Higgs inflation – consistency and possibilities
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Outline

1 Introduction – Standard Model and the reality of the Universe

2 Higgs inflation – mechanism and predictions
   - The model with non-minimal gravity coupling
   - Predictions: CMB parameters and Higgs boson mass

3 Consistency
   - Background dependent cut-off scale
   - Quantum corrections during inflation
   - Connection between inflationary and low energy physics

4 Summary
Standard Model – describes nearly everything

**Experimental problems:**

- **Laboratory**
  - Neutrino oscillations

- **Cosmology**
  - Baryon asymmetry of the Universe

- **Dark Matter**

- **Inflation**

- **Dark Energy**

**Describes**

- All laboratory experiments – electromagnetism, nuclear processes, etc.
- All processes in the evolution of the Universe after the Big Bang Nucleosynthesis ($T < 1\,\text{MeV, } t > 1\,\text{sec}$)

**Einstein gravity**

**Higgs inflation – consistency and possibilities**

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Can we describe everything with as small extension as possible?

- Minimal number of new particles
- No new scales before inflation/gravity
Standard Model – describes nearly everything

Einstein gravity

Experimental problems:

- Laboratory
  ✓ Neutrino oscillations

- Cosmology
  ✓ Baryon asymmetry of the Universe
  ✓ Dark Matter

- Inflation
- Dark Energy

with νMSM

- Right handed neutrinos
  - see-saw generation of active neutrino masses
  - keV scale DM
  - Baryogenesis via leptogenesis
Standard Model – describes nearly everything

with $\nu$MSM

- Right handed neutrinos
  - see-saw generation of active neutrino masses
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+ comological constant

Einstein gravity

Experimental problems:

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- Cosmology
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? Inflation

✓ Dark Energy
Chaotic inflation—a scalar field

\[ \mathcal{H}^2 \simeq \frac{1}{3M_p^2} \left( V(\phi) + \frac{\dot{\phi}^2}{2} \right), \quad \ddot{\phi} + 3\mathcal{H}\dot{\phi} + V'(\phi) = 0 \]

\[ V = \frac{\lambda(20M_p)^4}{4} \]

\[ \delta T/T \sim 10^{-5} \text{ normalization} \]

quartic coupling: \( \lambda \sim 10^{-13} \)

(or mass: \( m \sim 10^{13} \text{ GeV} \))

Can not be the SM Higgs field?
Non-minimal coupling to gravity solves the problem

Normally gravity couples to energy

\[ F = \frac{1}{8\pi M_P^2} \frac{m_1 m_2}{r^2} \quad E = mc^2 \]

action: \[ S = \int d^4x \sqrt{-g} \left\{ g^\mu\nu \frac{\partial_\mu h \partial_\nu h}{2} - V(h) \right\} \]

It is possible to couple also to the Higgs field background

\[ S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R - \frac{\xi}{2} h^2 R + g^\mu\nu \frac{\partial_\mu h \partial_\nu h}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right\} \]

This leads to the change of the Freedman equation

\[ H^2 \simeq \frac{1}{3M_P^2} \frac{V(h)}{(1 + \xi h^2 / M_P^2)^2} \]

(field equation is also modified)

[Zee’78, Smolin’79, Salopek, Bond, Bardeen’79], [FB, Shaposhnikov’08]
Conformal transformation – nice way to calculate

It is possible to get rid of the non-minimal coupling by the conformal transformation (change of variables)

\[ \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 \equiv 1 + \frac{\xi h^2}{M_p^2} \]

Redefinition of the Higgs field to get canonical kinetic term

\[ \frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2 / M_p^2}{\Omega^4}} \implies \begin{cases} h \simeq \chi & \text{for } h < M_p / \xi \\ \Omega^2 \simeq \exp \left( \frac{2\chi}{\sqrt{6} M_p} \right) & \text{for } h > M_p / \xi \end{cases} \]

Resulting action (Einstein frame action)

\[ S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_p^2}{2} \hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{\lambda}{4} \frac{h(\chi)^4}{\Omega(\chi)^4} \right\} \]
Potential – different stages of the Universe

\[ U = \frac{\lambda M_P^4}{4 \xi^2} \left( 1 - e^{-2\chi / \sqrt{6M_P}} \right)^2 \]

- Hot Big Bang
- Preheating
- Slow roll inflation

\[ \delta T/T \sim 10^{-5} \text{ normalization} \]

\[ \frac{\xi}{\sqrt{\lambda}} \approx 47000 \]

Small \( \lambda \) is traded for large \( \xi \)
CMB parameters are predicted

\[
\begin{align*}
\text{spectral index} & \quad n \approx 1 - \frac{8(4N+9)}{(4N+3)^2} \approx 0.97 \\
\text{tensor/scalar ratio} & \quad r \approx \frac{192}{(4N+3)^2} \approx 0.0033
\end{align*}
\]

\[
\frac{\delta T}{T} \sim 10^{-5} \quad \Rightarrow \quad \frac{\xi}{\sqrt{\lambda}} \approx 47000
\]
Higgs decouples from all fields during inflation

Action for the gauge fields and fermions is invariant under conformal transformations \((A_\mu \leftrightarrow A_\mu, \psi \leftrightarrow \Omega^{3/2} \psi)\) except for the mass terms

\[ L^J_A = g^2 h^2 A_\mu A_\mu \quad \rightarrow \quad L^E_A = g^2 \frac{h^2}{\Omega^2} A_\mu A_\mu = g^2 \frac{M_p}{\sqrt{\xi}} \left( 1 - e^{-\frac{2\chi}{\sqrt{6M_p}}} \right) A_\mu A_\mu \]

\[ L^J_Y = y h \bar{\psi} \psi \quad \rightarrow \quad L^E_Y = y \frac{h}{\Omega} \bar{\psi} \psi = y \frac{M_p}{\sqrt{\xi}} \left( 1 - \frac{1}{2} e^{-\frac{2\chi}{\sqrt{6M_p}}} + \ldots \right) \bar{\psi} \psi \]

In inflationary region \( h > M_p/\sqrt{\xi}: \)

\[ \Omega^2 \equiv 1 + \frac{\xi h^2}{M_p^2} \simeq \exp \left( \frac{2\chi}{\sqrt{6M_p}} \right) \]

Exponentially weak coupling of \( \chi \) to other matter

Non-minimal coupling made the Higgs potential flat and at the same time took care of the corrections from the other fields
Prescription to calculate potential with radiative corrections

1. Run all constants with SM RG equations from the EW scale up to $M_P/\sqrt{\xi}$

2. Run all constants $\lambda_i(\mu)$ with chiral EW theory RG equations up to scale $\mu$ equal to a typical particle mass for the given field background $\chi$

$$\mu^2 = \kappa^2 m_t^2(\chi) = \kappa^2 y_t(\mu)^2 M_P^2 \frac{M_P^2}{\xi(\mu)} \left( 1 - e^{-\frac{2\chi}{\sqrt{6}M_P}} \right).$$

3. Calculate the effective potential

$$U(\chi) = U_{\text{tree}}(\lambda_i(\mu), \chi) + U_{1-\text{loop}}(\lambda_i(\mu), \chi) + U_{2-\text{loop}}(\lambda_i(\mu), \chi)$$

4. Calculate the inflationary properties for the resulting potential

- Depends on UV completion!
Validity of the model up to inflation

Radiative corrections – “screening” of the Higgs self-interaction depending on scale

Higgs mass bounds

\[
126.1 \text{GeV} + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5 < m_H < 193.9 \text{GeV} + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.1176}{0.002} \times 0.1
\]

And nothing else seen at higher energies

[FB, Magnin, Shaposhnikov’09, FB, Shaposhnikov’09]
Consistency

Up to now we assumed that the model is a full model, and anything beyond it does not spoil the story.

Is this really the case?
Cut off scale today

Let us work in the Einstein frame for simplicity

Change of variables: \[
\frac{d\chi}{dh} = \frac{M_P \sqrt{M_P^2 + (\xi + 6\xi^2)h^2}}{M_P^2 + \xi h^2}
\] leads to the higher order terms in the potential

\[
V(\chi) = \lambda \frac{h^4}{4\Omega^4} \simeq \lambda \frac{h^4}{4} \simeq \lambda \frac{\chi^4}{4} + \# \frac{\chi^6}{(M_P/\xi)^2} + \cdots
\]

Unitarity is violated at tree level

in scattering processes (eg. \(2 \rightarrow 4\)) with energy above the cut-off

\[
E > \Lambda_0 \sim \frac{M_P}{\xi}
\]

Hubble scale at inflation is \(H \sim \lambda^{1/2} \frac{M_P}{\xi}\) – not much smaller than the today cut-off \(\Lambda_0\):

[Burgess, Lee, Trott’09, Barbon, Espinosa’09, Hertzberg’10]
Cut off is background dependent!

Classical background

\[ \chi(x, t) \rightarrow \bar{\chi}(t) \]

Quantum perturbations

\[ \delta \chi(x, t) \]

leads to background dependent suppression of operators of \( \text{dim } n > 4 \)

\[ \frac{O(n)(\delta \chi)}{[\Lambda(n)(\bar{\chi})]^{n-4}} \]

Example

Potential in the inflationary region \( \chi > M_P \):

\[ U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left( 1 - e^{-\frac{2\chi}{\sqrt{6}M_P}} \right)^2 \]

leads to operators of the form:

\[ \frac{O(n)(\delta \chi)}{[\Lambda(n)(\bar{\chi})]^{n-4}} = \frac{\lambda M_P^4}{\xi^2} e^{-\frac{2\bar{\chi}}{\sqrt{6}M_P}} \frac{(\delta \chi)^n}{M_P^n} \]

Leading at high \( n \) to the cut-off

\[ \Lambda \sim M_P \]
Cut-off grows with the field background

Jordan frame

Einstein frame

Relation between cut-offs in different frames:

\[ \Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}} \Omega \]

Reheating temperature \( \frac{M_P}{\xi} < T_{\text{reheating}} < \frac{M_P}{\sqrt{\xi}} \)

[FB, Magnin, Sibiryakov, Shaposhnikov'11]
Consistency  
Quantum corrections during inflation

Shift symmetric UV completion allows to have effective theory during inflation

\[ \mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - U_0 \left( 1 + \sum u_n e^{-n \cdot \chi / M} \right) \]
\[ = \frac{(\partial_\mu \chi)^2}{2} - U_0 \left( 1 + \sum \frac{1}{k!} \left[ \frac{\delta \chi}{M} \right]^k \sum n^k u_n e^{-n \cdot \bar{\chi} / M} \right) \]

Effective action (from quantum corrections of loops of \( \delta \chi \))

\[ \mathcal{L}_{\text{eff}} = f^{(1)}(\chi) \frac{(\partial_\mu \chi)^2}{2} - U(\chi) + f^{(2)}(\chi) \frac{(\partial^2 \chi)^2}{M^2} + f^{(3)}(\chi) \frac{(\partial \chi)^4}{M^4} + \ldots \]

All the divergences are absorbed in \( u_n \) and in \( f^{(n)} \sim \sum f_l e^{-n \chi / M} \)

**UV completion requirement**

Shift symmetry (or scale symmetry in the Jordan frame) is respected

\[ \chi \mapsto \chi + \text{const} \]
Connection of inflationary and low energy physics requires more assumptions on the UV theory

\[ \lambda U(\bar{\chi} + \delta \chi) = \lambda \left( U(\bar{\chi}) + \frac{1}{2} U''(\bar{\chi})(\delta \chi)^2 + \frac{1}{3!} U'''(\bar{\chi})(\delta \chi)^3 + \cdots \right) \]

in one loop: \( \lambda U''(\bar{\chi}) \bar{\Lambda}^2, \ \lambda^2 (U''(\bar{\chi}))^2 \log \bar{\Lambda} \),

in two loops: \( \lambda U^{(IV)}(\bar{\chi}) \bar{\Lambda}^4, \ \lambda^2 (U'''')^2 \bar{\Lambda}^2, \ \lambda^3 U^{(IV)}(U'')^2 (\log \bar{\Lambda})^2 \),

No power law divergences are generated

The loop corrections to the potential are arranged in a series in \( \lambda \)

\[ U(\chi) = \lambda U_1(\chi) + \lambda^2 U_2(\chi) + \lambda^3 U_3(\chi) + \cdots \]

A rule to fix the finite parts of the counterterm functions \( U_i(\chi) \)

Example – dimensional regularisation + \( \overline{\text{MS}} \)
Any “completely good” UV completions known today?

- “Perturbative” UV completion is not very nice – new heavy states introduce quadratic divergences (though it may easily respect shift invariance)
- Asymptotically safe gravity (if exists...)
- Classicalisation (if correct...)
- We don’t know yet!
Generalizations

We are not really constrained with the potential we get in the Einstein frame from the non-minimal coupling. Any one like this will do

\[ U(\chi) = U_0 \left(1 + u_1 F(\chi/M) + \cdots\right) \quad \text{with} \quad F(x) \to 0 \quad \text{at} \quad x \to +\infty \]

Example

Examples

- \( F(x) = e^{-x} \)
- \( F(x) = \frac{1}{x^\alpha} \)

a) For other SM particles one should also suppress Yukawas and gauge couplings in a similar way.

b) For good EFT at inflation \( F^{(k+1)} < F^{(k)}(x) \) for \( x \to \infty \) are needed.
Summary

Inflation is possible in the SM (+ non-minimal Higgs-gravity coupling)

- Is very nice
  - Spectral index is $n_s \approx 0.97$
  - Nearly no tensor perturbations $r \approx 0.003$

- Is not complete
  - Is ok...
    - Below the field dependent cutoff scale!
  - Is really ok – quantum corrections controlled during inflation.
    - If UV completion respects shift (scale) symmetry.
  - Is just great – can connect low energy and inflation parameters!
    - If UV completion has no quadratic divergences and a subtraction rule is fixed...


FB, M. Shaposhnikov, JHEP 0907 (2009) 089


Preheating

- Background evolution after inflation $\chi < M_P (h < M_P/\sqrt{\xi})$
  - Quadratic potential $U \simeq \frac{\omega^2}{2} \chi^2$ with $\omega = \sqrt{\frac{\lambda M_P}{3 \xi}}$
  - Matter dominated stage $a \propto t^{2/3}$
- Stochastic resonance
  - Particle masses $m_W^2(\chi) \sim g^2 \frac{M_P|\chi|}{\xi}$
  - W bosons are created (non-relativistic)
    - $\sqrt{\langle \chi^2 \rangle} \gtrsim 23 \left( \frac{\lambda}{0.25} \right) \frac{M_P}{\xi}$: non-resonant creation/W boson decay – slow
    - $\sqrt{\langle \chi^2 \rangle} \lesssim 23 \left( \frac{\lambda}{0.25} \right) \frac{M_P}{\xi}$: resonant creation/W boson annihilation – fast
  - Higgs creation – relativistic, less efficient $\sqrt{\langle \chi^2 \rangle} \sim 2.6 \left( \frac{\lambda}{0.25} \right)^{1/2} \frac{M_P}{\xi}$
- Radiation dominated stage starts

$3.4 \times 10^{13} \text{ GeV} < T_r < \left( \frac{\lambda}{0.25} \right)^{1/4} 1.1 \times 10^{14} \text{ GeV}$

- At Higgs amplitude $\sqrt{\langle \chi^2 \rangle} \lesssim \frac{M_P}{\xi}$ – exact SM. $T_{\text{reh}} > 1.5 \times 10^{13} \text{ GeV}$

[FB, Gorbunov, Shaposhnikov’09], [J.García–Bellido, D.Figueroa, J.Rubio’09]
Corrections to the potential

1-loop effective potential

\[ \Delta U(\chi) \sim \sum_{\text{particles}} \frac{m^4(\chi)}{64\pi^2} \log \frac{m^2(\chi)}{\mu^2} \quad | \quad \frac{m^4(\chi)}{64\pi^2} \log \frac{m^2(\chi)}{\mu^2/\Omega^2(\chi)} \]

In Einstein frame: \( m^2(\chi) \sim g^2 h^2(\chi)/\Omega^2(\chi) \)

- Correct by RG running
- Ambiguity in the theory definition in UV

Cutoff frame dependence and choice

<table>
<thead>
<tr>
<th>choice I</th>
<th>choice II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jordan frame</td>
<td>( M_P^2 + \xi h^2 )</td>
</tr>
<tr>
<td>Einstein frame</td>
<td>( M_P^2 )</td>
</tr>
</tbody>
</table>

[FB, Magnin, Shaposhnikov’09]
Future experiments—clue on Planck scale physics?

\[ m_t = 171.2 \text{ GeV}, \quad \alpha_s = 0.1176 \]

normalization prescription II

normalization prescription I

LHC & PLANCK precisions
Partial example: perturbative UV completion

Add one more field that generated the Planck mass as in induced gravity

\[ S_J = \int d^4x \sqrt{-g} \left\{ -\zeta \frac{\sigma^2}{2} R - \xi \frac{h^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} + g_{\mu\nu} \frac{\partial^\mu \sigma \partial^\nu \sigma}{2} \right. \]
\[ \left. - \frac{\lambda}{4} (h^2 - v^2)^2 - \frac{\kappa}{4} (\sigma^2 - v_\sigma^2 - \alpha h^2)^2 \right\} \]

Then there are always new perturbative state below the (field dependent) cut-off.
However, the inflation is governed by the new constants \(\zeta\) and \(\kappa\), \(\alpha\) in addition of the original combination \(\lambda/\sqrt{\xi}\).

Shift invariant, but is not free of quadratic contributions from the heavy state

[Giudice, Lee’11]