From Varying Constants to
Fundamental Physics
Lisboa, 6 September 2010

The Variation of G in a Negatively Curved Space-time

José P. Mimoso (Dep. Physics & CAAUL-U. Lisbon),

Francisco Lobo (CAUL, U. Lisbon)

September 6, 2010

Motivation

Gravitational effects of negative curvature?

Increasing interest in the features of large voids.

Possible relation with acceleration of late time unverse?

Unexpected **pecularities** of negative curvature?

Outline

Description of a certain static solution with surfaces of negative curvature

Derivation of the corresponding generalised ST exact solution

Discussion of difficulties regarding the PPN formalism

Cautionary note

Work still in progress…

The GR solution

We looked for a vacuum solution of the form

$$ds^2 = -e^{U(r)}dt^2 + e^{W(r)}dr^2 + r^2\left(du^2 + \sinh^2 u\, dv^2\right)$$

And this leads to

$$\boxed{ds^2 = -\left(\frac{2C}{r} - 1\right)dt^2 + \frac{dr^2}{\left(\frac{2C}{r} - 1\right)} + r^2\left(du^2 + \sinh^2 u\, dv^2\right)}$$

With regard to Petrov classification, this solution is a degenerate static solution of the class A2 of Ehlers and Kundt [J. Ehlers & W. Kundt, in Gravitation: an introduction to current research (1962) ed. L. Witten, pp 49-101]

$$\boxed{ds^2 = -\left(\frac{2C}{r} - 1\right)dt^2 + \frac{dr^2}{\left(\frac{2C}{r} - 1\right)} + r^2\left(du^2 + \sinh^2 u\, dv^2\right)}$$

Striking features are immediately apparent

$$\left(1 - \frac{2m}{r}\right)_{Schwarz} \Rightarrow \left(\frac{2C}{r} - 1\right)_{Neg-Schwarz}$$

Event horizon at $r = 2C$

Coordinates valid in the region $r < 2C$

Radial light rays $\frac{dt}{dr} = \frac{1}{\frac{2C}{r} - 1}$ $> 0 \leftarrow r < 2C$

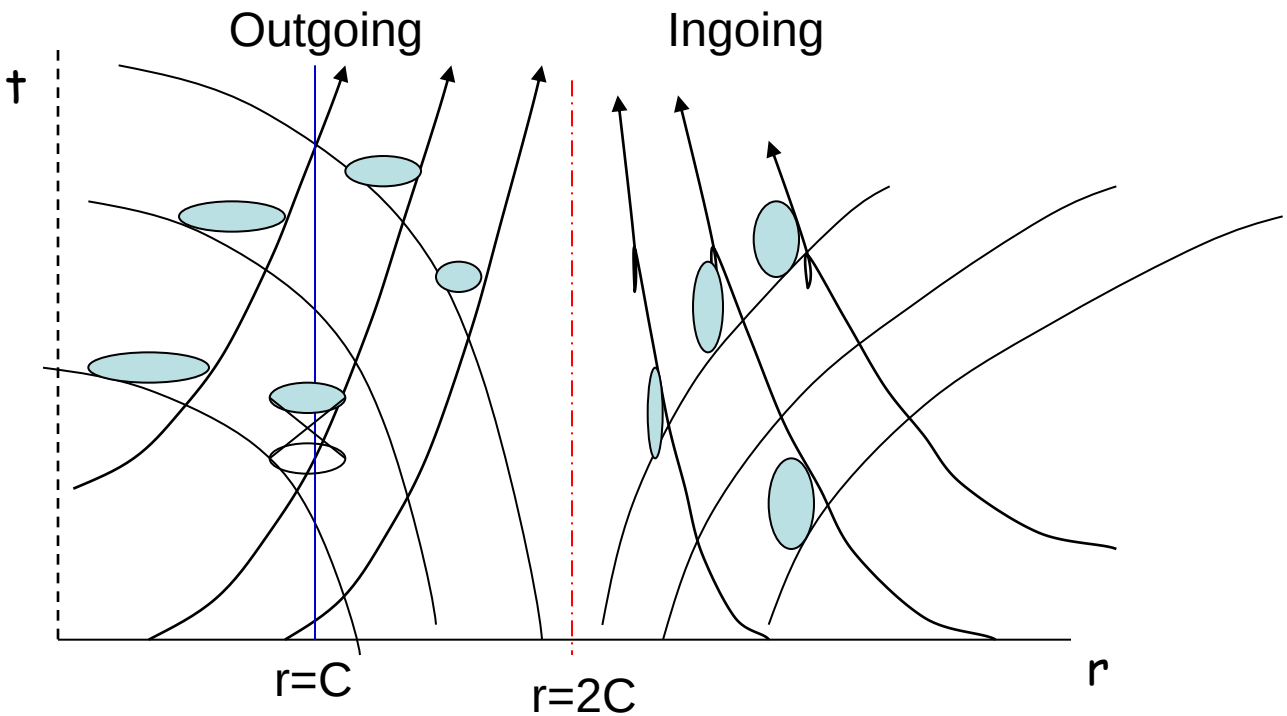
$< 0 \leftarrow r > 2C$

TABLE 2–3.1. Degenerate Static Vacuum Fields

1	2	3	4	5	6	7	8	9	10
Class	First Fundamental Form G	Coordinate Ranges	α	ν_S	ν_T	K_S	K_T	r	s
A1	$r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) + \frac{dr^2}{1 - b/r} - \left(1 - \frac{b}{r}\right) dt^2$	$0 \leqslant \vartheta \leqslant \pi$, φ mod 2π $0 < b < r < \infty$ or $b < 0 < r < \infty$	$-\frac{b}{r^3}$		$\left(\frac{\alpha}{b}\right)^{2/3}$	$\frac{1}{r^2}$			
A2	$z^2\,(dr^2 + \sinh^2 r\, d\varphi^2) + \frac{dz^2}{b/z - 1} - \left(\frac{b}{z} - 1\right) dt^2$	$0 \leqslant r < \infty$ φ mod 2π $0 < z < b$	$\frac{b}{z^3}$	0	$-\left(\frac{\alpha}{b}\right)^{2/3}$	$\frac{-1}{z^2}$	$-\alpha$		1S
A3	$z^2(dr^2 + r^2\, d\varphi^2) + z\, dz^2 - \frac{dt^2}{z}$	$0 \leqslant r < \infty$ φ mod 2π $0 < z < \infty$	$\frac{1}{z^3}$		0	0			
B1	$\frac{dt^2}{1 - b/r} + \left(1 - \frac{b}{r}\right) d\varphi^2 + r^2(d\vartheta^2 - \sin^2\vartheta\, dt^2)$	$0 < b < r < \infty$ or $b < 0 < r < \infty$ $0 < \vartheta < \pi$	$-\frac{b}{r^3}$	$-\left(\frac{\alpha}{b}\right)^{2/3}$			$\frac{1}{r^2}$	4	
B2	$\frac{dz^2}{b/z - 1} + \left(\frac{b}{z} - 1\right) d\varphi^2 + z^2(dr^2 - \sinh^2 r\, dt^2)$	$0 < z < b$ $0 < r < \infty$	$\frac{b}{z^3}$	$-\left(\frac{\alpha}{b}\right)^{2/3}$	0	$-\alpha$	$\frac{-1}{z^2}$		1T
B3	$z\, dz^2 + \frac{d\varphi^2}{z} + z^2(dr^2 - r^2\, dt^2)$	$0 < z < \infty$ $0 < r < \infty$	$\frac{1}{z^3}$	0			0		
C	$\frac{1}{(x + y)^2}\left(\frac{dx^2}{f(x)} + f(x)\, d\varphi^2 + \frac{dy^2}{\lvert f(-y)\rvert}\, \lvert f(-y)\rvert\, dt^2\right)$ $f(u) \equiv \pm(u^3 + au + b)$	$0 < x + y$ $f(-y) < 0$ $0 < f(x)$	$\pm(x + y)^3$	$f(x) - \alpha$	$-f(-y)$ $-\alpha$	ν_T	ν_s	2	0

Static Vacuum Fields 73

[From: J. Ehlers & W. Kundt, in Gravitation: an introduction to current research (1962) ed. L. Witten, pp 72]



The spatial metric is conformally flat, but at $r \to \infty$ it is not Minkowski

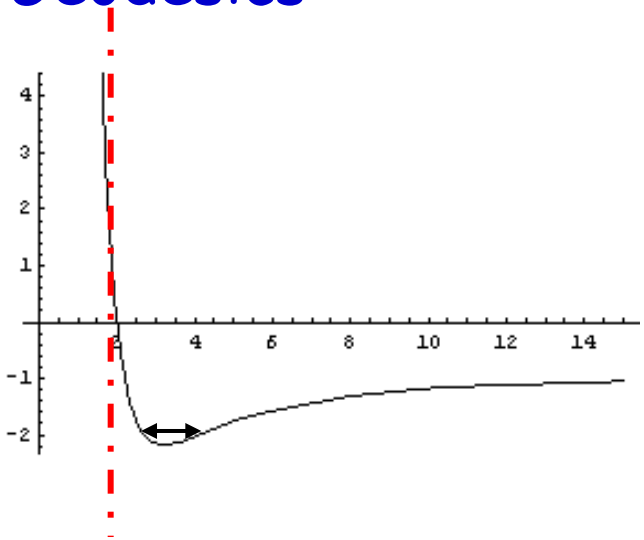
So it does not have the usual $r \to \infty$ weak field limit…

For r>2C, g_{00} and g_{rr} swap signs!

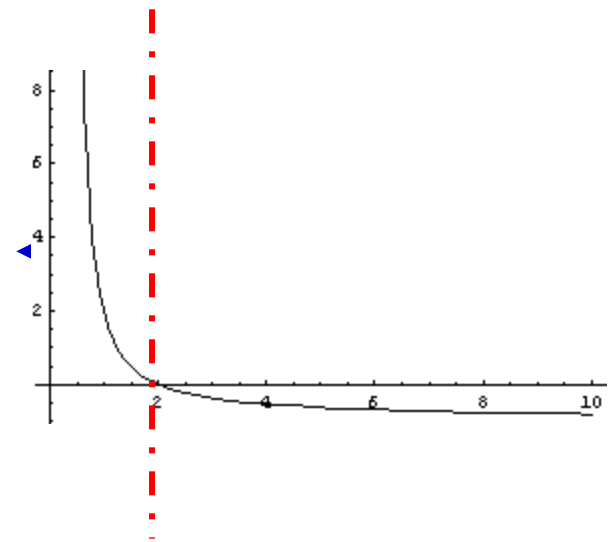
For r>C the metric becomes cosmological: Bianchi III

Geodesics

C=1



h=4



h=1

$$2V(r) = \left(\frac{2C}{r} - 1\right)\left(1 + \frac{h^2}{r^2 \sinh^2 u_0}\right)$$

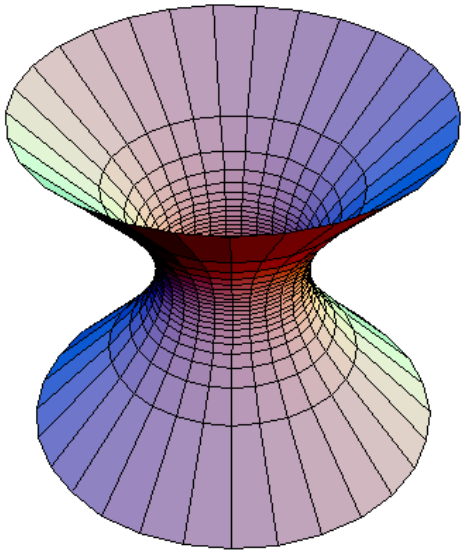
Stationary points at

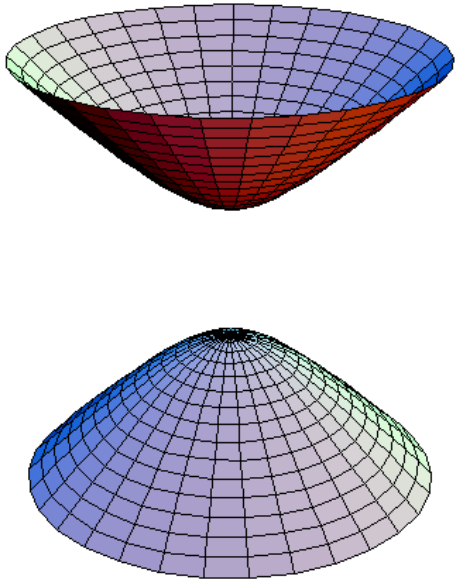
$$r_* = \frac{h \pm \sqrt{h^4 - 12C^2h^2}}{2C}$$

Beyond r =2C

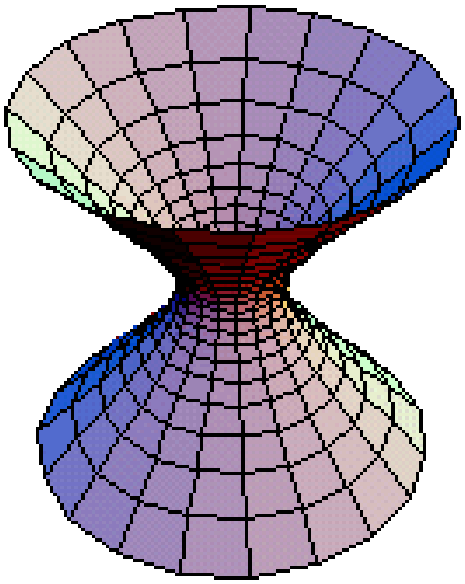
$$ds^2 = -\left(\frac{2C}{r} - 1\right)dt^2 + \frac{dr^2}{\left(\frac{2C}{r} - 1\right)} + r^2\left(\frac{d\chi}{1+\chi^2}^2 + \chi^2\, dv^2\right)$$

$$\frac{d\chi}{1+\chi^2}^2 + \chi^2\, dv^2 = \left(1 - \left(\sqrt{1+\chi^2}\right)'^2\right)d\chi^2 + \chi^2\, dv^2$$





$$x = a(r)\sinh u \cos v$$
$$y = a(r)\sinh u \sin v$$
$$z = a(r)\cosh u$$
$$w = b(r)$$



$$w^2 + x^2 + y^2 - z^2 = b^2(r) - a^2(r)$$

$$b'(r) - a'(r) = \left(\frac{2C}{r} - 1\right)^{-1}$$

Space-time tunneling ?

In [F. Lobo & J. P. Mimoso, Phys. Rev. D82:044034 (2010)]

We have shown that particular constraints are placed on the shape function, that differ significantly from the Morris-Thorne wormhole.

In particular, it is shown that the energy density is always negative and the radial pressure is positive, at the throat, contrary to the Morris-Thorne counterpart.

The static vacuum metrics can be written as

$$ds^2 = -e^{2U(x^\mu)} dt^2 + e^{-2U(x^\mu)} \bar{g}_{\mu\nu} dx^\mu dx^\nu$$

So that the EFE are

$$R_{\mu\nu} = \bar{R}_{\mu\nu} + 2U_{,\mu} U_{,\nu} = 0$$

[From: J. Ehlers & W. Kundt, in Gravitation: an introduction to current research (1962) ed. L. Witten, pp 72]

Solution of scalar-tensor Brans-Dicke Theory

Scalar-tensor theories can be derived from the action

$$\sqrt{-g}L = \sqrt{-g}\left[R - g^{ab}\nabla_a\phi\nabla_b\phi + 16\pi G L_m(\psi, m^\beta(\phi)g_{ab})\right]$$

Through a conformal transform $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(\phi)\, g_{\mu\nu}$

$$\sqrt{-g}L = \sqrt{-g}\left[\Phi R - \frac{\omega}{\Phi} g^{ab}\nabla_a\Phi\nabla_b\Phi + 16\pi G L_m(\psi)\right]$$

Buchdahl theorem (1959)

A theorem by Buchdahl [Phys. Rev. 113, 1325 (1959)] establishes the reciprocity between any static solution of Einstein's vacuum field equations and a one-parameter family of solutions of Einstein's equations with a (massless) scalar field. The so-called Einstein frame description of the scalar-tensor gravity theories fits precisely into the conditions of the Buchdahl theorem.

$$(g_{00,}\bar{g}_{\mu\nu}) \rightarrow (g_{00}{}^{B}, \bar{g}_{\mu\nu}{}^{1-B})$$

$$V = \lambda \ln(g_{00}), \quad B = \pm(1-2\lambda^2)^{1/2}$$

Solution of scalar-tensor Brans-Dicke Theory

$$ds^2 = -\left(\frac{2C}{r}-1\right)^{B-C} dt^2 + \frac{dr^2}{\left(\frac{2C}{r}-1\right)^{B+C}} + r^2\left(\frac{2C}{r}-1\right)^{1-B-C}\left(du^2+\sinh^2 u\, dv^2\right)$$

$$\Phi = \Phi_0\left(\frac{2C}{r}-1\right)^{C}$$

where

$$(2\omega+3)\,C^2 + B^2 = 1$$

Now the r=2C locus is a true point like singularity [Agnèse & La Camera PRD (1985)].

[Mimoso and Lobo: 2010 J. Phys.: Conf. Ser. 229 012078]

Geodesics

$$\varepsilon = \left(\frac{2C}{r} - 1\right)^{B-C} \dot{t}$$

Conserved

$$h / \sinh u_* = \left(\frac{2C}{r} - 1\right)^{1-B-C} r^2 \dot{v}$$

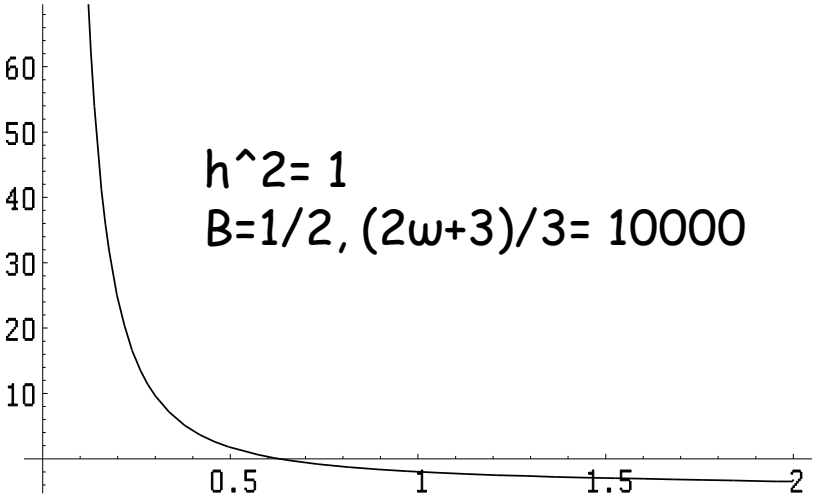
First integral

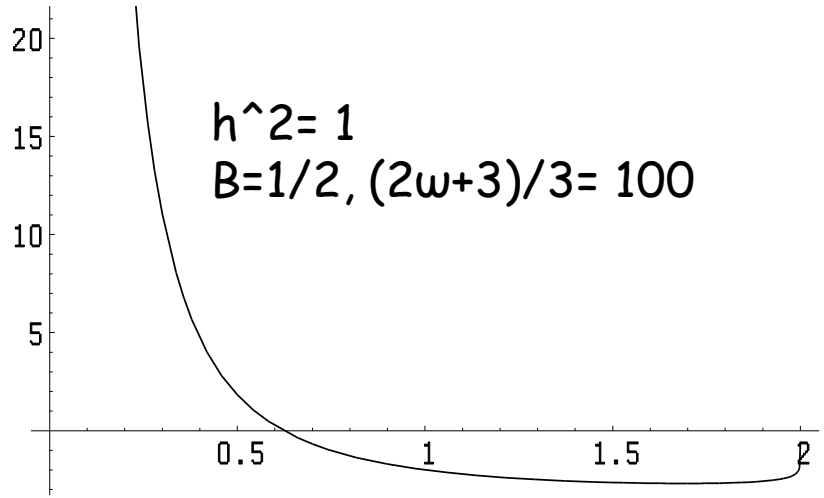
$$\left(\frac{2C}{r} - 1\right)^{-2C} \left(\dot{r}^2 + \left(\frac{2C}{r} - 1\right)^{B+C} \left(1 + \frac{h_*^2}{r^2 \left(\frac{2C}{r} - 1\right)^{1-B-C}} \right) \right) = \varepsilon^2$$

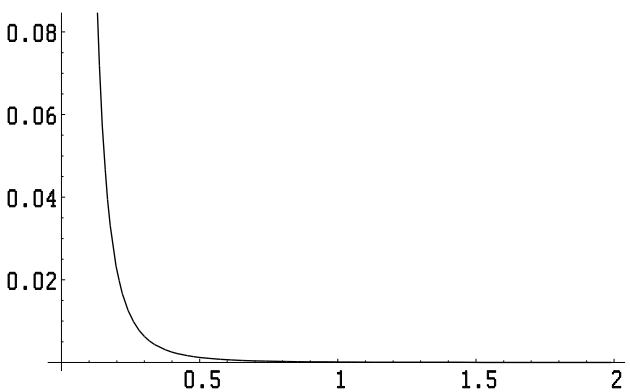
Some plots of the effective potential:

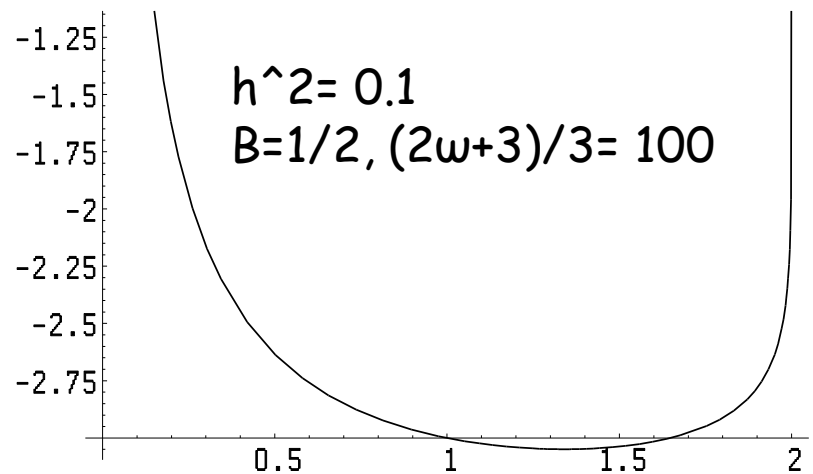
$$\dot{r}^2 = -2V(r)$$

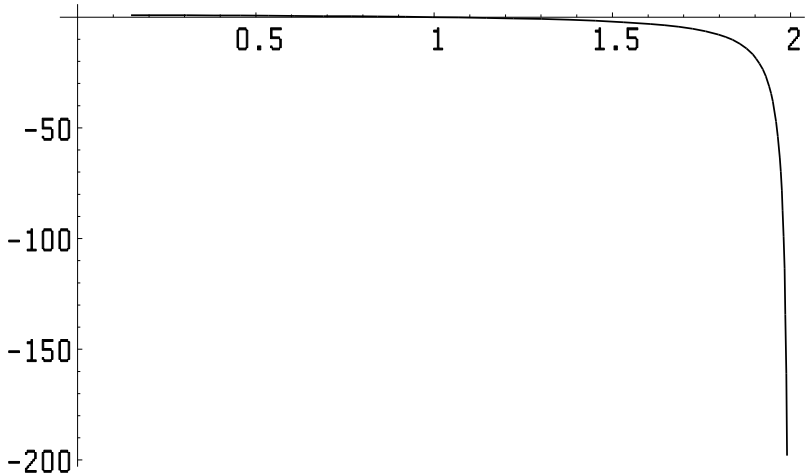
$$2V(r) = \left(\frac{2C}{r} - 1\right)^{B+C} \left(1 + \frac{h_*^2}{r^2\left(\frac{2C}{r} - 1\right)^{1-B-C}}\right) - \varepsilon^2 \left(\frac{2C}{r} - 1\right)^{2C}$$

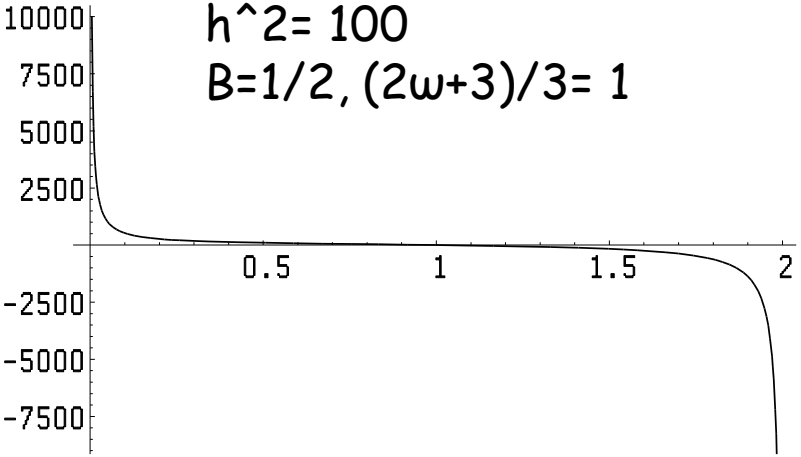

h^2= 1
B=1/2, (2ω+3)/3= 10000
60
50
40
30
20
10
0.5
1
1.5
2


h^2= 1
B=1/2, (2ω+3)/3= 100
20
15
10
5
0.5
1
1.5
2


h^2= 0.1
B=1/2, (2ω+3)/3= 1
0.08
0.06
0.04
0.02
0.5
1
1.5
2


h^2= 0.1
B=1/2, (2ω+3)/3= 100
-1.25
-1.5
-1.75
-2
-2.25
-2.5
-2.75
0.5
1
1.5
2


0.5
1
1.5
2
-50
-100
-150
-200


h^2= 100
B=1/2, (2ω+3)/3= 1
10000
7500
5000
2500
0.5
1
1.5
2
-2500
-5000
-7500

Is there a PPN limit?

No, since there is no weak field Newtonian limit for the GR solution

So, alternatively, one should to consider the strong field limit

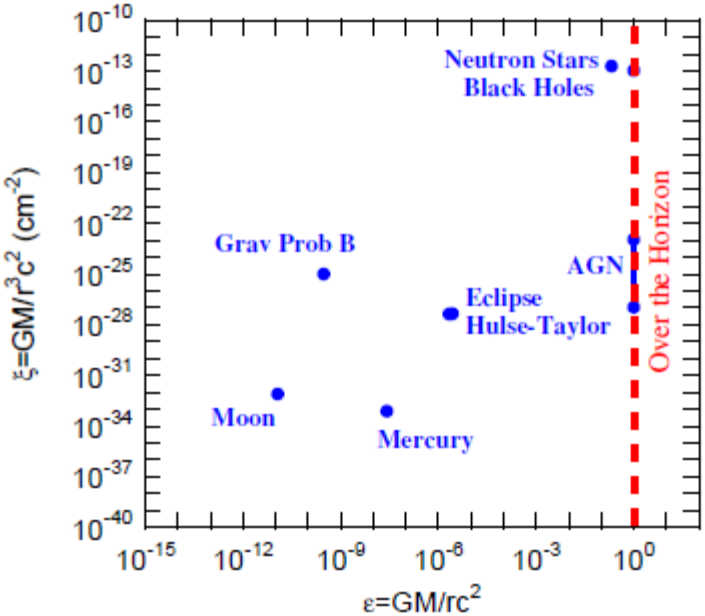


Figure 2: Tests of General Relativity placed on an appropriate parameter space. The long-dashed line represents the event horizon of Schwarzschild black holes.

[From: D. Psaltis, arXiv: 0806.1531]

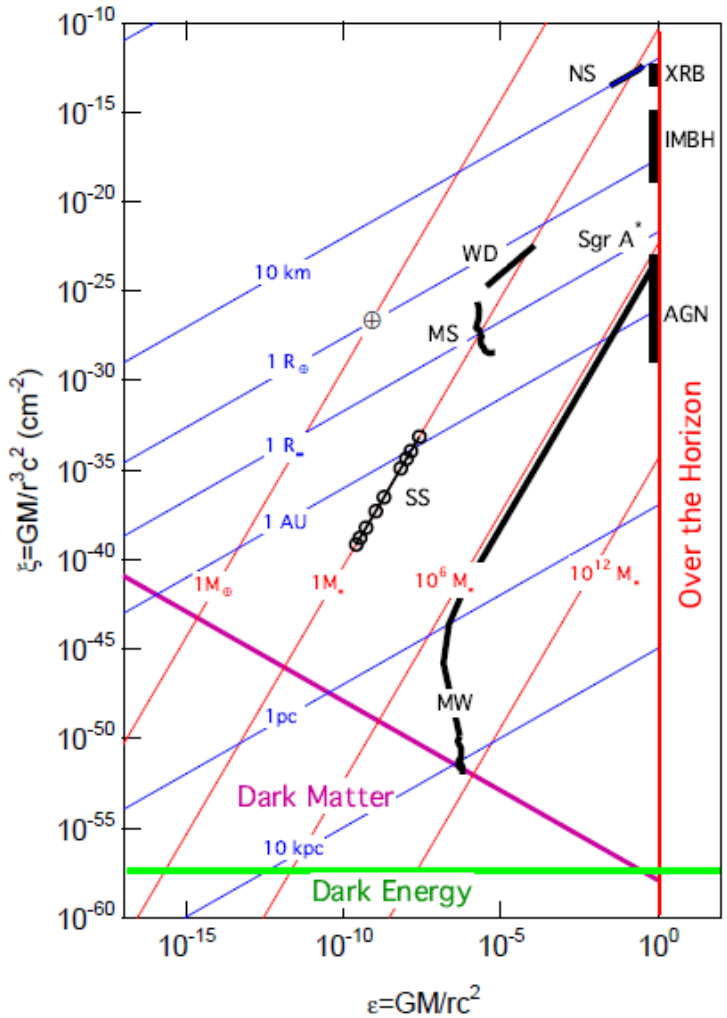


Figure 1: A parameter space for quantifying the strength of a gravitational field. The x-axis measures the potential $\epsilon \equiv GM/rc^2$ and the y-axis measures the spacetime curvature $\xi \equiv GM/r^3c^2$ of the gravitational field at a radius r away from a central object of mass M. These two parameters provide two different quantitative measures of the strength of the gravitational fields. The various curves, points, and legends are described in the text.

[From: D. Psaltis, arXiv: 0806.1531]

Somewhat inconclusive

Conclusions

Solutions with hyperbolic spatial sections exhibit repulsive gravity, naked singularities, and no newtonian weak field limit

The corresponding generalised ST solution has a larger parameter space, allowing some stationary, bound periodic orbits, but inherits the lack of a newtonian weak field limit of the GR solution.

One is thus driven to the need of considering the opposite strong field limit, and this means resorting to the investigation of tensor deformations of the vacuum metrics (gravitational waves) in a gravity theory dependent way. (In construction…)

THANKS FOR LISTENING

Acknowledgements: The authors acknowledge the financial support of FCT under the grants PTDC/FIS/102742/2008 and CERN/FP/109381/2009