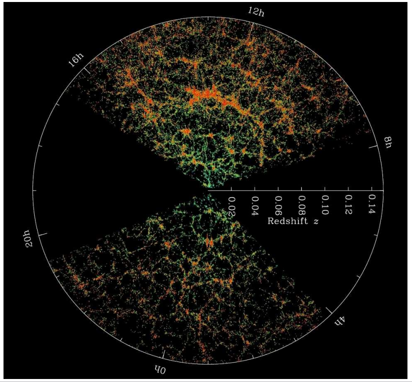

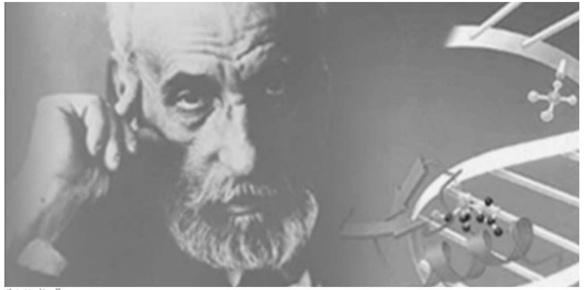

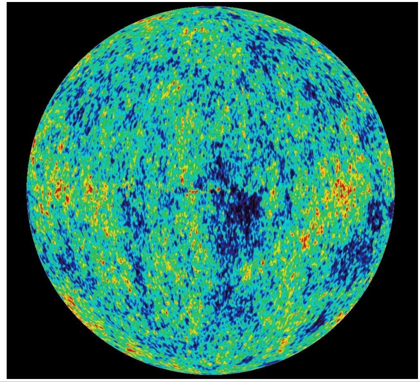

Can Renormalization Change the Observable Predictions of Inflation?

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- Typically, an inflationary model predicts the value of 3 parameters:
  - Scalar spectral index n_s $\Rightarrow$ $P_s(k) = P_s(k_0)\left(\frac{k}{k_0}\right)^{n_s-1}$
  - Tensor spectral index n_t $\Rightarrow$ $P_t(k) = P_t(k_0)\left(\frac{k}{k_0}\right)^{n_t}$
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- Consistency condition for (single-field) slow roll inflation: $r \equiv -8n_t$

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We shall argue that Quantum Field Renormalization significantly influences the predictions of primordial perturbations and hence the expected measurable imprint of inflation on the CMB. In particular, we will find a new consistency condition:

$$r = 4(1-n_s-n_t) + \frac{4n_t'}{n_t^2-2n_t'}\left(1-n_s-\sqrt{2n_t'+(1-n_s)^2-n_t'^2}\right)$$

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- **I will try to justify why renormalization is needed and how it affects the predictions of single field slow roll inflation.**

Tensorial Perturbations. Standard Approach.

- Consider the fluctuating tensorial modes $h_{ij}(\vec{x},t)$, where $g_{ij} = a^2(t)(\delta_{ij} + h_{ij})$ and $a(t)$ is the expansion factor in the background metric $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$.

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- In slow roll inflation ($\epsilon\equiv-\dot{H}/H^2\ll 1$) the form of the modes is

$$h_{\vec{k}}(\vec{x},t)=\sqrt{-\frac{16\pi G\tau\pi}{4(2\pi)^3a^2}}e^{i\vec{k}\vec{x}}H^{(1)}_{3/2+\epsilon}(-k\tau)$$

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- In pure de Sitter ($\epsilon = 0$) the modes take the simple form

$$h_{\vec{k}}(\vec{x},t) = \sqrt{\frac{G}{\pi^2 k^3}}(H - ike^{-Ht})e^{i\vec{k}\vec{x}} e^{ikH^{-1}e^{-Ht}}$$

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- The freezing amplitude is usually codified through the quantity $\Delta_h^2(k,t)$, defined in general by $\Delta_h^2 = 4\pi k^3 |h_{\vec{k}}|^2$ and evaluated at the horizon crossing time t_k (or a few Hubble times after it).

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- One finally obtains a nearly "scale free" tensorial power spectrum

$$P_t(k) \equiv 4\Delta_h^2 = \frac{8}{M_P^2}\left(\frac{H(t_k)}{2\pi}\right)^2$$

where $M_P = 1/\sqrt{8\pi G}$ is the reduced Planck mass.

Need for renormalization: variance of $h(\vec{x},t)$.

- In position space, the variance of the tensorial perturbations is defined as

$$\langle h^2\rangle = \int d^3\vec{k}|h_{\vec{k}}(\vec{x},t)|^2 = \int_0^\infty \frac{dk}{k}\Delta_h^2(k)$$

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- Due to the large k behavior of the modes the above integral is divergent:

$$\langle h^2(\vec{x},t) \rangle = \int_0^\infty \frac{dk}{k} \frac{16\pi G k^3}{4\pi^2 a^3} \left[\frac{a}{k} [1 + \frac{(2+3\varepsilon)}{2k^2\tau^2}] + \ldots \right]$$

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  - It is also common to consider $\langle h(x_1)h(x_2)\rangle$ as the basic object. However, in FRW $\langle h(x_1)h(x_2)\rangle = \int_0^\infty \frac{dk}{k}\Delta_h^2(k)\frac{\sin k|\vec{x}-\vec{x}'|}{k|\vec{x}-\vec{x}'|}$, so all the nontrivial information is contained in $\langle h^2\rangle$, which is ill defined.

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- Adiabatic renormalization removes the divergences by subtracting (second order) counterterms mode by mode in the integral

$$\langle h^2 \rangle_{ren} = \int_0^\infty \frac{dk}{k} \left[\Delta_h^2(k) - \frac{16\pi G k^3}{4\pi^2 a^3} \left(\frac{1}{w_k} + \frac{1}{2w_k^3} \left\{ \frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} \right\} \right) \right]$$

Renormalization of Tensorial Perturbations

- Calculations give

$$\langle h^2\rangle_{ren} = \int_0^\infty \frac{dk}{k}\, \frac{16\pi G k^3 (-\tau\pi)}{4\pi^2 2a^2} \left[|H_\nu^{(1)}(-k\tau)|^2 - \frac{2}{\pi(-k\tau)} \left(1 + \frac{(2+3\varepsilon)}{2k^2\tau^2}\right) \right]$$

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- Evaluated at the Hubble radius crossing time we get

$$P_t(k)_{ren} = \frac{8\alpha}{M_P^2} \left(\frac{H(t_k)}{2\pi} \right)^2 \varepsilon(t_k)$$

where $\alpha \approx 0.904$ and $\varepsilon \equiv -\dot{H}/H^2 \ll 1$.

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- Recall that the unrenormalized value is

$$P_t(k) = \frac{8}{M_P^2}\left(\frac{H(t_k)}{2\pi}\right)^2$$

Renormalization of Scalar Perturbations

- Scalar perturbations are commonly studied through the gauge-invariant quantity $\mathcal{R}$ (the comoving curvature perturbation) obeying

$$\frac{d^2\mathcal{R}_k}{d\tau^2} + \frac{2}{z}\frac{dz}{d\tau}\frac{d\mathcal{R}_k}{d\tau} + k^2\mathcal{R}_k = 0$$

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- Proceeding as before (with care to treat the adiabatic counterterms for the $\mathcal{R}_k$ field) we get

$$\langle \mathcal{R}^2 \rangle_{ren} = \int_0^\infty \frac{dk}{k} 4\pi k^3 \frac{-\pi\tau}{4(2\pi)^3 z^2}\left[|H_\mu^{(1)}(-\tau k)|^2 - \frac{2}{\pi(-k\tau)}\left(1 + \frac{(2+3(3\varepsilon-\eta))}{2(-k\tau)^2}\right)\right]$$

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- Assuming $n > 1$ but $n\varepsilon \ll 1$, we find that ($n =$ number of $e-$folds)
  - $\tilde{\Delta}^2_h(k,n) \approx \frac{2}{M_P^2}\left(\frac{H(t_k)}{2\pi}\right)^2 \varepsilon(t_k)(2n - 3/2)$
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- Though a better understanding of the decoherence processes is necessary to fully determine the time scale at which the quantum-to-classical transition really occurs, our predictions are accurate up to $n \sim 10-20$).

Testable Effects

- Renormalization also affects the spectral indices:

Standard Values

- $n_t = -2\epsilon$
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Renormalized Values

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- For more details and references see Phys.Rev. D81,043514(2010).

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- Only when the counterterms are evaluated at times well beyond the end of inflation does one recover the standard predictions. A deeper understanding of the quantum-to-classical transition is necessary.
- The new generation of high precision detectors may soon turn QFT in curved spacetimes into an experimental science. Our results indicate that renormalization may play an important role in the interpretation of the observational data.

- Motivation and Summary
- Tensor Modes
- Amplitude of TM
- Need for renormalization
- Renormalization Method
- Tensor Perturbations
- Scalar Perturbations
- Time Dependence
- Testable Effects
- Conclusions

The End

Thanks !!!