MOND: Transition or Bust

Ali Mozaffari

Theoretical Physics Group
Imperial College London

5th Iberian Cosmology Meeting Porto 2010

Phys.Rev. D73 (2006) 103513
arXiv:0912.0710v1

Imperial College London

Outline

- Setup of the problem and possible solutions
- Modified Newtonian Dynamics
- Free functions and physical motivations
- TeVeS
- Experimental Tests
- Conclusions

The Problem

Galactic dynamics cannot be adequately explained by conventional gravity, the visible matter in many galaxies being too small

The 'missing mass' problem is one that can seen in both Dopler velocities and from gravitational lensing

Can GR (and hence its weak field limit) explain these problems?

Possible Solutions

Dark Matter

f(R) Gravity

Bi-metric Gravity

Modified Newtonian Dynamics / TeVeS

Einstein Aether Theories

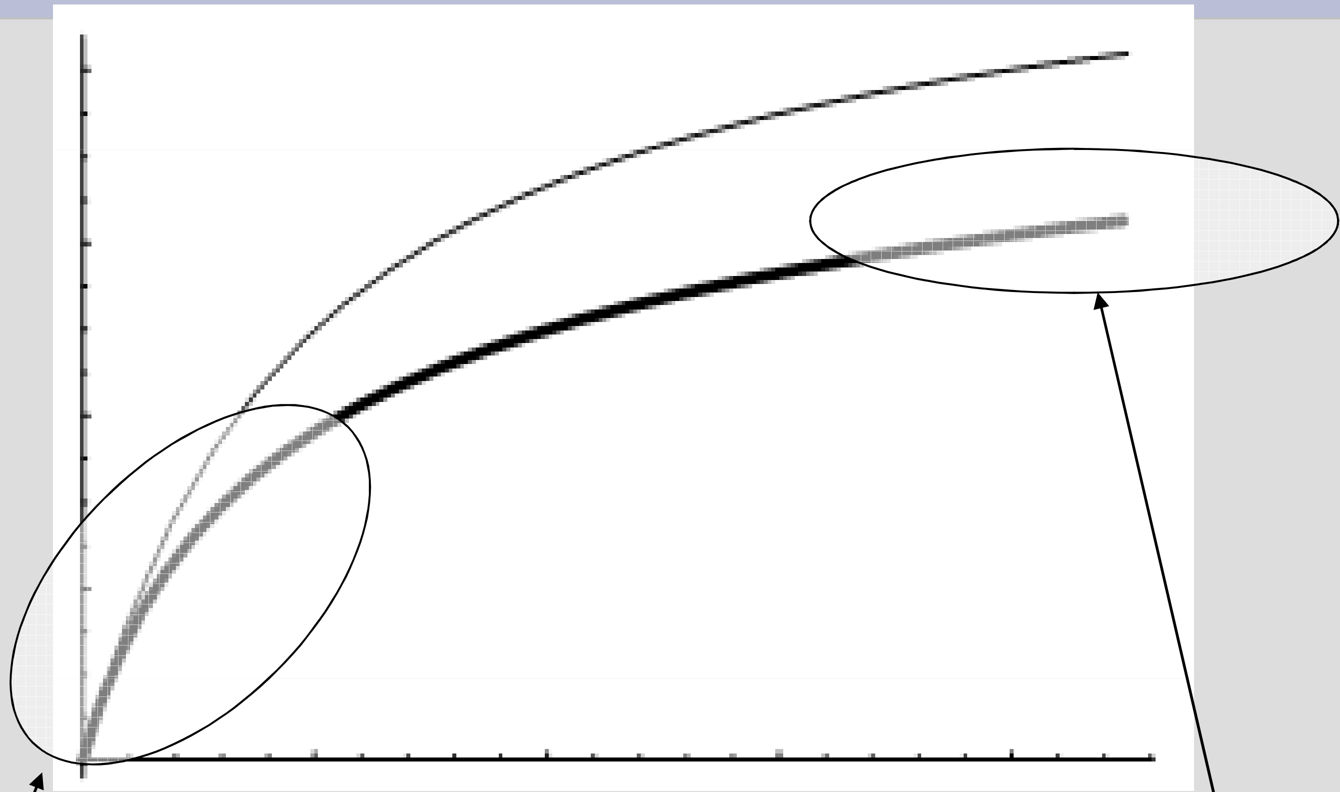
MOND

Modified Newtonian Dynamics, proposed by Milgrom in 1983

Theory has acceleration scale dependence, Milgrom's acceleration $a_0 \approx 10^{-10}$ ms^{-2}

Constructed to give flat galaxy rotation curves below a_0

MOND



Typical
Newtonian
Regime

Deep MOND
Regime

MOND Setup

$$\nabla^2 \Phi_N = 4\pi G_N \rho$$

$$\nabla . [\tilde{\mu} \nabla \Phi] = 4\pi G \rho$$

$$\Phi = \Xi \Phi_N + \phi$$

$$\tilde{\mu} = (\Xi + k/4\pi\mu)^{-1}$$

$$\nabla . [\mu \nabla \phi] = k G \rho$$

For $G = G_N$, we set $\Xi = 1$

MOND Basics

$$\tilde{\mu} = \tilde{\mu}\left(\frac{|a|}{a_0}\right)$$

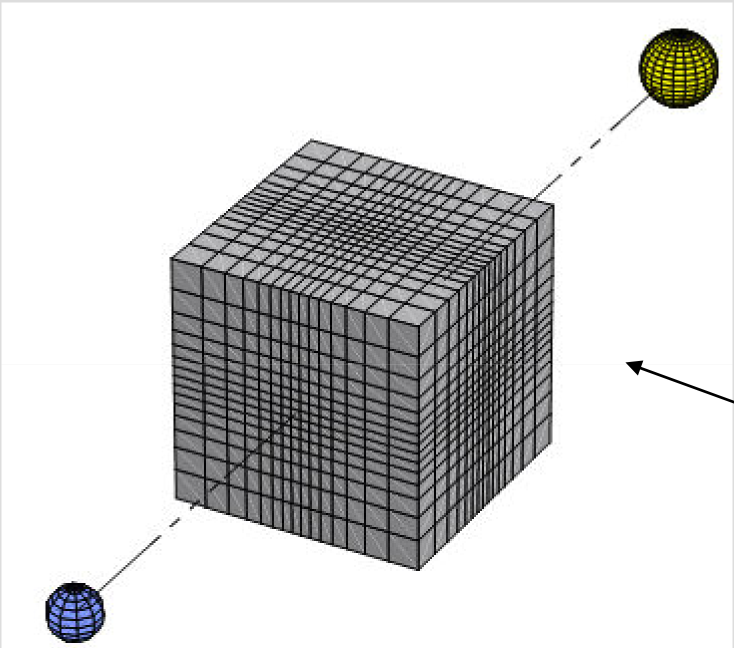
In the deep MOND regime $\mu \approx \frac{k}{4\pi}\frac{|\nabla\phi|}{a_0}$

We take the 'simple' form of $\tilde{\mu}(x) = \frac{x}{1+x}$

such that $\tilde{\mu}(x) = \begin{cases} 1 & x \ll 1 \quad \text{(Typical Newtonian regime)} \\ x & x \gg 1 \quad \text{(Galaxy Rotation regime)} \end{cases}$

MOND Applications

To examine the theory further, consider a 2 body problem



MOND Applications

Given that the theory is non-linear, we must first formally linearise

$$u = -\frac{4\pi\mu}{k}\nabla\phi$$

Such that we can now sum up all the sources

$$\sum_i u_i = u_{total}$$

$$\nabla\phi_{total} = -\frac{k}{4\pi}\frac{1}{\mu}u_{total}$$

MOND Applications

Now we just have to solve two equations

$$\nabla . u = -4\pi G \rho$$

$$\nabla \times \left(\frac{u}{\mu} \right) = 0$$

Can now define $u = F^{(N)} + \nabla \times h$

Find $\nabla \times h = 0$ for symmetric systems

MOND Applications

Applying this formalism at the Earth-Sun system, we find that around the saddle point of the gravitational potential i.e. whe $\nabla \Phi_N = 0$

We find there should exist deep MOND 'bubbles', characterised by their ellipsoidal shape in which modified gravity effects will be noticeable

For the Earth-Sun saddle, the semi-major axis of the MOND ellipsoid is predicted to be 766km long

Free Functions

The interpolating function   for the gravitational field is a free function in the theory

Whilst the asymptotics of the theory are set, the precise form is left open, bounded by some observation constraints

The transition function in the MOND field is also free

Physical Motivations

Physical bounds can be put on the transition function from anomalous physical effects – e.g. the Pioneer anomaly

However the exact form and behaviour of the function remain an open question

A relevant question is how dependent is the MOND field on the transition functions?

TeVeS

What about a relativistic theory of MOND?

TeVeS

Jacob Bekenstein (2004) developed a Tensor Vector Scalar gravity theory

Reduces to MOND in the weak-field limit

Constructed to fit with the standard tests of GR

TeVeS

$$S = \int d^4x \left[\frac{1}{16\pi G}(R - 2\Lambda) - \frac{1}{2}\{\sigma^2 (g^{\mu\nu} - U^\mu U^\nu)\phi_{,\alpha}\phi_{,\beta} + \frac{1}{2} G l^{-2} \sigma^4 F(kG\sigma^2)\} - \frac{1}{32\pi G}\{K\mathcal{F}^{\alpha\beta}\mathcal{F}_{\alpha\beta} - 2\lambda(U_\mu U^\mu + 1)\} \right](-g)^{1/2} + \mathcal{L}(\tilde{g}_{\mu\nu}, f^\alpha, f^\alpha_{|\mu}, \dots)(-\tilde{g})^{1/2}$$

Theory has free functions (here denoted *F*, sometimes called the Quilditch function)

Fails as a dynamical theory, as it can develop 'caustic' singularities [Phys.Rev.D78:044034 (2008)] (these can be rectified in an E-A theory)

Experimental Tests

An exciting prospect in testing modified gravity theories comes from the LISA pathfinder mission

It is hoped to observe the predicted 'bubbles' of deep MOND effects, around the saddle points of the gravitational field

Proving (or disproving) MOND on this scale would be a key result for the theory

Computational Results

Code developed to solve MOND in the solar system seems to suggest that for the Earth-Sun saddle point, very different classes of interpolating functions give the same results in the solar system regime

Results also suggest that detecting MOND on these scales will present considerable difficulties, even with sensitive instrumentation

Conclusions

The MOND/TeVeS theories of modified gravity suggest that gravity acts differently on different scales

We present the idea of testing these theories on solar system scales by examining the saddle points in gravitational potentials

We suggest that the current formulation of the theory appears to some extent to be model independent, but also the effects are hard to distinguish from Newtonian gravity