Beyond Bekenstein´s Theory

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Motivations

- Bekenstein's variable charge model uses these assumptions: covariance, gauge invariance, causality and time-reversal invariance of electromagnetism. So, it is guaranteed the applicability of the scheme to other gauge interactions such as the strong forces.
- Besides, it introduces a useful simplifying assumption; that the gravitational sector is unaffected by the scalar field introduced to vary the coupling constant .
- That is why it is interesting to explore first this simplified model before a similar exploration of more general theories.

Objetives

- We derive equations that govern the energy exchange between matter, the scalar field and the electromagnetic field. Although we do not analyze the precise mechanism of energy release, we assume that the work done by the scalar field is radiated away in an efficient way.
- We find how the energy flux of matter is modified by the scalar field.
- We estimate the total magnetic component of matter from “Sum rules techniques”.
- From studying the thermal history of the Earth in the presence of Bekenstein´s scalar field, we obtain a strict bound.

Time variation of α in Bekenstein's formalism

The foundational hypothesis

1. The theory must reduce to Maxwell's when $\alpha = \mathrm{Cte}$.
2. Changes in α are dynamical (generated by a dynamical field) ϵ.
3. The dynamics of the electromagnetic field as well as ϵ's can be obtained from a variational principle.
4. The theory must be local gauge invariant.
5. The theory must preserve causality.
6. The action must be time reversal invariant.
7. Planck's scale ℓ_P is the smallest length available in the theory.
8. Einstein's equations describe gravitation.

- String theories and the like in which there are other fundamental length scales, force us to set aside condition 7. These hypothesis uniquely lead to the following action:

$$S = S_{\rm em} + S_\epsilon + S_m + S_G, \tag{1}$$

$$\begin{aligned} S_{\rm em} &= -\frac{1}{16\pi}\int F^{\mu\nu}F_{\mu\nu}\sqrt{-g}d^4x, & (2)\\ S_\epsilon &= -\frac{\hbar c}{2\ell_B}\int \frac{\epsilon_{,}{}^{\mu}\epsilon_{,\mu}}{\epsilon^2}\sqrt{-g}d^4x. & (3)\end{aligned}$$

S_m, S_G are the matter and gravitational field actions, and the metric here is $(-1,1,1,1)$.

- The main difference between Maxwell's and Bekenstein's theories is the connection between the vector potential and the electromagnetic field

$$F_{\mu\nu} = \frac{1}{\epsilon}\left[(\epsilon A_\nu)_{,\mu} - (\epsilon A_\mu)_{,\nu}\right] \tag{4}$$

where the local value of the elementary electric charge (coupling constant)

$$e(\mathbf{r}, t) = e_0 \epsilon(\mathbf{r}, t) \tag{5}$$

that is

$$\epsilon = \left(\frac{\alpha}{\alpha_0}\right)^{\frac{1}{2}} \tag{6}$$

The energy exchange

- We will neglect the small spatial variations of α and focus on the cosmological variation, as we will be interested on any secular energy injection of the scalar field on a planet such as the Earth.
- In our approximation it is also enough to work in flat space-time.
- Using $c = 1$ and $\psi = \ln \epsilon$, we denote,

$$T_{\mathrm{f}}^{\mu\nu} = T_{\mathrm{em}}^{\mu\nu} + T_{\psi}^{\mu\nu} \tag{7}$$

$$\begin{aligned} T_{\mathrm{f}}^{\mu\nu} =& \frac{1}{4\pi}\left[F^{\mu\lambda}F^{\nu}{}_{\lambda} - \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right] \\ &+ \frac{\hbar}{\ell_B^2}\left(\psi^{,\mu}\psi^{,\nu} - \frac{1}{2}\eta^{\mu\nu}\psi_{,\alpha}\psi^{,\alpha}\right) \end{aligned} \tag{8}$$

- Calculating the divergence of T_{f} and using the conservation of the total energy-momentum tensor $T_{\mathrm{f}}^{\mu\nu}{}_{,\nu} + T_{\mathrm{m}}^{\mu\nu}{}_{,\nu} = 0$ we find,

$$T_{\mathrm{m}}^{\mu\nu}{}_{,\nu} = e^{\psi} j^{\alpha} F^{\mu}{}_{\alpha} - \psi_{,\nu} \left(\eta^{\mu\nu} \frac{\partial \sigma}{\partial \psi} + T_{\mathrm{em}}^{\mu\nu} - \frac{1}{16\pi} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right) \tag{9}$$

which is the source of any observable effect.

- Then, the component 0 reads,

$$T_{\mathrm{m}}^{0\nu}{}_{,\nu} = \mathbf{j}.\mathbf{E} - e^{-2\psi} \frac{\mathbf{B}^2 \dot{\psi}}{4\pi} - e^{-2\psi} \nabla \psi.\mathbf{S} + \dot{\psi} \frac{\partial \sigma}{\partial \psi} \tag{10}$$

- From the generalized Poynting theorem we discover,

$$T_{em}{}^{0\rho}{}_{,\rho} = \frac{\partial u_{em}}{\partial t} + \nabla . e^{-2\psi}\left(\frac{\mathbf{E}\times\mathbf{B}}{4\pi}\right) = -\mathbf{E}\cdot\mathbf{j} + \frac{e^{-2\psi}\mathbf{E}^2}{4\pi}\dot{\psi} + e^{-2\psi}\mathbf{S}.\nabla\psi \tag{11}$$

where $u_{em} = e^{-2\psi}(\mathbf{E}^2+\mathbf{B}^2)/(8\pi)$. Assuming that there is not scalar injection of energy,

$$2\dot{\psi}u_{em} = 2\dot{\psi}e^{-2\psi}\frac{(\mathbf{B}^2+\mathbf{E}^2)}{8\pi} = -\mathbf{j}\cdot\mathbf{E} + \dot{\psi}e^{-2\psi}\frac{\mathbf{E}^2}{4\pi} \tag{12}$$

or

$$\mathbf{j}\cdot\mathbf{E} = -\frac{\mathbf{B}^2}{4\pi}\dot{\psi}e^{-2\psi}. \tag{13}$$

- In this case the motion of matter is negligible, so the first index 0 is equivalent to project along the fluid four-velocity. Using some approximations we can write,

$$T_{\mathrm{m}\ ,\nu}^{0\nu} = \frac{\partial u}{\partial t} + \nabla \mathbf{J} \tag{14}$$

  where u is the internal energy density and $\mathbf{J}$ is the heat flux.

- We understand "internal energy" as the energy that can be exchanged by the system in the processes considered. The "rest mass" is the "non convertible energy". If the scalar field changes the effective electric charge, then it can alter the electromagnetic contribution to the rest mass; this contribution will be no longer "rest mass", but "internal energy".

- $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial t}|_{cooling} + \frac{\partial \sigma_\mu}{\partial t}$. This last term accounts for the dependence of the bulk of matter on the scalar field, which is given by the electromagnetic contribution to the nuclear mass.

$$\frac{\partial u}{\partial t}|_{cooling} + \frac{\partial \sigma_\mu}{\partial t} + \nabla \mathbf{J} = - \frac{\mathbf{B}^2}{4\pi}\dot{\psi} e^{-2\psi} - \frac{e^{-2\psi}\mathbf{B}^2\dot{\psi}}{4\pi} - e^{-2\psi}\nabla\psi.\mathbf{S} - \dot{\psi}\frac{\partial \sigma}{\partial \psi}. \quad (15)$$

- Then, assuming that the scalar field is space independent and supposing that the following condition $\frac{\partial \sigma}{\partial \psi} - \frac{\partial \sigma_\mu}{\partial \psi} \approx 0$ is fulfilled,

$$\nabla \mathbf{J} = -\frac{e^{-2\psi}\mathbf{B}^2\dot{\psi}}{2\pi} - \frac{\partial u}{\partial t}|_{cooling}. \quad (16)$$

- Assumption: *the cooling term is not modified by the scalar field*. Reasons:
  1. The electrostatic energy “injected” by the scalar field stays within the matter bulk.
  2. The thermal evolution should not change given the high thermal conductivity of the Earth considered in this work. We expect the magnetic energy excess to be radiated away, increasing the heat flux **J**.

The Earth’s heat flux

- Using “Sum rules techniques” we calculate the E_m of the nuclei. Then, the fractional contribution of the E_m to rest mass energy is

$$\zeta(A) \simeq \frac{E_{m_A}}{m_A c^2} \approx 8.60465 \times 10^{-6} A^{-1/3} \tag{17}$$

- The contribution of $\dot{\alpha}/\alpha$ to the heat flux can be calculated using the equation of heat conduction

$$\frac{1}{r^2}\frac{d}{dr}\left(Kr^2\frac{dT}{dr}\right) = -\varepsilon\rho \tag{18}$$

ε is the energy production per mass unit of any material.

$$\mathbf{J} = -K\frac{dT}{dr} \approx \bar{\varepsilon}\frac{m(r)}{4\pi r^2} \tag{19}$$

- Comparing the observed $\mathbf{J}$ with the one that follows the model,

$$\mathbf{J}_{\oplus} \simeq 2.6 \times 10^{4}\ \mathrm{W/m^2} \frac{\dot{\alpha}}{H_0 \alpha} \tag{20}$$

$$\mathbf{J}_{\oplus}^{\mathrm{obs}} \simeq (60 \pm 40)\ \mathrm{mW/m^2} \tag{21}$$

we find an upper bound

$$\left| \frac{\dot{\alpha}}{H_0 \alpha} \right|_0 < \frac{3\sigma}{2.6 \times 10^{4}\ \mathrm{W/m^2}} \simeq 5 \times 10^{-6} \tag{22}$$

which is the main result of our work.

- The weighted mean of the data from the geographic distribution of heat flux measurements is,

$$\bar{\mathbf{J}} = 69.6 \pm 3.3\,\mathrm{mW/m^2} \tag{23}$$

So, we can find an even tighter bound

$$\left|\frac{\dot{\alpha}}{H_0\alpha}\right|_0 < 4 \times 10^{-7} \tag{24}$$

which is the same order as Oklo´s $\left|\frac{\dot{\alpha}}{H_0\alpha}\right|_{\mathrm{Oklo}} < 1.4 \times 10^{-7}$.

Conclusions

- Eq.16 shows that there's an extra contribution to the heat current besides the cooling of matter.
- We justified our assumption that the matter cooling rate is not modified by ψ.
- We estimated the magnetic energy content of matter, thus permitting us to quantify the anomalous heat flux.
- Our best bound was obtained from the geothermal aspects of the Earth, $\left|\frac{\dot{\alpha}}{H_0\alpha}\right|_0 < 4 \times 10^{-7}$ is comparable to that of Oklo.
- This analysis may be applied to other theories with extra fields that introduce extra "internal energies" to matter. We will report further work on future publications.

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