Effects to consider on oscillation frequencies

The effect of rotation (perturbative approach)

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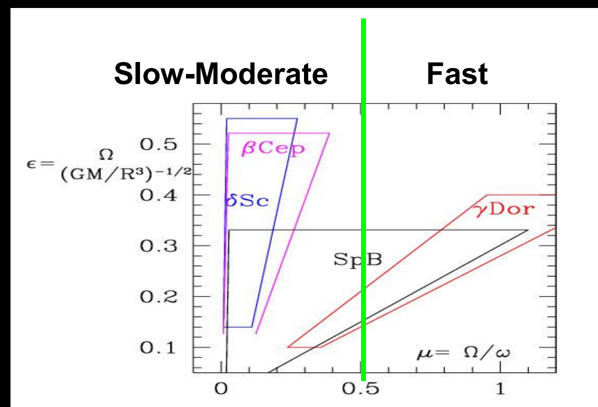
Plan

- Introduction
  - Slow, Moderate & Fast rotators
- Main effects (perturbative approach)
- Differential rotation
- Échèlle Diagrams
- Analysis of splitting asymmetries & rotation profile variations (RPV).

Introduction

- Type of rotators:
  - Slow ($\varepsilon,\mu \ll 1$)
  - Moderate ($\varepsilon,\mu \leq 0.5$)
  - Fast ($\varepsilon,\mu > 0.5$)



Perturbative approach

- Effect of rotation separated in orders O(μ)
  - O(μ): slow rotation
  - O(μ^2,μ^3): moderate rotation

$$\omega_{n\lambda m} = \omega_{0,n,\lambda} + m\Omega_s\left(C_L - 1 - J_0\right) + \frac{\Omega_s^2}{\omega_0}\left(D_0 + m^2 D_1\right) + \frac{\Omega_s^3}{\omega^2} T_{n\lambda m}$$

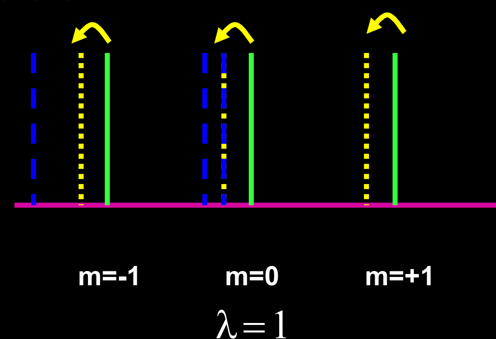
Dziembowski & Goode (1992), Soufi et al. 1998

Main effects

- First order (Coriolis, geometric)
  - Symmetry in multiplets
- Second order (Coriolis,Centrifugal)
  - Asymmetries in multiplets

$$\omega_{1} = m\Omega(C_{l0}-1) \qquad \omega_{2} = \frac{\bar{\Omega}^2}{\omega_0}(D_{0} + m^2 D_{1})$$

global shift / splitting | asymmetry



Near degeneracy

- Close – frequency modes
  - Avoided crossing, mixed modes
  - Selection rules: $\Delta m=0$, $\Delta\ell=0,2$

$$\omega = \omega_0 + d_1(\omega,\Omega) + d_2(\omega,\Omega^2) + \sum_j \frac{d_{3,j}(\omega,\Omega^3)}{(\omega_0 - \omega_j)^2} + \ldots$$

$\omega_0 \sim \omega_j$

Effects on adiabatic frequencies

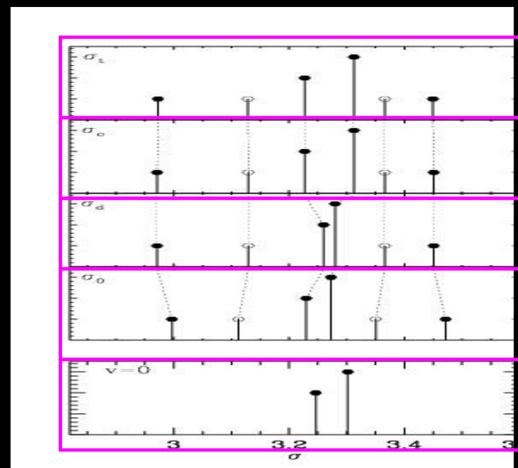
Cubic effects (3rd order)

Near degeneracy (2nd order)

Centrifugal (2nd order)

g_{eff} & Coriolis (1st order)

No rotation: $\omega=\omega_0^{(0)}$



Goupil al. 1999

$$\omega_{n\lambda m} = \omega_{0,n,\lambda} + m\Omega_s\left(C_L - 1 - J_0\right) + \frac{\Omega_s^2}{\omega_0}\left(D_0 + m^2 D_1\right) + \frac{\Omega_s^3}{\omega^2} T_{n\lambda m}$$

Differential rotation

- Shellular rotation gives rise to additional terms
  - Eigenfunctions (C_L, J_0, D_0, D_1)
  - Structure terms (J_0, D_0, D_1)

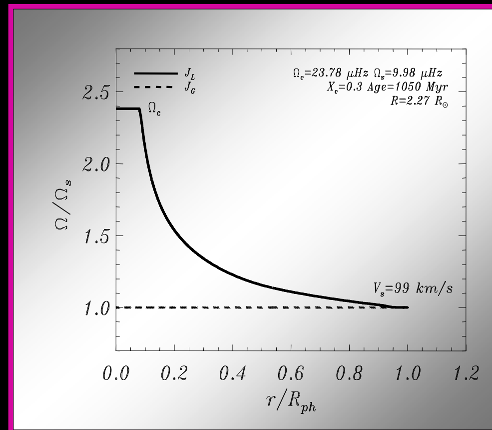
$$\Omega(r) = \Omega_s[1 + \eta_0(r)]$$

$$\omega = \omega_0 + \underbrace{m\Omega_s\left(C_l - 1 - J_0\right)}_{\omega_1} + \underbrace{\frac{\Omega_s^2}{\omega_0}\left(D_0 + m^2 D_1\right)}_{\omega_2}$$

Differential rotation

- Intermediate-mass, moderately rotating stars
- DR vs. UR models
- DR as shellular (local conservation of angular momentum)

$$\Omega(r) = \Omega_s[1+\eta_0(r)]$$



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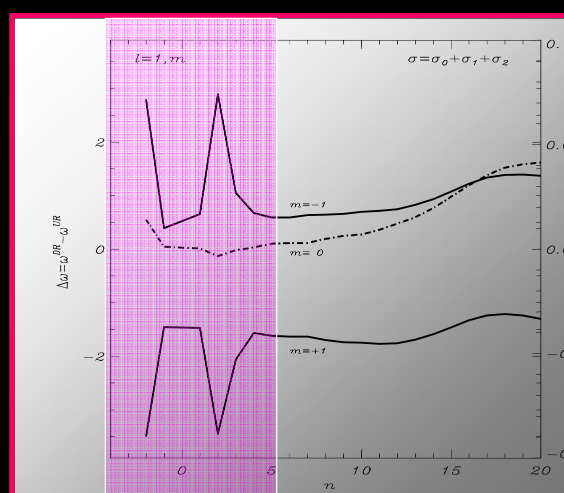
Differential rotation. The effects on frequencies

- Mainly affect ω_1 & ω_2
- Significant effects for g and mixed modes
- Up to 3μHz for *g-mixed* and 1μHz for *p* modes



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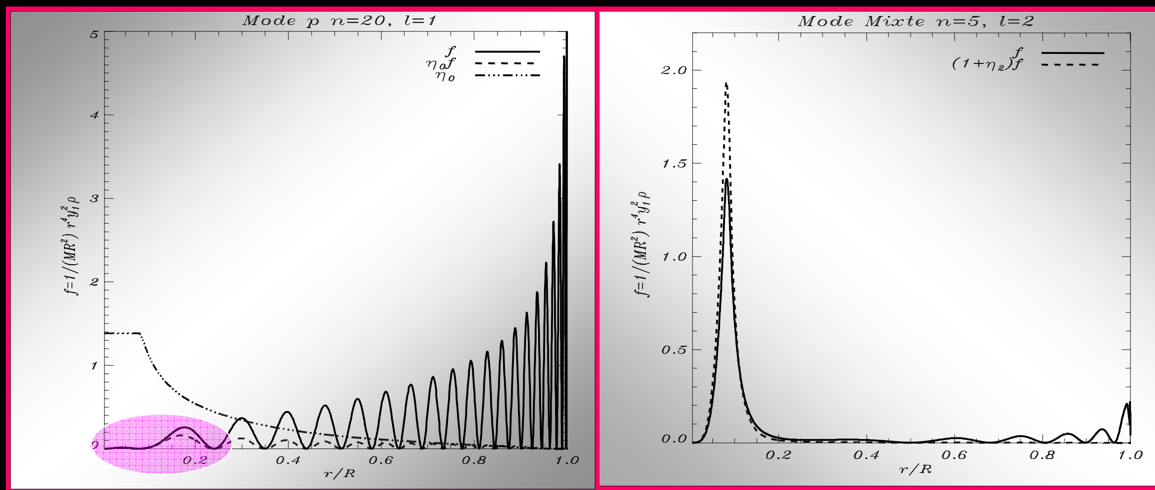
Differential rotation. The effects on eigenfunctions



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Rotation & Échèlle diagrams

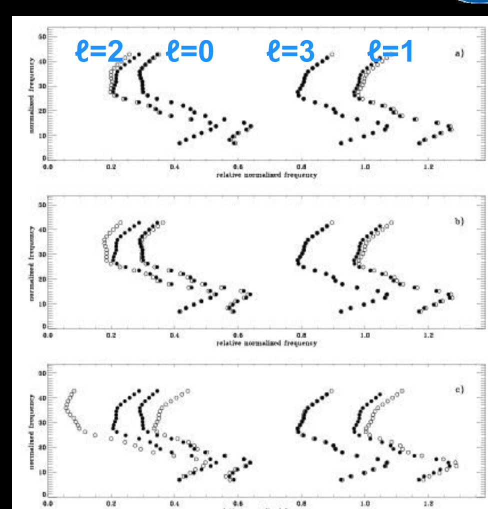
- Solar-like pulsators (high-frequency modes excited)
- Significant effects for large frequencies
- Near degeneracy naturally affects small separation. Largest effects for ℓ=(0,2) modes.
- *m* components may complicate diagnostics but amplitudes may alleviate the problem

20 km/s

30 km/s

50 km/s

$1.5\ M_{\odot}$



Lochard et al. 2006, in prep

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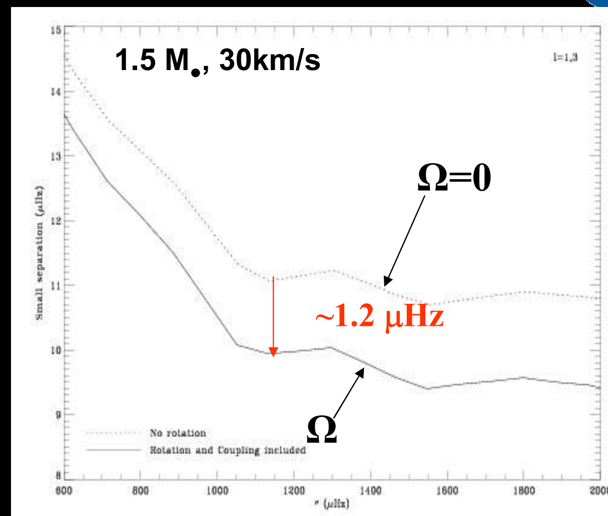
Rotation & Échèlle diagrams

- Seismic modelling based on δω fitting diagnostics generally neglect rotation.
- For a 1.5 $M_\bullet$ (MS, 1Gyr) model $\delta\omega^{(\Omega)} - \delta\omega^{(0)}$ ~1μHz
- But rotation effects can be "depolluted"



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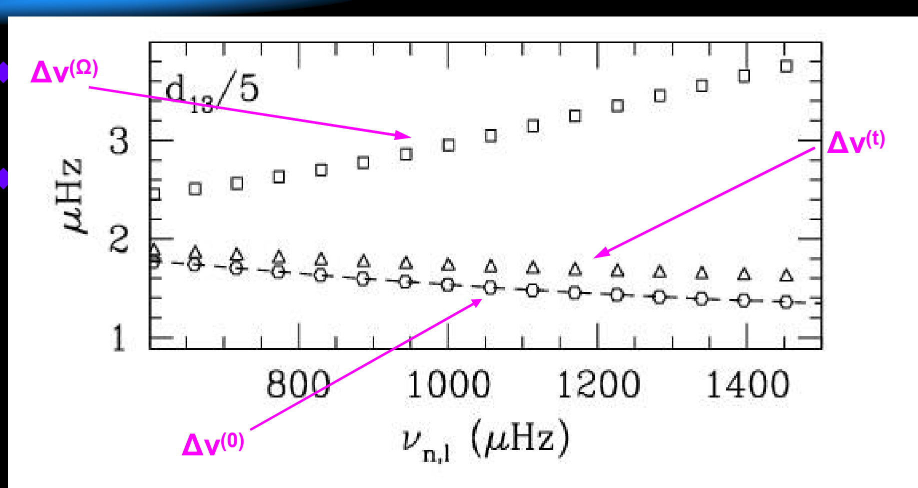
Rotation & Échèlle diagrams



$1.50M_\bullet, (\lambda_a, \lambda_b) = (1,3), \ v_s = 30\,km/s$ *Goupil et al. 2003*

Analysis of splitting asymmetries & RPV

- Asymmetries of rotation splittings (S_j) are observed in pulsating stars (δ Scuti, β Cephei, etc.)
- Rotation profile variations (RPV) affects the rotation splittings: asymmetries (A_j)
- Study of RPV through A_j is a promising tool to search for the true RP.

$$S_j = \frac{\omega_{-m} + \omega_{+m}}{2} \qquad A_j = \omega_{-m} + \omega_{+m} - 2\omega_{m=0}$$

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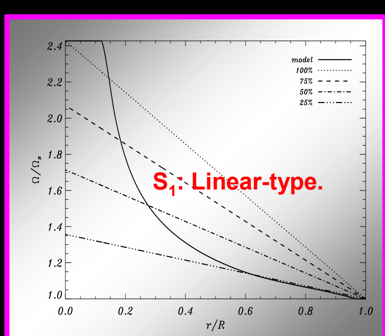
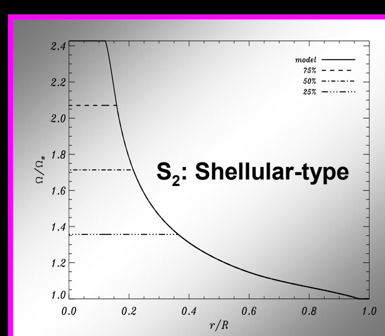
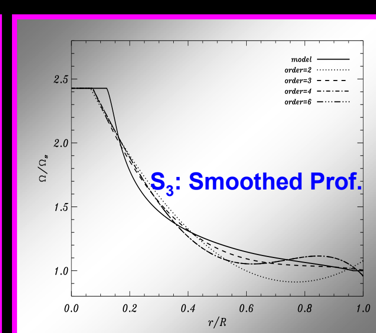
Analysis of splitting asymmetries & RPV

- A_j cannot be explained by UR models
- DR models in the approximation of shellular rotation.







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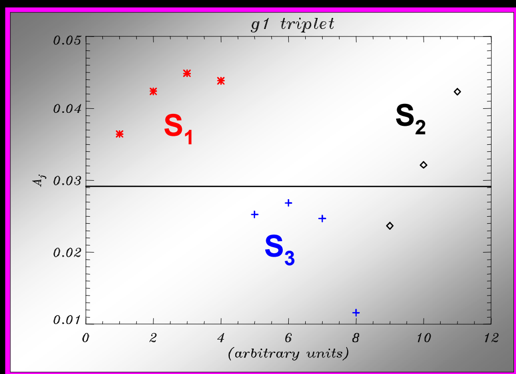
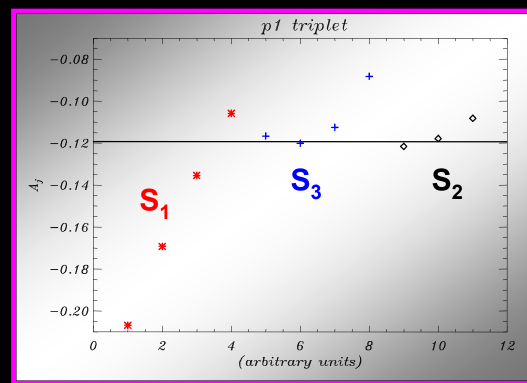
Analysis of splitting asymmetries & RPV

- $A_j(g)$ behaves differently to $A_j(p)$
- A_l of low-order g & p modes are the most affected RPV near the core (μ-gradient zone).





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Analysis of splitting asymmetries & RPV

- Asymmetries are directly proportional to the second –order term D_0
- Analysis of D_0 *Kernels (work in progress) will help to understand the behaviour of* A_j*:*
  - Mode dependency (n,ℓ)
  - RPV effects

$$A_j = \int_0^1 \Omega^2(x) K_{D_0}(x)\,dx$$

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Bibliography

- Zahn et al. 1992 A&A 265, 115
- Dziembowski & Goode 1992 A&A
- Goupil et al. 2003 ESA Eddington Conference, SP
- Soufi et al. 1998 A&A 334, 911
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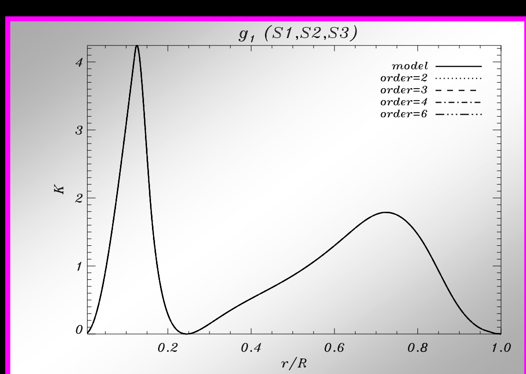
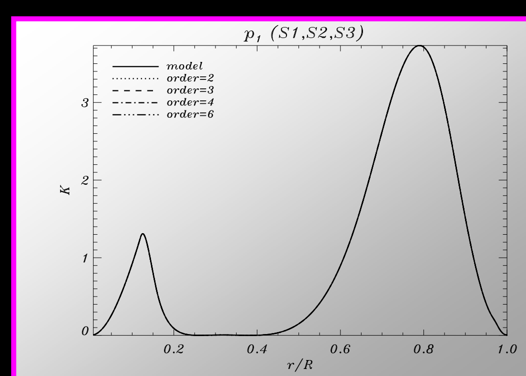
Analysis of splitting asymmetries & RPV

- Low order, low-degree modes (g & mixed) modes are the most affected by variations near the core & the μ-gradient zone





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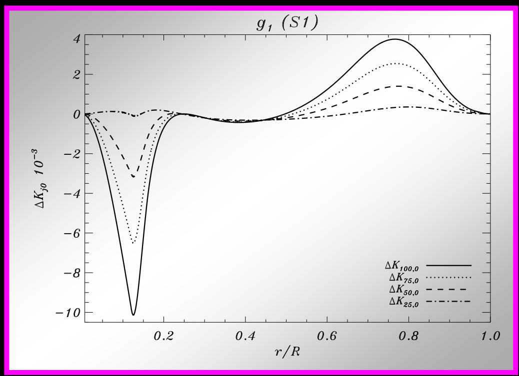
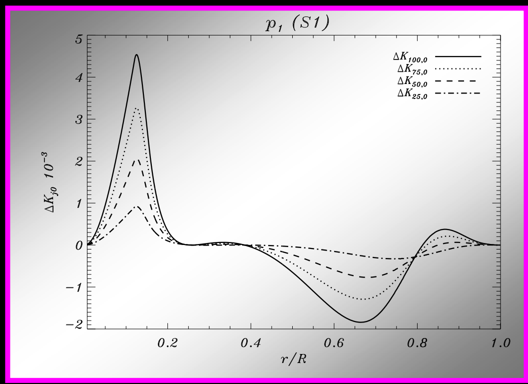
Analysis of splitting asymmetries & RPV

- RPV modify the asymmetry of rotationally split multiplets





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