Looking for variations with latitude of the base of the solar convection zone

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Abstract

The Sun’s m-averaged frequencies of global oscillations have been used successfully in determining the basic characteristics of the base of the solar convective region (e.g. Christensen-Dalsgaard et al. 1995, Basu 1997). In particular, it has been possible to constrain its location and more particularly the extent of convective overshoot. With higher quality data now available it is becoming possible to isolate the properties of this region at different latitudes. This is important for the modelling aspects such as mixing and convective overshoot occurring at the base of the convection zone. It can also have interesting interplay with the rotation profile around the region of the tachocline and the generation of magnetic fields. Such properties as deeper penetration of the convection at certain latitudes may be intimately connected with the angular velocity profile there, and the identification of such characteristics in the thermal stratification is therefore important to complement the information from the rotation inversions (Schou et al. 1998). Here we present an analysis of SOHO-MDI data to try to detect such latitudinal variations.

Key words: convection zone; convective overshoot; latitude; tachocline; solar cycle.

1. Introduction

The base of the Sun’s convection zone is key to understanding a number of aspects of solar and stellar physics. It links the convective envelope, which occupies the outer 30 per cent by radius and which extends essentially to the observable surface, to the unobservable radiative interior. Convective penetrative overshoot, turbulent mixing, meridional circulations and gravitational settling taking place in this region can mix material between the convection zone and the radiative interior, and so understanding this region and its dynamics is crucial for understanding the burning of lithium and the depletion of helium in the outer envelope relative to what is believed to be its protosolar value. (See e.g. Zahn 1995 for a discussion of transport processes; in the helioseismic context cf. also Gough et al. 1996.) The role of this interface region in the transport of angular momentum between envelope and interior is also of interest, particularly since the discovery by helioseismology (see Thompson et al. 1996; Schou et al. 1998 and references therein) that there is a rather abrupt transition — the so-called tachocline — between essentially surface-like latitudinal differential rotation in the convective envelope and latitudinally independent rotation beneath the convection zone: for theoretical discussion of the tachocline, see Spiegel & Zahn 1992 and Elliott 1997. It is also widely believed that this region is the seat of the solar dynamo (see Monteiro et al., these proceedings, and references therein).

The variation of the angle between the rotation axis and the local direction of gravity, the latitudinal variation of the rotation rate within the convection zone, and the preferred latitudes for the appearance of sunspots, all make it at least plausible that the properties of the convection and of the convective overshoot beneath the base of the convection zone might also depend on latitude.

Depending on whether convective overshoot is included or not, solar models indicate that at the base of the convection zone the adiabatic sound speed has a discontinuity in either its first or second derivative. Either way, the consequence for solar p modes, which are set up by acoustic waves propagating in the interior of the Sun, is that they see a possibly weak but nonetheless abrupt transition in the structure at that depth. Such a localized effect produces in the p-mode frequencies a signal which is periodic in mode frequency. This signal has been successfully used to detect the base of the convection zone and to place limits on the extent of convective overshoot (Basu et al. 1994; Monteiro et al. 1994; Roxburgh & Vorontsov 1994; Christensen-Dalsgaard et al. 1995).

Each p mode is labeled by its radial order n, its degree l and azimuthal order m. In a spherically sym-
metric star the frequency \( \nu_{nlm} \) of the mode would be independent of \( m \), but rotation and other departures from spherical symmetry raise this degeneracy, and the resulting difference in frequency between modes in the same \((n,l,m)\) multiplet but of different azimuthal orders is known as frequency splitting. The value of frequency \( \nu_{nlm} \) is sensitive to conditions in the acoustic cavity where that particular mode propagates. In the radial direction, the acoustic cavity extends from close to the surface down to a depth, which depends on \( \nu_{nlm}/(l+1/2) \), where the waves undergo total internal reflection. The extent of the cavity in the latitudinal direction depends on \( l \) and \( m \) in the combination \( m/(l+1/2) \): thus within the same multiplet, modes with different \( m \) values are sensitive to different regions in latitude of the Sun.

In this work we look for to see whether it is possible to detect variations with latitude at the base of the convection zone, using the sensitivity of the frequency splitting to different latitudes. As described below, we do this by isolating the contribution to the signal in the frequencies of the sharp transition occurring at the base of the envelope from each region in latitude.

2. ISOLATING IN THE FREQUENCIES THE PROPERTIES FOR A GIVEN LATITUDE

Data concerning the variation of the frequencies with \( n, l \) and \( m \) are available from SOHO-MDI in the form of mean multiplet frequencies \( \nu_n \) and so-called \( a \)-coefficients \( a_j(n,l) \). These provide a parametrized fit to the individual \( \nu_{nlm} \) frequencies:

\[
\nu_{nlm} = \nu_n + \sum_{j=1}^{j_{\text{max}}} a_j(n,l) P_j^0(m).
\]  

Here the \( P_j^0(m) \) are polynomials in \( m \) of degree \( j \). They are defined to be orthogonal, with normalization \( \int P_j^0(x) P_j^0(x) dx = 1 \). Explicit expressions for the polynomials are given by Pijpers (1997); but here we shall use the approximation

\[
P_j^0(m) \approx L P_j(m/L)
\]  

(the equality would be exact in the high \( l \) limit) where \( P_j \) is the Legendre polynomial of degree \( j \) and \( L = \sqrt{l(l+1)} \). Evidently it would in general be necessary to have \( j_{\text{max}} = 2l \) to describe completely the variation with \( m \) of the frequencies within a given \((n,l,m)\) multiplet, but for reasons of stability in fitting the data a smaller number of coefficients are normally used. The variation of the frequencies with \( m \) arises from rotation and any departure from spherical symmetry. The leading-order splitting due to rotation gives rise to non-zero \( a \)-coefficients for odd values of \( j \). These can be inverted to infer the internal rotation of the Sun. Non-zero even coefficients arise from structural asphericity, centrifugal distortion and other second-order rotational effects: other symmetry-breaking agents such as magnetic fields and non-rotational velocity fields may also contribute. For the time being we suppose that the only asphericity of relevance is structural and that its dominant effect is due to a small non-spherical but axisymmetric perturbation \( \delta c(r, \theta) \) to the otherwise spherically symmetric adiabatic sound speed \( c(r) \). Then in first-order perturbation theory the effect of the perturbation to the sound speed is to perturb the frequencies by an amount

\[
\delta \nu_{nlm} = \int_0^R K_{nl}(r) \left\{ \tilde{P}_l^m(\cos \theta) \right\}^2 \frac{\delta c}{c} \sin \theta d\theta d\phi,
\]  

so that

\[
\nu_{nlm} = \nu_n + \delta \nu_{nlm}.
\]  

Here \( R \) is the radius of the star; the functions \( K_{nl}(r) \) are functions of the hypothetical spherically symmetric star's structure and its global mode eigenfunctions (see Fig. 1); and the \( \tilde{P}_l^m \) are the usual Legendre functions, but normalized such that \( \int_0^1 [\tilde{P}_l^m(x)]^2 dx = 1 \). It may be noted that eq. (3) holds both for spherical and aspherical perturbations.
where
\[ \alpha_k = \frac{(-1)^k(2k)!}{2^{2k}(k!)^2} \],
we obtain
\[ \nu_{nlm} = \nu_{nl} + \sum_{k \geq 1} \left( \int_0^R K_{nl}(r) f_{2k}(r) dr \right) \frac{\alpha_k}{L} P_{2k}^{(l)}(m). \]
It follows that
\[ a_{2k}(n, l) = \left( \int_0^R K_{nl}(r) f_{2k}(r) dr \right) \frac{\alpha_k}{L}. \]

In our previous analysis (e.g., Christensen-Dalsgaard et al. 1995), we have studied the mean multiplet frequencies of a supposedly spherically symmetric model, to infer the properties of the region at the base of its outer convective envelope. In the present work, we wish to explore those properties in the Sun but as a function of latitude. In order to apply our earlier analysis, we therefore first construct frequencies \( \nu_{nl}^{(\theta_0)} \) of a hypothetical spherically symmetric star whose sound speed at radius \( r \) is equal to that of the Sun at the same radius but at the colatitude \( \theta = \theta_0 \) under study. Relative to the spherically symmetric average Sun, such a star would have a relative sound-speed perturbation
\[ \frac{\delta c}{c} = \sum_{k \geq 1} f_{2k}(r) P_{2k}(\cos \theta_0) \]
from eq. (5). Substituting this into eq. (3), and using eq. (9), the frequencies of such a star would be
\[ \nu_{nl}^{(\theta_0)} = \nu_{nl} + \sum_{k \geq 1} \frac{L}{\alpha_k} P_{2k}(\cos \theta_0). \]

In this way we separate the contribution to the frequencies from each region in colatitude since each set of frequencies constructed from the observational data using Eq. (11) will be representative of a Sun with the characteristics of the base of the convection zone as the actual Sun is at that particular value of the colatitude \( \theta \).

In this work we have used coefficients up to \( a_{68} \). The latitudinal sensitivity of the combinations (Eq. 11) obtainable from these coefficients is illustrated in Fig. 1. In brief, these were constructed as follows. The latitudinal sensitivity of individual \((n, l, m)\) modes to the sound speed is proportional to the square of the corresponding Legendre functions, as given by Eq. (3). Using the orthogonality of the \( P_{2k}^{(l)}(m) \) functions, Eq. (1) can be used to express each \( \alpha \)-coefficient as a linear combination of individual mode frequencies, and hence a kernel describing their sensitivity to the latitudinal variation of sound speed can be written down. Finally, Eq. (11) expresses \( \nu_{nl}^{(\theta_0)} \) as a linear combination of \( \alpha \)-coefficients, and so the kernel describing its latitudinal sensitivity can be obtained: these are shown, for four values of \( \theta_0 \), in Fig. 1. Note that the kernels are rather broad, indicating that it is not possible with these data alone to obtain highly localized information about latitudinal variations.

3. MEASURING THE SIGNAL IN THE FREQUENCIES

As shown in Christensen-Dalsgaard et al. 1995 the signal due to the base of the convection zone has a period associated with the acoustic depth \( \tau \) (with \( dr = -dr/c \), \( c \) is the sound speed and \( r \) the radius) of the base of the convection zone and an amplitude that depends on the stratification of such a transition layer. The expression for the expected signal in the frequencies \( \omega_{nl} \) (note that \( \nu_{nl} = \omega_{nl}/2\pi \), if a possible overshoot layer is assumed to be near adiabatically stratified, is to first order given by
\[ \delta \omega = A_{\omega, d} \cos \left[ 2\omega \tau_d - \gamma_d \frac{l(l+1)}{\omega} + 2\phi_0 \right], \]
where
\[ A_{\omega, d} = \frac{1-2\Delta}{(1-\Delta)^2} \sqrt{\frac{\omega_d^4}{(1-\Delta)^3} \frac{\omega}{\omega_d} + a_d^2 \left( \frac{\omega}{\omega_d} \right)^2}. \]

Here
\[ \Delta = \frac{l(l+1)}{(l+1)(l+2)} \Delta_d, \]
\[ \tau_d = \int_{r_d}^c \frac{dr}{c} + \Delta, \]
\[ \gamma_d = \int_{r_d}^R \frac{C}{r^2} \, dr + a_\gamma. \]  

The two constants \((a_\phi, a_\gamma)\) are corrections due to surface effects (see Monteiro et al. 1994 for details), while \(R\) is the radius of the Sun and \(r_d\) the radial location of the base of the convection zone. This expression is fitted to the data to determine the quantities \((a_\phi, a_\gamma, f_0, \eta, \alpha, \Delta \omega_d)\), as described in Monteiro et al. (1994). In calculating the results presented here we have also used the reference values \(I = 20\) and \(\omega' / 2\pi = 2500 \muHz\).

In order to compare the values of the amplitude we use again the value it takes at a specified frequency (in this case \(\omega\)), by defining

\[ A_d = A_{2\omega, 0} - (a_1^2 + a_2^2)^{1/2}. \]  

The value of \(A_d\) is associated with the sharpness of the transition, while \(\eta_d\) measures the acoustic depth of the base of the envelope. Note that \(\eta_d\) gives the true acoustic depth plus \(a_\phi\), the latter arising from a contribution from the surface layers where the modes are reflected. However, as discussed in Monteiro et al. (1994), \(a_\phi\) is not strongly dependent on the physics used to construct the near surface layers of the Sun indicating that we can use the relative differences in \(\eta_d\) as a measure of relative differences in radial position of the base of the convection zone.

Christensen-Dalsgaard et al. (1995) determined the variation of the amplitude \(A_d\) with the size \(\ell_{0\omega}\) of an overshoot layer when it is near adiabatically stratified (see Monteiro et al., these proceedings). Since the measured values for the signal in the frequencies agree very closely with the predicted values such a dependence was used to put an upper limit on \(\ell_{0\omega}\). However, that limit was calculated for \(m\)-averaged data and therefore gave only some kind of latitudinal average of the properties of the layer. Since we can now construct data combinations corresponding to different latitudes, we can investigate the behaviour of such a layer as a function of latitude.

4. THE FORWARD PROBLEM: SOLAR MODELS AND THE SUN

In order to test the method and its capability of detecting variations with latitude we consider a combination of two solar models. One is a standard solar model with the depth of the convection zone at 0.72876\(R_\odot\), while the other is a model incorporating overshoot as in Christensen-Dalsgaard et al. (1995) and extending the convective envelope down to 0.71128\(R_\odot\). From these two models and their frequencies of oscillation we construct a new set of frequencies corresponding to a model with a structure as sketched in Fig. 2. This was done using a perturbative approach. The frequencies of the composite model were considered as perturbations to the frequencies of one of the spherically symmetrical models, and assumed to arise purely from a perturbation to the sound speed, at fixed density. The 2D kernels for these perturbations were then taken to be the spherically symmetric sound-speed kernels multiplied by squares of Legendre functions to provide

![Figure 3. Results for the test model shown in Figure 2 (circles joined by solid lines). The solid lines to either side show the 1-\(\sigma\) effect of observational errors (from Monte Carlo simulations) when the quoted observational errors from the SOHO-MDI 144-day data are taken. Values obtained by fitting the mean multiplet frequencies of our test model are indicated by the arrows. Also shown with horizontal lines are the values obtained from fitting frequencies from the two spherically symmetric models from which our composite model was created: values for the standard non-overshoot model are indicated with the dashed line, while those for the overshoot model are indicated with the dot-dashed line.](image)

the latitudinal dependence. Equivalently, within our framework, we could have constructed the data from the \(a\)-coefficients of the two spherically symmetric models. Note that we have not constructed a complete composite model that is in hydrostatic equilibrium, but in terms of the analysis we are discussing here that is not necessary: what we have is a latitudinal variation in sound speed which mimics what might be found in a more complete model and which therefore allows us to test our technique for measuring the changes of the location and type of transition at different colatitudes.

The results for the amplitude \(A_d\) and the period \(\eta_d\) for this test model are shown in Fig. 3. It is possible to identify the higher amplitude (due to overshoot) and greater acoustic depth (deeper base of the convection zone) at mid-latitudes, compared with the equatorial values recovered at higher values of the colatitude. This is the signature of the presence in our test model of the overshoot region around colatitude 60°. As should be expected from the kernels in Figure 1, the value of the amplitude for colatitude 60° does not approach the values for the overshoot (dot-dashed lines in Fig. 3) as it would if the ker-
splitting of the modes to measure the contributions arising from each region of the base of the convective region located at different latitudes. The effect of the present observational errors is illustrated in Figure 3 for the test model considered.

The results for the solar data are shown in Fig. 4. Two sets of frequencies and $\alpha$-coefficients were used, one for 144 days and the other for a full year. The two sets of data give different behaviors for the amplitude and acoustic depth. The 144-day set shows a hint of a feature around 50°. In fact, results on a finer grid of target colatitudes show that both fitted amplitude and fitted $\tau_d$ are greater at points in the interval between colatitudes 40° and 50° than at the end-points of the interval. On the face of it, this suggests perhaps an enhancement of overshoot at mid-latitudes, perhaps near to where the rotation rates in the radiative interior and convective envelope coincide. It is also interesting that this is the latitudinal region where magnetic field storage, associated with the solar activity cycle, may be expected to take place (e.g. Kuhn 1996). The effect of either the coincidence of the rotation rates or of the presence of the magnetic field may be to extend the convection or convective overshoot (cf. Monteiro et al., these proceedings, and references therein).

However, based on the artificial experiments and the kernels (Fig. 1) we cannot naturally expect a narrowly localized feature. The 360-day set shows very little evidence for any latitudinal variation: indeed, within the error bars the results are consistent with there being no latitudinal variation at all.

5. DISCUSSION

In this work we have been mainly focus in presenting a method to measure the properties at the base of the solar convective envelope at different latitudes. A test model was used to show the possibilities of such a method and a preliminary application to MDI data has been presented. No firm conclusions can be drawn yet. However, the results presented indicate that the method is valid and can be used, if accurate data is available.

One of the aspects needing further effort is the inclusion of a larger number of $\alpha$-coefficients in order to have more localized kernels. In order to control better the propagated errors in the inversion of a larger set of $\alpha$-coefficients, we need to explore using 1-D latitudinal inversion techniques (cf. Schou et al. 1994) instead of the more naive but natural combinations given by Eq. (11). This work is in progress. The improvement in resolution that we anticipate is of particular interest at mid-latitudes, near the point where the rotation rates of the convective envelope and radiative interior appear to coincide, and where the 144-day results (and maybe theoretical expectations too) suggest that enhanced overshoot might occur.
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