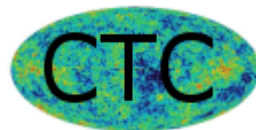


Accelerating Universe from String Field Theory

mainly based on [arxiv:0803.3484\[hep-th\]](https://arxiv.org/abs/0803.3484)
[arxiv:0804.xxxx\[hep-th\]](https://arxiv.org/abs/0804.xxxx)

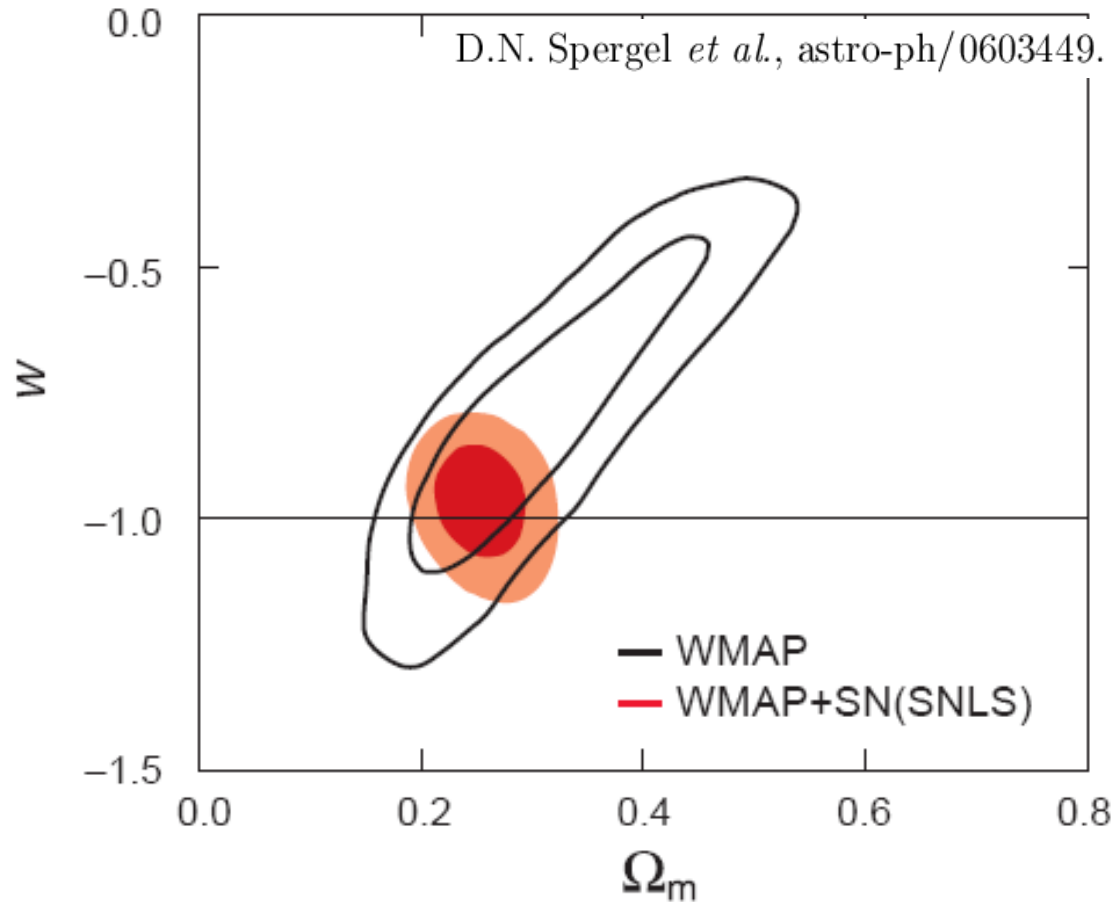
Liudmila Joukovskaya
Centre for Theoretical Cosmology,
DAMTP, Cambridge



Dark Energy State Parameter w

**WMAP3 - Wilkinson
Microwave Anisotropy Probe
+
SNLS - Supernova Legacy
Survey
+
Large scale structure**

$$w = -1.062^{+0.128}_{-0.079}$$



Melchiorri's talk $w = -1 \pm 0.1$

DE Models

Dark energy models

- $w = -1$: the cosmological constant;
- $w = \text{const} \neq -1$: the cosmic strings, domain walls, etc.;
- $w \neq \text{const}$: quintessence scalar field, chaplygin gas, k-essence, Dirac-Born-Infeld (DBI) action, braneworlds, etc.;
- $w < -1$: phantom models.

The most challenging case if $w < -1$.

In this case the weak energy condition ($\rho > 0, \rho + p > 0$) is violated.

Phenomenological DE Models with $w < -1$

- Fluid with $w < -1$ (the Universe at finite time ends in the singularity, “Big Rip”, Caldwell, Kamionkowski, Weinberg, PRL2003)
- Ghost scalar field (*Phantom*) – no “Big Rip”, but instability
- Lorentz-violating background
- Modifications of GR, in particular Brane Cosmology



Phantom as an Effective Theory

A possible way to overcome instability problem for models with $w < -1$ is to get a phantom model as an *effective* one, arising from a more fundamental theory without a negative kinetic term.

Model is based on String Field Theoretic formulation of a fermionic NSR string.

We study nonlocal dynamics of string tachyon in the cosmological Friedman metric.

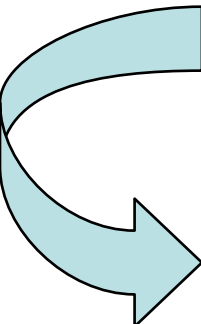
Level-truncated Action for Fermionic String

Witten;

Covariant String Field Theory

I. Aref'eva, P. Medvedev, Zubarev;
Sen, Berkovich, Zwiebach;

Level truncation


$$S = \int d^4x \left[\frac{M_s^4}{g_4} \left(\frac{\kappa^2}{2} \phi(\square_g/M_s^2) \phi + \frac{1}{2} \phi^2 - V(e^{k\square_g/M_s^2} \phi) - \Lambda' - T \right) \right]$$

$$S = \int dx \left[-\frac{\kappa^2}{2} \partial_\mu \phi(x) \partial^\mu \phi(x) + \frac{1}{2} \phi^2(x) - \frac{1}{4} \Phi^4(x) - \Lambda \right]$$

interacting term contains $\Phi(x) = (e^{m\square} \phi)(x)$, $m = \frac{1}{8}$,

$$e^{m\square} \equiv \sum_{n=0}^{\infty} \frac{(m)^{n\square}}{n!}, \quad \text{metric } \square \equiv -\frac{d^2}{dt^2} + \vec{\nabla}^2,$$

The operator $e^{m\square}$ contains infinite number of derivatives (*nonlocal*).

Spatially-Homogeneous Configurations

In the case of spatially-homogeneous configurations $\phi(x) = \phi(t)$ action takes the form

$$S[\phi] = \int dt \left[\frac{\kappa^2}{2} [\partial\phi(t)]^2 + \frac{1}{2}\phi(t)^2 - \frac{1}{4}\Phi^4(t) - \Lambda \right], \quad \Phi = e^{-m\partial^2}\phi, \quad m = \frac{1}{8}, \quad \partial \equiv \frac{d}{dt}.$$

Equation of motion takes the form

$$\left(-\kappa^2 \frac{d^2}{dt^2} + 1\right)(e^{a\partial^2}\Phi)(t) = \Phi(t)^3, \quad a = 2m$$

The operator $e^{a\partial^2}$ for $a > 0$ could be represented in integral form

$$(e^{a\partial^2}\varphi)(t) = \frac{1}{\sqrt{4\pi a}} \int e^{-\frac{(t-\tau)^2}{4a}} \varphi(\tau) d\tau$$

Stress Tensor and Energy Conservation

Definition from the theory of general relativity

$$T_{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\alpha\beta}}$$

Including metric in the action

$$S[\phi] = \int dx \sqrt{-g} \left[\frac{1}{2} \phi^2(x) + \frac{\kappa^2}{2} \phi(x) \square_x \phi(x) - \frac{1}{4} \Phi^4(x) - \Lambda \right],$$

where covariant D'Alembert operator has the form

$$\square_x = \frac{1}{\sqrt{-g}} \partial_{\mu_1} \sqrt{-g} g^{\mu_1 \nu_1} \partial_{\nu_1}.$$

To perform variation of the tiled fields we use the following identity

$$\frac{\delta e^{\hat{A}}}{\delta g^{\alpha\beta}(x)} = \int_0^1 d\rho e^{\rho \hat{A}} \left(\frac{\delta \hat{A}}{\delta g^{\alpha\beta}(x)} \right) e^{(1-\rho)\hat{A}}, \quad \hat{A} \text{ is some operator.}$$

Lemma. For any function $\psi(t)$, which has infinitely many derivatives and has Fourier transform, and for any $a > 0$ the following equality holds

$$e^{a \frac{d^2}{dt^2}} \psi(t) = \frac{1}{\sqrt{4\pi a}} \int_{-\infty}^{\infty} e^{-\frac{(t-\tau)^2}{4a}} \psi(\tau) d\tau$$

Proof. Let

$$\psi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikt} \tilde{\psi}(k) dk,$$

then

$$\begin{aligned} e^{a \frac{d^2}{dt^2}} \psi(t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ak^2} e^{-ikt} \tilde{\psi}(k) dk = \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ak^2} e^{-ikt} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ik\tau} \psi(\tau) d\tau dk = \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(\tau) d\tau \int_{-\infty}^{\infty} e^{-ak^2 + k(-it+i\tau)} dk = \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(\tau) d\tau \sqrt{\frac{\pi}{a}} e^{\frac{i^2(\tau-t)^2}{4a}} = \frac{1}{\sqrt{4\pi a}} \int_{-\infty}^{\infty} e^{-\frac{(\tau-t)^2}{4a}} \psi(\tau) d\tau \end{aligned}$$

this proves the statement.

Stress Tensor

The stress tensor takes the form

$$\begin{aligned}
 T_{\alpha\beta}(x) = & -g_{\alpha\beta} \left(\frac{1}{2}\phi^2 - \frac{\kappa^2}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}\Phi^4 - \Lambda \right) - \kappa^2\partial_\alpha\phi\partial_\beta\phi \\
 & -g_{\alpha\beta} m \int_0^1 d\rho \left[(e^{m\rho\Box_g}\Phi^3)(\Box_g e^{-m\rho\Box_g}\Phi) + (\partial_\mu e^{m\rho\Box_g}\Phi^3)(\partial^\mu e^{-m\rho\Box_g}\Phi) \right] \\
 & + 2m \int_0^1 d\rho (\partial_\alpha e^{m\rho\Box_g}\Phi^3)(\partial_\beta e^{-m\rho\Box_g}\Phi),
 \end{aligned}$$

Note that here and below integration over ρ understand as limit of the following regularization

$$\int_0^1 d\rho f(\rho) = \lim_{\epsilon_1 \rightarrow +0} \lim_{\epsilon_2 \rightarrow +0} \int_{\epsilon_1}^{1-\epsilon_2} d\rho f(\rho).$$

The Energy and Pressure

Energy and Pressure are defined as $E(t) = T^{00}$ and $P(t) = -T_i^i$ correspondingly

$$E(t) = E_k + E_p + \Lambda + E_{nl}, \quad E_{non-loc} = E_{nl_1} + E_{nl_2}$$

where

$$E_k = \frac{\kappa^2}{2} (\partial\phi)^2 \quad E_p = -\frac{1}{2}\phi^2 + \frac{1}{4}(\Phi)^4$$

$$E_{nl_1} = m \int_0^1 d\rho (e^{-m\rho\partial^2} \Phi^3) (e^{m\rho\partial^2} \partial^2 \Phi)$$

$$E_{nl_2} = -m \int_0^1 d\rho (e^{-m\rho\partial^2} \partial\Phi^3) (e^{m\rho\partial^2} \partial\Phi)$$

and

$$P(t) = E_k(t) - E_p(t) - \Lambda - E_{nl_1}(t) + E_{nl_2}(t)$$

or

$$P(t) = -E(t) + 2E_k(t) + 2E_{nl_2}(t)$$

Energy Conservation

Theorem

$$\frac{dE(t)}{dt} = 0,$$

where

$$E(t) = \frac{\kappa^2}{2}(\partial\phi)^2 - \frac{1}{2}\phi^2 + V(\Phi) + \Lambda + m \int_0^1 d\rho (e^{-m\rho\partial^2} \frac{\partial V}{\partial\Phi}) \overleftrightarrow{\partial} (e^{m\rho\partial^2} \partial\Phi),$$

where κ, m are positive constants,

and

$$(-\kappa^2 \frac{d^2}{dt^2} + 1)\phi = e^{-m\partial^2} \frac{\partial V}{\partial\Phi}.$$

Proof

Let us prove the energy conservation directly

$$\frac{dE(t)}{dt} = \kappa^2(\partial\phi)\partial^2\phi - \phi\partial\phi + \frac{\partial V}{\partial\Phi}\partial\Phi + m \int_0^1 d\rho(e^{-m\rho\partial^2} \frac{\partial V}{\partial\Phi}) \overleftrightarrow{\partial^2} (e^{m\rho\partial^2} \partial\Phi).$$

Now using the identity

$$-m \int_0^1 d\rho(e^{-m\rho\partial^2} \varphi) \overleftrightarrow{\partial^2} (e^{-m(1-\rho)\partial^2} \phi) = \varphi \overleftrightarrow{e^{-m\partial^2}} \phi,$$

the equation of motion, and the definition of the field Φ we get

$$\begin{aligned} \frac{dE(t)}{dt} &= \kappa^2(\partial\phi)\partial^2\phi - \phi\partial\phi + \frac{\partial V}{\partial\Phi}\partial\Phi - \frac{\partial V}{\partial\Phi} \overleftrightarrow{e^{-m\partial^2}} \partial\phi = \\ &= \partial\phi \left[\underbrace{(\kappa^2\partial^2 - 1)\phi + e^{-m\partial^2} \frac{\partial V}{\partial\Phi}}_{E.O.M} \right] = 0. \end{aligned}$$

More realistic case including gravity

In this section we would like to consider more realistic case including gravity

$$S = \int d^4x \sqrt{-g} \left(\frac{m_p^2}{2} R + \frac{\kappa^2}{2} \phi \square_g \phi + \frac{1}{2} \phi^2 - \frac{1}{4} \Phi^4 - \Lambda \right),$$

where ϕ is a dimensionless scalar field, $\Phi = e^{m \square_g \phi}$ and $m_p^2 = g_4 \frac{M_p^2}{M_s^2}$, $g_4 = \frac{g_0^2}{v_6} \left(\frac{M_c}{M_s} \right)^6$.

As a particular metric we will consider a FRW one

$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2),$$

for which the Beltrami-Laplace operator for spatially-homogeneous configurations takes the form $\square_g = -\partial^2 - 3H(t)\partial = -\mathcal{D}_H^2$.

Equation of motion for the scalar field Φ takes the form

$$(\xi^2 \square_g + 1) e^{-2m \square_g \Phi} = V'(\Phi).$$

The Friedmann equations have the following form

$$3H^2 = \frac{1}{m_p^2} \mathcal{E}$$
$$3H^2 + 2\dot{H} = -\frac{1}{m_p^2} \mathcal{P}.$$

Numerical Scheme for Solution Construction

For numerical calculations we operate with scalar field equation of motion and the difference of equations

$$(-\kappa^2 \mathcal{D}_H^2 + 1)e^{2k\mathcal{D}_H^2}\Phi = \Phi^3, \quad \dot{H} = -\frac{1}{2m_p^2} (\mathcal{P} + \mathcal{E}).$$

The outline of the numerical scheme is the following

- For equations we introduce lattice in t variable and then solve resulting system of nonlinear equations using iterative relaxation solver using discrete L_2 norm to control error tolerance.
- The nontrivial thing from computational point of view is efficient evaluation of $e^{2k\rho\mathcal{D}_H^2}\Phi$ for $\rho \in [0, 2]$. This operator could be interpreted in terms of initial value problem for the following diffusion equation with boundary conditions

$$\partial_\rho \varphi(t, \rho) = \partial_t^2 \varphi(t, \rho) + 3H(t) \partial_t \varphi(t, \rho),$$

$$\varphi(0, t) = \Phi(t), \quad \varphi(\rho, \pm\infty) = \Phi(\pm\infty).$$

Once solution of this equation is constructed we have $e^{2k\rho\mathcal{D}_H^2}\Phi(t) = \varphi(\rho, t)$.

Numerical Scheme for Solution Construction - continuation

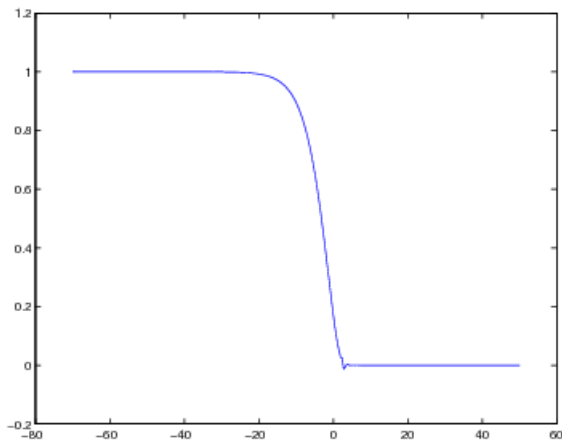
- To solve it we used second order Crank-Nicholson scheme which is based on approximation

$$e^{2k\Delta_\rho\tilde{\mathcal{D}}_H^2}\varphi = \left(1 + k\Delta_\rho\tilde{\mathcal{D}}_H^2\right)\left(1 - k\Delta_\rho\tilde{\mathcal{D}}_H^2\right)^{-1}\varphi + o(\Delta_\rho^2\|\tilde{\mathcal{D}}_H^2\|),$$

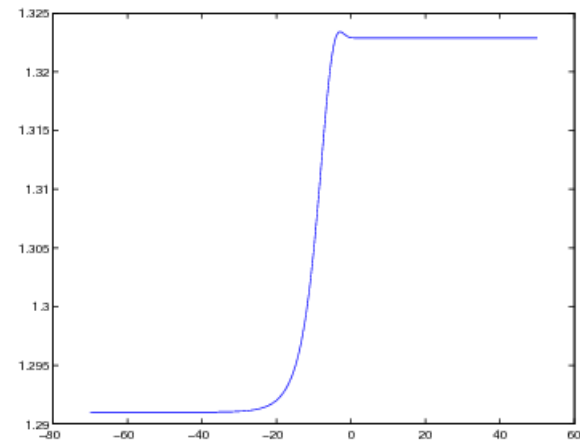
where $\tilde{\mathcal{D}}_H^2$ is a \mathcal{D}_H^2 operator on the t -lattice (it thus has a finite norm) and Δ_ρ is a step size along ρ variable. Derivatives in t variable were approximated using 4th order finite differences on uniform lattice (symmetric scheme).

- In order to exclude possible artifacts of this specific numerical scheme we tried Chebyshev-pseudospectral method which is known to have impressive exponential convergence. This scheme is known to have very different properties compared to finite difference scheme described above, but it produced the same results up to the approximation error which gives us confidence in the existence of the rolling solution reported in this work.

Numerical Solution for Friedman

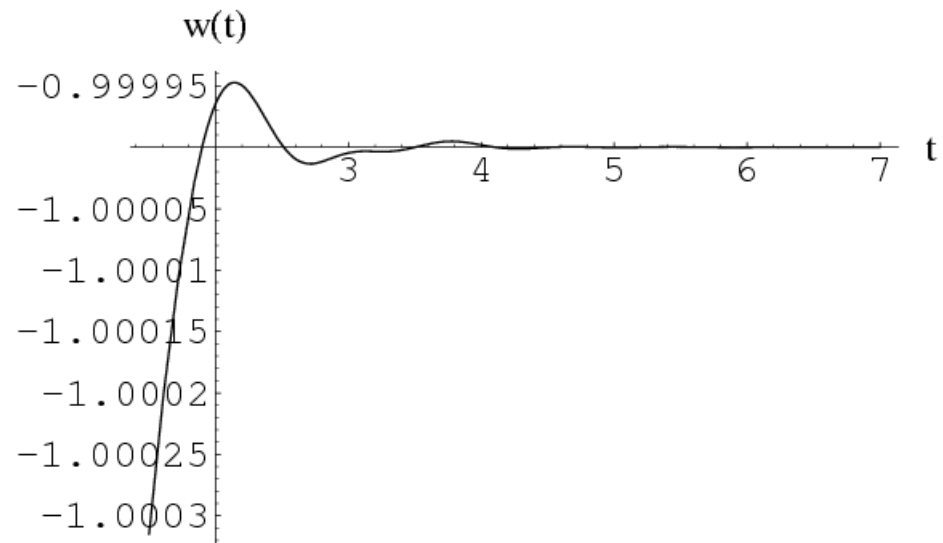
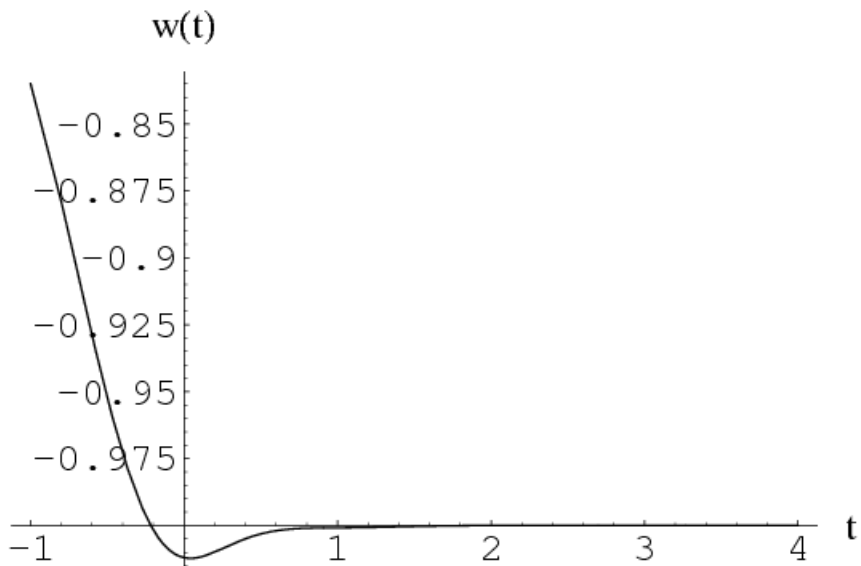
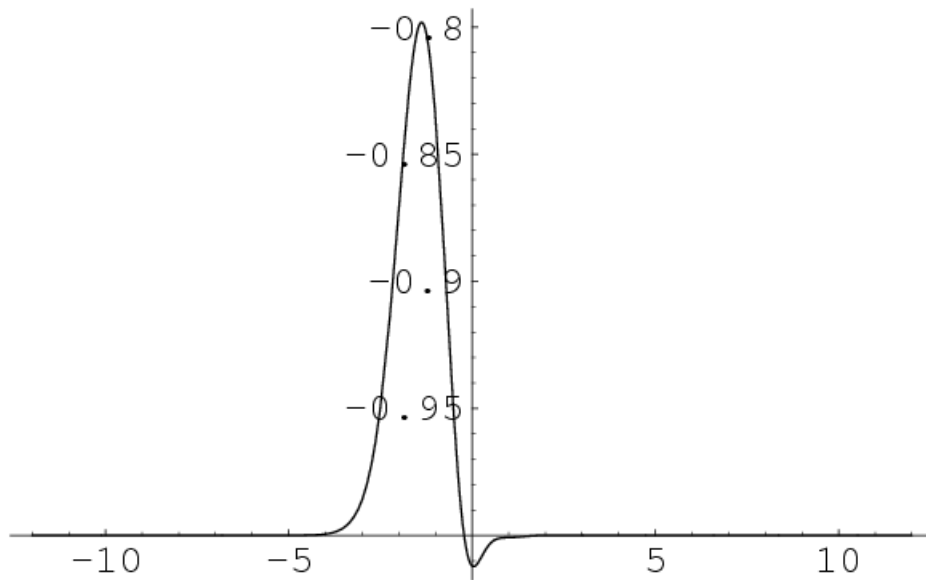


a).



b).

W



Hubble Parameter

$$H \propto m_p^{-2}$$

$$m_p^2 = \frac{M_p^2}{M_s^2} \frac{g_o^2}{V_6} \left(\frac{M_c}{M_s} \right)^6$$


$$M_c \propto M_p$$

$$H \propto M_p \left(\frac{M_s}{M_p} \right)^9$$

$$M_s \propto 10^{-6.6} M_p$$

$$H \propto 10^{-60} M_p$$

M_s – string scale, M_p – Planck scale



At the same time there have been a number of attempts to realize description of the early Universe via nonlocal cosmological models.

One example is p -adic inflation model **Barnaby, Biswas, Cline 2006** which is represented by nonlocal p -adic string theory coupled to gravity. For this model, a rolling inflationary solution was constructed and the interesting features were discussed and compared with cosmic microwave background (CMB) observations. The possibility of obtaining large nongaussian signatures in the CMB has also been considered in a general class of single field nonlocal hill-top inflation models **Barnaby, Cline 2007**.

Another example is investigation of the inflation near a maximum of the nonlocal potential when non-local derivative operators are included in the inflaton Lagrangian. It was found that higher-order derivative operators in the inflaton Lagrangian can support a prolonged phase of slow-roll inflation near a maximum of the potential **Lidsey, 2007**.

Cosmology effaces differences between diff. stringy approaches

Witten's Cubic Open String Field Theory

A. Sen, JHEP, 04 (2002) 048
G.W. Gibbons, Phys. Lett. B 537 (2002) 1,
Class. Quant. Grav. 20 (2003) S321

$$S = -\frac{1}{g_0^2} \int \left(\frac{1}{2\alpha'} \Phi * Q_B \Phi + \frac{1}{2} \Phi * \Phi * \Phi \right)$$

E. Witten, Nucl. Phys. B 268 (1986) 253
V.A. Kostelecky and S.Samuel, Phys. Lett. B 207 (1988) 169
V.A. Kostelecky and S.Samuel, Phys. Lett. B 207 (1988) 169
N. Moeller, A. Sen, B. Zwiebach, JHEP, 08 (2000) 039

$$S = \frac{1}{g_0^2} \int d^D x \left[\frac{1}{2\alpha'} \phi (\alpha' \partial_\mu \partial^\mu + 1) \phi - \frac{\lambda}{3} (e^{\frac{\ln \lambda}{3} \square} \phi)^3 - \Lambda \right]$$

I. Aref'eva, L.J., JHEP 2005
G. Calcagni, JHEP 05 (2006) 012
N. Barnaby, T. Biswas, J.M. Cline, JHEP, 2007
J.E. Lidsey, Phys. Rev.D, 2007
L.J., Phys. Rev.D, 2007
N. Barnaby, J.M. Cline, JCAP, 2007; arXiv: 0802.3218
D. Mulryne

N. Moeller, B. Zwiebach, JHEP, 10 (2002) 034
H. Yang, JHEP 11 (2002) 007
I. Aref'eva, L.J., A. Koshelev, JHEP, 09 (2003) 012
G. Calcagni, JHEP 05 (2006) 012
V. Forini, G. Grignani, G. Nardelli, JHEP 03 (2005) 079

Model / Minkowski case

$$S = \int d^4x \left(\frac{m_p^2}{2} R + \frac{1}{2} \phi \square_g \phi + \frac{1}{2} \phi^2 - \frac{\lambda}{3} \Phi^3 - \Lambda' \right)$$

where $\lambda = \frac{3^{9/2}}{2^6} \approx 2.19$, $\Lambda' = (6\lambda^2)^{-1}$, ϕ is a dimensionless scalar field,
 $\Phi = e^{k \square_g \phi}$, $k = \frac{\ln \lambda}{3} \approx 0.26$, $m_p^2 = g_4 \frac{M_p^2}{M_s^2}$ and $\square_g = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$.

Equation of motion

$$(\square + 1)e^{-2k \square} \Phi = \lambda \Phi^2$$

For spatially homogeneous configurations $\square_g = -\partial^2$.

Energy

The Energy is defined as $E(t) = T^{00}$ and for our model have the form

$$\mathcal{E} = \mathcal{E}_k + \mathcal{E}_p + \Lambda' + \mathcal{E}_{nl1} + \mathcal{E}_{nl2}$$

$$\mathcal{E}_k = \frac{1}{2}(\partial\phi)^2, \quad \mathcal{E}_p = -\frac{1}{2}\phi^2 + \frac{\lambda}{3}\Phi^3$$

$$\mathcal{E}_{nl1} = k \int_0^1 d\rho (e^{k\rho\Box} \lambda \Phi^2) (-\Box e^{-k\rho\Box} \Phi),$$

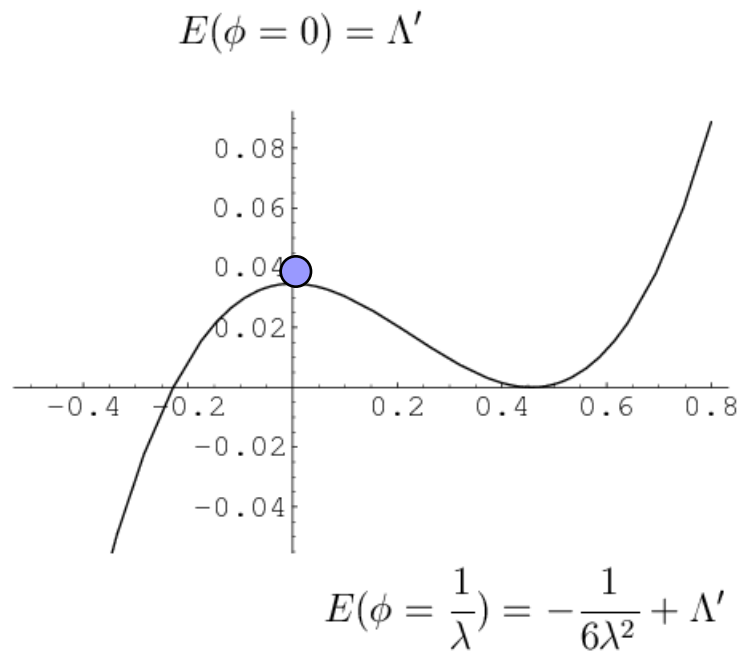
$$\mathcal{E}_{nl2} = -k \int_0^1 d\rho (\partial e^{k\rho\Box} \lambda \Phi^2) (\partial e^{-k\rho\Box} \Phi).$$

To avoid calculation of $e^{k\rho\Box}$ term which is much harder to compute than $e^{-k\rho\Box}$ ($k > 0$) as computation of the former results in an ill-posed problem we will use the following representation for nonlocal energy terms E_{nl1} and E_{nl2} on the equation of motion for the scalar field

$$\mathcal{E}_{nl1} = k \int_0^1 d\rho ((\Box + 1)e^{-(2-\rho)k\Box} \Phi) (-\Box e^{-k\rho\Box} \Phi),$$

$$\mathcal{E}_{nl2} = -k \int_0^1 d\rho (\partial(\Box + 1)e^{-(2-\rho)k\Box} \Phi) (\partial e^{-k\rho\Box} \Phi).$$

Does exist the rolling tachyon solution in this case?



I. Aref'eva, L.J., JHEP 2005
 G. Calcagni, JHEP 05 (2006) 012
 N. Barnaby, T. Biswas, J.M. Cline, JHEP, 2007
 J.E. Lidsey, Phys. Rev.D, 2007
 N. Barnaby, J.M. Cline, JCAP, 2007

Coupling to the gravity / FRW case

$$S = \int d^4x \sqrt{-g} \left(\frac{m_p^2}{2} R + \frac{1}{2} \phi \square_g \phi + \frac{1}{2} \phi^2 - \frac{\lambda}{3} \Phi^3 - \Lambda' \right)$$

Scalar field and Friedmann equations:

$$(-\mathcal{D}_H^2 + 1)e^{2k\mathcal{D}_H^2\Phi} = \lambda\Phi^2, \quad \mathcal{D}_H^2 = \partial_t^2 + 3H(t)\partial_t,$$

$$3H^2 = \frac{1}{m_p^2} \mathcal{E}, \quad 3H^2 + 2\dot{H} = -\frac{1}{m_p^2} \mathcal{P},$$

$$\mathcal{E} = \mathcal{E}_k + \mathcal{E}_p + T + \Lambda' + \mathcal{E}_{nl1} + \mathcal{E}_{nl2},$$

$$\mathcal{P} = \mathcal{E}_k - \mathcal{E}_p - T - \Lambda' - \mathcal{E}_{nl1} + \mathcal{E}_{nl2},$$

$$\mathcal{E}_k = \frac{\xi^2}{2} (\partial\phi)^2, \quad \mathcal{E}_p = -\frac{1}{2}\phi^2 + \frac{\lambda}{3}\Phi^3,$$

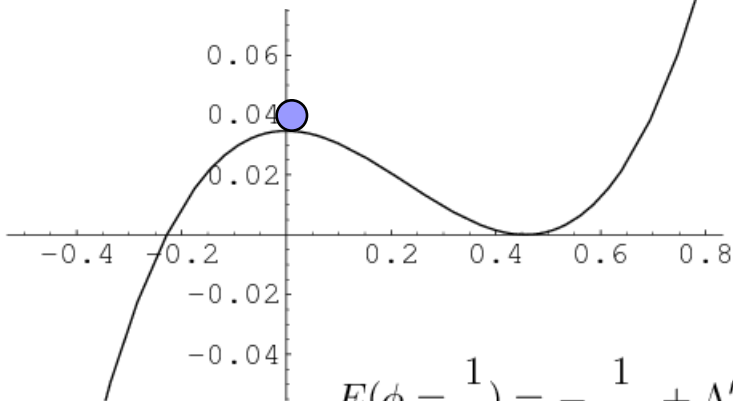
$$\mathcal{E}_{nl1} = k \int_0^1 d\rho \left((-\mathcal{D}_H^2 + 1)e^{(2-\rho)k\mathcal{D}_H^2\Phi} \right) \left(\mathcal{D}_H^2 e^{k\rho\mathcal{D}_H^2\Phi} \right),$$

$$\mathcal{E}_{nl2} = -k \int_0^1 d\rho \left(\partial(-\mathcal{D}_H^2 + 1)e^{(2-\rho)k\mathcal{D}_H^2\Phi} \right) \left(\partial e^{k\rho\mathcal{D}_H^2\Phi} \right).$$

Do we have the rolling tachyon solution in this case?

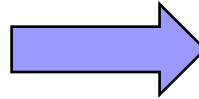
$$H(\phi = 0) = \sqrt{\frac{\Lambda'}{3m_p^2}}$$

$$E(\phi = 0) = \Lambda'$$



$$E\left(\phi = \frac{1}{\lambda}\right) = -\frac{1}{6\lambda^2} + \Lambda'$$

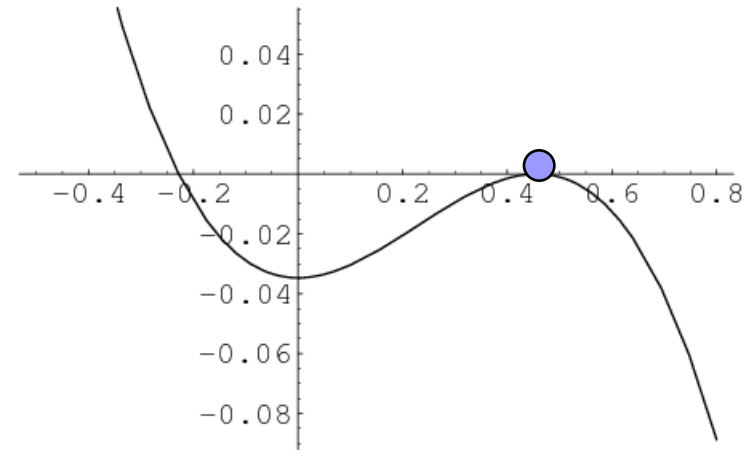
$$H\left(\phi = \frac{1}{\lambda}\right) = \sqrt{\frac{-\frac{1}{6\lambda^2} + \Lambda'}{3m_p^2}} = 0$$



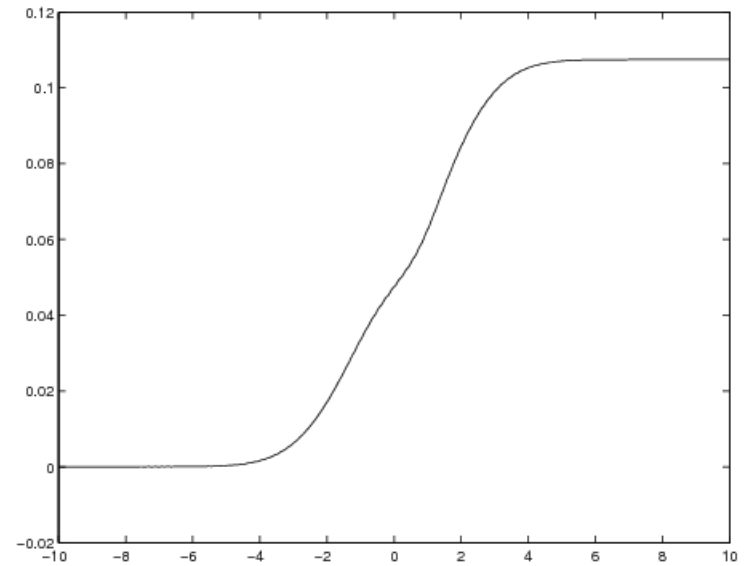
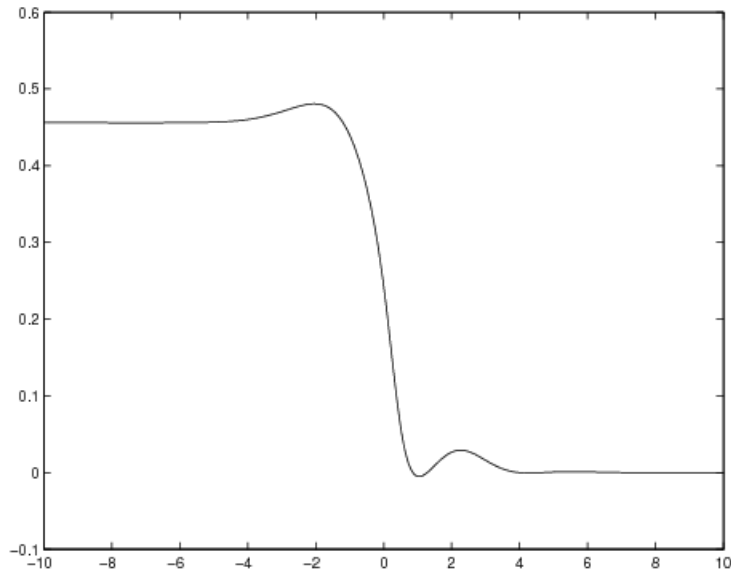
$$(-\mathcal{D}_H^2 + 1)e^{2k\mathcal{D}_H^2\Phi} = \lambda\Phi^2, \quad \mathcal{D}_H^2 = \partial_t^2 + 3H(t)\partial_t$$

$$\partial^2\Phi = -\frac{\Phi - \lambda\Phi^2}{(2k-1)} - 3H\partial\Phi$$

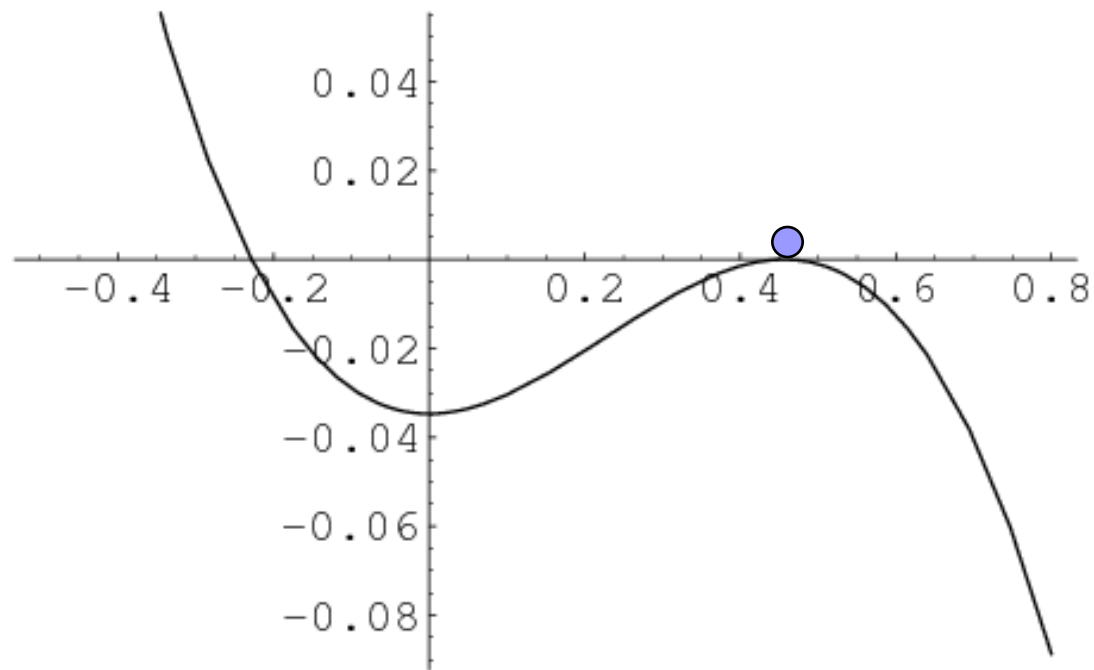
$$V(\Phi) = \frac{-\frac{1}{2}\Phi^2 + \frac{\lambda}{3}\Phi^3}{(1-2k)}$$



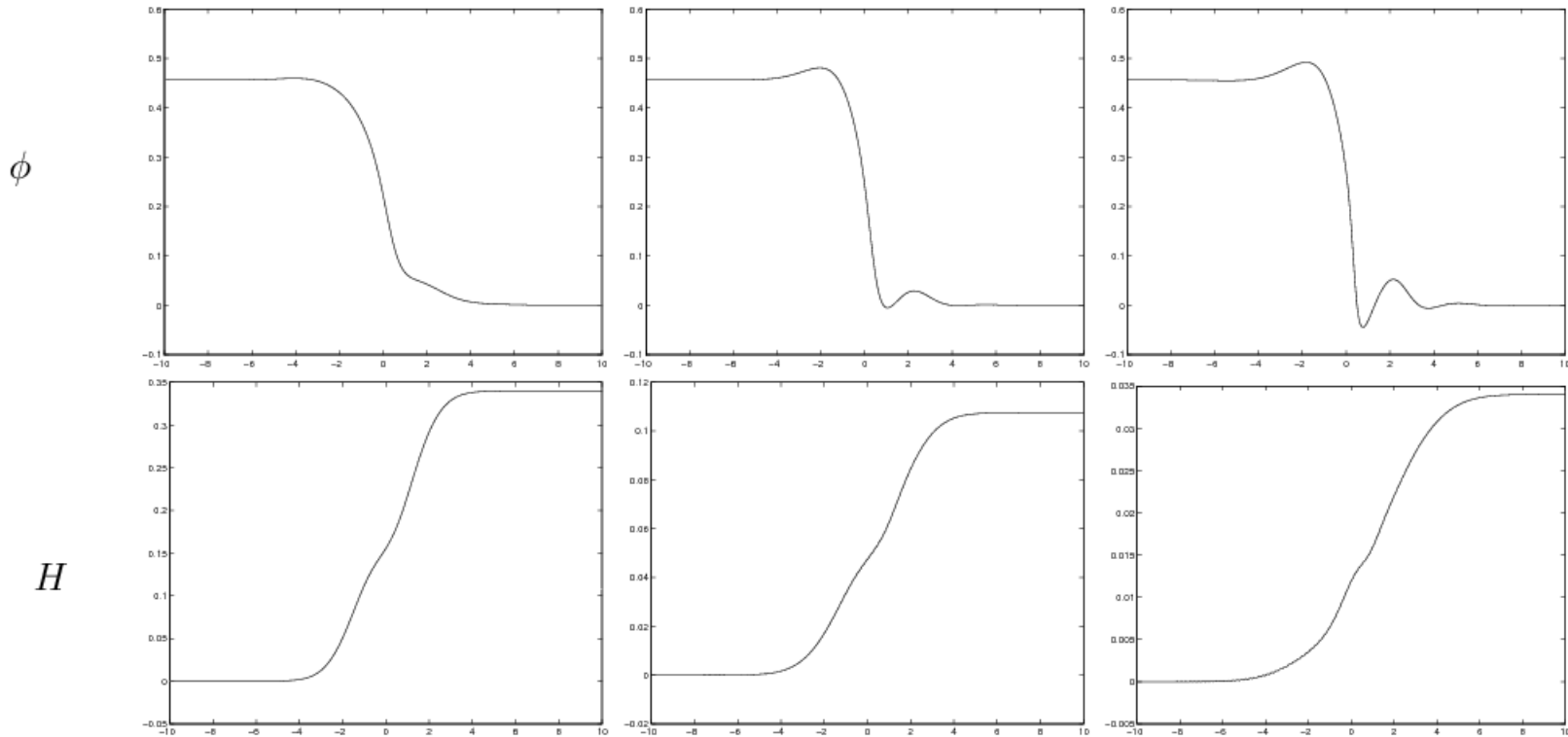
Numerical solutions



Dynamics of the scalar field for different parameters of the system



Numerical solutions for different parameters of the system



$$m_p^2 = 0.1.$$

$$m_p^2 = 1.$$

$$m_p^2 = 10.$$

Conclusions

- We have studied the properties of nonlocal cosmological models driven by String Field Theory in the Friedmann space-time
- We obtain classical solutions of the corresponding Friedmann equations which can be considered as a first approximation to the quantum solutions and might be useful for the study of ways to avoid the cosmological singularity problem.
- Model assumptions
 - level truncation approximation
 - direct generalization of the tachyon nonlocal action to Friedmann space-time
- It would be interesting to try to find an analog of these solutions to a full SFT theory (without level truncation).
- Development of the perturbation theory on the models with infinitely many derivatives



Thank you for the attention!