Modelling the Big Bang in String Theory

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Fundamental questions

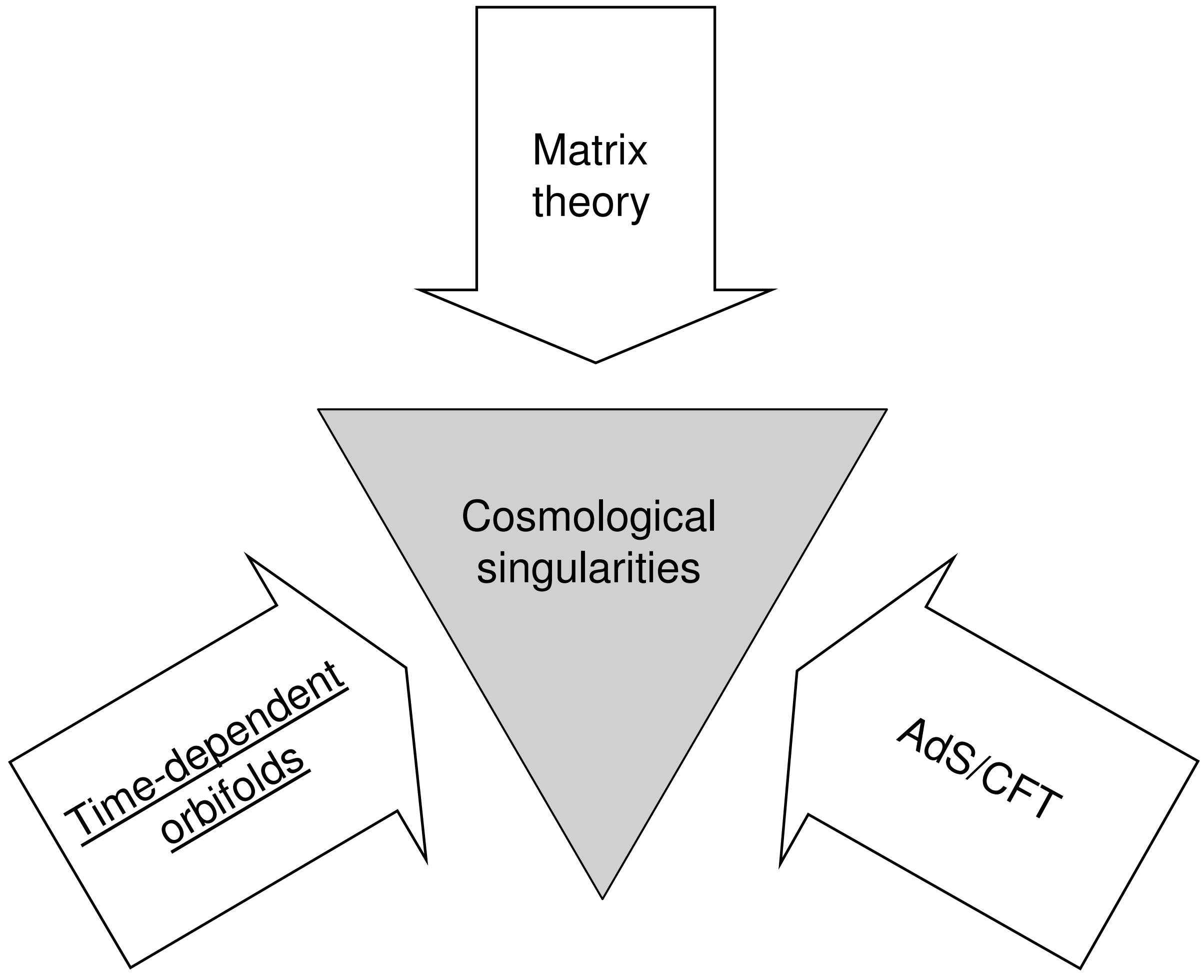
1. What replaces space-time/gravity near the Big Bang?
2. Did space-time exist before the Big Bang? Veneziano; Khoury, Ovrut, Seiberg, Steinhardt, Turok; …
3. If so, how did perturbations propagate through the singularity?
4. If not, what determined the initial state of the universe?

Will look for answers in a few string theory models. (Many other interesting models exist!)

Strategy:

- Start with space-time with space-like (or light-like) singularity
- Embed in string theory and find out whether "reasonable" extrapolations of usual rules of string theory give sensible/consistent results

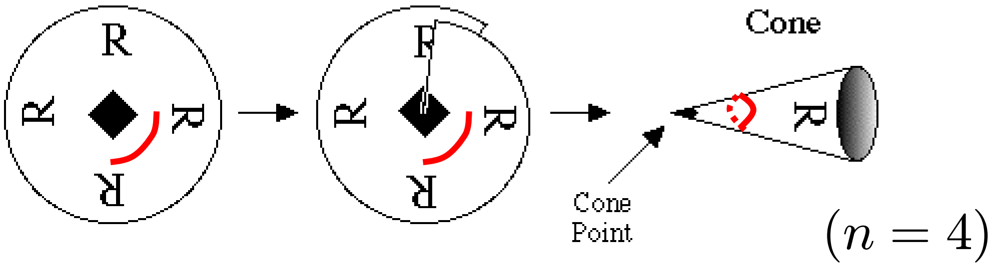
Cosmological singularities: three approaches



String theory resolves static orbifold singularities

$$\mathbb{C}/\mathbb{Z}_n\colon\ z \sim e^{2\pi i/n} z$$

Conical singularity at z=0



Twisted closed strings make string perturbation theory smooth, and thus resolve the singularity perturbatively.

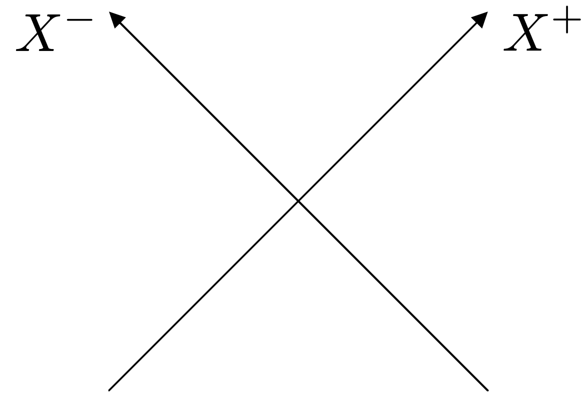
Dixon, Harvey, Vafa, Witten

The Milne-orbifold: a model of a big crunch/big bang singularity

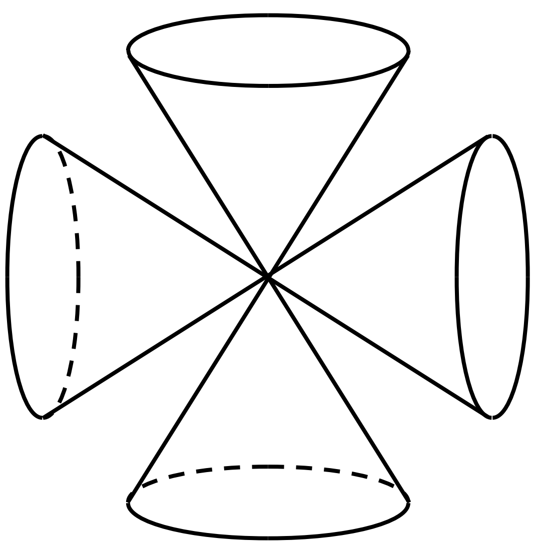
$$ds^2_{10} = -2dX^+dX^- + (dX^i)^2$$

Boost identification: $X^\pm \sim e^{\pm 2\pi\tilde{Q}} X^\pm$

Singularity at $X^\pm = 0$

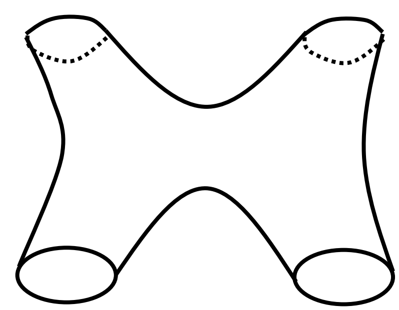


Does propagation through big crunch/big bang singularity make sense?



Study propagation of strings from past to future cone through singularity using standard string perturbation theory.

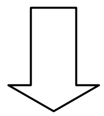
E.g. tree-level 2 → 2 scattering amplitude:



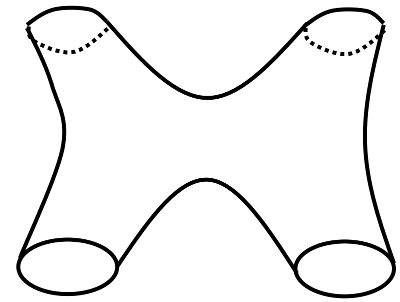
Khoury, Ovrut, Seiberg, Steinhardt, Turok; Cornalba, Costa; Nekrasov; Tolley, Turok; Berkooz, BC, Kutasov, Rajesh; …

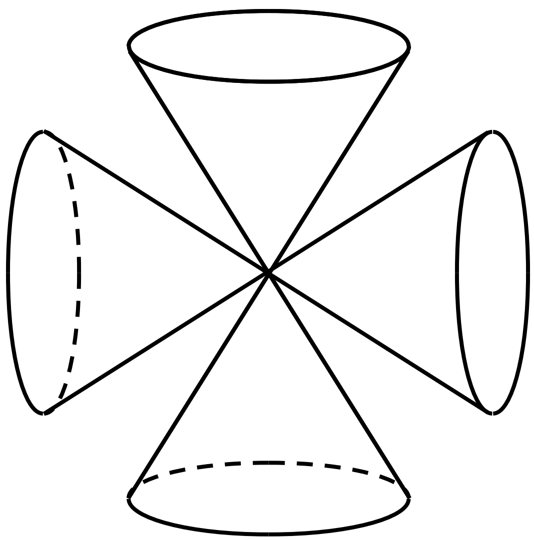
Gravitational backreaction spoils the perturbative picture

$2 \rightarrow 2$ tree level scattering amplitude diverges



the singularity is not resolved in perturbative string theory





As circle shrinks, infinite blue-shift of perturbations creates large gravitational field.

Tree-level gravitational interaction with the second perturbation causes the divergence.

A non-perturbative manifestation of gravitational backreaction: formation of a large black hole

Liu, Moore, Seiberg; Horowitz, Polchinski; Berkooz, BC, Kutasov, Rajesh

Can we make sense of time-dependent orbifolds?

- Tree-level contribution diverges: breakdown of string perturbation theory!

- Other examples: tree-level divergences not always accompanied by large black hole formation
  Cornalba, Costa

- Dynamics of winding modes? Berkooz, Pioline and collaborators

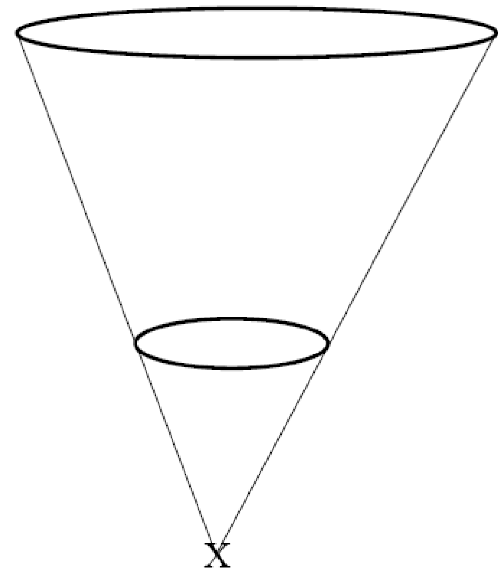
- Eikonal resummation of divergences? Cornalba, Costa

- Models with winding tachyons whose condensation excises “bad” regions in spacetime
  Costa, Herdeiro, Penedones, Sousa; McGreevy, Silverstein; …

- Use D-brane probes to study condensation of twisted sector modes
  Berkooz, Komargodski, Reichmann, Shpitalnik; She; Hikida, Tai;…

Most approaches: no clear answers yet to fundamental questions.
Will discuss one specific model where computations have been claimed to lead to a definite picture.

A variation of the Milne orbifold with localized closed string tachyons



Take fermions anti-periodic around Milne circle → tachyonic winding modes when Milne circle smaller than string length.

Tachyon wave function grows exponentially as one goes back in time towards the singularity. What is the effect of an exponentially large tachyon condensate T near the Milne singularity?

The tachyon appears in the string world-sheet as

$$S = \int d\tau d\sigma \left(-G_{\mu\nu}(X)\partial_\alpha X^\mu \partial^\alpha X^\nu - T(X)\right)$$

Comparing with the world-line action of a relativistic particle

$$S = \int d\tau \left(-G_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu - m^2\right)$$

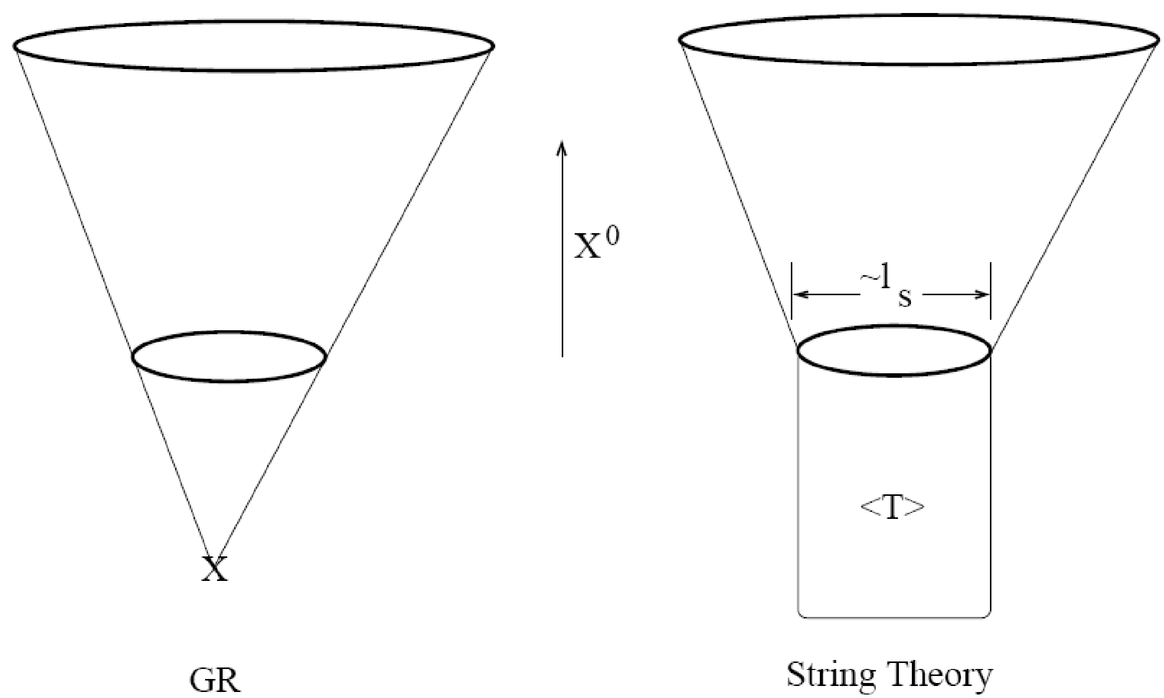
we see that a large tachyon condensate makes all closed string states very heavy.

McGreevy, Silverstein

A variation of the Milne orbifold with localized closed string tachyons

Tachyon condensate → closed strings, including gravity, lifted before singularity is reached.

"Nothing" phase from which spacetime emerges. Perturbative string theory claimed to be valid.



McGreevy, Silverstein

Winding tachyon model: resulting picture

- No bouncing universe
- Space-time replaced by “nothing” phase: no gravity, no other closed string modes
- No unique initial state

Non-perturbative string theory: holography

Holographic principle: a higher-dimensional gravitational theory is described at the fundamental level by a lower-dimensional gauge theory.

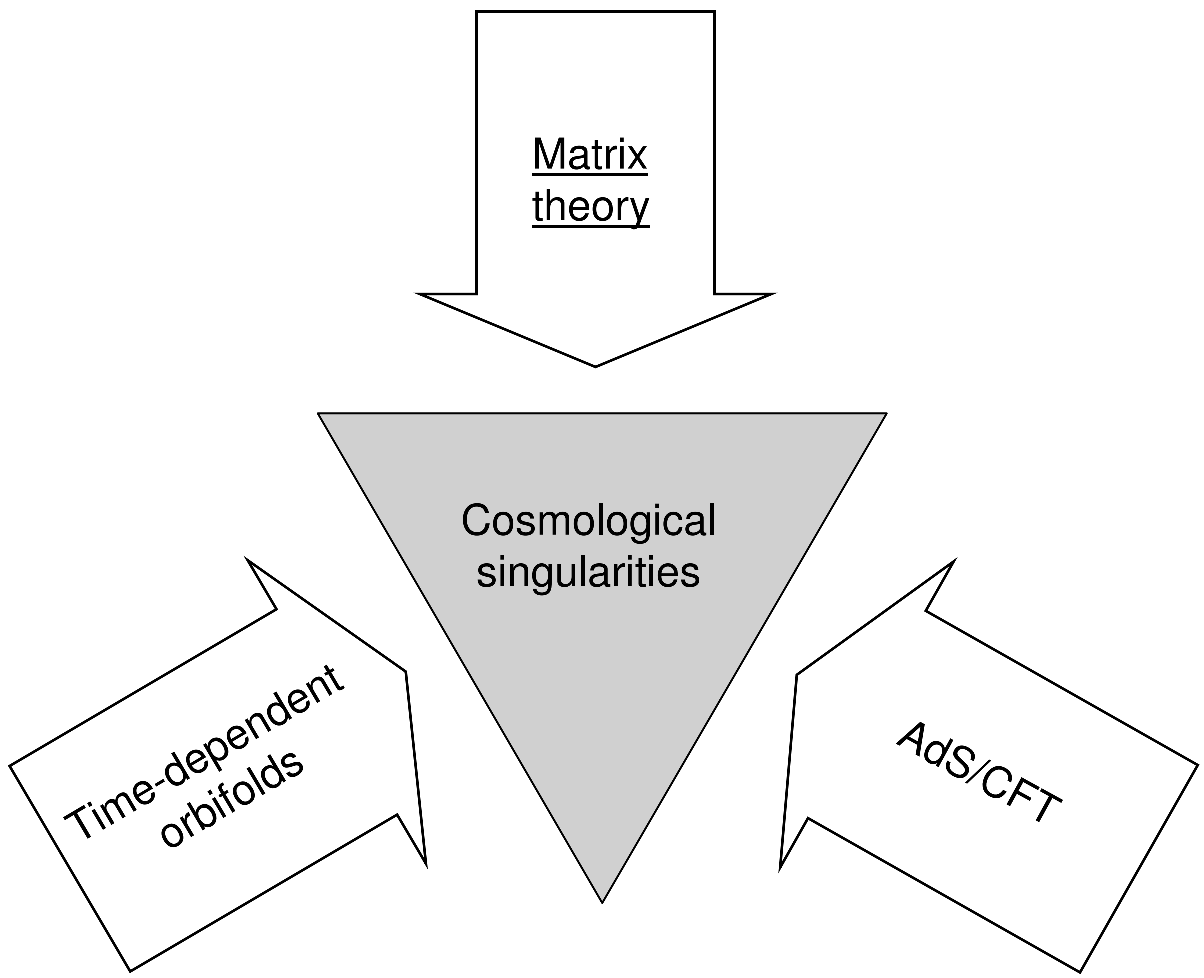
't Hooft, Susskind

Example: black hole entropy is proportional to the horizon area, not to the volume within the horizon.

Bekenstein, Hawking

The known non-perturbative formulations of string theory (in certain background spacetimes) are holographic, e.g. matrix theory (asymptotically flat spacetimes, $\Lambda = 0$) and AdS/CFT (asymptotically AdS spaces, $\Lambda < 0$).

Cosmological singularities: three approaches



Matrix (string) theory: holography in asymptotically flat space-time

Matrix string theory: non-perturbative formulation of type IIA string theory in 10d Minkowski space. Described by $\mathcal{N}=8$ U(N) super-Yang-Mills theory in 2d, in a large N limit:

$$S = \int d\tau d\sigma \, \mathrm{Tr} \left((D_\mu X^i)^2 + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2}[X^i, X^j]^2 + \text{fermions} \right)$$

World-sheet: infinite cylinder. Coordinates (τ, σ), where $\sigma \sim \sigma + 2\pi$.

X^i: N x N hermitean matrices; index i labels 8 spacetime directions transverse to worldsheet.

<u>At weak string coupling</u> $g_s \to 0$: $[X^i, X^j] = 0$

- Eigenvalues of X^i correspond to coordinates of (pieces of) superstring.
- Off-diagonal matrix elements: very massive, can be integrated out.
- Space-time arises dynamically from the “moduli space” of vacua.

Banks, Fischler, Shenker, Susskind; Motl; Banks, Seiberg; Dijkgraaf, Verlinde, Verlinde

Concrete model: light-like linear dilaton

Simple time-dependent solution of 10d (type IIA) string theory: flat space with a light-like linear dilaton (preserves ½ susy):

$$ds_{10}^2 = -2dX^+dX^- + (dX^i)^2$$

$$\Phi = -QX^+$$

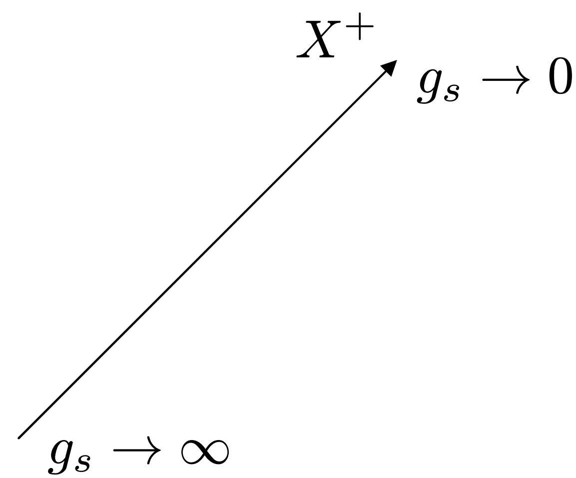
The dilaton Φ is a scalar field that appears in the low-energy effective action as

$$S \sim \int d^{10}x\,\sqrt{G}\,e^{-2\Phi}\,(R + 4\partial_\mu\Phi\partial^\mu\Phi + \ldots)$$

Therefore, the exponential of the dilaton can be viewed as the string coupling "constant":

$$g_s = e^{\Phi}$$

Strong coupling singularity for $X^+ \to -\infty$ corresponds in Einstein frame to curvature singularity at finite affine parameter



BC, Sethi, Verlinde

Matrix description of the light-like linear dilaton: “matrix big bang”

$$S = \int d\tau d\sigma \, \mathrm{Tr} \left((D_\mu X^i)^2 + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2}[X^i, X^j]^2 + \text{fermions} \right)$$

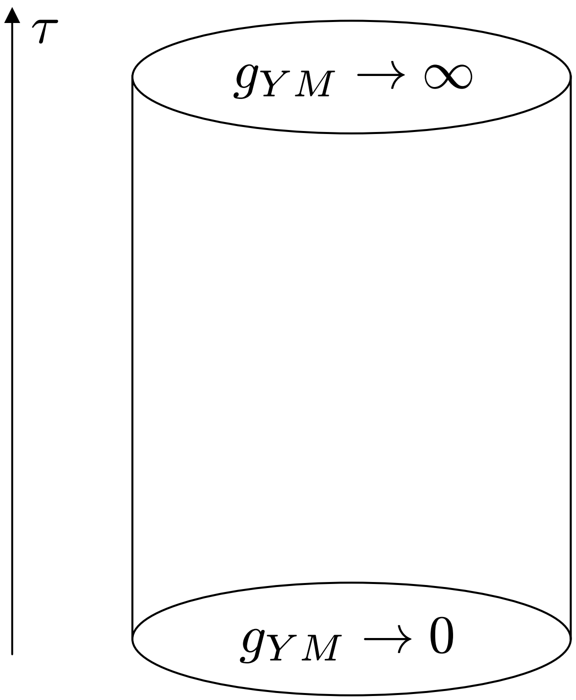
Turns out: τ is related to X^+ by $X^+ = \frac{\tau}{R}$, where R is a parameter related to the total light-cone momentum p^+ of the system under consideration: $p^+ = \frac{N}{R}$.

Result: simply plug in $g_s = e^{-QX^+} = e^{-\frac{Q\tau}{R}}$, leading to (1+1)-dimensional SYM on the cylinder, with coupling

$$g_{YM} = \frac{1}{\ell_s} \exp\left(\frac{Q\ell_s\tau}{R}\right)$$

BC, Sethi, Verlinde

Cosmological evolution: emergence of spacetime



$[X^i, X^j] = 0$: spacetime emerges
(weakly coupled strings)

free SYM: non-commuting matrices
(new light degrees of freedom)

BC, Sethi, Verlinde

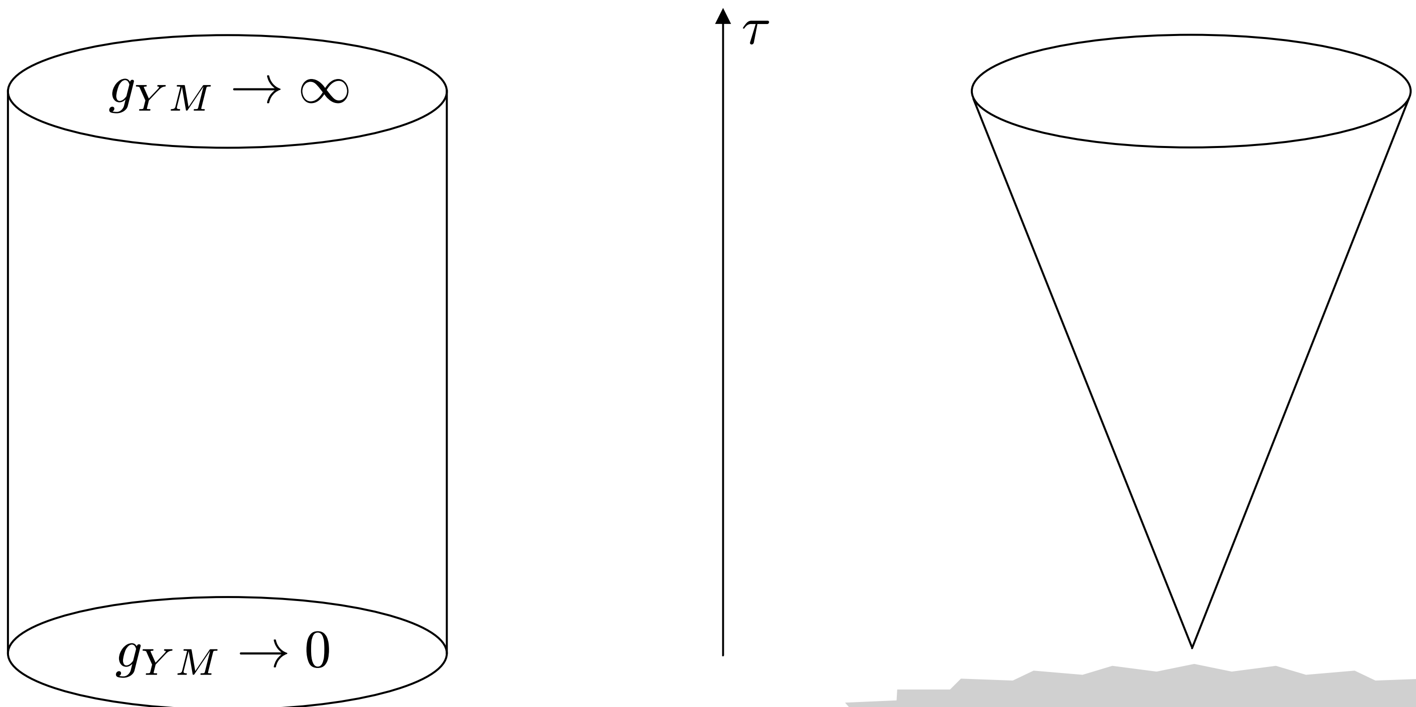
Cosmological evolution: two equivalent pictures

SYM on the cylinder with

$$g_{YM} = \frac{1}{\ell_s} \exp\left(\frac{Q\ell_s\tau}{R}\right)$$

SYM with constant coupling on future Milne cone

$$ds^2 = e^{\frac{2Q\tau}{R}}(-d\tau^2 + d\sigma^2)$$



Can time evolution be defined beyond the Milne singularity?

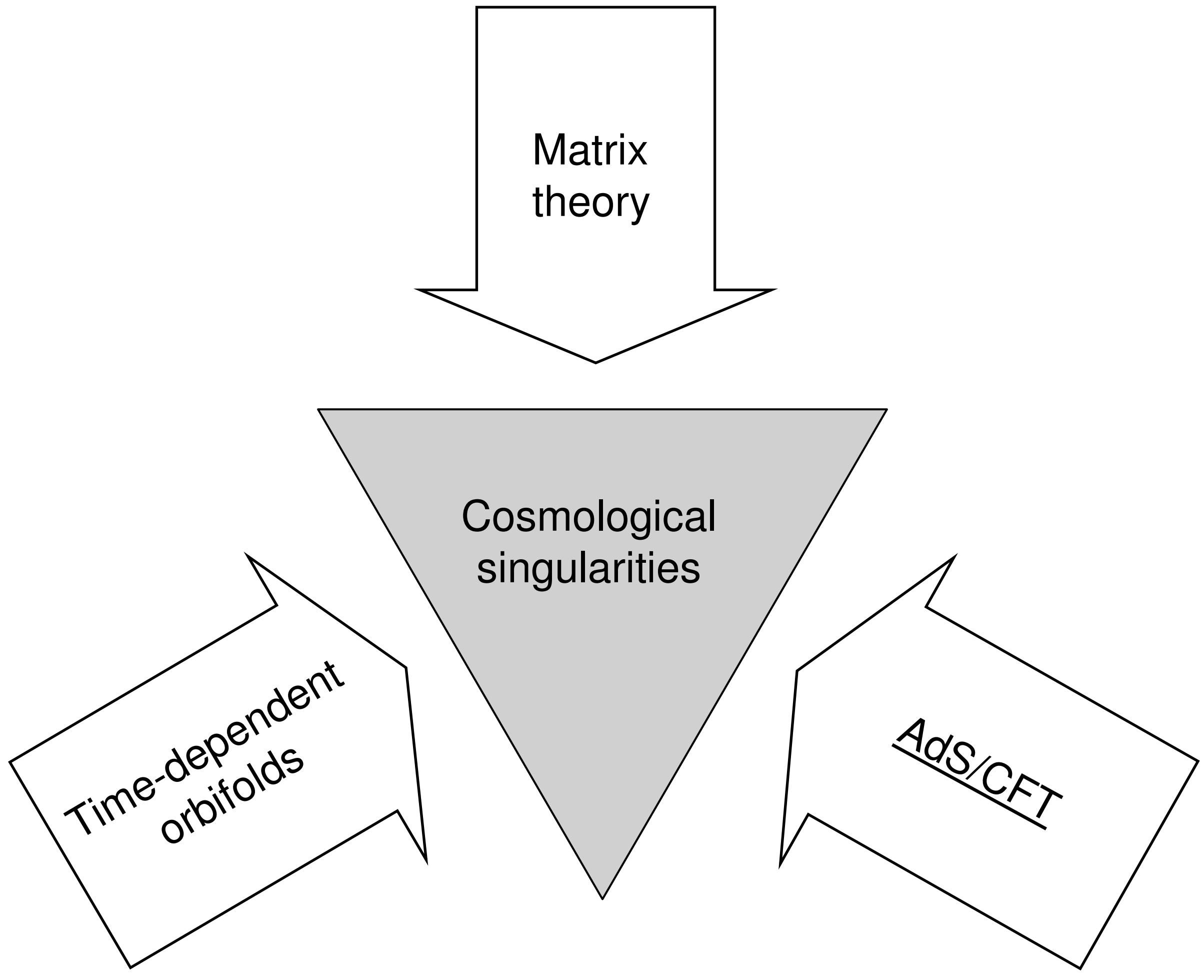
BC, Sethi, Verlinde

BC, De Roo, Evnin, in progress

Matrix Big Bang model: resulting picture

- Spatial coordinates replaced by non-commuting matrices near singularity
- Not clear yet whether time can be continued beyond the singularity

Cosmological singularities: three approaches



AdS/CFT: holography with negative cosmological constant

Type IIB string theory on $AdS_5 \times S^5$ with string coupling g_s and radius of curvature R	dual ↔	$\mathcal{N}$=4 SU(N) super-Yang-Mills theory with Yang-Mills coupling g_{YM} on the 3+1 dimensional boundary of AdS_5
g_s	↔	$(g_{YM})^2$
$(RM_s)^4$	↔	$(g_{YM})^2 N$

The AdS/CFT correspondence gives a non-perturbative definition of string theory in (asymptotically) anti-de Sitter space.

The weak curvature (supergravity) limit of the string theory corresponds to the strong ’t Hooft coupling limit of the gauge theory.

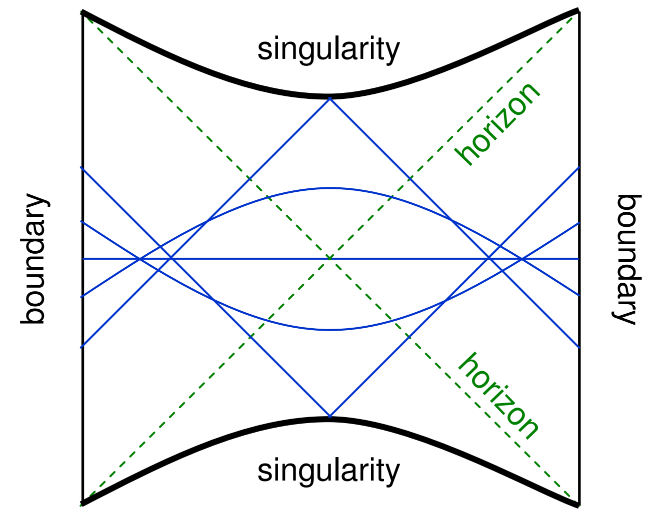
Maldacena

AdS Schwarzschild black hole: subtle signatures of the singularity

Schwarzschild black hole in 5d
anti-de Sitter space

↕ AdS/CFT

entangled state in tensor product of two
N=4 super-Yang-Mills theories
(one at each boundary).



Analytically continued gauge theory correlation functions: divergences from spacelike geodesics that come close to singularity → subtle signature of black hole singularity.

Goal: use gauge theory to determine if singularity is resolved in string theory.

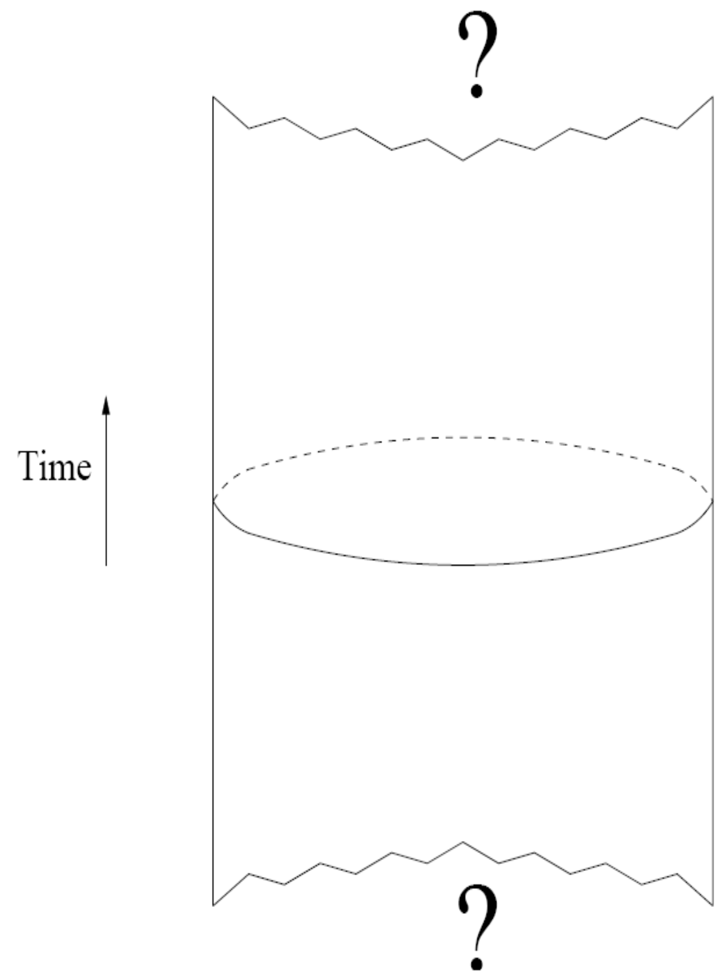
Challenging because black hole hidden behind horizon.

Kraus, Ooguri, Shenker; Fidkowski, Hubeny, Kleban, Shenker; …

AdS cosmologies: basic idea

Starting point: supergravity solutions in which smooth, asymptotically AdS initial data evolve to a big crunch singularity in the future (and to a big bang singularity in the past).

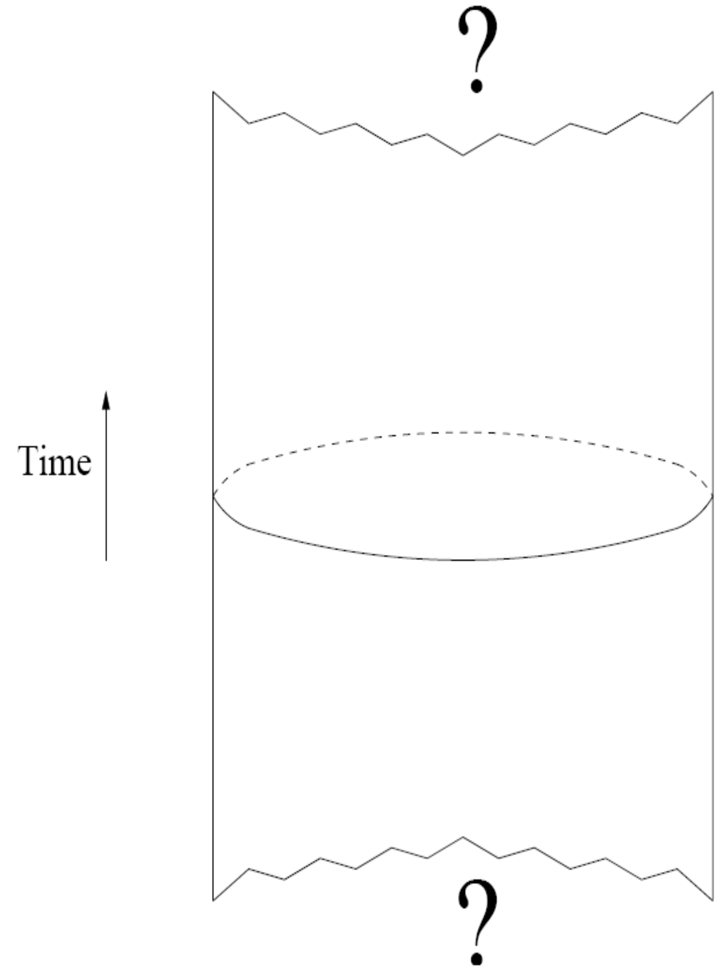
Can a dual gauge theory be used to study the singularity in quantum gravity?



Hertog, Horowitz

AdS cosmologies: basic idea

- AdS: boundary conditions required
- Usual supersymmetric boundary conditions: stable
- Modified boundary conditions: smooth initial data that evolve into big crunch (which extends to the boundary of AdS)
- AdS/CFT relates quantum gravity in AdS to field theory on its conformal boundary
- Modified boundary conditions → potential unbounded from below in boundary field theory; scalar field reaches infinity in finite time
- Goal: learn something about cosmological singularities (in the bulk theory) by studying unbounded potentials (in the boundary theory)



Hertog, Horowitz

Quantum mechanics with unbounded potentials

Consider $\hat{H} = -\frac{d^2}{dx^2} + V(x)$ with $V(x) = -a^2 x^p$ for $x > 0$ and $p > 2$. For such potentials, classical trajectories can reach infinity in finite time. So do quantum mechanical wavepackets, which would seem to lead to loss of probability/unitarity.

Unitarity can be restored by restricting the domain of allowed wavefunctions such that the Hamiltonian is self-adjoint ("self-adjoint extension"):

$$(\hat{H}\phi_1, \phi_2) = (\phi_1, \hat{H}\phi_2) \qquad \leftrightarrow \qquad \left[\frac{d\phi_1^*}{dx}\phi_2 - \phi_1^*\frac{d\phi_2}{dx}\right]_{x=\infty} = 0$$

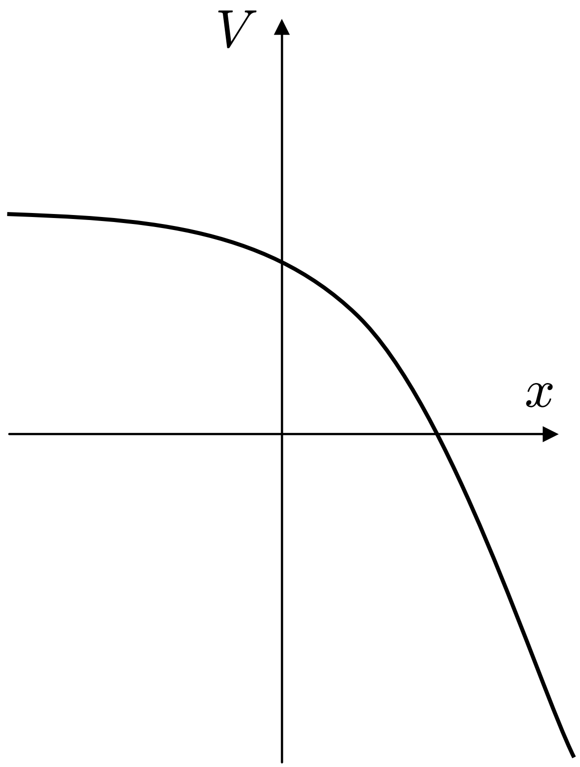
The WKB energy eigenfunctions $[2(E + a^2 x^p)]^{-1/4} \exp\left(\pm i \int_0^x \sqrt{2(E + a^2 y^p)}dy\right)$

are an increasingly good approximation for large x. Unitarity can be achieved by only allowing the linear combination that for large x behaves as

$$\psi_E^\alpha(x) \sim (2a^2 x^p)^{-1/4} \cos\left(\frac{\sqrt{2}ax^{p/2+1}}{p/2+1} + \alpha\right)$$

Reed, Simon

Interpretation of the self-adjoint extensions

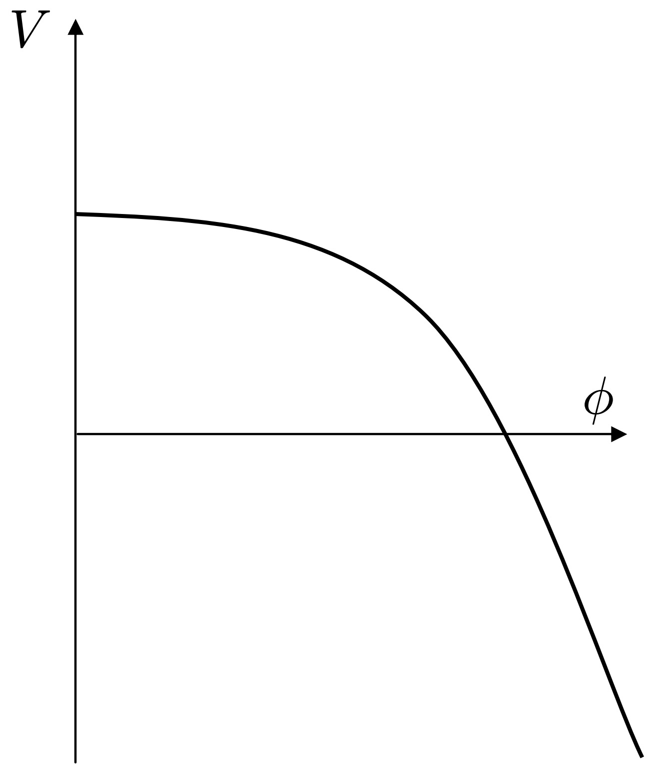


Right-moving wave packet disappearing at infinity is always accompanied by left-moving wave packet appearing at infinity (think of brick wall at infinity)

Carreau, Farhi, Gutmann, Mende

Energy spectrum consists of bound states (energy levels depend on phase α) as well as scattering states (if potential is bounded from above)

Does the universe bounce?



Consider the homogeneous mode $\phi_0(t)$.

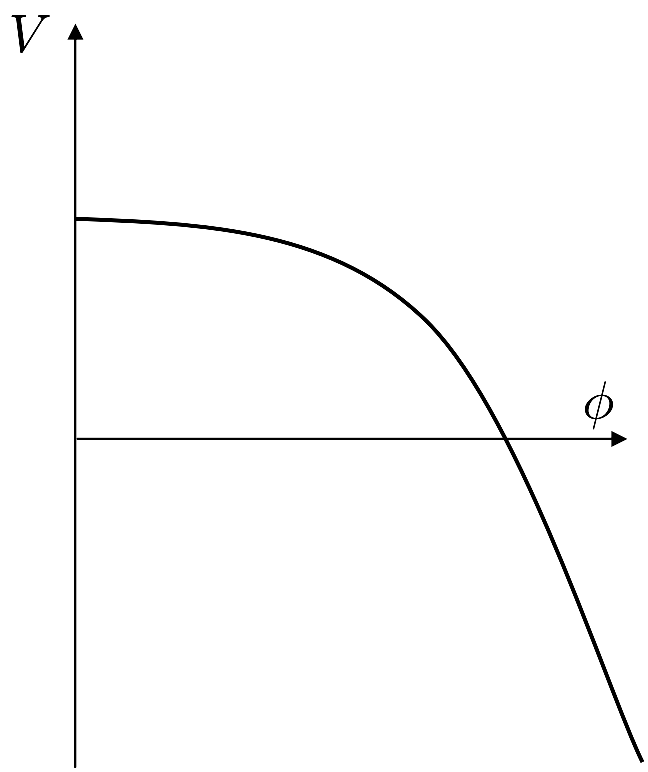
Self-adjoint extension would seem to imply that after rolling to infinity, it rolls up the hill again, returning to the original configuration → bounce in space-time.

However, inhomogeneous modes may be created and may drain energy out of the homogeneous mode.

Thus we need to compute the energy in created particles and see how far the homogeneous mode can roll up the hill again.

Note: this is particle creation in the boundary field theory, not in space-time!

Does the universe bounce: results



If the homogeneous mode were classical, particle creation would drain too much energy out of it for it to bounce back up the hill.

However, the field theory lives in a finite volume space, so the homogeneous mode spreads quantum mechanically. This turns out to suppress particle production.

If the initial spread is large enough (yet small enough for the wave packet to be well-localized), the most likely outcome is a bounce.

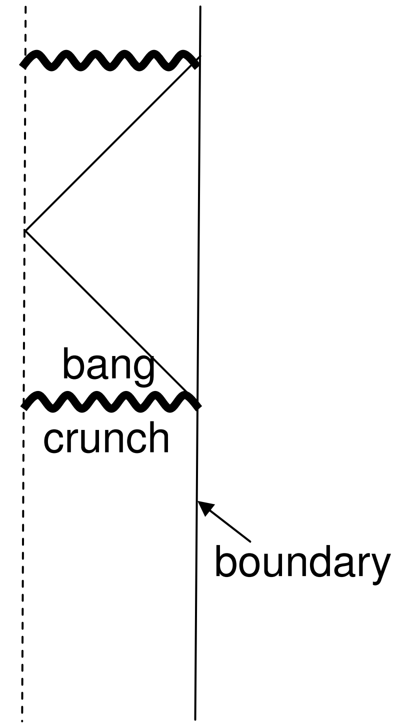
Caveat: computations done for small ’t Hooft coupling.

Reconstructing bulk perturbations from the boundary

AdS/CFT: state of boundary theory $\leftrightarrow$ bulk state

boundary data $\leftrightarrow$ asymptotic behavior of bulk fields

Bulk configuration before crunch → state in boundary theory
→ propagate beyond singularity → bulk configuration after bang



Relevant modes in boundary theory are frozen in near the singularity

→ quantum fluctuations become classical

→ classical bulk metric perturbations (cosmological perturbations)

Precise correspondence yet to be worked out

Intriguingly, boundary correlators are approximately scale invariant, reflecting the slightly broken conformal invariance of the boundary theory!

(Keep in mind: not a realistic model!)

AdS cosmology: resulting picture

- Computations suggest that bounce is possible (modulo caveat)
- Space-time singularity replaced by field reaching infinity in dual field theory. Dynamics described by self-adjoint extension; possible underlying degrees of freedom unclear.

Conclusions

- Winding tachyon model: space-time near singularity replaced by "nothing" phase. No pre-big-bang space-time

- Matrix big bang: space-time near singularity replaced by non-commuting matrices. Can propagation through the singularity be described?

- AdS cosmology: singularity is dual to field reaching infinity in field theory with unbounded potential. Evidence that a bounce is possible.