

Planetary Magnetic Fields and Fluid Dynamos

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Douglas Gough 70th Birthday Celebration Gargano, 20th June 2011

Internal Structure of the Earth

The Earth's magnetic field is generated in the fluid outer core.

Liquid iron with some light elements: inner core is solid iron.





As core cools, inner core grows, releasing light material at the inner core boundary (ICB), and also latent heat. May also be radioactive heat release. Only thermal convection before inner core formation.

The buoyancy drives convection which stirs the core at a typical velocity of 5×10^{-4} metres/sec. Sufficient to generate magnetic field by dynamo action.

Earth's magnetic field: Core Mantle Boundary

Field at Core-Mantle boundary, CMB, can be reconstructed from satellite and observatory data. Strength $\sim 8\times 10^{-4} {\rm Tesla} = \sim 8$ Gauss.



Field at the CMB in year 2000, units 10^{-3} T. Note the intense flux patches near Canada and Siberia. Patches under Africa are moving westward. Field at poles surprisingly low. Reversed flux patches under S. Atlantic are growing: field about to reverse?

Earth's Core Flow

The secular variation, together with an assumption such as tangential geostrophy,





20 km per year is 6×10^{-4} metres/sec. Slower than a snail! Note westward drift in S. Atlantic, Indian Ocean. Pacific has low secular variation. Waves or flow? Small scales filtered out by crustal magnetism.

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Convection in Mars?

Mars also doesn't seem to have mantle convection at present.

However, Mars has two very different hemispheres



Left Topographic map with Tharsis region prominent: Right Magnetic field of Mars, indicating the hemispheric structure is deep-seated. (Note longitude plotted differently! Hellas basin (blue) is nonmagnetic.). Mantle convection enhances heat flux and can create core convection.

Collapse of the Martian dynamo

Mars Global Surveyor discovered martian rocks are strongly magnetised.

Suggests Mars had a dynamo originally, like Earth.

Dating the rocks suggests martian dynamo stopped working 350 Myr after formation.

Numerical models suggest that dynamos are often subcritical: this means there is hysteresis. If the driving is slowly reduced, the dynamo suddenly switches off.

More driving is required to start the process than to maintain it.

Spherical geometry models



Rotating about z-axis. Gravity radially inward, $g = g_0 r$. Centrifugal acceleration small. Length scale dis gap-width from inner to outer boundary. Convection onsets outside tangent cylinder

Boussinesq fluid.

No heat sources in shell: on average same total heat flux goes in as comes out. Constant temperature drop across the layer.

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Dimensionless equations of the model

$$\frac{E}{Pm}\frac{D\mathbf{u}}{Dt} + 2\hat{\mathbf{z}} \times \mathbf{u} = -\nabla p + \mathbf{j} \times \mathbf{B} + E\nabla^2 \mathbf{u} + \frac{ERaPm}{Pr}\theta\mathbf{r}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\frac{\partial \theta}{\partial t} = \frac{Pm}{Pr} \nabla^2 \theta - \mathbf{u} \cdot \nabla \theta + \beta(r) u_r$$

 $\nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{u} = 0$

 $\beta(r)$ - Basic state temperature profile. Temperature unit $\kappa \Delta T/\eta$. No-slip and stress-free boundaries considered, but assume $\theta = 0$ there. 7

Dimensionless Parameters

Ekman number $E=\nu/\Omega d^2$

Rayleigh number $Ra = \frac{g_0 \alpha \beta d^3}{\Omega \kappa}$

Prandtl number $Pr = \nu/\kappa$

Magnetic Prandtl number $Pm=\nu/\eta$

 η magnetic diffusivity, u turbulent kinematic viscosity,

 κ is the turbulent thermal diffusivity, $d = r_o - r_i$.

In all runs here, Pm = Pr = 1, $E = 5 \times 10^{-5}$, small but not so small as too make numerical simulation inconvenient.

Nonmagnetic rapidly rotating convection at onset



Contours of axial vorticity. Sequence with $E = 3 \times 10^{-5}$, 10^{-5} , $E = 3 \times 10^{-6}$, 10^{-6} (E. Dormy).

Note convection onsets first near tangent cylinder (where field is strongest in the dynamo). Convection columns get thinner at smaller E, width $\sim E^{1/3}$.

Nature of the nonlinear dynamo solutions



 $E = 5 \times 10^{-5}$, Pr = Pm = 1, Ra = 400 with stress-free boundaries.

Left: radial magnetic field at the CMB. Right: radial velocity at $r = r_i + 0.8$.

This is a standard dipolar solution which persists for all time. Flow has the columns predicted by linear theory, though they are now time-dependent.



Red is primary flow, which convects the heat out. Blue is secondary flow which provides helicity $\mathbf{u} \cdot \boldsymbol{\zeta}$, where $\boldsymbol{\zeta}$ is the vorticity. Helicity is important for **dynamo action**.

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B_{ϕ} from a dynamo model



 $<\!B_{\phi}\!>$

Azimuthal average of B_{ϕ} for a moderately supercritical dynamo.

Antisymmetric about the equator, so $B_{\phi} = 0$ on the equator. Note $B_z > 0$ near tangent cylinder, usually gives $B_{\phi} > 0$ in N. hemisphere. Field generally increases with s near ICB.

Mechanism for generating B_z from B_ϕ

Start with $B_{\phi} > 0$ in northern hemisphere where $u_z > 0$ in anticyclones, A, $u_z < 0$ in cyclones, C



Green magnetic field line initially in ϕ -direction is displaced parallel to rotation axis z by secondary flow.

Primary flow sweeps positive B_z towards the ICB, negative B_z towards the CMB.

Net effect is to create positive B_z inside, negative B_z outside, just as in a dipolar field. In S. hemisphere B_ϕ and u_z are reversed, so effect is same.

Mechanism for generating B_{ϕ} from B_z





Northern Hemisphere. $u_z > 0$ in anticyclones, A, $u_z < 0$ in cyclones, C.

Left: u_{ϕ} flow resulting from u_z profile stretches out red positive B_z to give B_{ϕ} .

Right: constant z section in Northern Hemisphere viewed from above. Positive B_{ϕ} is moved outward in radius, negative B_{ϕ} moved inward in radius by the vortex circulations. This reinforces the original B_{ϕ} configuration and allows the magnetic field to grow.

Why dipolar? Why subcritical?

This mechanism is an α^2 dynamo. But this can also produce quadrupolar fields. Why are dipolar modes strongly preferred in planets?

Maybe the form of convection makes a dipolar field grow faster than a quadrupolar field (Kinematic Dynamo explanation). Not much evidence for this.

Could the magnetic field affect the convection to enhance its helicity?

Perhaps a dipolar field gives more helicity than a quadrupolar field?

If the helicity is enhanced by magnetic field, this might make the dynamo subcritical.

Evolution of magnetic energy with time



 $E = 10^{-4}$, Pr = Pm = 1 and stress-free boundary conditions. (i) Ra = 400, dipole dominated solution. (ii) Ra = 600 strong field dipole dominated solution. (iii) Ra = 600, initial small field solution grows into a relatively weak quadrupolar solution. (iv) as case (iii) but with a different small initial field. (v) Ra = 500, an initial small field decays away. (vi) Ra = 550, an small initial field grows, eventually resulting in a quadrupolar field

Linear rapidly rotating magnetoconvection

It is possible to develop an asymptotic theory of rapidly rotating convection in the limit of small E.

When the convection onsets close to the tangent cylinder perturbations scale as

$$\frac{1}{s}\frac{\partial}{\partial\phi} \sim O\left(E^{-1/3}\right), \quad \frac{\partial}{\partial s} \sim O\left(E^{-2/9}\right), \quad \frac{\partial}{\partial z} = O(1) \quad \text{as} \quad E \to 0.$$

$$\mathbf{u} = u_z \mathbf{\hat{z}} + \nabla \times \psi \mathbf{\hat{z}}, \quad \mathbf{u} \sim \exp i m \phi$$

and u_z and u_s are the same order of magnitude in the $E \to 0$ limit but u_{ϕ} is $O(E^{1/9})$ smaller. Tall thin columns.

z-structure given by a 2nd order ODE in z. ϕ -component of magnetic field dominates. Need to assume a form of $b_{\phi}(z)$.

The radial structure is a modulated wave, which can be worked out.

Asymptotic system: Roberts-Busse equations

The z-vorticity equation is

$$E(a^2 - i\omega Pm^{-1})a^2\psi - 2\frac{du_z}{dz} + \frac{imERaPm}{Prr_o}\theta = -\Lambda \left(\frac{b_\phi}{s}\right)^2 \frac{m^2a^2}{a^2 - i\omega}\psi$$

where ω is the frequency, ψ the horizontal flow streamfunction, u_z is z-velocity, θ is temperature perturbation, Λ is the Elsasser number, $a^2 = m^2/s^2$.

The equations for the z-velocity and temperature perturbation

$$E(a^2 - i\omega Pm^{-1})u_z - 2\frac{d\psi}{dz} - \frac{zERaPm}{Prr_o}\theta = -\Lambda \left(\frac{b_\phi}{s}\right)^2 \frac{m^2}{a^2 - i\omega}u_z,$$

$$(a^2 PmPr^{-1} - i\omega)\theta = \frac{im\psi + zu_z}{r^3}r_ir_o.$$

Boundary conditions

$$im\psi + zu_z = 0$$
, (stress – free at $z = \pm h$)

Helicity in the model

Using these scalings, the dominant contributions to the helicity are from $u_s \zeta_s$ and $u_z \zeta_z$.

$$\mathbf{u} \cdot \boldsymbol{\zeta} = \frac{1}{s^2} \left(\frac{\partial u_z}{\partial \phi} \frac{\partial \psi}{\partial \phi} - u_z \frac{\partial^2 \psi}{\partial \phi^2} \right), \qquad (3.6)$$

so both the s and z contributions to the helicity are equal. Averaged over the short azimuthal wavelength

$$H(z) = \langle \mathbf{u} \cdot \boldsymbol{\zeta} \rangle = \frac{m^2}{2s^2} \left(u_z \bar{\psi} + \bar{u}_z \psi \right). \tag{3.7}$$

Dipole b_ϕ : choose $b_\phi=3\sqrt{3}z(h^2-z^2)/2h^3$

Quadrupole
$$b_\phi$$
: choose $b_\phi = (h^2 - z^2)^2/h^4$



The solution of the Roberts-Busse system, together with the helicity H. Both the real and imaginary parts of ζ , ζ_r and ζ_i are shown, and of w, w_r and w_i . Left: $\lambda = 0$, the nonmagnetic solution, Right: $\lambda = 5$, solution influenced by dipolar magnetic field. Note the massive increase in helicity when magnetic field is present.

Helicity: quadrupole field, Ekman suction 1.0 0.5 0.5 0.00.0Re{ ζ_} $\operatorname{Re}\{\zeta\}$ -0.5 -0.5 -lm{ ζ_] $\operatorname{Im}\{\zeta_{}\}$ Re{ u_} Re{ u_} -1.0 [Im{ u_} ____ lm{ u_] -1.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 -1.0 -0.5 0.0 0.5 1.0 Ζ Ζ

The solution of the Roberts-Busse system (3.1-3.5) is shown, together with the helicity H. Both the real and imaginary parts of ζ , ζ_r and ζ_i are shown, and of w, w_r and w_i . Left: $\lambda = 5$, with the quadrupolar field. Some increase in helicity over nonmagnetic case, but much less than with dipolar field. Right: Nonmagnetic, but with the Ekman suction (no-slip) boundary condition at $E = 10^{-4}$. There is a modest increase in helicity over the stress-free case.

Effect of different kinds of B_{ϕ} ?

Why does the dipolar field give much more helicity?

The real part of the z-vorticity equation is the dominant part,

$$\frac{d\hat{u}_z}{dz} \approx a^2\hat{\zeta} + \lambda b_\phi^2\hat{\zeta} - \frac{s^2\hat{R}\hat{\zeta}}{r_0a^2r^3}.$$

 $u_z = 0$ at the equator, and the real part of u_z is small at the boundary, so the z-integral of this must be small. The buoyancy must balance the magnetic and viscous dissipation.

With quadrupolar fields, the magnetic dissipation and the buoyancy are both strongest at the equator, so u_z is small everywhere.

With dipolar fields the magnetic dissipation is zero at the equator, strongest in mid-latitudes. So du_z/dz is large and negative near the equator and large and positive in mid-latitudes. Hence u_z is much larger with dipolar magnetic field.

Helicity of the strong and weak field solutions



 $E = 5 \times 10^{-5}$, Pr = Pm = 1, with stress-free boundaries. Helicity is antisymmetric about equator.

Left: is the helicity for the strong field solution on the plane z = 0.5, Ra = 400. Right: is the helicity without magnetic field at Ra = 500. At Ra = 500 an initial seed field decays.

Note that in the pure convection case, there is helicity, but little net helicity. With magnetic field, there is far more net helicity, as predicted by linear theory.

Radial velocity of strong and weak field solutions



 $E = 5 \times 10^{-5}$, Pr = Pm = 1, with stress-free boundaries.

Left: is the radial velocity for the strong field solution on the equatorial plane z = 0, Ra = 400. Right: radial velocity on equatorial plane at Ra = 500 with negligible magnetic field.

With magnetic field, strong convection extends far from the tangent cylinder, but with no magnetic field convection is strongest close to tangent cylinder, where it first onsets. Why?

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Azimuthal velocity of strong and weak field solution



 $E = 5 \times 10^{-5}$, Pr = Pm = 1, with stress-free boundaries.

Left: is the azimuthal velocity for the strong field solution on the equatorial plane z = 0, Ra = 400. Right: azimuthal velocity on equatorial plane at Ra = 500 with negligible magnetic field.

With no magnetic field there is a strong mainly axisymmetric differential rotation, but with magnetic field this disappears, and differential rotation is of thermal wind type.

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Conclusions

 \bullet Convection occurs in columns in rapidly rotating convection. The dipolar dynamos are of α^2 type, and the generation mechanism is reasonably well-understood.

Helicity production much stronger in the presence of a dipolar magnetic field.
Leads to subcritical behaviour and preference for dipoles over quadrupoles.
Connection with martian dynamo?

• In the absence of magnetic field, strong differential rotation can arise. This can suppress convection and helicity. This also promotes subcriticality.

• Columnar convection maintains the dipolar field. At high Ra convection columns may break down: possible explanation for reversals? Geodynamo maybe operating near the threshold of breakdown. Long term variations of heat flux are likely, due to fluctuations on the mantle convection timescale.