Report on Nice Oscillation Code

J. Provost and G. Berthomieu
OCA, Dépt. Cassiopée, UMR CNRS 6202

Plan

1. Presentation and properties of NOC
   - ⋆ Physics
   - ⋆ Numerics
   - ⋆ Strategy

2. Lagrangian / Eulerian code

3. NOC Internal accuracy tests
Applications to Sun, cases Task2/Step1 & Step1b
   - ⋆ Accuracy
   - ⋆ Internal consistency
   - ⋆ Sensitivity to mesh distribution

4. Conclusions

1. NOC Properties : $\star$ Physics

• linear adiabatic oscillations
(non adiabatic with frozen convective flux)

• NOC1 : Eulerian code
$\star$ system of 4 linear first order differential equations with variables (cf Unno et al. 1989):

$$y1 = \frac{\xi_r}{r}, y2 = \frac{(p'/\rho + \phi')}{gr}, y3 = \frac{\phi'}{gr}, y4 = \frac{1}{g}\frac{d\phi'}{dr}$$

$\star$ Eigenvalue problem with 4 boundary conditions:
at center $C1\omega^2 y1 = \ell y2$
$\ell y3 = y4$
at surface – $(\ell + 1)y3 + y4 = 0$
– mechanical condition:
either fit with isothermal atmosphere
either $\delta p = 0$ $(y1 - y2 + y3 = 0)$

$\star$ Coefficients depend on model through:
$\mathcal{N}$ (Brunt-Väissälä frequency),
U, V, g and Γ_1
$(\mathcal{N}^2 = gA/r$, $A = \frac{1}{\Gamma_1}\frac{dlogp}{dlogr} - \frac{dlog\rho}{dlogr}$, $V = \frac{dlogp}{dlogr}$, $U = \frac{dlogM_r}{dlogr})$

⋆ **Physics**

- NOC2 Lagrangian code

$$\star \quad \tilde{y}1 = \frac{\xi_r}{r}, \tilde{y}2 = \frac{\delta p}{p}, \tilde{y}3 = \frac{\phi'}{gr}, \tilde{y}4 = \frac{1}{g}\frac{d\phi'}{dr} + U\frac{\xi_r}{r}$$

⋆ Boundary conditions: derived fom NOC1

⋆ Coefficients: same as NOC1,
EXCEPT $\mathcal{N}$ Brunt-Väissälä

- When to use NOC2 ?

  * discontinuities of ρ and v_{sound}
  (Jupiter's solid core)

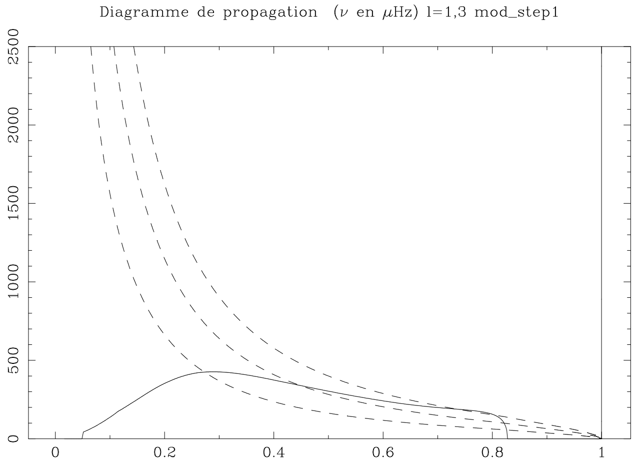
  * rapid variations of ρ and v_{sound}
  (frontier of convective core)

  * difficulties related to $\mathcal{N}$

Why we prefer to use NOC2?

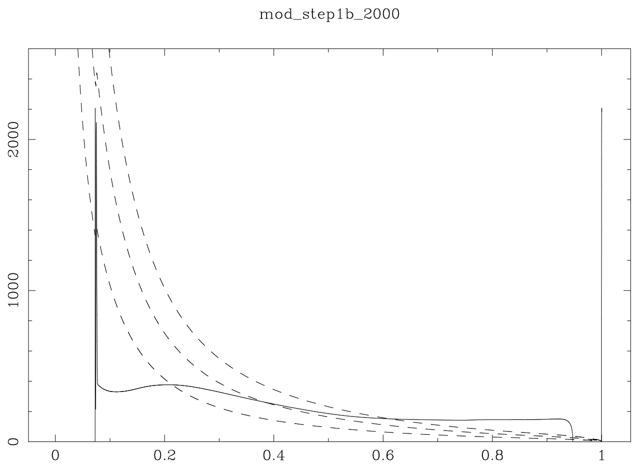
•Propagation diagram - case Task2 - step1

$1.2M_\odot$, X_c=0.69 $r_{cc} \sim 0.05R_\star$ $r_{ZC} \sim 0.83R_\star$

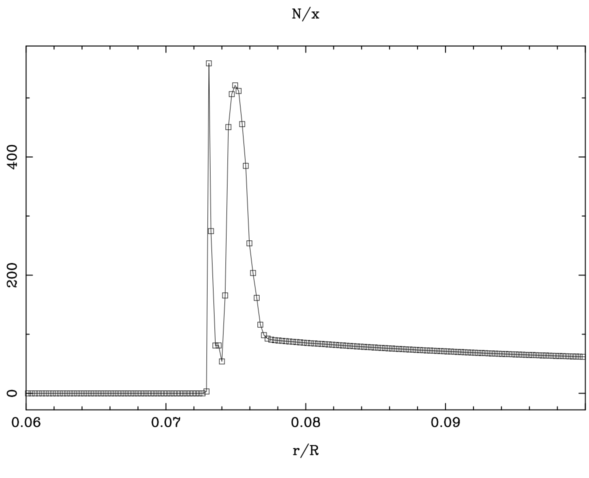


• case Task2 - step1b

$1.5M_\odot$, X_c=0.40 $r_{cc} \sim 0.07R_\star$ $r_{ZC} \sim 0.95R_\star$ (N$\sim$ 2000)



$\star\, \mathcal{N}/r$



1. NOC properties: ⋆ Numerics

(cf Unno et al. 1989)

- Method to solve eigenvalue problem

–discretization of the four differential equations and four boundary conditions at N mesh points for the independent variable r

–solve the system setting aside the surface mechanical condition (for ex $\delta p = 0$) with an arbitrary eigenvalue and a normalisation of ξ_r at the surface

–search for $\delta p_{surface}$ small enough for two such arbitrary values

–use a relaxation method to converge on eigenvalue and eigenfunctions

- Richardson extrapolation (Shibahashi & Osaki 1981)

Difference scheme of second order induces truncation errors in eigenfrequency and eigenfunctions in N^{-2}:

$$\nu_{Ri}^2(N) = \frac{1}{3}\left(4\nu^2(N) - \nu^2(N/2)\right)$$

1. NOC properties: ⋆ Strategy

Perform various internal checks of accuracy:

- Effects of number N and distribution of mesh points?

  – accuracy : $\delta\nu_N = \nu_{Ri}(N) - \nu_{Ri}(N/2)$
  – eventually $\delta\nu_{2N}$ using interpolation

- Internal consistency?

  – Comparison of ν and ν^{var}:

$$(\nu^{var})^2 = \frac{\int \xi^* \mathcal{L}(\xi) \rho dv}{\int \xi^* \xi \rho dv}$$

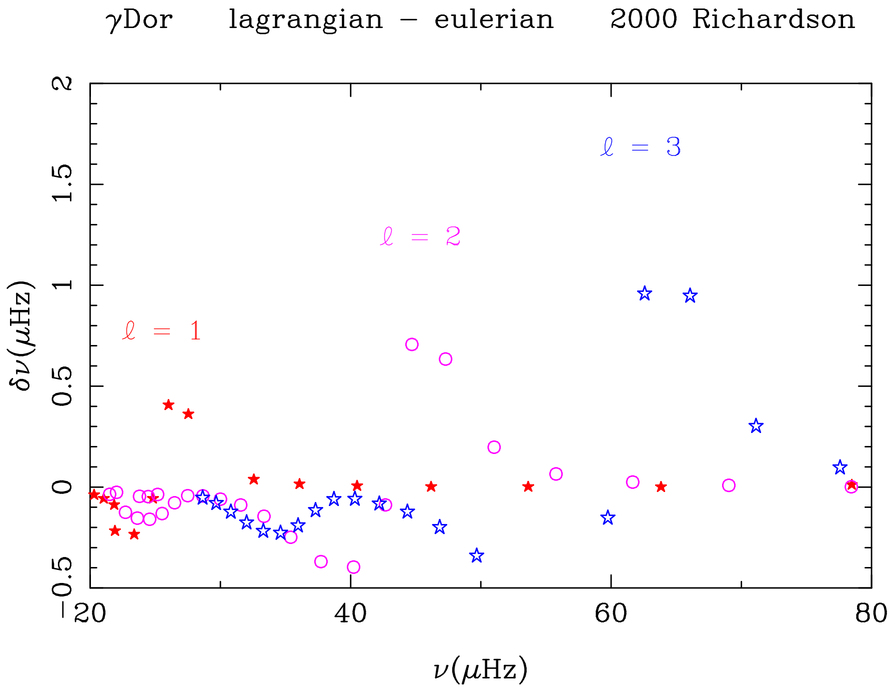
  – Check of computation for both model and oscillations : $\nu - \nu^{var}$

2. **Lagrangian/ Eulerian codes**

NOC2 - NOC1 : Task2 - Step1b – N$\sim$ 2000

1.5M$_\odot$, X$_c$=0.40

γDor oscillations - 20 to 80 μHz

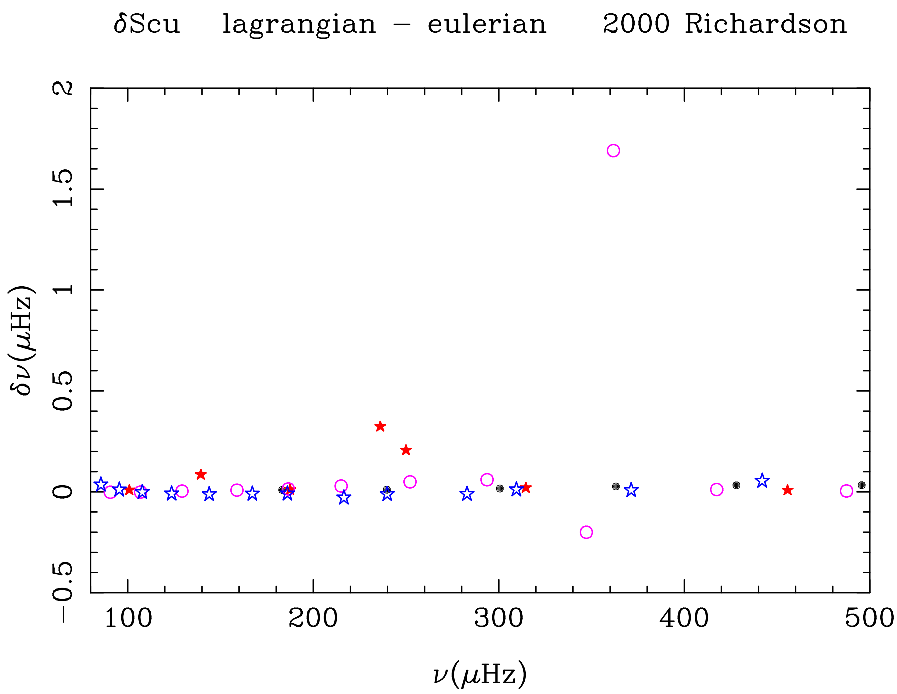


2. **Lagrangian/ Eulerian codes**

NOC2 - NOC1 : Task2 - Step1b – N∼ 2000

1.5$M_\odot$, X_c=0.40

δ Scuti oscillations – 80 to 500μHz

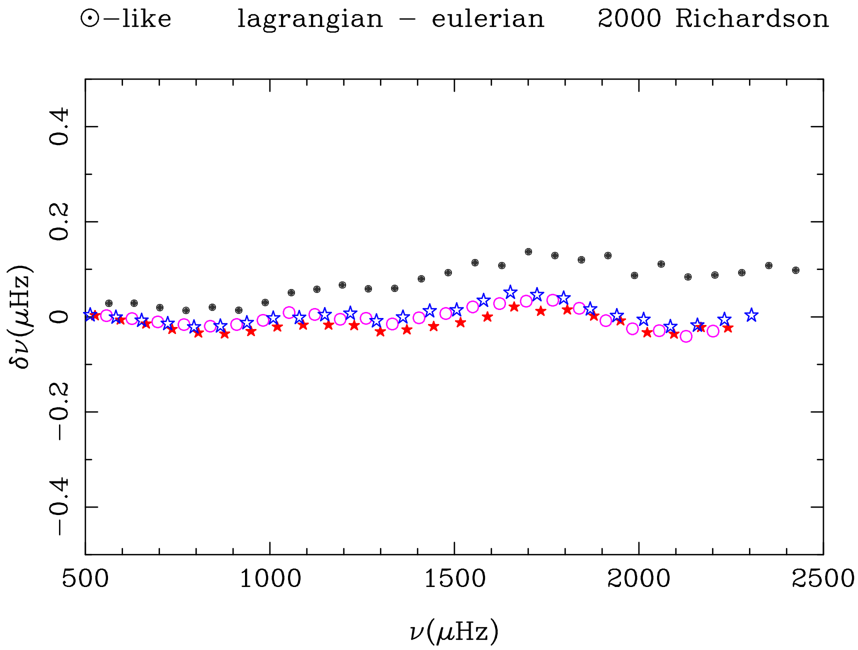


2. **Lagrangian/ Eulerian codes**

NOC2 - NOC1 : Task2 - Step1-b - N$\sim$ 2000

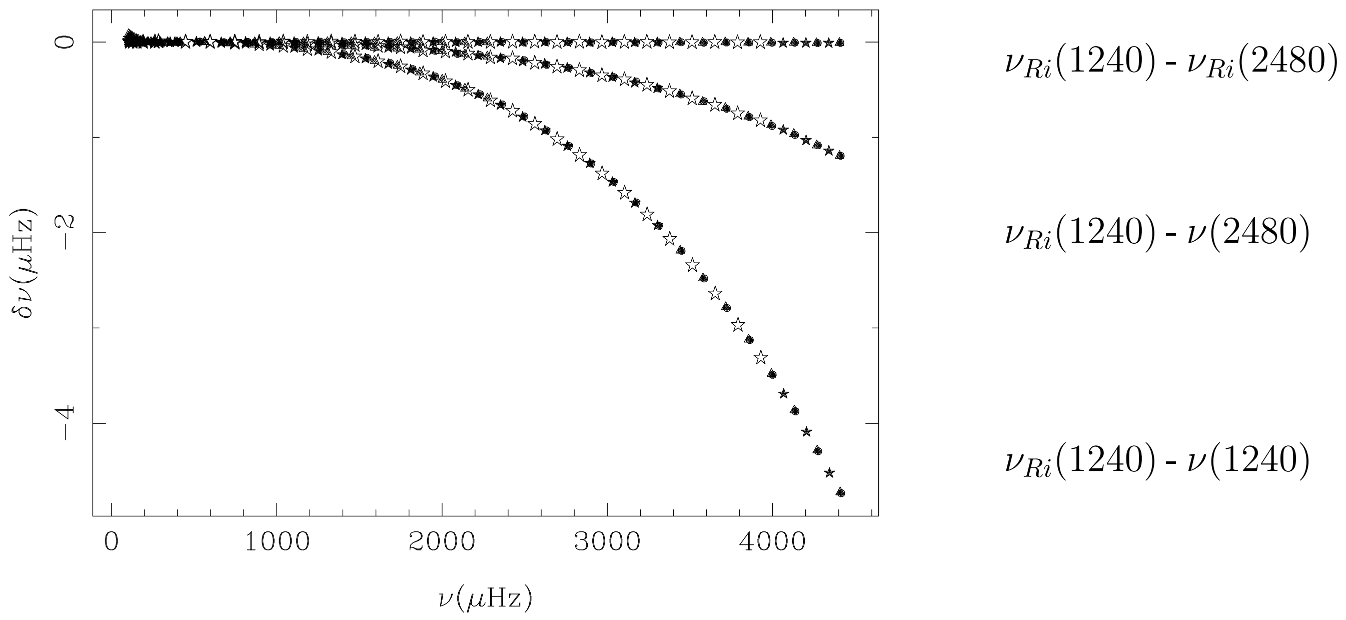
1.5$M_\odot$, X_c=0.40

solar-like oscillations – 500 to 2500μHz

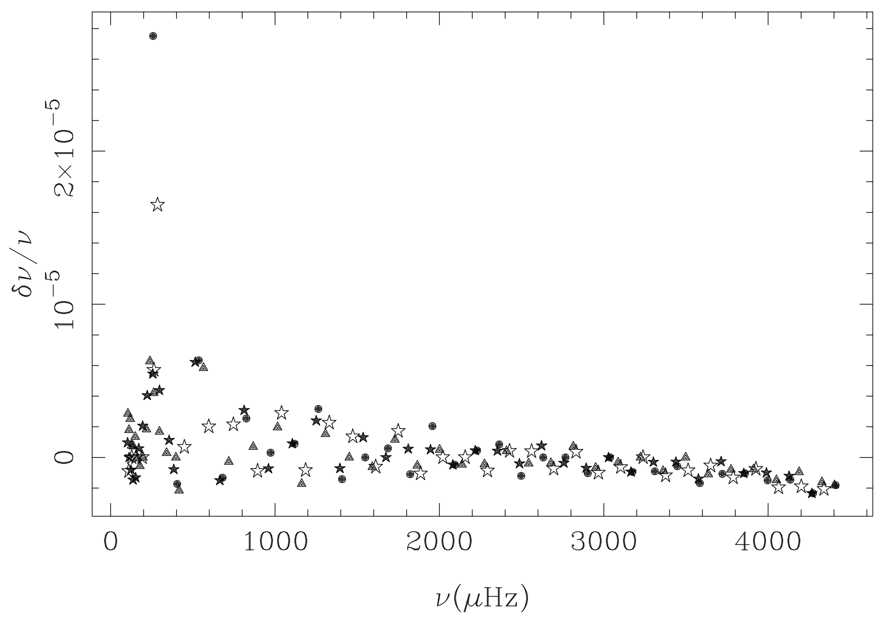


3 Internal tests:
$\star$ Accuracy – $\delta\nu_N = \nu_{Ri}(N) - \nu_{Ri}(N/2)$

Solar model S (J. Christensen-Dalsgaard) – N=2480



- $|\nu_{Ri}(1240) - \nu_{Ri}(2480)|$ smaller than 0.01μHz



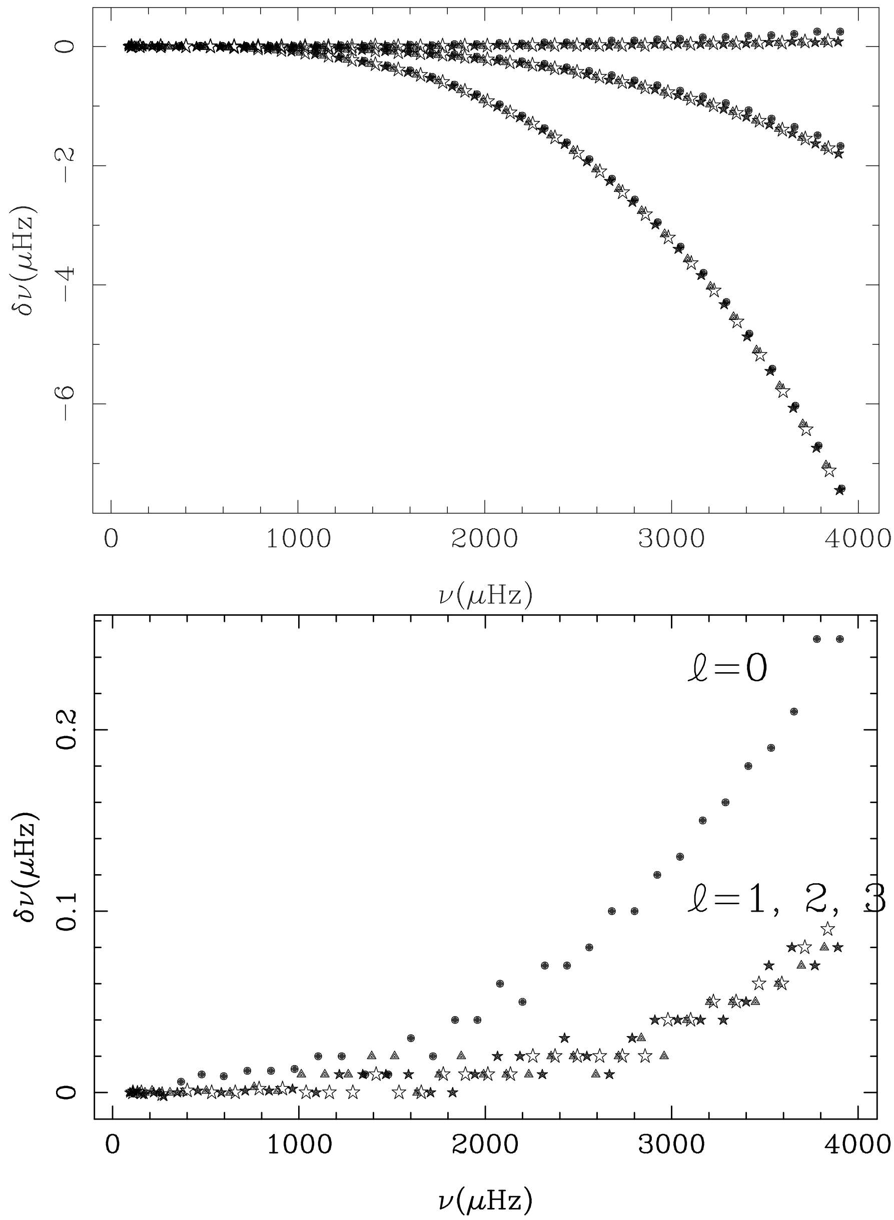
- large variation of the computed frequency with N
- with Richardson, N∼2000 leads to $\delta\nu/\nu \sim 10^{-5}$

3 Internal tests:

⋆ **Accuracy** – $\delta\nu_N = \nu_{Ri}(N) - \nu_{Ri}(N/2)$

Effect of number of mesh points - Task2 - Step1 – N∼900

$1.2M_\odot$, X_c=0.69



$\nu_{Ri}(N)$ - $\nu_{Ri}(2N)$

$\nu_{Ri}(N)$ - $\nu(2N)$

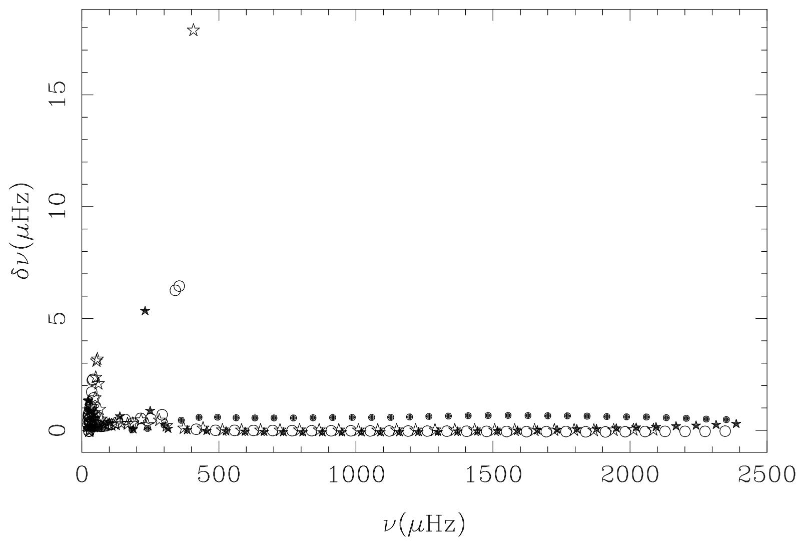
$\nu_{Ri}(N)$ - $\nu(N)$

- $|\nu_{Ri}(N) - \nu_{Ri}(2N)| \leq 0.2\mu$Hz small but still significant hence N ∼ 900 is too small
- different behavior of $\ell = 0$: mesh distribution?
- $\nu(2N)$ is obtained by interpolation of the model.....

3 Internal tests:
$\star$ **Accuracy** – $\delta\nu_N = \nu_{Ri}(N) - \nu_{Ri}(N/2)$

1.5M$_\odot$, X$_c$=0.40 Task2 - Step1b – N$\sim$2000

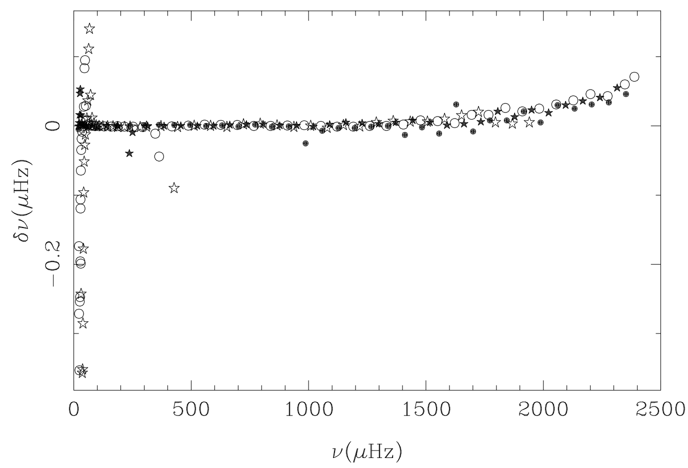
$\star$ NOC1 - eulerian:
$\delta\nu_N = |\nu_{Ri}(N) - \nu_{Ri}(N/2)| \leq 15\mu$Hz

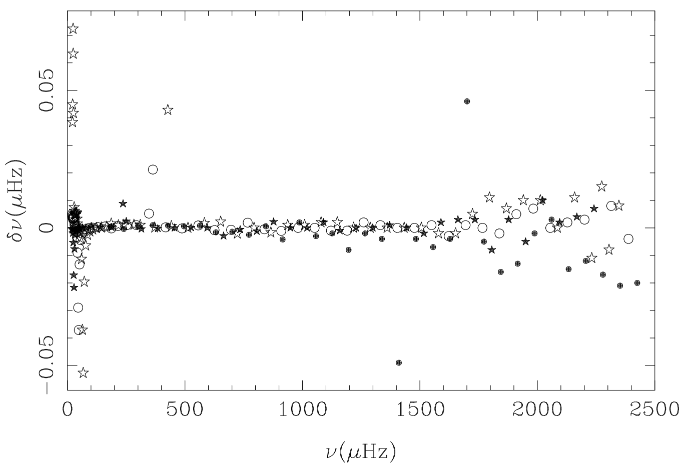


$\star$NOC2 - lagrangian

$\delta\nu_N \leq 0.3\mu$Hz (left)

$\delta\nu_{2N} \leq 0.05\mu$Hz (right)

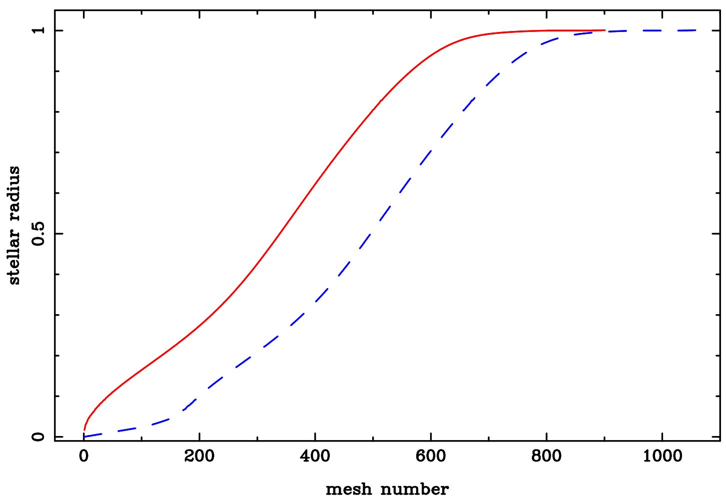


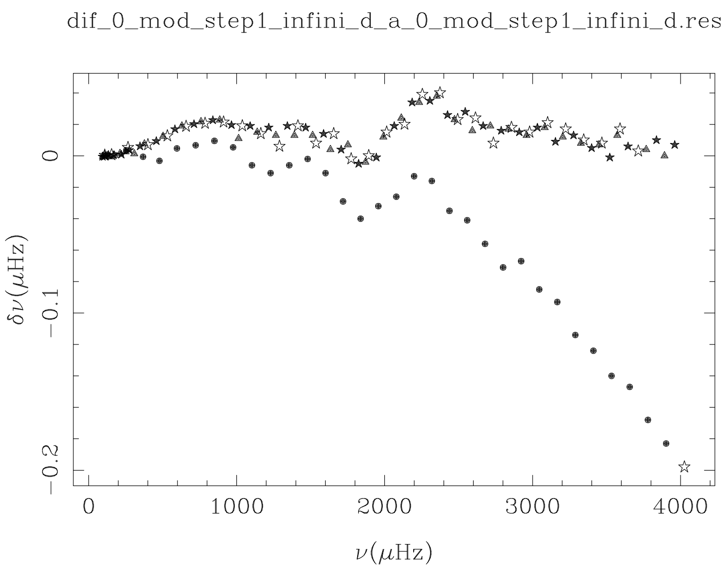


⋆ **Sensitivity to mesh points distribution**

Task2-Step1 – N=900 – 1.2M$_\odot$, X$_c$=0.69

- Adding points in central part to insure enough mesh points over a wavelength new/initial distribution: blue/red line





- Main effect: change of frequency for ℓ= 0

Symbols: ℓ =0 circle, 1 open star, 2 full star, 3 triangle Joint HELAS and

⋆ **Internal consistency** $\nu_{Ri}(N) - \nu_{Ri}^{var}(N)$

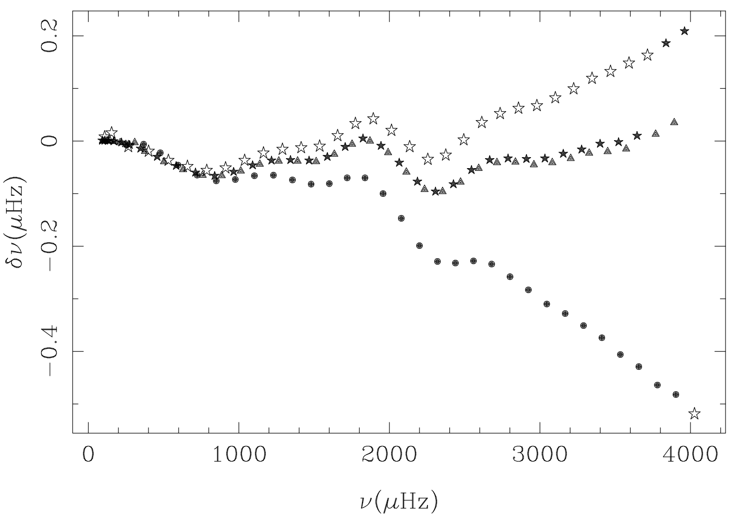
Task2-Step1 – N=900 – 1.2M$_\odot$, X$_c$=0.69

• Direct eigenfrequency/ its "variational" expression

⋆ $|\nu_{Ri}(N) - \nu_{Ri}^{var}(N)| \leq 2\mu$Hz

⋆ $|\nu_{Ri}(2N) - \nu_{Ri}^{var}(2N)| \leq 0.5\mu$Hz

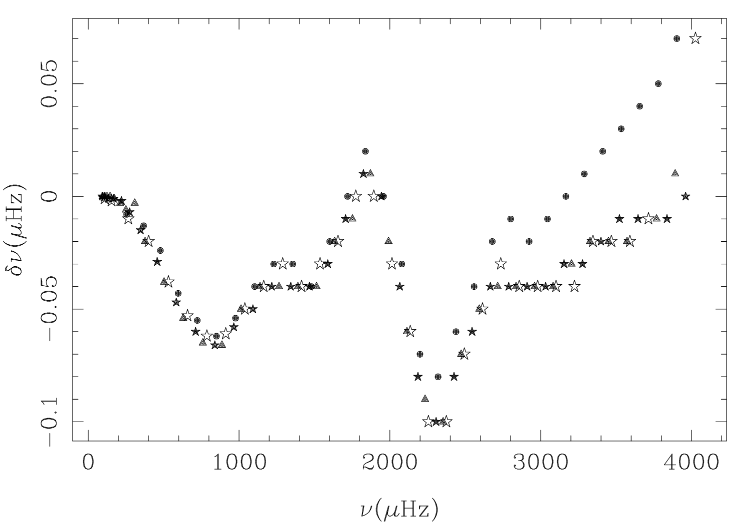
dif_0_mod_step1_infini_d _0_var_mod_step1_infini_d.res



• Adding points in central stellar part results in a better internal consistency:

⋆ $|\nu_{Ri}(2N) - \nu_{Ri}^{var}|(2N) \leq 0.1\mu$Hz (adding points)

dif_a_0_mod_step1_infini_d_a_0_var_mod_step1_infini_d.res



Conclusions

⋆ Improvement of initial code by changing from eulerian to lagrangian variables
– very important for g- and mixed modes in "evolved" ⋆

⋆ NOC has been successfully compared with other codes:
– Aarhus & M. Gabriel's codes for ⊙ g- and p- modes
– FILOU & Roxburgh's codes in COROT context (Milestone 2000)
– ESTA Task2 frequency comparisons

⋆ Development of specific tools for stellar oscillations and of various internal tests of accuracy:
very important to check internal consistency comparing ν and ν_{var}

⋆ Accurate frequency computations if:
– consistent equilibrium model
– large enough number of mesh points (N≤∼2000) with good distribution along radius
– using lagrangian code