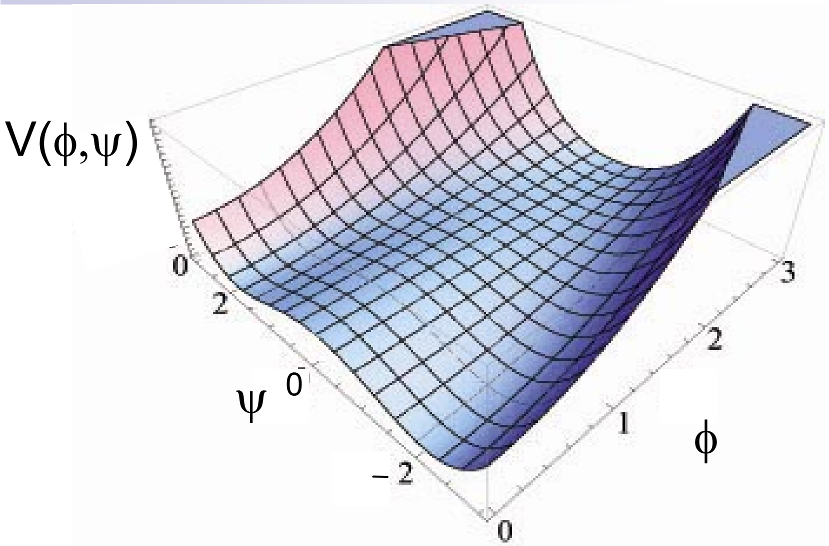
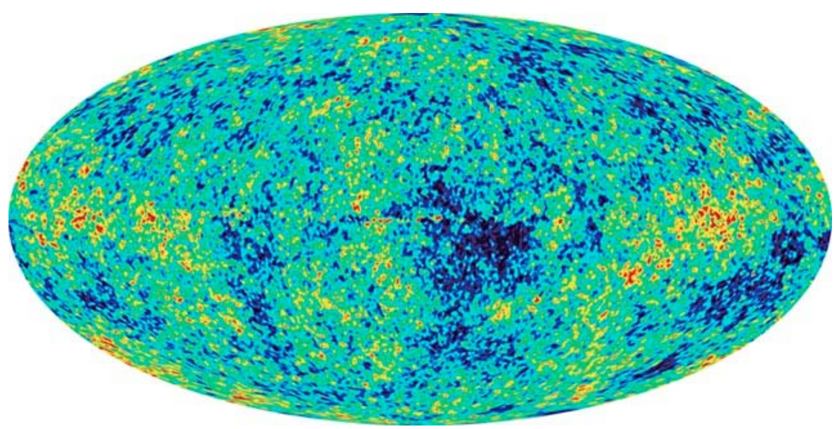
ESF Exploratory Workshop, Porto

March 28th, 2008

From HEP & inflation to the CMB and back…

Jonathan Rocher (ULB - Brussels)





Outline

Introduction : Grand Unified Theories and Inflation

Part A : Constraints on inflation from CMB

1. Constraints on F-term inflation
2. D-term inflation and constraints
3. Some ways out …

Part B : Initial conditions for hybrid inflation

1. Dynamics of inflation
2. Is there a fine tuning of the Initial Conditions ??

Part C : Constraints from the spectral index n_S

1. Naïve approach
2. Difficulties to improve the calculation of radiative corrections

Conclusions and open questions

Intro : hybrid (F-term) inflation

Superpotential (F-term case): $W^F = \kappa S(\Phi_+\Phi_- - M^2)$

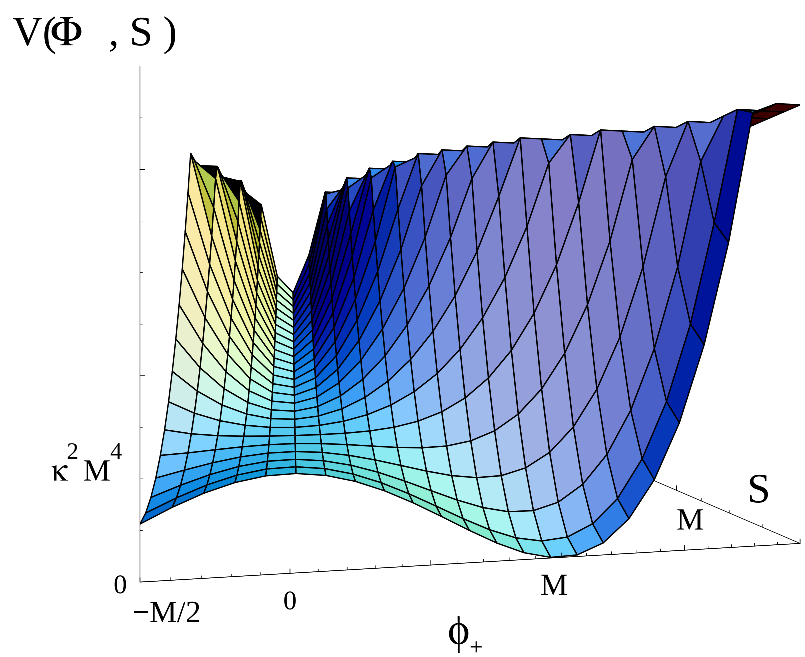
S : inflaton field

Φ_+, Φ_- : pair of Higgs fields

κ : superpotential coupling

M : energy scale of inflation and SSB

Important property : The inflationary phase ends with the SSB.



Motivations :

- Take into account coupling with other fields.
- Perfectly flat potential. The radiative corrections induce a tilt for slow roll.
- No additional symmetry nor extra fields.
- The SUGRA corrections don't spoil the inflation [Copeland et al (1994)].
- No fine tuning needed to generate the anisotropies ? See later.

Models of SUSY GUT

- Models based on SU(n), SO(10), E_6.
- Most standard phenomenological ingredients :
- Proton life-time measurements (Super-Kamiokande) $\tau_P(p \to e^+\pi^0) > 6\times10^{33}$ yr

  Motivates :
  - Supersymmetry (M_{GUT} sufficiently high)
  - Z_2 of R-parity unbroken at low energy (Bonus = dark matter)

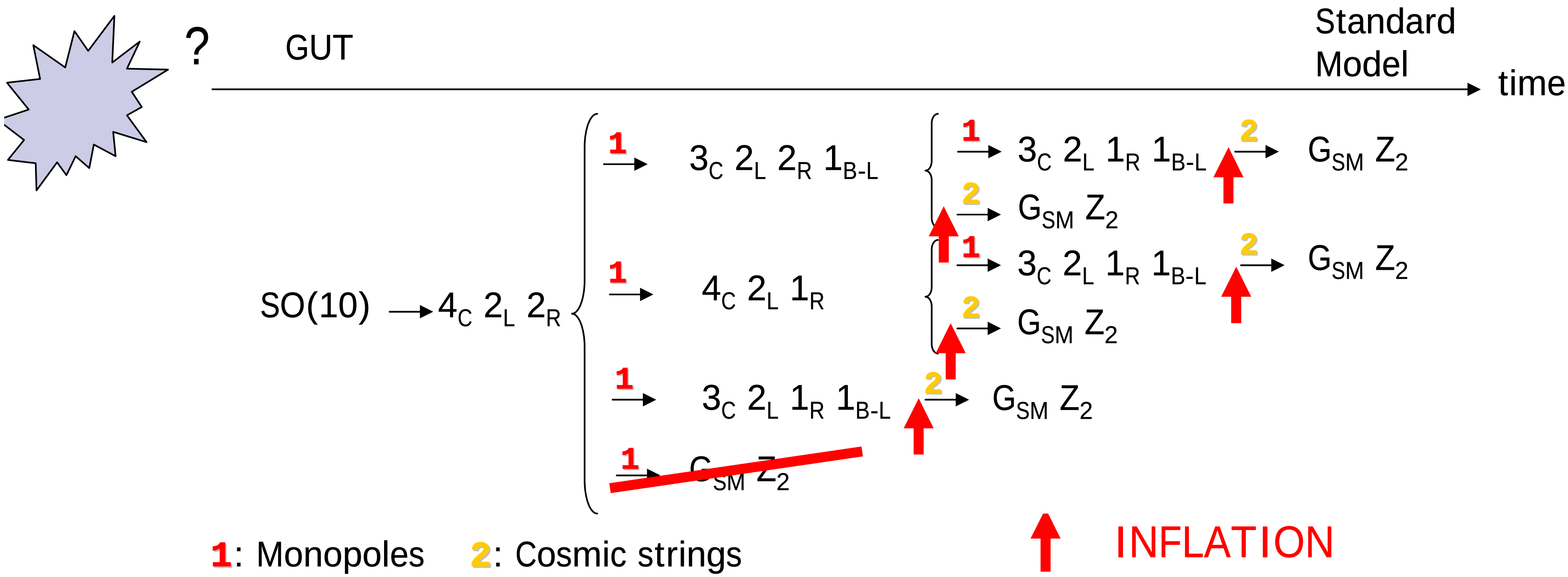
- Oscillations of solar and atmospheric neutrinos [Super-K, (1998)], …

  Requires mass to neutrinos (via See-saw)

  Requires B-L in G_{GUT} and broken at high energy (Bonus = leptogenesis)

- To explain the CMB data, and solve the monopole problem (and others), a phase of inflation (hybrid)

An example : SO(10)



Conclusions : [Jeannerot, J.R., Sakellariadou (2003)]

• 34 schemes compatibles with all hypothesis for SO(10),

String formation always after inflation

• Same conclusion for [E_6, SU(8), SU(9), SO(14)] : strings are generic and generically form at the end of hybrid inflation $\mu \approx M_{infl}^2$

- Are these strings consistent with CMB data ? What is their influence on temperature anisotropies ?

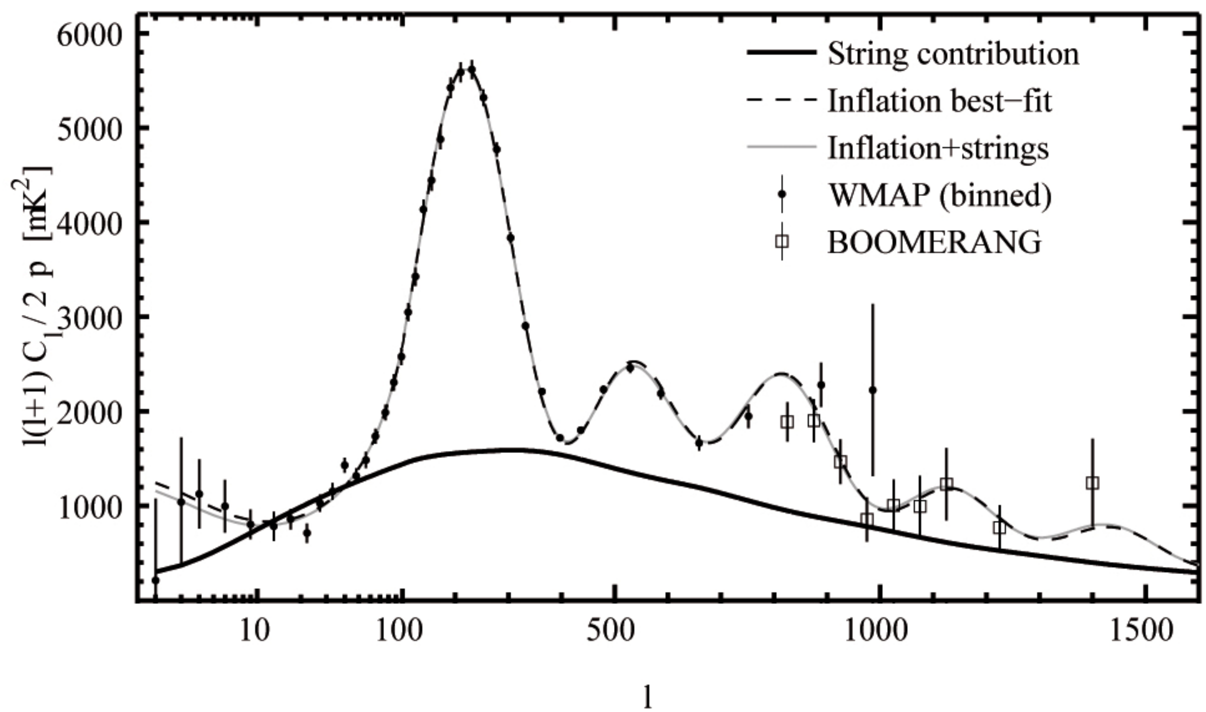
- What can we learn from their low influence ?

CS not (yet) observed !

$C_l = A_{CS} C_l^{CS} + (1 - A_{CS}) C_l^{infl}$

WMAP 3 : $A_{CS} < 11\%$

[Bevis et al. (2007)]



Part A: Constraining F-term inflation

Superpotential : $W^F = \kappa S(\Phi_+\Phi_- - M^2)$

[Dvali, et al. (1994)]

SUSY

$$V_{\rm eff}(S) = \kappa^2 M^4 \left\{ 1 + \frac{\kappa^2 N}{32\pi^2}\left[2\ln\frac{|S|^2\kappa^2}{\Lambda^2} + \underbrace{(z+1)^2\ln(1+z^{-1}) + (z-1)^2\ln(1-z^{-1})}_{f(z)} \right] \right\}$$

Ex. The Higgs field responsible for SSB : N=**126** if it corresponds to B-L in SO(10).

Contributions to CMB quadrupole anisotropies :

$$\left(\frac{\delta T}{T}\right)^2_{Q-\rm infl} = \left(\frac{\delta T}{T}\right)^2_{Q-\rm scal} + \left(\frac{\delta T}{T}\right)^2_{Q-\rm tens}$$

$$\left(\frac{\delta T}{T}\right)_{Q-\rm strings} \sim \alpha G\mu \quad \text{with} \quad \mu \approx 2\pi \langle h \rangle^2$$

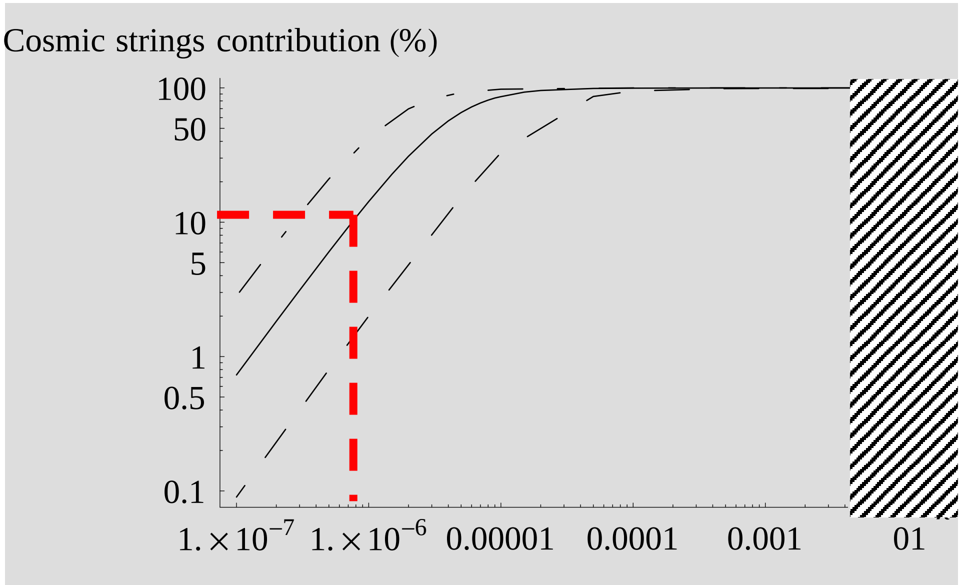
α=(9-10) [Landriau & Shellard '03]

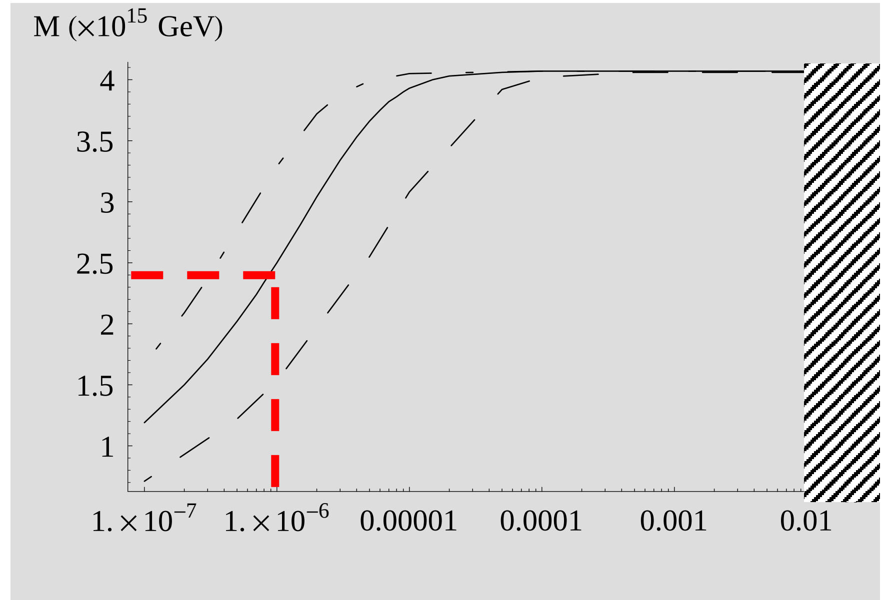
Solution to the horizon problem :

✓ $N_Q \approx 60$

Normalization to COBE/WMAP :

✓ $(\delta T/T)_Q \sim 6\times10^{-6}$.





- Coupling to see-saw mechanism, gravitino overproduction imposes $\kappa < 10^{-2}$.
- WMAP3 : $A_{CS} < 11\%$ (95% CL). [Bevis et al. (2007)]

Conclusions for SO(10) : $\kappa < 10^{-6}$ ← "Reduced" fine tuning

$M < 2\times10^{15}$ GeV ← Constraints on the SSB scale !

- For E_6, the limit on M and the conclusion unchanged.
- $\mathcal{N}$ gives a window on G_{GUT} ! [J.R., Sakellariadou (2005)]
- This SSB related to neutrino masses (via see-saw) in this framework.

Constraints on standard D-term inflation

Motivated by HEP :

- Can be embedded in SUSY/SUGRA GUTs [Jeannerot '97]

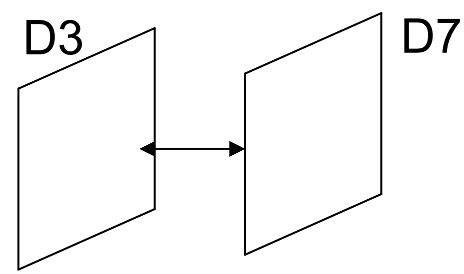
G x U(1) -> HxU(1) -> H -> SM

- No η-problem in SUGRA framework
- Effective low energy description of Brane inflation [Dasgupta et al '02]

Singlet inflaton field -> brane separation

SSB driven by Higgs fields -> Tachyon condensation

Cosmic strings formed -> D1-brane formed



- Can be embedded in weakly coupled string theory where anomalous U(1)s generate FI term [Lyth & Riotto '99]

$$\frac{\xi}{M_p^2} \propto \mathrm{Tr}\, Q$$

$\mathcal{D}$-term inflation : the model

Introduction of an additional U(1) factor, with gauge coupling g + FI term $\xi \neq 0$.

Charges under U(1) : Q(S)=0, $Q(\Phi_\pm)=\pm 1$ (modified if SuperConformal origin of SUGRA).

Superpotential : $$W^D = \lambda S \Phi_+ \Phi_-$$ [Binétruy & Dvali 96, Halyo 96, Binétruy et al 2004]

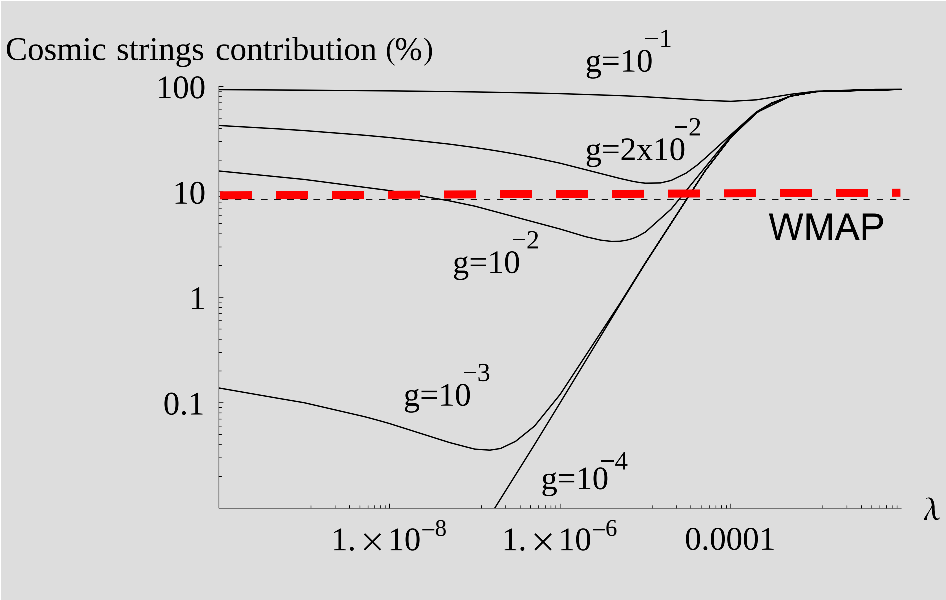
Minimal SUGRA : $K_{min}=|S|^2+|\phi_+|^2+|\phi_-|^2$ or including first order non-renorm terms

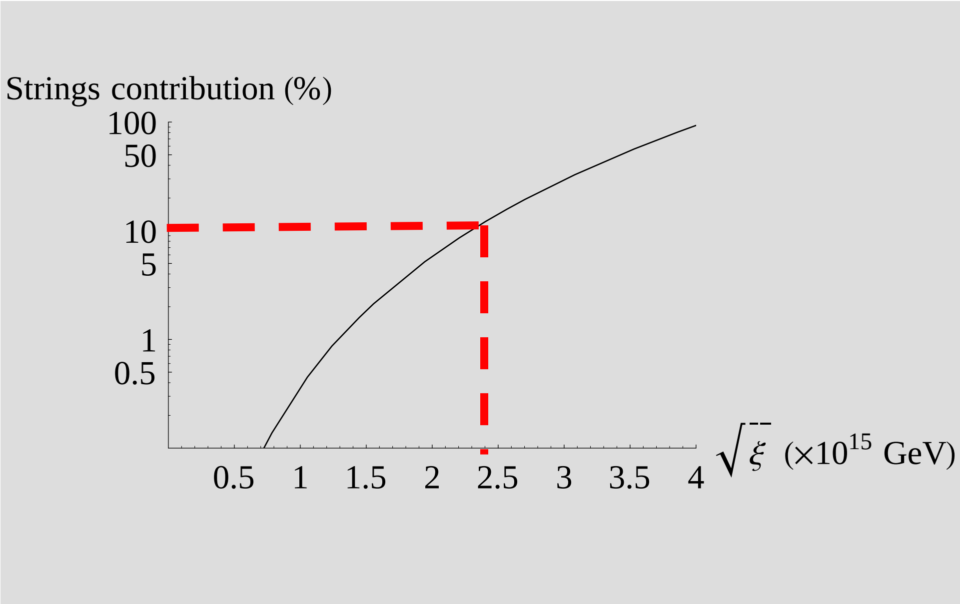
$$K = K_{\min} + \frac{1}{M_p^2}\left(c_+|S|^2|\phi_+|^2 + c_-|S|^2|\phi_-|^2 + b|S|^4\right)$$

S U G R A

$$V_{\text{eff}}^{\text{D}}(S) = \frac{g^2\xi^2}{2}\left\{1+\frac{g^2}{16\pi^2}\left[2\ln\left(z\frac{g^2\xi}{\Lambda^2}\right)+f(z)\right]\right\}$$

We get A_{CS} as a function of 3 parameters : g, λ, ξ.





- The WMAP3 constraint on A_{CS} to the CMB data imposes

$$g < 2.2 \times 10^{-2}$$

$$\lambda < 3 \times 10^{-5} \quad \longleftarrow \text{“Reduced” fine tuning}$$

$$\sqrt{\xi} < 2 \times 10^{15}\ \mathrm{GeV} \quad \longleftarrow \text{Same constraint}$$

- The SUGRA corrections imply a lower limit on λ.

[J.R. & Sakellariadou (2005a), (2005b)]

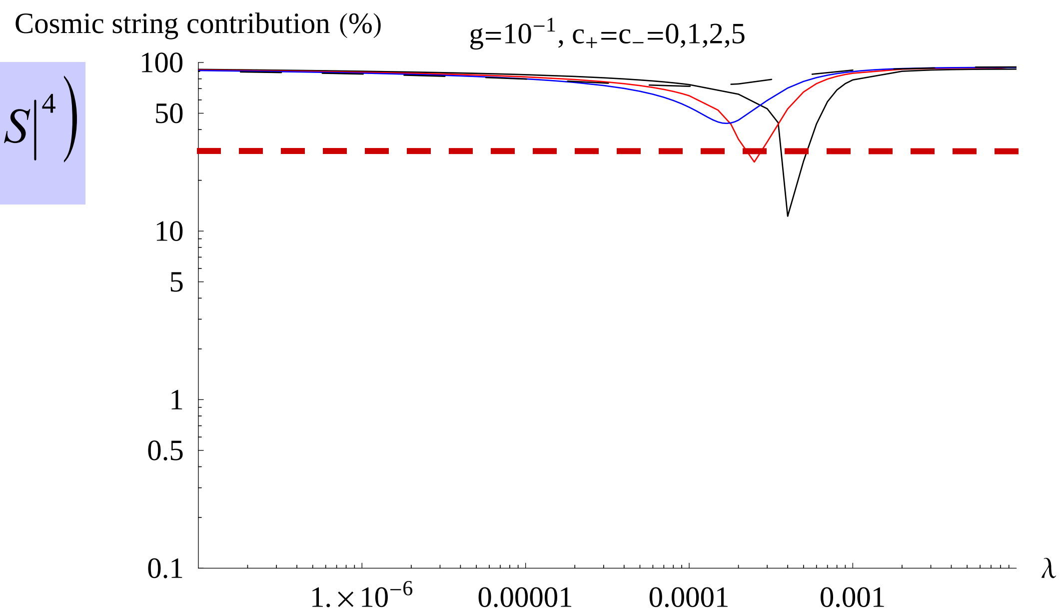
Ways out ... ?

- Invoke a curvaton mechanism (cost = one more field + 1 more free parameter) [J.R. Sakellariadou 2005]
- Making the CS unstable
  - Introducing some additional chiral superfields [Urestilla et al 2004 ; Binétruy et al. 2004]
  - Assuming additional symmetries [Binétruy et al. 2004] :

  Consider ϕ_- = triplet under some gauge SU(2). The SSB is then
  SU(2) x U(1) -> U(1)' (as in electroweak SSB)
- Can we solve the fine tuning problem with non-flat kähler geometry [Seto & Yokoyama (2005)] ? NO !
- …

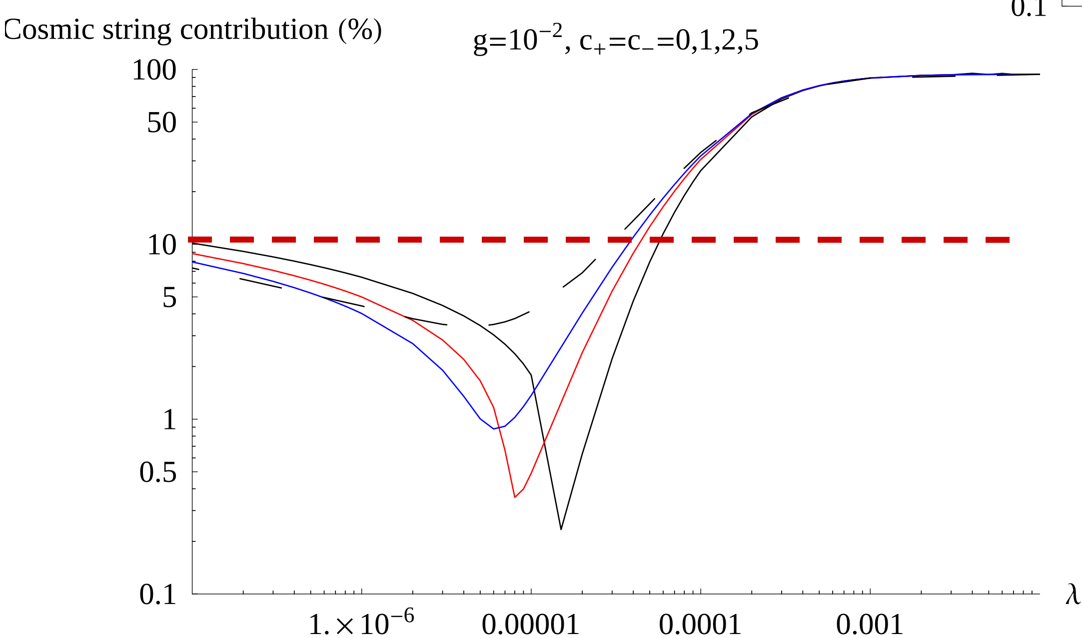
D-term inflation and non-flat Kahler Geometry

$$K = K_{\min} + \frac{1}{M_p^{\ 2}}\left(c_+|S|^2|\phi_+|^2 + c_-|S|^2|\phi_-|^2 + b|S|^4\right)$$



Case where b=0 :

$$z = \frac{\lambda}{g^2\xi}\frac{|S|^2}{(1+f_+)(1+f_-)}Exp\left(\frac{|S|^2}{M_p^{\ 2}}\right)$$

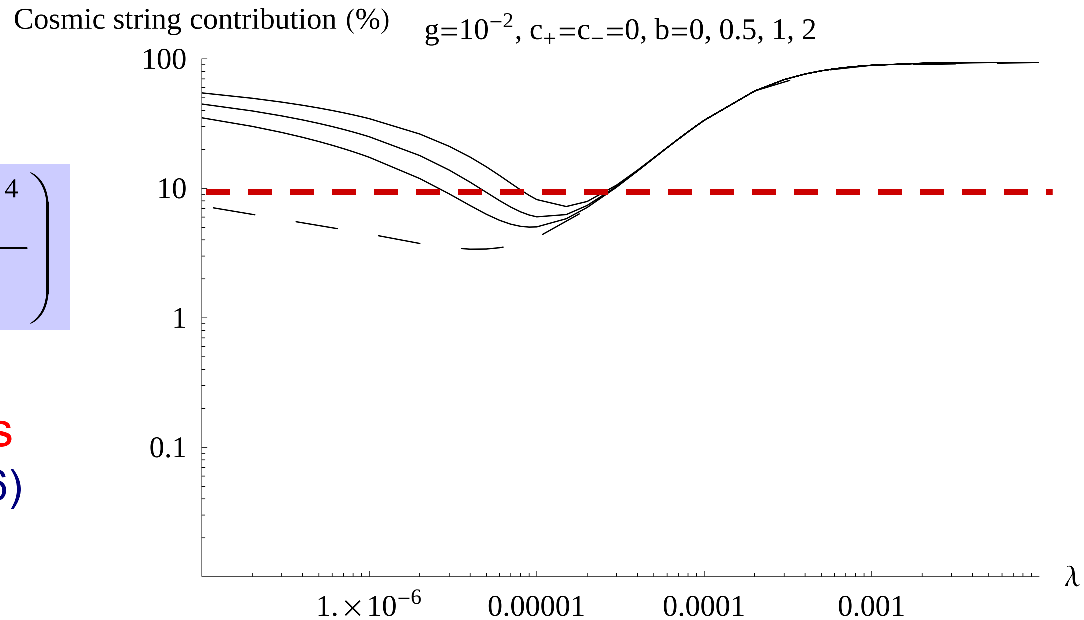


Conclusions :

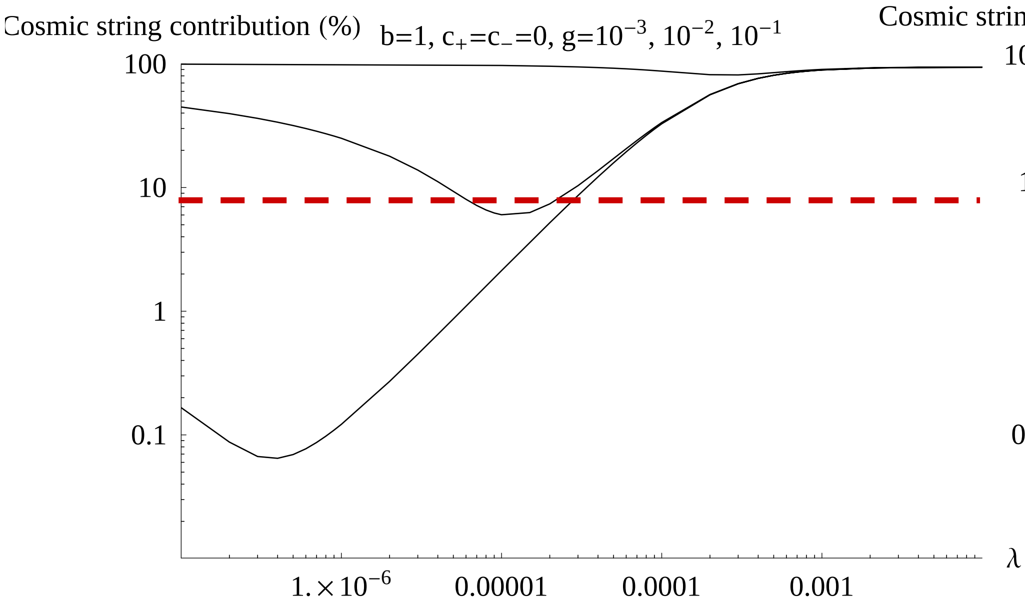
- Constraints on ξ, g and λ ~unchanged.
- Fine tuning for λ still necessary

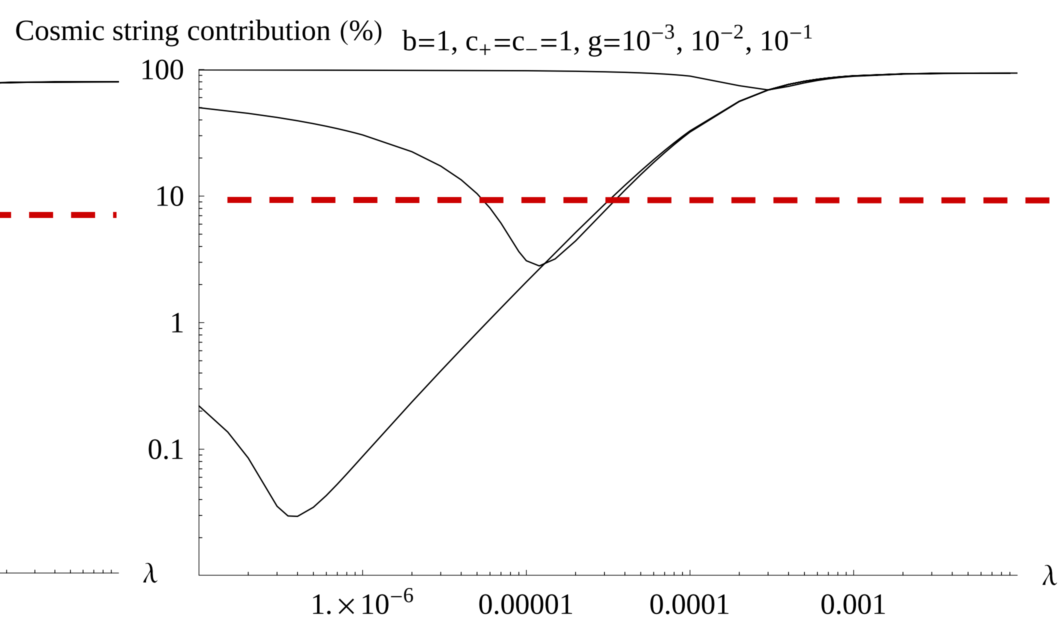
Case where b≠0 :

$$z = \frac{\lambda}{g^2 \xi} \frac{|S|^2}{(1+f_+)(1+f_-)} \mathrm{Exp}\left(\frac{|S|^2 + b|S|^4}{M_p^2} \right)$$



Conclusion : the fine tuning is always necessary ! J.R., Sakellariadou (2006)





- The coupling constant, the energy scale and the inflaton value « measured » from the data.
- What INITIAL CONDITIONS (I.C.) lead to enough efolds ?
- Is there fine tuning on these I.C. ?

Part B : Fine tuning on Initial Conditions ?

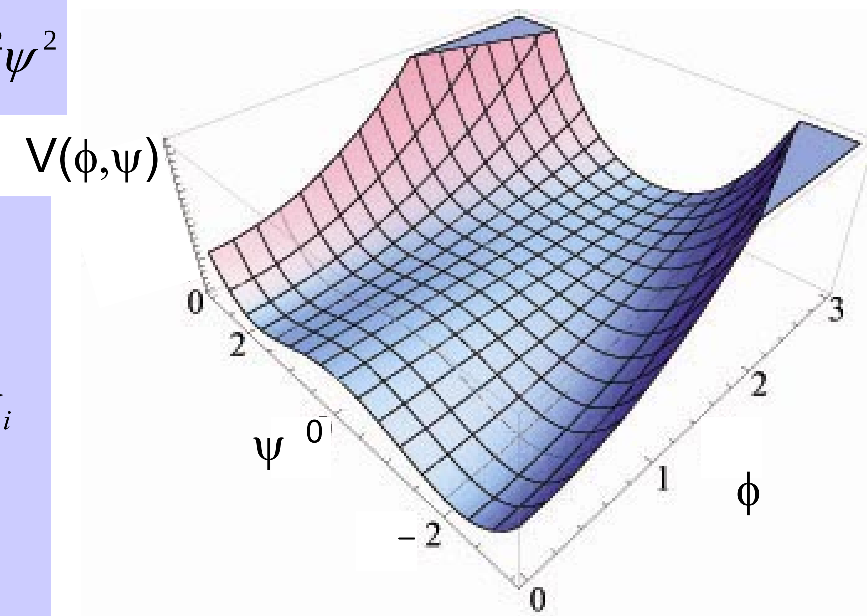
Study dynamics of non SUSY hybrid model (= toy model for many realistic models) [Linde (1991)]:

$$V(\phi,\psi)=\frac{1}{4}\lambda\left(\psi^2-M^2\right)^2+\frac{1}{2}m^2\phi^2+\frac{1}{2}\lambda'\phi^2\psi^2$$

Equations of motion :

$$\begin{cases} \ddot{\phi}+3H\dot{\phi}+\dfrac{\partial}{\partial\phi}V(\phi,\psi)=0, \quad \text{with} \quad \phi(0)=\phi_i \\ \ddot{\psi}+3H\dot{\psi}+\dfrac{\partial}{\partial\psi}V(\phi,\psi)=0, \quad \text{with} \quad \psi(0)=\psi_i \end{cases}$$

$$\text{with} \quad H^2=\frac{1}{3M_P^2}\left[\frac{1}{2}\dot{\phi}+\frac{1}{2}\dot{\psi}+V(\phi,\psi)\right]$$

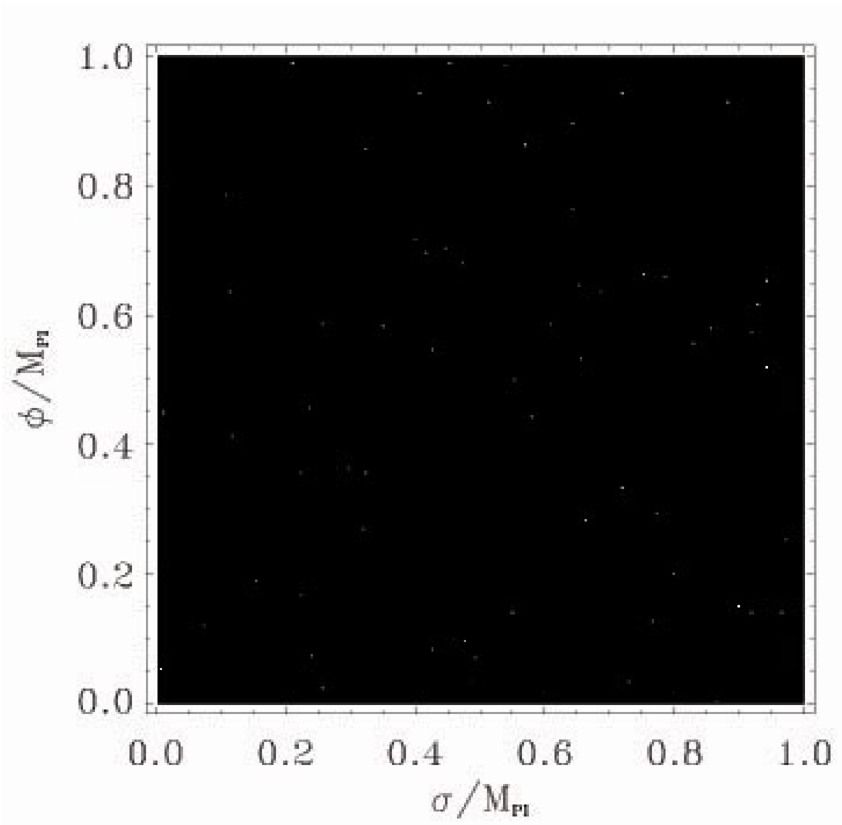


Goal : assuming some random initial conditions for the fields φ and ψ, we study the trajectory and if the slow roll conditions are realized + calculate N(t).

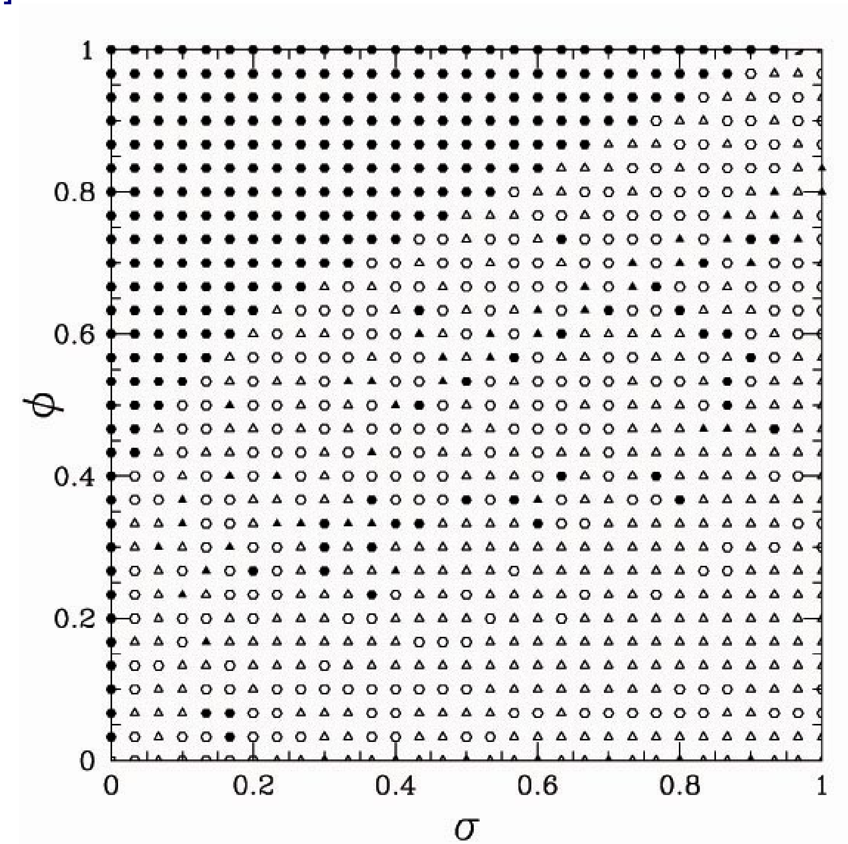
Definition : successful inflation if

- Normalization to COBE
- N_{tot} > 60

Known situation : fine tuning on initial values of ϕ and ψ to have a sufficient slow roll and N_{tot}>60 [Mendes & Liddle 2000, Tetradis 1998].



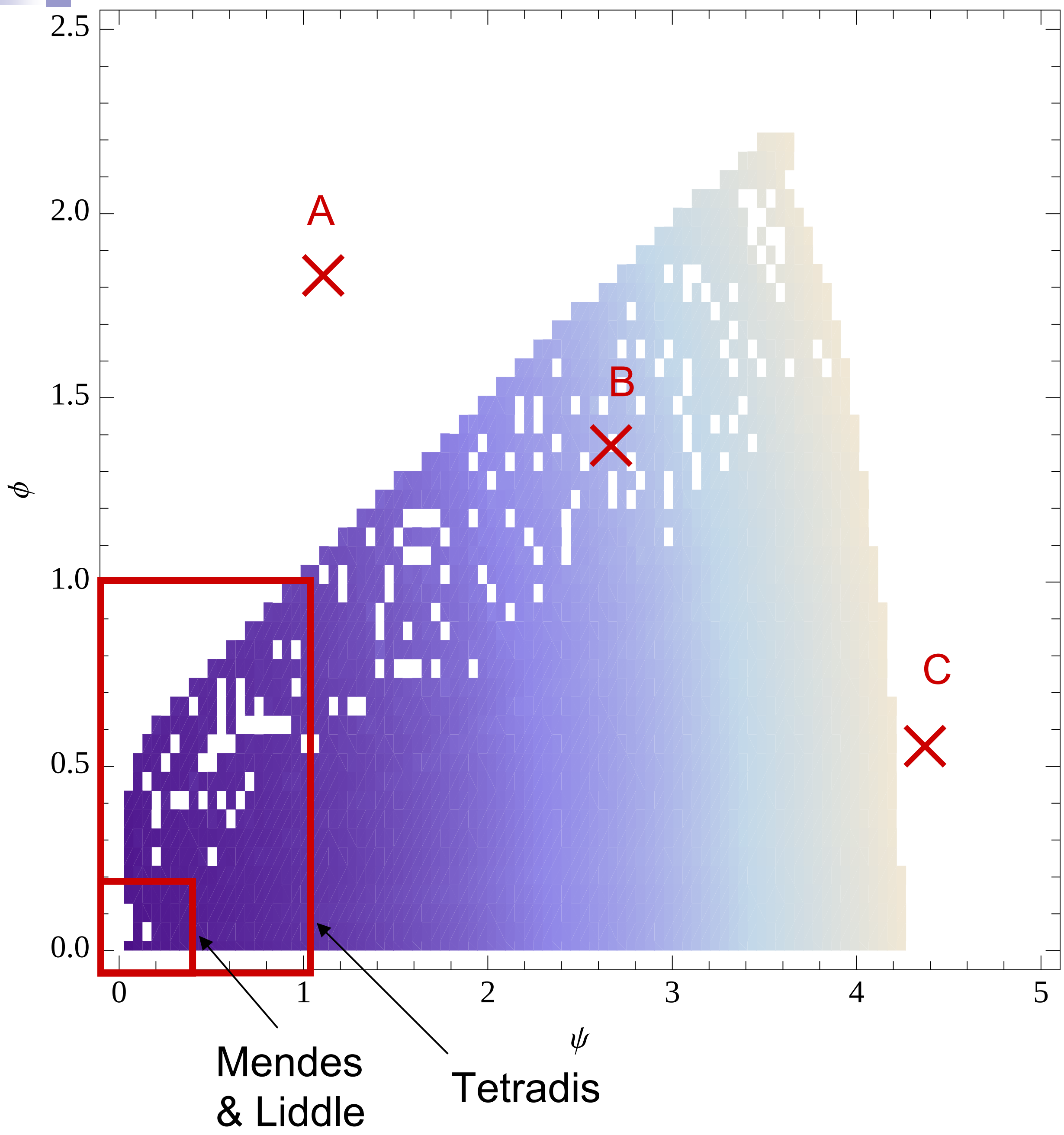
Mendes & Liddle 2000



Tetradis 1998

Questions :

- Is this true in the rest of the parameter space ?
- How to interpret/understand the isolated successful dots ?



$\lambda=1$, $\lambda_2=1$

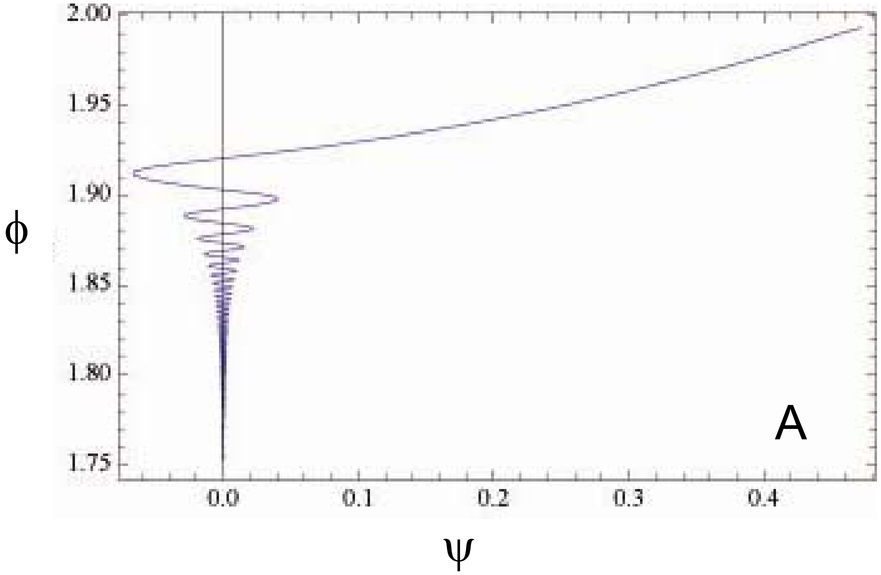
$M=3\times10^{-2}M_{pl}$, $m=10^{-6}M_{pl}$

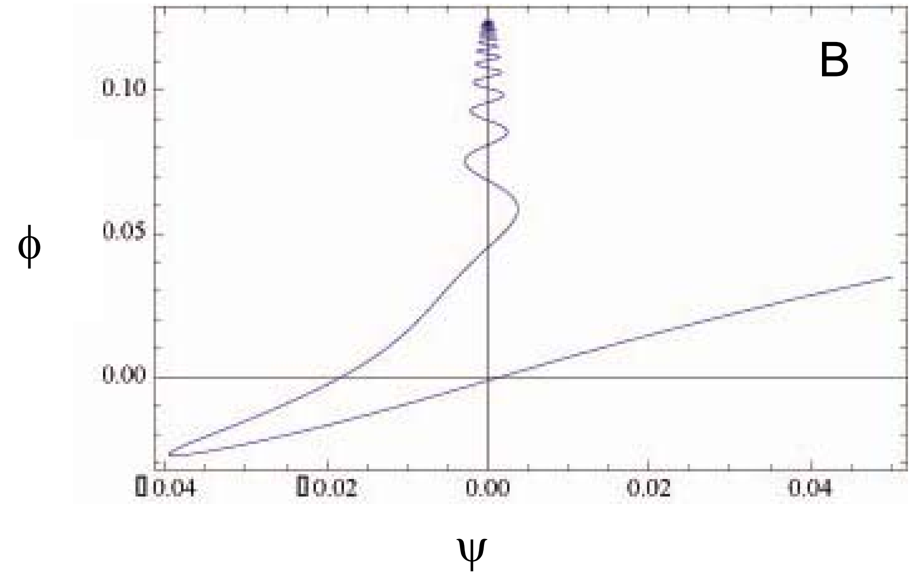
Our results :

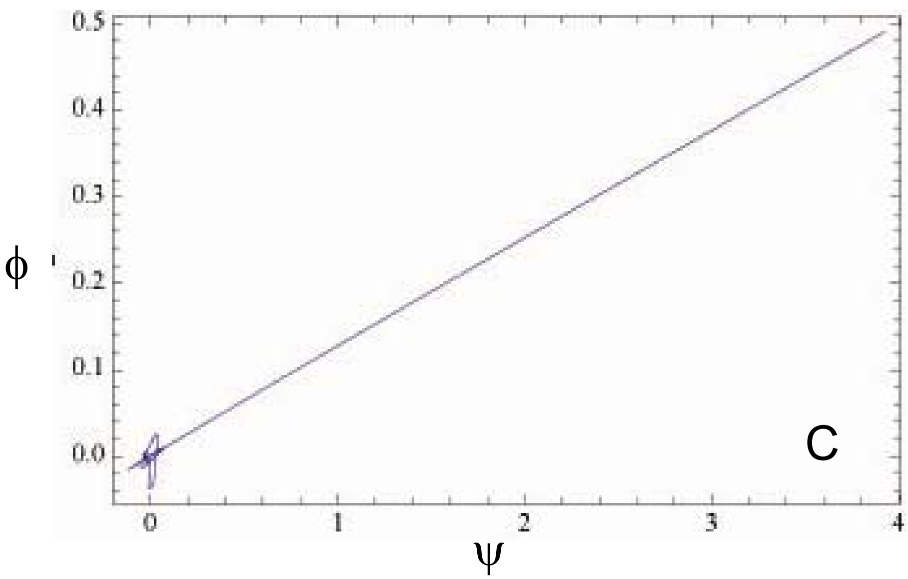
- In agreement with Tetradis.
- show some isolated solutions to the horizon problem
- Most of the parameter space is successful if one allows for transplanckian values

[Clesse & J.R. to appear].

Interpretation with some successful trajectories :







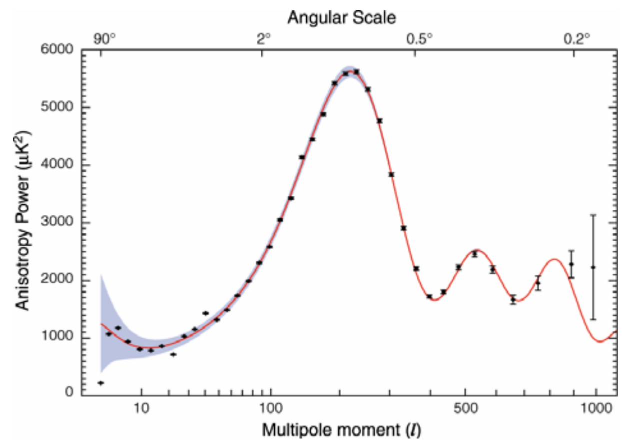
Conclusions :

- A type trajectories are oscillation + slow roll in ϕ direction
- B type trajectories are « miraculous » : the system climb up the valley and slide back down
- C type trajectories are mostly chaotic inflation in the ψ direction

- Can we constrain more D-term inflation with the spectral index ?
- How to improve predictions for n_S for D-term inflation ?

Part C : Constraints from the spectral index ?

Power spectrum of primordial fluctuations $P_k \propto k^{n_S - 1}$



Observations WMAP 5 : $n_s \approx 0.96 \pm 0.015$

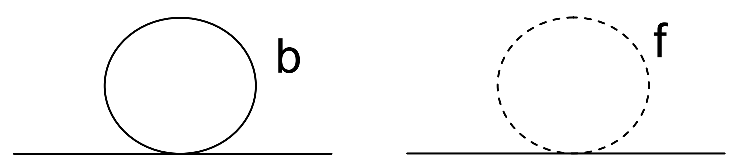
Predictions ?

If slow roll, $n_S \approx 1 - 6\varepsilon + 2\eta$ with $\varepsilon \equiv \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2$ and $\eta \equiv M_{pl}^2\left(\frac{V''}{V}\right)$

1) Naïve approach :

1-loop potential given by perturbative formula [Coleman-Weinberg '73],

$$V_{\text{eff}}(S) = V_0\left\{1 + \alpha \sum_i (-1)^i m^4(\Phi_\pm)\log\left(\frac{m^2(\Phi_\pm)}{\mu^2}\right)\right\}$$



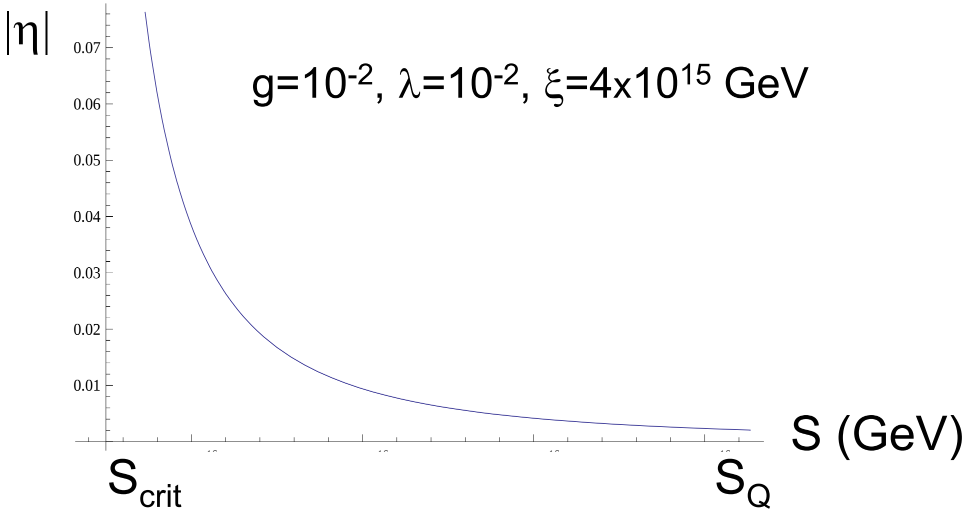
⇒ calculate V', V'' in S_Q, and then n_S

⇒ Confrontation to the data

Problem 1 : at critical point, $m^2(\Phi_-)=0$, => $\log(m^2)$ diverge

⇒ Divergence of V''/V for $S \sim S_{crit}$.

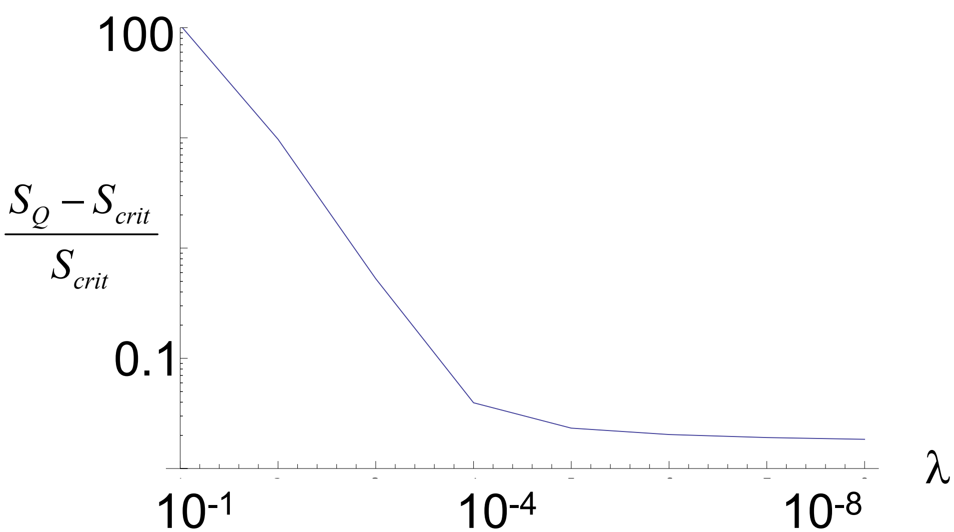
⇒ Artificial behaviour of n_S close to the critical point !



→ Divergence of $V''(S)$ close to S_{crit}

⇒ Inflection point in $V(S)$

⇒ Need improved potential when S_Q close to S_{crit}



→ Problem 2 : in the allowed parameter space, S_Q very close to S_{crit}

2) Renormalization group methods :

Renormalisation group can resum the (sub)leading log.

ex : $V=\lambda\phi^4$ is replaced by $V=\bar{\lambda}\phi^4$

Perturb.

$$\bar{\lambda}(\phi) = \lambda + \frac{3\lambda^2}{16\pi^2}\ln\left(\frac{\phi}{\mu}\right)$$

$\longrightarrow$

R.G.E.

$$\bar{\lambda}(\phi) = \frac{\lambda}{1 - \frac{3\lambda}{16\pi^2}\ln\left(\frac{\phi}{\mu}\right)}$$

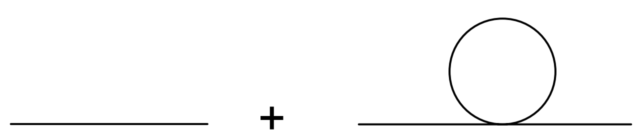
Toy model : $V(\phi)$= massive $\lambda\phi^4$ in the broken symmetry phase

Eq of Callan-Symanzick applied to the physical quantities : $V(\phi)$, $\Gamma^{(n)}$

$$\frac{dV}{d\ln\mu} = \left(\mu\frac{\partial}{\partial\mu} + \beta(\lambda)\frac{\partial}{\partial\lambda} + \beta(m^2)\frac{\partial}{\partial m^2} + \beta(\Lambda)\frac{\partial}{\partial\Lambda} + \beta(\phi)\frac{\partial}{\partial\phi}\right)V(\phi,\lambda,m,\Lambda,\mu) = 0$$

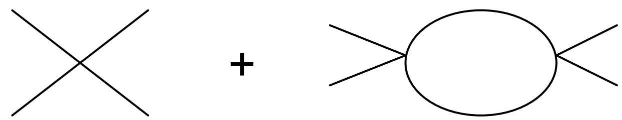
Eq of Callan-Symanzick applied to proper vertices to get β-functions :

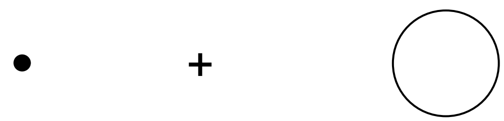
$\Gamma^{(2)}$:



$\longrightarrow$

$$\beta(m^2) = \hbar\frac{\lambda m^2}{16\pi^2} + O(\hbar^2)$$

$\Gamma^{(4)}$:  $\longrightarrow$ $\beta(\lambda) = \hbar \dfrac{3\lambda^2}{16\pi^2} + O(\hbar^2)$

$\Gamma^{(0)}$:  $\longrightarrow$ $\beta(\Lambda) = \hbar \dfrac{m^4}{32\pi^2} + O(\hbar^2)$ CC required if m ≠ 0 !!

Solution for V_{RG}: we introduce running parameters to account for a change in μ :

$$\frac{\mathrm{d}\overline{c_i}(t)}{\mathrm{d}t} = \beta(\overline{c_j}), \quad \text{with} \quad \bar{c}(t=0) = c \quad \text{and} \quad t = \ln \overline{\mu} / \mu$$

Theorem : then the RG improved potential is constructed by [Bando et al. (1993)]

- a perturbative solution $V_{pert}(\phi, c_i)$ to the CS equation at L-loop
- β functions at (L+1)-loop
- $V^{RG}(\overline{c_i}) = V_{pert}(\overline{c_i})$

in order to resum the leading (L=0), sub-leading (L=1), … logs.

Application to massive $\lambda\phi^4$

$$V_{\text{tree}} = \Lambda \pm \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4$$

$$V_{\text{cw}} = V_{\text{tree}} + \frac{\hbar}{64\pi^2} \left(\pm m^2 + \frac{1}{2} \lambda \phi^2 \right)^2 \left[\ln \left(\frac{\pm m^2 + \frac{1}{2} \lambda \phi^2}{\mu^2} \right) - \frac{3}{2} \right]$$

[Coleman-Weinberg '73],

$$V_{\text{RG}} = \overline{\Lambda} + \frac{1}{2} \overline{m^2} \phi^2 + \frac{1}{4!} \overline{\lambda} \phi^4 + \frac{\hbar}{64\pi^2} \left(\pm \overline{m^2} + \frac{1}{2} \overline{\lambda} \phi^2 \right)^2 \left[\ln \left(\frac{\pm \overline{m^2} + \frac{1}{2} \overline{\lambda} \phi^2}{\overline{\mu}^2} \right) - \frac{3}{2} \right]$$

with $\overline{\lambda}(t) = \dfrac{\lambda_0}{1 - 3\kappa\lambda_0 t}$, $\quad \overline{m^2} = \dfrac{m_0^2}{(1 - 3\kappa\lambda_0 t)^{1/3}}$, $\quad \overline{\Lambda} = \Lambda + \dfrac{m_0^4}{2\lambda_0} \left[1 - (1 - 3\kappa\lambda_0 t)^{1/3} \right]$

$\Rightarrow$ t cannot diverge thus $\overline{\mu}$ cannot vanish !

Conclusion : Don't forget the CC !!

The improvement ?

• μ = cst ⟶ running $\overline{\mu}(\phi)$ which can follow m_{eff}^2 => Log stay small in a bigger range.

• CW contains only $\hbar$ terms ⟶ RG contains infinite powers of $\hbar$.

• All the leading divergences in the Logs are resumed.

• For t=0 or at order $\hbar$, the perturbative potential MUST be recovered.

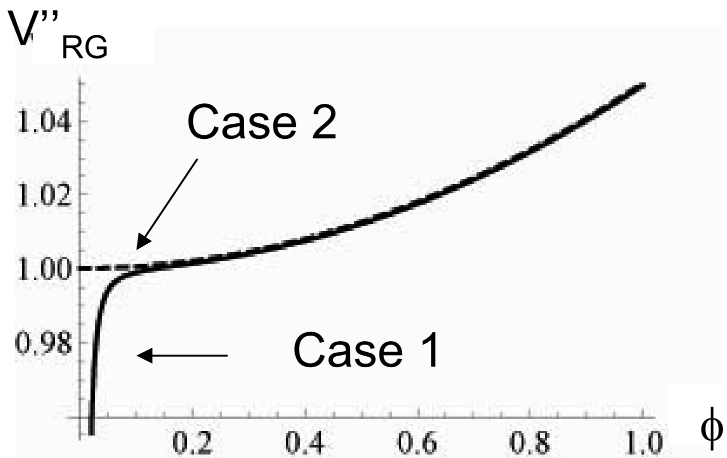
How to choose V_{pert} and $\overline{\mu}^2$?

1) Restored symmetry case ($m^2>0$)

Here $m_{eff}^2 = m^2 + \lambda/2\ \phi^2$

Case 1 : $\overline{\mu}^2 = \phi^2$ and $V_{pert}=V_{CW}$

Case 2 : $\overline{\mu}^2 = m^2 + \lambda/2\ \phi^2$ and $V_{pert}=V_{CW}$



In the case 2, $\overline{\mu}^2$ is IR regulated by m^2 (and don't vanish anymore).

2) Broken symmetry case ($m^2<0$)

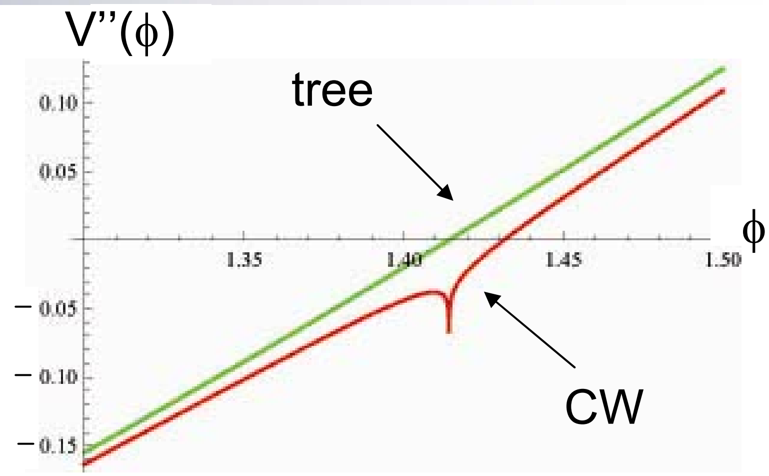
Here $m_{eff}^2 = -m^2 + \lambda/2\ \phi^2$. V'' diverges at $\phi_{crit}^2=2m^2/\lambda$.

Study of $V''(\phi)$ (all choices below give a non-singular V).

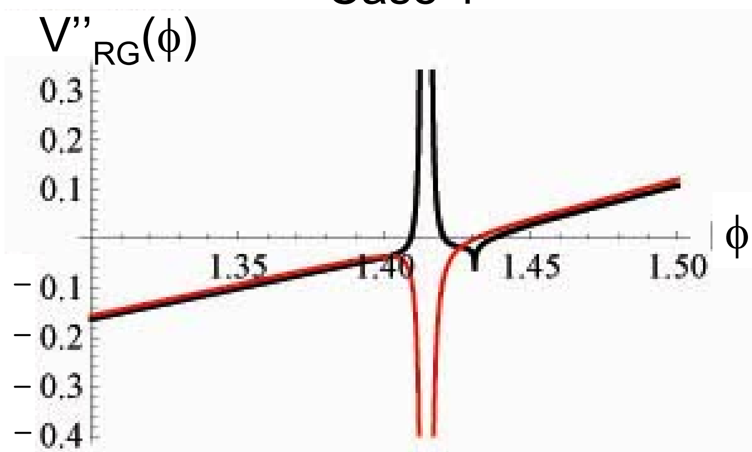
Case 1 : $\bar{\mu}^2 = -m^2 + \lambda/2\ \phi^2$, $V_{pert}=V_{tree}$ or V_{CW}.

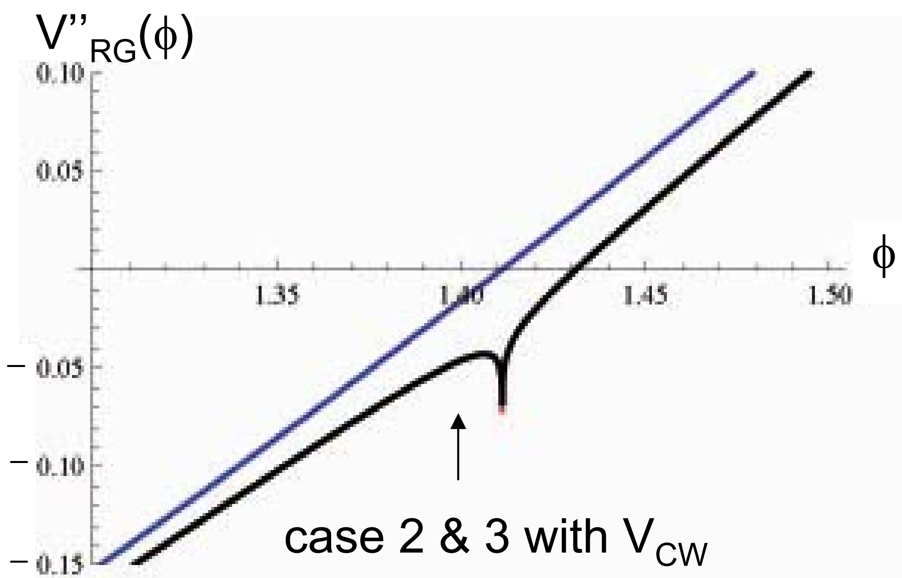
Case 2 : $\bar{\mu}^2 = m^2 + \lambda/2\ \phi^2$, $V_{pert}= V_{CW}$.

Case 3 : $\bar{\mu}^2 = m^2 + \lambda/2\ \phi^2$, $V_{pert}=V_{CW}$ and β-functions @ 2 loop.



Case 1





[J.R., Greene in prep (2008)]

Conclusions

- Cosmic strings are powerful objects to constrain models of inflation motivated by HEP.
- D-term inflation suffers from fine tuning on its coupling constant due to Cosmic Strings formation ($\lambda<10^{-5}$). Situation unchanged if consider non-minimal Kahler or SUGRA from SCFT.
- When embedded in SUSY GUTs, F-term inflation generically produces Cosmic Strings at the end. Similar fine-tuning imposed on its coupling.
- The dynamic shows NO fine-tuning on the Initial Conditions of hybrid inflation if transplanckian field values allowed. The fine-tuned successful points are due to fortuitous trajectories in field space.
- To study predictions of D-term inflation about spectral index, one needs to improve the calculation of radiative correction. The usual renormalization group methods don't work so far.

  This improvement is motivated for many HEP models of inflation (containing an inflection point) as well as other aspects of cosmology (preheating, …).

Many open questions

Phenomenology of SUSY GUTs and orthogonal constraints

• Constraints from neutrinos mass for F-term inflation within SUSY GUTs + see-saw.

• What constraint on M from gauge unification ?

• Constraints from proton decay on other SSBs ? Phenomenology of GUT models with multiple SSB not well known.

Nature of strings in SUSY GUTs

• Generic nature of strings : are the strings abelian or Z_2 or both ? More realistic SSB pattern will involve additionnal discrete symmetries. Formation of hybrid defects or Y-junctions ? Consequence for cosmology ?

RG improved potential for theories with inflection point

• How to construct an improved potential whose second derivative is non singular ?

• Application to MSSM inflation & some string theory models

• Application to (p)reheating of hybrid inflation

Hybrid inflation & initial conditions

• More of the parameter space to study.

• For what models of inflation is it safe to consider super-Planckian values of the fields ?

The End