B}\) dictates the tensor structure. Note that the pre-factor of \(P_S\) is even under parity while the one of \(P_A\) is odd under parity.
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$P_A \propto |B_+|^2 - |B_-|^2$ determines the helicity of the magnetic field. Its integral is the helicity density while the integral of $P_S \propto |B_+|^2 + |B_-|^2$ determines the energy density in magnetic fields.

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On small scales the magnetic field is damped by the viscosity of the cosmic plasma, \(P_S = P_A = 0\) for \(k > k_d(t)\). Here \(k_d(t)\) is a time-dependent damping scale.
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Depending on whether or not the magnetic field is helical it evolves differently in the cosmic plasma: if the field is not helical, it involves mainly via viscosity damping on small scales. On large scales the spectrum is not modified. However, if the field is helical, **helicity conservation leads to an inverse cascade** which moves the correlation scale to larger and larger scales (numerical simulations by Banerjee & Jedamzik ’04, Campanelli, ’07)
If the magnetic field is generated during a phase transition, its correlation length is finite. It is typically of the size of the largest bubbles when they coalesce and the phase transition terminates. This is a fraction of the Hubble scale at the transition. On scales larger than the Hubble scale, correlation vanish by causality.
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For the energy density per log-$k$-interval this implies

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Hence the question is still open...
As we now show, this already implies very stringent limits on magnetic fields from phase transitions. Let $\epsilon = \Omega_B^*/\Omega_r^*$ be the ratio of the magnetic field to the radiation energy density at the moment of formation and $k_*$ the cutoff scale. Since radiation and magnetic fields scale the same way, at later times and scales larger than the cutoff, the magnetic field to radiation density is given by

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For the electroweak phase transition with $k_* > \mathcal{H}_* \simeq 10^{-4}\text{Hz}$ and $k_1 = 1/(0.1\text{Mpc}) \simeq 10^{-13}\text{Hz}$ this yields the following limit for the field at scale $k_1$:

$$\left( \frac{B(k_1)}{10^{-6}\text{Gauss}} \right)^2 \simeq \Omega_r^{-1} \frac{d\Omega_B}{d\log(k_1)} < \epsilon \times 10^{-46}$$

Requiring $\epsilon < 1$ this implies $B(k_1) < 10^{-29}\text{Gauss}$. Using slightly more model dependent but also more realistic numbers (e.g. $k_* \simeq 100\mathcal{H}_*$ one arrives at $B(k_1) < 10^{-31}\text{Gauss}$.)
Limits on magnetic fields from phase transitions

As we now show this already implies very stringent limits on magnetic fields from phase transitions. Be \( \epsilon = \Omega_B^*/\Omega_r^* \) the ratio of the magnetic field to the radiation energy density at the moment of formation and \( k_* \) the cutoff scale. Since radiation and magnetic fields scale the same way, at later times and scales larger than the cutoff, the magnetic field to radiation density is given by

\[
\frac{d\Omega_B}{d \log(k)} = \epsilon \Omega_r \left( \frac{k}{k_*} \right)^5
\]

For the electroweak phase transition with \( k_* > \mathcal{H}_* \approx 10^{-4} \text{Hz} \) and \( k_1 = 1/(0.1 \text{Mpc}) \approx 10^{-13} \text{Hz} \) this yields the following limit for the field at scale \( k_1 \):

\[
\left( \frac{B(k_1)}{10^{-6} \text{Gauss}} \right)^2 \approx \Omega_r^{-1} \frac{d\Omega_B}{d \log(k_1)} < \epsilon \times 10^{-46}
\]

Requiring \( \epsilon < 1 \) this implies \( B(k_1) < 10^{-29} \text{Gauss} \). Using slightly more model dependent but also more realistic numbers (e.g. \( k_* \approx 100 \mathcal{H}_* \) one arrives at \( B(k_1) < 10^{-31} \text{Gauss} \).

The limits from the QCD phase transition are somewhat less stringent but still discouraging, \( B(k_1) < 10^{-28} \text{Gauss} \).
If the magnetic field is helical, the inverse cascade which moves power from small to larger scales can help. But a detailed calculation (Caprini, RD, Fenu, ’09) shows

\[ B(k_1) < 5 \times 10^{-26} \text{ Gauss} \] for the electroweak phase transition.
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and

\[ B(k_1) < 10^{-21} \text{ Gauss} \] for the QCD transition. This result could be marginally sufficient dynamo amplification, but is still several orders of magnitude below the Neronov-Vovk-bound.
Limits on magnetic fields from phase transitions

Phase transitions, $n=2$

(from: Caprini 2011)
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Let us discuss a simple case where we couple the inflation to the electromagnetic field.

\[
S = \int d^4x \sqrt{-g} \left[ R + \frac{1}{2} \partial_{\mu}\phi\partial^{\mu}\phi - V(\phi) + \frac{f(\phi)}{4} F^2 \right]
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With this modification in the action, the modified evolution equation for the 'renormalized' electromagnetic potential \( \mathcal{A} = a f(\phi) A \) in Fourier space becomes (in Coulomb gauge)

\[ \ddot{\mathcal{A}} + \left( k^2 - \frac{\ddot{f}}{f} \right) \mathcal{A} = 0 \]
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This is a wave equation with a time-dependent mass term. We know how to calculate the generation of its modes out of the vacuum. This case has been discussed for the first time in (Ratra '92).
For example if $f \propto a^{\gamma}$ is a simple power law, we can compute the resulting magnetic fields spectrum to

$$P_S \propto k^n \quad \text{with} \quad n = \begin{cases} 1 + 2\gamma & \text{if } \gamma \leq 1/2 \\ 3 - 2\gamma & \text{if } \gamma \leq 1/2 \end{cases}$$
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During inflation we cannot assume that the Universe is highly conducting and the electric field is damped. We therefore also have to compute the electric field spectrum. One finds (Martin & Yokoyama '08, Subramanian '10)

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P_E \propto k^m \quad \text{with} \quad m = \begin{cases} 
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Since there is no infrared cutoff, the spectral index should not be less than $-3$ otherwise $\frac{d\rho_B}{d\log(k)} \propto k^3 P_S \propto k^{3+n}$ or $\frac{d\rho_E}{d\log(k)} \propto k^3 P_E \propto k^{3+m}$ diverges. This limits $-2 \lesssim \gamma \lesssim 2$.

$\gamma \simeq -2$ gives a scale invariant spectrum for the magnetic field!
On the other hand, if the spectrum is too blue, the fact that magnetic fields should not dominate the energy density of the Universe leads to very stringent constraints on small scales. Since the Hubble scale at the end of inflation is so small, the spectrum needs not be very blue for this to happen.
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For this result we have normalized $f_0 = 1$ at the end of inflation. Since $f$ is growing rapidly during inflation this means that $f_i \ll 1$ for most of the time during inflation. But since charged particles couple to the canonically normalized field $\sqrt{f} F_{\mu\nu}$, their charge during inflation has the renormalized value $e_N = e/\sqrt{f} \gg e$. 

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Hence during inflation the electron charge was much larger than 1. In this regime we cannot trust perturbation theory and our calculation does actually not apply... (Demozzi et al. ’09).

This problem can probably be avoided by modifying not only electrodynamics but also the Dirac equation. But there is worse...
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$$\frac{\text{Weyl}}{\text{Ricci}} \sim \frac{k^2 \Psi}{H^2} \sim \frac{\rho_B}{\rho} \ll 1$$

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Hence the formation of such magnetic fields is in contradiction with the homogeneity and isotropy of the Universe!
Conclusions

- Magnetic fields are observed on all cosmological scales (galaxies, clusters, filaments and even voids) with significant amplitudes. Intergalactic fields with coherence length of about 1Mpc and amplitudes of $10^{-16}$ Gauss (Neronov-Vovk-bound) are required.
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Magnetic fields from inflation can have many different spectra. They can actually be scale invariant leading to sufficient fields on large scales, but in this case, the generate perturbation of the metric which become very large in the subsequent radiation era, destroying the homogeneity and isotropy of the Universe.
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