THE COMPLEMENTARY ROLES OF INTERFEROMETRY AND ASTEROSEISMOLOGY

O. L. Creevey\textsuperscript{1,2,+,3}, M. J. P. F. G. Monteiro\textsuperscript{2,4}, T. S. Metcalfe\textsuperscript{1}, and T. M. Brown\textsuperscript{5}

\textsuperscript{1}High Altitude Observatory/NCAR, 3080 Center Green, Boulder 80301, CO, USA
\textit{Email:} creevey,travis@hao.ucar.edu
\textsuperscript{2}Centro de Astrofísica da Universidade do Porto, Rua das Estrelas 4150-762 Porto, Portugal
\textit{Email:} mjm@astro.up.pt
\textsuperscript{3}Instituto de Astrofísica de Canarias, C/ Vía Láctea, s/n E38205, La Laguna, Tenerife, Spain
\textsuperscript{4}Departamento de Matemática Aplicada, Faculdade de Ciências da Universidade do Porto, Portugal
\textsuperscript{5}Las Cumbres Observatory Inc., 6740 Cortona Dr. Goleta, CA 93117, USA. Email: tbrown@lcogt.net

\textsuperscript{+}Visitor

\section*{ABSTRACT}

How important is an independent diameter measurement for the determination of stellar parameters of a solar-type star? If we can determine the radius of the star to between 1\% and 4\% how does this effect the theoretical uncertainties? Interferometry can provide this independent measurement and it has been suggested that we should expect at least a 4\% precision on this measurement for solar-type stars. This study aims to provide both qualitative and quantitative answers to the posed questions for a star such as our sun, one to which seismology can be applied. We find that the radius is fundamental for the determination of all stellar parameters and in particular the mass and the initial hydrogen abundance. Its influence depends not only on the size of the error in the radius but also on the errors in the seismic observables. The combination of observables available is also important for determining how influential the radius measurement can be.

Key words: interferometry; asteroseismology; solar-type stars.

\section{1. INTRODUCTION}

Asteroseismology is the interpretation of a star’s oscillation frequency spectrum to characterize its internal structure. By probing the interior, we are in effect testing our knowledge of fundamental physics. This interpretation is however not usually entirely independent, because we use the frequencies to model the mass of the star. If we could obtain an independent measure of mass, then we could use the oscillation frequencies to probe the stellar interior.

Recent studies have shown that an independent measurement using interferometry allows us to measure the radius of a star with a precision of about 1\% for the brightest stars and it has been suggested that we could obtain up to 4\% for most solar-type stars whose diameters we can measure [1, 2, 3]. Our interest is exploring how an interferometrically determined radius measurement can complement oscillation frequency information. Can this radius determine the mass of the star? And if it can, can we then use the frequencies to probe the physics of the star? How important is the radius for other parameters such as age and chemical composition? And what is the effect on the theoretical uncertainties?

This study follows the theoretical approach presented in [4] and [5] and is summarized in section 2 along with the parameters, physical assumptions of the models and the observables. Section 3 summarizes the theoretical uncertainties, and presents some results from simulations that support trends consistent with theory.

\section{2. ANALYSIS}

\subsection{2.1. Mathematical Description}

For an elaborate description of the problem we refer the readers to [4] and [6] and summarize here the main concepts for the purposes of understanding our work.

Taylor’s Theorem allows us to approximate any differentiable function near a point by a polynomial that depends only on the derivatives of the function at that point. To first order this can be written as

$$f(x) = f(x_0) + f'(x_0)(x - x_0)$$

(1)

where $x = \{x_j\}_{j=1}^N$ are the $N$ parameters defining the system (“input” parameters) and $f = \{f_j\}_{j=1}^M$ are the $M$ expected outputs of the system that depend on $x$. These
are the expected measurements or observables. To distinguish between both the expected observables and real observations we denote the latter by \( O = \{ O_i \}_{i=1}^M \).

We measure \( f \) and would like to find the set \( x \) that produce these measurements. This problem is a typical inverse problem whose solution \( x \) (in the pure linear case) falls neatly out from

\[
x = x_0 + VW^{-1}UT^{\dagger}f,
\]

where

\[
\delta f = \frac{O - f(x_0)}{\epsilon}.
\]

Here

\[
f'(x_0)e^{-1} = D = UWV^{T}.
\]

is the Singular Value Decomposition (SVD) of the derivative matrix \( D \) and \( \epsilon = \{ \epsilon_i \}_{i=1}^M \) are the measurement errors.

SVD is the factorization of any \( M \times N \) matrix \( D \) into 3 components \( U, V^{T} \) and \( W \) (Eq. (4)). \( V^{T} \) is the transpose of \( V \) which is an \( N \times N \) orthogonal matrix that contains the input basis vectors for \( D \), or the vectors associated with the parameter space. \( U \) is an \( M \times N \) orthogonal matrix that contains the output basis vectors for \( D \), or the vectors associated with the observable space. \( W \) is a diagonal matrix that contains the singular values of \( D \).

We use SVD in our analysis because it provides a method to investigate the information content contained in our observables and their impact on each of the parameters. Eqs. (3) and (4) were defined in terms of the observed or expected error so then SVD can be used to study the unexpected uncertainties in each of the parameters via the covariance matrix

\[
C_{jk} = \sum_{i=1}^{N} \frac{V_{ji}V_{ki}}{W_{ii}}.
\]

For clarity, we shall denote the parameter uncertainties by \( \sigma \), reminding readers that \( \epsilon \) denotes the observational errors.

2.2. Parameters and Observables

A solar-type star can be described by five parameters: mass \( M \), age \( \tau \), initial hydrogen \( X \) (or helium) and metallicity \( Z \) abundances, and a mixing-length parameter \( \alpha \) to describe convection in the outer envelope. We use the codes ASTEC [7] for stellar structure and evolution, and ADIPLS for the pulsations. The physics used are the following: The equation of state (EOS) is given by the Eggleton, Faulkner & Flannery EOS [8]. The opacities are the OPAL 1995 tables [9], supplemented by Kurucz opacities at low temperatures. Convection is described by the classical mixing-length theory [10]. We ignore overshoot effects and do not include Coulomb corrections nor diffusion. The parameters of the model star are \( M = 1.03 \, M_{\odot}, \tau = 1.00 \, \text{Gyr}, X = 0.740, Z = 0.018 \), and \( \alpha = 1.5 \). The results however are similar for stars around these ranges of parameters.

The stellar codes, given the five input parameters, produce some observables such as luminosity \( L \), radius \( R \), effective temperature \( T_{\text{eff}} \), and oscillation frequencies \( \nu_{n,\ell} \). We also calculate the observable \( [M/\mathcal{H}] = \log(Z/X) - \log(Z/X)_{\odot} \) and assume that we observe it using spectroscopy.

For this study we concentrated on two sets of observables, one using 5 observables and one using 29:

\[ OS1 = \{ R, T_{\text{eff}}, [M/\mathcal{H}], \Delta\nu, \delta\nu \} \]

where \( \Delta\nu \) and \( \delta\nu \) denote the mean values of the large and small frequency separations respectively, and

\[ OS2 = \{ R, T_{\text{eff}}, [M/\mathcal{H}], \Delta\nu_{n,1}, \delta\nu_{n,0} \} \]

for a range of \( n \) and \( l \), where \( \Delta\nu_{n,\ell} = \nu_{n,\ell} - \nu_{n-1,\ell} \) and \( \delta\nu_{n,0} = \nu_{n,0} - \nu_{n-1,2} \).

3. RESULTS

3.1. Theoretical Uncertainties

We calculated the theoretical uncertainties in each of the parameters using the diagonal elements of Eq. 5. We varied the error on radius from 0.5 to 9% and fixed \( \epsilon(\nu) = 0.5 \, \mu\text{Hz} \) and \( 1.3 \, \mu\text{Hz} \), \( \epsilon(T_{\text{eff}}) = 50 \, \text{K} \) and \( \epsilon([M/\mathcal{H}]) = 0.05 \). For the observables \( \Delta\nu \) and \( \delta\nu \) we used the propagated error (i.e. \( \sqrt{2\epsilon(\nu)^2} \)).

Fig. 1 presents the theoretical uncertainties in each of the parameters as a function of radius error for the 0.5\,\mu\text{Hz} frequency error (top panel) and the 1.3\,\mu\text{Hz} error (bottom) while using OS1 (5 observables only).

The uncertainties in \( M \), \( Z \) and \( X \) show similar dependence on \( \epsilon(R) \) for both \( \epsilon(\nu) = 0.5 \) and 1.3\,\mu\text{Hz}, indicating that \( \nu \) has no significant role to play for the determination of the parameters at this precision on radius. \( \sigma(\alpha) \) and \( \sigma(\tau) \) are most affected by the worse frequency errors. \( \epsilon(\nu) \) is important for constraining \( \alpha \) only where \( \epsilon(R) \) is less than \( \sim 4\% \), while for a worse precision in radius, \( \sigma(\alpha) \) has a similar value in both panels. As expected, \( \sigma(\tau) \) suffers most from having worse frequency errors. For \( M \) and \( X \) we find that at \( \epsilon(R) = 8\% \) we manage to...
constrain these parameters to only 25 and 20% respectively. But improving the precision on radius causes a drastic reduction in the uncertainties of these parameters, reaching below ~ 4% for \( \epsilon(R) = 1\% \) at worse frequency error and 2-3% for better frequency error. Note also that improving \( R \) also improves the determination of \( Z \) where we could expect that this value is constrained mainly by \([M/R]\).

For the larger frequency error (lower panel) the importance of the radius is suppressed at a much higher value of about 6%, where then the frequency information begins to dominate and we see no noticeable increase in the precision of each of the parameters after this value. The importance of the radius is most apparent at lower \( \epsilon(R) \). For \( \epsilon(R) < 3\% \), it is only \( R \) that can get the uncertainties to a very high precision. At \( \epsilon(R) = 1\% \), we can constrain \( M \) and \( X \) to less than 2.5%.

![Figure 1](image1.png)

**Figure 1.** The theoretical uncertainties expected for each of the parameters using OS1 as a function of \( \epsilon(R) \). Top: \( \epsilon(\nu_{\text{ini}}) = 0.5\mu\text{Hz} \). Bottom: \( \epsilon(\nu_{\text{ini}}) = 1.3\mu\text{Hz} \).

![Figure 2](image2.png)

**Figure 2.** The theoretical uncertainties expected for each of the parameters using OS2 as a function of \( \epsilon(R) \). Top: \( \epsilon(\nu_{\text{ini}}) = 0.5\mu\text{Hz} \). Bottom: \( \epsilon(\nu_{\text{ini}}) = 1.3\mu\text{Hz} \).

### 3.2. Simulations

We simulate observations by adding random gaussian errors to the observables calculated with the codes ASTEC and ADIPLS. We do this for a range of radius and frequency errors, and fix \( \epsilon(T_{\text{eff}}) = 50\text{K} \) and \( \epsilon([M/H]) = 0.05 \). The simulated observable takes the form

\[
B_{\text{sim}} = B_{\text{ori}} + r_g \times \epsilon(B),
\]

where the subscripts “sim” and “ori” denote the simulated and original observable respectively; \( r_g \) is a random gaussian error with \( \sigma = 1 \); \( \epsilon(B) \) is the error on observable \( B \). For the frequencies we add a systematic error.

For each radius and frequency error, we simulated 100 sets of the OS1 observables. Given that the parameters
with these observables show a more stable (linear) system than using for example individual frequencies, we wanted to see how well we could recover the input model parameters using only Eq. (2) once. Using a grid of models, we found sets of parameters whose observables fell to within 3\sigma of the observed values, and chose the first set in this possible list to be our initial parameters. In most cases the initial mass value was \(0.90 \, M_\odot\).

Fig. 3 shows the mean and standard deviations of the fit (inverted) mass as a function of radius error for \(\epsilon(\nu) = 0.5 \mu Hz\) (upper panel), \(\epsilon(\nu) = 1.3 \mu Hz\) (center panel), and \(\epsilon(\nu) = 2.5 \mu Hz\) (lower panel). The envelope dashed lines are the theoretical uncertainties.

The results of the simulations are consistent with theory for all values of \(\epsilon(\nu)\) and \(\epsilon(R)\), although at lower \(\epsilon(\nu)\) and \(\epsilon(R)\), the simulations provide uncertainties on the mass that are better than the theoretical ones (Fig. 1 top and center panels).

The main difference between the upper and center panel is at \(\epsilon(R) < 2\%\), where \(R\) is complemented well by the frequency information to constrain the mass to \(-1\%\). At higher \(\epsilon(R)\), there is no difference between the theoretical mass uncertainties for the different panels, indicating that it is the radius that is the dominant observable contributing to the theoretical uncertainty. The worse frequency errors contribute mainly to the accuracy of the results as the lower panel indicates, however the mean value still falls to within the error bar.

Is the linearization in Eq. (1) a good approximation? While we may be wary that at higher values of \(\epsilon(R)\) we fail to constrain the mass well, it is encouraging to see that both at low radius errors using 5 observables Eq. (1) does provide a good approximation: the mass is determined quite well and the trends in the precision of the results are consistent with theory. Increasing the number of observables and/or using a minimization method to fit rather than just inverting the derivative matrix will improve the determination of the parameters especially for larger errors in all observables.

4. CONCLUSIONS

The aims of this study were to determine how important an independent radius measurement can be for the determination of stellar parameters of a solar-type star and what are the effects on the theoretical uncertainties. Our work provides some qualitative and quantitative answers to these posed questions, most notably:

- The combinations of observables available are important to diagnose how important an interferometrically measured diameter is (Figs. 1,2).
- The size of the errors on the observable other than the radius is also important to determine how useful \(R\) is (Figs. 1,2,3).

ACKNOWLEDGMENTS

This work was supported in part by a grant (OC) within the project POCI/CTE-AST/57610/2004 from FCT and POCI 2010 with funds from the European programme FEDER. Travel grant for SOHO 18/GONG 2006/HELAS I provided by US SOHO Project.
REFERENCES