Time dependent Convection
vs.
frozen convection approximations

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Introduction

- Perturbation of energy conservation equation:

$$i\omega T\delta s = \delta\varepsilon - \frac{1}{\rho}\nabla \bullet \left[\vec{F} + \xi\left(\nabla.\vec{F}\right) - (\xi.\nabla)\vec{F}\right]$$

- In the stellar interior, the radiative energy transfer can be described by the diffusion approximation:

$$\vec{F} = -\frac{4ac}{3\rho\kappa}T^3\nabla T$$

a Radiative pressure constant

c The speed of light

κ The opacity of the matter

References

- Baker & Kippenhahn, 1962, 1965
- Ando & Osaki, 1975
- Saio & Cox, 1980
- Cox et al., 1987
- Li, 2000
- Löffler, 2003
- Grigahcène, Phd thesis, Univ. Granada, 2004.

Frozen convection approximations

- Ignore the eulerian variation of the convective luminosiy $L'_c = 0$
- Ignore the lagrangian variation of the convective luminosiy $\delta L_c = \delta(4\pi F_{c,r}) = 0$
- Ignore the lagrangian variation of the convective flux $\delta \vec{F}_c = 0$
- Ignore the lagrangian variation of the convective flux $\delta(\nabla . \vec{F})_c = 0$

Ignoring the eulerian variation of convective luminosity:

$$L'_c = 0$$

Perturbed Equation:

$$i\sigma\overline{T}\delta s = -\frac{d(\delta L_R)}{dm} + \varepsilon\left[\frac{\delta\varepsilon}{\varepsilon} + \lambda(\lambda+1)\frac{\xi_h}{r}\right] - \frac{d}{dm}\left(\frac{dL_c}{dr}\xi_r\right) + \frac{\lambda(\lambda+1)}{4\pi r^3\overline{\rho}}\left\{L_R\left[\frac{\delta T}{r\left(\frac{dT}{dr}\right)} - \frac{\xi_r}{r}\right] - L_c\frac{\xi_h}{r}\right\} + \frac{\lambda(\lambda+1)}{\overline{\rho}r}\delta F_{c,h} + \delta\left(\overline{\varepsilon_2} + \frac{1}{\overline{\rho}}\overline{\vec{V}.\nabla p_{th}}\right)$$

$\delta F_{c,h} = 0$

Equation

$$i\sigma\overline{T}\delta s = \delta\varepsilon - \frac{d(\delta L_R)}{dm} + \lambda(\lambda+1)\frac{\xi_h}{r}\frac{dL}{dm} + \frac{\lambda(\lambda+1)}{4\pi r^3\overline{\rho}}L_R\left\{\frac{\delta T}{r\left(\frac{dT}{dr}\right)} - \frac{\xi_r}{r}\right\} + \frac{\lambda(\lambda+1)}{\overline{\rho}r}L_c\frac{\xi_h}{r} - \frac{d}{dm}\left(\frac{dL_c}{dr}\xi_r\right)$$

Work integral (radial case):

$$W = -\int_0^M (\Gamma_3 - 1)\Re\left\{\frac{\delta\rho^*}{\rho\sigma}\left[\frac{d(\delta L_R)}{dm} - \delta\varepsilon - \frac{d}{dm}\left(\frac{dL_c}{dr}\xi_r\right)\right\}dm$$

$F'_{c,h} = 0$

$$i\sigma\overline{T}\delta s = \delta\varepsilon - \frac{d(\delta L_R)}{dm} + \lambda(\lambda+1)\frac{\xi_h}{r}\frac{dL}{dm} + \frac{\lambda(\lambda+1)}{4\pi r^3\overline{\rho}}L_R\left\{\frac{\delta T}{r\left(\frac{dT}{dr}\right)} - \frac{\xi_r}{r}\right\} - \frac{d}{dm}\left(\frac{dL_c}{dr}\xi_r\right)$$

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Ignoring the lagrangian variation of convective luminosity:

$$\delta L_c = 0$$

Perturbed Equation:

$$i\sigma \bar{T}\delta s = -\frac{d(\delta L_R)}{dm} + \varepsilon\left[\frac{\delta\varepsilon}{\varepsilon} + \lambda(\lambda+1)\frac{\xi_h}{r}\right] + \frac{\lambda(\lambda+1)}{4\pi r^3\bar{\rho}}\left\{L_R\left[\frac{\delta T}{r\left(\frac{dT}{dr}\right)} - \frac{\xi_r}{r}\right] - L_c\frac{\xi_h}{r}\right\} + \frac{\lambda(\lambda+1)}{\bar{\rho} r}\delta F_{c,h} + \delta\left(\overline{\varepsilon_2} + \frac{1}{\bar{\rho}}\vec{V}.\nabla p_{th}\right)$$

Equation (Pesnell, 1990)

$\delta F_{c,h} = 0$

$$i\sigma\bar{T}\delta s = \delta\varepsilon - \frac{d(\delta L_R)}{dm} + \lambda(\lambda+1)\frac{\xi_h}{r}\frac{dL}{dm} + \frac{\lambda(\lambda+1)}{4\pi r^3\bar{\rho}}L_R\left\{\frac{\delta T}{r\left(\frac{dT}{dr}\right)} - \frac{\xi_r}{r}\right\} - \frac{\lambda(\lambda+1)}{4\pi r^3\bar{\rho}}L_c\frac{\xi_h}{r}$$

$F'_{c,h} = 0$

$$i\sigma\bar{T}\delta s = \delta\varepsilon - \frac{d(\delta L_R)}{dm} + \lambda(\lambda+1)\frac{\xi_h}{r}\frac{dL}{dm} + \frac{\lambda(\lambda+1)}{4\pi r^3\bar{\rho}}L_R\left\{\frac{\delta T}{r\left(\frac{dT}{dr}\right)} - \frac{\xi_r}{r}\right\}$$

Work integral (radial case):

$$W = -\int_0^M (\Gamma_3 - 1)\Re\left\{\frac{\delta\rho^*}{\rho\sigma}\left[\frac{d(\delta L_R)}{dm} - \delta\varepsilon\right]\right\}dm$$

$$W = -\int_0^M (\Gamma_3 - 1)\Re\left\{\frac{\delta\rho^*}{\rho\sigma}\left[\frac{d(\delta L_R)}{dm} - \delta\varepsilon\right]\right\}dm$$

Ignoring the lagrangian variation of convective flux:

$$\delta\vec{F}_c = 0$$

In this case

$$\delta L_c = 2L_c\frac{\xi_r}{r}$$

Perturbed Equation:

$$i\sigma\bar{T}\delta s = \delta\varepsilon - \frac{d(\delta L_R)}{dm} + \lambda(\lambda+1)\frac{\xi_h}{r}\frac{dL}{dm} + \frac{\lambda(\lambda+1)}{4\pi r^3\bar{\rho}}L_R\left[\frac{\delta T}{r\left(\frac{dT}{dr}\right)} - \frac{\xi_r}{r}\right] - \frac{\lambda(\lambda+1)}{4\pi r^3\bar{\rho}}L_c\frac{\xi_h}{r} - 2\frac{d}{dm}\left(L_c\frac{\xi_r}{r}\right)$$

Work integral (radial case):

$$W = -\int_0^M (\Gamma_3 - 1)\Re\left\{\frac{\delta\rho^*}{\rho\sigma}\left[\frac{d(\delta L_R)}{dm} - \delta\varepsilon + 2\frac{d}{dm}\left(L_c\frac{\xi_r}{r}\right)\right]\right\}dm$$

Ignoring the lagrangian variation of convective flux:

$$\delta\left(\nabla.\vec{F}_c\right)=0$$

In this case

$$\delta L_c = 2L_c\frac{\xi_r}{r}$$

Perturbed Equation:

$$i\sigma\bar{T}\delta s=\delta\varepsilon-\frac{d\left(\delta L_R\right)}{dm}+\lambda(\lambda+1)\frac{\xi_h}{r}\frac{dL_R}{dm}$$
$$+\frac{\lambda(\lambda+1)}{4\pi r^3\bar{\rho}}L_R\left[\frac{\delta T}{r\left(\frac{dT}{dr}\right)}-\frac{\xi_r}{r}\right]$$
$$-\frac{dL_c}{dm}\frac{1}{r^2}\frac{d}{dr}\left(r^2\xi_r\right)$$

Work integral (radial case):

$$W=-\int_0^M(\Gamma_3-1)\Re\left\{\frac{\delta\rho^*}{\rho\sigma}\left[\frac{d\left(\delta L_R\right)}{dm}-\delta\varepsilon+\frac{dL_c}{dm}\frac{1}{r^2}\frac{d}{dr}\left(r^2\xi_r\right)\right]\right\}dm$$

Effects on periods (LI, 1991)

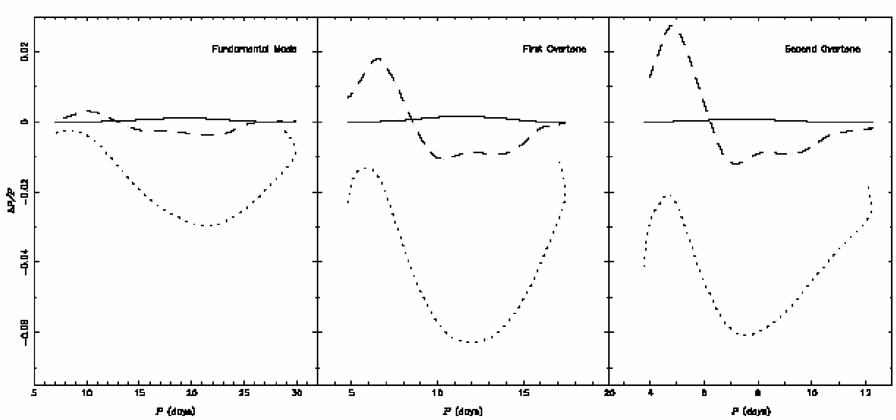


FIG. 3.—Differences in the periods of the pulsation modes by use of the four frozen-convection approximations for the $9M_\odot$ model. The solid lines are for DLC to DFC, dashed lines for DNFC to DFC, and dotted lines for LCP to DFC.

Effects on instability coefficients (Li, 1991)

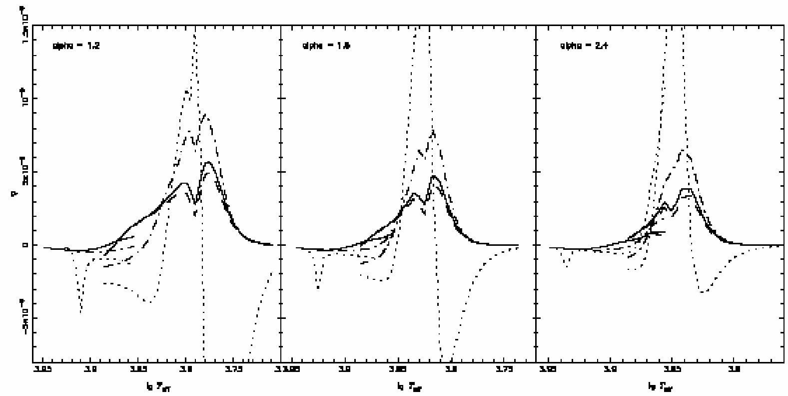


FIG. 4.—Instability coefficients vs. the effective temperature of the pulsation modes by use of the four frozen-convection approximations for the fundamental mode of the 2 $M_\odot$ star. The solid lines are for DFC, dashed lines for DLC, dash-dotted lines for DNFC, and dotted lines for LCP.

More effects

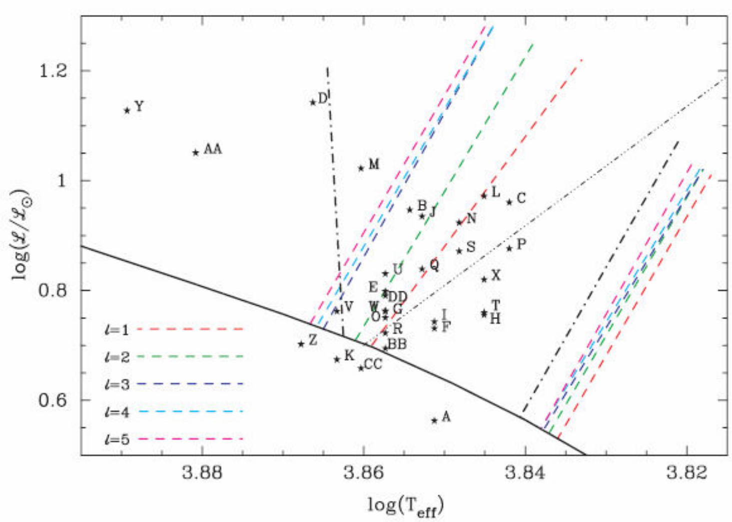


FIG. 1.—Calculated instability strip (shown in color for various ℓ) and calculated main sequence (*solid line*) compared with theoretical the red edge of the δ Scuti instability strip (Breger & Pamyatnykh 1998; *triple-dot–dashed line*) and the observational γ Dor instability strip (Handler & Shobbrook 2002; *dot-dashed line*). Also shown are all 30 bona fide γ Dor stars (see Table 2). See text for details.

Warner et al., 2003

Pesnell oscillations code

Time dependent convection

Hydrodynamic equations

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{v}) = 0$$

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \bullet (\rho \vec{v}\vec{v}) = -\rho \nabla \Phi + \nabla \bullet P$$

$$\frac{\partial (\rho U)}{\partial t} + \nabla \bullet (\rho U \vec{v}) = \rho \varepsilon_N - \nabla \bullet \vec{F}_R - P \otimes \nabla \vec{v}$$

$$P = P_G + P_R$$
$$p = p_G + p_R$$
$$P_X = p_X \; 1 - \beta_X$$

P: Pressure tensor ; p : its diagonal component.

$\vec{F}_R$ Radiative Flux

Grigahcène et al., 2005

Convective Fluctuation

Splitting the variables

$$y = \overline{y} + \Delta y$$
$$\vec{v} = \vec{u} + \vec{V}$$

Mean Equations

$$\frac{d\overline{\rho}}{dt} + \overline{\rho} \nabla \bullet (\vec{u}) = 0$$

$$\overline{\rho} \frac{d\vec{u}}{dt} = -\overline{\rho} \nabla \overline{\Phi} - \nabla \left(\overline{p}_G + \overline{p}_R + \underline{\underline{\overline{p}_T}} \right) + \nabla \bullet \left(\overline{\beta}_G + \overline{\beta}_R + \underline{\underline{\overline{\beta}_T}} \right)$$

$$\overline{\rho} \overline{T} \frac{d\overline{s}}{dt} = -\nabla \bullet \left(\overline{\vec{F}}_R + \underline{\underline{\overline{\vec{F}}_C}} \right) + \overline{\rho}\, \overline{\varepsilon}_N + \underline{\underline{\overline{\rho}\, \overline{\varepsilon}_2}} + \underline{\underline{\overline{\vec{V} \bullet \nabla (p_G + p_R)}}}$$

$$\underline{\underline{\overline{\rho} \frac{d}{dt} \left(\frac{1}{2} \frac{\overline{\rho V^2}}{\overline{\rho}} \right)}} = -\underline{\underline{\overline{\rho}\, \overline{\varepsilon}_2}} - \underline{\underline{\overline{\vec{V} \bullet \nabla (p_G + p_R)}}} - \underline{\underline{\nabla \bullet \vec{F}_2}}$$

$\overline{\rho \vec{V}\vec{V}} = \overline{p}_T \; 1 - \overline{\beta}_T$ Tensor of Reynolds

$p_T = \overline{\rho V_r^2}$ Turbulence pressure

$\vec{F}_2 = \frac{1}{2} \overline{\rho V^2 \vec{V}}$ Flux of the kinetic energy of turbulence

M. Gabriel's Theory

Fluctuation Equations

$$\bar{\rho}\frac{d}{dt}\left(\frac{\Delta\rho}{\bar{\rho}}\right)+\nabla\bullet\left(\rho\vec{V}\right)=0$$

$$\bar{\rho}\frac{d\vec{V}}{dt}=\frac{\Delta\rho}{\bar{\rho}}\nabla\bar{p}-\nabla\Delta p-\frac{8\rho\vec{V}}{3\tau_C}-\rho\vec{V}\bullet\nabla\vec{u}$$

$$\frac{\Delta\left(\rho T\right)}{\overline{\rho T}}\frac{d\bar{s}}{dt}+\frac{d\Delta s}{dt}+\vec{V}\bullet\nabla\bar{s}=-\frac{\Gamma^{-1}+1}{\tau_C}\Delta s$$

$$\rho\varepsilon_2=\left(\beta_G+\beta_R\right)\otimes\nabla\vec{V}$$

Dissipation rate of kinetic energy of turbulence into heat per unit volume.

Approximations of Gabriel's Theory

$$\frac{\Delta\rho}{\bar{\rho}}\nabla\bullet\left(\bar{\beta}_G+\bar{\beta}_R+\bar{\beta}_T\right)-\nabla\bullet\left(\Delta\beta_G+\Delta\beta_R+\Delta\beta_T\right)=\frac{8}{3}\frac{\rho\vec{V}}{\tau_C}$$

$$\rho\varepsilon_2-\overline{\rho\varepsilon_2}+\rho T\nabla s\bullet\vec{V}-\overline{\rho T\nabla s\bullet\vec{V}}=\frac{\left(\nabla\bullet\vec{F}_R-\overline{\nabla\bullet\vec{F}_R}\right)}{\left(\overline{\rho T}\right)}-\frac{\Delta s}{\tau_C}$$

$$\frac{\left(\nabla\bullet\vec{F}_R-\overline{\nabla\bullet\vec{F}_R}\right)}{\left(\overline{\rho T}\right)}=-\omega_R\,\Delta s$$

ω_R The inverse of the characteristic time of radiative energy lost by turbulent eddies.

τ_C Life time of the convective elements.

$\Gamma^{-1}=\omega_R\tau_C$ Convective efficiency.

In the static case, assuming constant coefficients (Hp>>l !), we have solutions which are plane waves identical to the ML solutions.

Perturbation of the mean equations → Linear pulsation equations

Equation of mass conservation

$$\frac{\delta\rho}{\rho}+\frac{1}{r^2}\frac{d\left(r^2\delta r\right)}{dr}-\frac{l\left(l+1\right)}{\sigma^2 r^2}\left(\delta\Phi+\frac{\delta p}{\rho}\right)=0$$

Radial component of the equation of momentum conservation

$$\sigma^2\delta r=\frac{d\delta\Phi}{dr}-\frac{1}{\rho}\frac{d\delta p}{dr}+g\frac{\delta\rho}{\rho}+\underline{\underline{\frac{2A-1}{A}\frac{\bar{p}_T}{r}\frac{\partial\delta r}{\partial r}-\delta\left(\nabla_j\overline{\beta_T^{rj}}\right)}}$$

Transversal component of the equation of momentum conservation

$$\sigma^2\delta r_H=\frac{1}{r}\left(\delta\Phi+\frac{\delta p}{\rho}+\underline{\underline{\frac{r\,ViscH}{\bar{\rho}}+\frac{2A-1}{A}\frac{\bar{p}_T}{\bar{\rho}}\left(\frac{\delta r}{r}-\frac{\delta r_H}{r}\right)}}\right)$$

Equation of Energy conservation

$$i\sigma T\delta s = \delta\varepsilon_N + \left(\frac{\delta\rho}{\rho} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\delta r\right)\right)\frac{dL}{dm} - \frac{d\delta\left(L_R + \underline{\underline{L_C}}\right)}{dm}$$
$$+ \frac{l(l+1)}{4\pi r^3\rho}\left(L_R\left(\frac{\delta T}{r(dT/dr)} - \frac{\delta r}{r}\right) - \underline{\underline{L_C}}\,\frac{\delta r_H}{r}\right)$$
$$+ \underline{\underline{\frac{l(l+1)}{\bar{\rho} r}FCH}} + \underline{\underline{\delta\varepsilon_2}} + \underline{\underline{\delta\left(\overline{\vec{V} \bullet \frac{\nabla\left(p_G + p_R\right)}{\bar{\rho}}}\right)}}$$

FCH Amplitude of the horizontal component of the convective flux

ML Perturbation

The main source of uncertainty in any ML theory of convection-pulsation interaction is in the way to perturb the mixing-length.
In the results presented below, we used :

$$\frac{\delta l}{l} = \begin{cases} \dfrac{1}{1+(\sigma\tau_C)^2}\dfrac{\delta H_P}{H_P} & \text{Time-dependent treatment 1} \\[2ex] \dfrac{\delta H_P}{H_P} & \text{Time-dependent treatment 2} \end{cases}$$

$$\frac{1}{1+(\sigma\tau_C)^2} \rightarrow \begin{cases} 1 & \text{when} \quad \sigma\tau_C \ll 1 \\ 0 & \text{when} \quad \sigma\tau_C \gg 1 \end{cases}$$

σ Angular pulsation frequency

H_P Pressure scale

τ_C Life time of the convective elements

Radial Modes – 1.8 M_0, α=1.5

Frozen Convection

Time-dependent convection

$$\frac{\delta l}{l} = \frac{1}{1+(\sigma\tau_C)^2}\frac{\delta H_P}{H_P}$$

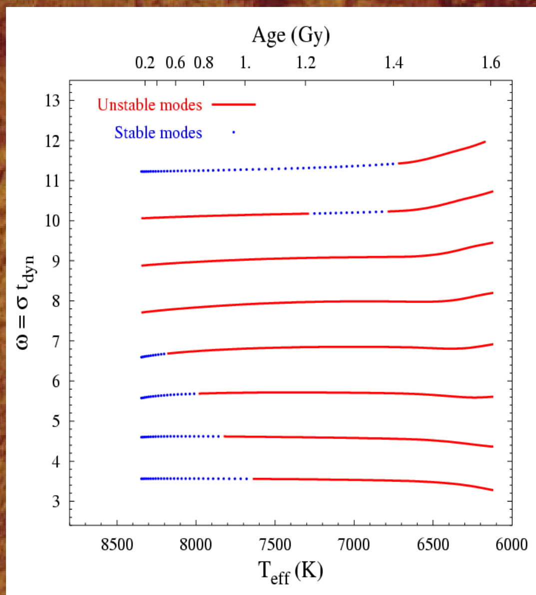


Figure 3

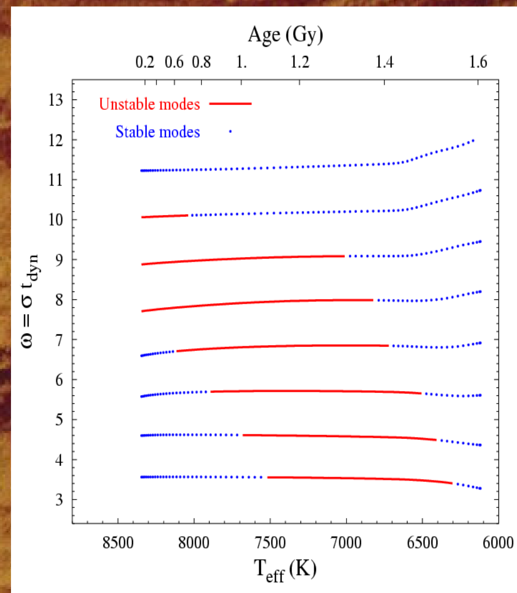


Figure 4

λ=2 modes – 1.8 M_0, α=1.5

Frozen Convection

Time-dependent convection

$$\frac{\delta l}{l} = \frac{1}{1+(\sigma\tau_C)^2}\frac{\delta H_P}{H_P}$$

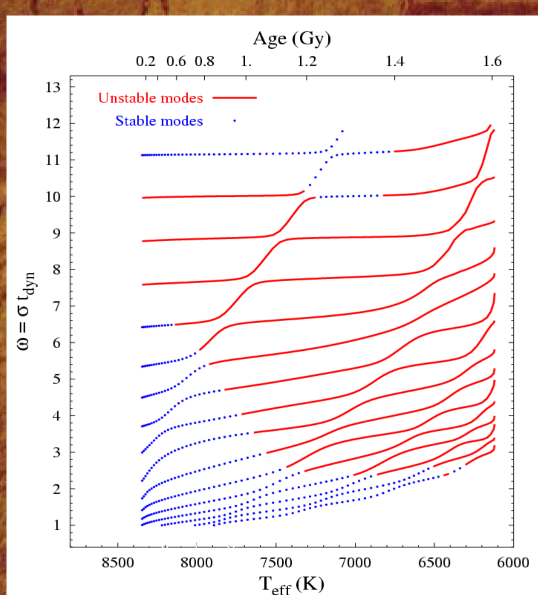


Figure 6

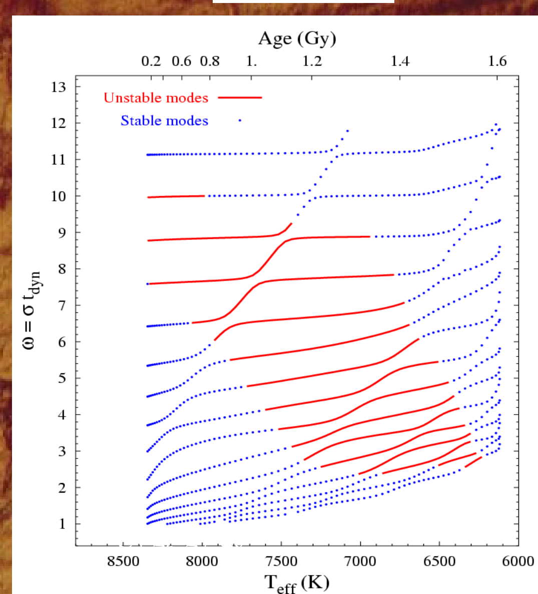


Figure 7

δ Scuti Instability Strips

M=1.4-2.2 M_0, α=1.5, λ=0

$$\frac{\delta l}{l} = \frac{1}{1+(\sigma\tau_C)^2}\frac{\delta H_P}{H_P}$$

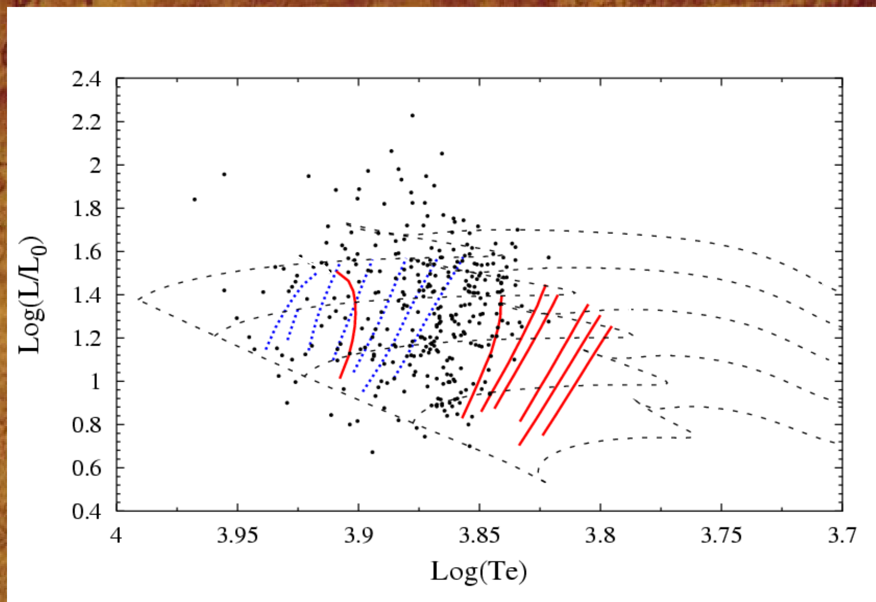


Figure 9

δ Scuti Instability Strips

M=1.4-2.2 M_0, α=1.5, λ=2

$$\frac{\delta l}{l} = \frac{1}{1+(\sigma\tau_C)^2}\frac{\delta H_P}{H_P}$$

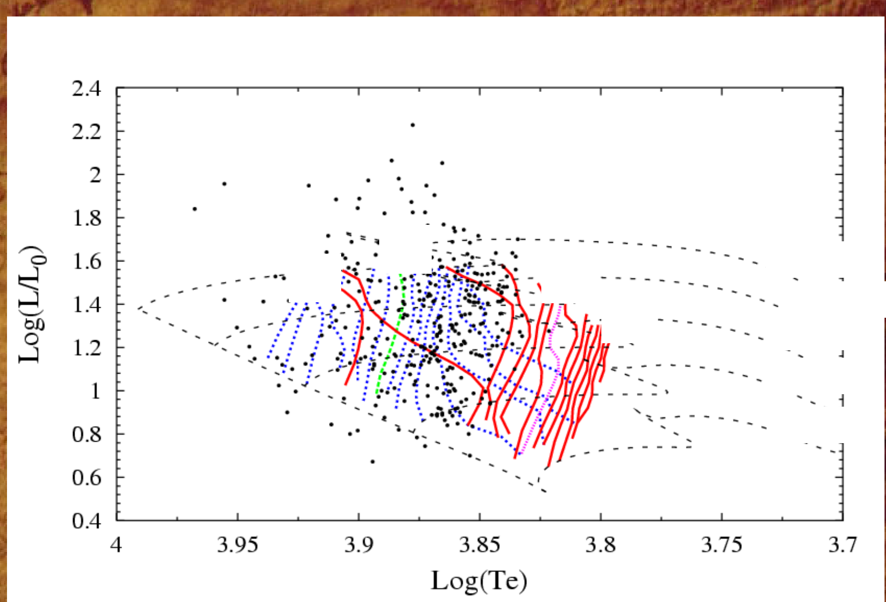


Figure 12

γ Dor Instability modes
M=1.5 M_0, α=1, λ=1

$$\frac{\delta l}{l} = \frac{1}{1+(\sigma\tau_C)^2}\frac{\delta H_P}{H_P}$$

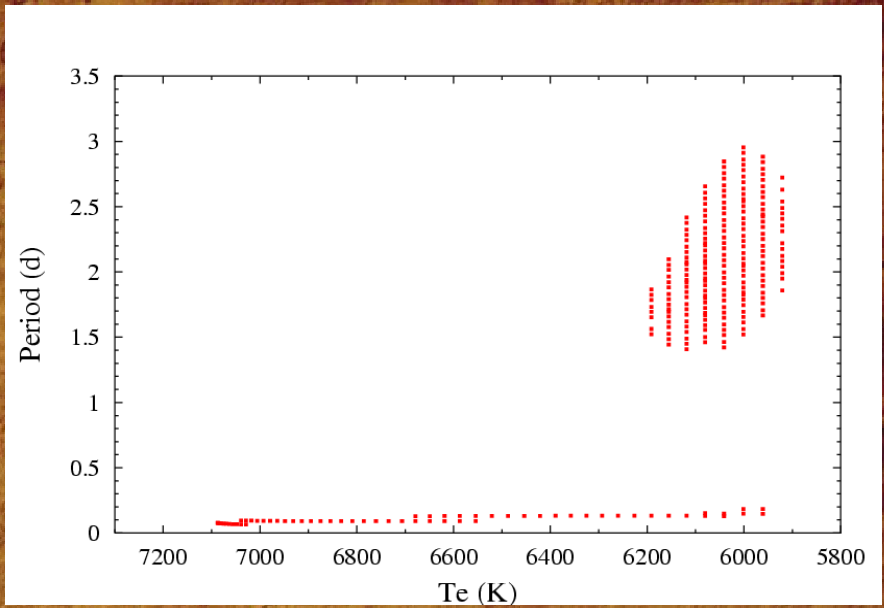


Figure 13

Comparison between γ Dor Instability Strips (λ=1) for α=1 and α=1.5

$$\frac{\delta l}{l} = \frac{1}{1+(\sigma\tau_C)^2}\frac{\delta H_P}{H_P}$$

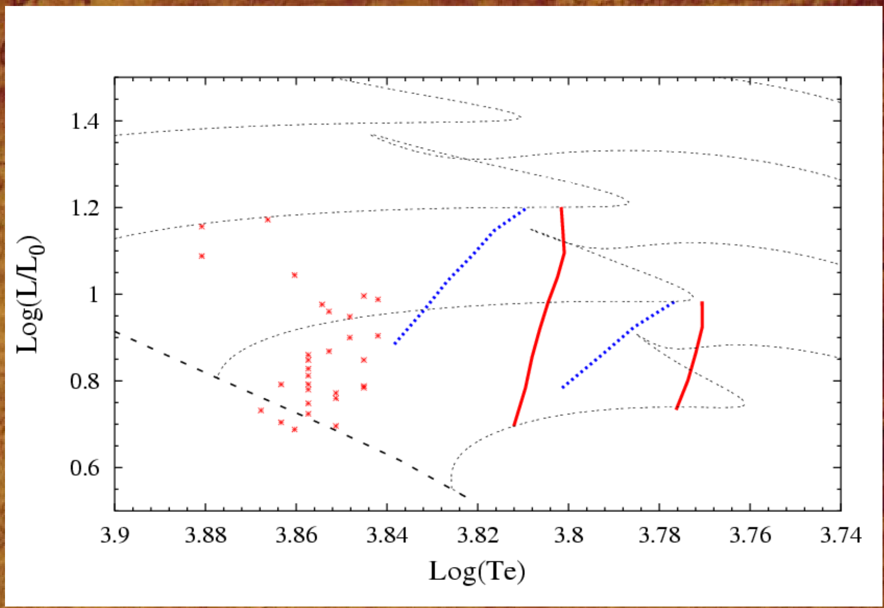


Figure 16

Comparison between δ Scuti Instability Strip (λ=1, P_1) and γ Dor Instability Strip (λ=1) for α=1.5

$$\frac{\delta l}{l} = \frac{1}{1+(\sigma\tau_C)^2}\frac{\delta H_P}{H_P}$$

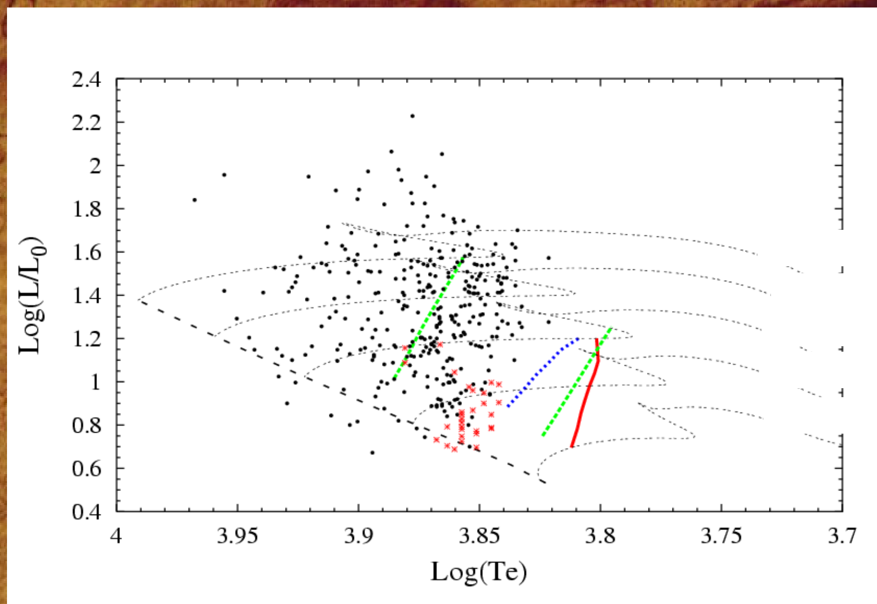
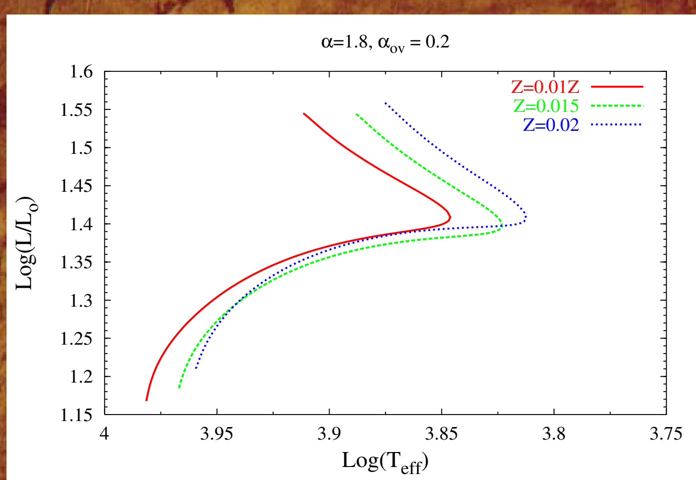


Figure 17

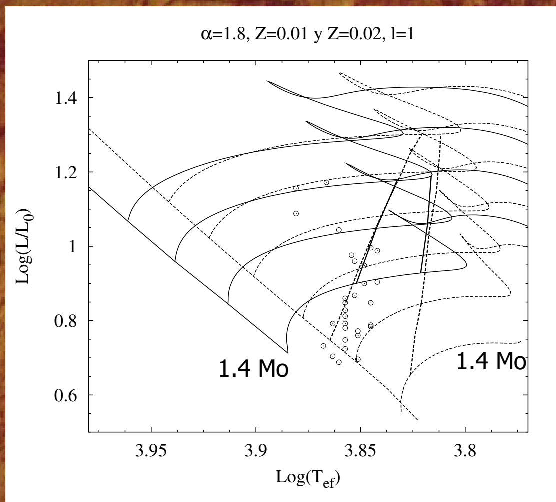
Metallicity



In the internal structure the metallicity has the following effects:

- Great sensibility of ZAMS points to Z.
- Shift of all the track to high effective temperature with larger Z
- The width of the main sequence reduces a little with larger Z.

Effects of different values of metallicity on g Dor Instability strip



- The **γ** Doradus instability strip is not influenced by metallicity.
- We have nearly the same Instability Strip for models with different matallicities.

Comparison: Effects on frequencies

ℓ (1)	n (2)	f_{ad} (3)	Δf_{FC} (4)	$\Delta f_{\delta F_c}$ (5)	$\Delta f_{\delta p_t}$ (6)	$\Delta f_{\delta(p_t,\epsilon_2)}$ (7)	$\Im(\omega_{FC})$ (8)	$\Im(\omega_{\delta F_c})$ (9)	$\Im(\omega_{\delta p_t})$ (10)	$\Im(\omega_{\delta(p_t,\epsilon_2)})$ (11)
					δ Sct: Model 7 of Table 1, $t_{dyn} = 3460$ s					
0	1	1.631E+02	2.322E-02	2.321E-02	2.324E-02	2.322E-02	-6.449E-07	-7.699E-07	-9.807E-07	-8.291E-07
	4	3.125E+02	1.214E-02	-6.325E-03	-7.519E-04	-3.993E-03	-9.658E-05	-3.475E-05	-1.192E-04	-3.061E-05
1	-2	1.197E+02	2.765E-02	2.765E-02	2.765E-02	2.765E-02	-1.170E-09	-9.845E-10	-6.680E-10	-6.906E-10
	3	2.723E+02	9.861E-03	4.281E-03	6.390E-03	5.196E-03	-3.894E-05	-2.907E-05	-6.623E-05	-3.432E-05
2	-4	1.081E+02	1.372E-02	1.372E-02	1.372E-02	1.372E-02	-1.021E-09	-7.092E-10	-1.555E-10	-2.415E-10
	2	3.005E+02	7.918E-03	-4.078E-03	-1.708E-04	-2.358E-03	-6.793E-05	-3.305E-05	-8.948E-05	-2.932E-05
3	-5	1.205E+02	1.919E-02	1.919E-02	1.919E-02	1.919E-02	-2.942E-09	-2.611E-09	-1.329E-09	-1.364E-09
	2	3.172E+02	5.659E-03	-1.603E-02	-9.262E-03	-1.296E-02	-1.106E-04	-3.456E-05	-1.137E-04	-1.155E-05
					γ Dor: Model 5 of Table 1, $t_{dyn} = 3800$ s					
1	-82	3.811E+00	2.039E-04	-2.608E-04	-2.760E-04	-2.619E-04	-1.855E-06	2.166E-07	5.067E-07	-5.007E-08
	-22	1.365E+01	3.082E-03	3.082E-03	3.082E-03	3.082E-03	-6.447E-09	-6.134E-10	3.925E-09	-1.545E-09
	-2	1.181E+02	3.343E-02	3.343E-02	3.343E-02	3.343E-02	-7.644E-09	-8.936E-09	-1.097E-08	-9.551E-09
2	-94	5.752E+00	-1.270E-03	-1.729E-03	-1.746E-03	-1.709E-03	-1.289E-06	6.185E-07	8.926E-07	-1.631E-07
	-23	2.251E+01	4.231E-03	4.230E-03	4.230E-03	4.230E-03	-1.010E-08	-2.919E-09	1.291E-08	-4.308E-09
	-4	1.065E+02	1.800E-02	1.800E-02	1.800E-02	1.800E-02	-5.679E-09	-5.231E-09	-6.481E-09	-4.628E-09
3	-99	7.730E+00	-1.274E-03	-1.687E-03	-1.693E-03	-1.631E-03	-8.644E-07	6.347E-07	1.058E-06	-4.975E-07
	-26	2.852E+01	3.816E-03	3.814E-03	3.813E-03	3.814E-03	-3.132E-08	-9.095E-09	4.206E-08	-9.212E-09
	-6	1.071E+02	1.733E-02	1.733E-02	1.733E-02	1.733E-02	-2.221E-09	-2.226E-09	-2.774E-09	-1.702E-09

Table 4. Frequencies ($f = \Re\{\sigma\}/(2\pi)$) and dimensionless damping rates ($\Im\{\omega\} = \Im\{\sigma\} t_{dyn}$) for different modes of a δ Sct model (top) and a γ Dor model (bottom). Column 3 gives the adiabatic frequency (in μHz). Column 4 gives the difference between the non-adiabatic frequencies of FC models and the adiabatic frequencies. Columns 5 to 7 give the difference between the non-adiabatic frequencies of TDC models and the adiabatic frequencies. Columns 8 to 11 give the dimensionless damping rate of FC models (Column 8) and of TDC models (Columns 9 to 11); it is negative for unstable modes and positive for stable modes. The perturbation of convective flux $\delta \boldsymbol{F}_c$ is taken into account in all the TDC models. The perturbation of turbulent pressure δp_t is taken into account in Columns 6, 7, 10 and 11. The perturbation of turbulent kinetic energy dissipation $\delta \epsilon_2$ is taken into account in Columns 7 and 11.

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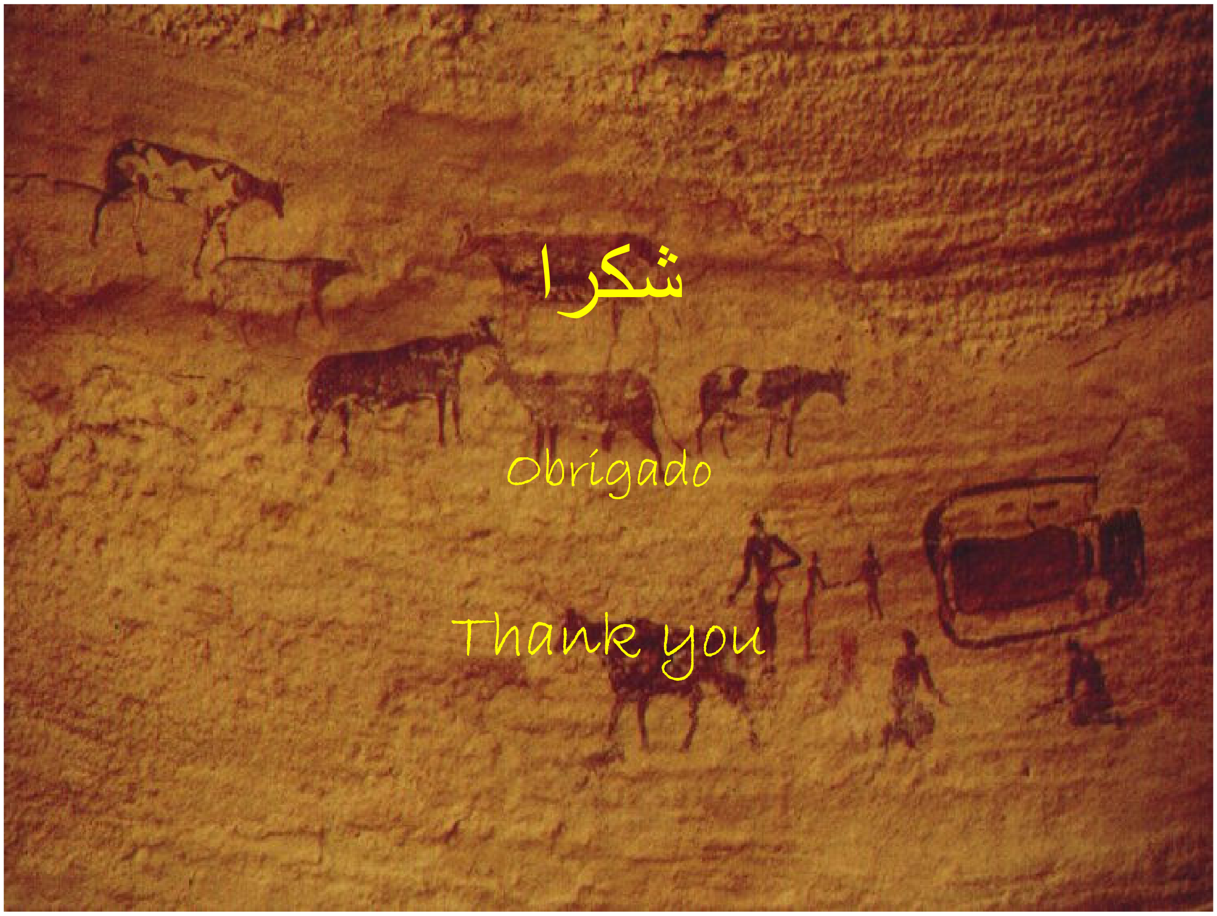
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Obrigado
Thank you