Models of Dark Energy Accretion onto Black Holes

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- Back reaction analysis [Faraoni and Jacques, 2007]

Accretion models with no back reaction

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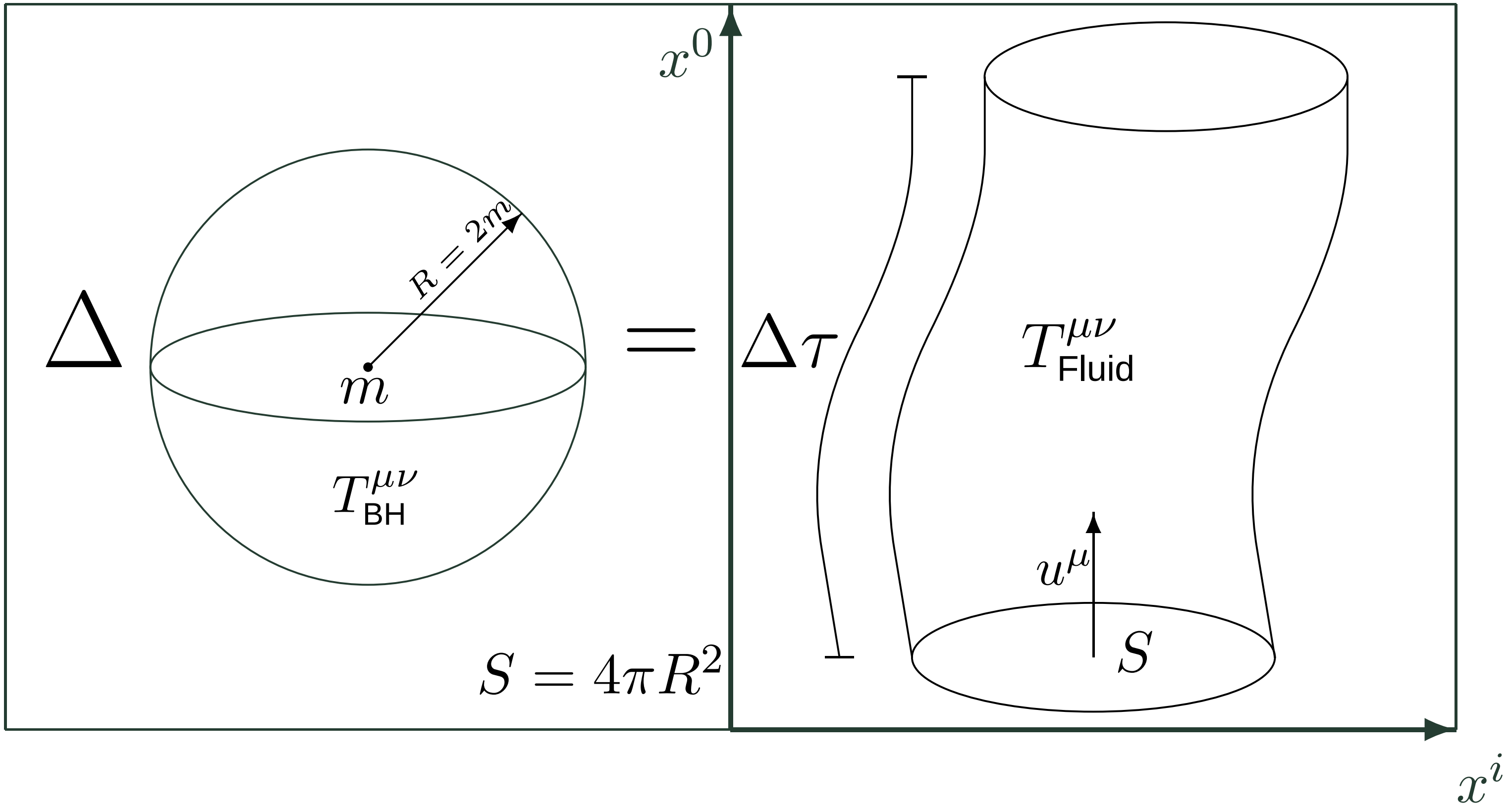
$$\boxed{\frac{\mathrm{d}m}{\mathrm{d}t} = S {T_0}^1} \qquad (1)$$

Energy-momentum flow accross a 3-surface



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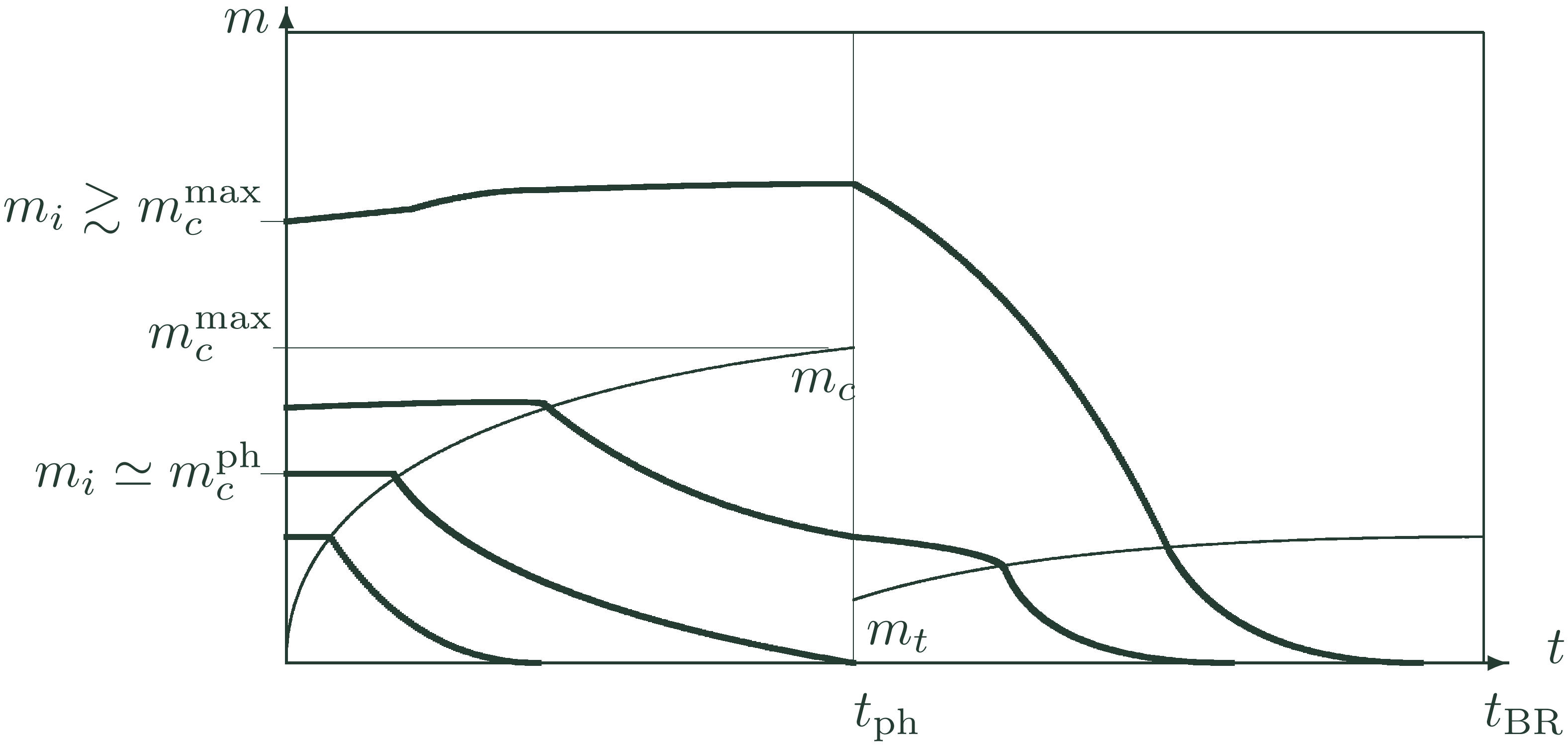
$$m_t = \left[\frac{c^3}{G^2}\frac{A(m)}{|1+\omega|\,\rho_{\mathrm{ph}}}\right]^{1/4}$$

Model summary



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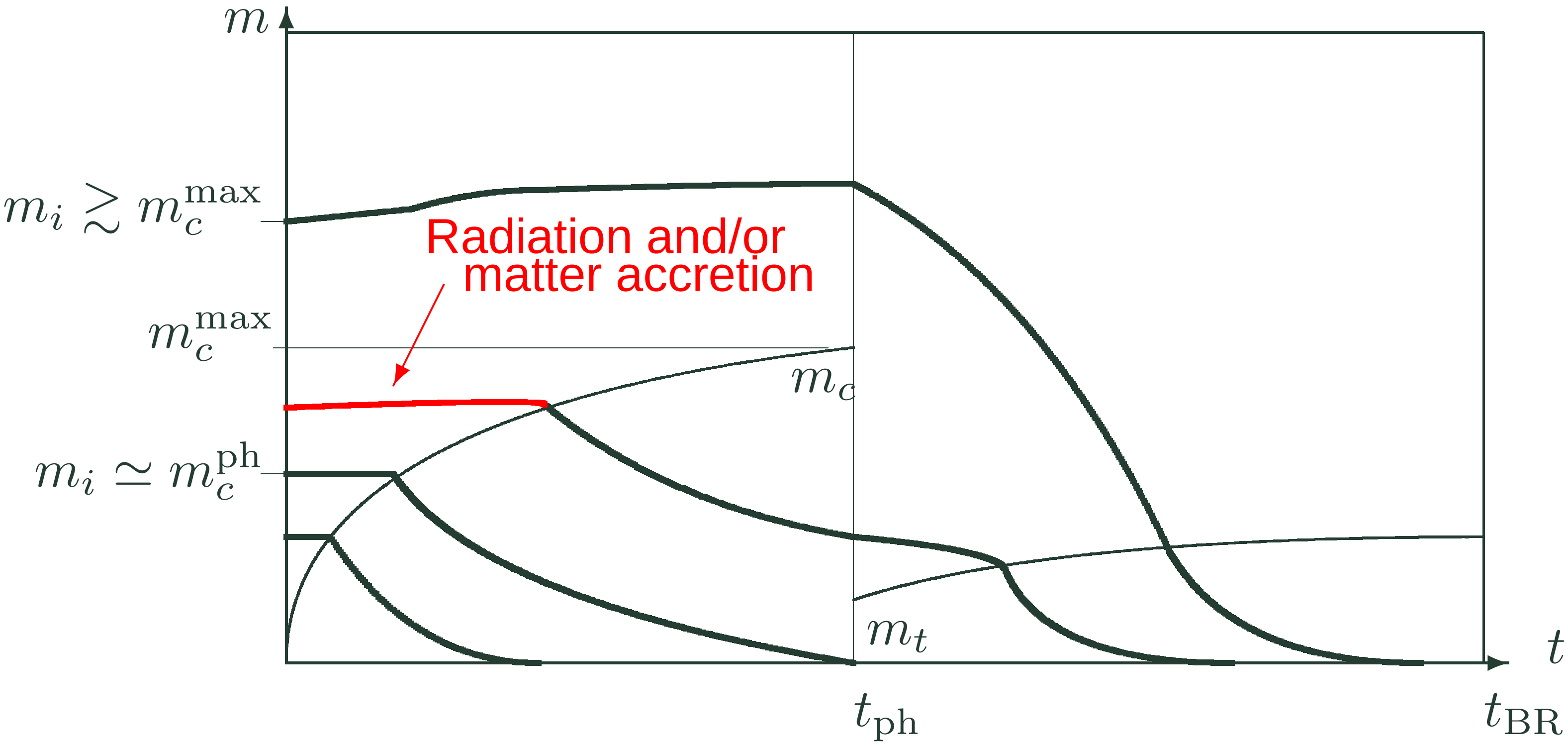

Radiation and/or matter accretion

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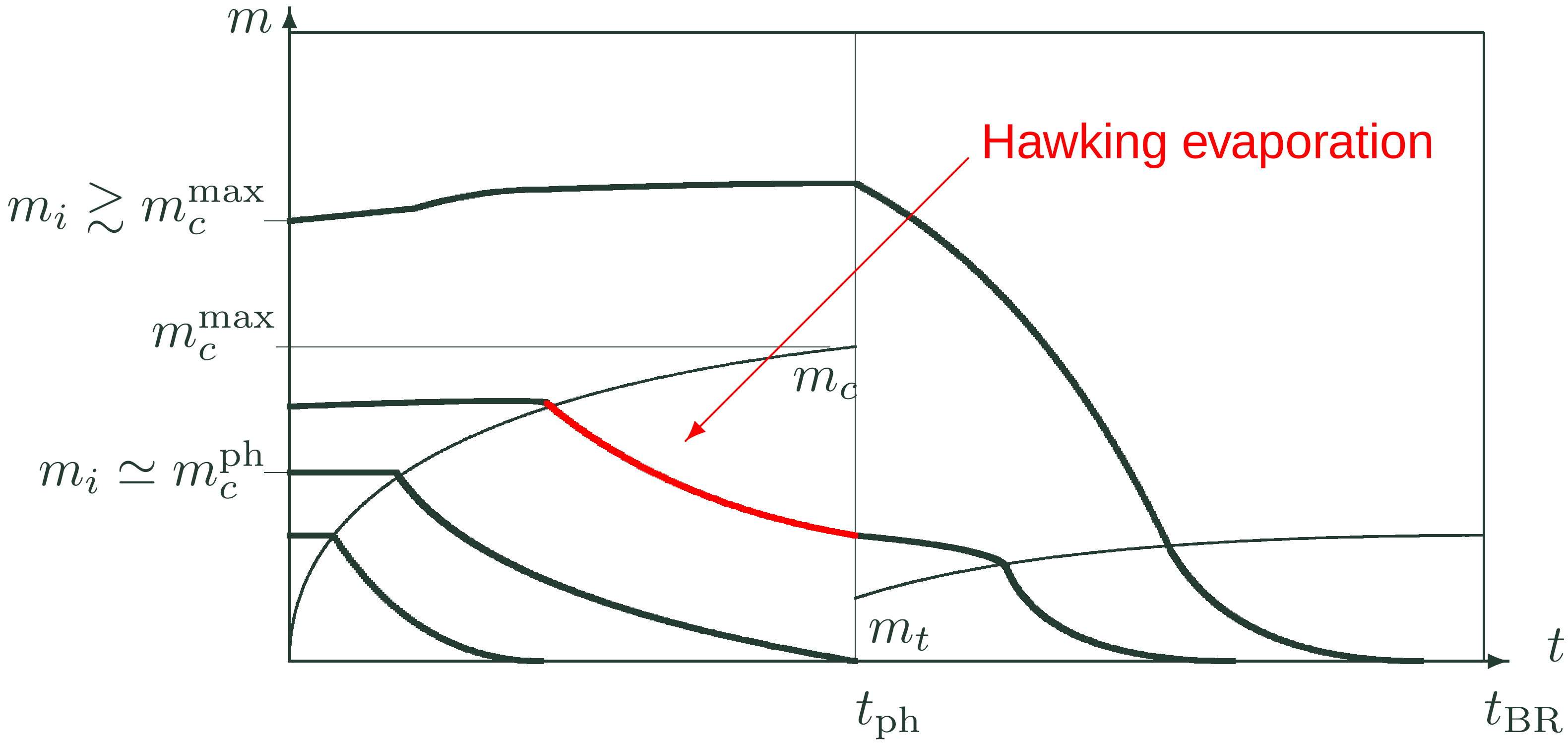

Hawking evaporation

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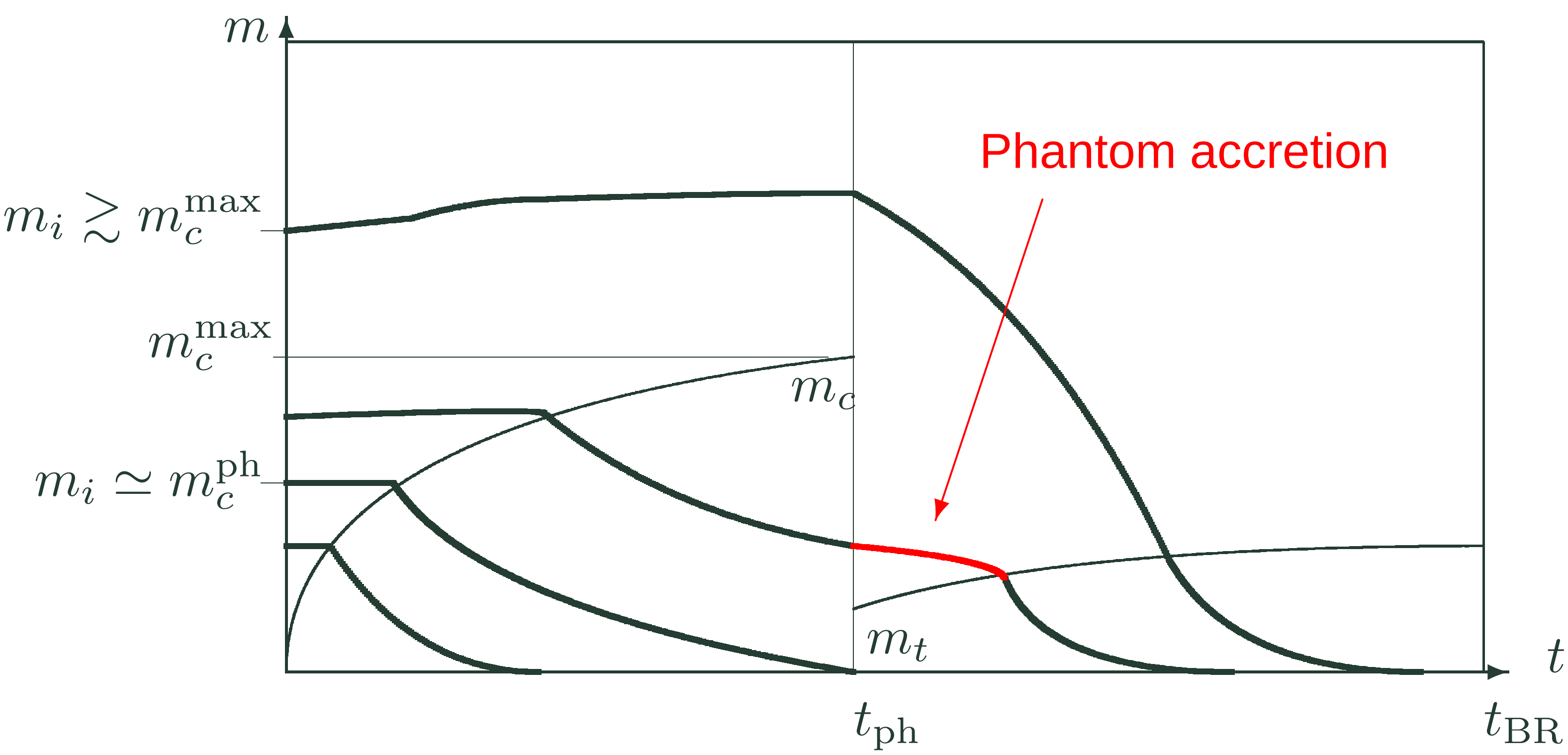

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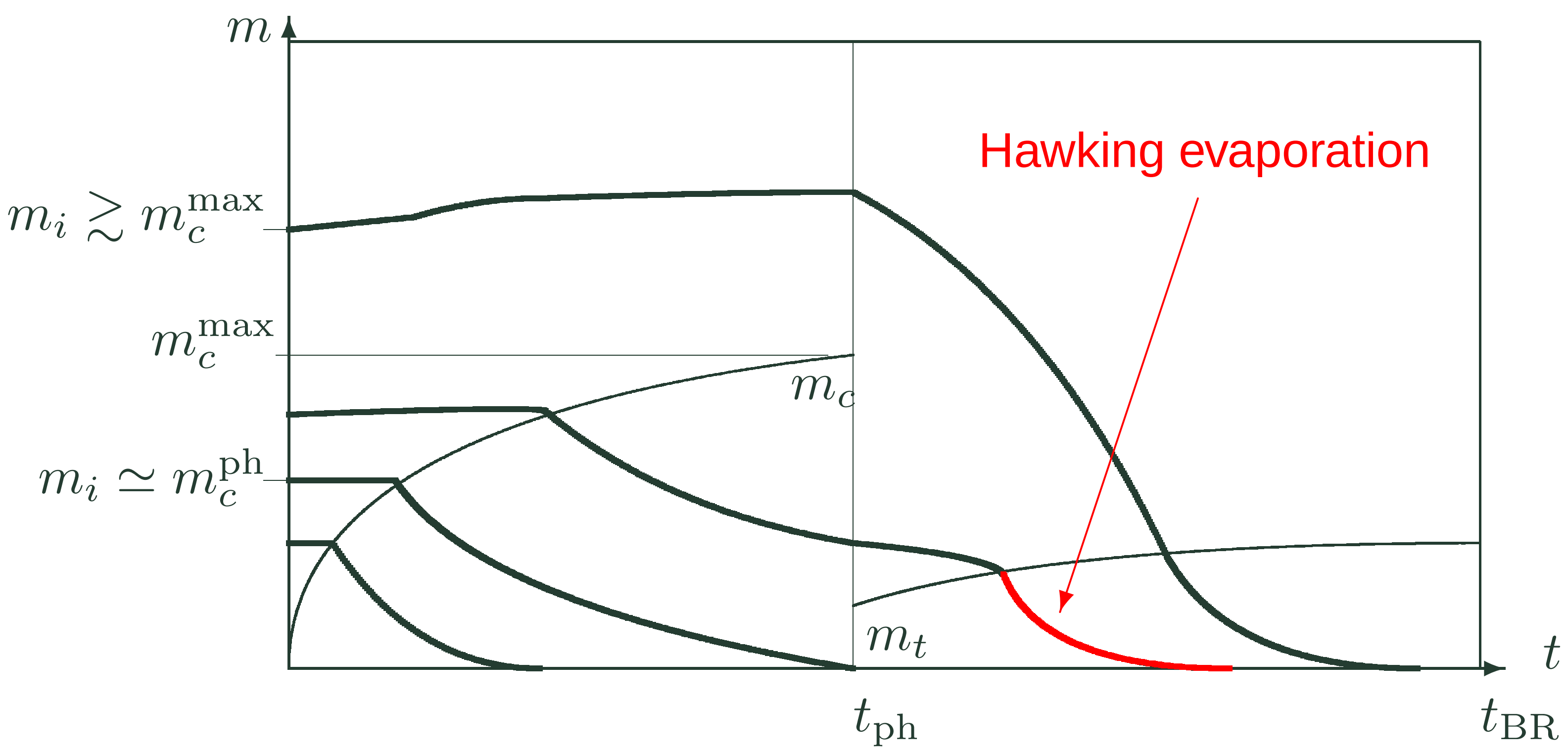

Back Reaction analysis

The consistency problem

- Simplest non-vacuum solution: Vaidya metric

$$\mathrm{d}s^2 = \left(1 - {}^{2m(\nu)}/_{r}\right)\mathrm{d}\nu^2 - 2\mathrm{d}\nu\mathrm{d}r - r^2\mathrm{d}\Omega^2$$

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- Cosmological black holes: generalized McVittie solution [Faraoni and Jacques, 2007]

$$\mathrm{d}s^2 = \frac{\left(1 - \frac{m(t)}{2a(t)r}\right)^2}{\left(1 + \frac{m(t)}{2a(t)r}\right)^2}\mathrm{d}t^2 - a^2(t)\left(1 + \frac{m(t)}{2a(t)r}\right)^4\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right)$$

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  - ☐ Two perfect dust-like fluids [Sultana and Dyer, 2005]

$$\frac{m(t)}{a(t)} = m_0$$

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- Black hole mass evolution

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -S \left(\frac{1 - \frac{m}{2ar}}{1 + \frac{m}{2ar}} \right)^4 \Bigg[(\rho_1 + p_1) \left(k^0\right)^2 + \rho_2 \sqrt{ (k^0)^4 \frac{(\rho_1 + p_1)^2}{\rho_2^2} - \frac{\left(1 + \frac{m}{2ar}\right)^2}{\left(1 - \frac{m}{2ar}\right)^2} (k^0)^2 \frac{(\rho_1 + p_1)}{\rho_2} } \Bigg] \tag{3}$$

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- Work in progress

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