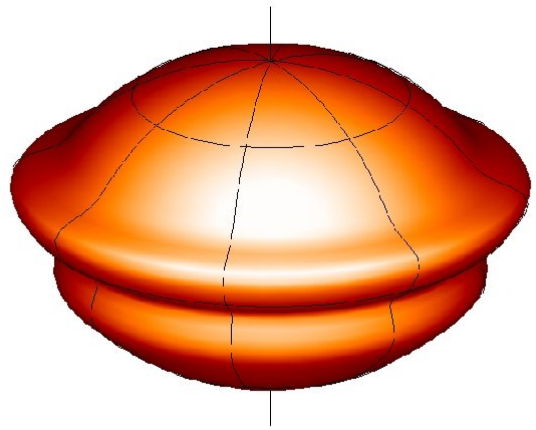
The effects of stellar rotation and magnetism on oscillation frequencies

Daniel Reese



Joint HELAS and CoRoT/ESTA Workshop
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Introduction

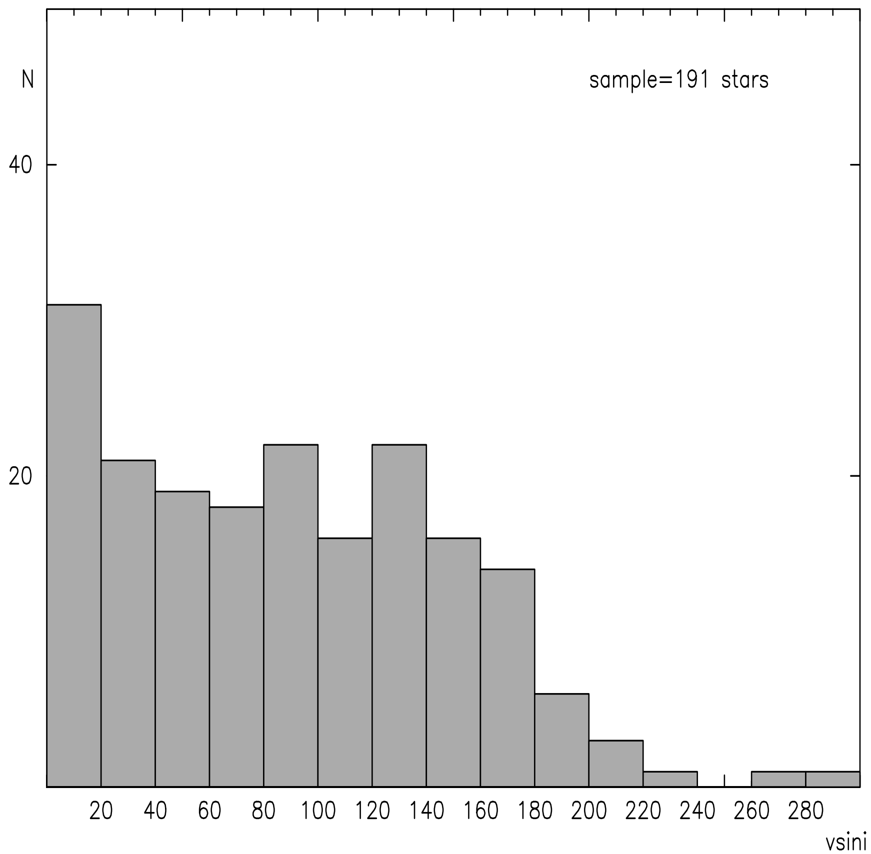
- ✘ traditionally, stellar pulsations are calculated assuming spherical symmetry
- ✘ however, neither stellar rotation nor stellar magnetism respect this symmetry
- ✘ Oscillation modes are no longer described by a single spherical harmonic
- ✘ No longer $1D$ calculations, but $2D$ or $3D$

Outline

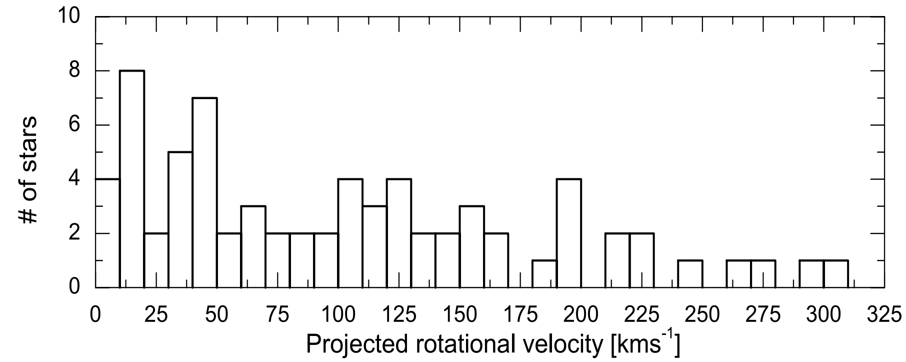
1. **The effects of stellar rotation**
2. The effects of stellar magnetism
3. Conclusion

Incidence of stellar rotation

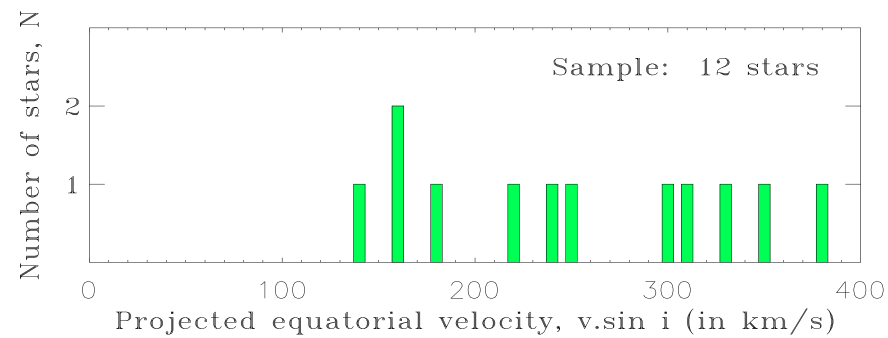
A few statistics :



δ Scuti (Rodríguez et al., 2000)



β Cephei (Stankov & Breger, 2005)



ζ Oph stars(based on Balona & Dziembowski, 1999)

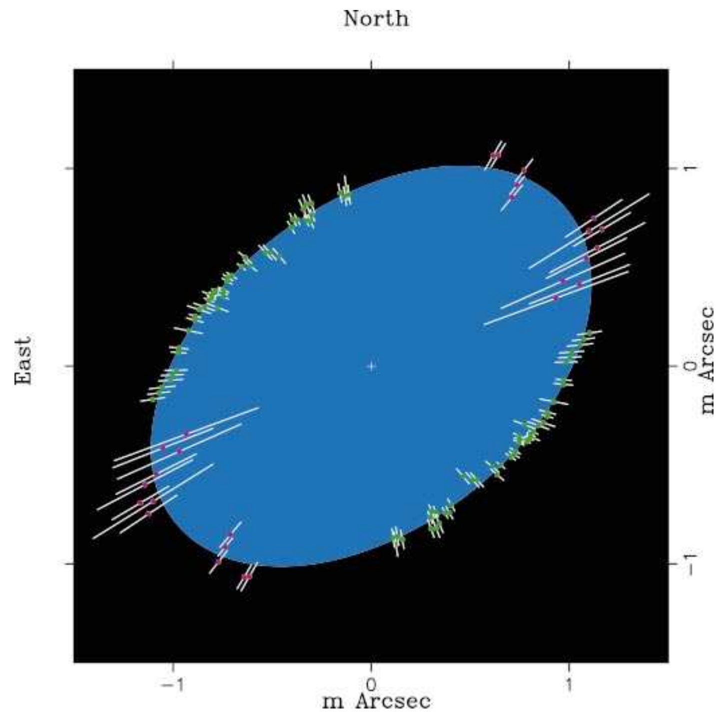
Targets for space missions

Identification	Name	Type	$v \cdot \sin i$ (in $\mathrm{km} \cdot s^{-1}$)	Mission
HD 187642	Altair	δ Scuti	230	WIRE
HD 149757	ζ Oph	ζ Oph	380	MOST
HD 181555		δ Scuti	170	CoRoT
HD 49434		γ Doradus	90	CoRoT
HD 171834		γ Doradus	72	CoRoT
HD 170782		δ Scuti	198	CoRoT
HD 170699		δ Scuti	$>$ 200	CoRoT
HD 177206		δ Scuti	$>$ 200	CoRoT

Effects of rotation

✘ Two forces appear because of rotation

- centrifugal force : stellar deformation and modification of equilibrium quantities
- Coriolis force : intervenes in all dynamical processes

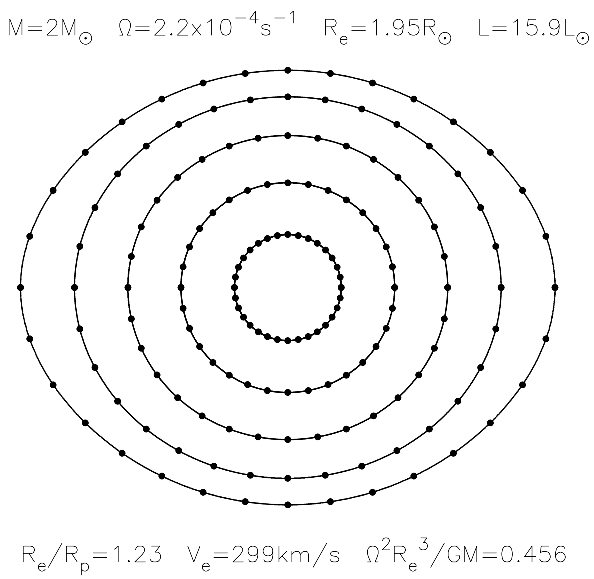


Domiciano de Souza et al. (2003)

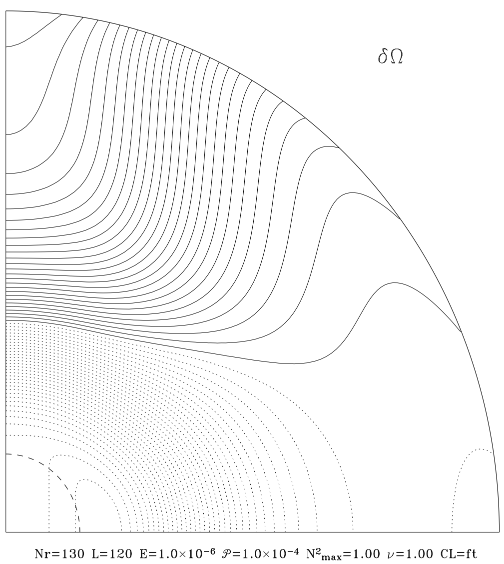
Models of rapidly rotating stars

A few references :

- ✘ Meynet and Maeder (1997-2000)
- ✘ Roxburgh (2004, 2006)
- ✘ Jackson et al. (2005), MacGregor et al. (2006)
- ✘ ESTER (Rieutord et al., 2005, Rieutord, 2006)



Roxburgh (2004)



Rieutord (2006)

Rotation and oscillations

Two basic approaches to take the effects of rotation into account :

Perturbative approach

✘ the rotation rate Ω is considered to be small

✘ equilibrium model and oscillation modes :

$$\begin{aligned} \vec{v} &= \vec{v}_0 + \vec{v}_1\Omega + \vec{v}_2\Omega^2 + \ldots\mathcal{O}\left(\Omega^{n+1}\right) \\ \omega &= \omega_0 + \omega_1\Omega + \omega_2\Omega^2 + \ldots\mathcal{O}\left(\Omega^{n+1}\right) \end{aligned}$$

Complete approach

✘ the rotation rate Ω is not considered small

✘ equilibrium model and oscillation modes = a solution to a 2D problem which fully includes the effects of rotation

A few references...

Perturbative approach

✘ 2^{nd} order methods :

- Saio (1981)
- Gough & Thompson (1990)
- Dziembowski & Goode (1992)

✘ 3^{rd} order methods :

- Soufi et al. (1998)
- Karami et al. (2005)

Complete approach

✘ Clement (1981-1998)

✘ Dintrans et al. (1999), Dintrans & Rieutord (2000)

✘ Espinosa et al. (2004)

✘ Lignières et al. (2006), Reese et al. (2006)

Slow rotation rates

Perturbative expression of pulsation frequencies :

$$\omega = \omega_0 - m(1-C)\Omega + \left(D_1 + D_2 m^2\right)\Omega^2 + m\left(T_1 + T_2 m^2\right)\Omega^3 + \mathcal{O}\left(\Omega^4\right)$$

No rotation

Slow rotation rates

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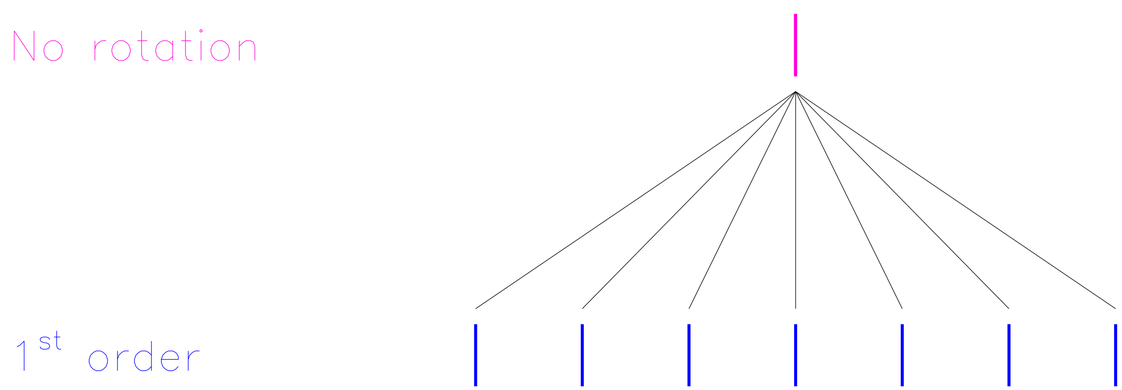

Slow rotation rates

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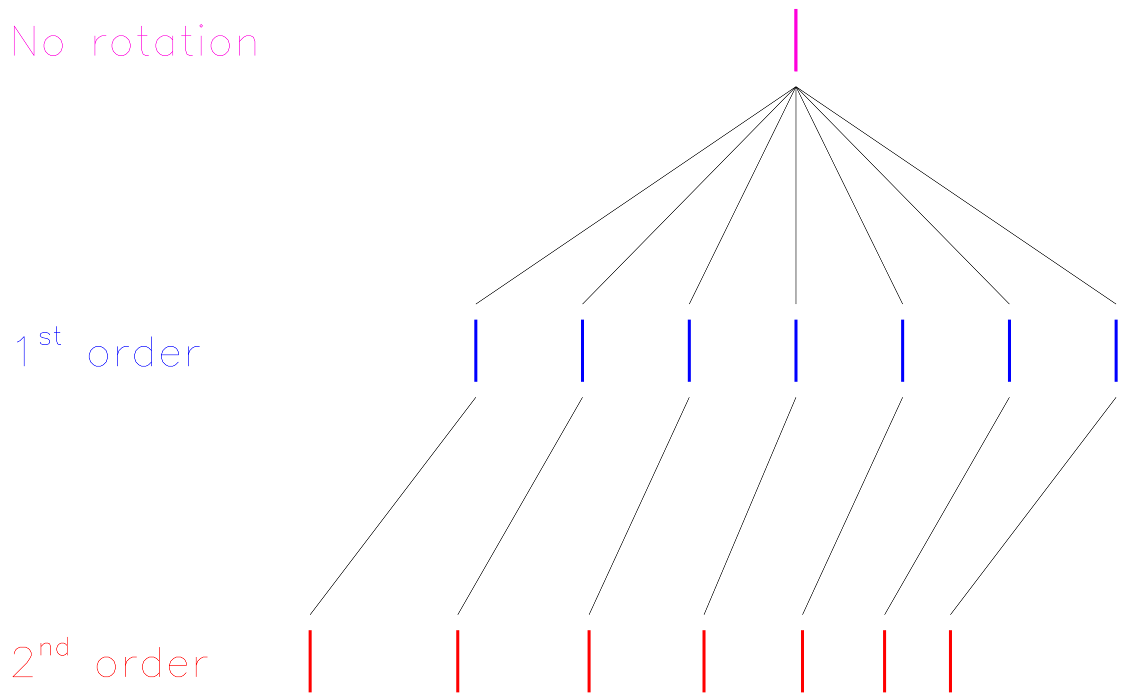
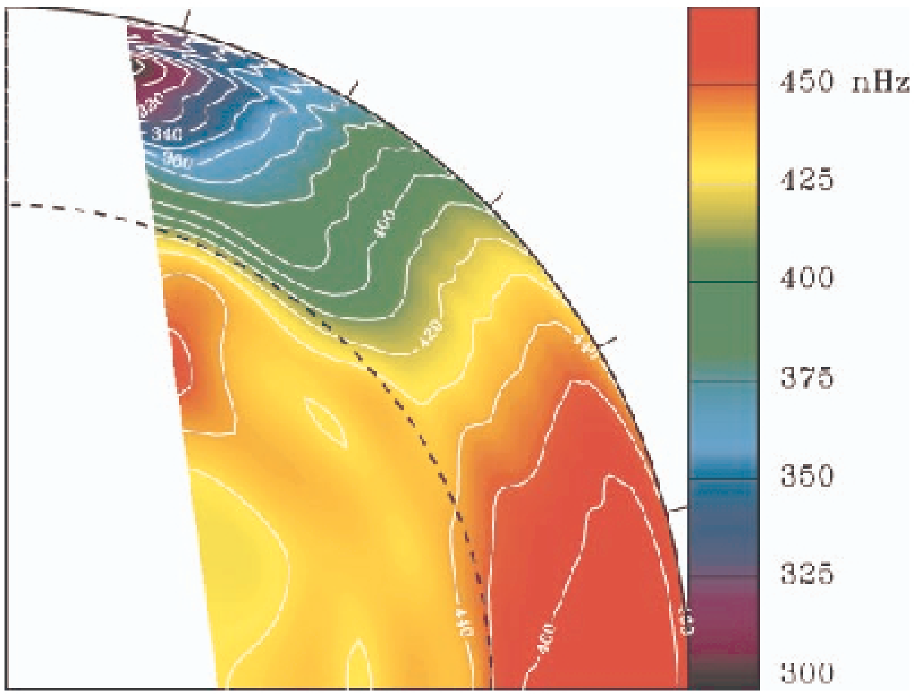


Solar rotation profile

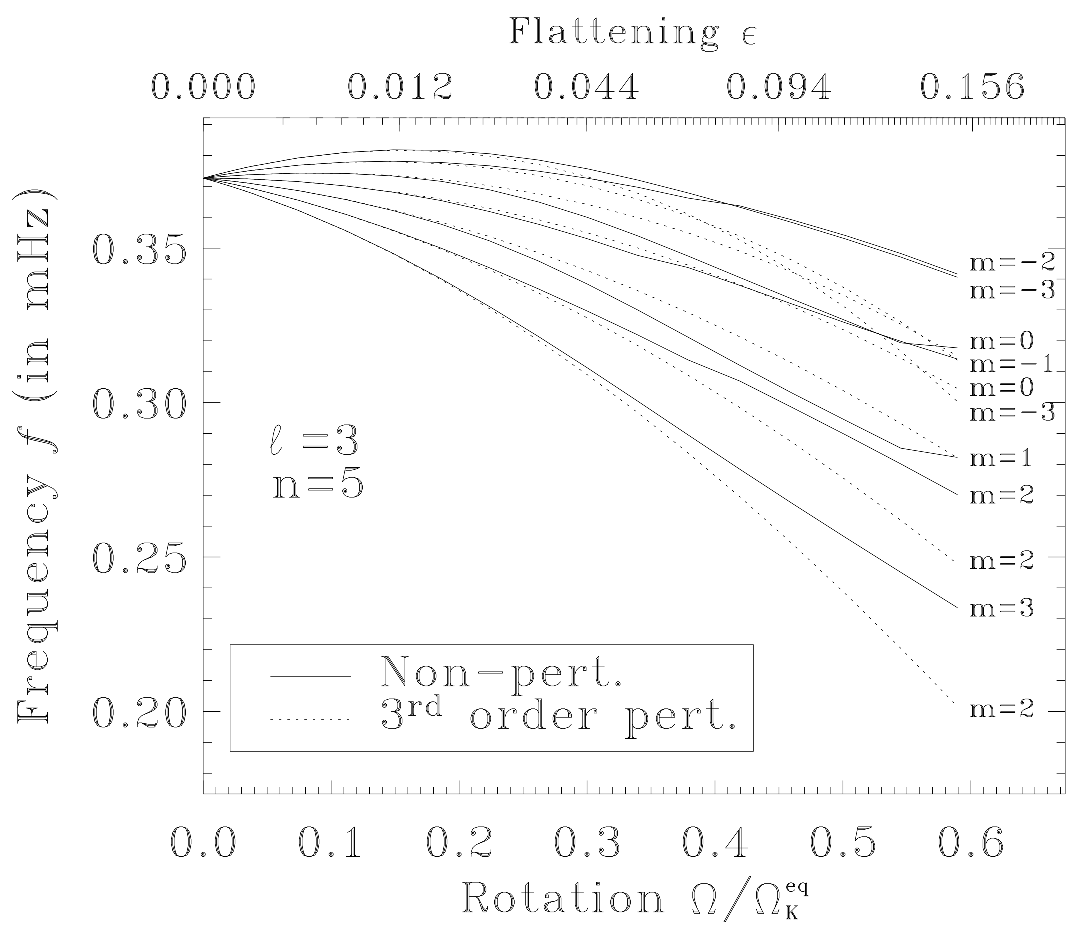
- ✘ use of 1^{st} order methods
- ✘ inversion techniques



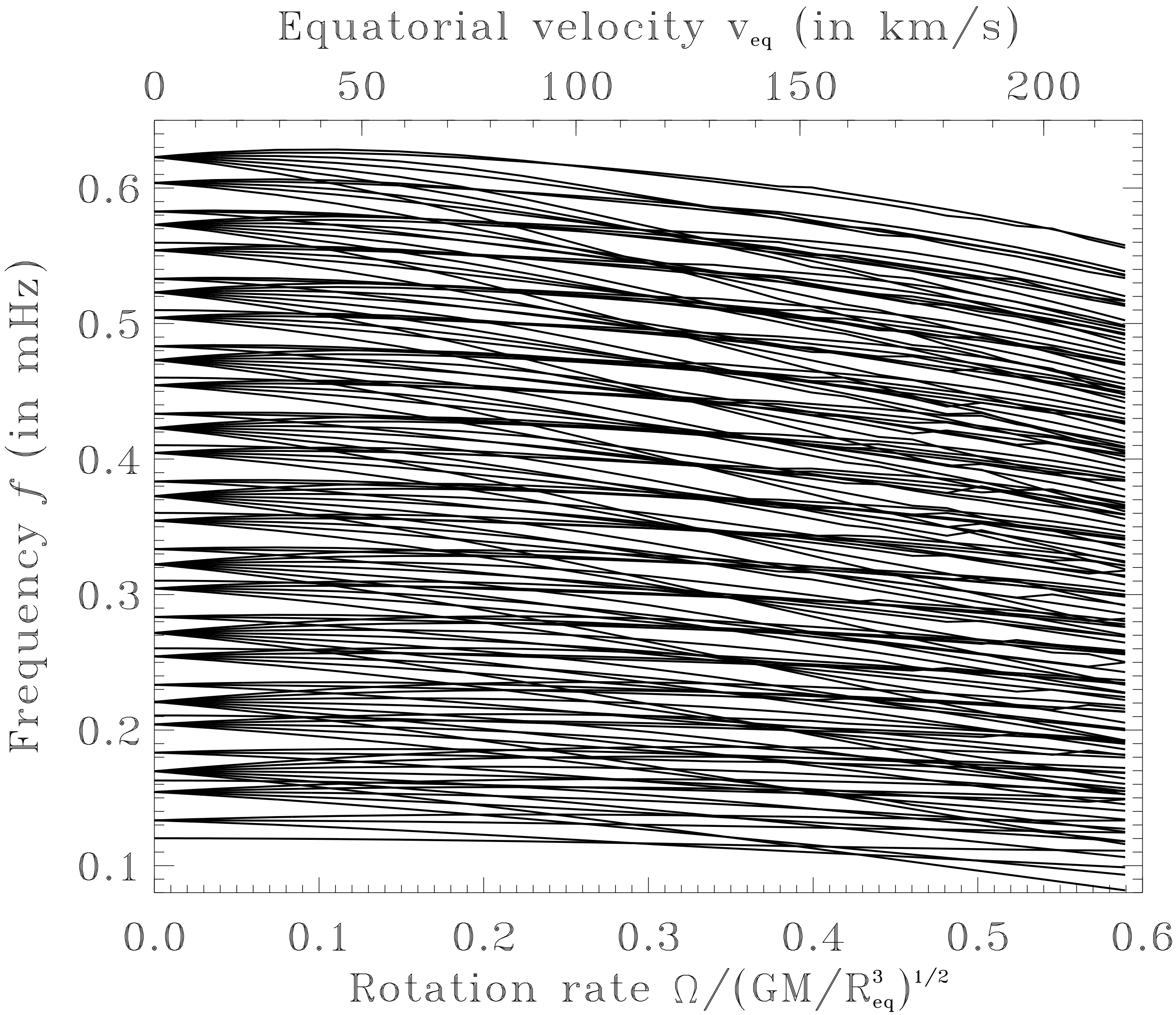
Schou et al. (1998), Thompson et al. (2003)

High rotation rates

A multiplet :



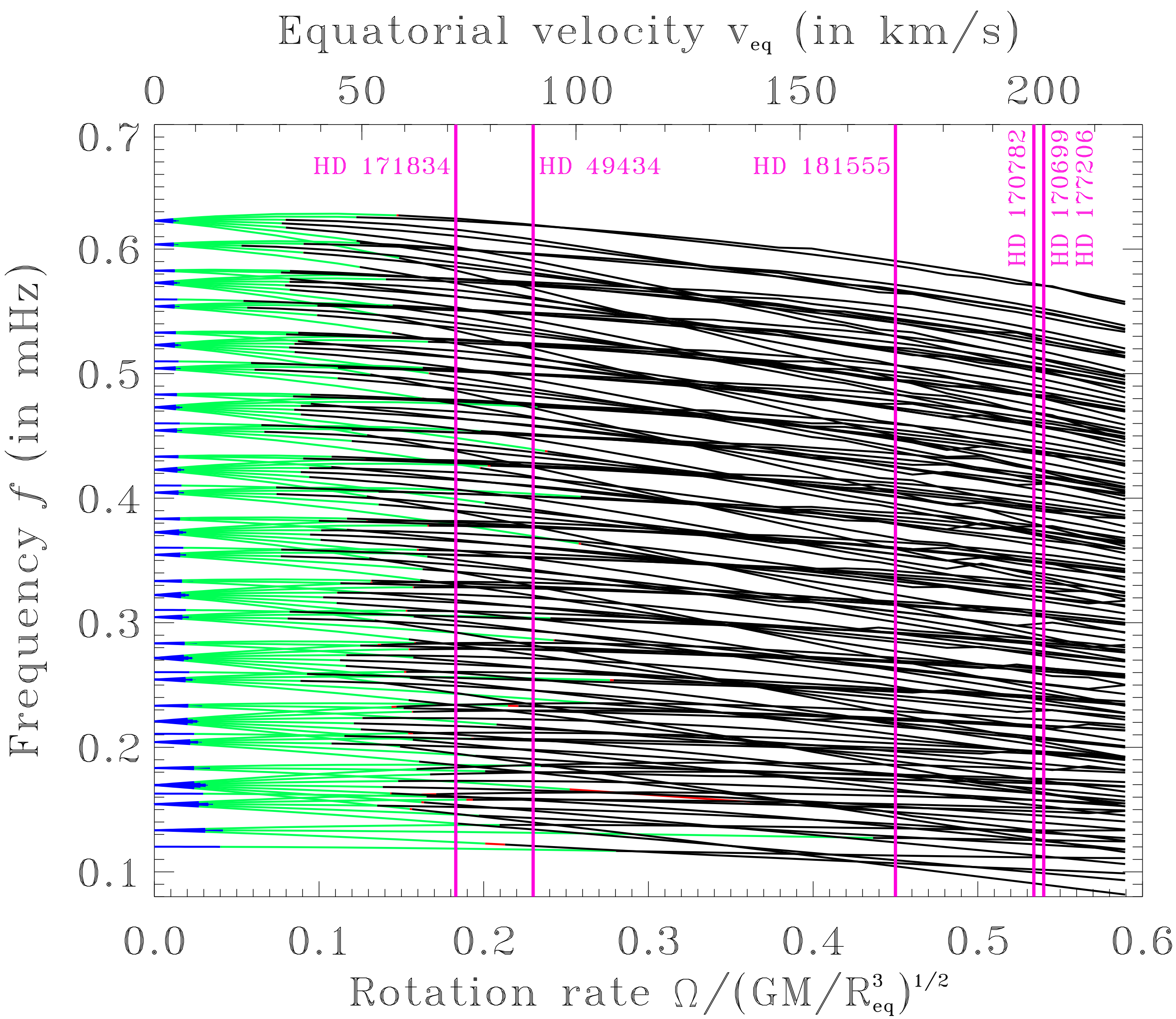
High rotation rates



Modes :

$$n = 1 \text{ to } 6$$
$$\ell = 0 \text{ to } 3$$
$$m = -\ell \text{ to } \ell$$

High rotation rates

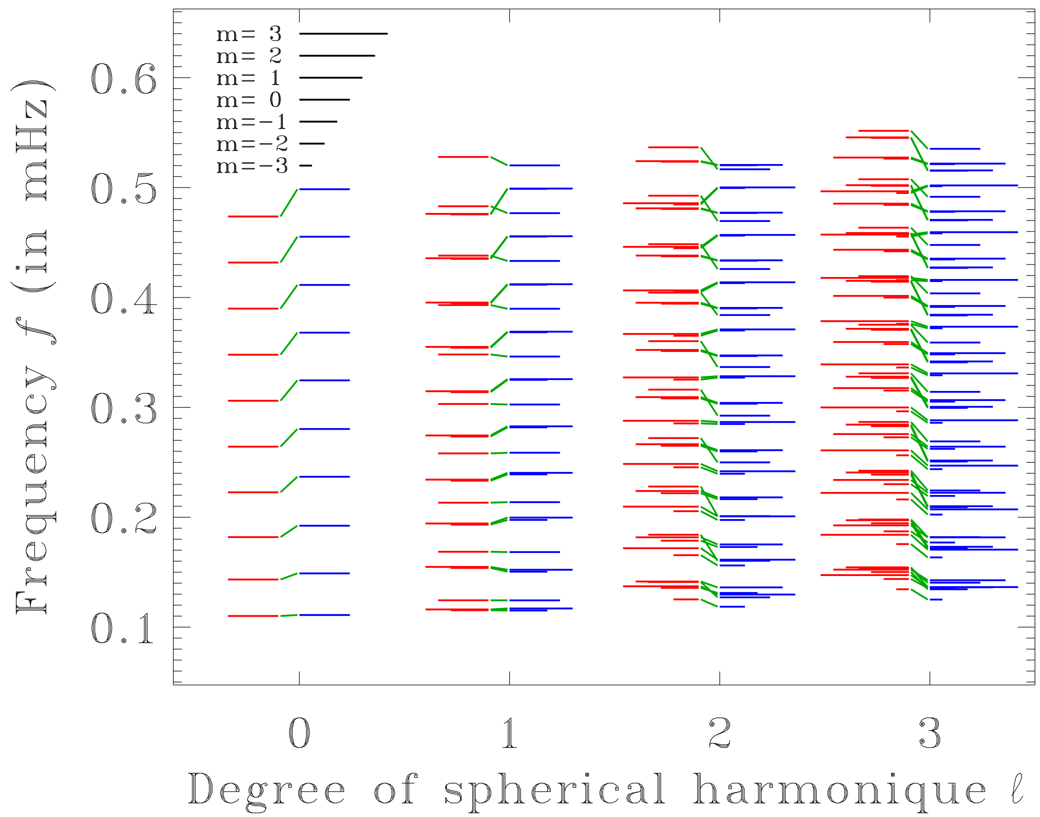


Validity domain for 150 days of observation ($\Delta\omega = 0.08$ μHz)

- 1^{st} order
- 2^{nd} order
- 3^{rd} order

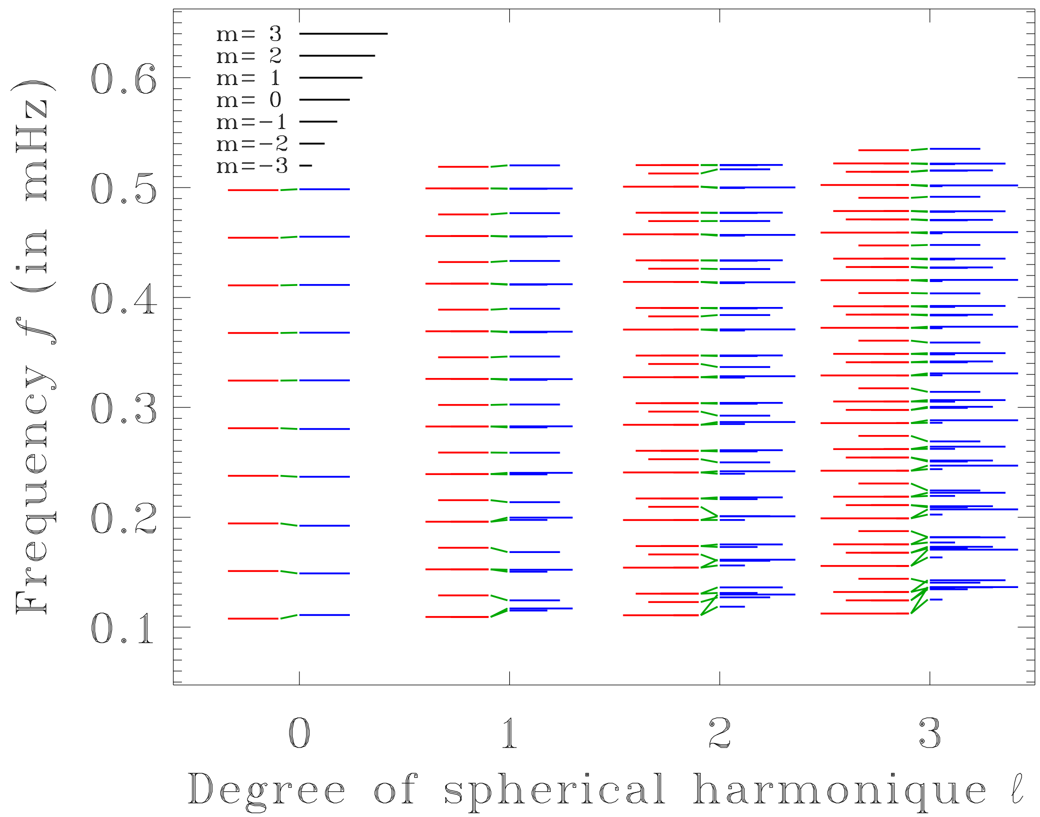
(see Reese et al., 2006)

Organisation of frequency spectrum



$$f_{n\,\ell\,m} = f^0_{n\,\ell\,m} + f^1_{n\,\ell\,m}\Omega + f^2_{n\,\ell\,m}\Omega^2 + f^3_{n\,\ell\,m}\Omega^3 + \mathcal{O}(\Omega^4)$$

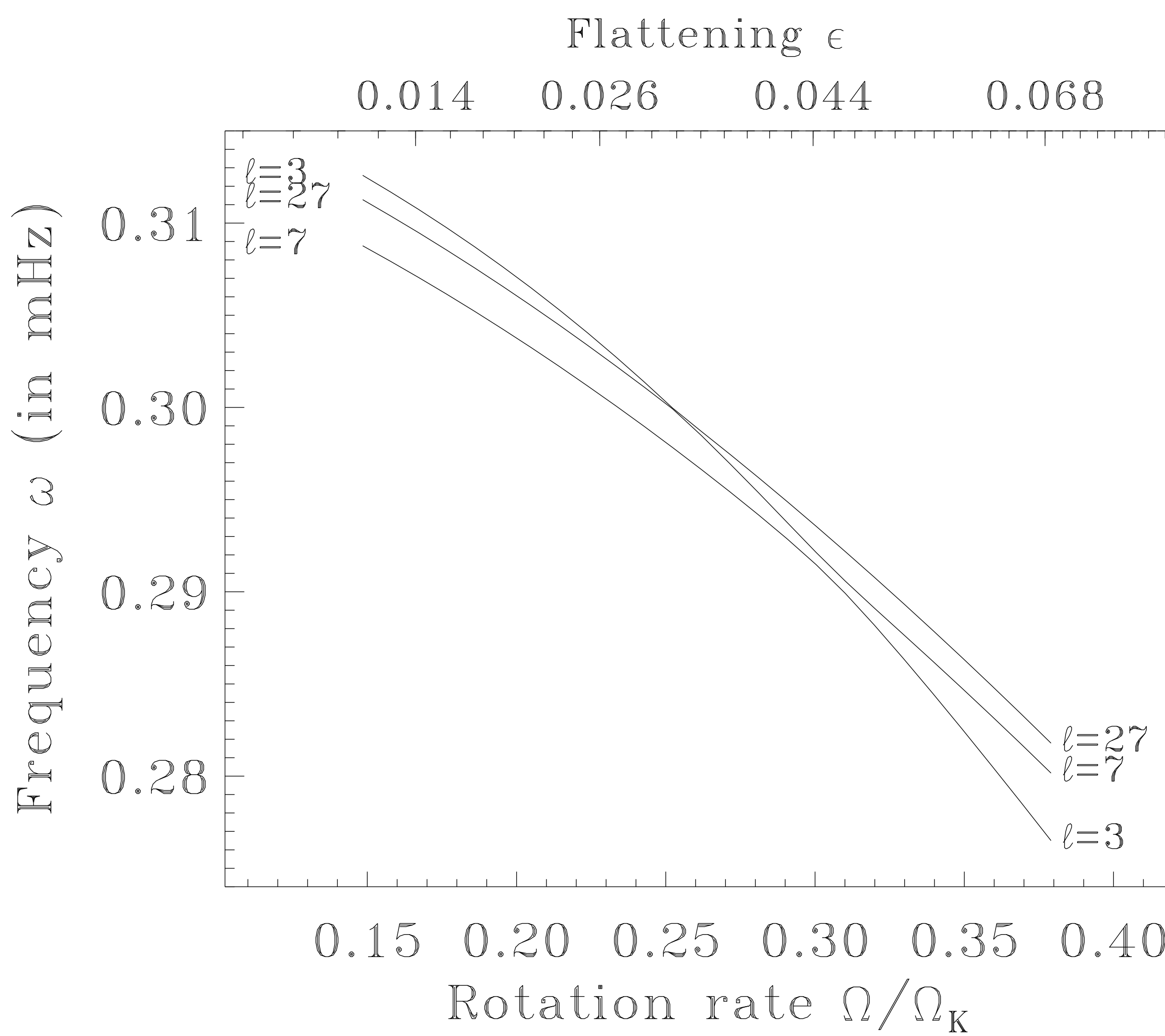
Organisation of frequency spectrum



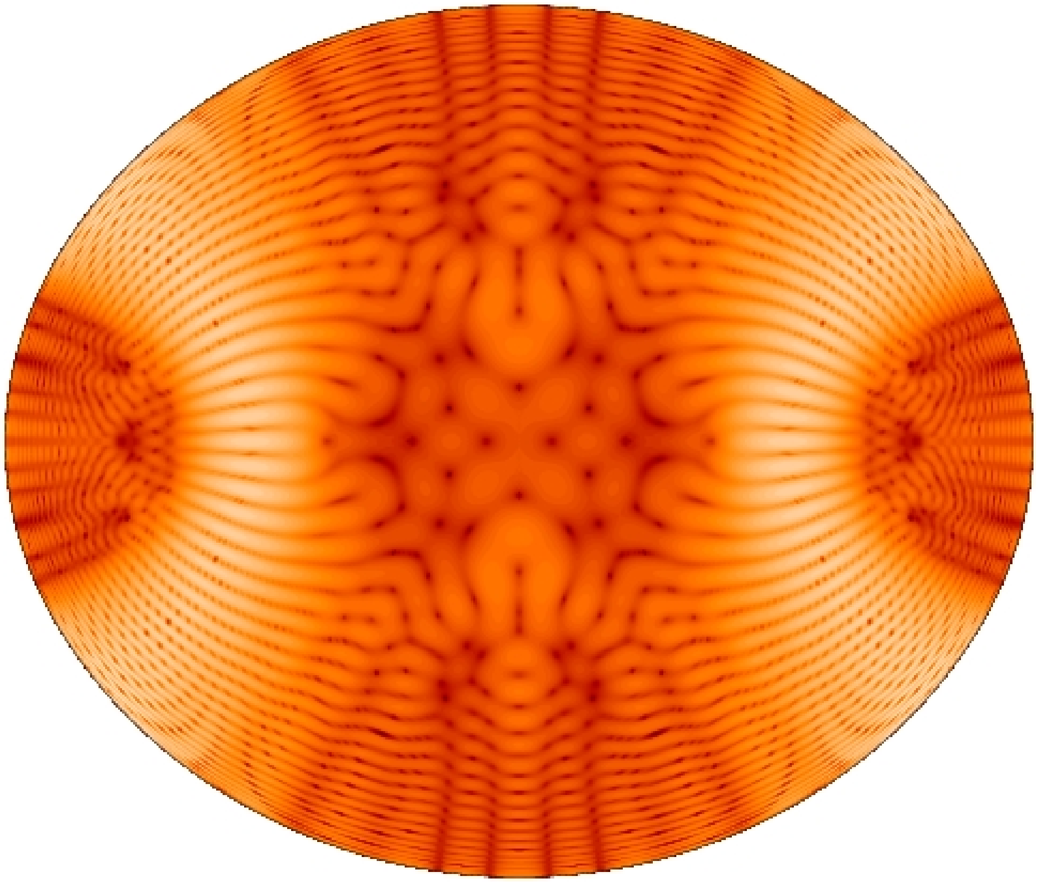
$$f_{n\,\ell\,m} \simeq \Delta_n n + \Delta_\ell \ell + \Delta_m |m| + \alpha^{\pm}$$

(see Lignières et al., 2006, and Reese, 2006)

Avoided crossings



Mode identification



$n = ? \quad \ell = ?$

Outline

1. The effects of stellar rotation
2. **The effects of stellar magnetism**
3. Conclusion

roAp stars

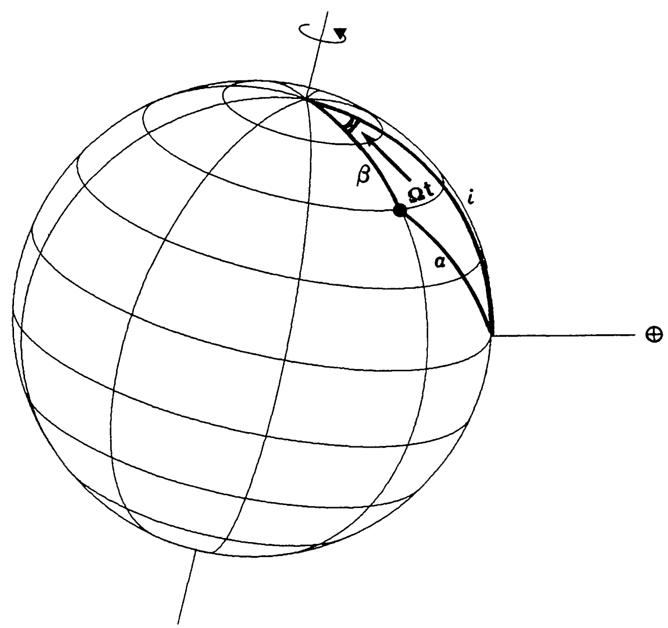
✘ Discovered by Kurtz in 1978

✘ Characteristics :

- peculiar chemical composition,
- strong dipolar magnetic field,

✘ Pulsation modes :

- luminosity variations with periods ranging from 5 to 15 min.
- well described by the oblique pulsator model (*e.g.* Kurtz, 1990)



Magnetism and oscillations

A few references :

- ✘ Roberts & Soward (1983), Campbell & Papaloizou (1986)
- ✘ Dziembowski & Goode (1996), Bigot et al. (2000), Bigot & Dziembowski (2002)
- ✘ Cunha & Gough (2000), Cunha (2006)
- ✘ Balmforth et al. (2001), Théado et al. (2005)
- ✘ Rincon & Rieutord (2003), Reese et al. (2004)
- ✘ Saio & Gautschy (2004), Saio (2005)

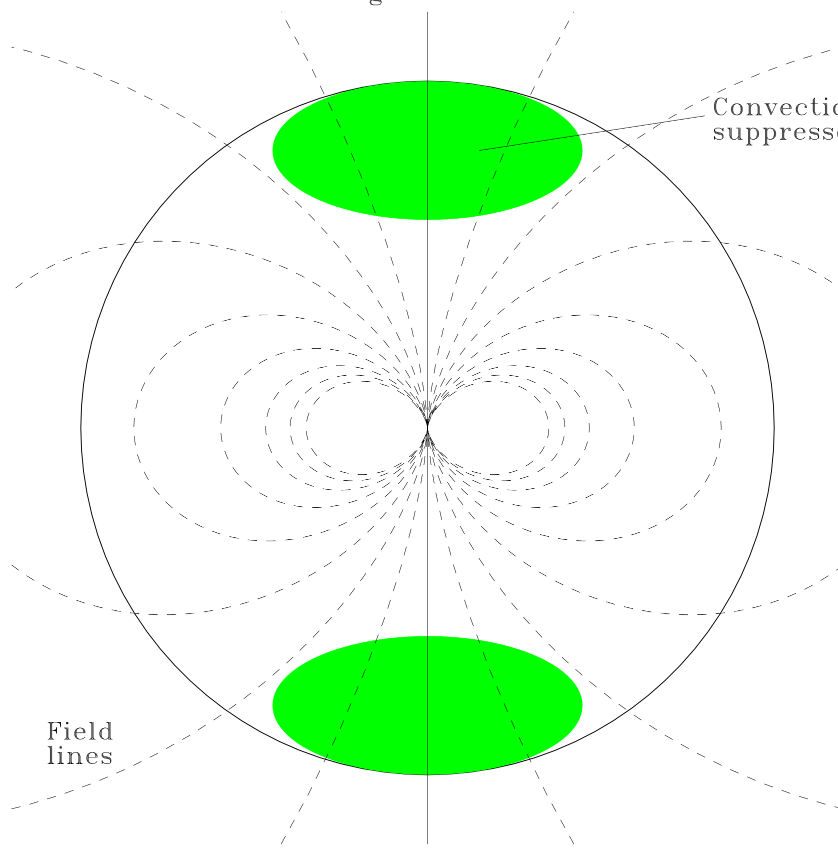
Effects of magnetism :

- ✘ suppression of convection near magnetic poles $\longrightarrow$ diffusion
- ✘ cyclic behaviour of frequency shifts
- ✘ self-similar structure in frequency spectrum
- ✘ magnetic shear layers
- ✘ magnetic oscillations and different frequency spectrum structure

Magnetism, convection and diffusion

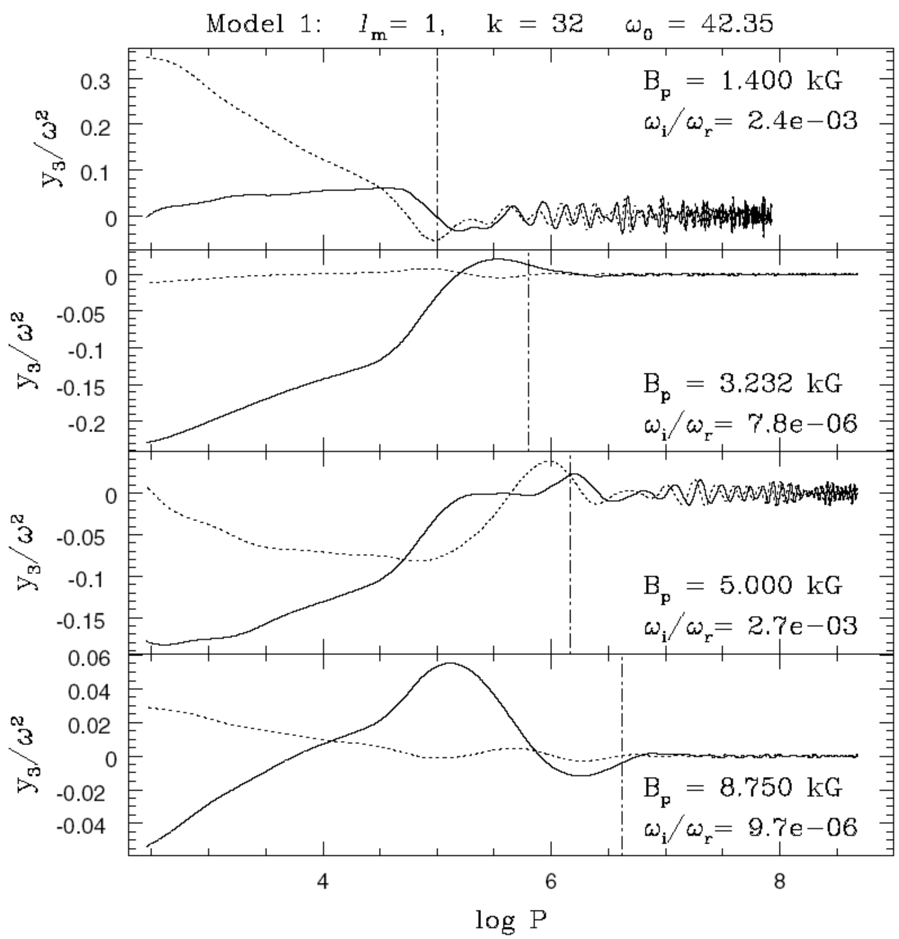
Balmforth et al. (2001) :

- ✘ convection suppressed in polar regions due to vertical $\mathbf{B}$
- ✘ chemical diffusion in polar regions
- ✘ enable κ mechanism in the hydrogen ionisation zone operating in polar regions



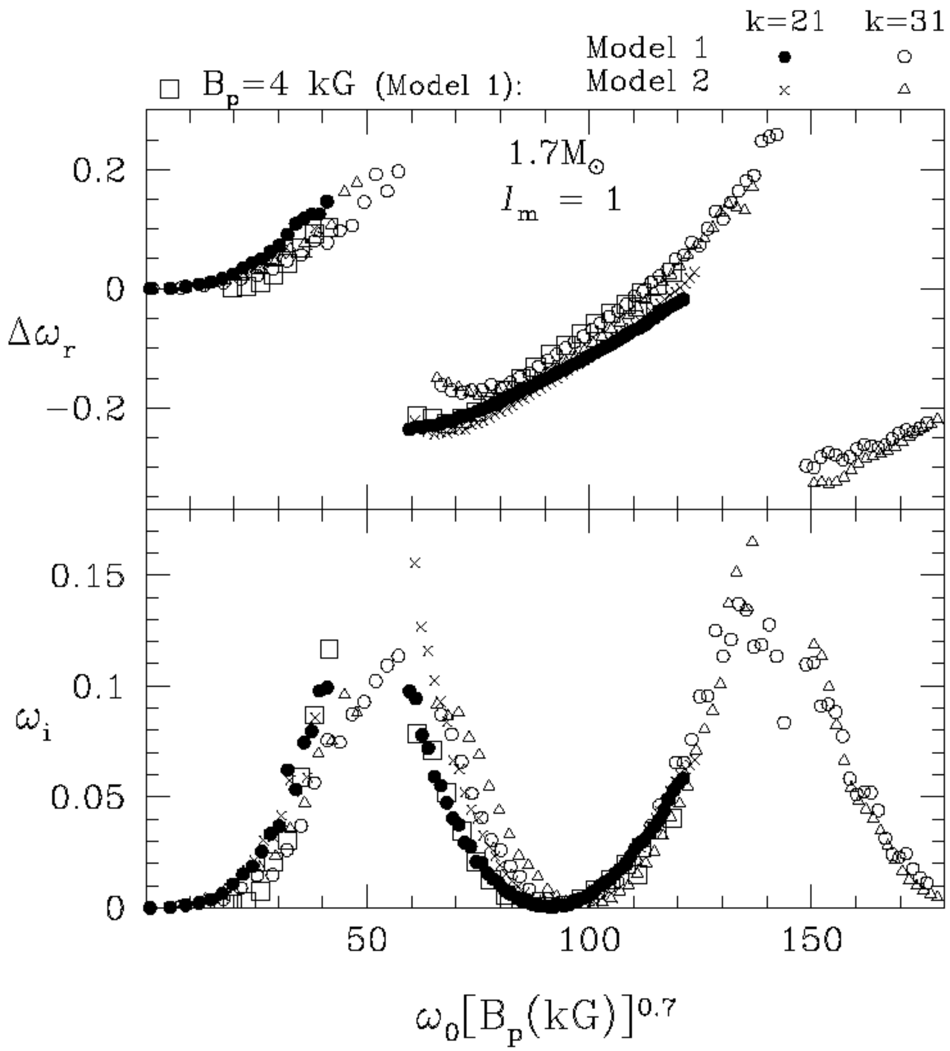
Trapping of magnetic waves

- ✘ coupling of acoustic and magnetic waves in outer region, and decoupling below $v_A \approx c$
- ✘ dissipation of slow magnetic waves below
- ✘ high damping rate when wave has an antinode near $v_A \approx c$
- ✘ low damping rate when wave has a node near $v_A \approx c$



Saio & Gautschy (2004), see also Cunha & Gough (2000)

Self similarity of frequency shifts



Saio (2004)

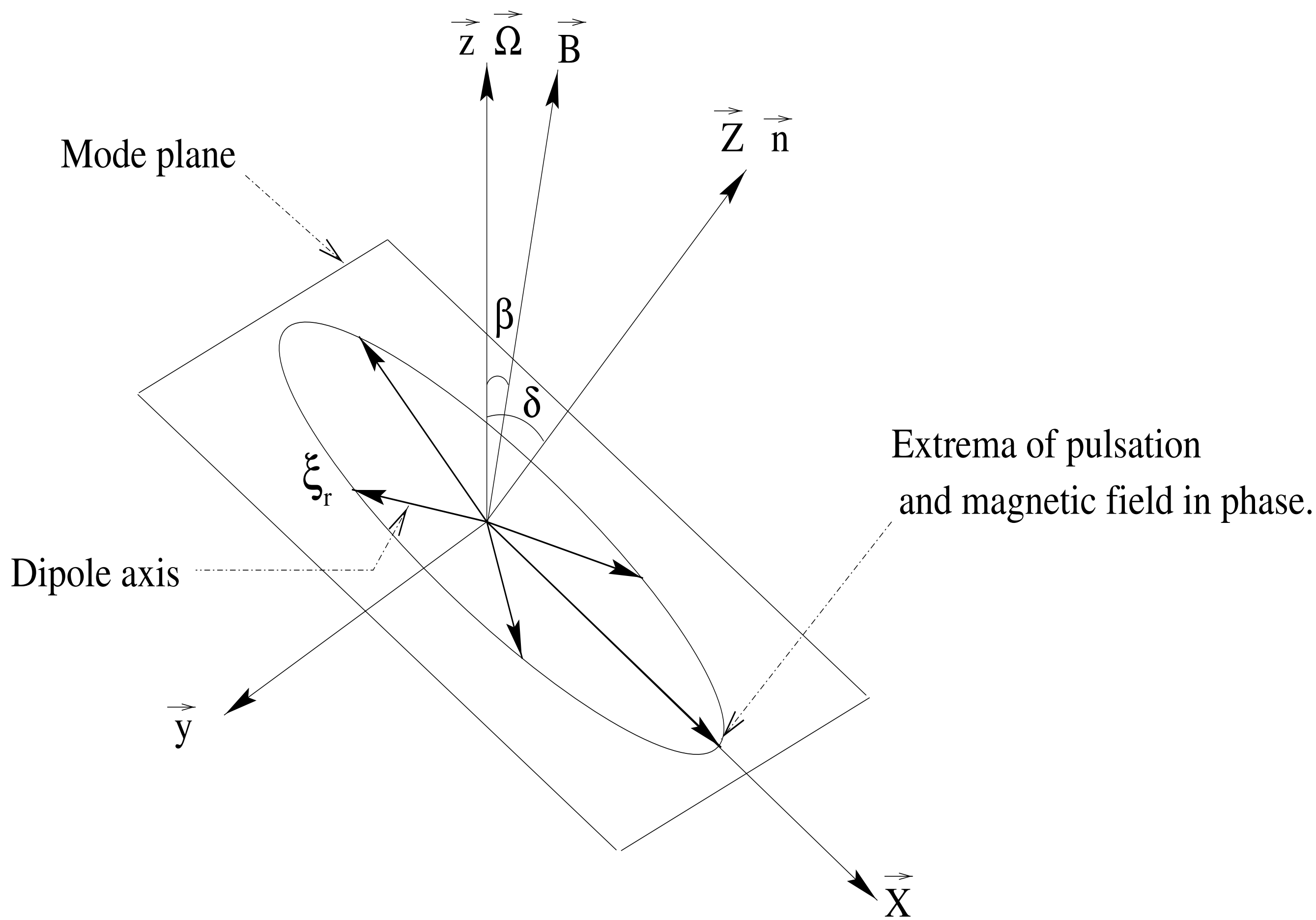
$$\Delta\omega = f(\omega_0 B_p^\alpha)$$

(*e.g.* Cunha & Gough, 2000, Saio & Gautschy, 2004)

$\alpha = 1/(1+N)$ for polytropes

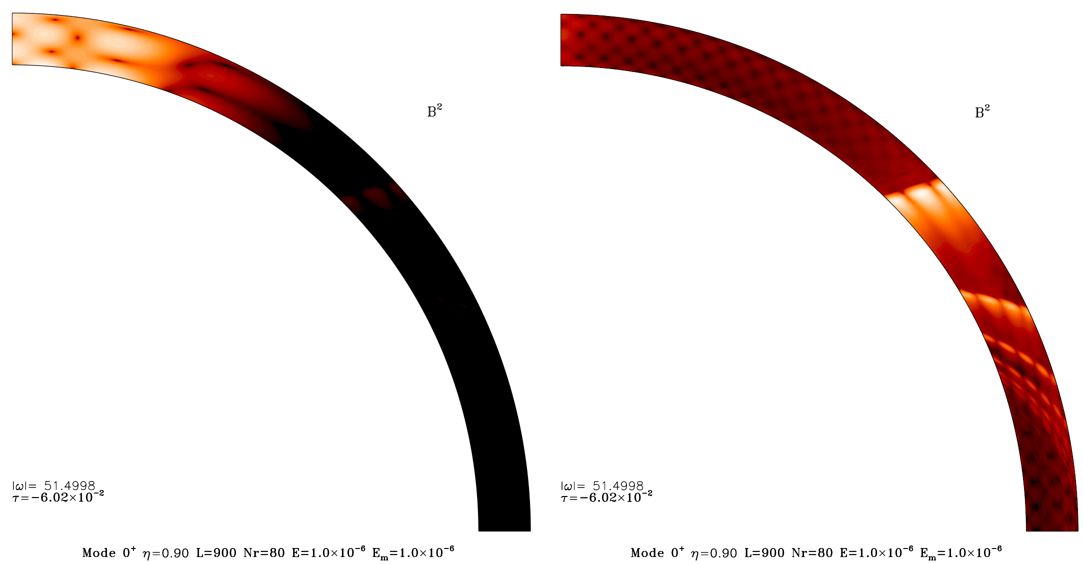
Axis of pulsation

Bigot & Dziembowski (2002) predict that the pulsation axis is located somewhere between the magnetic axis and the rotation axis.



Magnetic shear layers

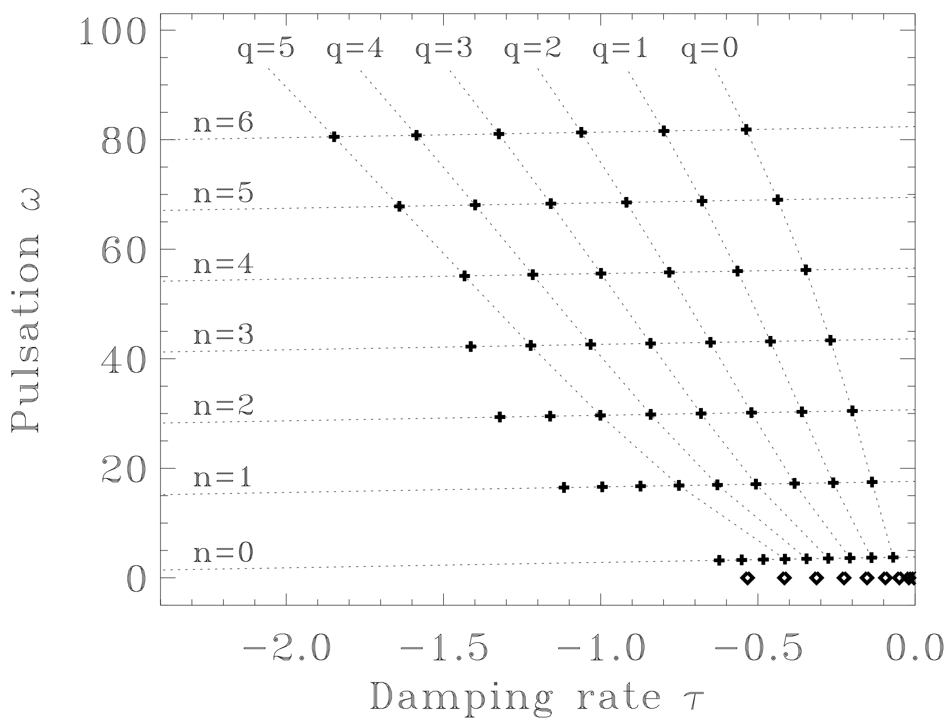
- ✘ include viscosity and/or magnetic diffusivity
- ✘ magnetic shear layers
- ✘ may intervene in mode selection



(Rincon & Rieutord, 2003)

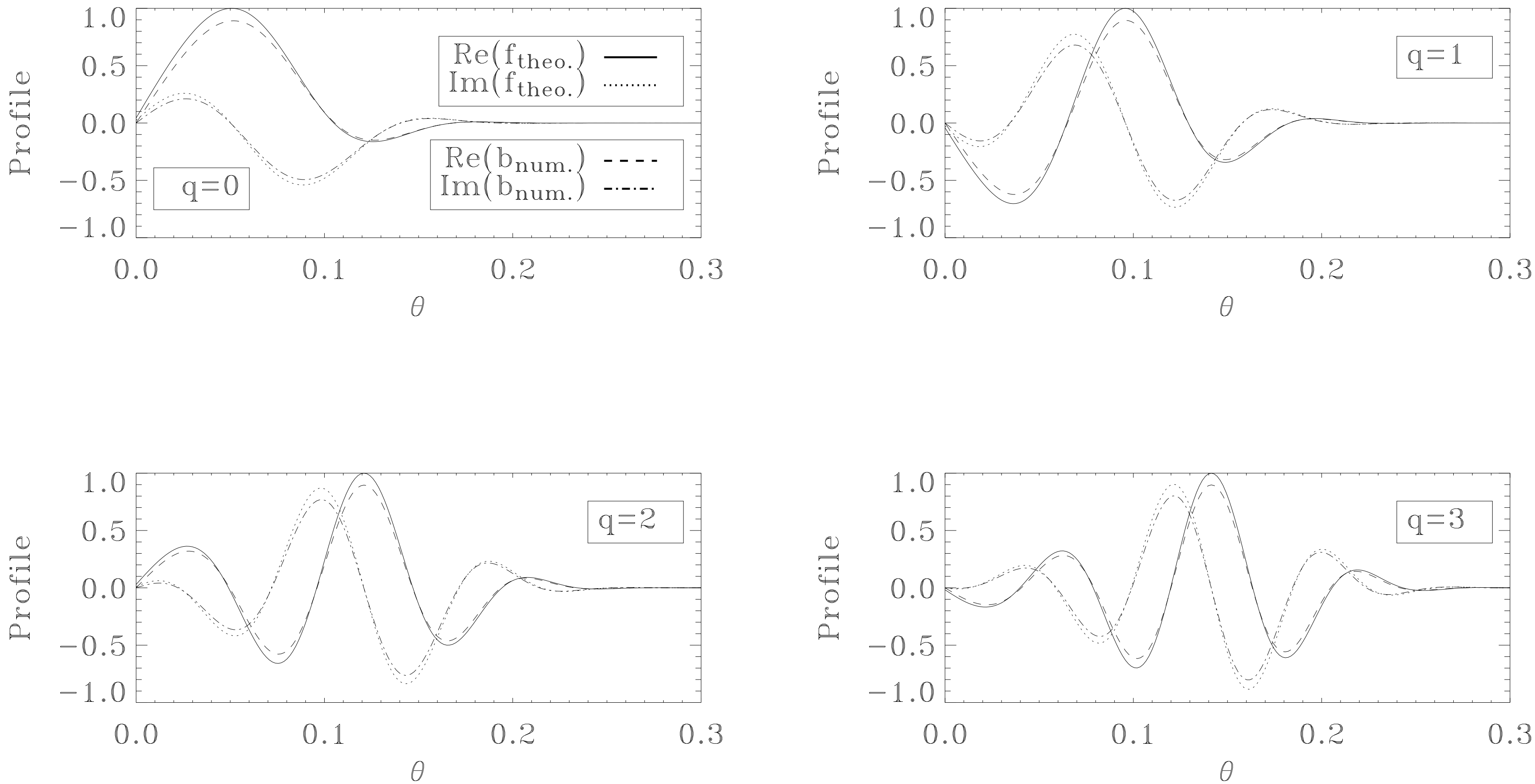
Alfvén waves

- ✘ different frequency spectrum
- ✘ different structure to pulsation modes
- ✘ certain types become singular in the ideal (inviscid) limit



(Reese et al., 2004)

Latitudinal structure and quantification



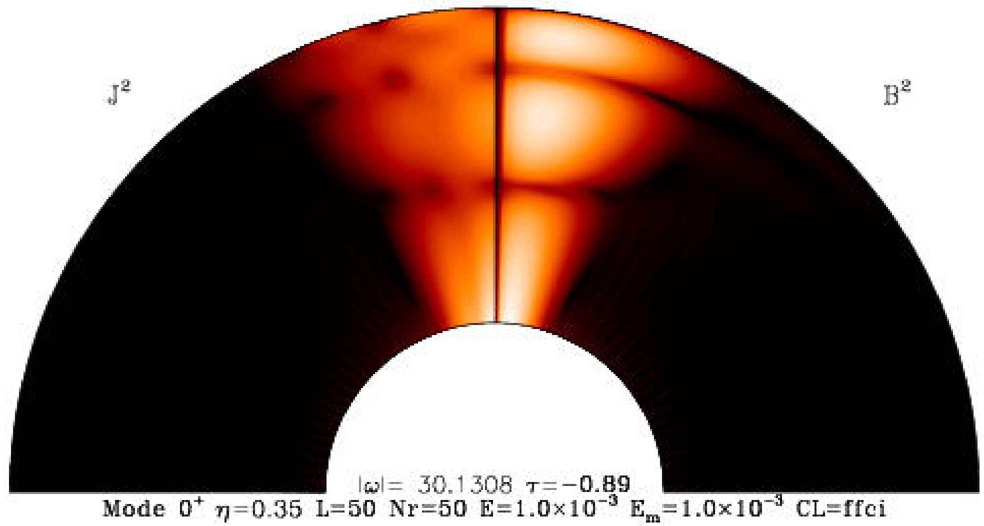
Outline

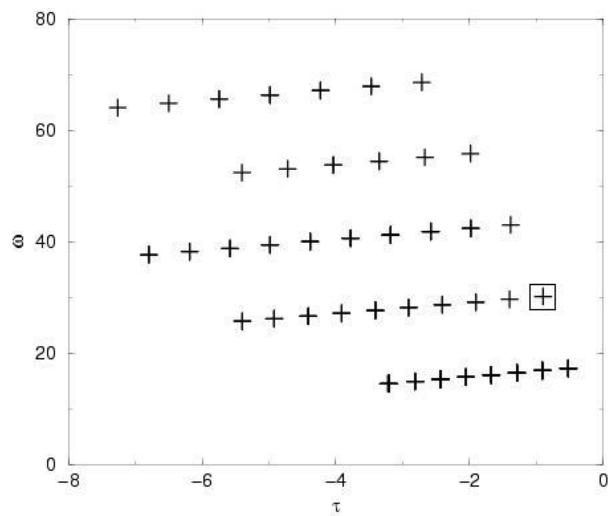
1. The effects of stellar rotation
2. The effects of stellar magnetism
3. **Conclusion**

Conclusion

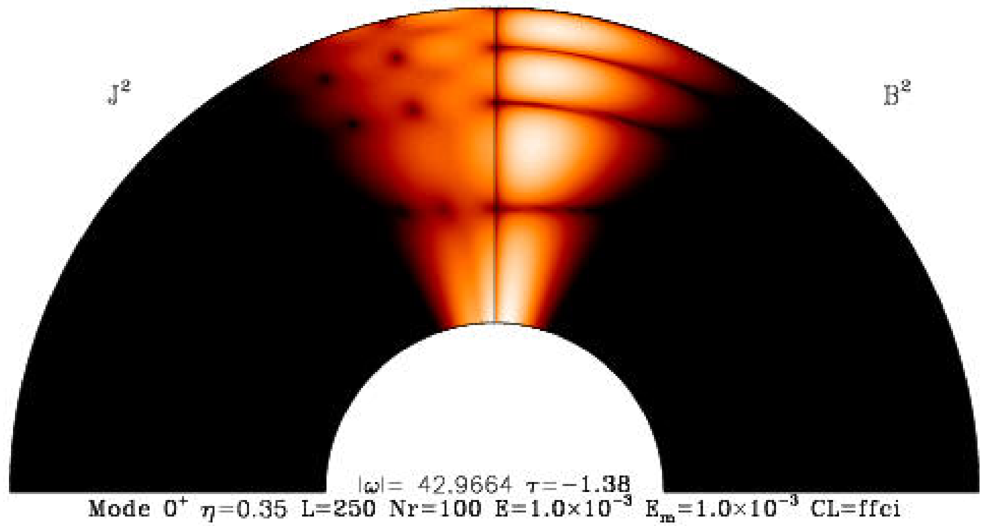
- ✘ stellar rotation and magnetism introduce many new phenomena
- ✘ increased difficulty for calculating pulsation modes
- ✘ need for powerful numerical and theoretical methods in order to interpret observed pulsations
- ✘ exciting prospects for stellar physics

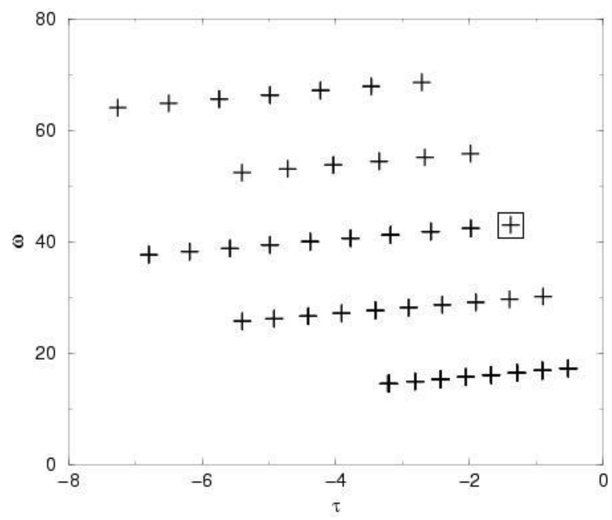
Radial structure



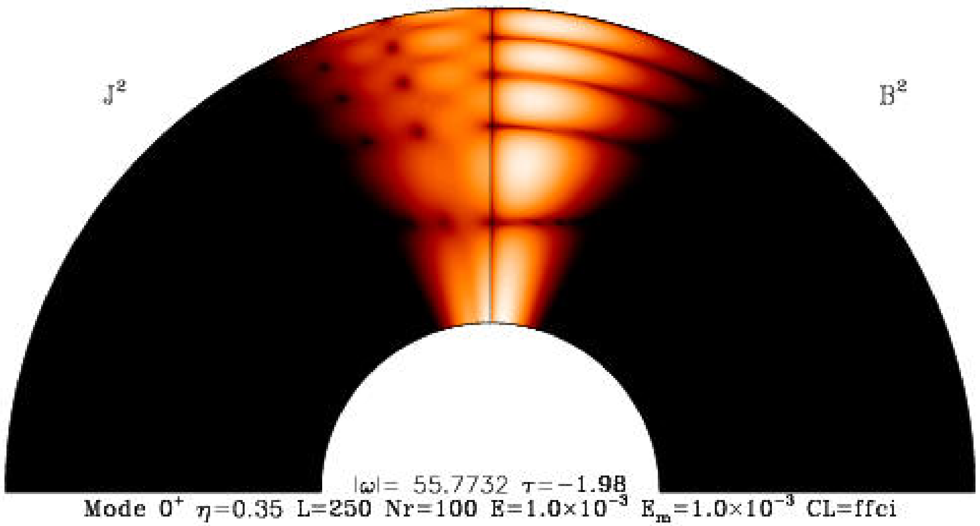


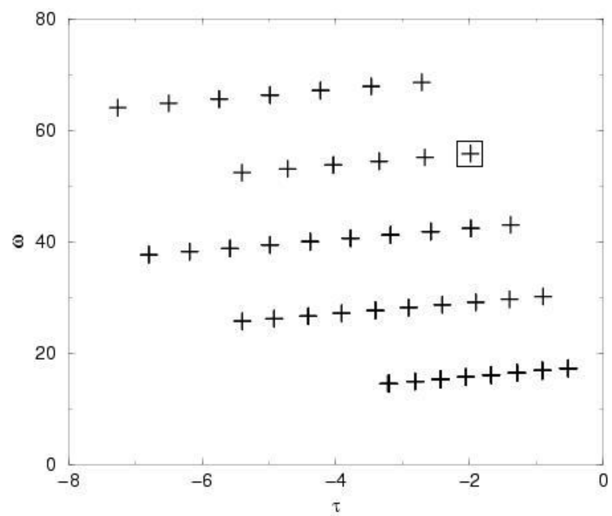
Radial structure



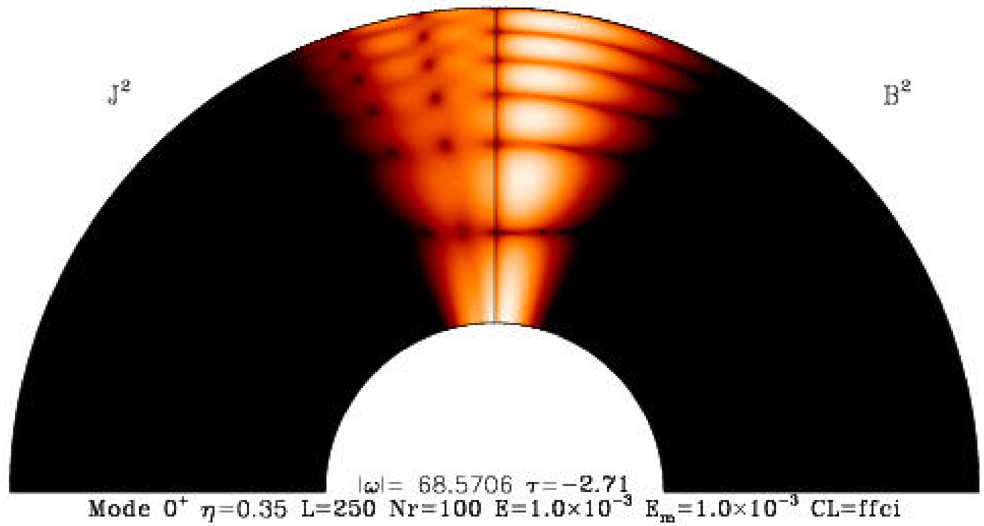


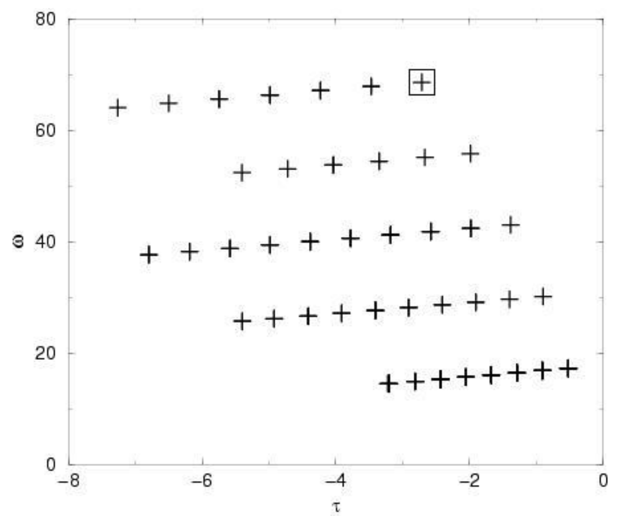
Radial structure



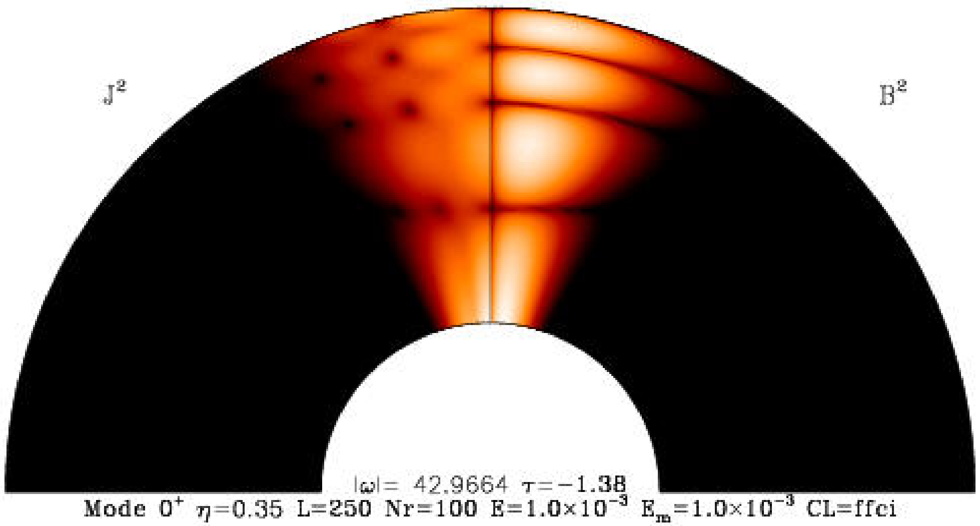


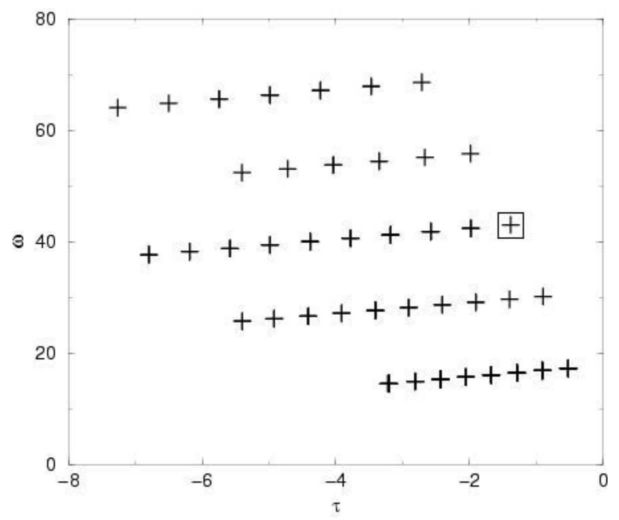
Radial structure





Latitudinal structure





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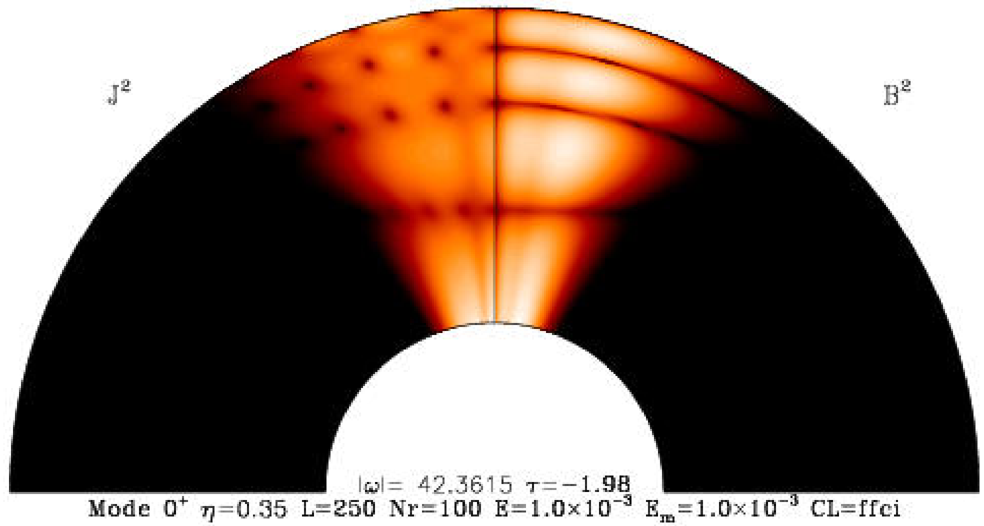
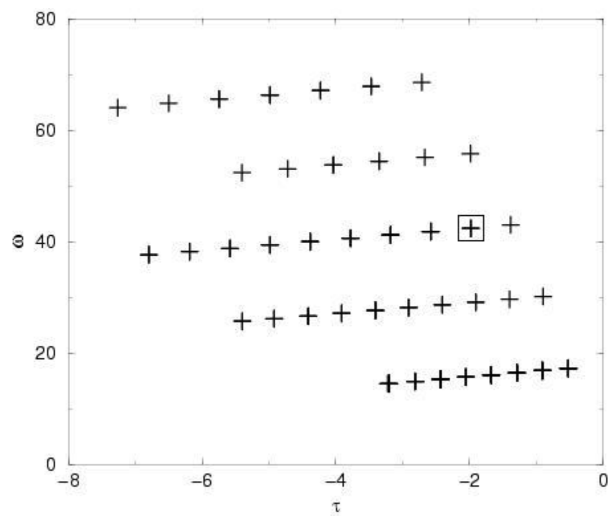
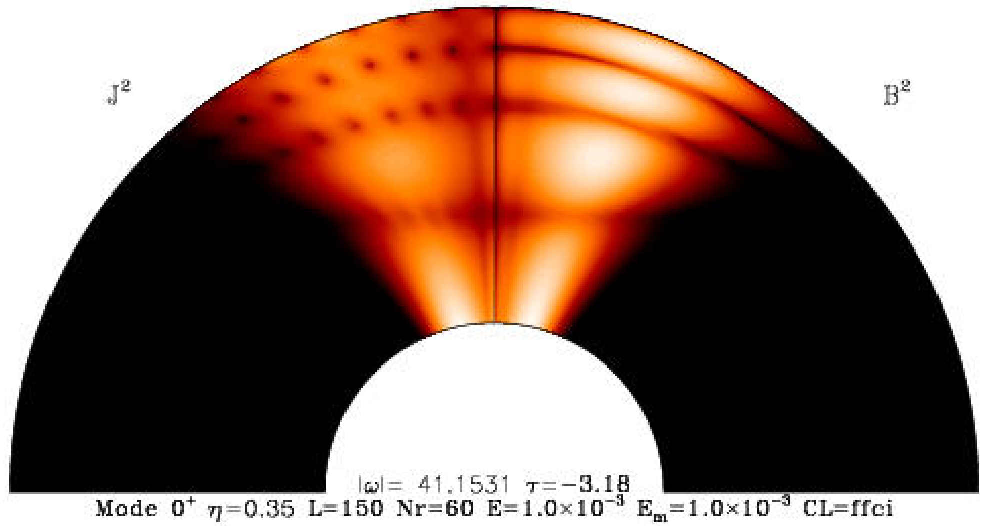
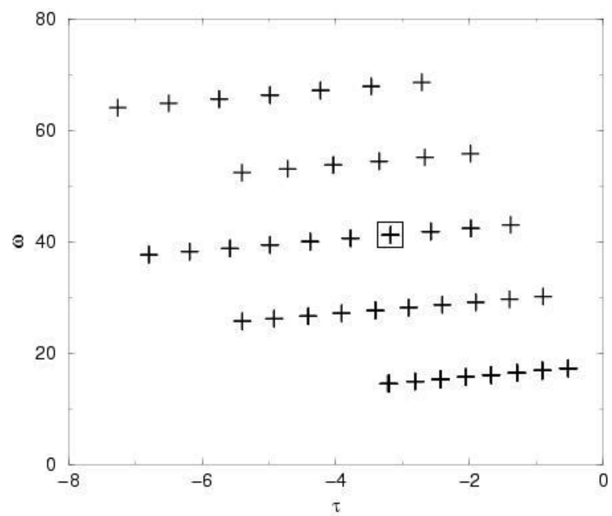


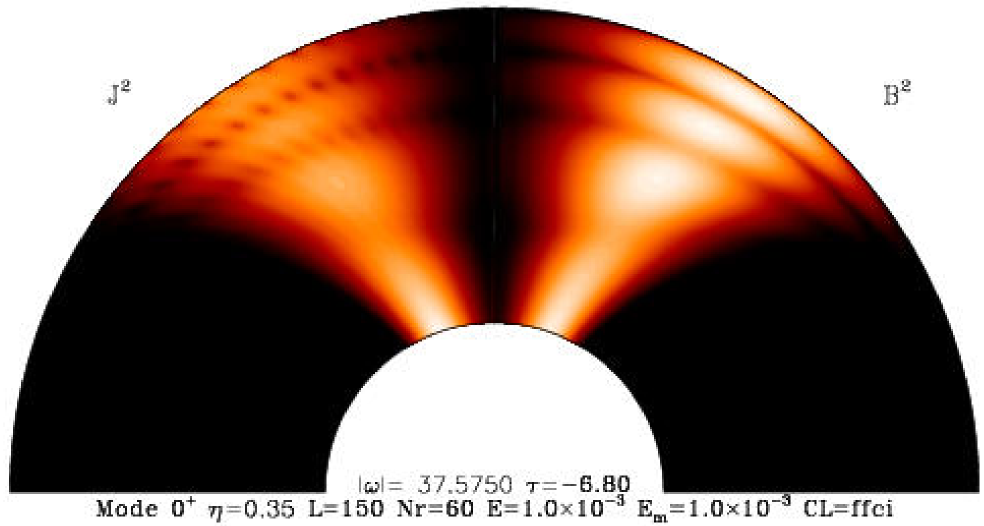


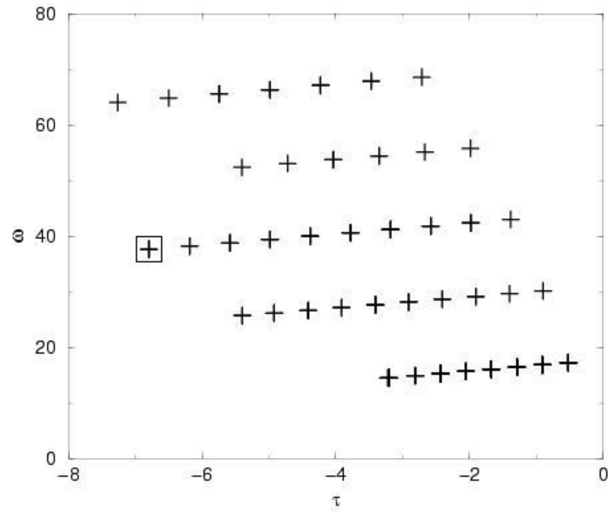
Latitudinal structure



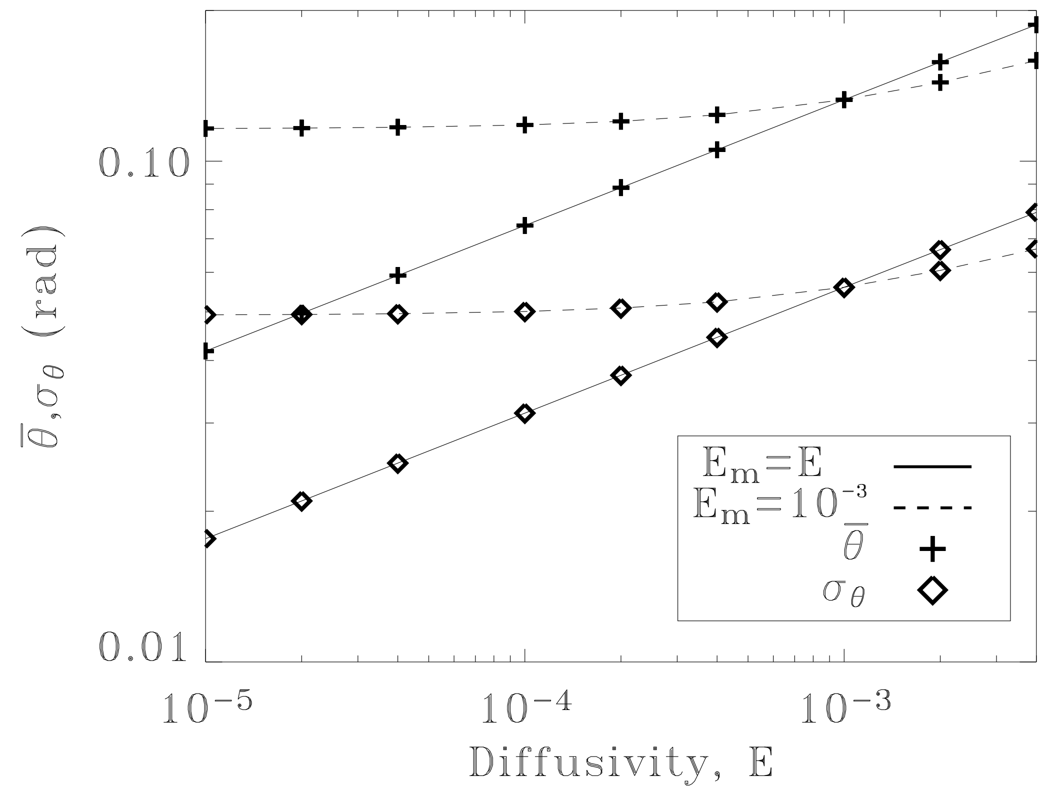


Latitudinal structure





Effects of viscosity and magnetic diffusivity



Empirical law (for $E = E_m$) :

$$\begin{array}{lcl} \text{position} & \propto & E^{1/4} \\ \text{thickness} & \propto & E^{1/4} \end{array}$$

Asymptotic formulas

- Analytical solutions for small diffusivities :

$$E = K\varepsilon, \quad E_\mathrm{m} = K_\mathrm{m}\varepsilon, \quad \varepsilon \to 0$$

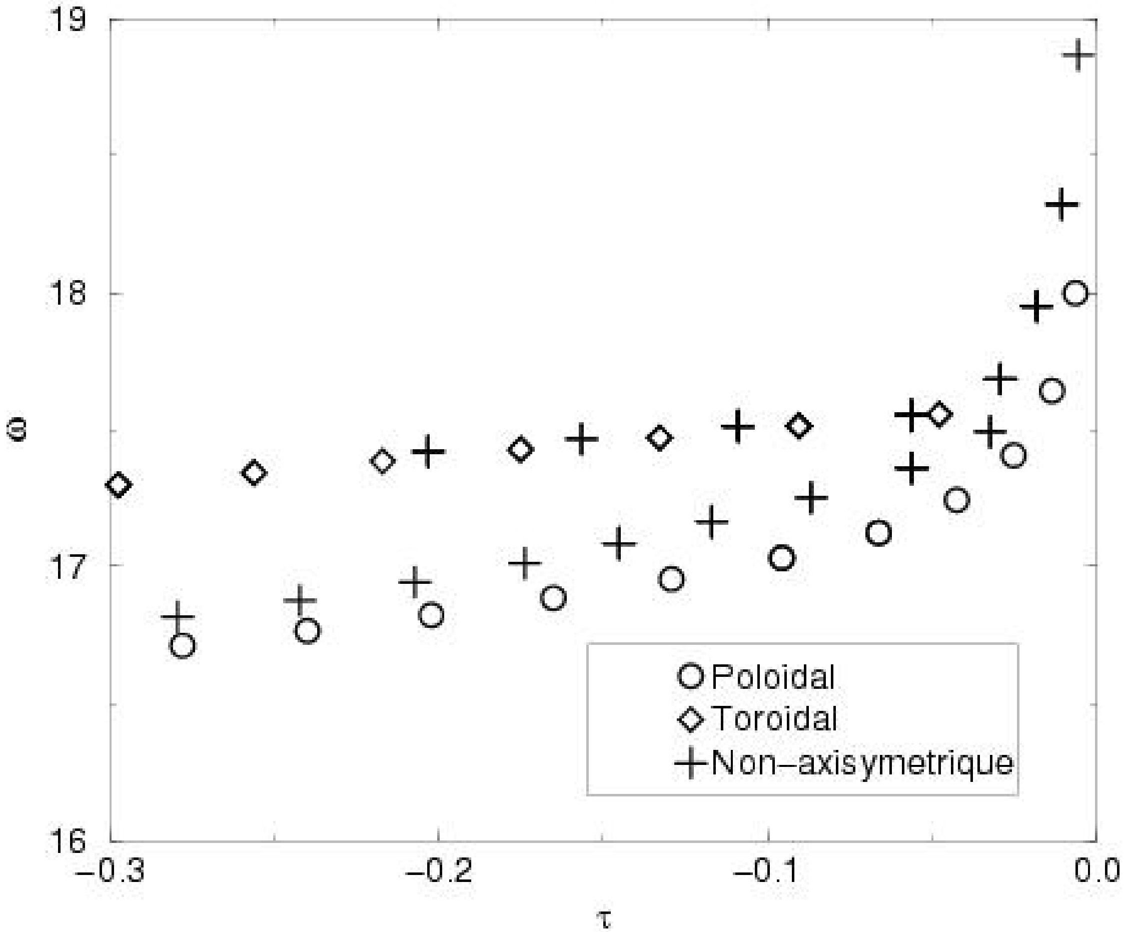
- Form of solutions :

$$\begin{aligned} b(r,\nu) &= b_n(r) f_{n,q}(\varepsilon^{-1/4}\nu) + \mathcal{O}(\varepsilon^{1/2}) \\ v(r,\nu) &= v_n(r) f_{n,q}(\varepsilon^{-1/4}\nu) + \mathcal{O}(\varepsilon^{1/2}) \\ \lambda_{n,q} &= \lambda_n^0 + \varepsilon^{1/2}\lambda_{n,q}^1 + \mathcal{O}(\varepsilon) \end{aligned}$$

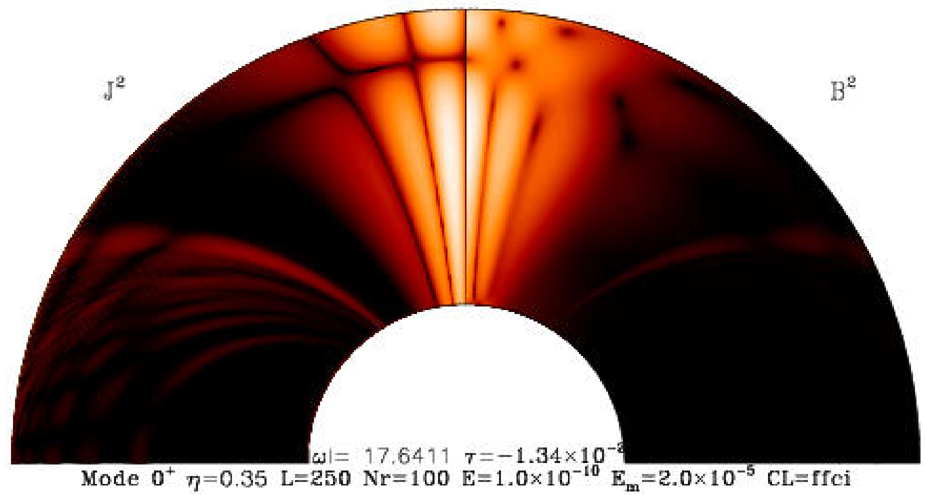
- Zeroth order : radial structure (b_n, v_n) and mode quantification (n)
- Next order : latitudinal structure $(f_{n,q})$ and mode quantification (q)
  - use of adjoint system to obtain $f_{n,q}$

Non-axisymmetric modes

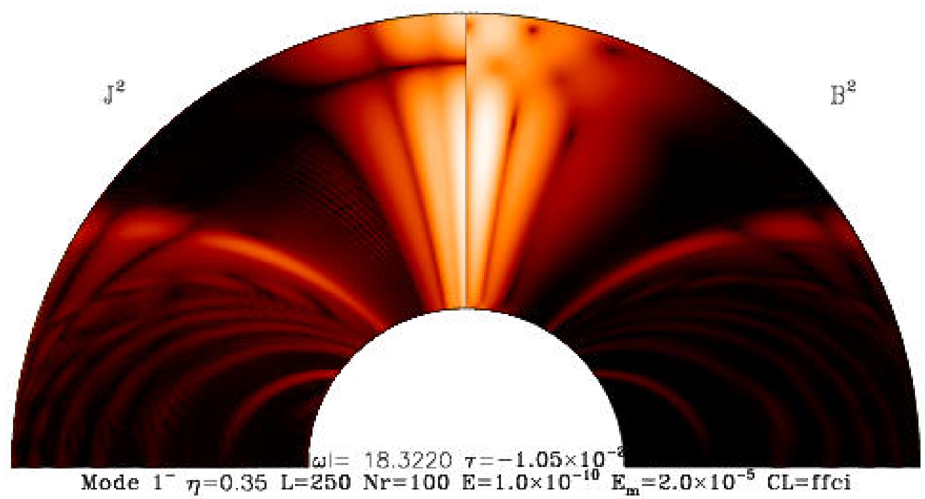
- poloidal and toroidal components are now coupled



Comparison with axisymmetric modes

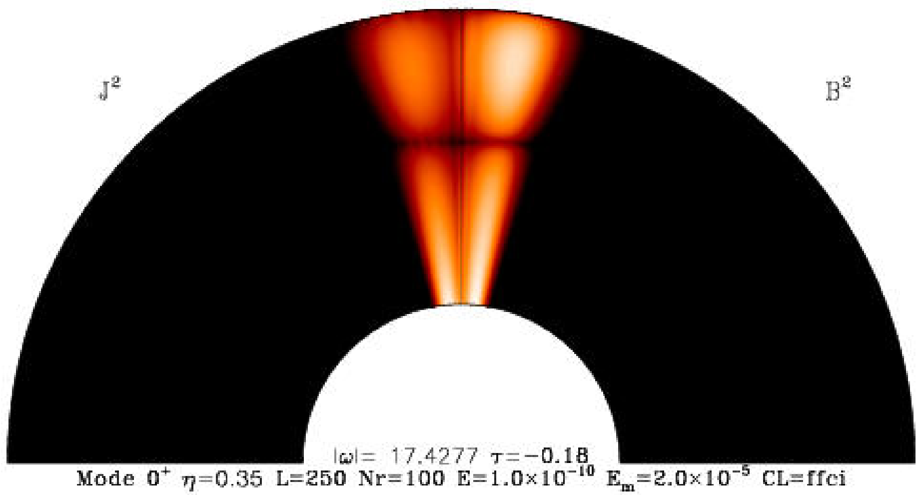


Poloidal

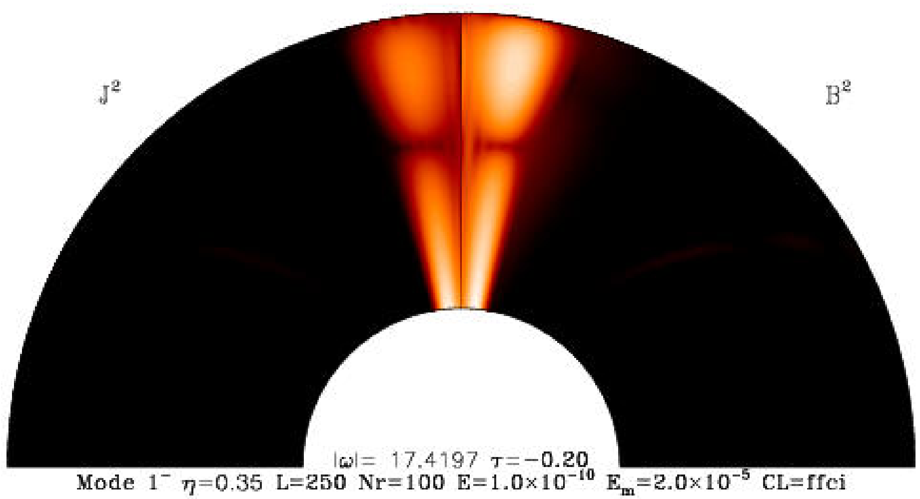


m = 1

Comparison with axisymmetric modes



Toroidal



$m = 1$

Conclusion

- Toroidal modes : singular
- Non-axisymmetric modes : poloidal or toroidal characteristics
- Prospects
  - study of magneto-acoustic waves
  - study of magneto-inertial waves
  - understanding/constraining the interior of planets such as Jupiter

Asymptotic developments

- Change of variables $(r, \nu = \frac{\sin\theta}{\sqrt{r}}, \varphi)$
- Scale change $\mathsf{E}^{1/4} = (K\,\varepsilon)^{1/4}$ and $\mathsf{E}_{\mathrm{m}}^{1/4} = (K_{\mathrm{m}}\,\varepsilon)^{1/4}$ where $\varepsilon \to 0$

$$\begin{aligned}
\lambda b &= \left(1 - \frac{1}{2}\varepsilon^{1/2} r \hat{\nu}^2\right)\left[\frac{1}{r^3}\frac{\partial v}{\partial r} - \frac{3v}{2r^4}\right] + \frac{\varepsilon^{1/2} K_{\mathrm{m}}}{r^3}\Theta[b] \\
\lambda v &= \left(1 - \frac{1}{2}\varepsilon^{1/2} r \hat{\nu}^2\right)\left[\frac{1}{r^3}\frac{\partial b}{\partial r} + \frac{3b}{2r^4}\right] + \frac{\varepsilon^{1/2} K}{r^3}\Theta[v]
\end{aligned}$$

where $\Theta[b] = \dfrac{\partial^2 b}{\partial \hat{\nu}^2} + \dfrac{1}{\hat{\nu}}\dfrac{\partial b}{\partial \hat{\nu}} - \dfrac{b}{\hat{\nu}^2}$

Asymptotic developments

- at zeroth order, we have :

$$\begin{aligned} \lambda^0 b &= \frac{1}{r^3}\frac{\partial v}{\partial r} - \frac{3}{2r^4}v, \\ \lambda^0 v &= \frac{1}{r^3}\frac{\partial b}{\partial r} + \frac{3}{2r^4}b, \\ v(\eta) &= 0, \quad b(1) = 0. \end{aligned} \Rightarrow \left\{ \begin{aligned} b(r,\nu) &= b_n(r) f(\hat{\nu}) + \mathcal{O}(\varepsilon^{1/2}) \\ v(r,\nu) &= v_n(r) f(\hat{\nu}) + \mathcal{O}(\varepsilon^{1/2}) \\ \lambda &= \lambda_n^0 + \mathcal{O}(\varepsilon^{1/2}) \end{aligned} \right.$$

- at next order, we get :

$$\begin{aligned} \lambda_n^0 b^1 - \frac{1}{r^3}\frac{\partial v^1}{\partial r} + \frac{3v^1}{2r^4} &= -\lambda^1 b_n^0 f - \frac{\lambda_n^0 r \hat{\nu}^2 b_n^0 f}{2} + \frac{b_n^0 \Theta[f]}{r^3}, \\ \lambda_n^0 v^1 - \frac{1}{r^3}\frac{\partial b^1}{\partial r} - \frac{3b^1}{2r^4} &= -\lambda^1 v_n^0 f - \frac{\lambda_n^0 r \hat{\nu}^2 v_n^0 f}{2} + \frac{v_n^0 \Theta[f]}{r^3}, \\ b^1(r=1,\hat{\nu}) &= 0, \quad v^1(r=\eta,\hat{\nu}) = 0. \end{aligned}$$

This is of the form $\mathcal{L}_0\Psi_1 = \mathcal{L}_1\Psi_0$: solution of adjoint problem

$$\Rightarrow \left\{ \begin{aligned} f(\hat{\nu}) &= f_{n,q}(\hat{\nu}) \\ \lambda &= \lambda_n^0 + \varepsilon^{1/2}\lambda_n^1 + \mathcal{O}(\varepsilon^{1/2}) \end{aligned} \right.$$