Convective Envelopes in Solar-Type Stars: What Can We Learn from Their Seismic Study?

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1. Introduction

Seismic studies of the base of the solar convective envelope have constrained the properties at that transition region (Christensen-Dalsgaard et al. 1995, and references therein), which is of relevance to mixing, rotation and magnetic activity (e.g. Monteiro et al. 1998c, and references therein). An important implication is that the overshoot layer is not nearly adiabatic stratified, as in simple models: there is growing evidence that the effect on the temperature stratification is small in spite of an extended penetration region (e.g. Singh, Roxburgh & Chan 1995).

There is little doubt that convective overshoot, to some extent, occurs at the borders of convective regions in stars. However, there is no reliable theory of overshooting in stars. To be able to constrain the type and intensity of overshoot in other stars is of great importance. Extending the work of Monteiro et al. (1998b), we consider the effect of overshoot on the properties of the signal in the frequencies for stars of mass between 0.9 and 1.2 $M_\odot$.

2. Signal in the frequencies of solar-type stars

At the edge of a convective region the change in the temperature gradient from being radiative to the adiabatic value causes a discontinuity in the second derivative of the sound speed. Also, overshoot may be present; a simple, but extreme, model of overshoot is that the region of adiabatic stratification is extended into the convectively stable region, possibly with a discontinuous jump in the temperature gradient to its radiative value. The presence of either kind of discontinuity makes a small contribution to the global frequencies (Monteiro et al. 1994) of the form

$$\delta \omega_p = \left[ A_1^2 \omega^{-4} + A_2^2 \omega^{-2} \right]^{1/2} \cos \left[ 2 (\omega \tau_d + \phi_0) \right],$$

for low-degree modes. Here $A_1$ and $A_2$ are amplitudes that depend weakly on frequency $\omega$: $A_1$ is always present in general, but $A_2$ will be non-zero only if
there is overshoot, of the nature discussed above. Also, \( \tau_d \) is essentially the
acoustical depth \( \tau \) (i.e., the sound travel time measured from the photospheric
radius) of the edge of the convection, and \( \phi_0 \) is a constant related to the phase
of the eigenfunctions.

To facilitate the comparison between different stars, we consider the
amplitude of the periodic signal evaluated at a fiducial frequency \( \tilde{\omega} \) by defining
\( A_4 \equiv (A_2^4 \tilde{\omega}^{-4} + A_3^2 \tilde{\omega}^{-2})^{1/2} \). For the Sun at its present age, we use \( \tilde{\omega}/2\pi=2500 \mu \text{Hz} \)
since this frequency is in the range where the signal from the base of the convec-
tion zone is clearest. We scale this value for other stars (using homology scaling
for frequencies) to find \( \tilde{\omega}/2\pi=2500 \mu \text{Hz} \times (M/M_\odot)^{1/2} (R/R_\odot)^{-3/2} \). According
to Christensen-Dalsgaard et al. (1995) and Monteiro et al. (1998a),

\[
A_1 \propto \left( \frac{d\nabla_r}{d\tau} \right)_{\tau_d} \quad \text{and} \quad A_2 \propto (\nabla_r - \nabla_a)_{\tau_d}.
\]  

These give an estimate of the dependence of the amplitude \( A_2 \) on the properties
of the transition. Note that the amplitude of the signal changes along the life of
the star due to the changes in the internal structure.

3. Stellar models and their seismic properties

In order to find the limits for the amplitude of the signal for models with different
mass and amount of overshoot, we have computed several stellar models and
their oscillation frequencies. The computations were essentially as described by
Christensen-Dalsgaard (1982), although with somewhat updated physics.

<table>
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<th>( M/M_\odot )</th>
<th>( \tau )</th>
<th>( \tau_d )</th>
<th>( \tau_d/R )</th>
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Table 1. Characteristics of some of the ZAMS models considered in this
work. The total acoustic radius \( \tau_a \) and the acoustic depth \( \tau_d \) of the transition
in the temperature gradient are in seconds. The overshoot layer has been
modelled as being (essentially) adiabatically stratified and having a size \( \ell_{ov} \),
here given in units of the local pressure scale height \( H_p \).

We first consider several models on the zero-age main sequence (ZAMS),
with masses between 0.9 and 1.2 \( M_\odot \), both without and with overshoot (Table 1).
Near our upper mass limit, the models show a small convective core. As long as
the core has a small acoustic size, we can neglect the signal originating from its
border (see Monteiro et al. 1998a, for the effect of convective cores).

From the expressions for the amplitude (Eqs 2) we may determine the expected
amplitude of the signal. This is shown in Fig. 1 for the 1 \( M_\odot \) models
considered here. A more realistic representation of the expected observational
signal is obtained by fitting computed frequencies to Eq. (1). The fitting pro-
cedure used is a simplified version of the method described by Monteiro et al.
(1994). Here we have considered modes with degree \( l \leq 4 \), in a range of frequency
corresponding approximately to the modes observed in the Sun. Note that the
Figure 1. Expected amplitudes (dotted line) for ZAMS models, and the values found by fitting the signal in the frequencies (filled symbols). The values shown are for zero-age (circles) and evolved (diamonds) stellar models of one solar mass having different overshoot layers at the bottom of the convective envelope, illustrating the change due to evolution from the zero age up to an age of $4.53 \times 10^9$ years.

signal amplitude is smallest for models that have no convective overshoot. Any extension of the convection zone by overshooting will give a larger amplitude: the actual value depends on the type of stratification imposed and extent of the overshoot layer.

Christensen-Dalsgaard et al. (1995) calculated the variation of $A_d$ with the extent of the overshoot layer if an adiabatic stratification is assumed for this region. It was shown that the fitted parameters to the signal in the frequencies reproduced the theoretically expected values for $\ell_{ov} \geq 0.1 H_P$. However, as also seen in Fig. 1, the theory does not agree well with the fitted values for small overshoot layers (say $\ell_{ov} < 0.1 H_P$): our theoretically expected amplitude (dotted line in Fig. 1) decreases slightly for small amounts of overshooting (see also Roxburgh & Vorontsov 1994). However, we must emphasize that the fitted amplitudes show no such behaviour: overshooting increases the amplitude of the signal obtained by fitting the frequencies.

We also note that the behaviour would be different if the assumption of having an adiabatically stratified overshoot layer were relaxed. Therefore the results found for the overshoot models presented here must be taken as an upper limit for the amplitude of the signal which might be measured for such stars.

4. Dependence of the amplitude on mass, overshoot depth and age

To illustrate in more detail the dependence of the signal on stellar mass and overshoot we have also calculated amplitudes and acoustic depths from fits to frequencies of the computed ZAMS models. These values are shown in Fig. 2. It is evident that, for each mass, models with overshoot have a substantially larger amplitude. Thus we may hope to be able to discriminate between these cases if the mass of the star is known with sufficient accuracy.

Furthermore, to investigate the dependence on age, frequencies for a $1 M_\odot$ model at close to solar age have also been analysed. The resulting amplitudes satisfy $A_{d,\odot}(\text{now})/A_{d,\odot}(\text{ZAMS}) \approx 0.6$. Such decrease in amplitude with age must also be taken into account when testing the models against observations.
Figure 2. Results found from interpolating the values of the parameters from the fit of Eq. (1) to the frequencies of the computed ZAMS models. The continuous line joins values for models with an overshoot layer of size $\ell_{ov}=0.2H_p$, while the dashed line is for models with $\ell_{ov}=0.1H_p$ and the dotted line for $\ell_{ov}=0.0H_p$. Panel (a) gives the behaviour of the amplitude, while panel (b) shows the acoustic depth of the base of the convection zone as measured from the signal. Note that $\tau_d=\tau_d+\alpha_r$, where $\alpha_r$ is a shift due to the reflection of the modes at the surface of the star (Monteiro et al., 1994).

If enough modes (more than three for each $l$ value) for degrees up to $l=4$ are available then the signal in the frequencies can be measured if we have sufficiently accurate observations. Several space missions have been planned for observing solar-type oscillations in other stars. These are expected to provide data on a few stars with the accuracy required by our analysis.

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References