Cosmological constraints on variations of fundamental constants

Eloisa Menegoni
ICRA, University of Rome "La Sapienza"

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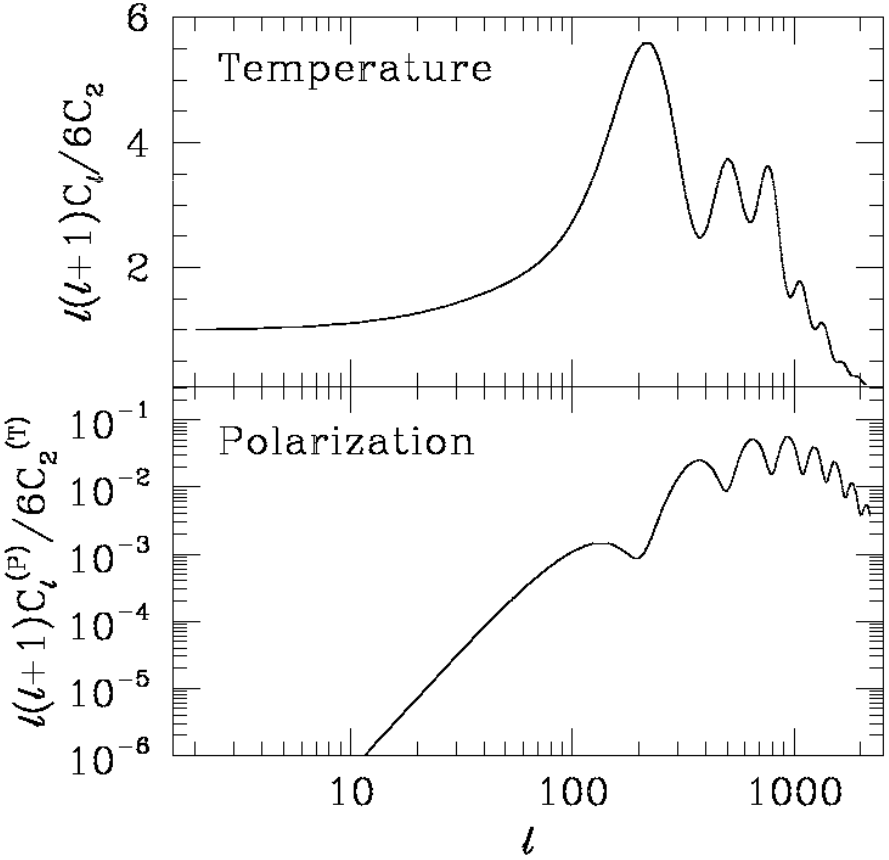
Outline

- The CMB anisotropies
- The Recombination Epoch
- New Constraints on the fine structure constant
- Relation between the fine structure constant and the equation of state parameter w and the gravitational constant G
- Conclusions ☺

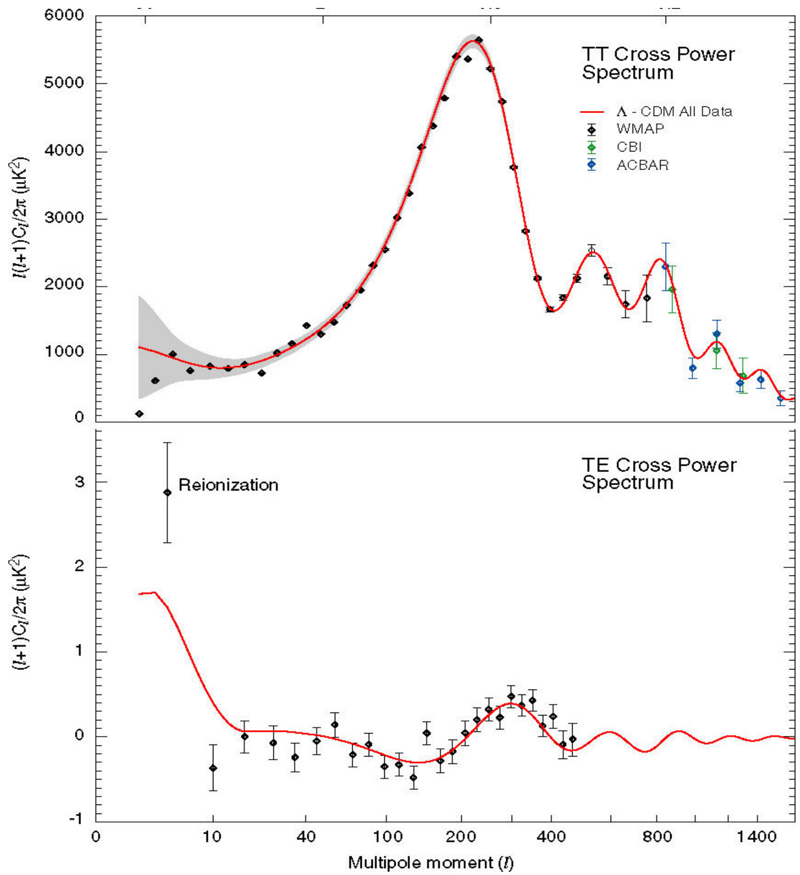
CMB theoretical predictions are in good agreement with the experimental data (in particular with the temperature angular power spectrum).

$$\left\langle \frac{\Delta T}{T}(\vec{\gamma}_1)\frac{\Delta T}{T}(\vec{\gamma}_2)\right\rangle = \frac{1}{2\pi}\sum_{\ell}(2\ell+1)C_\ell P_\ell(\vec{\gamma}_1\cdot\vec{\gamma}_2) \qquad \text{con} \quad l \approx 1/\vartheta$$

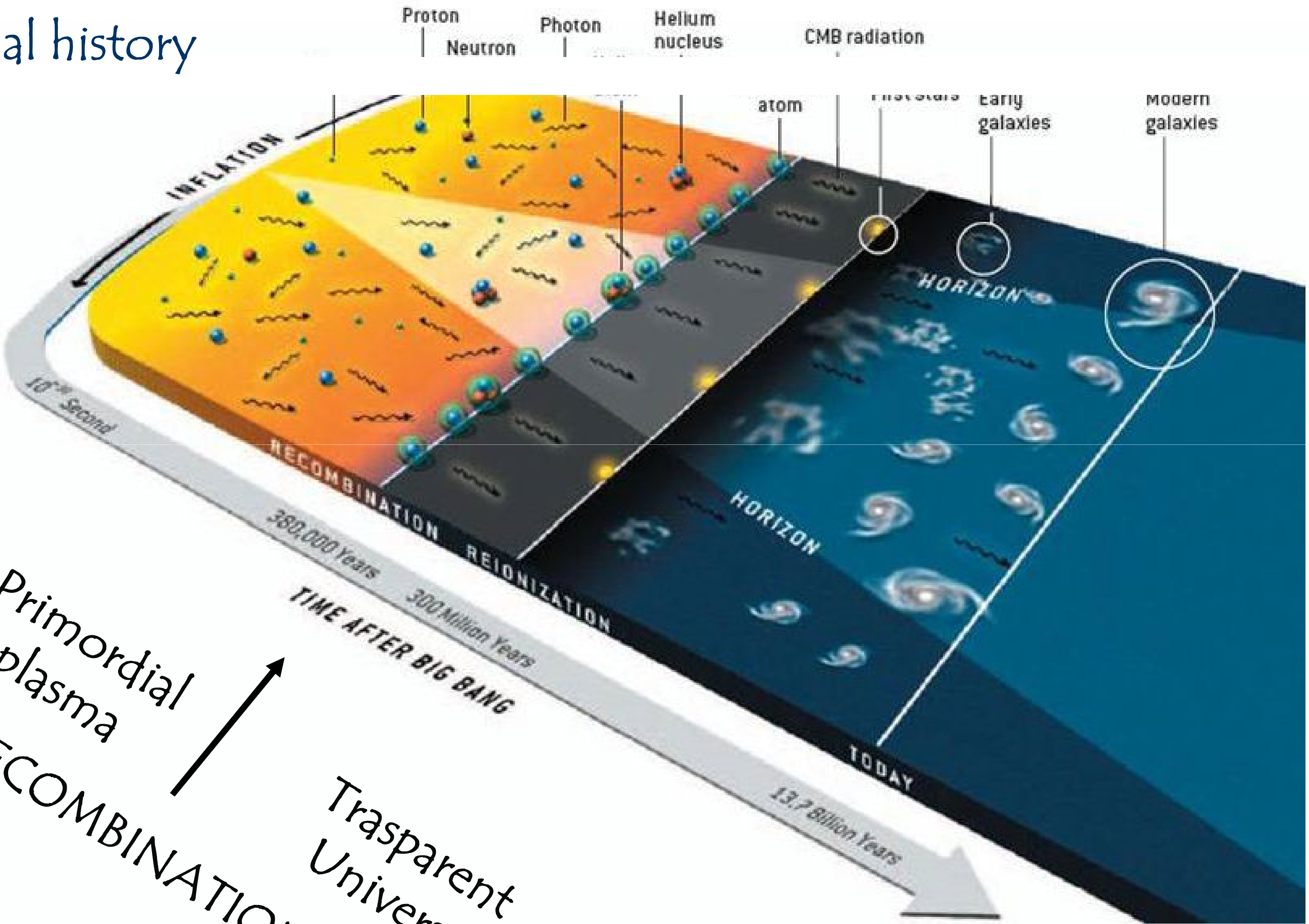
Theory (1994)



Data (2004)



Thermal history



Primordial plasma

Trasparent Universe

RECOMBINATION

Physical Processes to Induce CMB Fluctuations

The primary anisotropies of CMB are induced by three principal mechanisms:

- **Gravity (Sachs-Wolfe effect, regions with high density produce big gravitational redshift)**
- **Adiabatic density perturbations (regions with more photons are hotter)**
- **Doppler Effect (peculiar velocity of electrons on last scattering surface)**

The anisotropies in temperature are modulated by the visibility function which is defined as the probability density that a photon is last scattered at redshift z:

$$\frac{\Delta T}{T}(\vec{n}) \doteq \int_0^\infty \left[g(z) \left(\Psi + \Theta_0 + \vec{n} \cdot \vec{\mathrm{v}}_{\mathrm{b}} \right) \right] dz$$

Gravity — Adiabatic — Doppler

Visibility function and fine structure constant

$$g(\eta) = \dot{\tau}\, e^{-\tau}$$

Rate of Scattering

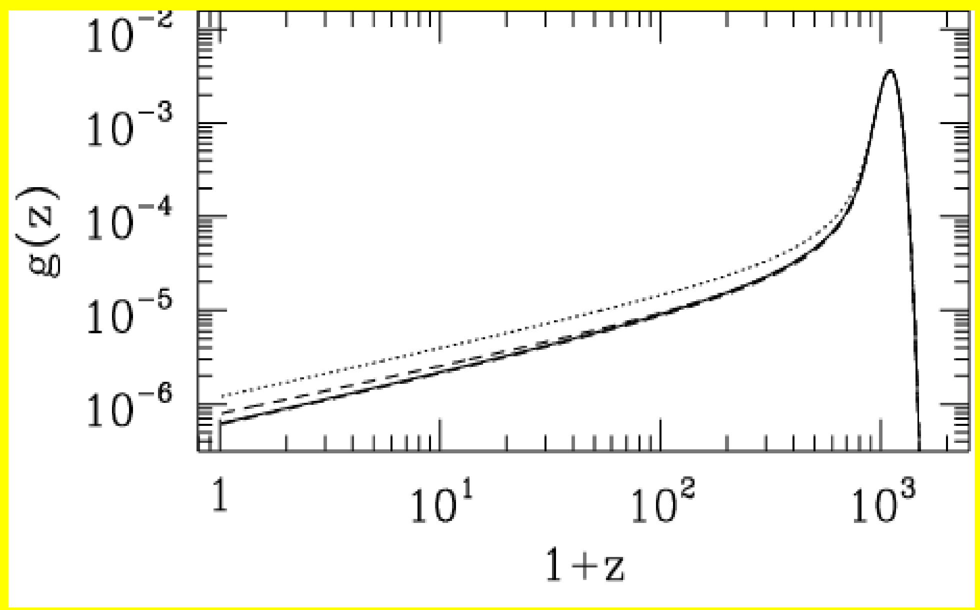
$$\dot{\tau}(\eta) = n_e\, x_e\, a\sigma_T$$

Optical depth

$$\tau(\eta) = \int_{\eta}^{\eta_0} d\eta' n_e\, x_e\, a\sigma_T$$

$$x_e = \frac{n_e}{n_e + n_H}$$

We can see that the visibility function is peaked at the Epoch of Recombination.



Thomson scattering cross section

$$\sigma_T = \frac{8\pi}{3}\frac{\hbar^2}{m_e^2 c^2}\alpha^2$$

Evolution of the free electron fraction with time

ionization coefficient

$$\beta_H \equiv R_H \left(\frac{2\pi m_e K_B T}{h^2} \right) e^{-B_2 / K_B T}$$

recombination coefficient

$$R_H \approx \sigma_{nl} f(B_n, T)$$

cross section of ionization

$$\sigma_{nl} \propto \alpha^{-1} m_e^{-2} f(h\nu / B_1)$$

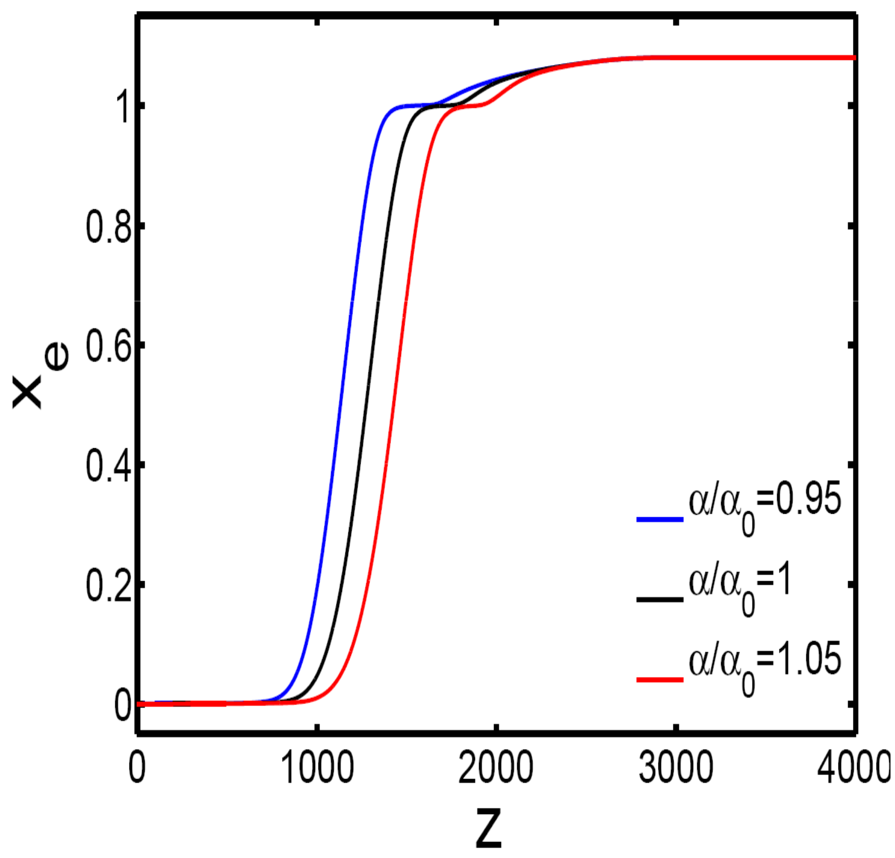
$$\frac{dx_e}{dt} = C_H \left[\beta_H (1 - x_e) e^{-\frac{B_1 - B_2}{K_B T}} - R_H n_p x_e^2 \right]$$

$$C_H = \frac{1 + K\Lambda_{2s}(1 - x_e)}{1 + K(\beta_H + \Lambda_{2s})(1 - x_e)}$$

- Rate of decay 2s a 1s: $\Lambda_{2s} \propto m_e \alpha^8$
- Constant K: $K = n_e \lambda^3 / (8\pi H) \propto m_e^{-3} \alpha^{-6}$
- Lyman-alpha: $\lambda_\alpha = 16\pi\hbar / (3 m_e c \alpha^2)$

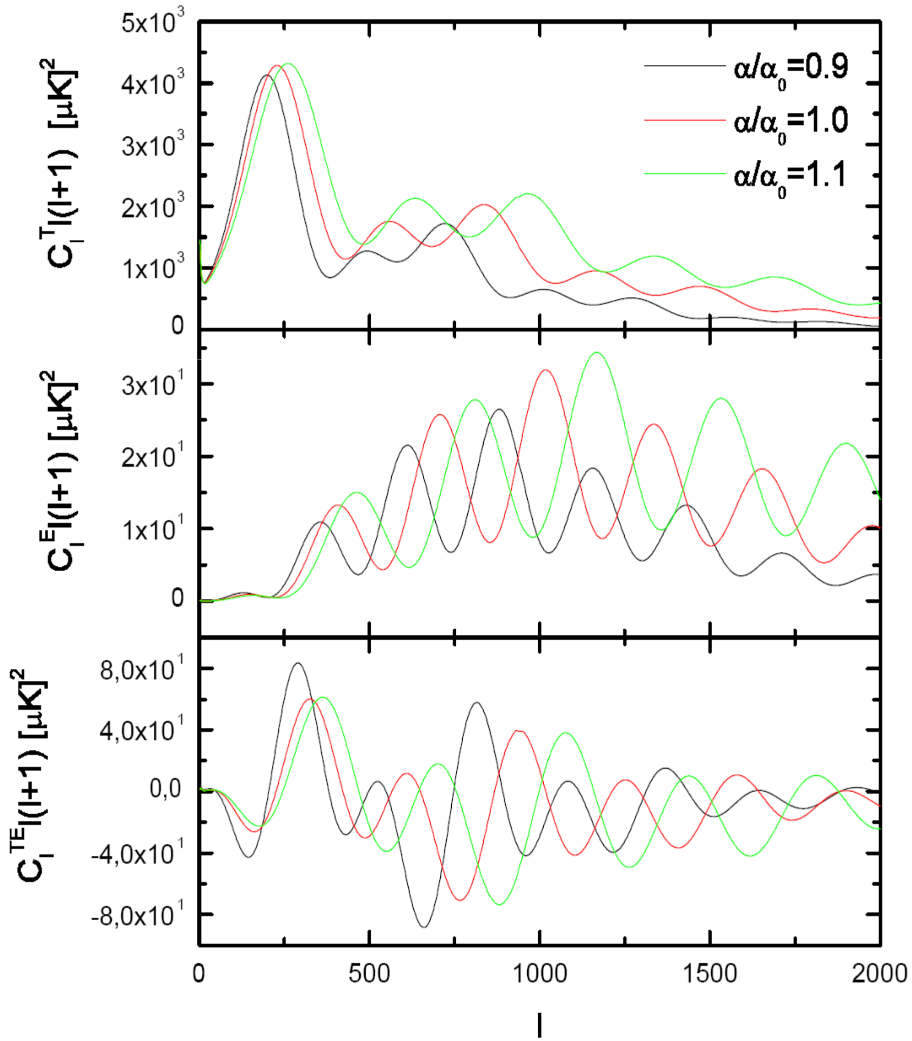
Variation of free electron fraction

If we plot the free electron fraction versus the redshift, we can notice a different epoch of Recombination for different values of alpha. In particular if the fine structure constant α is smaller than the present value, then the Recombination takes place at smaller z.



(see e.g. Avelino et al., Phys.Rev.D64:103505,2001)

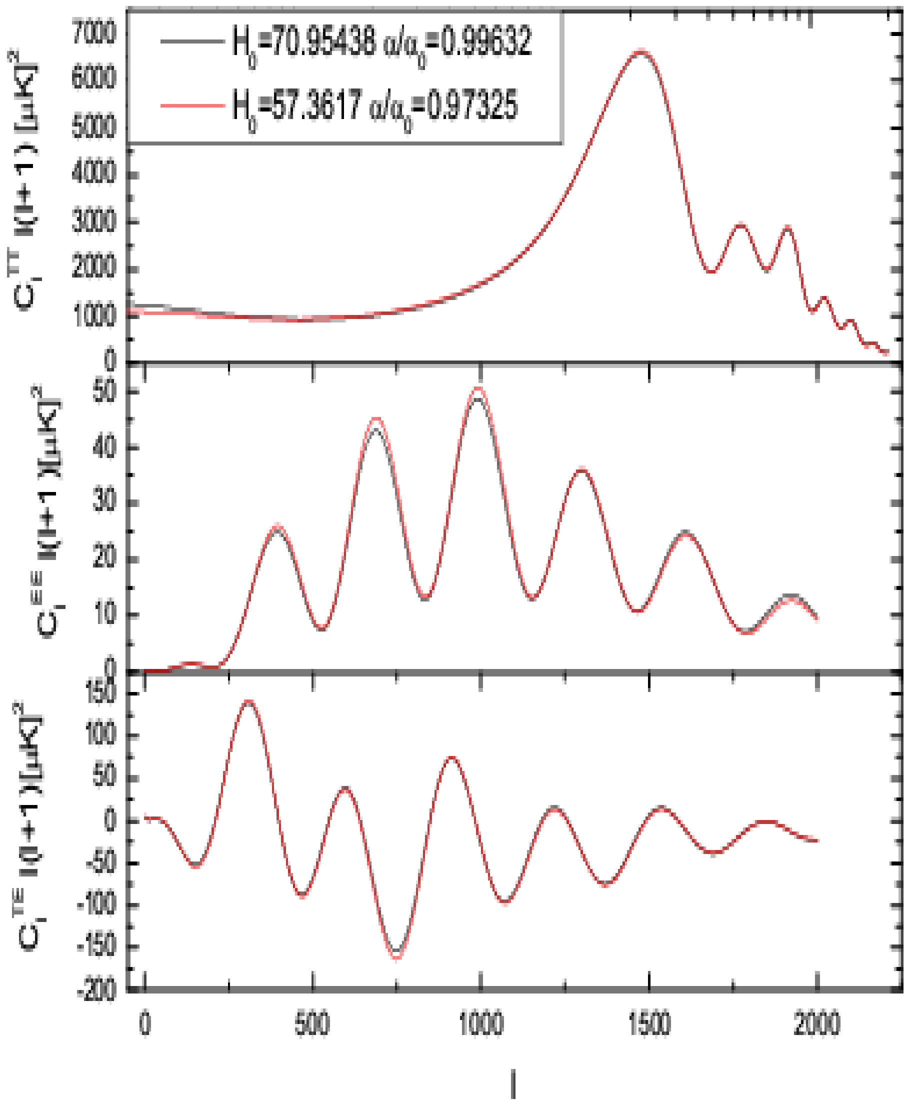
Modifications caused by variations of the fine structure constant



If the fine structure constant is $\alpha / \alpha_0 < 1$ recombination is delayed, the size of the horizon at recombination is larger and as a consequence the peaks of the CMB angular spectrum are shifted at lower l (larger angular scales).

Therefore, we can constrain variations in the fine structure constant at recombination by measuring CMB anisotropies !

Caveat: is not possible to place strong constraints on the fine structure constant by using cmb data alone !



A "cosmic" degeneracy is cleary visible in CMB power spectrum in temperature and polarization between the fine structure constant and the Hubble constant.

The angle that subtends the horizon at recombination is indeed given by:

$$\theta_H \approx c_s H^{-1}(z_r) / d_A(z_r)$$

The horizon size increases by decreasing the fine structure constant but we can compensate this by lowering the Hubble parameter and increasing the angular distance.

New constraints on the variation of the fine structure costant

Menegoni, Galli, Bartlett, Martins, Melchiorri, arXiv:0909.3584v1
Physical Review D *80 081302 (2009)*

We sample the following set of cosmological parameters from WMAP-5 years observations:

Baryonic density	$\Omega_b h^2$
Cold dark matter density	$\Omega_c h^2$
Hubble parameter	H_0
Scalar spectrum index	n_S
Optical depth	τ
Overall normalization of the spectrum	A_s
Variations on the fine structure constant	α / α_0

We also permit variations of the parameter of state w and on the gravitational constant G.

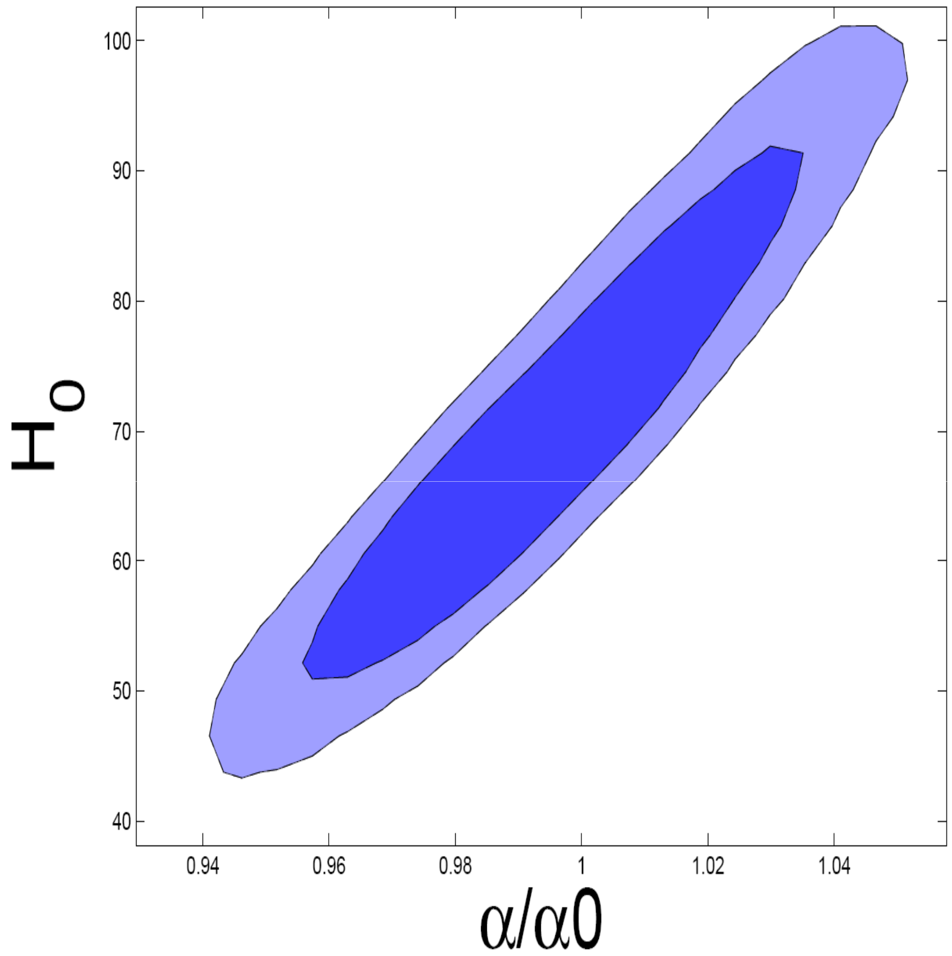
We use a method based on Monte Carlo Markov Chain (the algorithm of Metropolis-Hastings).

The results are given in the form of likelihood probability functions.

We are looking for possible degeneracies between the parameters.

We assume a flat universe.

Constraints from WMAP-5



! External prior on the Hubble parameter:

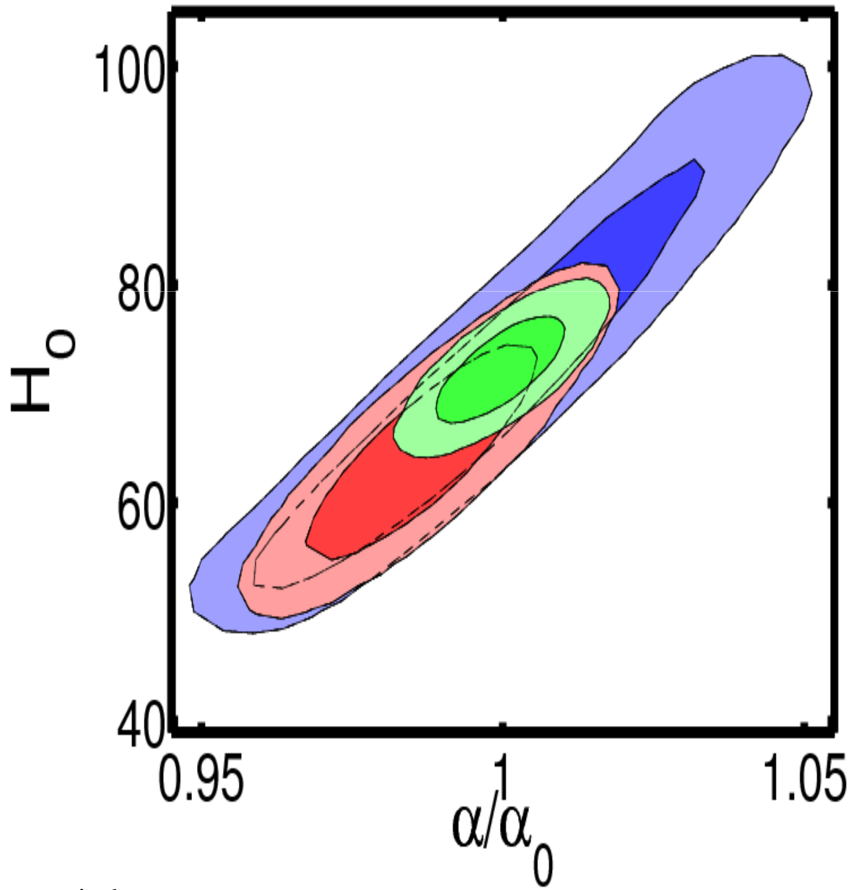
$40\ km/s/Mpc < H_0 < 100\ km/s/Mpc$

Constraints on the fine structure constant

In this figure we show the 68% and 95% c.l. constraints on the α/α_0 vs Hubble constant for different datasets .

Experiment	α/α_0	68% c.l.	95% c.l.
WMAP-5	0.998	± 0.021	$^{+0.040}_{-0.041}$
All CMB	0.987	± 0.012	± 0.023
All CMB+ HST	1.001	± 0.007	± 0.014

TABLE I: Limits on α/α_0 from WMAP data only (first row), from a larger set of CMB experiments (second row), and from CMB plus the HST prior on the Hubble constant, $h = 0.748 \pm 0.036$ (third row). We report errors at 68% and 95% confidence level.



Menegoni, Galli, Bartlett, Martins, Melchiorri, arXiv:0909.3584v1
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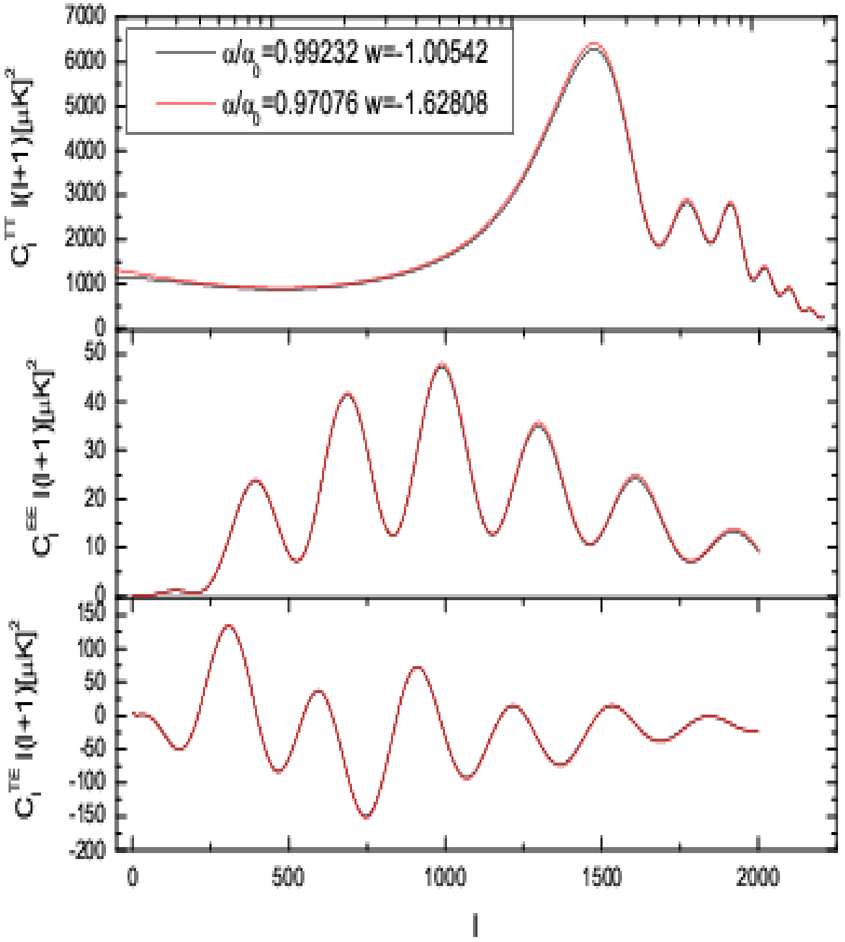
What about dark energy ?

The degeneracy between the fine structure constant with the dark energy equation of state w



If we vary the value of w we change the angular distance at the Recombination. Again this is degenerate with changing the sound horizon at recombination varying the fine structure constant.

$$d_A = \frac{cH_0^{-1}}{(1+z)} \int_0^{1100} \frac{dz'}{E(z')}$$

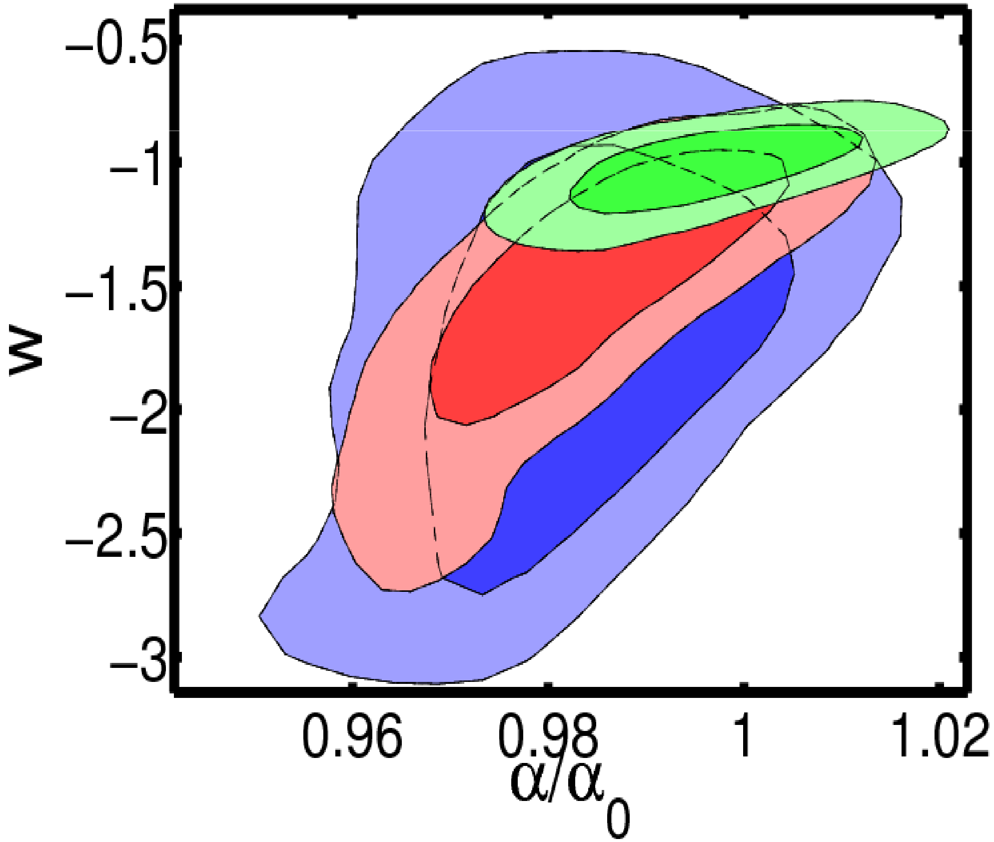
$$E(z) = \left[\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_X (1+z)^{[3(1+w)]}\right]^{1/2}$$

Constraints on the dark energy parameter

Constraints on the dark energy equation of state in presence of a varying fine structure constant. Menegoni, et al .(In press), 2010

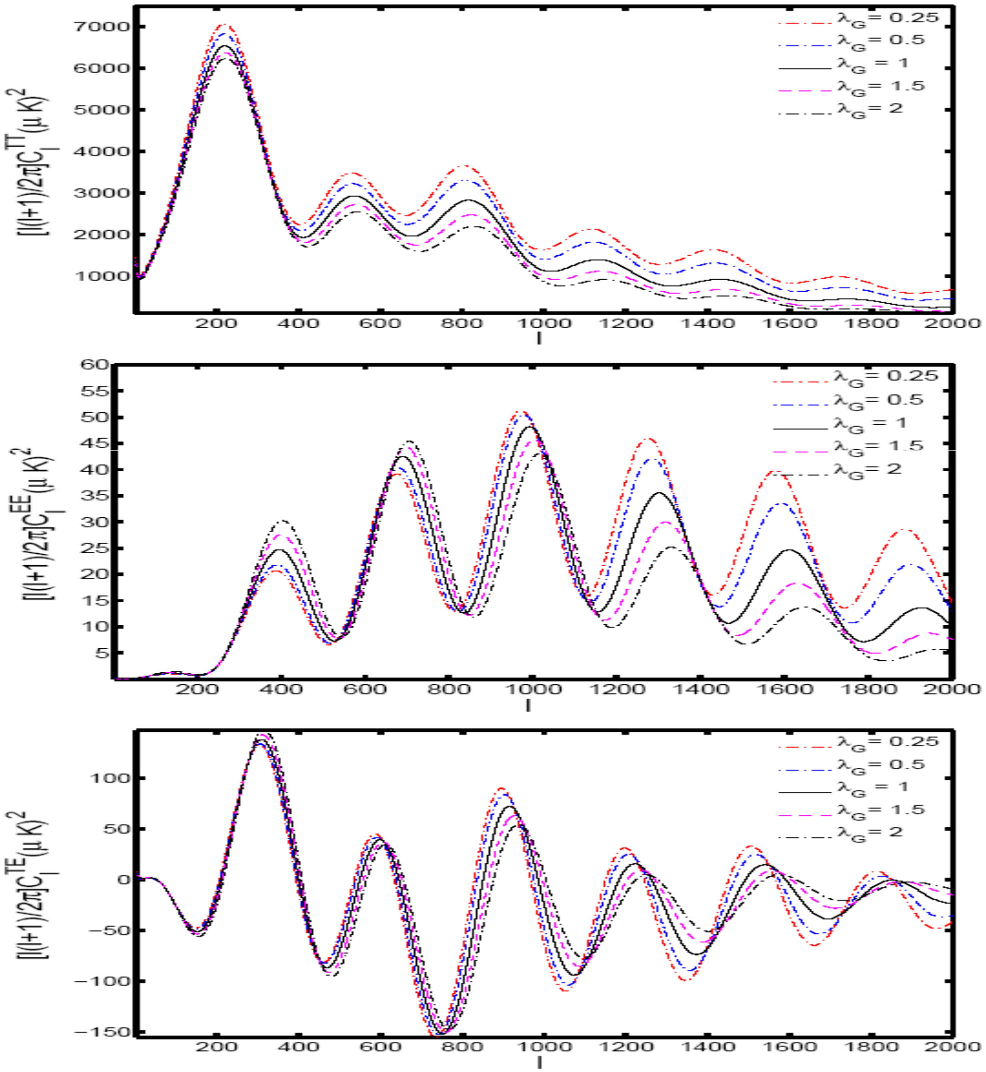
Datasets	α/α_0	w
CMB	0.983 ± 0.012	-1.74 ± 0.53
CMB+ HST	0.983 ± 0.011	-1.52 ± 0.39
CMB+ HST+SN-Ia	0.996 ± 0.009	-1.02 ± 0.11

TABLE I: Limits on w and α/α_0 from CMB experiments (first row), from CMB plus the HST prior on the Hubble constant, $h = 0.748 \pm 0.036$ (second row), and from CMB+HST plus luminosity distances of supernovae type Ia from the UNION catalog. We report errors at 68% confidence level.



What about other constants ?

We can introduce a possible variations of the Newton gravitational constant G



Galli, et al.PhysRevD.80.023508

We can describe variatons in the gravitational constant by a dimensionless parameter λ :

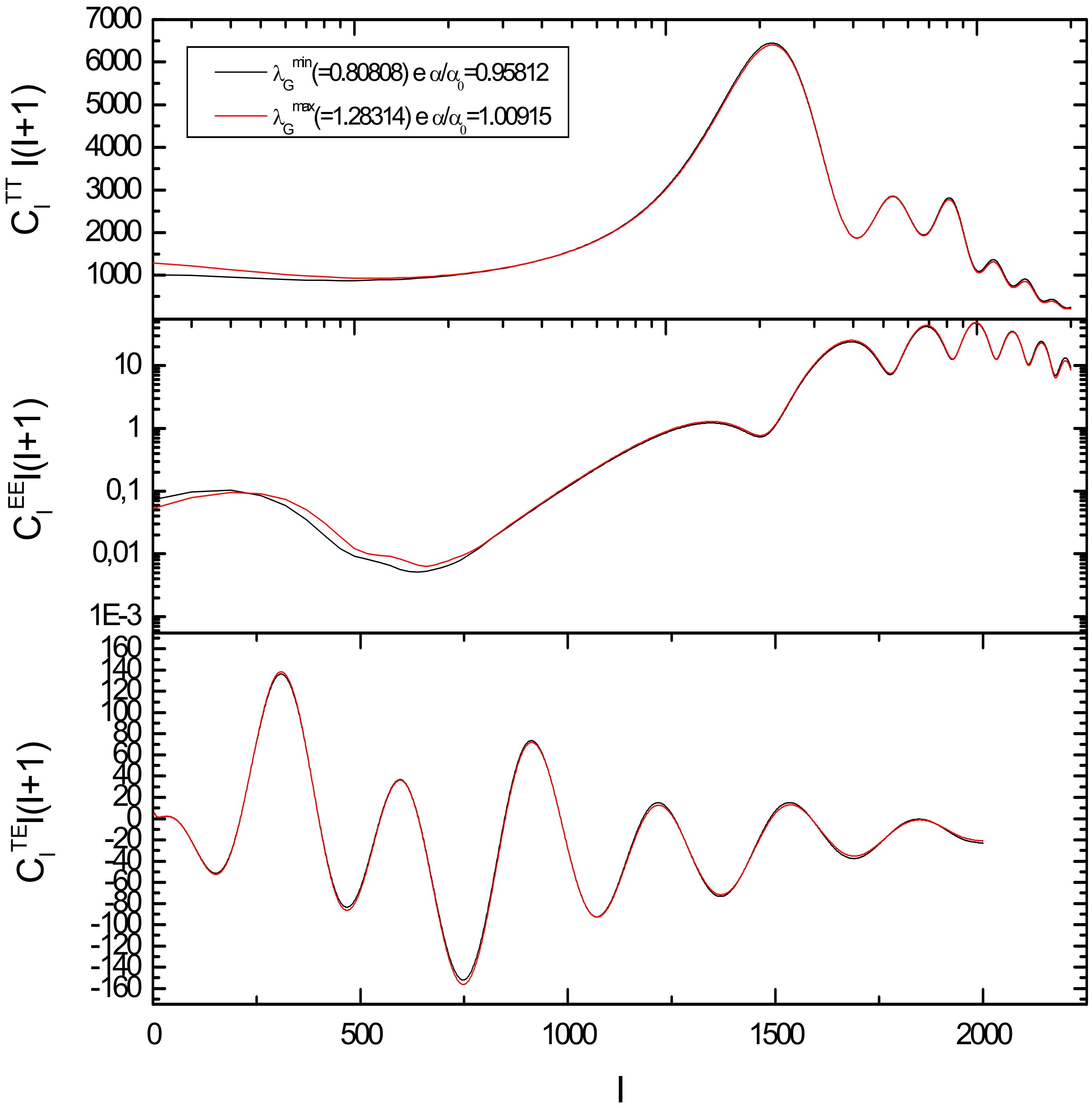
$$G \rightarrow \lambda^2 G$$

The expasion rate H now satisfies at the relation:

$$H(a,\lambda) = \lambda f(a)$$

We can modify the Friedmann equation, and so we find:

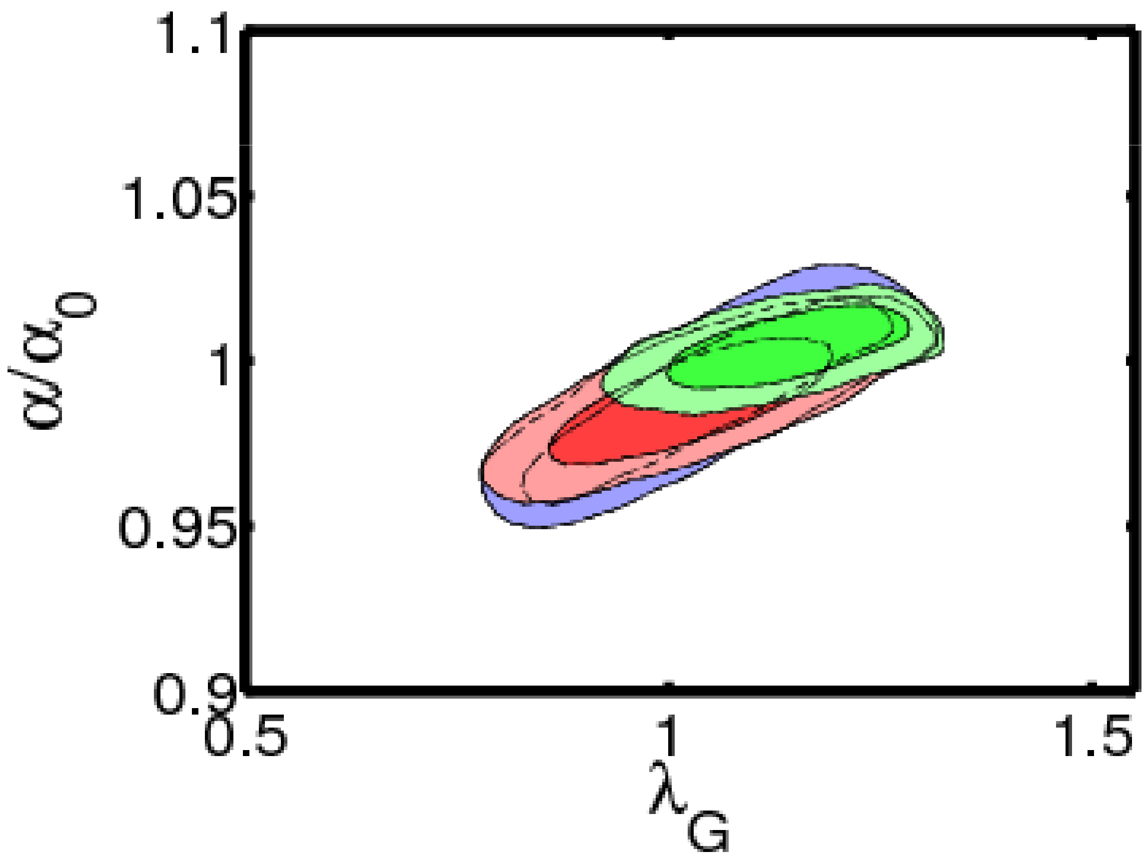
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho \rightarrow \frac{8\pi}{3}\lambda^2 G\rho$$



λ_G^{min}(=0.80808) e α/α_0=0.95812
λ_G^{max}(=1.28314) e α/α_0=1.00915
C_l^{TT} l(l+1)
C_l^{EE}l(l+1)
C_l^{TE}l(l+1)
l

There is a degeneracy between the fine structure constant and gravitational constant

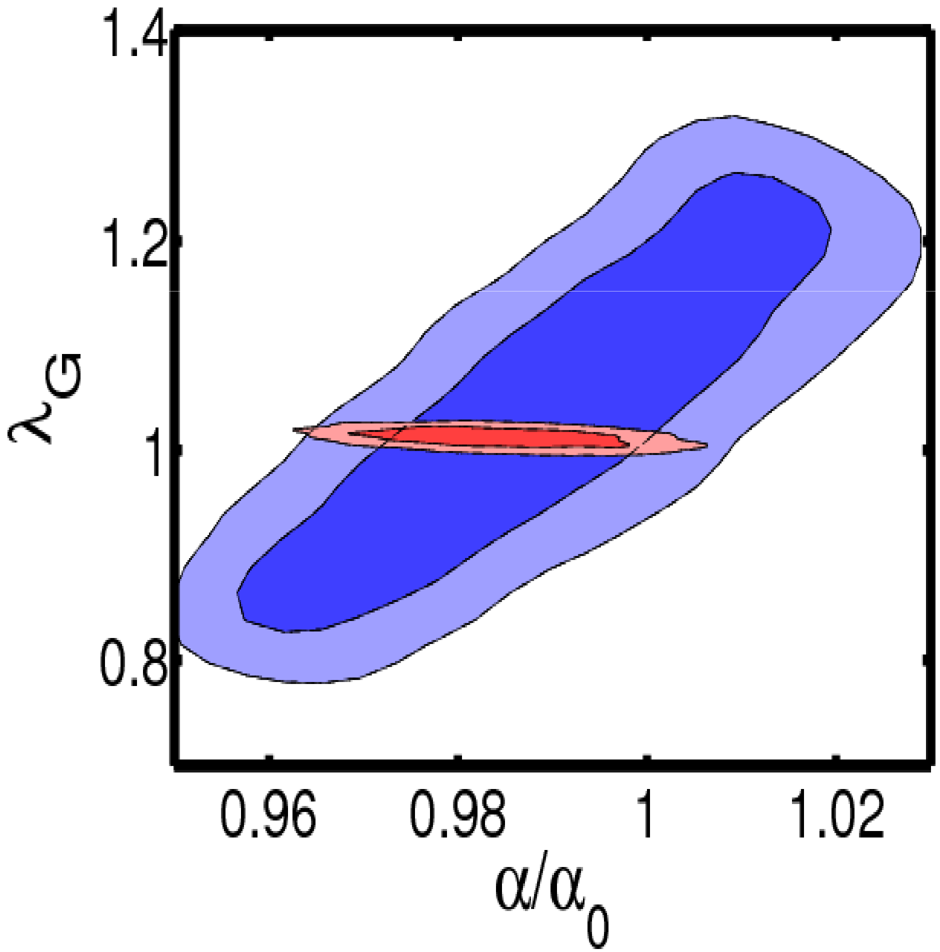
C.J.A.P. Martins, Menegoni, Galli, Mangano, Melchiorri arXiv:1001.3418v3



Experiment	α/α_0	68% c.l.	λ_G	68% c.l.
All CMB	0.999	±0.017	1.04	±0.12
All CMB+SN-Ia	0.989	±0.012	1.04	±0.11
All CMB+HST	1.003	±0.008	1.13	±0.09
ALL CMB+BBN	0.985	±0.009	1.01	±0.01
Planck only	1.000	±0.015	1.02	±0.09

TABLE I: Limits on α/α_0 and λ_G from CMB data only (first row), from CMB+SN-Ia (second row), from CMB plus the HST prior on the Hubble constant, $h = 0.748 \pm 0.036$ (third row), from CMB plus BBN (fourth row) and for simulated mock data for the Planck experiment. We report errors at 68% confidence level.

If we include the BBN data the degeneracy between G and the fine structure constant can be broken



If we assume that the fine structure constant and G don't vary from BBN to recombination we can combine the CMB results with BBN analysis. Differently than for CMB, in case of BBN, variations of the fine structure constant and G are negatively correlated, since both and Deuterium are increasing functions of both parameters: this implies that the likelihood countours for BBN and CMB are almost orthogonal in that plane, thus leading to a tighter bound, in particular on λ_G.

Conclusions:

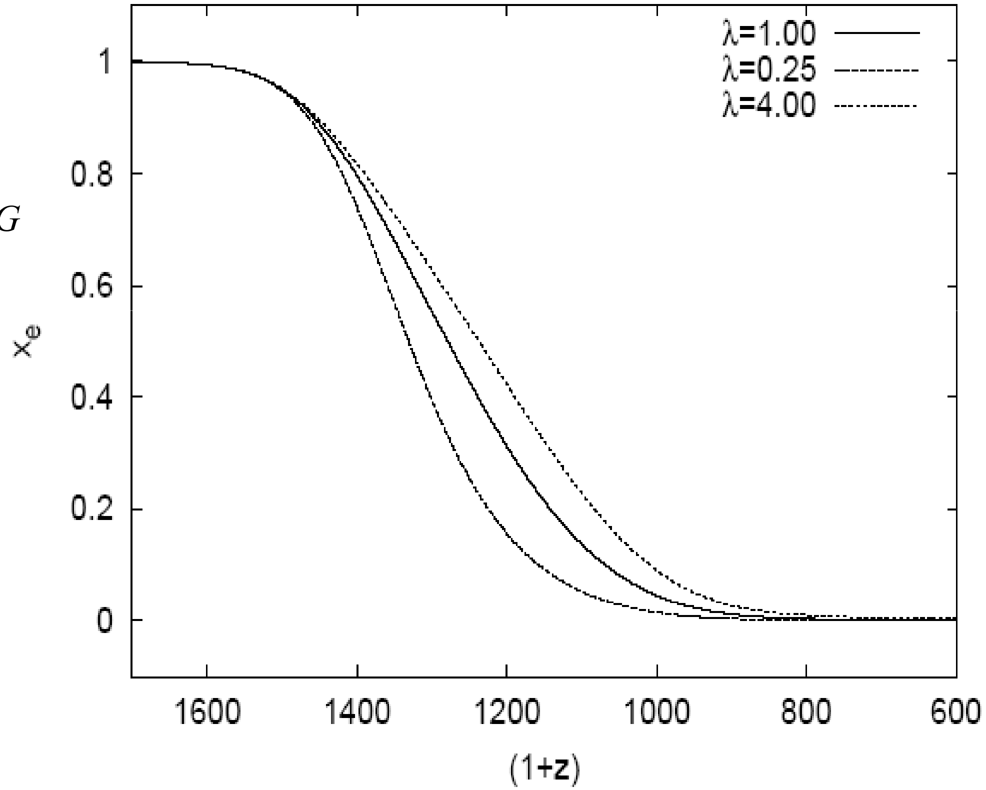
- We found a substantial agreement with the present value of the fine structure constant (we constrain variations at max of 2,5% at 68% level of confidence from WMAP-5 years and less than 0.7% when combined with HST observations).
- When we introduce also variations on G, we found that the current data gives no clear indication about the relative sign of the variations, but already prefers that any relative variations in the fine structure constant should be of the same sign of G for 1% variations. We found much tighter constraints by adding BBN data.
- When we consider an equation of state parameter w, again we notice a degeneracy that can alters the current constraints on w significantly (by 10%).

The temperature and the redshift in which Recombination takes place doesn't vary after a variation of the gravitational constant's value.

The free electron fraction depends on the value of λ_G

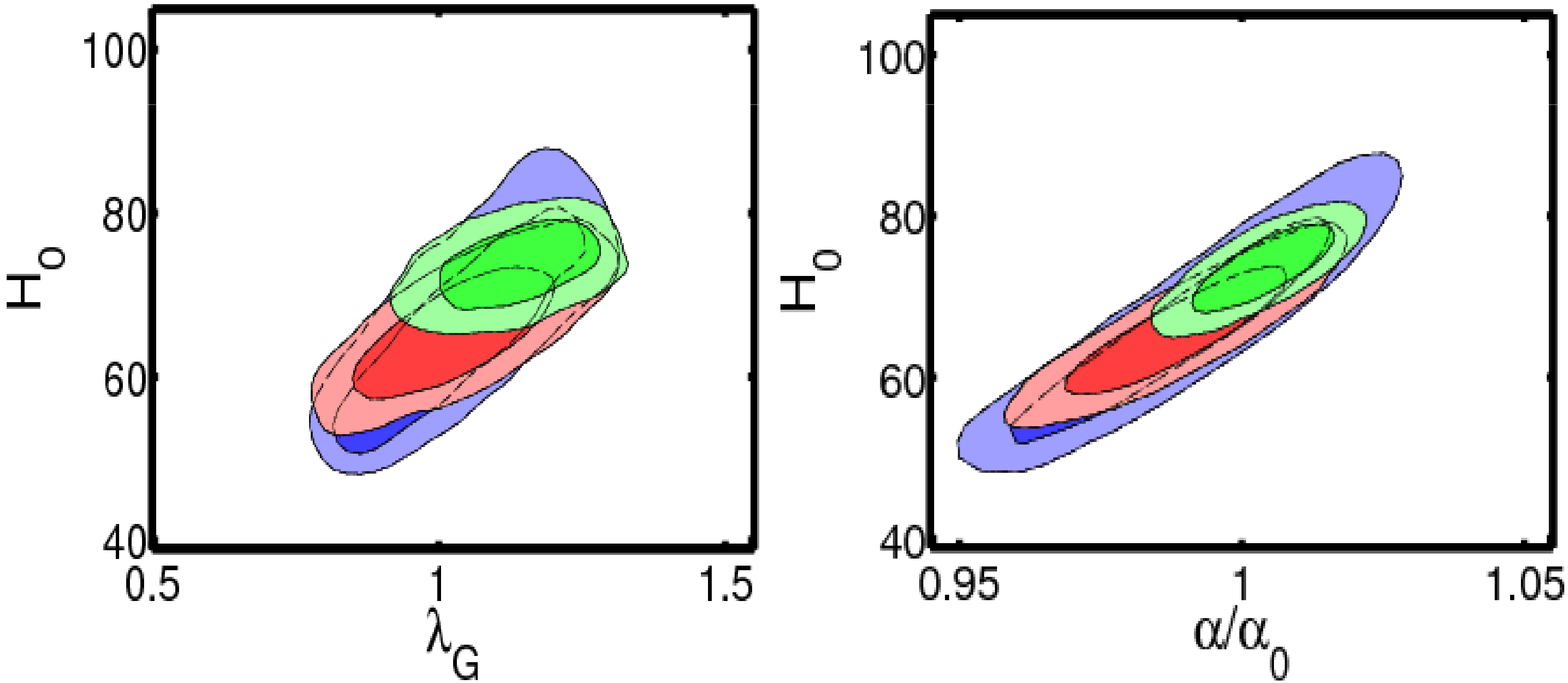
In fact the faster the universe is expanding at a given redshift (i.e the larger λ_G),

the more difficult it's for hydrogen to recombine and hence the larger is the free electron fraction: so the free electron fraction at a given redshift after start of recombination increases.



Zahn, Zaldarriaga, Phys.Rev, D67 (2003) 063002

Constraints on the Newton gravitational constant and the fine structure constant



Likelihood fucntion and marginalization method

To analyse the CMB anisotropies we use the likelihood function which is definied as the probability that an experiment'll give the number of the theoretical model (θ). We use the Bayes's theorem:

conditional probability

$$P(\theta \mid \vec{x}) = \frac{P(\vec{x} \mid \theta) P(\theta)}{P(\vec{x})}$$

prior

Likelihood Function

$$P(\theta \mid \vec{x}) \propto L(\theta) \equiv \exp\left(-\frac{\chi^2(\theta)}{2} \right)$$

with $\chi^2(\theta) = \left(\vec{x} - \vec{x}_\theta^{\,pred}\right) C^{-1} \left(\vec{x} - \vec{x}_\theta^{\,pred}\right)$

If we have a N-dimensions likelihood function L, we had to integrate on the correlated distribution function.

This method is called marginalization:

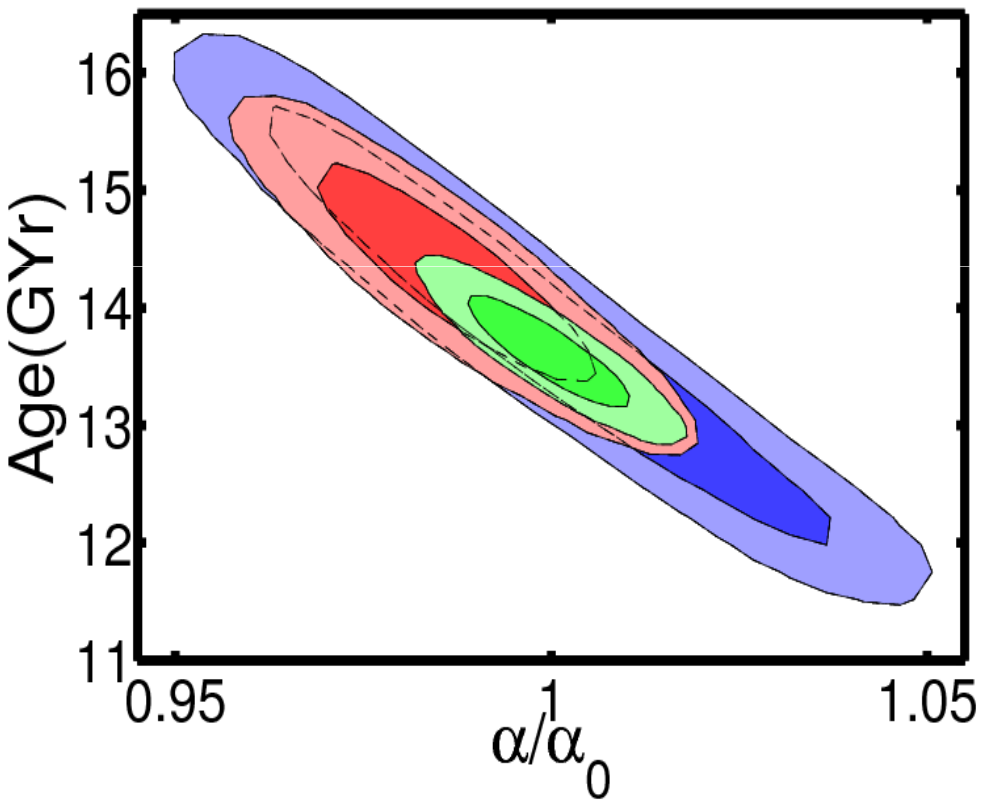
$$L(\beta,\gamma) = \int_{V_\Omega} L(\beta,\gamma,\delta,.......) d\Omega$$

$$\theta = (\beta,\gamma,\delta.....)$$

$$\Omega = \{\delta,.....\}$$

We use a method based on the Markov chain MCMC (the algorithm of Metropolis-Hastings).

Age of the Universe



We indeed found that if one allows for variations in the fine stucture constant, the WMAP five years data bounds the age of the Universe to

$$t_0 = 13.9 \pm 1.1 Gyrs$$

(at 68% c.l.) with an increase in the error of a factor 3 respect to the quoted standard constraint.

68% and 95% c.l. constraints on the α / α_0 vs the age of Universe for different datasets. The countour regions come from the WMAP-5 data (blue), all current CMB data (red), and CMB+HST (green).

Recombination: standard Model

Direct Recombination

NO net recombination

$$H_{1s} + \gamma \leftrightarrow H^{+} + e^{-}$$

Decay to 2 photons from 2s levels metastable

$$H^{+} + e^{-} \leftrightarrow H_{2s} + \gamma$$

$$H_{2s} \leftrightarrow H_{1s} + 2\gamma$$

Cosmological redshift of **Lyman alpha's photons**

$$H^{+} + e^{-} \leftrightarrow H_{2p} + \gamma$$

$$H_{2p} \leftrightarrow H_{1s} + \gamma$$