f(R) Brane Cosmology
based on JCAP 0911, 011 (2009)

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Outline

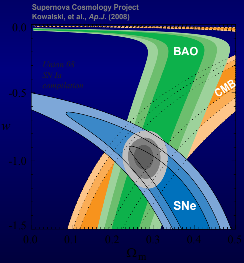
1. Introduction
2. The DGP scenario
3. Can we make the normal branch self-accelerating?
   - Modifiying the bulk action: simplest option a GB term
   - Modifying the brane action: simplest option an f(R) term
4. Conclusions

Outline

Introduction

- The universe seems accelerating
- Observational evidence: SN, CMB, BAO, ISW ...
- How to describe this acceleration or (encode our ignorance)?
  - Dark energy
  - Modified gravity: DGP scenario, f(R) models...
  - Other possibilities: Multiverse, LTB universes.
- Our ignorance can be encoded

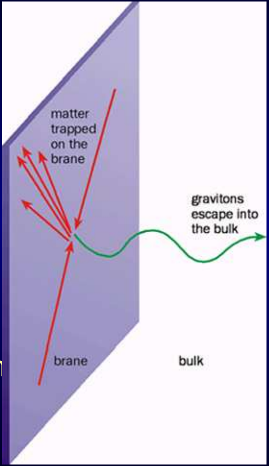
  on an effective equation of state



Outline

Dvali, Gabadadze and Porrati Model

- One brane embedded in a 5d Minkowski space-time
- Brane has no tension
- Induced gravity brane-world model
- Extra dimension is infinite
- 2 ways of embedding the brane in the bulk
- 4d FLRW cosmology is recovered at high en

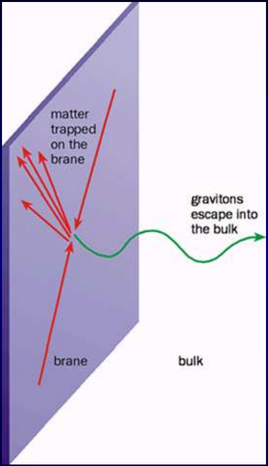


Dvali, Gabadadze and Porrati '00

Expansion on the self-accelerating DGP branch

- The expansion of the brane
  $H^2 - \frac{1}{r_c} H = \frac{\kappa_4^2}{3}\rho_m, \quad r_c = \frac{\kappa_5^2}{2\kappa_4^2}$
- The self-accelerating branch:
  - Transition from a 4d regime to a self-accelerating regime.
  - Dark energy is not needed to describe the late-time acceleration
  - The brane is asymptotically de Sitter
  - Theoretical problem: Ghost issue

Deffayet '00, Koyama '05

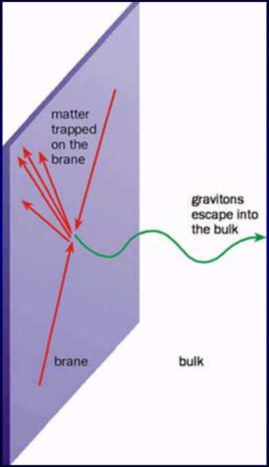


Expansion on the normal DGP branch

- The expansion of the brane

$$H^2 + \frac{1}{r_c} H = \frac{\kappa_4^2}{3}(\Lambda + \rho_m), \quad r_c = \frac{\kappa_5^2}{2\kappa_4^2}$$

- The normal branch:
  - Transition from a 4d regime to a 5d regime.
  - Dark energy is needed to describe the late-time acceleration
  - No Ghost issue
  - **Can we make the normal branch self-accelerating?**
  - That is the main question I will address in this talk



Sahni, Shtanov '02, Lue, Starkman '04

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Strategy

- The strategy to reply the question “Can we make the normal branch self-accelerating?” is based on modifying the gravitational action.
- The simplest modifications of the bulk action involves a Gauss-Bonnet term
  Brown et al ’05, Bouhmadi-López and Vargas Moniz ’08
- The simplest modifications of the brane action involves an $f(R)$ term
  Nojiri, Odintsov ’06, Capozziello, Francaviglia ’07, De Felice, Tsujikawa ’10

Outline

DGP model with GB effect

- The bulk action contains a Gauss-Bonnet term and no cosmological constant
- The brane action contains an induced gravity term
- Cosmological evolution: Kofinas, Maartens, Papantonopoulos '03

$$\left(1+\frac{8}{3}\tilde{\alpha}H^2\right)^2 H^2 = \left(r_c H^2 - \frac{\kappa_5^2}{6}\rho\right)^2$$

- Comparison with DGP:

$$H^2 \mp \frac{1}{r_c}\left(1+\frac{8}{3}\tilde{\alpha}H^2\right) H = \frac{\kappa_4^2}{3}\rho$$

- The set of solutions that contains the normal DGP branch for $\tilde{\alpha} \to 0$, $\tilde{\alpha}$ GB parameter, is the one with "$+$" sign

Solutions of the modified Friedmann eq. of the DGP-GB model (normal branch)

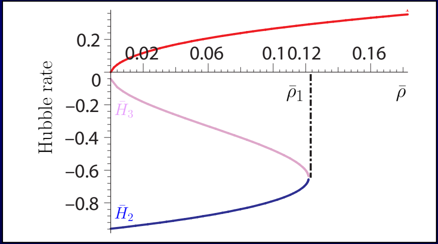
- Modified Friedmann equation

$$\bar{H}^3 + \bar{H}^2 + b\bar{H} - \bar{\rho} = 0$$

$$b = \frac{8}{3}\frac{\tilde{\alpha}}{r_c^2},$$

$$\bar{H} = \frac{8}{3}\frac{\tilde{\alpha}}{r_c}H,$$

$$\bar{\rho} = \frac{32}{27}\frac{\kappa_5^2\tilde{\alpha}^2}{r_c^3}\rho$$



- There are no de Sitter solutions in absence of matter. Therefore, we can conclude that there is no self-acceleration in this case.

Brown et al ’05, Bouhmadi-López and Vargas Moniz ’08

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Can we make the normal branch self-accelerating? Yes: f(R) brane model

- We consider a 5d induced gravity brane-world model (IGBWM) with an $f(R)$ brane action (simplest option).
- The brane splits the bulk in two symmetric pieces.
- The gravitational action

$$\mathcal{S} = \int_{\mathcal{B}} d^5X \sqrt{-g^{(5)}} \left\{ \frac{1}{2\kappa_5^2} R[g^{(5)}] \right\} + \int_b d^4X \sqrt{-g} \left\{ \alpha f(R[g]) + \mathcal{L}_m \right\}$$

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Cosmology of an f(R) brane-1-

- The expansion of the FLRW brane

$$H^2 = \frac{1}{6\alpha f'}\left(\rho^{(m)} + \rho^{(c)}\right) \pm \frac{1}{\kappa_5^2 \alpha f'} H$$

where $f' = \frac{df}{dR}$, $\rho_c = 2\alpha f'\left[\frac{1}{2}\left(R - \frac{f}{f'}\right) - 3H\dot{R}\frac{f''}{f'}\right]$

- The effective gravitational constant $8\pi G_{\mathrm{eff}} = 1/(2\alpha f')$
- The crossover scale is evolving with time $r_c^{(r)} = \kappa_5^2 \alpha f'$
- Comparison with the DGP model: (i) an evolving crossover scale and (ii) the effective gravitational “constant” is not constant
- Comparison with standard 4d $f(R)$ model: the last term on the Friedmann equation is absent.

Cosmology of an f(R) brane-2-

- The expansion of the FLRW brane

$$H^2 \pm \frac{1}{\kappa_5^2 \alpha f'} H = \frac{1}{6\alpha f'} \left(\rho^{(m)} + \rho^{(c)}\right)$$

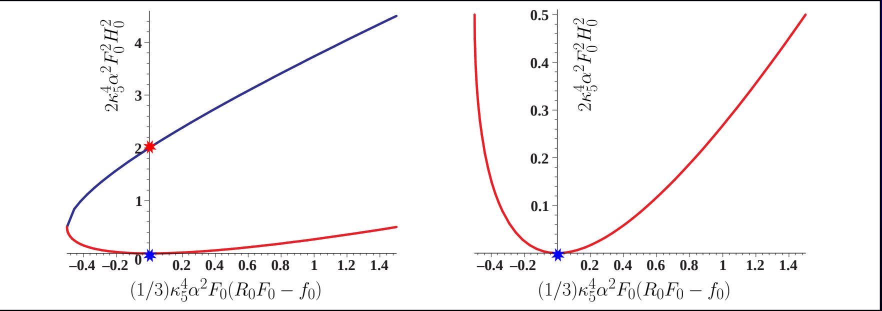
where $f' = \frac{df}{dR}$, $\rho_c = 2\alpha f' \left[\frac{1}{2}\left(R - \frac{f}{f'}\right) - 3H\dot{R}\frac{f''}{f'}\right]$

- The normal branch (+ sign):
  - Transition:
    - from a 4d regime with evolving gravitational constant
    - to a self-accelerating regime through geometrical effects $\rho_c \neq 0$ (next slide)
  - Dark energy is not needed to describe the late-time acceleration in this branch!!

BL '09

Self-acceleration of an f(R) brane on the normal branch

- The expansion of the brane: $H^2 \pm \frac{1}{r_c f'} H = \frac{\kappa_4^2}{3f'}(\rho_m + \rho_c)$
- The self-accelerating solutions = de Sitter space-times (blue/red -/+ sign, stars DGP sols):



Stability analysis-1-

- The stability of de Sitter solutions under homogeneous perturbations up to first order on $\delta H = H(t) - H_0$ (we follow the approach of Faraoni and Nadeau '05)
- The evolution equation for δH:

$$\delta \ddot{H} + 3H_0\delta\dot{H} + m_{\rm eff}^2\delta H = 0,$$

- The effective mass of the perturbations

$$m_{\rm eff}^2 = \frac{1}{3}\frac{F_0}{f_{RR}} - 4H_0^2 - \frac{1}{\kappa_5^4\alpha^2}\frac{1}{f_{RR}(f_0 - 6H_0^2F_0)}.$$

- In the 4d case $H_{(4)}^2 = \frac{1}{6}\frac{f_0}{F_0}$ and H_0^2 can be rewritten as

$$H_0^2 = H_{(4)}^2 + \frac{1 - \sqrt{1 + \frac{2}{3}\alpha^2\kappa_5^4F_0f_0}}{2\alpha^2\kappa_5^4F_0^2}$$

Stability analysis-2-

- It is useful to rewrite $m^2_{\rm eff}$ as

$$m^2_{\rm eff} = m^2_{(4)} + m^2_{\rm shift} + m^2_{\rm pert},$$

where $m^2_{(4)}$ is the analogous quantity to $m^2_{\rm eff}$ in a 4d f(R) model

$$m^2_{(4)} = \frac{1}{3}\left(\frac{F_0}{f_{RR}} - 2\frac{f_0}{F_0}\right)$$

$$m^2_{\rm back} = -\frac{2}{\alpha^2\kappa_5^4 F_0^2}\left[1 - \sqrt{1+\frac{2}{3}\alpha^2\kappa_5^4 F_0 f_0}\right]$$

$$m^2_{\rm pert} = \frac{F_0}{3f_{RR}}\left[1 - \sqrt{1+\frac{2}{3}\alpha^2\kappa_5^4 F_0 f_0}\right]^{-1}$$

- Therefore, the extra-dimension induces a shift on $m^2_{\rm eff}$ caused by:
  - a purely background effect due to the shift on the Hubble parameter encoded on $m^2_{\rm back}$
  - a purely perturbative extra-dimensional effect described by $m^2_{\rm pert}$

Stability analysis-3-

- If the de Sitter branes are close to the standard 4d regime ($H_0^2 \sim H_{(4)}^2$)

$$1 \ll \kappa_5^4 \alpha^2 F_0 f_0$$

  then
  - $m^2_{\rm back} > 0$: the shift on the brane Hubble parameter respect to the standard 4d case tends to make the perturbation heavier, i.e. the de Sitter universe would be more stable
  - $m^2_{\rm pert} < 0$: the perturbative effect encoded on $m^2_{\rm pert}$ would make the perturbations lighter and therefore the de Sitter space-time would be less stable than in the pure 4d case
  - The extra-dimension has a *benigner* effect in the 4d f(R) model; i.e. $m^2_{\rm eff} > m^2_{(4)}$, as long as $F_0^2 < 4 f_0 f_{RR}$
- In the general case (not necessarily close to the 4d standard case), the brane is stable if $0 < m^2_{\rm eff}$

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Conclusions

- The normal DGP branch requires dark energy to describe the late-time acceleration but it is free from the ghost problem.
- Can we make the normal branch self-accelerating?
- No if we consider a GB term on the bulk action
- Yes if we consider an $f(R)$ term on the brane action. For this case:
  - We have obtained all the self-accelerating (de Sitter) solutions and analysed their stability under homogeneous perturbations.
  - Open questions: more realistic models, which f(R), stability, solar system constraints ...

More details on JCAP 0911, 011 (2009)