Inflation within MSSM

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Inflaton is a great idea …but many open questions

particle theory origins?

-usually assumed to be gauge singlet

(radiative corrections can spoil flatness)

- Not motivated by theory: no known gauge singlets

coupling to baryons?

- reheating/origin of matter

If singlet, no testable consequences in the lab

- how can ever make sure inflation is correct?

Allahverdi, KE, Garcia-Bellido, Mazumdar;
Allahverdi, KE, Garcia-Bellido, Jokinen, Mazumdar

Inflation from MSSM

- MSSM has scalars: sleptons, squarks, higgses
- low scale → low H; field values << M_P
- perturbations ~ H/ε → need very flat potential
- but then: radiative corrections can spoil everything

need extra protection

susy, gauge symmetry → directions in MSSM scalar field space with V=0 in the limit of exact susy

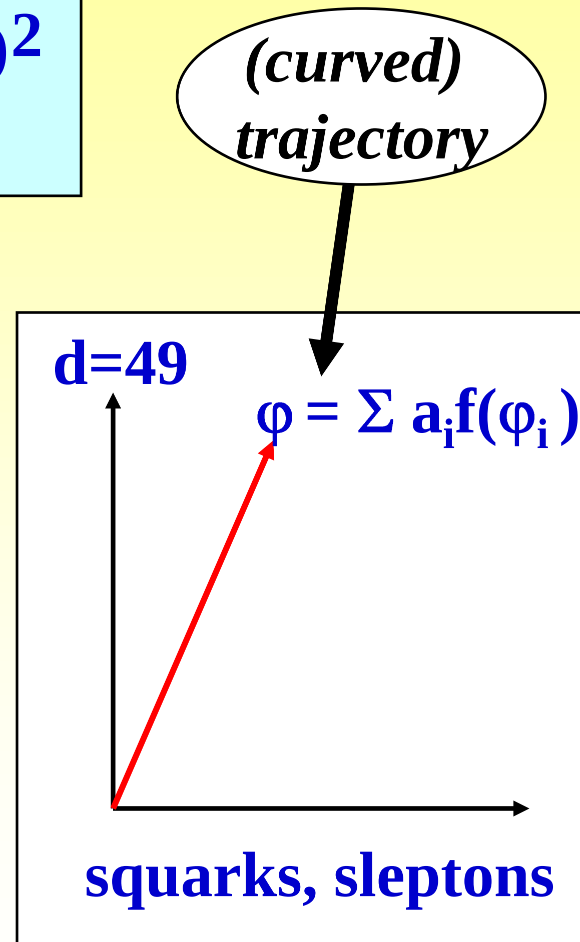
Superpotential: $F = \lambda LHe + \lambda' QHu + \ldots$

unbroken susy:

$$V = \Sigma\, |F_i|^2 + \tfrac{1}{2} D^2$$

$$F_i = \frac{\partial F}{\partial \varphi_i} = 0$$ F-flat

$$D^a = \varphi^\dagger_i T^a \varphi_i = 0$$ D-flat



global U(1): $\varphi_i \to \varphi_i e^{i\theta}$

flat directions typically carry B and/or L

MSSM flat directions all classified:

$H_u H_d$ — ***B=L=0***
LH_u — ***purely leptonic***
$u^c d^c d^c$ — ***purely baryonic***
LLe^c
$QQQL$
$u^c u^c d^c e^c$
$QQu^c u^c e^c$
$u^c u^c u^c e^c e^c$
$QQLLd^c d^c$
$QQQQLLd^c$
$QLu^c QQd^c d^c$

Dine, Randall, Thomas

etc

review: P.Rep. 380 (2003) 99

example: udd

colour

$$u_1^{\alpha} = d_2^{\beta} = d_3^{\gamma} = \frac{1}{\sqrt{3}}\phi \quad (\alpha \neq \beta \neq \gamma)$$

generation (+perm)

FLAT DIRECTIONS AND GRAVITY

- Inflation means gravity
- Gravity is not renormalizable
- MSSM + gravity is a non-renormalizable effective theory valid at scales << M_P

→ *must include all the non-renormalizable superpotential terms*

- for each flat direction, the first non-renormalizable term allowed by gauge and supersymmetry has a definite dimension *n*
- in addition, supersymmetry is broken softly in the Nature

flat directions "lifted": $V \neq 0$

Lifting flat directions

soft susy breaking

$$V = \tfrac{1}{2} m^2 |\phi|^2$$

non-renormalizable terms

$$F = \phi^n / M^{n-3}, \psi\phi^{n-1} / M^{n-3}$$

$$V = |F_k|^2 = \phi^{2(n-1)} / M^{2(n-3)}$$

soft breaking A-terms

$$V = \tfrac{1}{2} \mathrm{Re}\, AF / M^{(n-3)}$$

All flat directions lifted by operators dim ≤ 9

example: udd, lifted by dim=6 operators

$$w \supset \frac{1}{M_P^3}(uud)(uud)$$

Flat direction potential

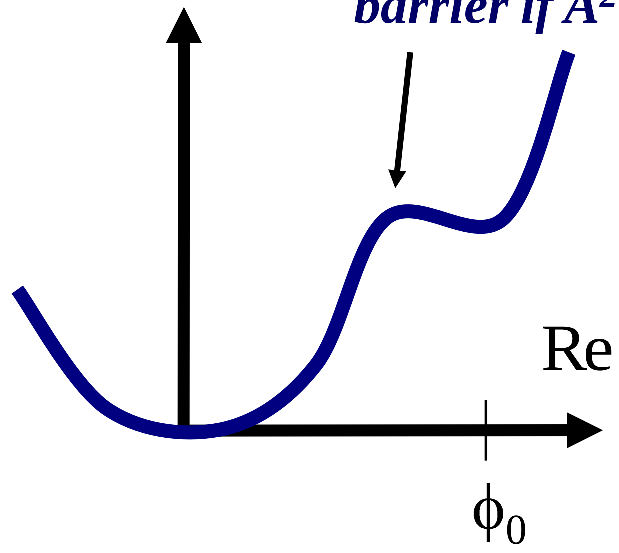
$$\phi = \phi e^{i\theta}$$

$$V = \tfrac{1}{2} m_\phi^2 \phi^2 + A\cos(n\theta + \theta_A)\frac{\lambda_n \phi^n}{n M_P^{n-3}} + \lambda_n^2 \frac{\phi^{2(n-1)}}{M_P^{2(n-3)}}$$

A-term → n-fold set of minima when cos = -1 provided

$$A^2 \geq 8(n-1) m_\phi^2$$

(but not too large A: otherwise true minimum breaking colour)



inflation never ends

$$V(\mathrm{Im}\,\phi) \propto m_\phi^2 \phi^2$$

barrier vanishes if

$$A^2 = 8(n-1)m_\phi^2$$

saddle point

then

$$V'(\phi_0) = V''(\phi_0) = 0$$

$$V'''(\phi_0) = 2(n-2)^2 \frac{m_\phi^2}{\phi_0}$$

➡ ***very flat!***

at the saddle point

$$\phi_0 = \left(\frac{m_\phi M_P^{n-3}}{\lambda_n \sqrt{2n-2}} \right)^{1/(n-2)}$$

$$H = \frac{n-2}{\sqrt{6n(n-1)}} \frac{m_\phi \phi_0}{M_P}$$

saddle point ⟺ eternal inflation regime

⇒ 2-point correlator spreads in time

⇒ classical slow roll takes over when

$$(\phi_0 - \phi) = \left(\frac{m_\phi \phi_0^2}{M_P^3} \right)^{1/2} \phi_0$$

⇒ ***business as usual***

the model:

$L_iL_je_k$ or ud_id_j flat direction (others: wrong amplitude)

$$m_\phi \approx O(1)\ TeV; n=6; A=\sqrt{40}m_\phi; \lambda \approx O(1)$$

$$H_{\inf} \approx O(1)\ GeV; \phi_0 \approx O(10^{14})\, GeV$$

slow roll → saddle point condition must hold very precisely

$$(A/m_\phi)^2 = 40(1+O(10^{-16}))$$

# of total e-folds

$$N = \int H_{\inf} d\phi / \dot{\phi} \approx 10^3$$

# of observationally relevant e-folds (immeadiate decay)

$$N_{COBE} \approx 47$$

amplitude of perturbations

$$\delta_H = \frac{1}{5\pi} \frac{H^2}{\dot{\phi}} \approx 1.9 \times 10^{-5}$$

slow roll parameters

$$\varepsilon \approx 1 / N^4_{COBE} \, ; \eta \approx -2 / N_{COBE}$$

spectral tilt

$$n_s = 1 - 4 / N_{COBE} \approx 0.92$$

$$\frac{dn_s}{d \ln k} = -4 / N_{COBE} \approx -0.002$$

no gravitational waves

REHEATING

after inflation oscillation frequency ~ m_ϕ ~ 10^3H

⇒ expansion negligible

udd breaks SU(3) U(1)
Lle breaks SU(2)×U(1) ⇒ susy conserving masses ~ gϕ

⇒ ***instant preheating***

ϕ → gauge bosons, gauginos → squarks/quarks
within 1 Hubble time

details remains to be worked out

Measuring inflaton parameters at colliders

CMB amplitude ⇒ $m_\phi(\phi_0) = 340\, GeV \lambda^{-1}$

run down to TeV:

for LLe:

ξ (gaugino/flat)	m_ϕ(TeV)
2	$(1.9)^{1/2}$ 340 GeV
1	$(1.3)^{1/2}$ 340 GeV
0.5	$(1.1)^{1/2}$ 340 GeV

LHC slepton mass limits can rule the model out!

WHY SADDLE POINT?

What fixes A/m?

In MSSM just parameters put in by hand

must go beyond MSSM

KE,Mether, Nurmi
Nurmi

SUPERGRAVITY CHANGES THINGS

expect corrections

$$\delta V_{SUGRA} = H^2 M^2 \left(\frac{\phi}{M} \right)^2$$

$$H \approx \frac{m_\phi \phi_0}{M} \Rightarrow \delta V_{SUGRA} << V_{MSSM}$$

small, but the saddle point is finetuned:

$$\delta \left(\frac{m}{A} \right)^2 \approx \frac{\phi_0^2}{M^2} \approx 10^{-10} >> 10^{-16}$$

depends on the Kähler potential

$$V_{SUGRA} = e^{G}(G_i G_{i\bar{j}} G^{i\bar{j}} - 3)$$

D=0

$$G_{i\bar{j}} = G_{i\bar{j}}(h, \phi)$$

Kähler metric

hidden

observable

$$m^2 = \frac{\partial^2}{\partial\phi\partial\bar{\phi}} V_{SUGRA}(h)$$

$$A_6 = V^{(6)}{}_{SUGRA}(h)$$

related?

Require:

$$A = \sqrt{40} m_{\phi}$$

$$\phi = \phi_0$$

not accidental but holds for **all values** of hidden sector fields

⇒ ***Kähler potential, moduli fields***

KE, Mether, Nurmi

Let us assume

$$W = \hat{W}(h_m) + \frac{\hat{\lambda}(h_m)\phi^6}{6M_P^3}$$

hidden sector

flat direction

Kähler potential

$$G = K + \ln|W|^2$$

$$K = \hat{K}(h_m, \bar{h}_m) + \sum_{n=1}^{\infty} \hat{Z}_{2n}(h_m, \bar{h}_m)\phi^{2n}$$

does there exist K such that identically? $A = \sqrt{40}m_\phi$

YES

to lowest order

$$V = (V_0 + V_2\phi^2 + V_6\phi^6 + V_{10}\phi^{10})(1 + O(\phi^2))$$

$$V_2 = e^{\bar{K}} \left|\hat{W}\right|^2 \hat{Z}_2(\hat{K}^m \hat{K}_m + \hat{K}^m \hat{K}^{\bar{n}} (\hat{Z}_2^{-2} \hat{Z}_{2m} \hat{Z}_{2\bar{n}} - \hat{Z}_2^{-1} \hat{Z}_{2m\bar{n}}) - 2)$$

etc. saddle point condition

$$\left|\hat{K}^m \hat{K}_m - 6\hat{Z}_2^{-1} \hat{K}^m \hat{Z}_{2m} + 3\right|^2$$
$$= 20(\hat{K}^m \hat{K}_m + \hat{K}^m \hat{K}^{\bar{n}} (\hat{Z}_2^{-2} \hat{Z}_{2m} \hat{Z}_{2\bar{n}} - \hat{Z}_2^{-1} \hat{Z}_{2m\bar{n}}) - 2)$$

= partial differential equation of two unknown functions

assume: $V_0 = 0$

simplest case: only one hidden sector field

$$\partial_h \hat{K} \partial_{\bar{h}} \hat{K} = -\beta \partial_h \partial_{\bar{h}} \hat{K} \Rightarrow \hat{K} = \beta \log(h+\bar{h})$$

no-scale
sugra

$$\hat{Z}_2 = (h+\bar{h})^{-2/9+\beta/6} \left[c_1 (h+\bar{h})^{\omega} + c_1 (h+\bar{h})^{-\omega} \right]^{5/9}$$

$$\omega = \frac{1}{2}\sqrt{-17-6\beta}$$

MSSM inflaton potential as the leading order several hidden fields: try the Ansatz

$$K = \sum_m \beta_m \log(h_m + \bar{h}_m) + \kappa \prod_m (h_m + \bar{h}_m)^{\alpha_m} \phi^2 + O(\phi^4)$$

"modular weights"

like abelian orbifold compactification of the heterotic string

$-\beta$ = number of moduli

saddle point if

$$\alpha\,(36\alpha + 16 - 12\beta) + (\beta + 7)^2 = 0$$

$$\alpha = \sum_m \alpha_m, \quad \beta = \sum_m \beta_m$$

look for solutions that are rational numbers:

saddle point if

$\beta = \sum_m \beta_m$	$\alpha = \sum_m \alpha_m$
-3	$-\frac{4}{9}$
-7	0
-7	$-\frac{25}{9}$
-11	$-\frac{1}{9}$
-11	-4

these values not found in abelian orbifold compactification

HIGHER ORDER

$$K = O(\phi^2) \quad \Rightarrow \quad V = V_{MSSM}\,(1 + O(\phi^2))$$

do these spoil flatness?

order by order

$$\Delta V_1 = V_4 \phi^4 + V_8 \phi^8 + V_{12} \phi^{12}$$

$$\Delta V_2 = V_6 \phi^6 + V_{10} \phi^{10} + V_{14} \phi^{14}$$

third and higher order negligible

must consider

$$K = \sum_m \beta_m \log(h_m + \bar{h}_m) + \hat{Z}_2\phi^2 + \hat{Z}_4\phi^4 + \hat{Z}_6\phi^6 + \ldots$$

saddle point, flatness maintained if

$$\Delta V_1' = \Delta V_1'' = 0$$
$$\Delta V_2' = 0$$

⇒

$$\hat{Z}_4 \propto \hat{Z}_2^2$$
$$\hat{Z}_6 \propto \hat{Z}_2^3$$

only solutions

with coefficients determined by α_m, β_m

K expansion in canonically normalized field d=6 flat MSSM inflaton potential guaranteed by

$$K=\sum_m \beta_m \log(h_m+\bar{h}_m)+\kappa\prod_m (h_m+\bar{h}_m)^{\alpha_m}\phi^2$$

$$+\mu\left(\kappa\prod_m (h_m+\bar{h}_m)^{\alpha_m}\right)^2\phi^4+\nu\left(\kappa\prod_m (h_m+\bar{h}_m)^{\alpha_m}\right)^3\phi^6$$

$\beta=\sum_m\beta_m$	$\alpha=\sum_m\alpha_m$	$\gamma=\sum_m\alpha_m^2/\beta_m$	μ	ν
-3	$-\frac{4}{9}$	$\frac{29}{81}$	$\frac{19}{108}$	$\frac{1507}{91854}+\delta\frac{1}{21}$
-7	0	$\frac{20}{9}$	$-\frac{7}{36}$	$-\frac{19}{378}+\delta\frac{1}{9}$
-7	$-\frac{25}{9}$	$-\frac{770}{81}$	$\frac{26}{27}$	$\frac{1158182}{203391}+\delta\frac{29}{186}$
-11	$-\frac{1}{9}$	$\frac{7613}{2673}$	$-\frac{559}{1782}$	$\frac{39281159}{671623326}+\delta\frac{31}{282}$
-11	-4	$-\frac{47}{3}$	$\frac{31}{12}$	$\frac{35507}{1458}+\delta\frac{13}{81}$.

$$\delta=\sum_m \alpha_m^3/\beta_m^2$$ free parameter

in shorthand:

$$K = \ln Z(\beta) + \sum_{m=1} a_m (Z(\alpha)|\phi|^2)^m$$

known, several solutions

$$Z(x) = \prod_n Z_n^{x_n}$$

$$Z_n = h_n + h_n^*$$

Constraints relaxed if one does not require

$$\phi = \phi_0$$

Nurmi hep-th/0710.1613

⇨ saddle point shifted by sugra

⇨ constraints on the Kähler potential parameters slightly changed

⇨ some particularly simple logarithmic solutions

$$K = -\ln\left(\prod_m (h_m + h_m^*)^{-\beta_m} - \kappa \prod_m (h_m + h_m^*)^{\alpha_m - \beta_m} |\phi|^2\right)$$

$\beta=\Sigma\beta_m$	$\alpha=\Sigma\alpha_m$	$\gamma=\Sigma\alpha_m^2/\beta_m$	$\delta=\Sigma\alpha_m^3/\beta_m^2$
-7	0	-5/4	free
-7	-25/9	-145/81	-985/720
-11	-1/9	-89/81	-721/720
-11	-4	-17/8	-91/64

Example of a solution for $\alpha = 0$, $\beta = -7$:

$$\beta_m = -1, \alpha_1 = 1, \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = -\frac{1}{4}, \alpha_6 = \alpha_7 = 0$$

proof of existence of a solution

supergravity corrections to inflation

- non-trivial Kähler potential → non-minimal kinetic terms
→ normalize to get canonical

- corrections from e^K

⇒ small (to lowest order) linear term

⇒ small, does not spoil MSSM inflation

⇒ small change in the spectral index: n = 0.92 … 0.94

N.B. for K=log(…) corrections vanish

STABILITY: RADIATIVE CORRECTIONS IN MSSM

Allahverdi,KE, Garcia-Bellido Jokinen, Mazumdar

1-loop radiative corrections:

$$A^2 \Rightarrow A^2 \times \left(1 + K_1 - \frac{2}{3} K_2 + \frac{1}{5} K_3\right)$$

from RGEs, depend on the flat direction

→ *seemingly fine-tuning required; e.g. LLe with unification:*

$$K_1 \approx -0.017\xi^2; K_2 \approx -0.009\xi; K_3 \approx -0.029;$$

$$\xi = M_g / m_0$$

→ ***A/m shifted by O(10^-2^)***

A/m shifted by O(10^{-2})

a saddle point remains

but: assumes A and m_0 are independent parameters instead of functions of the moduli fields

stability under radiative corrections in the observable sector

wave function renormalization $|\phi| \to Z^{1/2}|\phi|$

can be absorbed by scaling the moduli: e.g.

$$K = -\ln(x^{-\beta_1} y^{-\beta_2} - \kappa x^{\alpha_1-\beta_1} y^{\alpha_2-\beta_2} |\phi|^2)$$

$$\to K(Z^{1/2}|\phi|) = K$$

$$x \to Z^{-\beta_2/(\alpha_2\beta_1)} x$$

$$y \to Z^{-\beta_1/(\alpha_1\beta_2)} y$$

in the superpotential

$$\lambda \to Z^{-3}\lambda$$

vertex corrections?

moduli dynamics?

Lalak, Turzynski

if moduli shifted during inflation → inflation affected

what stabilizes the moduli?

superpotential for the moduli?

(cosmological constant)

open questions:

initial condition

Kähler potential fixed → implications for sparticle phenomenology

e.g. non-flat direction ψ

$$W = \frac{1}{3}\hat{\lambda}\psi^3$$

assume non-flat directions have all the same "modular weights" as the inflaton direction

⇒ trilinear A-term

phases

$$A_3 = \frac{4\cos\xi(\alpha - \beta/3)}{\sqrt{\alpha-\beta-2}}\, m_\psi$$

at scale ϕ_0

⇒ ***in principle testable***

Conclusions

- ***n=6 MSSM flat directions have all the ingredients for successful inflation***
- ***inflaton is a gauge invariant combination of squarks or sleptons: couplings to matter known***
- ***requires saddle point: but fine tuning can be a consequence of sugra (string theory?)***
- ***proof of existence of a class of Kähler potentials that give rise to the saddle point: expansion in terms of the canonically normalized inflaton***
- ***parameters of the inflaton potential (e.g. inflaton mass) can in principle be determined in laboratory***
- ***many open questions***