Can we Design a Simple Dark Energy Engine?

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- Since ancient times, earth's gravity has been used to harness energy, converting gravitational potential energy kinetic energy using systems such as the water wheel

- Comparatively, an attempt to harness dark energy in a simple engine of two tethered balls is conducted. The concept is similar to that of a waterfall converting potential energy to kinetic energy

Two identical balls falls in the earth's gravitational field with an initial separation. This separation will increase as a function of time, and a tether will turn a motor.

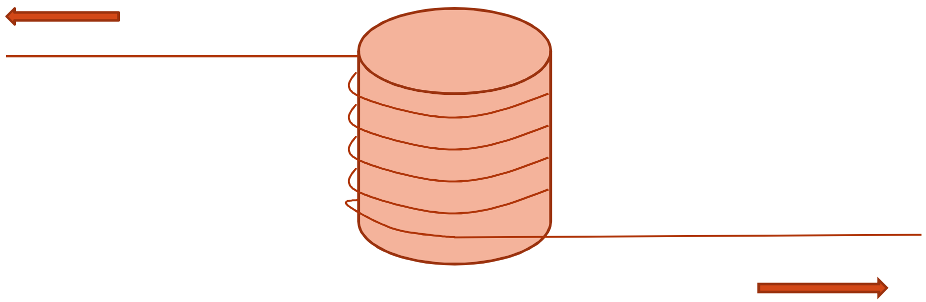
As the ball falls the height changes as

$$x = \left[x_o - \frac{R_\oplus}{2} \right] \cosh\left(\sqrt{\frac{2g_o}{R_\oplus}} t \right) + \frac{R_\oplus}{2}$$

For $x << R_\oplus$

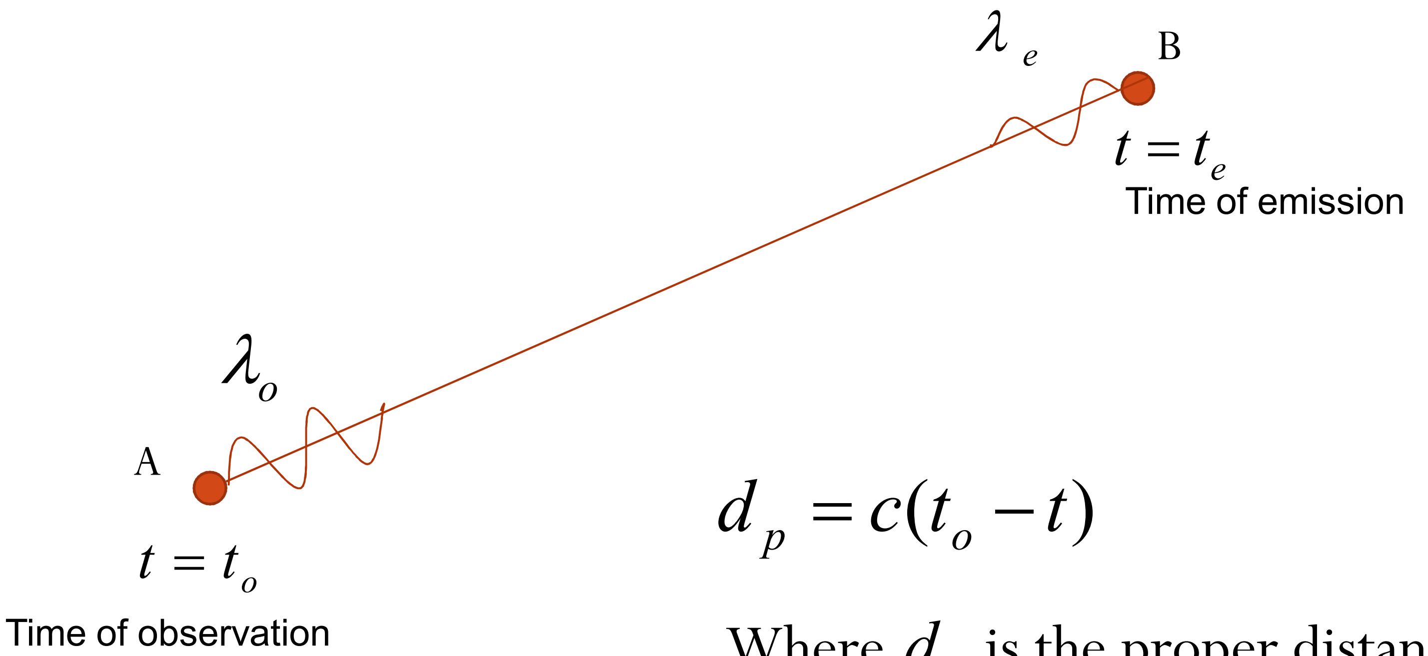
$$x(t) \approx x_o - \frac{2g}{R_\oplus} \left[\frac{R_\oplus}{2} - x_o \right] t^2$$

- As the two balls fall, Δx is continuously increasing



- The concept is approached in a cosmological setting where the distances are small (We are envisioning a photon-tractor)

Consider a photon falling from a distant galaxy



$$d_p = c(t_o - t)$$

Where d_p is the proper distance and $z <<< 1$

We use the following form for the FRW Metric

$$ds^2 = c^2 dt^2 - R^2(t)\left[d\chi^2 + S^2(\chi)\{d\theta^2 + \sin^2\theta d\varphi^2\}\right]$$

Where $(r = S(\chi), \theta, \varphi)$ are comoving coordinates,

$$\chi = c\int_t^{t_o} \frac{dt}{R(t)}$$

In general, we find that the cosmological fluid obeys

$$\frac{\dot{a}^2}{a^2} = H_o^2\left[\frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} + \Omega_\Lambda + \frac{\Omega_k}{a^2}\right]$$

where

$$a = \frac{R}{R_o} = \frac{1}{1+z}$$

In a vacuum domination $\Omega_\Lambda \approx 1$

$$H = \frac{\dot{a}}{a} = H_o \sqrt{\Omega_\Lambda}$$

$$a = \left(\frac{a_o}{e^{H_o \sqrt{\Omega_\Lambda} t_o}} \right) e^{H_o \sqrt{\Omega_\Lambda} t}$$

This can be related to the scale factor, R, as

$$R = \left(\frac{R_o}{e^{H_o \sqrt{\Omega_\Lambda} t_o}} \right) e^{H_o \sqrt{\Omega_\Lambda} t}$$

Iff $\Omega_\Lambda \approx 1$

Thus

$$\chi R_o = \frac{c}{H_o}\frac{1}{\sqrt{\Omega_\Lambda}}\left[e^{H_o\sqrt{\Omega_\Lambda}(t_o - t)} - 1\right]$$

Thus the proper distance to this object which has a look back time of $(t_0 - t)$, where $\Omega_\Lambda \approx 1$ is $d_p = R(t_0)\chi$, or

$$d_p = \frac{c}{H_o}\frac{1}{\sqrt{\Omega_\Lambda}}[e^{H_o\sqrt{\Omega_\Lambda}(t_o - t)} - 1]$$

In the limiting case for $t_o - t \approx 0$ where the objects are close together,

$$d_p \approx c(t_o - t)$$

As is expected.

For $\Omega_\Lambda \approx 1$, the proper velocity of B is

$$v_p = \frac{dd_p}{dt_o}$$

$$v_p = c\left[1 - \frac{1}{1+z}\right]e^{H_o\sqrt{\Omega_\Lambda}(t_o - t)}$$

For all z where $\Omega_\Lambda \approx 1$,

$$(t_o - t) = \frac{1}{H_o\sqrt{\Omega_\Lambda}}\ln(1+z)$$

For $z << 1$, the proper velocity becomes

$$v_p = cz$$

as expected.

The proper velocity can be written as

$$v_p = cH_o\sqrt{\Omega_\Lambda}(t_o - t)$$

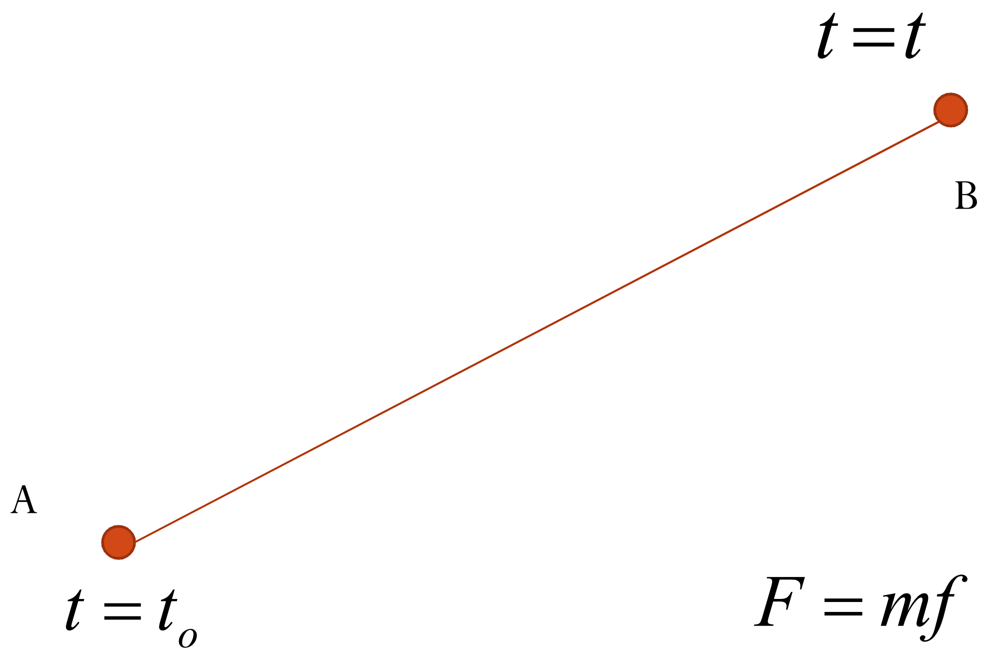
Where $t_o \approx t$ and $z << 1$

We then calculate the acceleration of B with respect to A where $t_o \approx t$, $z <<< 1$, and $\Omega_\Lambda \approx 1$.

$$f = \frac{dv_p}{dt_o}$$

$$f \approx cH_o^2 \Omega_\Lambda (t_o - t)$$

Now consider two balls separating where B is experience a force.



$$F = mf$$

$$F \approx mcH_o^2\Omega_\Lambda(t_o - t)$$

for small z, where $t_o \approx t$ and $\Omega_\Lambda \approx 1$

The potential energy of the system is

$$-d\phi = F\, dd_p$$

Where $d_p \approx c(t_o - t)$

And $dd_p \approx c\, d(t_o - t)$

Thus, the potential becomes

$$\phi = \phi_o - \frac{1}{2} mc^2 H_o^2 \Omega_\Lambda (t_o - t)^2$$

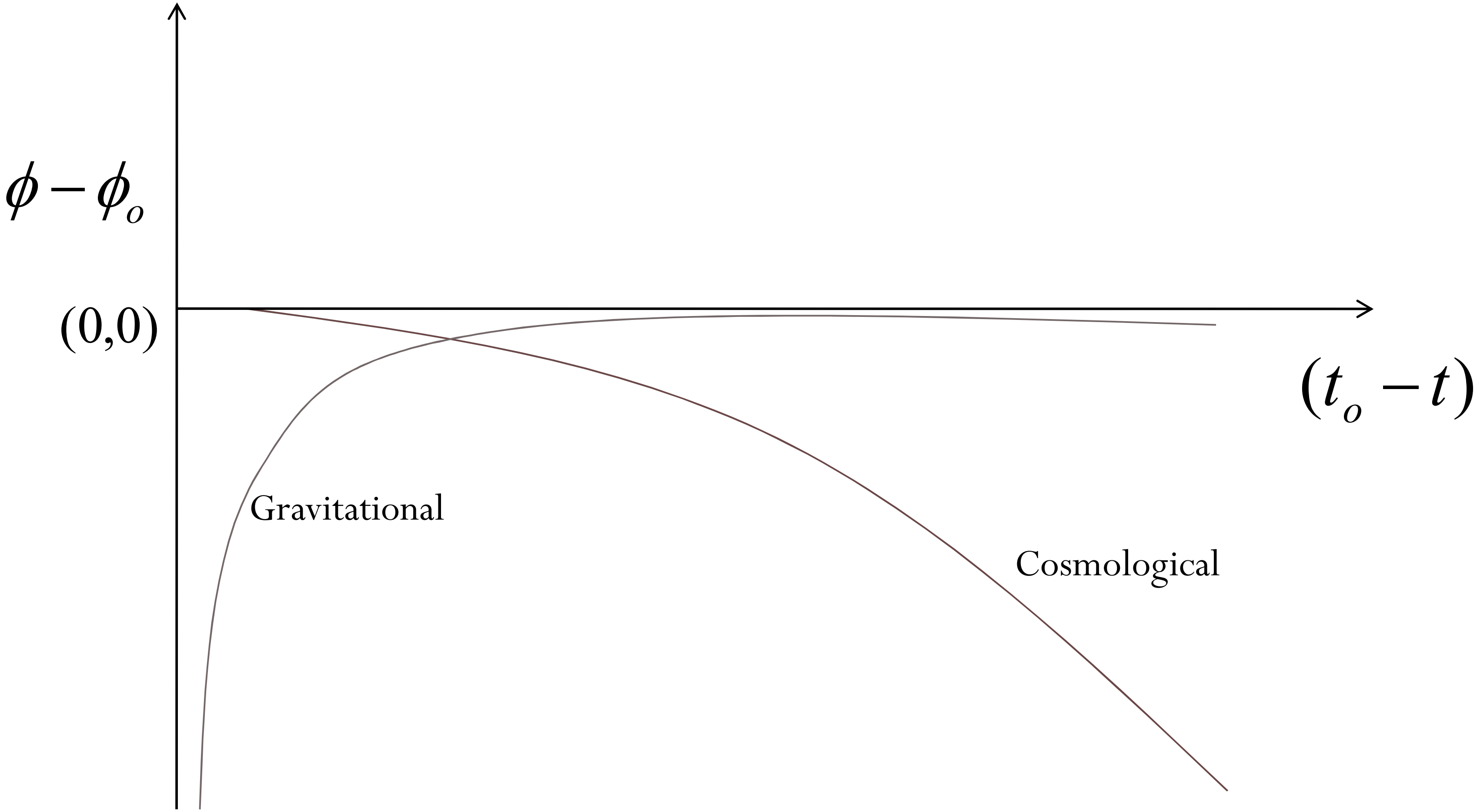
When $d_p = 0$, or $t_o = t$, A and B are together. The potential energy is then ϕ_o. As the balls separate, the potential decreases, which is the opposite of the gravitational potential.

$$\phi \rightarrow -\infty$$

As $(t - t_o) \rightarrow \infty$

Or as $t \rightarrow \infty$

Or as the balls diverge to ∞


$\phi - \phi_o$
(0,0)
$(t_o - t)$
Gravitational
Cosmological

Because the potential energy diverges to $-\infty$ from ϕ_o, power should be generated.

$$-P = \frac{d\phi}{dt_o}$$

$$P = mc^2 H_o^3 \Omega_\Lambda^{3/2} (t_o - t)^2$$

Infinite power can be generated as the proper distance between the spheres increases. That is, as $t_o - t$ increases.

In an approximation where $m \equiv M_{sun}$

$$P \approx M_{sun} c^2 H_o^3 \Omega_\Lambda^{3/2} (t_o - t)$$

where $\Omega_\Lambda \approx 1$

$$P \approx 2.5\ W$$

Where $t_o - t \approx 10^3$, which is 1000 light seconds away, or 1000 x the distance to the moon. Thus, the power generated is very small.

In order for us to do this, the dark energy should dominate the local gravity.

In the weak field limit, general relativity gives:

$$\nabla^2 \phi \approx 4\pi G\rho - \Lambda c^2$$

$$g \approx -\frac{GM}{r^2}\hat{r} + \frac{c^2 \Lambda r}{3}\hat{r}$$

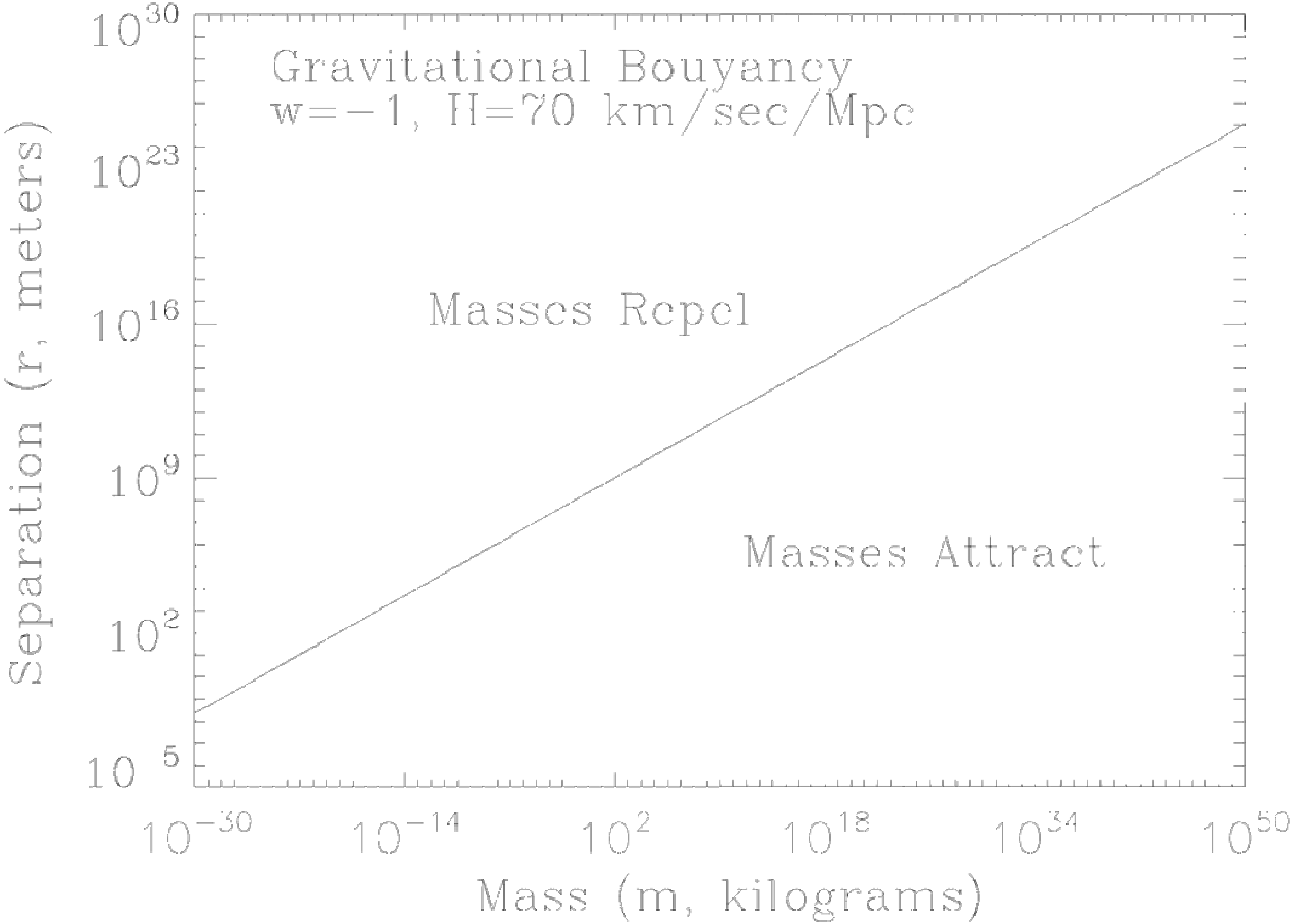
Therefore, there must exist a point at which g=0, occurring at

$$r_* = \left(\frac{3GM}{c^2 \Lambda}\right)^{1/3}$$

In geometrized units for the sun, this distance is

$$r_* = \left(\frac{3\times 1477}{10^{-52}}\right) \approx 103\ pc$$

Thus in this case the length of the tether (photon-tractor!) should be > 103 pc! It is not possible to achieve such a length.



Gravitational Bouyancy
w=-1, H=70 km/sec/Mpc
Masses Repel
Masses Attract
Separation (r, meters)
Mass (m, kilograms)
10^{30}
10^{23}
10^{16}
10^{9}
10^{2}
10^{-5}
10^{-30}
10^{-14}
10^{2}
10^{18}
10^{34}
10^{50}

In a cosmological setting, we can differentiate

$$\frac{\dot{a}^2}{a^2} = H^2 = H_o^2 \left[\frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} + \Omega_\Lambda + \frac{\Omega_K}{a^2} \right]$$

At $a = a_*,\ \ddot{a} = 0$

This infers a coasting period in the history of the universe when $a \approx a_*$. Using WMAP data, this happens at

$$a_* = \left(\frac{\Omega_m}{2\Omega_\Lambda} \right)^{1/3} \Rightarrow z_* = \frac{1}{a_*} - 1 \approx 0.7$$

Thus the engine is huge here and it becomes impractical.

Discussion

- While conceptually the prospect of harnessing dark energy is appealing, the practical applications are not currently plausible.

- In order to create the large power outputs desired, large masses must be considered and separated by very large distances

Discussion

- This would require a very long tether to connect the object, the mass of which could dwarf that of the objects
- Instead we envision a photon-tractor [beam]
- My point is to make a suggestion about a scenario in which this energy could be harnessed, though not practical presently