Cosmology and astrophysics of extra dimensions

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Astrophysical tests of fundamental physics
Porto, 27-29 March 2007

Why extra dimensions?

Often appear in the context of unifying gravitation with other interactions

Th. Kaluza and O. Klein

geometric unification of electromagnetism and gravitation

Th. Kaluza (1921) - O. Klein (1926)

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu + 2\, A_\mu\, dx^\mu\, dx^5 + \varphi(x)\, dx^5\, dx^5$$

$A_\mu = g_{\mu 5}$

new field?

Einstein on Kaluza-Klein:

1919 to Kaluza: « At first glance I like your idea enormously »
1926 to Lorentz: « It appears that the union of gravitation and Maxwell's theory is achieved in a completely satisfactory way by the 5-dimensional theory. »
1931 to Ehrenfest: « It is anomalous to replace the 4-dimensional continuum by a 5-dimensional one and then subsequently to tie up artificially one of these dimensions to account for the fact that it does not manifest itself. »

(see A. Pais book)

A matter of scales (1):
unifying gravitation and other gauge interactions

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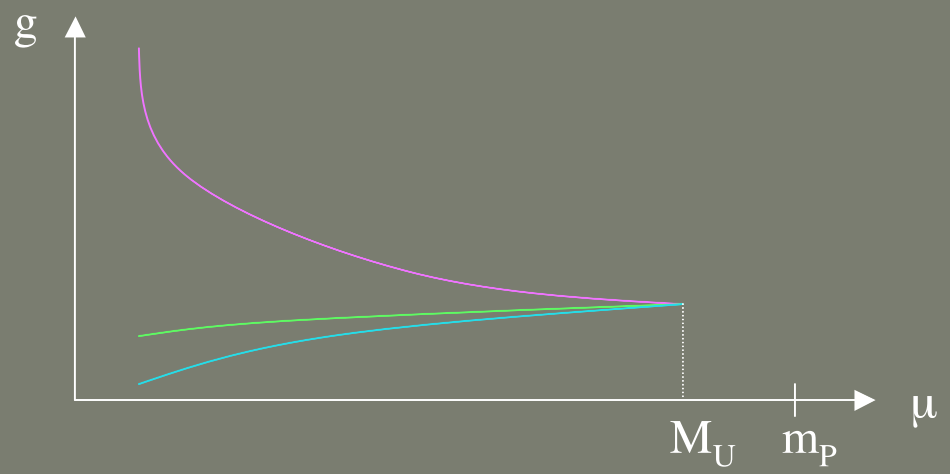
$M_W \ll m_P$

Two roads to unification



$M_W \rightarrow m_P$

Grand unification

requires supersymmetry

$M_W \leftarrow m_P$

Large extra dimensions

For a theory in D=4+n dimensions with n dimensions compactified on a circle of size L :

$$G_N = G_f / L^n$$

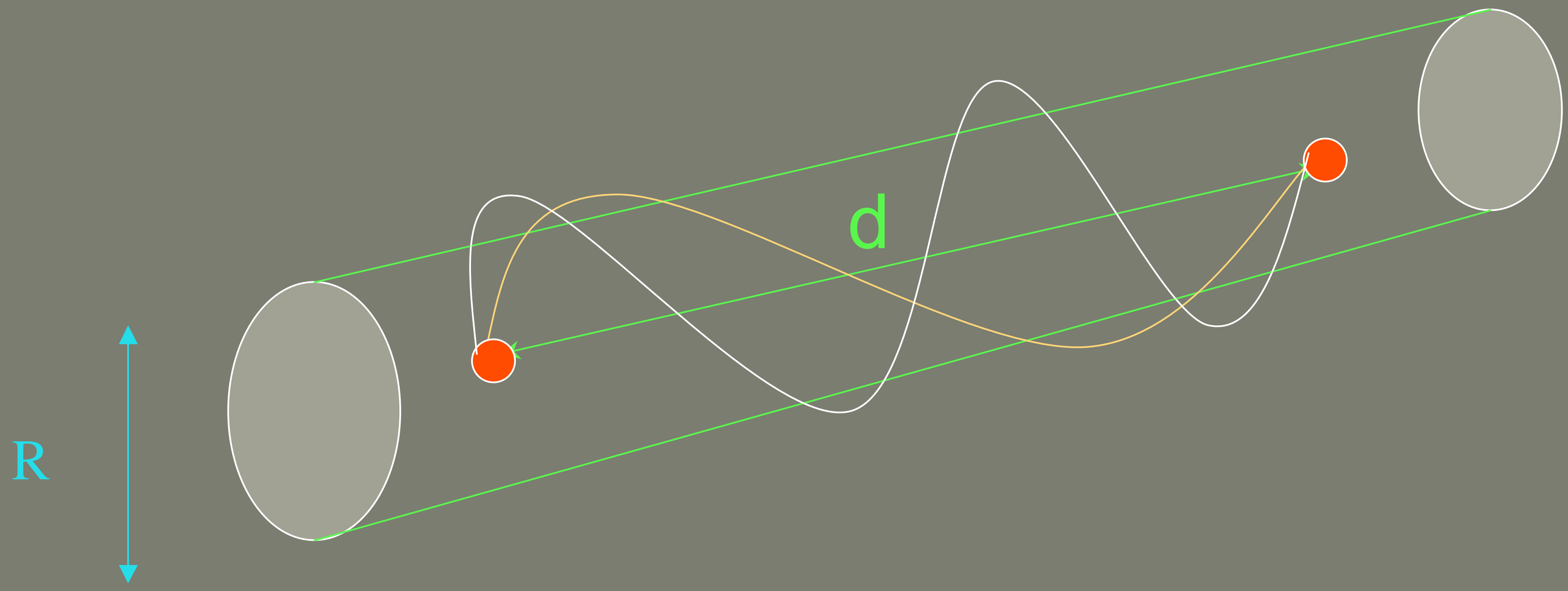
$G_f \equiv M_f^{-(D-2)}$ gravitational constant of the fundamental D-dim. theory

$$m_{Pl}^2 = M_f^{2+n} L^n$$

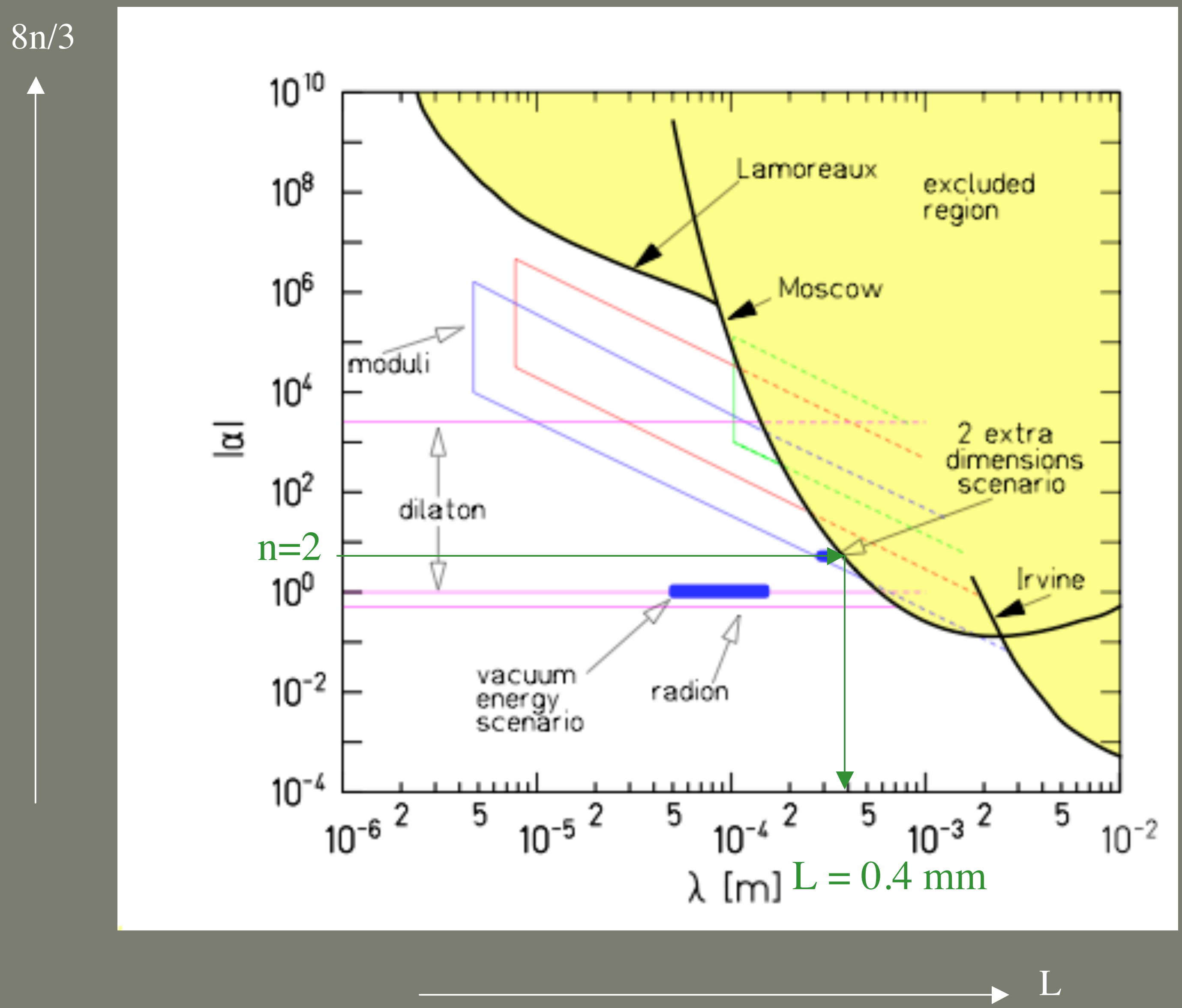
M_f fundamental scale of gravity in D dimensions

Explaining why gravity is so weak ($m_{Pl} \gg M_W$) might amount to explain why some dimensions are large ($M_f \sim M_W$).

At distance d»L, one recovers the standard law in $1/d^2$ for 3 infinite spatial dimensions and 1 or more microscopic dimension.



Hence test gravitational interaction at distances smaller than L.

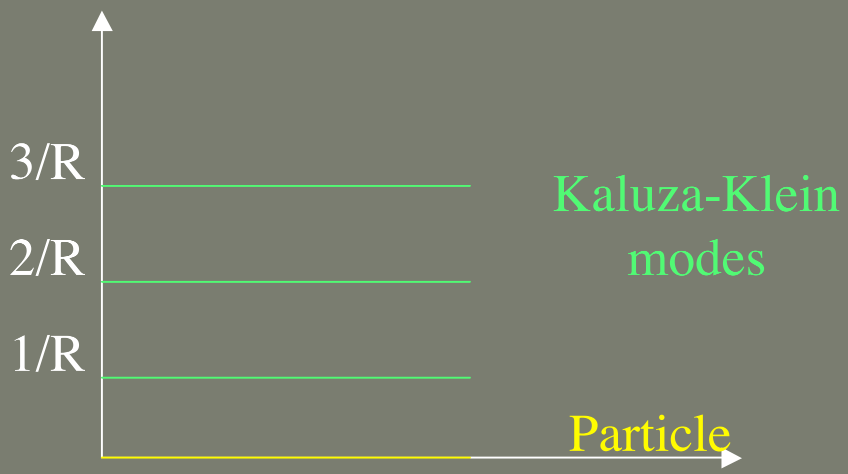


$$\delta V = -G_N \frac{m_1 m_2}{r} \alpha e^{-r/\lambda}$$

Hoyle et al.
hep-ph/0405262

However if L is macroscopic or mesoscopic, how do we reconcile this with the fact that no sign of extra dimensions has been observed in colliders?

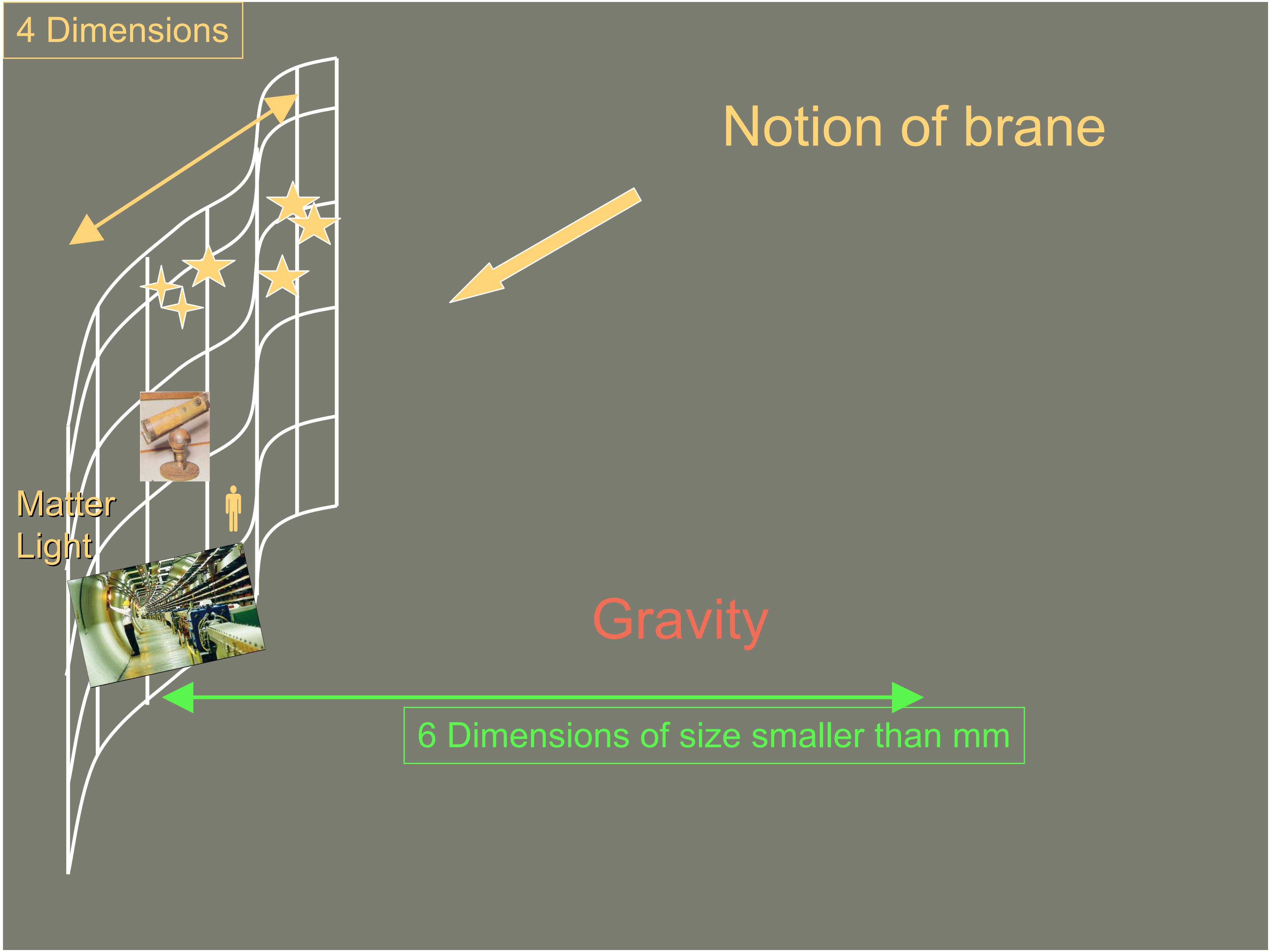
Phenomenology of extra dimensions at high energy colliders:
production of Kaluza-Klein modes



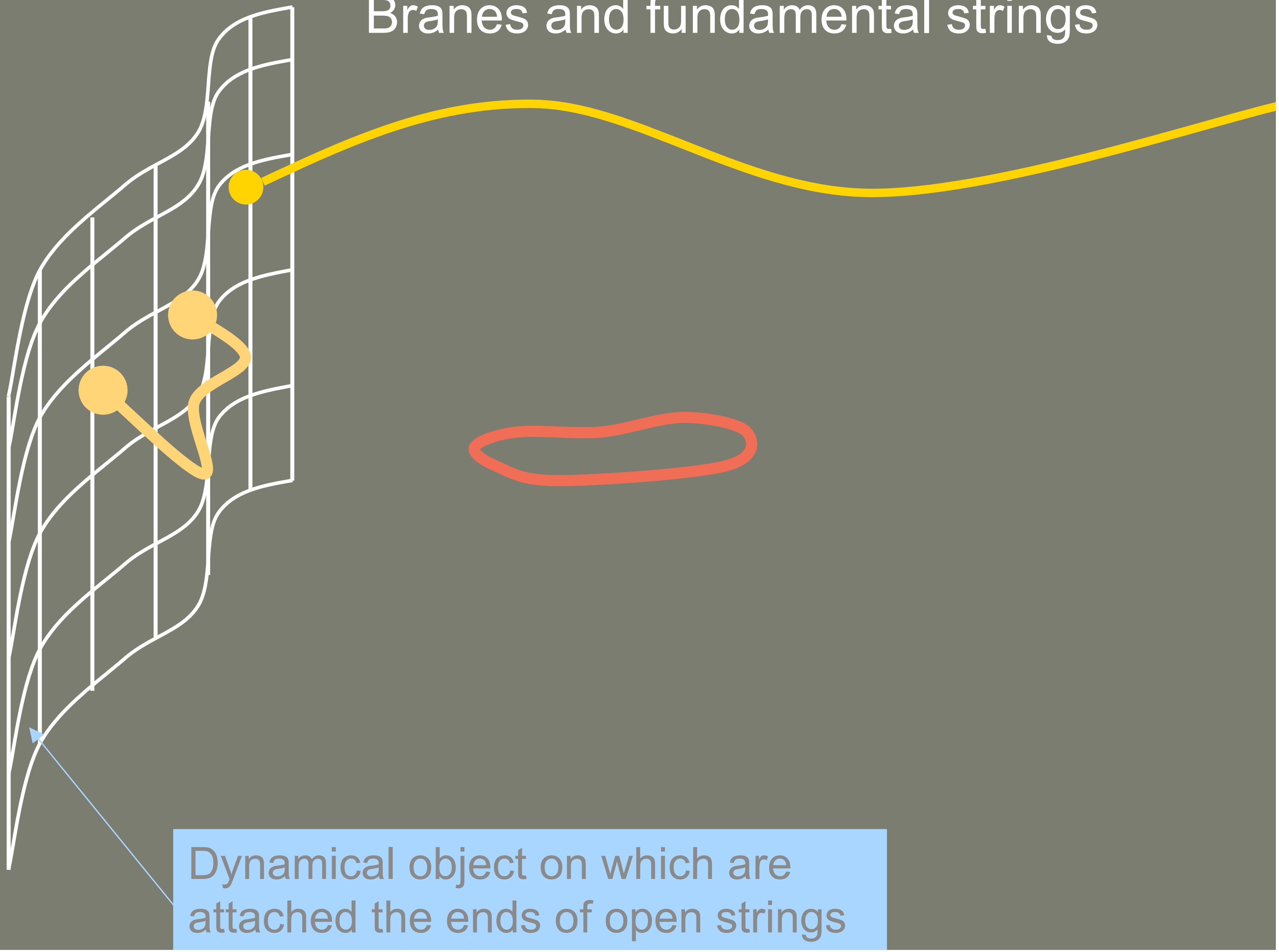
$L^{-1} > 100$ GeV

$L < 10^{-3}$ fm

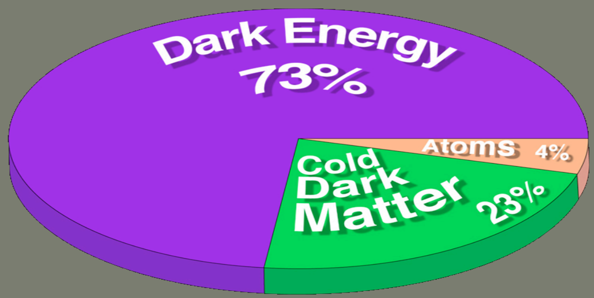
Solution: extra dimensions are only felt by gravitational interactions.


4 Dimensions
Notion of brane
Matter
Light
Gravity
6 Dimensions of size smaller than mm

Branes and fundamental strings



A matter of scale (2): dark energy



If dark energy is the main component in the present energy budget

$$H_0^2 \sim \rho_{DE} / m_P^2$$

$$\boxed{\rho_{DE} \sim H_0^2 m_P^2 \sim (10^{-3}\ \mathrm{eV})^4}$$

If this scale is a fundamental scale, then it is of « gravitational » nature

Note that :

- $$\frac{\hbar c}{10^{-3}\ \mathrm{eV}} \sim 0.1\ \mathrm{mm}$$
- low cut-off ($\Lambda \sim 10^{-3}$eV) may be welcome in the context of the cosmological cst problem

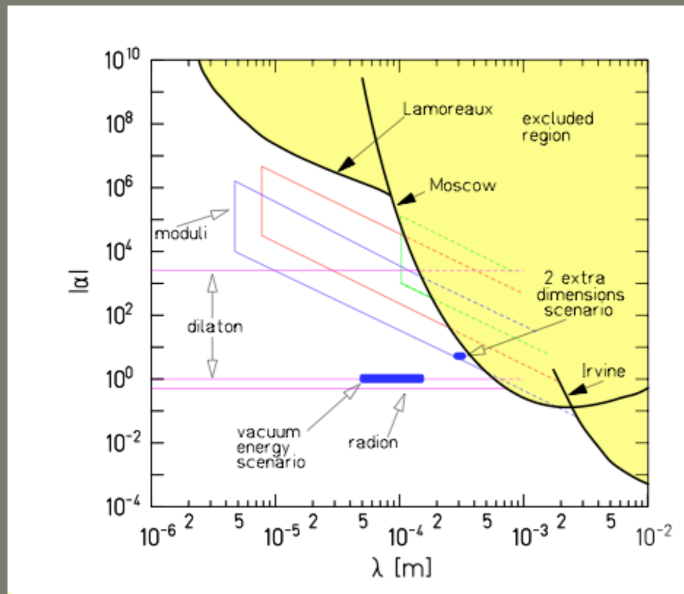
Similar analysis if expansion acceleration is explained by modification of gravity.

If dark energy is a scalar field φ (quintessence), then

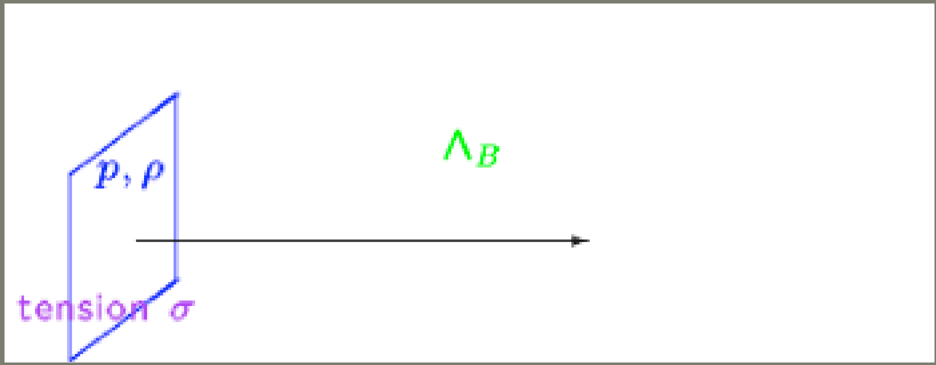
$$m_\varphi \sim H_0 \sim 10^{-33}\ \mathrm{eV}$$

Exchange of φ leads to new long range force (range ~ $H_0 \sim 10^{26}$m)

φ has couplings of subgravitational size to ordinary matter



Cosmological evolution on the brane



5-dimensional Einstein equation + Israel junction condition on the brane
→ Generalized Friedmann equation on the brane:

$$H^2 = \frac{1}{6M_5^3}\Lambda_B + \frac{1}{36M_5^6}\sigma^2 + \frac{1}{18M_5^6}\sigma\rho + \frac{1}{36M_5^6}\rho^2 + \frac{\mathcal{C}}{a^4} - \frac{k}{a_0^2}$$

P.B., Deffayet, Ellwanger, Langlois; Csaki, Graesser Kolda, Terning; Cline, Grojean, Servant

with $8\pi G_5 = M_5^{-3}$

To be compared with the standard Friedmann equation:

$$H^2 = \frac{1}{3}\lambda + \frac{1}{3M_P^2}\rho - \frac{k}{a^2}$$

When should one recover the 4-dimensional picture?

When the physics on the brane is 4-dimensional, i.e.

- if the extra dimension is compact and its radius is stabilized
  e.g.Randall-Sundrum I set up

- if the extra dimension is noncompact but the 4-dimensional graviton is localized   Rubakov,Shaposhnikov,Akama,Gogberashvili,Randall, Sundrum

e.g. in Randall-Sundrum (RSII) case, the extra dimension is warped

$$a(y) = e^{-|y|/\ell}, \quad \ell^2 = \frac{6M_5^3}{|\Lambda_B|} \quad AdS_5 \text{ curvature radius}$$

Minkowski (M_4) constraint: $\frac{1}{3}\lambda = \frac{1}{6M_5^3}\Lambda_B + \frac{1}{36M_5^6}\sigma^2 = 0$

Planck scale : $M_P^2 = M_5^3 \int_{-\infty}^{+\infty} e^{-2|y|/\ell} dy = M_5^3 \ell$

Agrees with the cosmological evaluation:

$$M_P^2 = \frac{6M_5^6}{\sigma} = M_5^3 \left(\frac{6M_5^3}{|\Lambda_B|}\right)^{1/2} = M_5^3 \ell$$

- The importance of the non-conventional ρ^2 term increases as one goes back in time $\rightarrow$ possible role in the early universe

inflation models

- The determination of the Planck scale using the effective low energy and the Friedmann equation may differ!

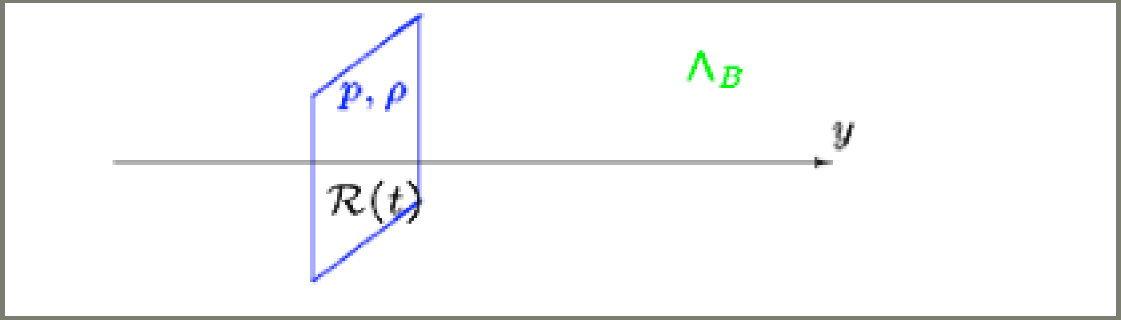
e.g. dS_4 or AdS_4 branes

for a recent discussion, see Mukohyama and Kofman

- the system of 4-dimensional effective gravitational theory is not closed because of bulk gravitational waves and bulk scalars

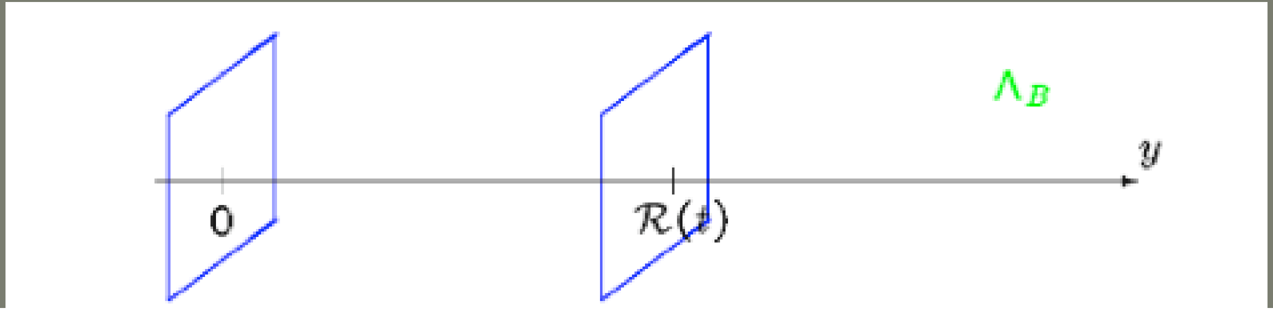
Moduli fields

In an AdS_5 background, choose static coordinates : the cosmological evolution on the brane is purely due to the motion in the bulk



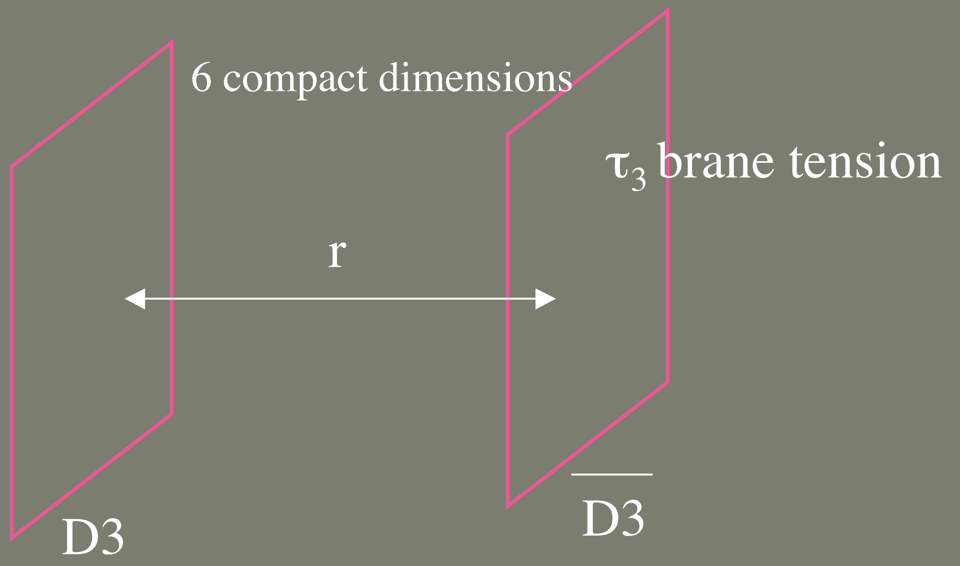
$\mathcal{R}(t)$ is the cosmic scale factor on the brane : the corresponding modulus field provides the semi-classical notion of time on the brane.

In the set up with two branes, the interbrane distance is described by a scalar field or radion.



Brane inflation

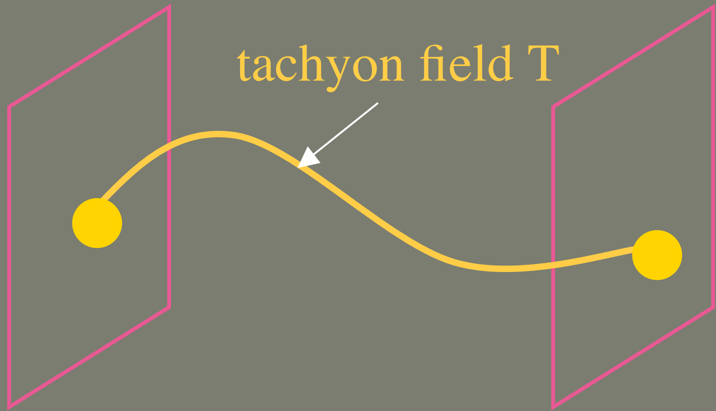
Brane inflation : Brane-antibrane system



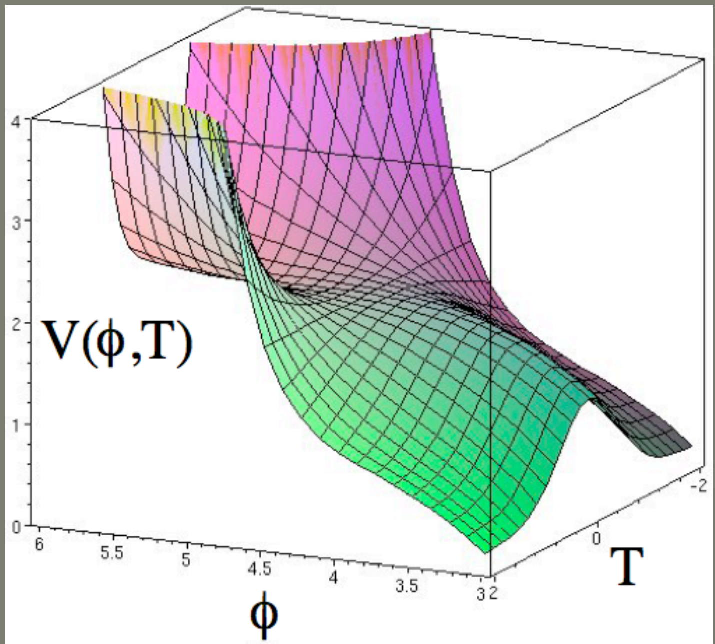
Brane gravitational potential $\propto r^{-4}$

Take the radion as the inflaton field : $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 + 2\left(\tau_3 - \frac{c}{\phi^4}\right)$ $\quad c = 4G_{10}\,\tau_3^{\,4}/\,\pi^2$

But, as $\phi \to 0$, the brane-antibrane system becomes unstable to annihilation into closed strings.



$$m_T^{\,2} = M_S^{\,2} \left(\frac{r^2 M_S^{\,2}}{2\pi^2} - 1 \right)$$

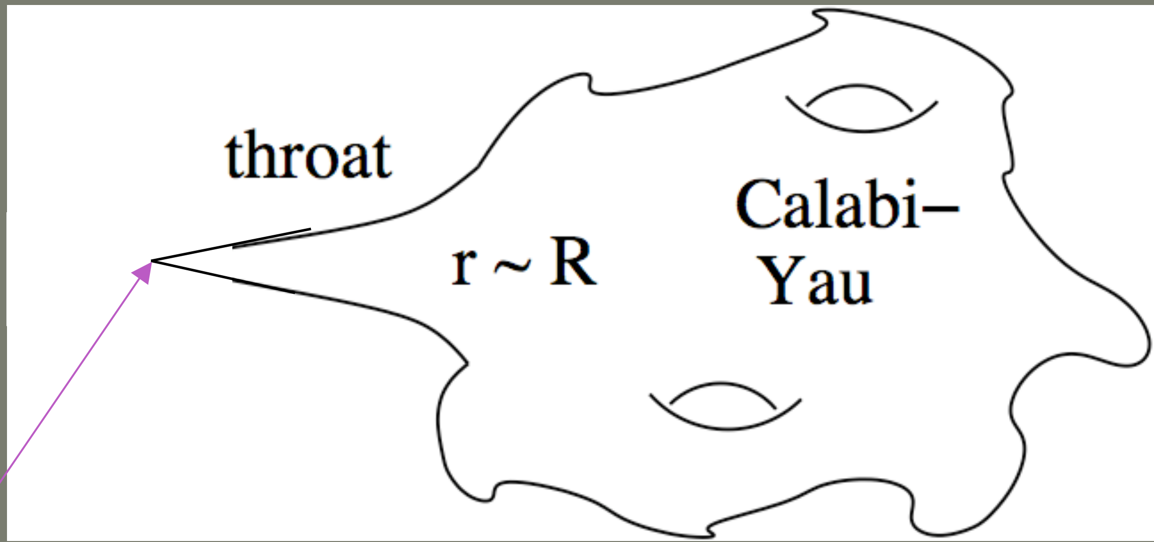


Similar to hybrid inflation

But $\eta = M_S^{12} V_6 / \phi^6 \ll 1$ for $\phi / M_S \gg V_6^{1/6} M_S$

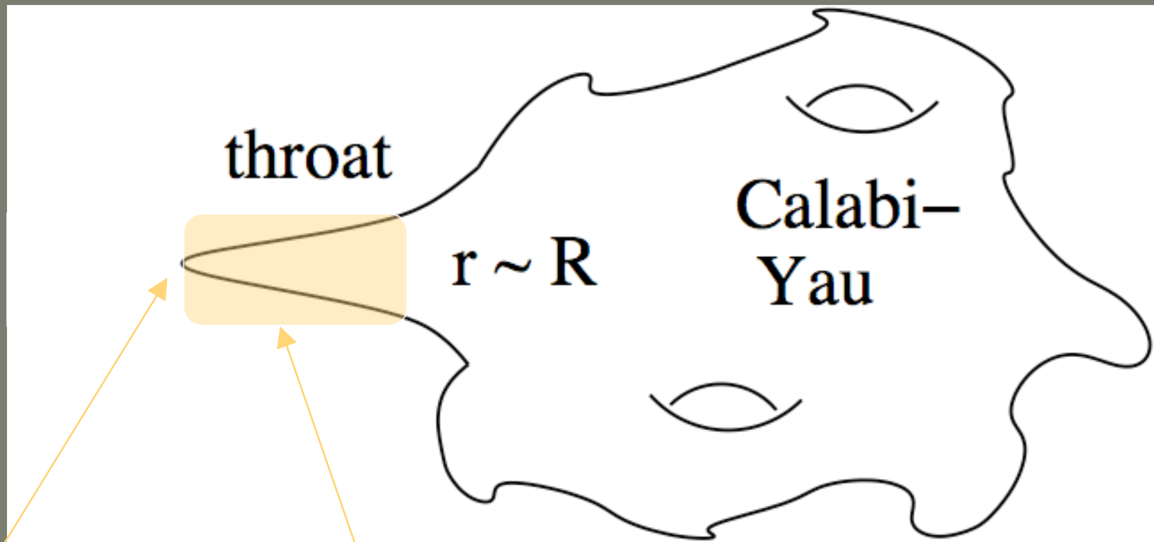
i.e. for r » size of the compact dimension

Go to warped compactification :



Conifold singularity
at r = 0

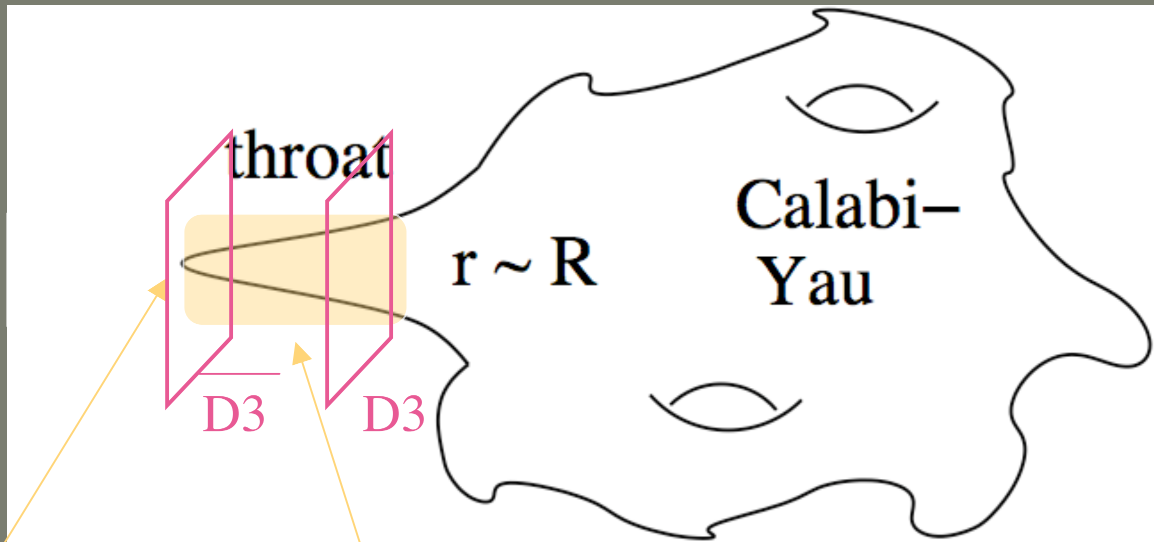
Turn on fluxes



Singularity smoothed out
$r = r_0$

warping $a(r)$

Turn on fluxes



Singularity smoothed out
$r = r_0$

warping $a(r)$

Then warping helps : $\eta \rightarrow a(r_0)^4 \, \eta$

Giddings, Kachru, Polchinski
KKLMMT

BUT the Kahler modulus ρ which fixes the radius of the Calabi-Yau model has not been stabilized.

Kahler potential $K = -3 \ln (\rho + \bar{\rho} - |\phi|^2)$

Scalar potential $V \sim e^K \ldots \Rightarrow m_\phi^2 \sim H^2$

Then famous η problem : $\eta = V''/V = m_\phi^2 / 3H^2 = 2/3$

How to get out of it?

- DBI inflation: new regime where the brane goes as fast as it can in the throat
  power law inflation, requires very high fluxes, non-gausianities
  Alishahiha, Silverstein, Tong
- shift symmetry: $\phi \rightarrow \phi + c$ Firouzjahi, Tye
- D3-D7 brane inflation Dasgupta, Herdeiro, Hirano, Kallosh

D3-D7 system leads to D-term inflatin which avoids the η problem: no e^K factor

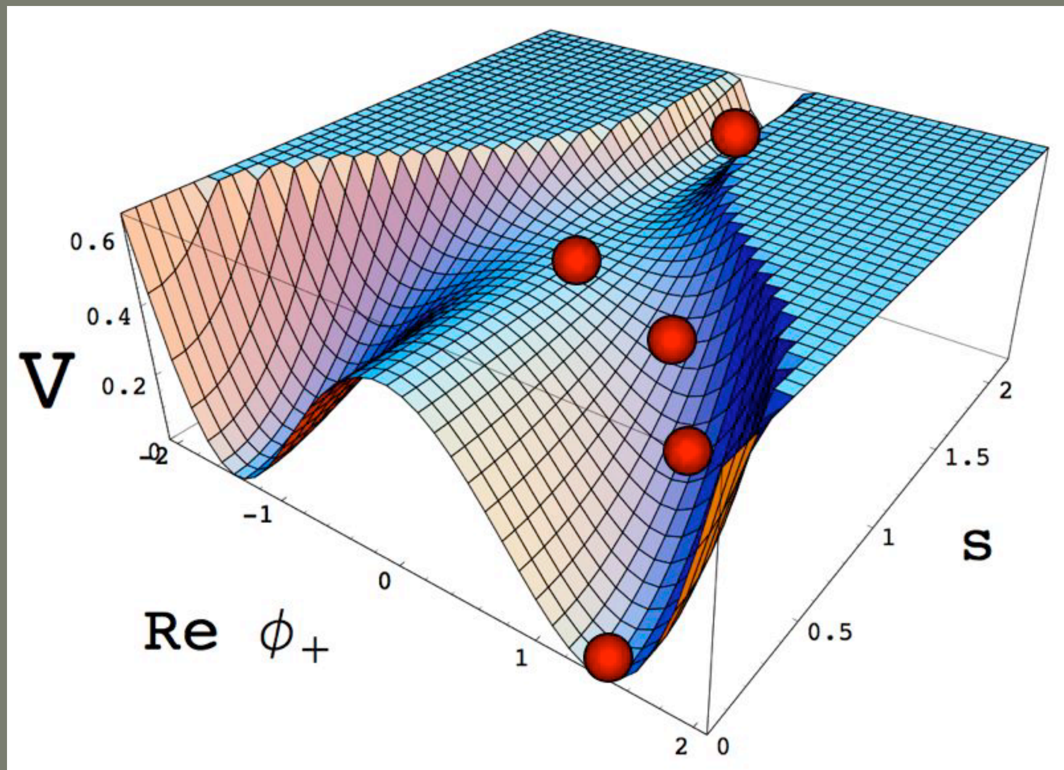
PB, G. Dvali

$K = - 3 \log (\rho + \rho) - (\phi - \bar{\phi})^2/2$

$s = \mathrm{Re}\, \phi$ distance between D3 and D7 brane

Shift symmetry: $s \rightarrow s + c$

Add hypermultiplet of fields $\phi_{\pm}$



Modification of gravity

DGP model

$$\mathcal{S} = \int d^5x\, \sqrt{-g} M_5^3 \frac{1}{2} R^{(5)} + \int_{\text{brane}} d^4x\, \sqrt{-h} M_{\text{Pl}}^2 \frac{1}{2} R^{(4)} + \int_{\text{brane}} d^4x\, \sqrt{-h}\mathcal{L}_m + \mathcal{S}_{GH}$$

For distances $r > r_c$, one recovers the 5-dim. $1/r^3$ behavior:

$$r_c = \frac{M_{\text{Pl}}^2}{2M_5^3}$$

gravity "leakage" into the extra dimension

For all these models, the critical distance must be cosmological.

Cosmology of induced gravity model

Deffayet; Deffayet, Dvali, Gabadadze

$$H^2 = \left(\sqrt{\frac{\rho}{3M_{\rm Pl}^2} + \frac{1}{4r_c^2}} + \frac{1}{2r_c}\right)^2 - \frac{k}{a^2}$$

Hence acceleration at late time, without a need for a cosmological constant!

More precisely, taking flat space, this may be written

$$H^2 - \frac{\epsilon}{r_c}H = \frac{\rho}{3M_{\rm Pl}^2} \ , \epsilon = \pm 1 \ .$$

As long as $H^{-1} \ll r_c$, we have the standard Friedmann equation

$$H^2 = \frac{\rho}{3M_{\rm Pl}^2} \ .$$

But when H^{-1} becomes larger than r_c,

- $\epsilon = +1$

Self-accelerating branch

the final regime is $H \rightarrow H_\infty = 1/r_c$.

- $\epsilon = -1$

the final regime is

$$H^2 = \rho^2 \frac{r_c^2}{9M_{\rm Pl}^4} = \frac{\rho^2}{36 M_5^6} \ .$$

Comparison with observational data

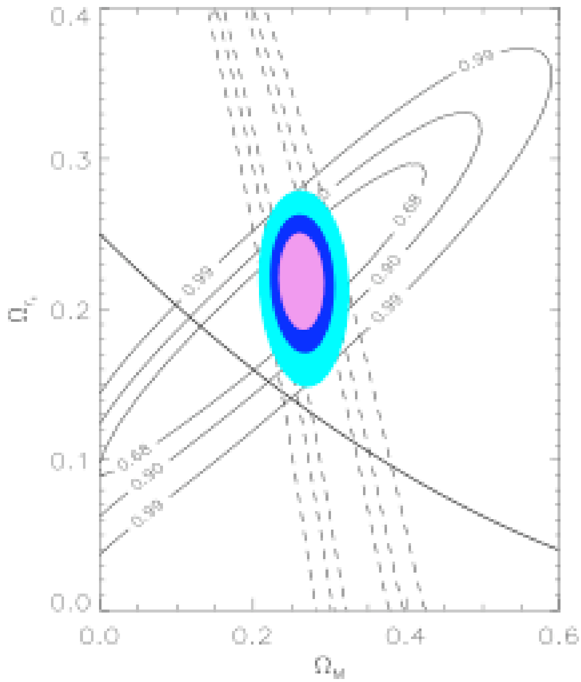
Write the Friedmann equation

$$H(z) = H_0 \left[\sqrt{\Omega_{r_c}^{(0)}} + \sqrt{\Omega_{r_c}^{(0)} + \Omega_m^{(0)}(1+z)^3} \right]$$

with $\Omega_{r_c}^{(0)} = 1/4 r_c^2 H_0^2$.

If we set $z = 0$, we have the normalisation condition:

$$\Omega_{r_c}^{(0)} = (1 - \Omega_m^{(0)})^2)/4 \ .$$



DGP predicts nonstandard gravity at much shorter scales

Deffayet, Dvali, Gabadadze, Vainshtein

Extra scalar graviton polarization changes the Newtonian potential:

For a source of mass M (gravitational radius $r_g = 2M/M_{Pl}^2$), gravity is modified at distances

$$R > r_* = (r_g\, r_c^{\,2})^{1/3} = (2MM_{Pl}^2/M_5^3)^{1/3}$$

$V(r) =$ $-GM/r$ $-4GM/(3r)$ $-4GMr_c/3r^2$

But self-accelrating branch of DGP is plagued with problems

(see e.g. R. Gregory 0801.1603 for a recent discussion)

- ghosts present in perturbation theory
- pressure catastrophies and energy unbounded from below
- destabilisation of 5D vacuum by unsuppressed tunneling processes

Conclusions

Not covered: cyclic universe, BH production at colliders…

Interesting numerical coincidence in favor of large extra dimensions: should be probed in the coming decade

At this stage, more than predictions, the braneworld idea provides a framework for new cosmological models.

Many arenas where such models could show up: cosmological observations, collider searches, tests of gravity…