SEISMOLOGY OF SOLAR-TYPE STARS

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ABSTRACT

Seismology of solar-type stars other than the Sun becomes a realistic possibility with the upcoming MOST, COROT and MONS missions, and the ESA mission Eddington. We report on the results of experiments with artificial data from models of solar-type stars to assess further what we might learn about the internal structure of such stars.

Key words: asteroseismology; solar-type stars.

1. INTRODUCTION

Seismology of solar-type stars besides the Sun is expected in the not-too-distant future to provide information of great relevance for understanding stellar evolution. Although helioseismology has shown itself to be a rich source of information on the Sun, we need to develop tools that will allow us to use to the full potential the information provided by seismic observations of other stars. Here we address this effort by establishing a hare-and-hounds type of exercise. One of us provided the "observational" data on two stars. The rest of us then established as much as possible about the actual characteristics of the "observed" stars, without having in the process any indication of what type of stars had been used.

We have combined different methods of using the data in order to constrain as well as currently possible the nature of the stars considered. The tools reported here include the helioseismic HR diagram (christened elsewhere the CD Diagram), inversions and the seismic analysis of convective boundaries.

2. THE "OBSERVED" FREQUENCIES

We assume here that all the information we have about each of the two test stars is a set of frequencies from their spectrum (with known \(n\) and \(l\)). The frequencies correspond to modes of degree \(l\) equal to 0, 1 and 2 for one star (S1), and \(l=0\)−3 for the other star (S2). No other information was known to the remaining authors in order to establish what constraints could be given on the properties of these stars. Some details will be provided below, in a comparison between the actual and inferred model properties.

<table>
<thead>
<tr>
<th>Star</th>
<th>Degree (l)</th>
<th>Order (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0</td>
<td>11-27</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>12-28</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>15-27</td>
</tr>
<tr>
<td>S2</td>
<td>0</td>
<td>14-32</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>13-29</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>15-30</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>16-28</td>
</tr>
<tr>
<td>Sun</td>
<td>0</td>
<td>12-33</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>12-33</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11-32</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11-32</td>
</tr>
</tbody>
</table>

3. THE "MEASURED" PROPERTIES

As an aid for comparison, we use a standard solar model incorporating settling of heavy elements (Model S of Christensen-Dalsgaard et al. 1996).

3.1. Large and small frequency separations

The so-called large and small separations are easily identifiable characteristics in the power spectrum of a solar-type star which may provide information on the star's mass and age when combined together in a CD Asteroseismic Diagram (Christensen-Dalsgaard et al. 1996).
1993). The large separation is defined as
\[ \Delta_{1}(\nu) = \nu_{n+1,1} - \nu_{n,1}, \] (1)
while the small separation is
\[ d_{1}(\nu) = \nu_{n,1} - \nu_{n-1,1} + 2. \] (2)
We also discuss here the diagnostic potential of a small separation calculated from modes with consecutive degrees, as defined by Gough (1998),
\[ d_{1/2}(\nu) = \frac{3}{2} \left( \nu_{n,0} - 2\nu_{n,1} + \nu_{n+1,0} \right). \] (3)
The small separations are sensitive to evolutionary changes in the structure of the core. The frequencies entering the definition of \( d_{1/2} \) have significantly different values (of the order of the large separation) which makes this quantity more sensitive to variations of the sound-speed derivative in the core. However, this spread of frequencies also implies that frequency-dependent surface effects may be imperfectly removed, so that there may be some confusion from the surface layers. The reason we still consider \( d_{1/2} \) is that it is expected to change in a significantly different way with age for a given frequency, when compared with \( d_{0} \). This can provide a complementary test to the structure of the core.

\[ \frac{\partial \log \Delta v_0}{\partial \log M} = -1.26, \quad \frac{\partial \log D_0}{\partial \log M} = -0.0242, \]
\[ \frac{\partial \log \Delta v_0}{\partial \Delta X_c} = 0.621, \quad \frac{\partial \log D_0}{\partial \Delta X_c} = 0.221. \] (4)

Using these derivatives, and comparing the separations of S1 and S2 with those of the solar model, we estimate masses and central hydrogen abundances for the mystery stars. The final estimates of mass and central hydrogen abundance for S1 and S2 are shown in Table 2.

**Table 2. Frequency separations as determined from the frequencies (see the text). The estimated values for the mass and the central hydrogen abundance are also given as determined from a CD Diagram. The estimated age \( t \) is given in units of \( 10^9 \) years.**

<table>
<thead>
<tr>
<th>Star</th>
<th>( \Delta v_0 )</th>
<th>( D_0 )</th>
<th>( M/M_\odot )</th>
<th>( \Delta X_c )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>134.4</td>
<td>10.29</td>
<td>1.00</td>
<td>0.34</td>
<td>4.6</td>
</tr>
<tr>
<td>S1</td>
<td>117.7</td>
<td>7.19</td>
<td>1.00</td>
<td>0.10</td>
<td>7.0</td>
</tr>
<tr>
<td>S2</td>
<td>99.2</td>
<td>8.99</td>
<td>1.20</td>
<td>0.40</td>
<td>2.2</td>
</tr>
</tbody>
</table>

There is a clear indication that star S2 has a higher mass than the Sun and is at an evolutionary stage similar to that of the Sun. The other star (S1) seems to have a significantly lower abundance of hydrogen at the centre, and is therefore a very old Sun. We note, however, that the actual values determined depend on the calibration implicit when constructing the CD Diagram. This only gives a unique solution because we ignore other effects, such as the heavy-element abundance and convective-core overshoot. Those will affect the frequency separations of the stars. The precise definition of the quantities used in the CD Diagram will also affect the result when other physical aspects are in error. Here we have used those proposed by Christensen-Dalsgaard (1993), but others including simple averages of the separations illustrated in Fig. 1 could be used instead.

### 3.2. Sound-speed inversions

The reference model was used to compute the mode kernels relating frequency differences to the difference \( \delta u/u \) between the reference and mystery models, where \( u \equiv p/\rho \) is the ratio of pressure to density.
Averaging kernels given by

$$ \mathcal{K}(r, r_0) = \sum_{n,l} c_{nl}(r_0) K^{nl}(r) $$

(5)

were then constructed, where the coefficients $c_{nl}$ were determined using the SOLA method under the assumption that the errors in the data were uniform.

The relative frequency differences between the reference and the test models were large and almost constant. This indicated that the mass or radius (or both) of the reference model differed from that of the test model (to be precise, the combination $M/R^2$ was different). The average value of the frequency differences allowed us to estimate the factor by which the reference model frequencies had to be scaled such that the combination $(M/R^2)$ for both the reference and test models was the same. Note that the information given by this scale factor is closely related to what we have already determined using the frequency separations. We recomputed the frequency differences after scaling the reference model frequencies to the $(M/R^2)$ value of the test model and used these frequency differences to compute

$$ \frac{\delta u}{u} = \sum_{n,l} c_{nl} \frac{\delta \nu_{nl}}{\nu_{nl}}. $$

(6)

3.3. Borders of the convection zones

As shown before (Basu et al. 1994; Monteiro et al. 1994; Christensen-Dalsgaard et al. 1995) the signal due to the base of the convection zone has a period related to the acoustic depth $r$ of the base of the convection zone and an amplitude that depends on the nature of the transition layer. Such a signal has been shown to give information on the internal structure of a star (Monteiro et al. 2000), providing complementary constraints extracted directly from the observed frequencies.

The expression for the expected signal in the frequencies $\omega_{nl}$ (note that $r_{nl} = \omega_{nl}/2\pi$), is of the first order given by

$$ \delta \omega = A(\omega) \cos \left(2(\omega r_d + \phi_0) \right), $$

(7)

where $r_d$ is essentially the acoustic depth at which the base of the convection zone is located and $\phi_0$ is a phase depending on the surface layers. The amplitude $A$ varies slowly with frequency, and so for convenience we quote its value $A_d$ at a fiducial frequency. The value of $A_d$ is associated with the sharpness of the transition. All the parameters $r_d$, $\phi_0$ and $A_d$ are determined from the observational data. The fitting is described by Monteiro et al. (1994).

Note that $r_d$ gives the true acoustic depth plus a contribution from the surface layers where the modes are reflected. However, as discussed by Monteiro et al. (1994), the surface contribution is not strongly dependent on the particular physics of the star’s surface layers so we can use the relative differences in $r_d$ as a measure of relative differences in radial position of the base of the convection zone.

By fitting the above expression to the mean multiplet frequencies, we measure the properties of the base of the convection zone for each of the two stars. The resulting values are given in Table 3.

![Figure 2. SOLA averaging kernels for the modesets provided for S1 (dotted lines) and S2 (dashed lines), for two target radii as indicated on the panels. As can be seen, the kernels are almost the same for the two modesets, and can essentially only be constructed at one location. For the deductions in the text, the kernels in the bottom panel have been used, since these are the cleanest in the near-surface layers.](image)

Table 3. Parameters for the signal fitted to the frequencies of each star. The acoustic depth $r_d$ is in seconds; the amplitude $A_d$ is in $\mu$Hz. The differences quoted for both parameters are calculated relative to the solar values.

<table>
<thead>
<tr>
<th>Star</th>
<th>$r_d$</th>
<th>$\delta r_d$</th>
<th>$A_d/2\pi$</th>
<th>$\delta A_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>2633</td>
<td>18%</td>
<td>0.053</td>
<td>-35%</td>
</tr>
<tr>
<td>S2</td>
<td>2408</td>
<td>8%</td>
<td>0.043</td>
<td>-48%</td>
</tr>
<tr>
<td>Sun</td>
<td>2243</td>
<td>-</td>
<td>0.082</td>
<td>-</td>
</tr>
</tbody>
</table>

As discussed by Monteiro et al. (2000), the amplitude decreases with age, while the value of $r_d$ increases with time. However, the amplitude is lower.
for higher mass stars, while $\tau_d$ is larger for higher values of the mass (up to 1.2 $M_\odot$). These changes can be estimated, relatively to the present Sun, as being (see Monteiro et al. 2000),

$$\delta(X_c) \approx -40\% \quad \Rightarrow \quad \begin{cases} \delta(A_d) \approx -60\% \\ \delta(\tau_d) \approx +20\% \end{cases} \tag{8}$$

$$\delta(M/M_\odot) \approx +20\% \quad \Rightarrow \quad \begin{cases} \delta(A_d) \approx -60\% \\ \delta(\tau_d) \approx +10\% \end{cases} \tag{9}$$

Taking into account these two dependencies, and using the values given in Table 2, we find that the expected values would be:

$$S1: \quad \delta(\tau_d) \approx +12\% , \quad \delta(A_d) \approx -33\%$$

$$S2: \quad \delta(\tau_d) \approx +7\% , \quad \delta(A_d) \approx -51\% .$$

By comparing the two sets of values (Eqs 9 with Table 3) it seems that there is an indication, if we assume that there is no overshoot, that the star S1 is slightly older than inferred from the CD Diagram. However, the values given in Eqs (8) are not good estimates for very old stars, as appears to be the case for star S1. In spite of the uncertainties, we may say that there seems to be no adiabatically stratified overshoot present in this star.

For star S2, we get about the expected acoustic depth and amplitude. Again, there seems to be a quite strong case for no overshoot affecting the stratification at the boundary corresponding to the base of the convective envelope.

4. COMPARISON WITH THE "REAL" VALUES

The actual parameters of the models used to compute the 'observed frequencies' are listed in Table 4; they can be compared with the inferred values in Table 2. Interestingly, the hounds determined correctly the masses of both stars as well as $X_c$ for S1, while for S2 $X_c$ was slightly lower than inferred. Also, the models had no adiabatically stratified overshoot from the convective envelopes, as was correctly inferred from the analysis of the oscillatory signal in the frequencies. The results of the inversion to determine $\delta u$ was somewhat less successful; it was correctly inferred that the value is small, but due to the broad kernels a more detailed comparison is difficult. This aspect of the exercise needs further analysis.

<table>
<thead>
<tr>
<th>Star</th>
<th>$M/M_\odot$</th>
<th>$X_c$</th>
<th>Overshoot</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>1.00</td>
<td>0.10</td>
<td>N</td>
<td>7.0</td>
</tr>
<tr>
<td>S2</td>
<td>1.20</td>
<td>0.33</td>
<td>N</td>
<td>2.4</td>
</tr>
</tbody>
</table>

The present experiment is not yet a fully realistic simulation of the observational situation. The unknown models and those employed by the hounds had essentially the same physics and chemical composition; differences in these aspects may clearly introduce systematic errors in the inferences (e.g. Gough 1987, Christensen-Dalsgaard 1993). Furthermore, the artificial data included no random errors, a fact that was noted by the hounds and which has significantly made their life easier in finding the correct answer.

On the other hand, the present analysis included only information from the oscillation frequencies. In reality, other information about the stars such as photometric and spectroscopic determinations of their luminosities and effective temperatures, would be used as constraints.

5. DISCUSSION

This work has demonstrated, with some success, the ability of oscillation frequencies, such as will be obtained from coming space missions, to constrain the properties of stars. Further experiments of this nature, taking into account also realistic simulations of the expected error properties of the data, are required to improve our understanding of the information contained in the frequencies and develop optimal techniques for extracting this information.

Hare-and-hounds exercises are very powerful tools for the development of analysis methods, as has also been shown clearly for helioseismic data. In addition, they provide a great deal of excitement for the hounds, and undoubtedly rather more fun for the hare than a real hunt.

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