Lucio Paternò

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(not yet retired, not yet 70 year old *)

The Solar Differential Rotation:

a Historical View

**** Birthday on December 22nd, this year***

Rome 24 June 2009

Lucio Paternò

I.N.P.D.A.P. – National Institute for Social Security of Civil Servants

The Solar Differential Rotation: a Historical View

Baia delle Zagare 23 June 2011

PRE-HELIOSEISMOLOGY ERA

20 years of work in 25 minutes

Observations of differential rotation

-Sunspots motions

Scheiner (1630): *first noticed that the equatorial sunspots had a shorter rotation period than the polar ones*

Carrington (1863): *first established a dependence of rotation on latitude*

Spörer (1874): *established the law of rotation*

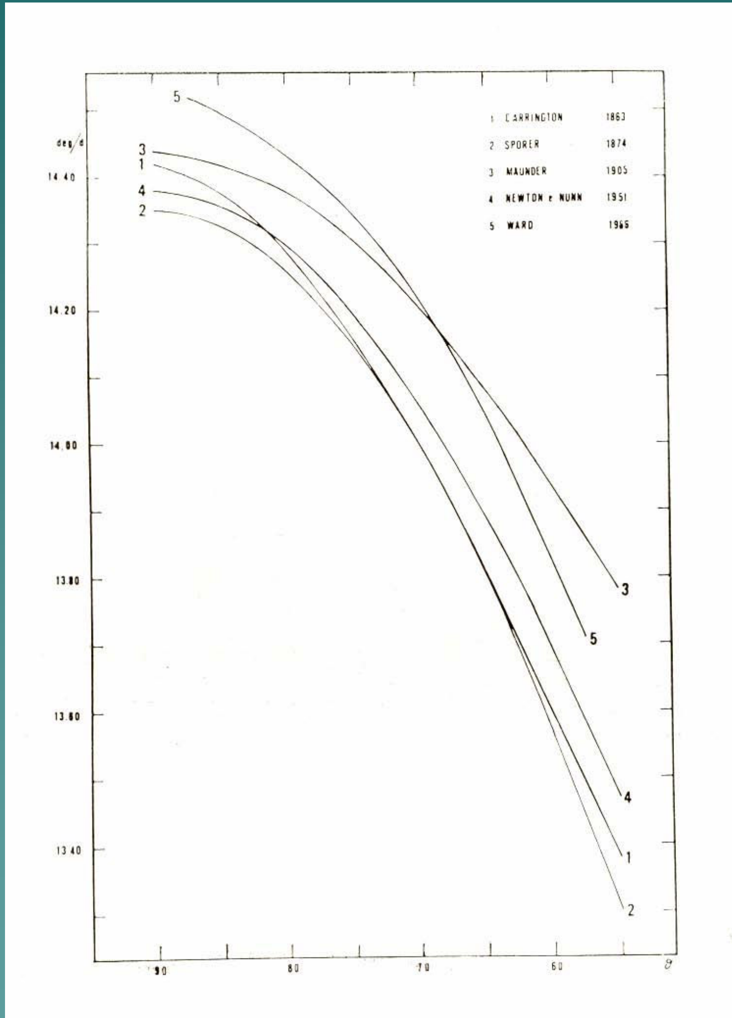
Maunder (1905): *discovered the famous minimum of sunspots between 1645 and 1715 by analyzing the old available data, and established a law of rotation*

Newton & Nunn (1951): *established the modern law of rotation by analyzing some 10 years of sunspot recurrent group motions*

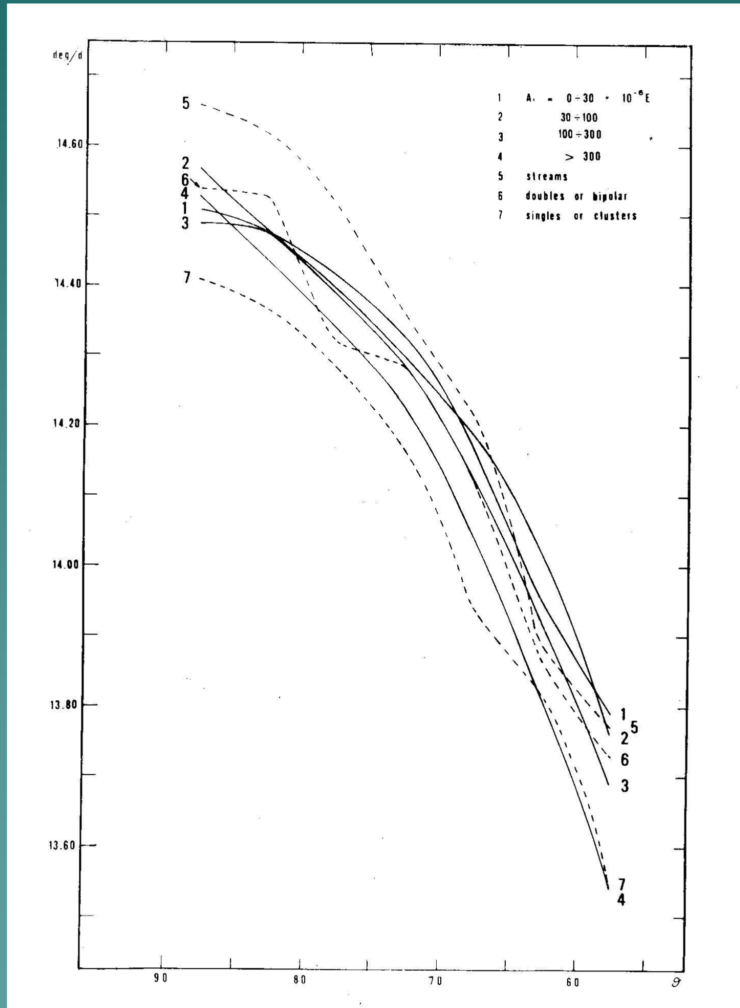
$$\Omega(\varphi) = 14.38 - 2.44 \sin^2 \varphi \text{ deg/day}$$

Ward (1965): *considered sunspots of different areas and types*

All determinations



Ward's determinations



Spectroscopic observations (Doppler effect)

Adams (1909)

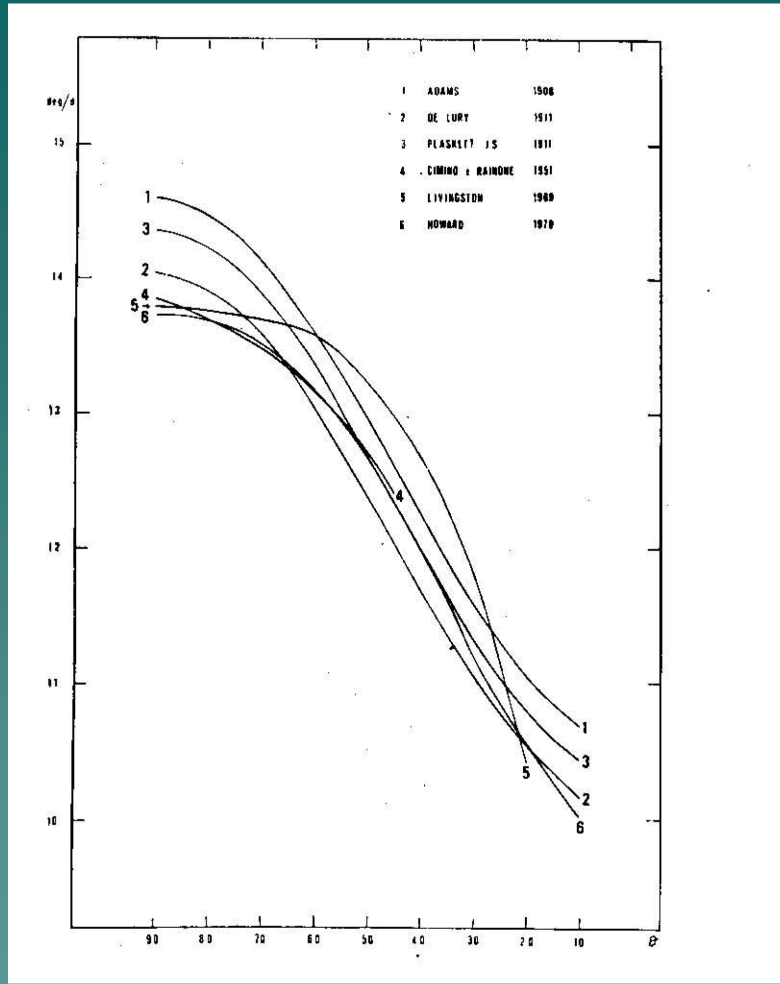
Plaskett (1915)

De Lury (1916)

Cimino & Rainone (1951)

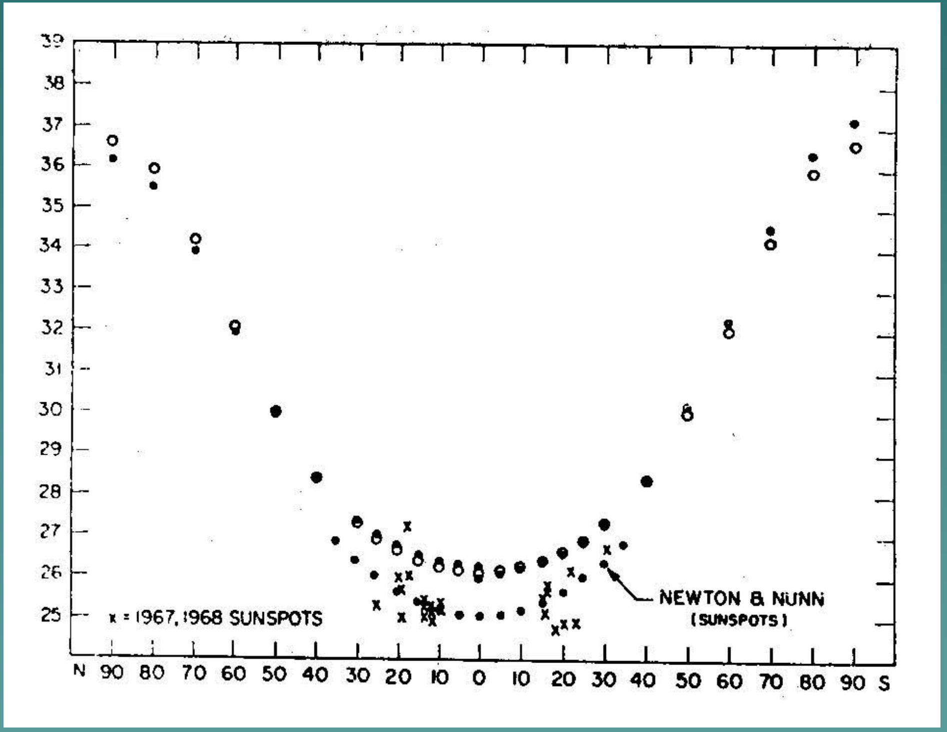
Livingston (1969)

Howard & Harvey (1970) *



$$* \ \Omega(\varphi) = 13.76 - 1.74 \sin^2 \varphi - 2.19 \sin^4 \varphi \ \text{deg/day}$$

Rotation periods determined by sunspot motions are shorter than those determined by spectroscopic measurements



Howard & Harvey vs Newton & Nunn

Secular variation of equatorial velocity

LONG TERM VARIATION OF THE SOLAR EQUATORIAL VELOCITY AND ITS RELATION TO NON-AXISYMMETRIC CONVECTION

G. BELVEDERE
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and

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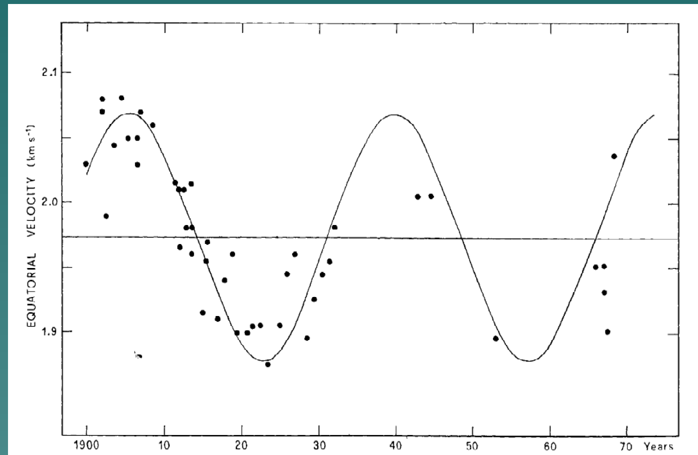
(Received 22 January; in revised form 21 December, 1974)



$P_{obs} \approx 34$ years

$$\omega_c = \omega^L_L - \omega^L_{L+2} = \Omega_\odot \left(\frac{1}{L+1} - \frac{L}{(L+2)(L+3)} \right).$$

$P \approx 20$ years, for $L \approx 29$ ($LH_p \approx 2R_\odot$)

No pole-equator temperature differences

Appenzeller & Schröter (1967)	*< 6 K*
Plaskett (1970)	*≈ 5 %*
Caccin et al. (1970)	*< 1 %*
Altrock & Canfield (1972)	*< 1.5 K*
Rutten (1973)	*< 3 K*
Noyes et al. (1973)	*< 7 K*
Falciani et al. (1974)	*≈ 0 K*

The observations of the surface rotation and pole-equator temperature differences were the only information the scientists of the '60s and '70s had in the hands for explaining the mechanism that generates the solar differential rotation.

Plus a theoretical knowledge of the internal stratification of the Sun as given by the evolutionary models that indicated a depth of the convection zone of about 0.8 $R_\odot$, shallower than that derived from helioseismology and from the modern theoretical models.

Scarce computing facilities for approaching complex fluid dynamic problems.

The proved existence of a quasi-stationary equatorial "acceleration" implies the action of a mechanism that transfers angular momentum toward the equator balanced by an equal amount of viscous momentum transferred from the equator to the poles.

Plausible mechanisms

- ***Large scale meridian circulation***
- ***Reynolds stress***

Large scale meridian circulation

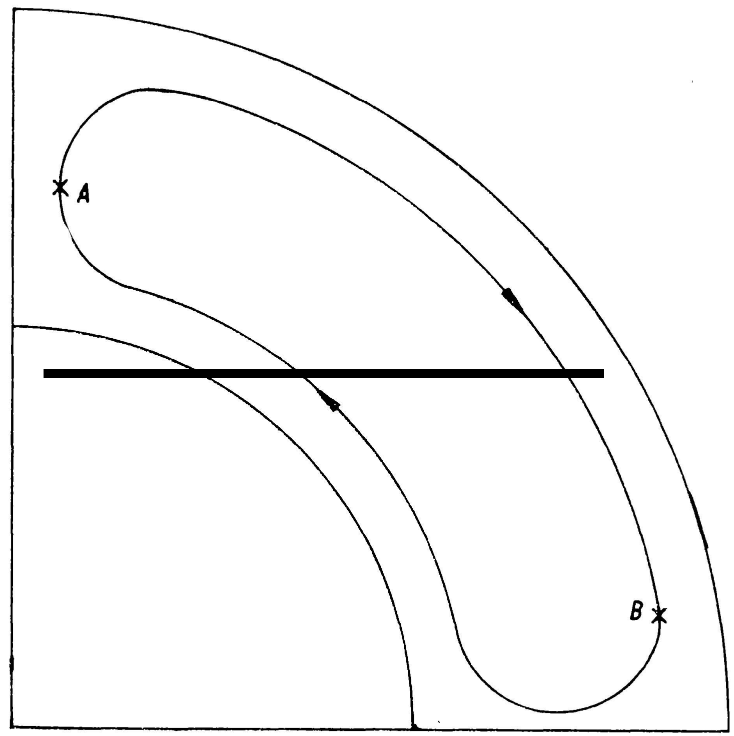
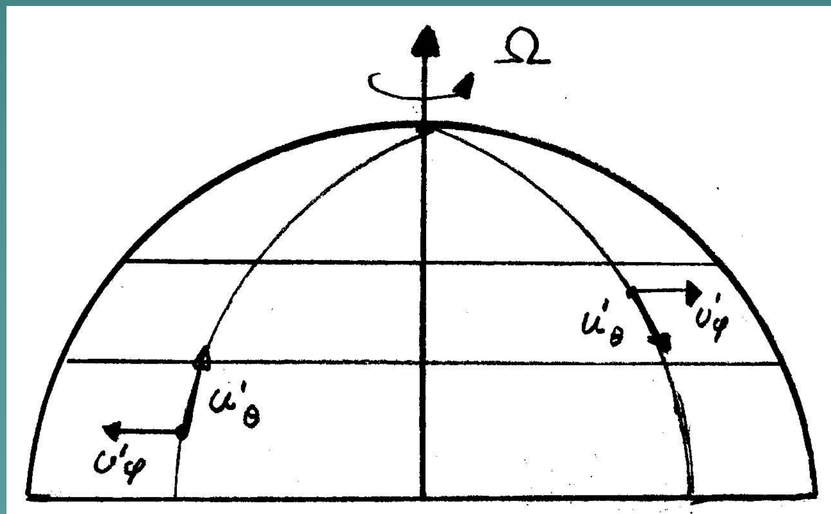


Fig. 1. Meridional circulation in a convective zone

Reynolds stresses

-Correlation of perpendicular velocities:

$\langle u_\theta' u_\varphi' \rangle$; $\langle u_r' u_\theta' \rangle$; $\langle u_r' u_\varphi' \rangle$



Astron. Astrophys. 252, 337–342 (1991)

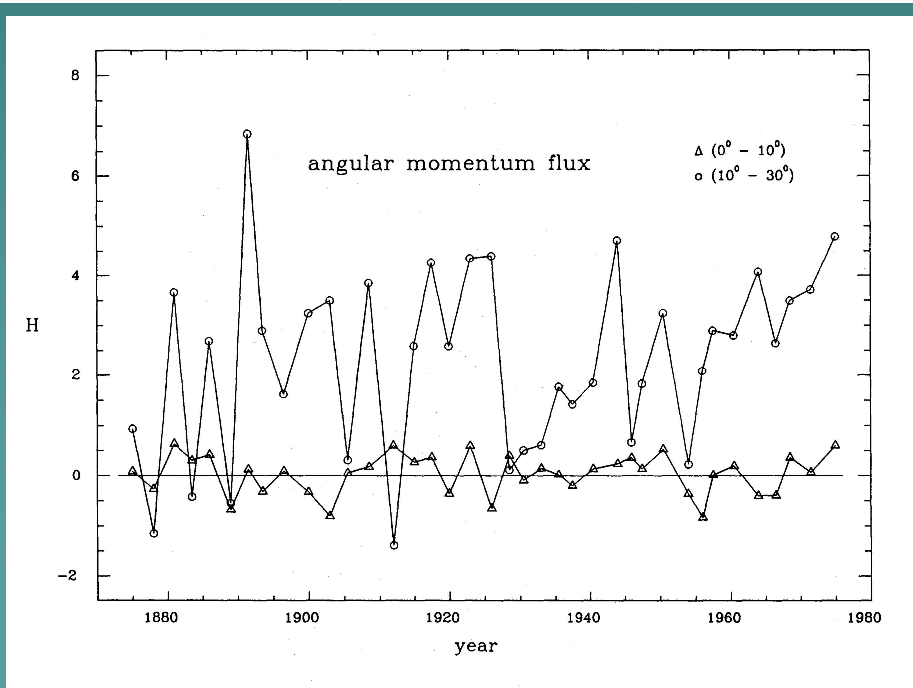
ASTRONOMY
AND
ASTROPHYSICS

Angular momentum transport by Reynolds stresses determined from the analysis of 100-year sunspot motions and its variations with solar cycle

L.釘Paternò[1,3], D. Spadaro[2,3], R.A. Zappalà[2,3], and F. Zuccarello[1,3]

[1] Istituto di Astronomia dell'Università di Catania, città Universitaria, V. le A. Doria 6, I-95125 Catania, Italy
[2] Osservatorio Astrofisico di Catania, città Universitaria, V. le A. Doria 6, I-95125 Catania, Italy
[3] CNR-Gruppo Nazionale di Astronomia, UdR di Catania, città Universitaria, V. le A. Doria 6, I-95125 Catania, Italy

Received March 18, accepted April 12, 1991



Anisotropy of viscosity

The pioneer: Ludwig Biermann 1951

Zeitschrift für Astrophysik, Bd. 28, S. 304—309 (1951).

Bemerkungen über das Rotationsgesetz in irdischen und stellaren Instabilitätszonen.

Von

L. Biermann,

Max-Planck-Institut für Physik, Göttingen.

Mit 1 Textabbildung.

(Eingegangen am 9. Oktober 1950.)

Zusammenfassung: Es wird gezeigt, daß in thermisch bedingten Instabilitätszonen auf rotierenden Körpern, wie sie in der Atmosphäre der Erde und auf Sternen auftreten, die Anisotropie der Geschwindigkeiten der Turbulenzelemente bewirkt, daß der Austausch (anders als die molekulare Reibung) nicht auf einen Ausgleich der Winkelgeschwindigkeit hinarbeitet; vielmehr sind die Verhältnisse komplizierter, und im stationären Zustand wird das Rotationsmoment in radialer Richtung im allgemeinen ausgeglichener sein, als dem Zustand starrer Rotation entspricht. Die Konsequenzen dieses Sachverhalts für die dynamische Meteorologie und die Dynamik der Sternmaterie und des interstellaren Gases werden kurz skizziert.

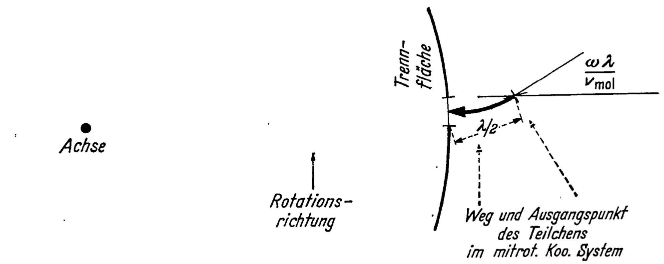


Abb. 1a. Molekülbahn in einer rotierenden Gaskugel.

PAPER 27

ON MERIDIONAL CIRCULATIONS IN STELLAR CONVECTIVE ZONES

L. BIERMANN

Max Planck Institut für Physik, Göttingen, Germany

ABSTRACT

It is shown in outline by a discussion of the equation of motion in all its components and by taking regard of the geometrical properties of turbulent exchange, that in general the state of pure rotation (without meridional circulations) is not a possible stationary state of motion for convective zones of stars.

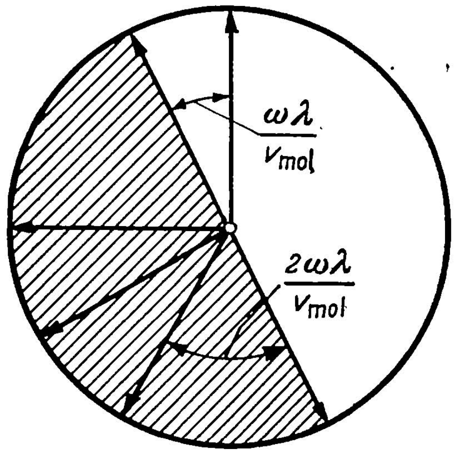


Abb. 1b. Geschwindigkeitsraum (schraffiert der Raum der Teilchen, welche die Trennfläche durchsetzen) (mitrot. Koo. System)

Rudolf Kippenhahn 1961

DIFFERENTIAL ROTATION IN STARS WITH CONVECTIVE ENVELOPES

RUDOLF KIPPENHAHN
Princeton University Observatory, Princeton, New Jersey
Received August 4, 1962

ABSTRACT

The rotation of a viscous shell is investigated in which the viscosity is caused by convection and is not isotropic. With a plausible assumption for the viscosity tensor, the problem can be solved with an approximation method that is valid for sufficiently high viscosity (or sufficiently slow rotation). The solution shows meridional circulation and differential rotation in the shell. Numerical solutions for constant density ρ and constant viscosity η in the shell are found and compared with the motions in the hydrogen convective zone of the sun.

Anisotropy of viscosity

given by a tensor H which has a diagonal form in a system of spherical co-ordinates r, ϑ, φ:

$$H = \begin{pmatrix} \eta_{rr} & 0 & 0 \\ 0 & \eta_{\vartheta\vartheta} & 0 \\ 0 & 0 & \eta_{\varphi\varphi} \end{pmatrix} . \qquad (1)$$

The radial component η_{rr} may be given by the usual expression in the mixing length theory

$$\eta_{rr} = \eta = \rho v_t l , \qquad (2)$$

while the other two components may be given by

$$\eta_{\vartheta\vartheta} = \eta_{\varphi\varphi} = s\eta_{rr} = s\eta , \qquad (3)$$

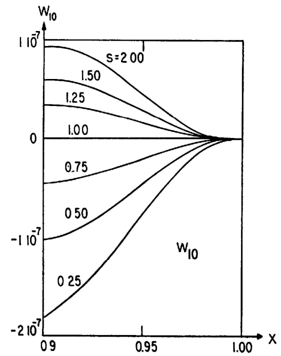


FIG. 2.—The function w_{10} for different parameters s and for $r_i = 0.9$

Following Kippenhahn with anisotropic viscous momentum transfer:

Sakurai 1966 - Cocke 1967

THE ASTROPHYSICAL JOURNAL, Vol. 150, December 1967

ON THE SOLAR DIFFERENTIAL ROTATION AND MERIDIONAL CURRENTS

W. J. COCKE*
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and Aerospace Corporation, El Segundo, California

Received March 1, 1967; revised May 22 and July 19, 1967

ABSTRACT

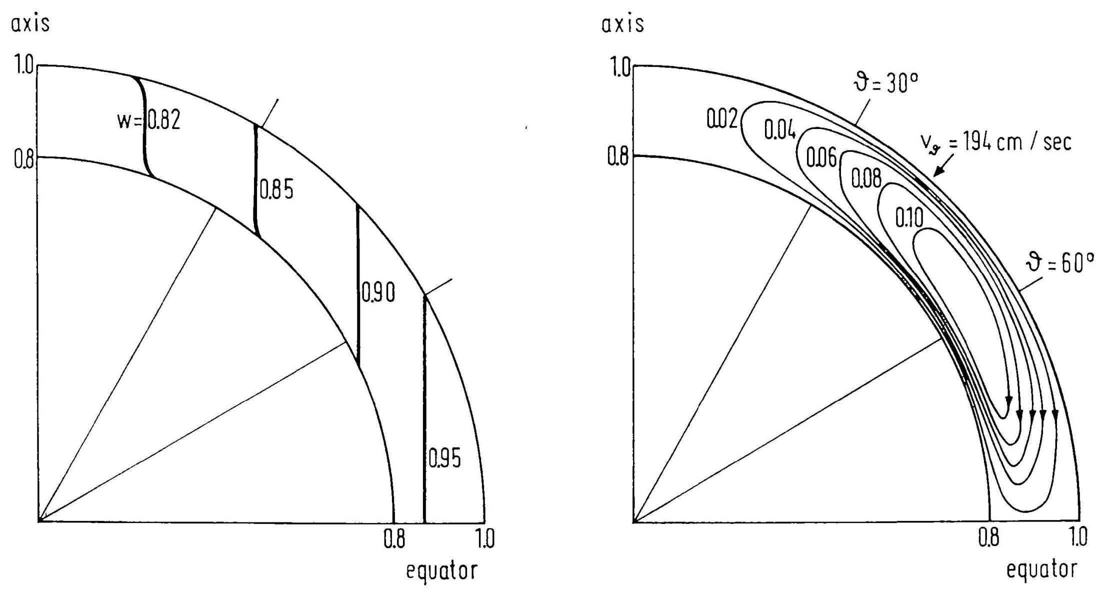
The solar differential rotation, and large-scale meridional currents are investigated with the axially symmetric, time-independent hydrodynamic equations of motion, including anisotropic convective viscosity forces. The ϕ-component of the equations of motion is integrated to give a linear, ordinary differential equation determining the angular velocity distribution. The Reynolds number for the differential rotation in the convection zone is shown to be large, and an approximation based on this fact is used to solve the equations of motion to first order, under the assumption that the polar heating effects are negligible and that the convection zone is barytropic. A good fit to the observed differential rotation is obtained if the anisotropy parameter satisfies $s - 1 \simeq \frac{1}{5}$, and the differential rotation is then approximately independent of the magnitude of the dynamic convective viscosity η. The circulational velocities near the surface at high latitudes are $\sim (s - 1)\rho^{-1}$ grad η. Reasonable agreement with observed values is shown.

Then Köhler 1970

DIFFERENTIAL ROTATION CAUSED BY ANISOTROPIC TURBULENT VISCOSITY*

H. KÖHLER
Universitäts-Sternwarte, Göttingen, Germany

(Received 23 January, 1970)



$\nu = \nu_\odot \quad s = 1.2$

Rüdiger 1974 – Λ effect

229

Astron. Nachr., Bd. 295, H. 5 (1974)

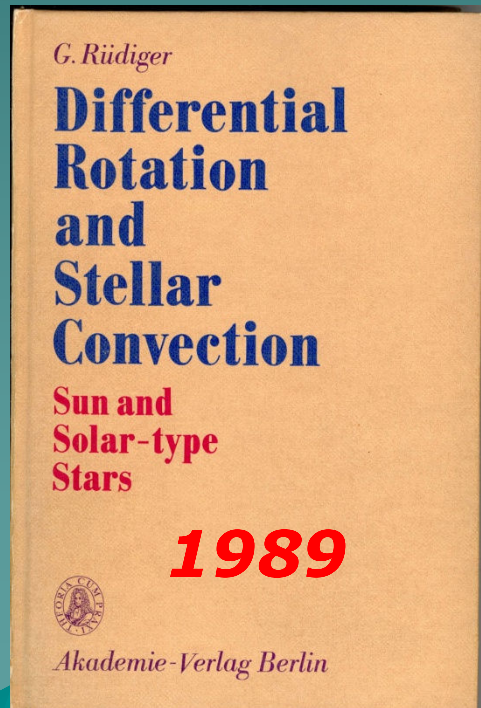
On the REYNOLDS Stresses in Mean-Field Hydrodynamics
III. Two-dimensional Turbulence and the Problem
of Differential Rotation

G. RÜDIGER, Potsdam

Zentralinstitut für Astrophysik der Akademie der Wissenschaften der DDR

With 1 Figure. (Received 1974 January 18)

Dominance of latitudinal Reynolds stresses with respect to the radial ones



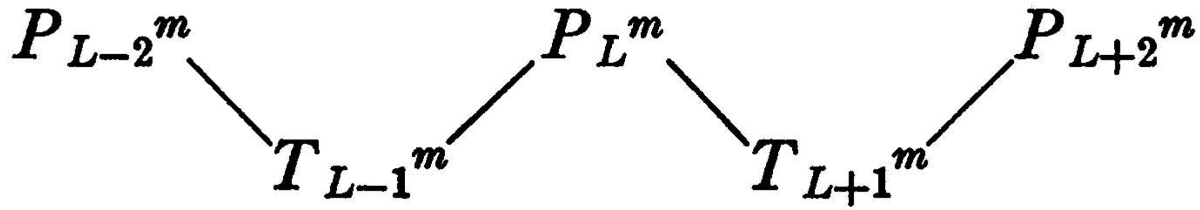
Global convection

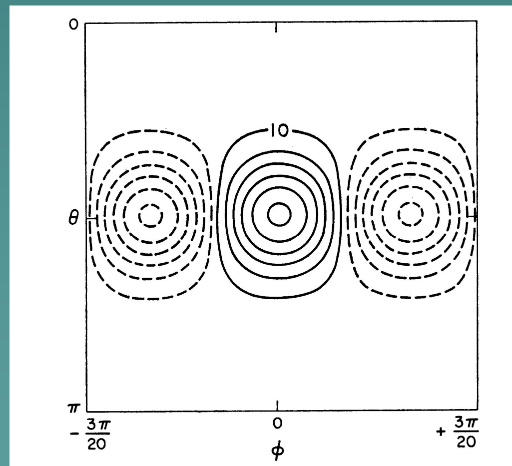
Durney 1970 – Global convection
(see also Busse 1970, Yoshimura 1971)

THE ASTROPHYSICAL JOURNAL, 161: 1115–1127, September 1970

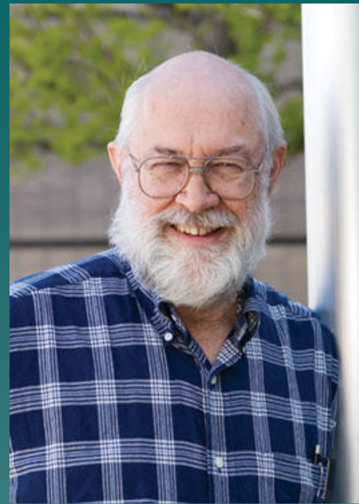
NONAXISYMMETRIC CONVECTION IN A ROTATING SPHERICAL SHELL

B. DURNEY
High Altitude Observatory, National Center for Atmospheric Research,* Boulder, Colorado
Received 1969 December 16; revised 1970 March 2





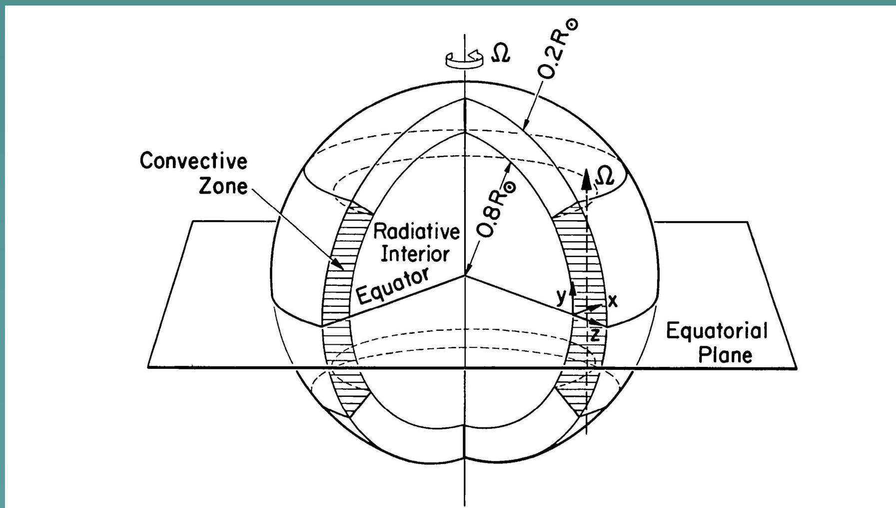
Gilman 1972 – later Gilman & Gatzmaier

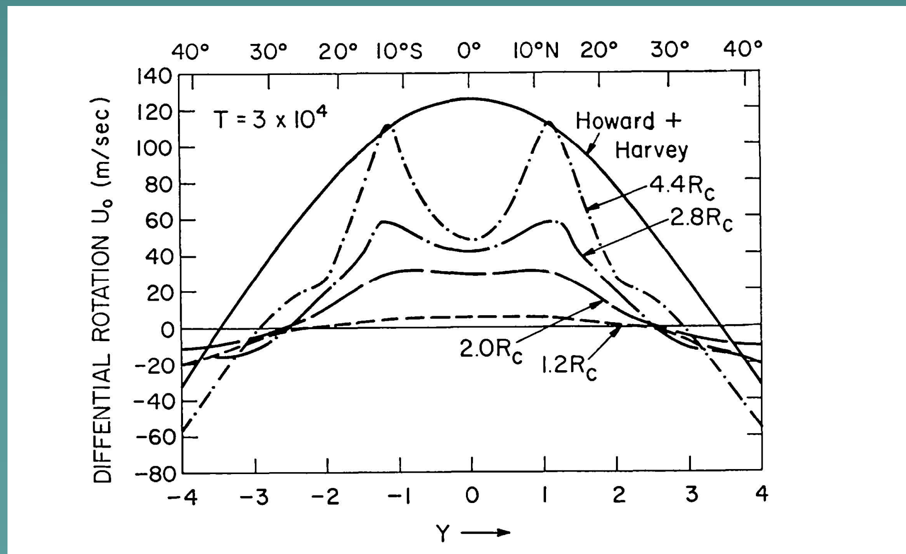


NONLINEAR BOUSSINESQ CONVECTIVE MODEL FOR LARGE SCALE SOLAR CIRCULATIONS

PETER A. GILMAN
*Advanced Study Program, National Center for Atmospheric Research**,
Boulder, Colo., U.S.A.

(Received 13 December, 1971; revised 15 May, 1972)





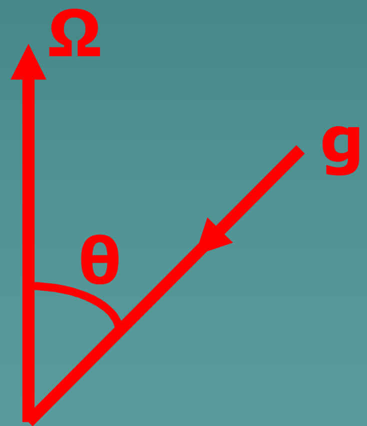
Interaction of rotation with turbulent convection

Nigel Weiss 1965 (The Observatory)

Convection and the Differential Rotation of the Sun

GENTLEMEN,—

Criteria for estimating the superadiabatic temperature gradient required to produce a given convective energy flux in a rotating system have been discussed elsewhere[1]. An attempt is made here to apply these criteria to the solar convective zone. It appears that, although the effect of rotation is negligible in the photosphere, it may double the superadiabatic gradient in the deep convective zone. Consequently, if convective heat transport is the same at the poles as at the equator of the Sun, the polar regions should be cooler by an amount of order 20 °K. As Plaskett[2, 3] has pointed out, the associated meridional pressure gradient might explain the differential rotation of the Sun.



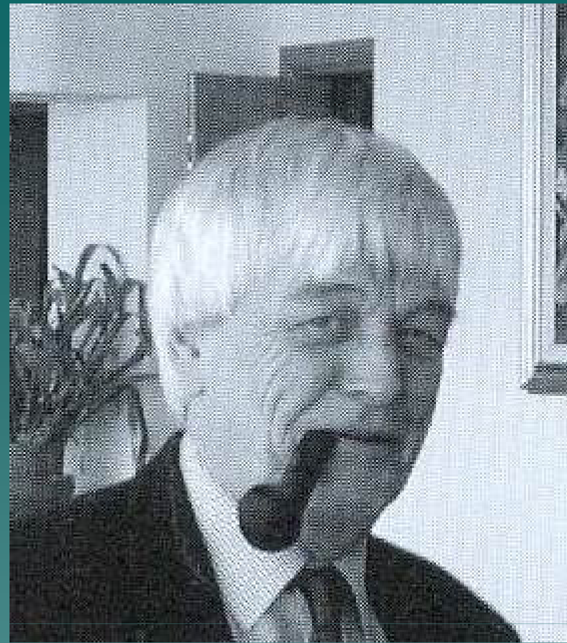
The latitudinal variation of the angle between Ω and g affects the convective flux thus determining a meridian circulation

$$\Omega^2 \leq g\beta / 4\Theta$$

$$K(r,\theta) = K_o(r)\,[1 + \varepsilon T_a P_2(\cos\theta)]$$

$$T_a = 4\Omega^2 L^4 / \nu$$

Durney & Roxburgh 1971



INHOMOGENEOUS CONVECTION AND THE EQUATORIAL ACCELERATION OF THE SUN

B. R. DURNEY

High Altitude Observatory, Boulder, Colo., U.S.A.

and

I. W. ROXBURGH

Queen Mary College, London

(Received 19 March 1970; in revised form 14 August, 1970)

Belvedere & Paternò 1976 -1977

Then B & P + Stix 1980

LARGE SCALE CIRCULATION IN THE CONVECTION ZONE AND SOLAR DIFFERENTIAL ROTATION

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(Received 29 September, 1975; in revised form 10 February, 1976)

CONVECTION IN A ROTATING DEEP COMPRESSIBLE SPHERICAL SHELL: APPLICATION TO THE SUN

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Osservatorio Astrofisico di Catania, Italy
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(Received 14 March; in revised form 16 May, 1977)

Geophys. Astrophys. Fluid Dynamics, 1980, Vol. 14, pp. 209–224
0309-1929/80/1403-0209 $4.50/0

Printed in Great Britain

Differential Rotation Set Up By Latitude-Dependent Heat Transport†

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LUCIO PATERNÒ
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MICHAEL STIX
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(Received March 15, 1979)

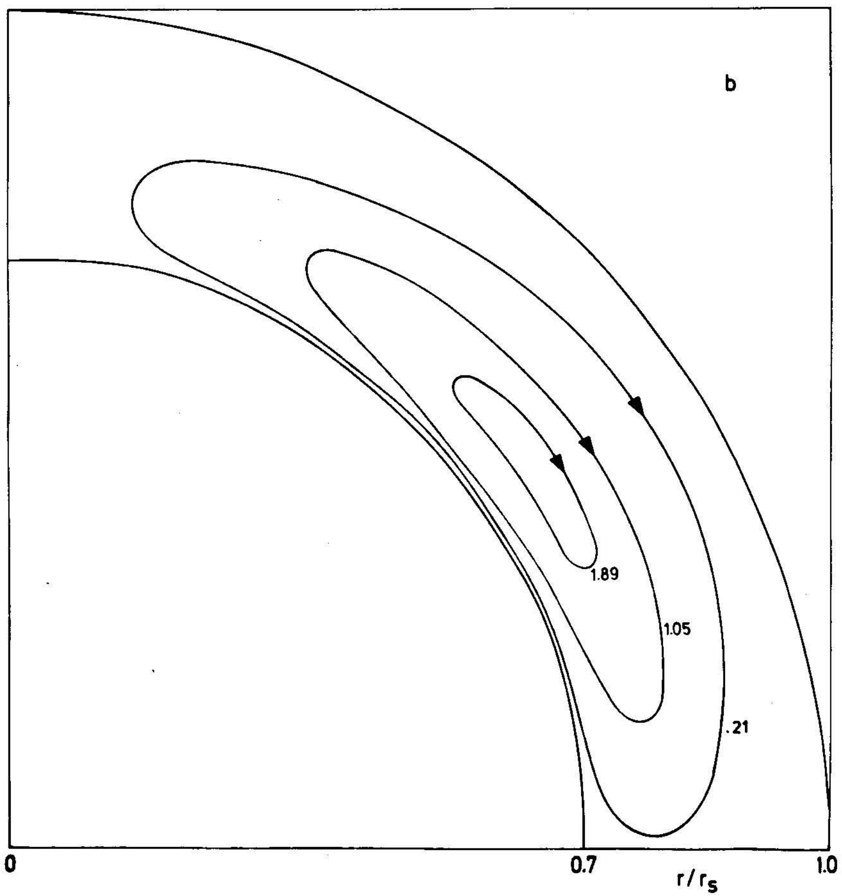

FIGURE 4 (b) Streamlines of the meridional circulation, in units of 10^{17} kg/s, for a model with $r_b/r_s = 0.7$ and a thermal diffusivity and kinematic viscosity of $\frac{1}{3} \cdot 10^{10}$ m^2/s.

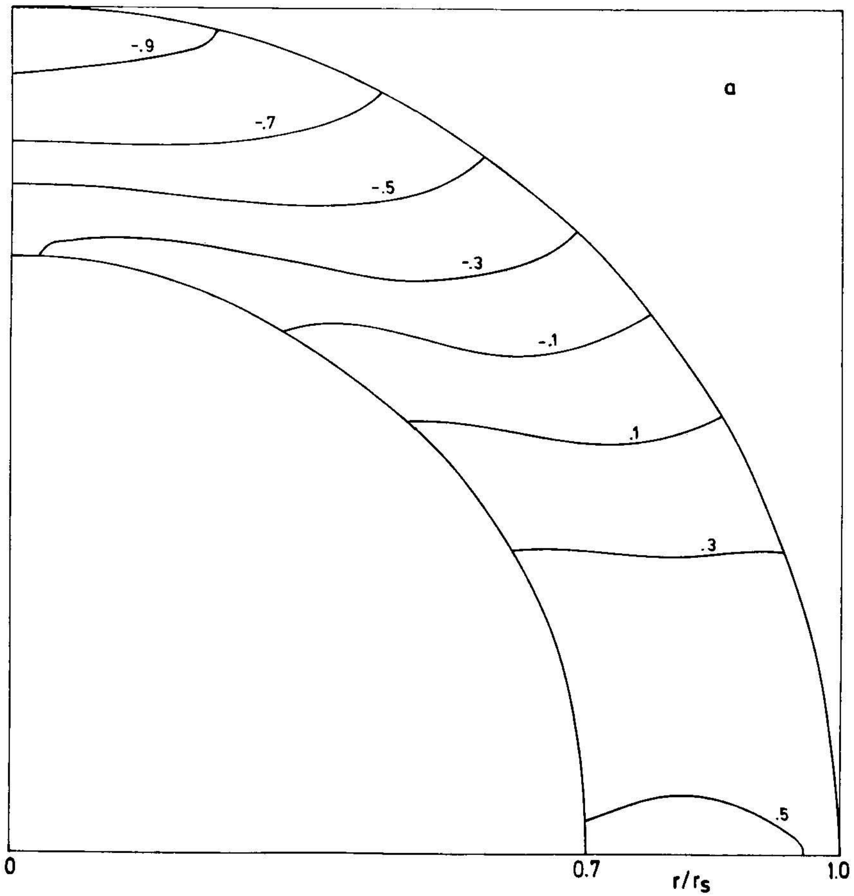

FIGURE 4 (a) Lines of constant angular velocity, $[u_o + u_2 \cdot P_2(\theta)]/0.189$;

Pidatella, Stix, Belvedere, Paternò 1986

Astron. Astrophys. 156, 22–32 (1986)

ASTRONOMY AND ASTROPHYSICS

The role of inhomogeneous heat transport and anisotropic momentum exchange in the dynamics of stellar convection zones: application to models of the Sun's differential rotation*

R.M. Pidatella[1], M. Stix[2], G. Belvedere[1,3], and L. Paterno'[1,4]

[1] CNR-Gruppo Nazionale di Astronomia, Unita' di Ricerca di Catania, c/o Osservatorio A
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[2] Kiepenheuer-Institut für Sonnenphysik, D-7800 Freiburg im Br., Federal Republic of Germ
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Received February 25, accepted July 8, 1985

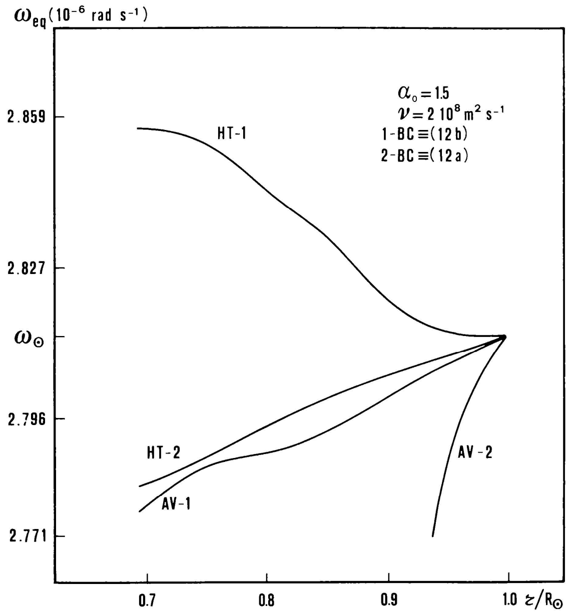
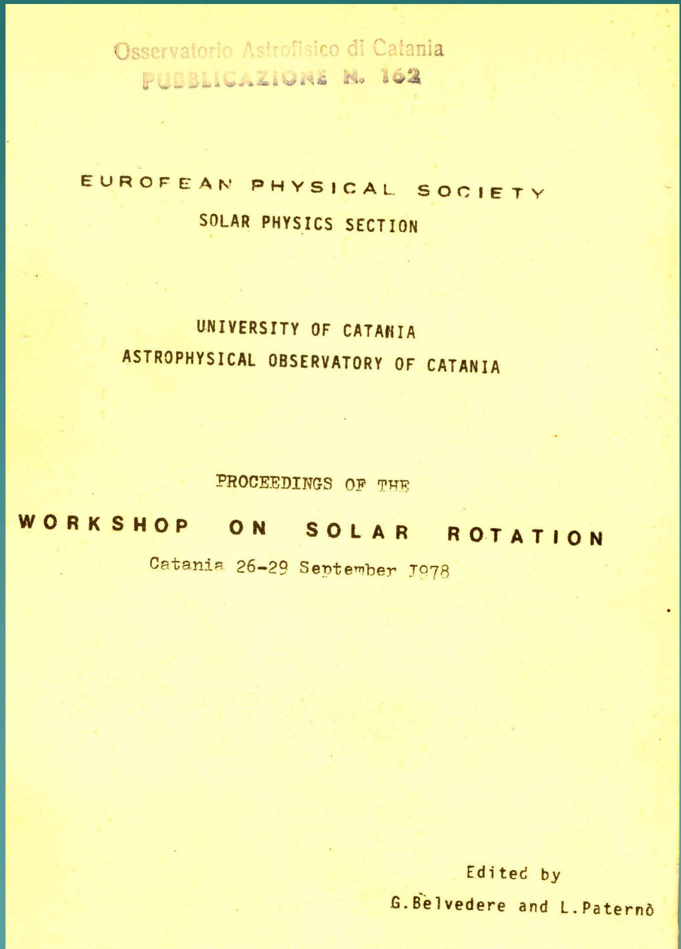


Fig. 5. The behaviour of the equatorial angular velocity as a function of the fractional radius $r/R_\odot$ for models with deep convection zones ($\alpha_0 = 1.5$) and kinematic viscosity $2\,10^8\,m^2\,s^{-1}$. The models are HT and AV with boundary conditions of the type (12a) and (12b), respectively (2-BC) and (1-BC)

Two milestones in solar differential rotation research

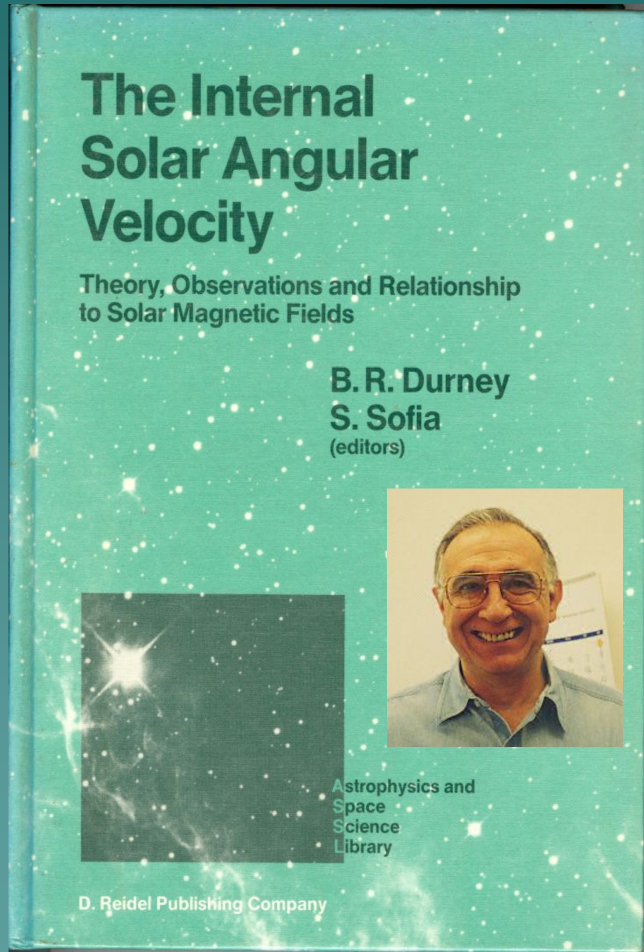

Osservatorio Astrofisico di Catania
PUBBLICAZIONE N. 162

EUROPEAN PHYSICAL SOCIETY
SOLAR PHYSICS SECTION

UNIVERSITY OF CATANIA
ASTROPHYSICAL OBSERVATORY OF CATANIA

PROCEEDINGS OF THE
WORKSHOP ON SOLAR ROTATION
Catania 26-29 September 1978

Edited by
G. Belvedere and L. Paternò



Differential rotation and dynamo problem in the pre-helioseismology era

In order to have a correct dynamo action (sunspot drift toward the equator):

$\alpha(\partial\omega/\partial r) < 0$ in the northern hemispere (NH)

but $\alpha > 0$ in the NH from poloidal-toroidal field phase relation observed at the surface

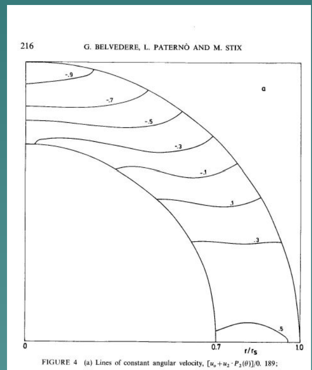
therefore $\partial\omega/\partial r < 0$ an angular velocity increasing inwards is required

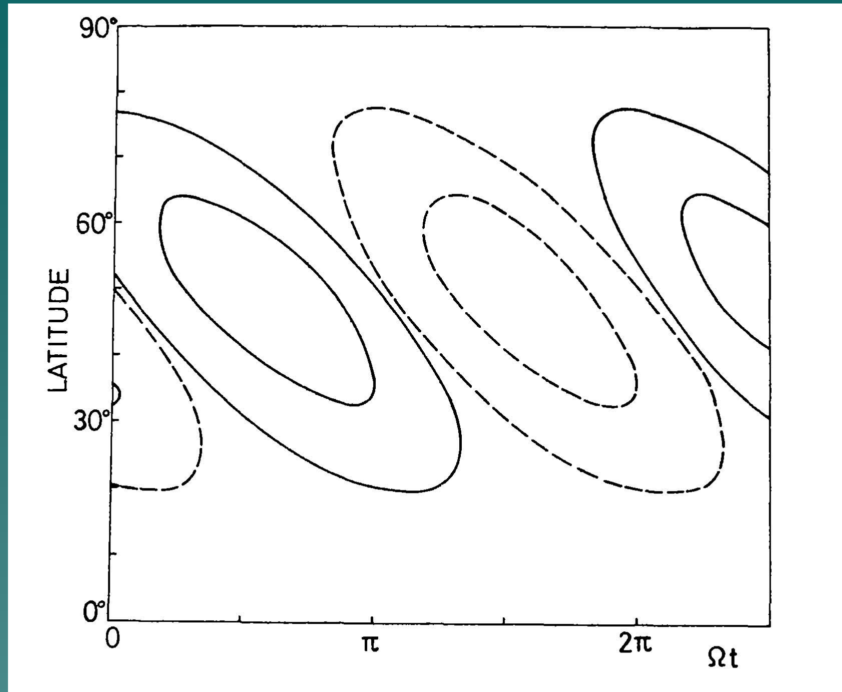
but $\partial\omega/\partial r > 0$ in the convection zone from most hydrodynamic models (rotation in cylinders)

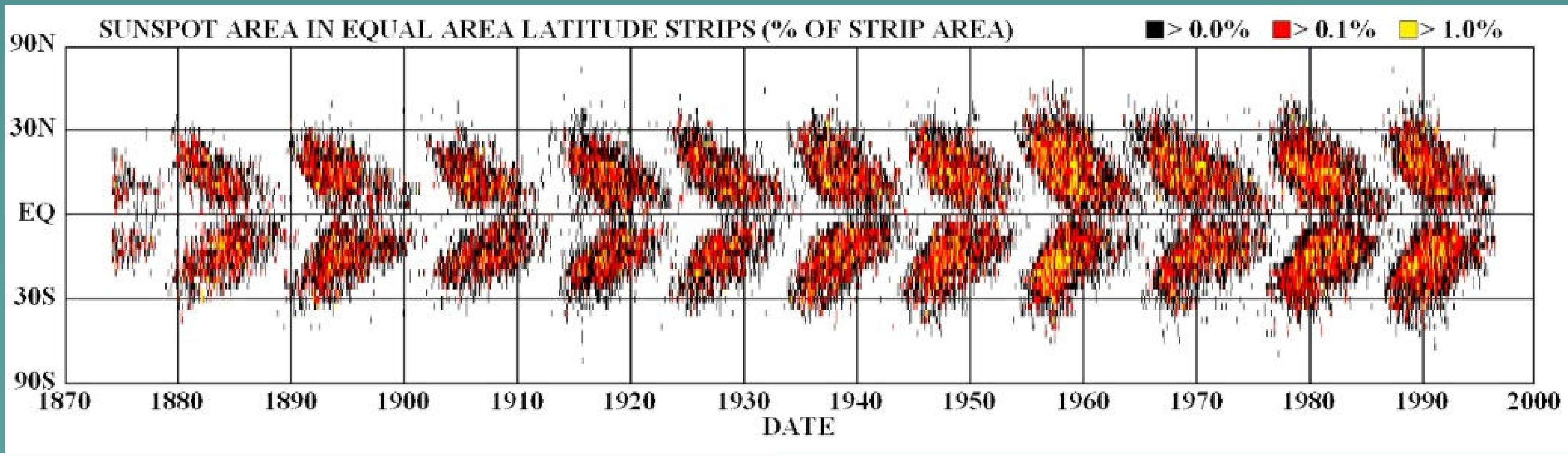
and $\alpha(\partial\omega/\partial r) > 0$ in the NH (dynamo cannot work correctly)

Belvedere, Paternò, Stix 1980

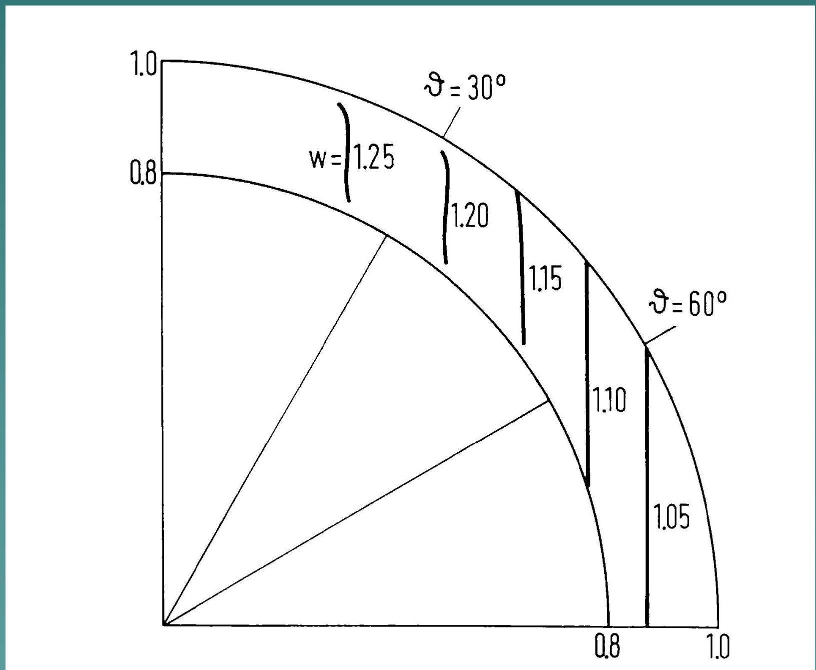
Dynamo model with calculated differential rotation

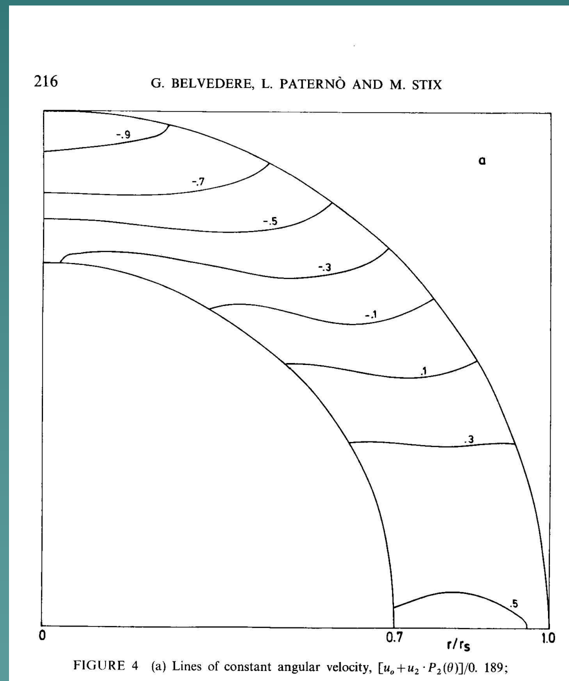






Two possible scenarios predicted after 20 years of efforts in solar differential rotation research





wrong dynamo action

bad dynamo action

Douglas Gough 1978 – The birth of helioseismology for probing internal rotation.

EPS Workshop on Solar Rotation – Catania, September 1978

ON THE POWER OF FIVE MINUTE OSCILLATIONS TO RESOLVE THE SOLAR ANGULAR VELOCITY

D.O. Gough
Observatoire de Nice; Institute of Astronomy and Department of Applied Mathematics and Theoretical Physics, University of Cambridge

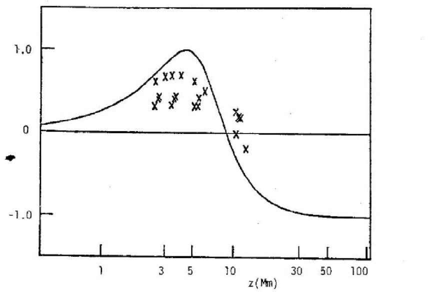
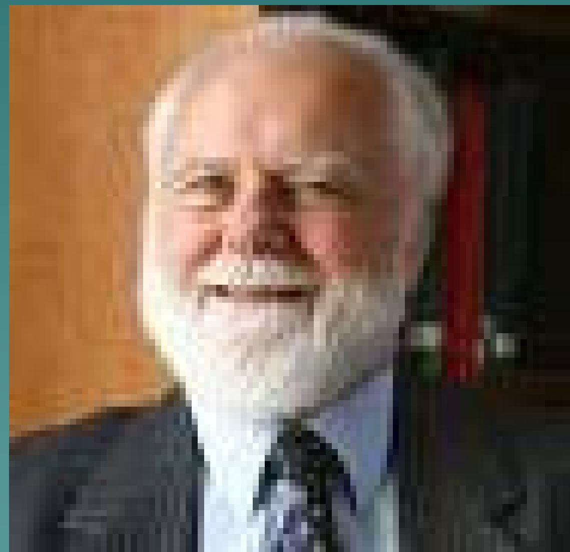


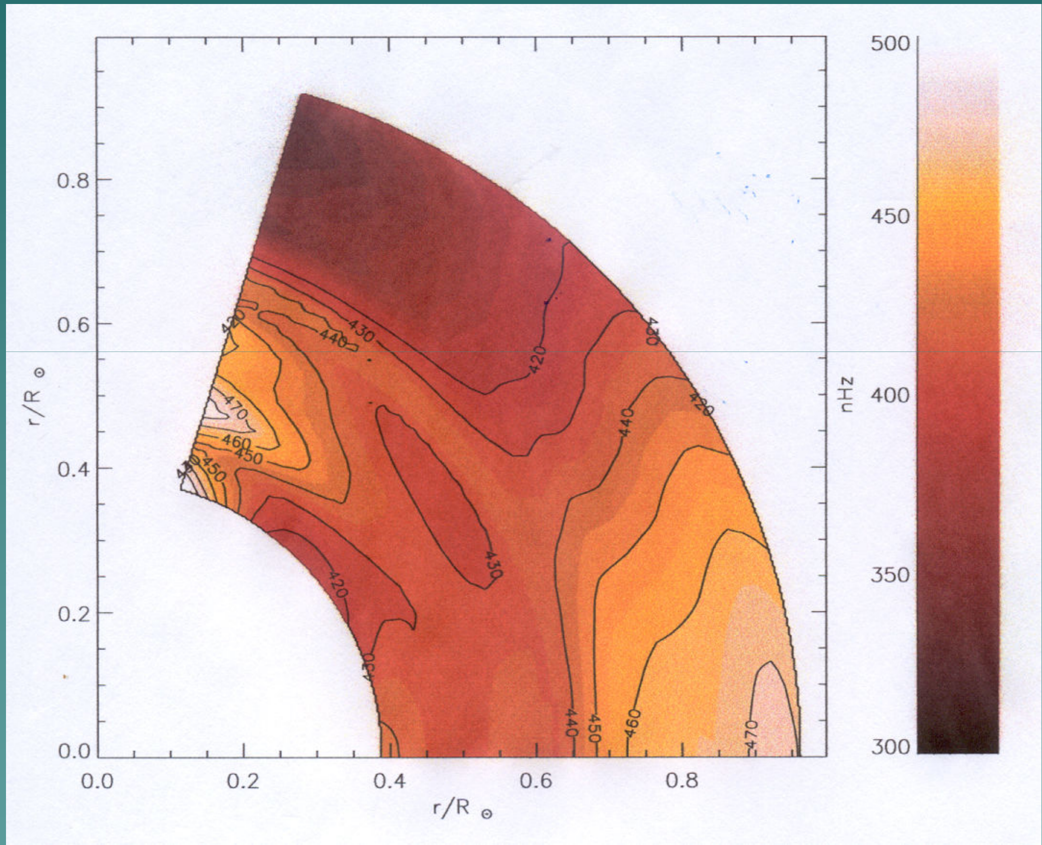
Figure 2. An example of an analysis of differential rotation by the method employed by Deubner et al. (1978). The continuous line represents the actual angular velocity $\Omega_1(z)$ of the model relative to the surface angular velocity Ω_0, measured in units of its maximum value. The crosses are $\Omega_1^*(z_{eff})$ plotted at $z = z_{eff}$ for the f modes and first four p modes with l = 100, 200, 300 and 400 of the polytrope with $\mu = 2.5$ and $\gamma = 1.2$. These are the estimates of $\Omega_1(z)$.



$$\Omega_1^*(z_{eff}) = \frac{\Delta\omega}{m} = \int_0^\infty z\,B(z)\,\Omega_1(z)\,d\ln z$$

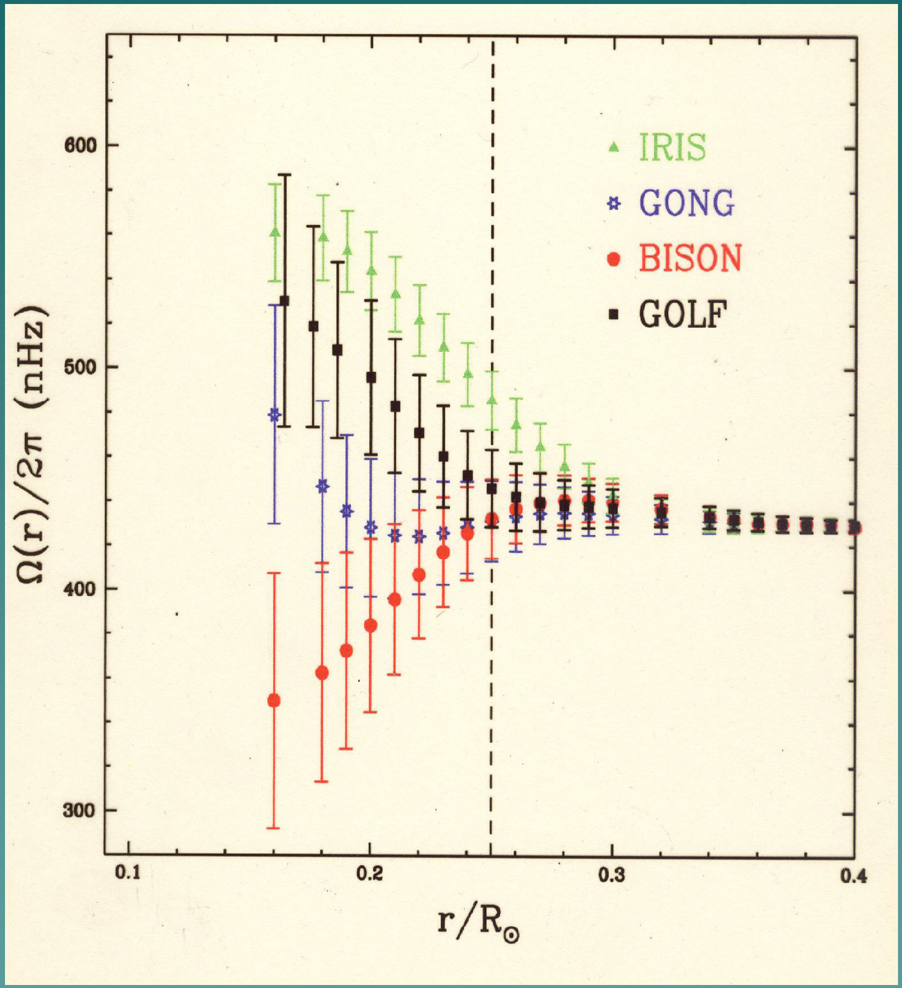
The response of the true Sun to the research efforts

Helioseismology era – the true dynamical Sun



Di Mauro, Dziembowski, Paternò 1998

Core rotation ?



Di Mauro 1998

This is the end of story

The continuation is left to the fantasy, application and love for scientific research of the young scientists, who will soon replace those who worked for more than 40 years and are on the way of retirement or definitively retired, like me

My grandsons

Mathieu 6.4 ys

Lucio 5.3 ys

Gianmarco 2.5 ys

THANK YOU