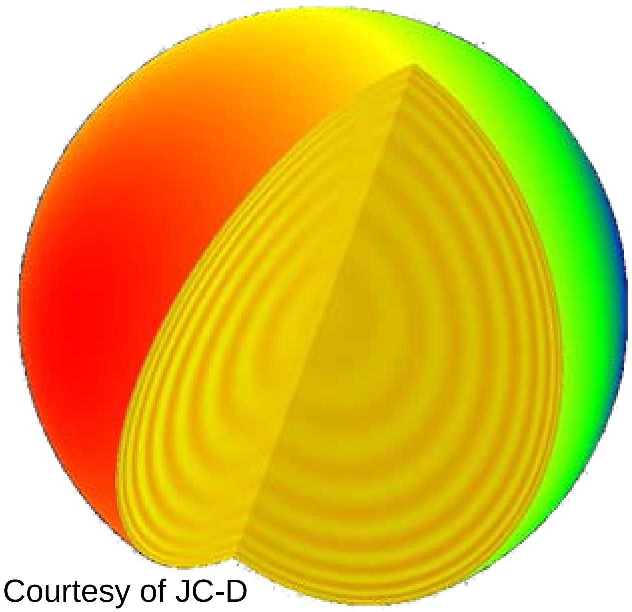
On the seismic age and heavy-element abundance of the Sun

submitted to MNRAS

Günter Houdek

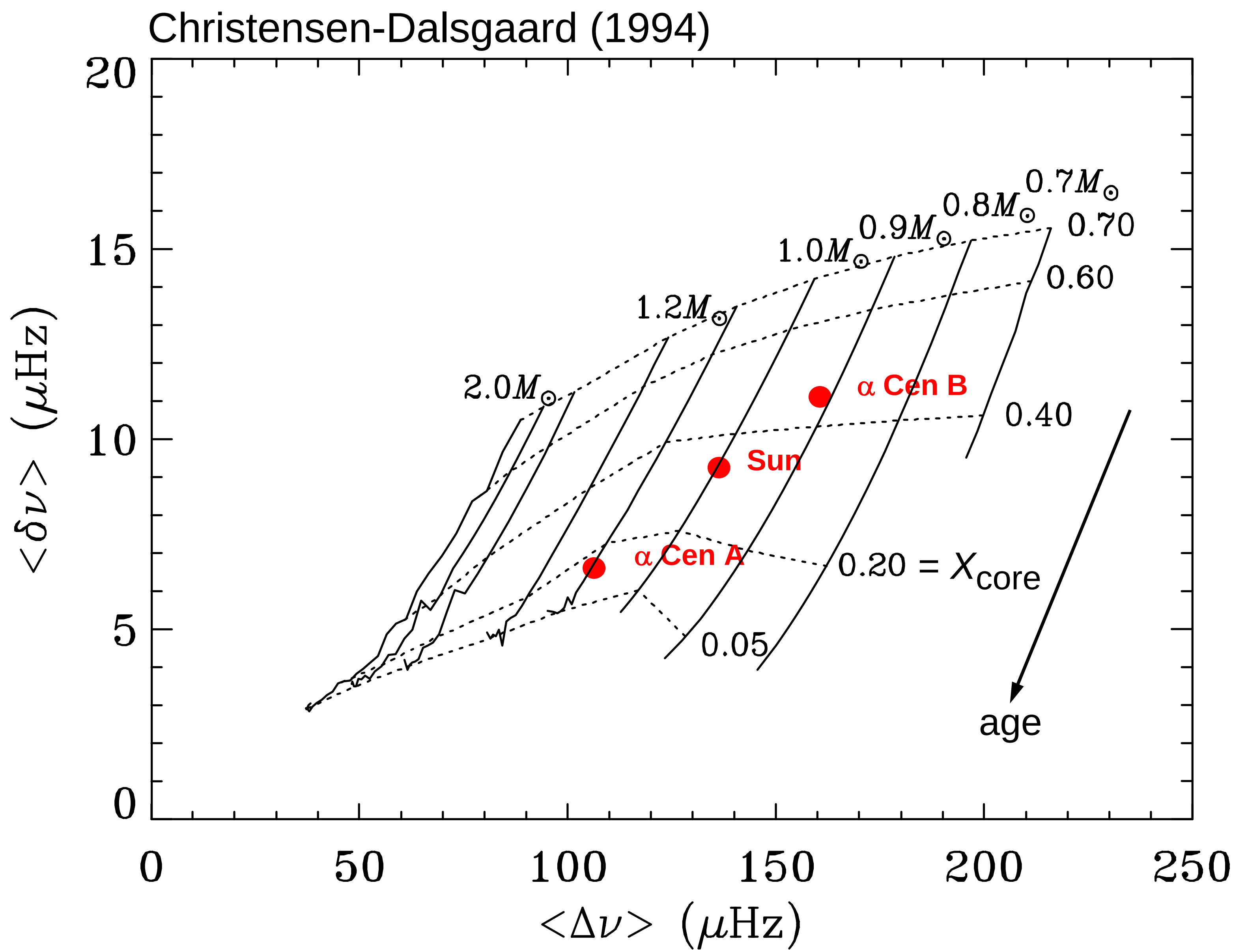
In collaboration with
Douglas Gough



Courtesy of JC-D

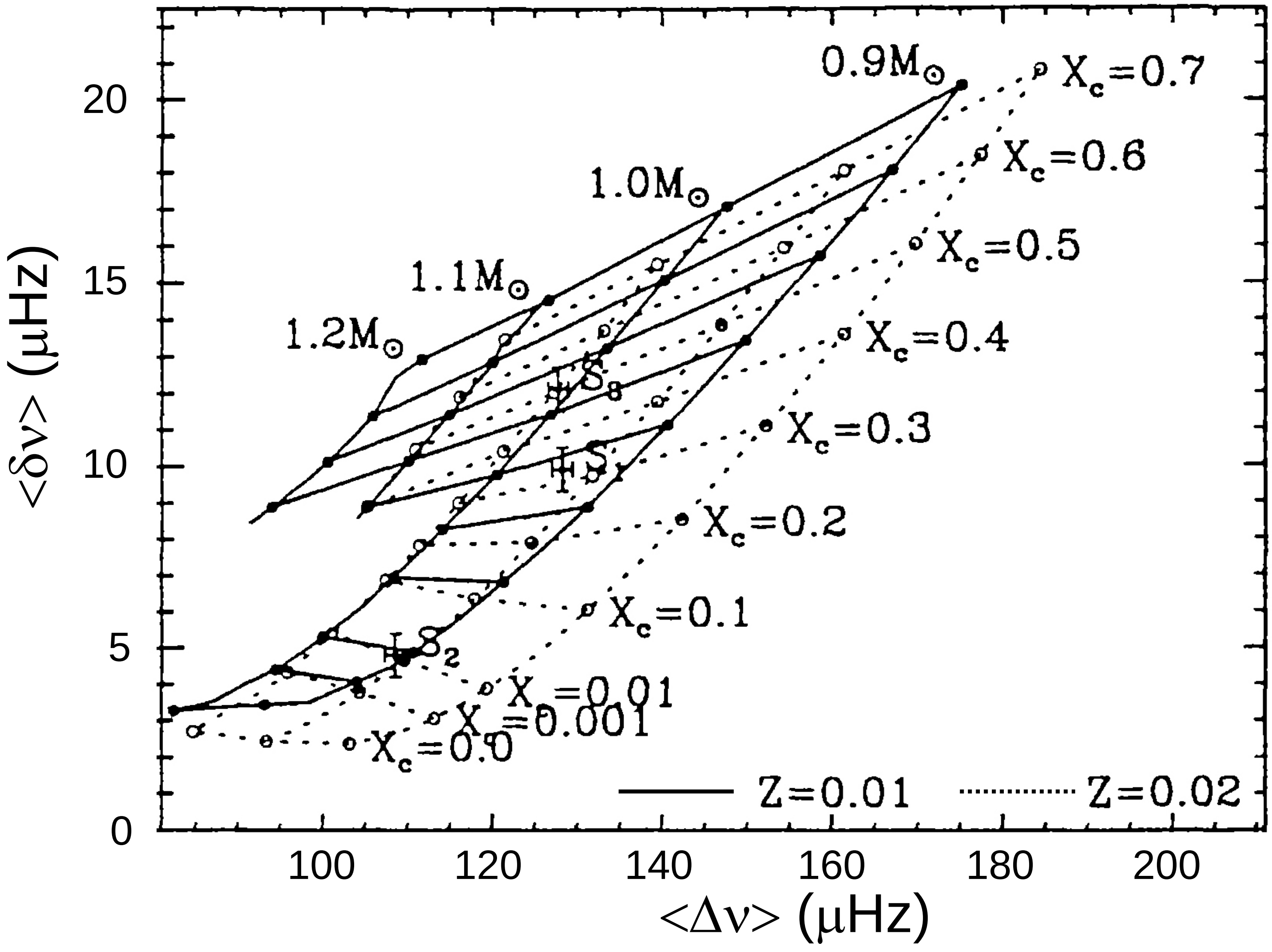
DOG Fest – Gargano – June 2011

Asteroseismic (Δν,δν) diagram



Asteroseismic (Δν,δν) diagram

Monteiro et al. (2002)



Points to consider in an age calibration procedure

- match seismic signatures of theoretical frequencies to observed frequencies.

- signatures are chosen to reflect principally the properties of energy-generating core.

- but such core signatures are also susceptible to e.g. zero-age chemical abundances and are contaminated by contributions produced by the surface layers.

- we need additional diagnostic to measure abundance (e.g., helium) independently and to separate the surface from the core signatures.

- here we use abrupt variation of the first adiabatic exponent γ_1 induced by He ionization.

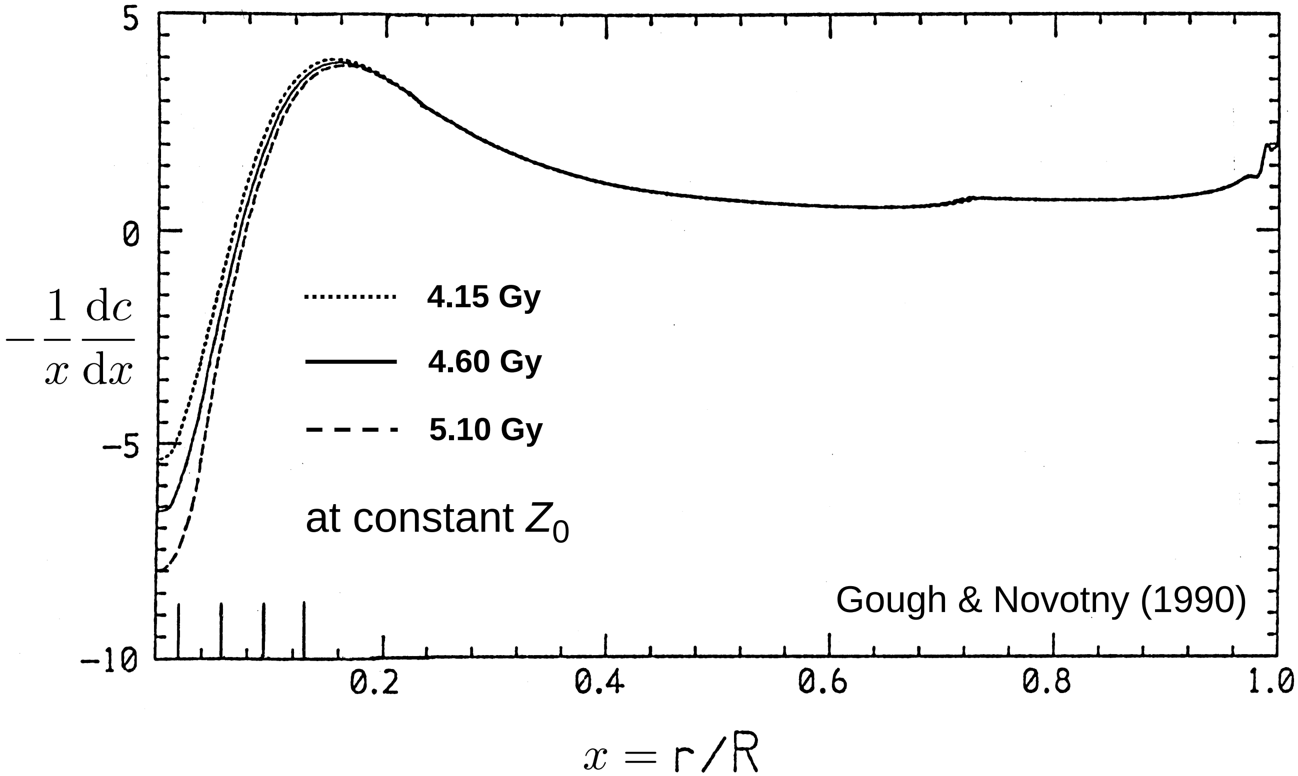
Age-sensitive diagnostics of the stellar structure

Age-sensitive diagnostics of the stellar structure

-asymptotic p-mode frequency behaviour (*n*>>*l*): $[L^2 = l(l+1)]$

$$\nu \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu}\,\nu_0^2 \qquad A = \frac{1}{4\pi^2\nu_0}\left[\frac{c(R)}{R} - \int_0^R \boxed{\frac{1}{r}\frac{\mathrm{d}c}{\mathrm{d}r}}\mathrm{d}r\right]$$

- evolutionary computations depend on 3 initial parameters: e.g., Y_0, Z_0 and α_c
- Calibrated (L, R) models: e.g. $Z_0(Y_0,\alpha_c)$ @ any $t_\odot$ ➡ 2-parameter set of models (Z_0,$t_\odot$)



Age-sensitive diagnostics of the stellar structure

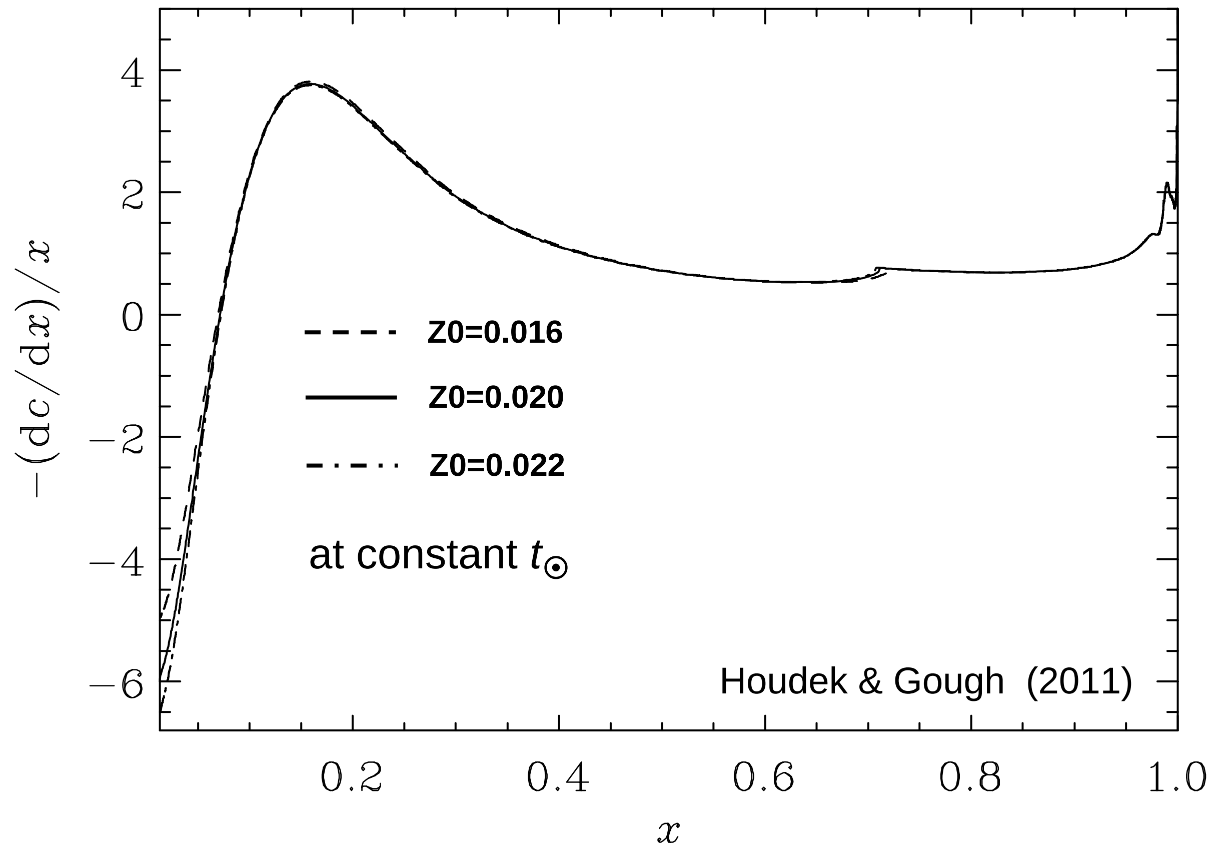
-asymptotic p-mode frequency behaviour (*n*>>*l*): $[L^2 = l(l+1)]$

$$\nu \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu}\,\nu_0^2$$

$$A = \frac{1}{4\pi^2\nu_0}\left[\frac{c(R)}{R} - \int_0^R \boxed{\frac{1}{r}\frac{\mathrm{d}c}{\mathrm{d}r}}\mathrm{d}r\right]$$

- evolutionary computations depend on 3 initial parameters: e.g., Y_0, Z_0 and α_c

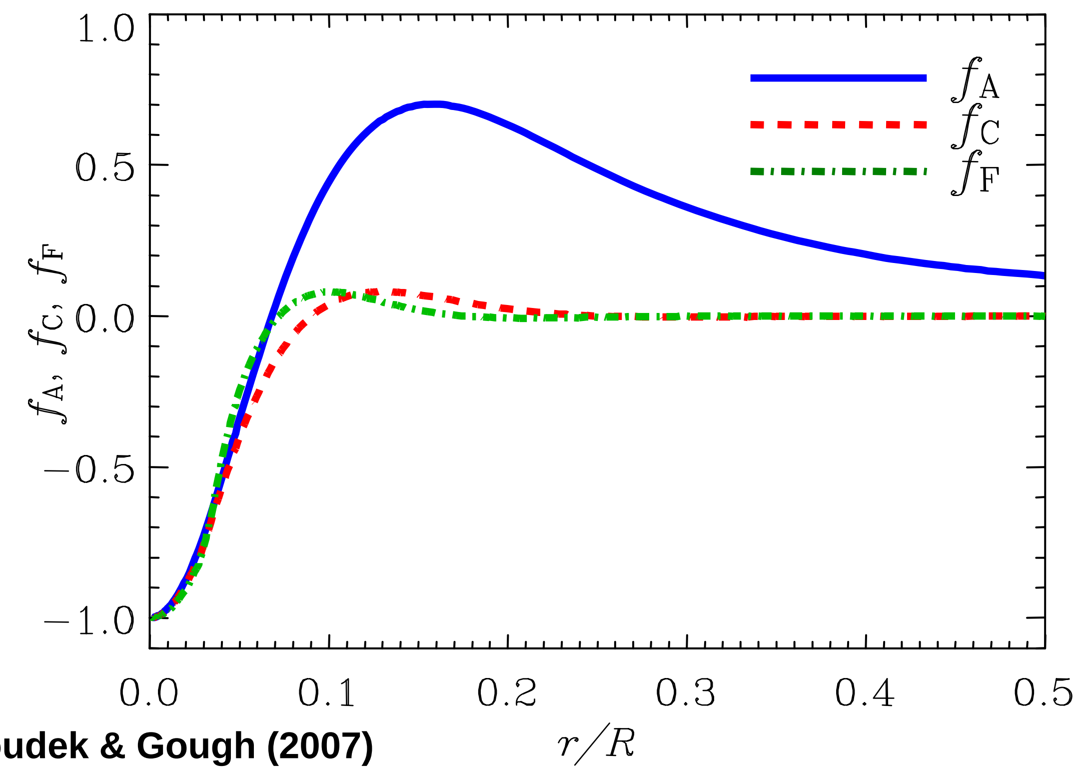
- Calibrated (*L*, *R*) models: e.g. $Z_0(Y_0,\alpha_c)$ @ any $t_\odot$ ➡ 2-parameter set of models ($Z_0, t_\odot$)



Age-sensitive diagnostics of the stellar structure

-asymptotic p-mode frequency behaviour (*n*>>*l*): $[L^2 = l(l+1)]$

$$\nu \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu}\,\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{\nu^3}\,\nu_0^4 - \frac{F\,L^6 - G\,L^4 + H\,L^2 - I}{\nu^5}\,\nu_0^6$$



Houdek & Gough (2007)

Age-sensitive diagnostics of the stellar structure

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_{\rm s} \simeq (n+\frac{1}{2}l+\epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu_{\rm s}}\,\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{{\nu_{\rm s}}^3}\,\nu_0^4 - \frac{F L^6 - G L^4 + H L^2 - I}{{\nu_{\rm s}}^5}\,\nu_0^6$$

Gough (2011): $A = \frac{-1}{2\omega_0}\left(\int_0^{r_\star} \frac{1}{r}\frac{{\rm d}c}{{\rm d}r}\,{\rm d}r \;+\; \text{other term}\right)$

$$C = \frac{1}{24\omega_0^3}\left[\int_0^{r_\star} \frac{1}{r}\frac{\rm d}{{\rm d}r}\left(\frac{1}{r}\frac{{\rm d}c^3}{{\rm d}r}\right){\rm d}r \;+\; \text{other term}\right]$$

$$F = \frac{-1}{240\omega_0^5}\left\{\int_0^{r_\star} \frac{1}{r}\frac{\rm d}{{\rm d}r}\left[\frac{1}{r}\frac{\rm d}{{\rm d}r}\left(\frac{1}{r}\frac{{\rm d}c^5}{{\rm d}r}\right)\right]{\rm d}r \;+\; \text{other term}\right\}$$

integrands of $\mathcal{A}_\alpha = A, C, F$: $\simeq \frac{(-1)^\alpha}{(2\alpha-1)2^\alpha \alpha!}\left(\frac{1}{r}\frac{\rm d}{{\rm d}r}\right)^\alpha \left(\frac{c}{\nu_0}\right)^{2\alpha-1} + \text{other term(s)}$

$\alpha = 1, 2, 3$

make integrands independent of ν_0: $\widehat{\mathcal{A}}_\alpha := \nu_0^{2\alpha-1}\mathcal{A}_\alpha$

→ $\widehat{A} = \nu_0 A\,,\qquad \widehat{C} = \nu_0^3 C\,,\qquad \widehat{F} = \nu_0^5 F$

Age-sensitive diagnostics of the stellar structure

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_{\rm s} \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu_{\rm s}}\,\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{{\nu_{\rm s}}^3}\,\nu_0^4 - \frac{F L^6 - G L^4 + H L^2 - I}{{\nu_{\rm s}}^5}\,\nu_0^6$$

- calibration using combinations of the seismically determined parameters

$$\xi_\alpha = (\underbrace{\widehat{A}, \widehat{C}, \widehat{F}}_{\text{age}}, \underbrace{\quad ? \quad}_{X_i}), \qquad \alpha = 1, 2, 3, 4\,,$$

$$\widehat{A} = \nu_0 A\,, \quad \widehat{C} = \nu_0^3 C\,, \quad \widehat{F} = \nu_0^5 F$$

Age-sensitive diagnostics of the stellar structure

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_{\rm s} \simeq (n+\frac{1}{2}l+\epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu_{\rm s}}\,\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{{\nu_{\rm s}}^3}\,\nu_0^4 - \frac{F L^6 - G L^4 + H L^2 - I}{{\nu_{\rm s}}^5}\,\nu_0^6$$

- calibration using combinations of the seismically determined parameters

$$\xi_\alpha = (\underbrace{\widehat{A}, \widehat{C}, \widehat{F}}_{\text{age}}, \underbrace{-\delta\gamma_1/\gamma_1}_{Y}), \qquad \alpha = 1,2,3,4\,, \qquad \widehat{A} = \nu_0 A\,,$$

$$-\delta\gamma_1/\gamma_1 \propto Y \qquad \widehat{C} = \nu_0^3 C\,,$$

$$\widehat{F} = \nu_0^5 F$$

- asymptotic formula approximates adiabatic ν_S only if scale height $H \gg k_v^{-1}$

→ glitch-free ν_S are frequencies of a "smoothed" stellar model

→ **we need a diagnostics for the acoustic glitch contributions to estimate $-\delta\gamma_1/\gamma_1$ and to construct ν_S**

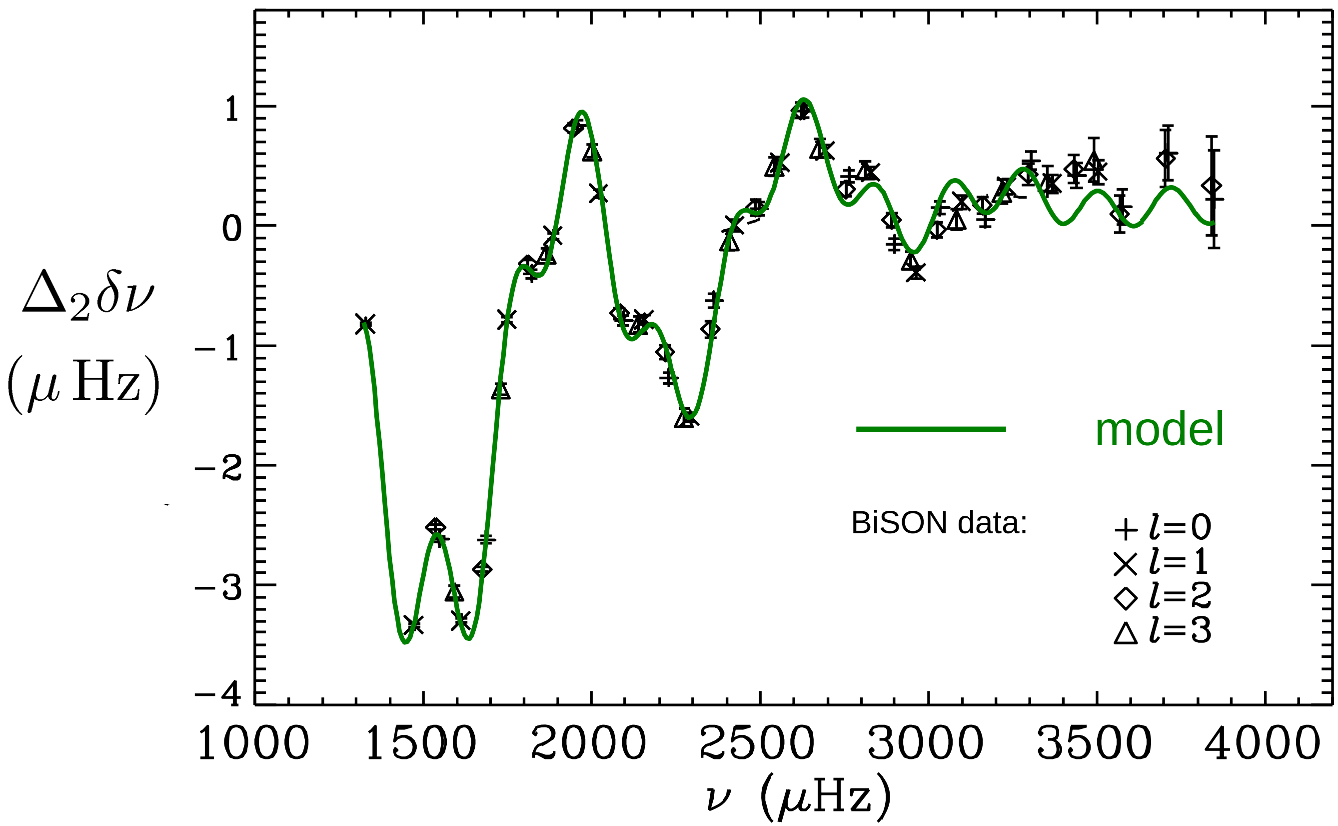
Glitch contributions $\delta\nu = \nu - \nu_{\rm s}$

(for $H \ll k_v^{-1}$)

Glitch contributions

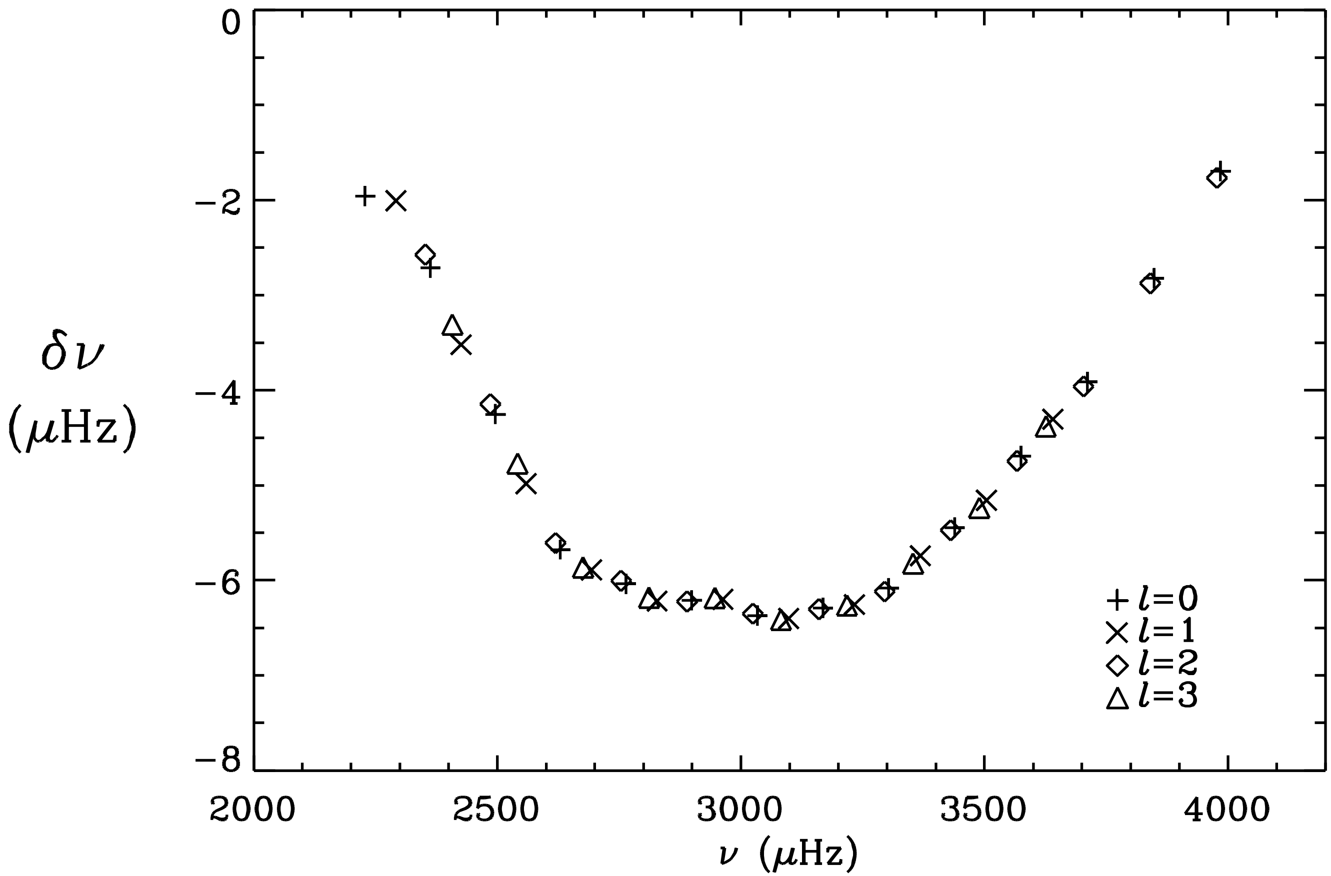
$$\delta\nu = \nu \; - \; \nu_{\rm s}$$

Gough (1990): $2^{\rm nd}$ frequency differences : $\Delta_2\nu \equiv \nu_{n-1} - 2\nu_n + \nu_{n+1}$



Glitch contributions

$$\delta\nu = \nu - \nu_{\rm s}$$



A model for glitch contributions
(A seismic diagnostics)

Seismic diagnostics: variational principle in (nonrotating) stars

Linearized, adiabatic, wave equation:

$$\omega^2 \boldsymbol{\xi} = \mathcal{L}(\boldsymbol{\xi})$$

operator $\rho_0 \mathcal{L}$ is hermitian for $\nabla p_0 = 0$ at boundary:

$$\omega^2 = \frac{\int_V \rho_0 \boldsymbol{\xi}^* \cdot \mathcal{L}(\boldsymbol{\xi})\, \mathrm{d}V}{\int_V \rho_0 \boldsymbol{\xi}^* \cdot \boldsymbol{\xi}\, \mathrm{d}V} = \frac{\mathcal{K}}{I}$$

$$\mathcal{K} \simeq \int_V (\mathcal{K}_1 + \mathcal{K}_2 + \mathcal{K}_3 + \mathcal{B})\, \mathrm{d}V \simeq \int_V \mathcal{K}_1\, \mathrm{d}V = 4\pi \int c^2 \rho (\mathrm{div}\boldsymbol{\xi})^2 r^2\, \mathrm{d}r$$

Seismic diagnostics

we approximate: $$\boxed{\delta\omega \simeq \frac{\delta_\gamma \mathcal{K}}{2\omega I}}$$ with $$\delta_\gamma \mathcal{K} = \int (\delta\gamma_1) p (\mathrm{div}\boldsymbol{\xi})^2 r^2 \,\mathrm{d}r$$

asymptotic limit (JWKB): $$(\mathrm{div}\boldsymbol{\xi})^2 = \left(\frac{\delta p}{\gamma_1 p}\right)^2 \simeq \frac{\pi\omega^3}{\gamma_1 p c r^2 \kappa} |x|^{1/2} |\mathrm{Ai}(-x)|^2$$

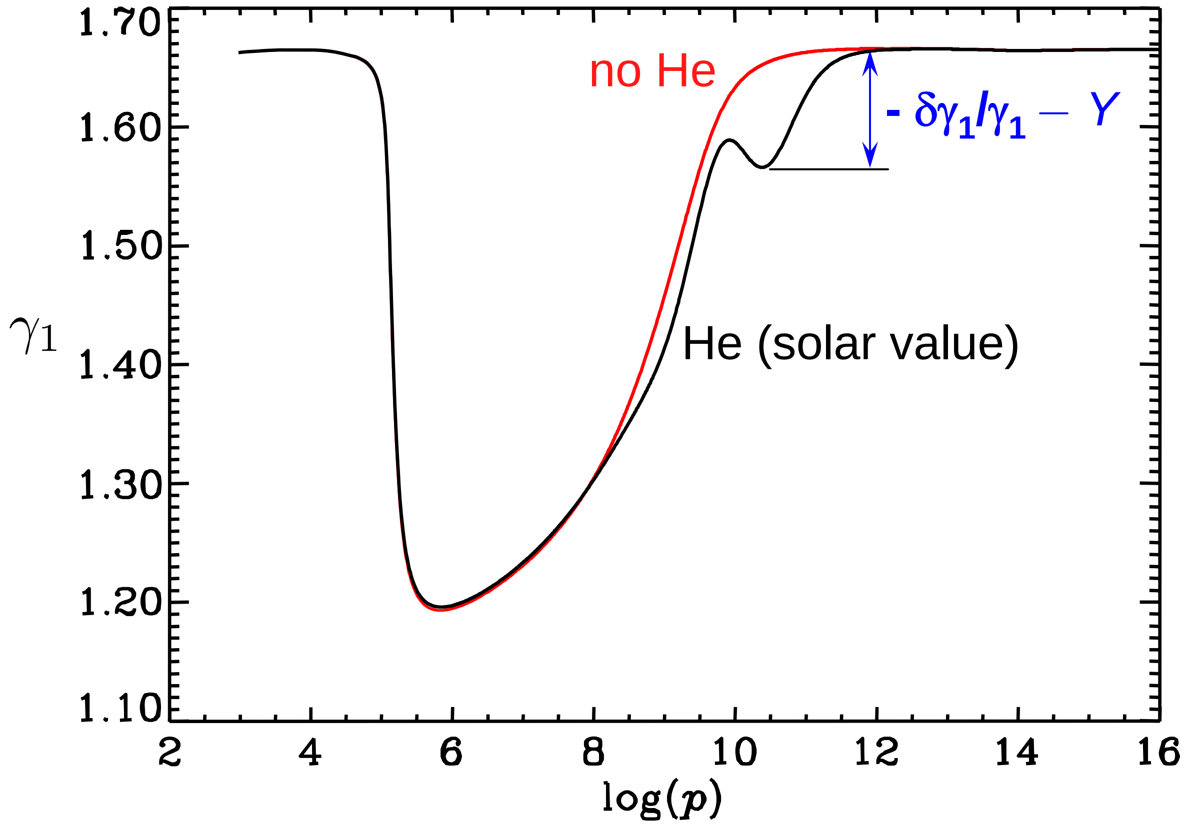
$$\boxed{\delta_\gamma \mathcal{K} \simeq \pi\omega^3 \int \kappa^{-1} \boxed{\frac{\delta\gamma_1}{\gamma_1}} |x|^{1/2} |\mathrm{Ai}(-x)|^2 \,\mathrm{d}\tau}$$

and $$\boxed{I} = \int_0^T \rho\, \boldsymbol{\xi}\cdot\boldsymbol{\xi}\, r^2 \,\mathrm{d}r \simeq \boxed{\frac{1}{2}\omega T - \frac{1}{4}(m+1)\pi}$$

Seismic diagnostics

Squared adiabatic sound speed $c^2 = \gamma_1 \, p/\rho$ $\gamma_1 = (\partial \ln p / \partial \ln \rho)_s$

Adiabatic exponent γ_1



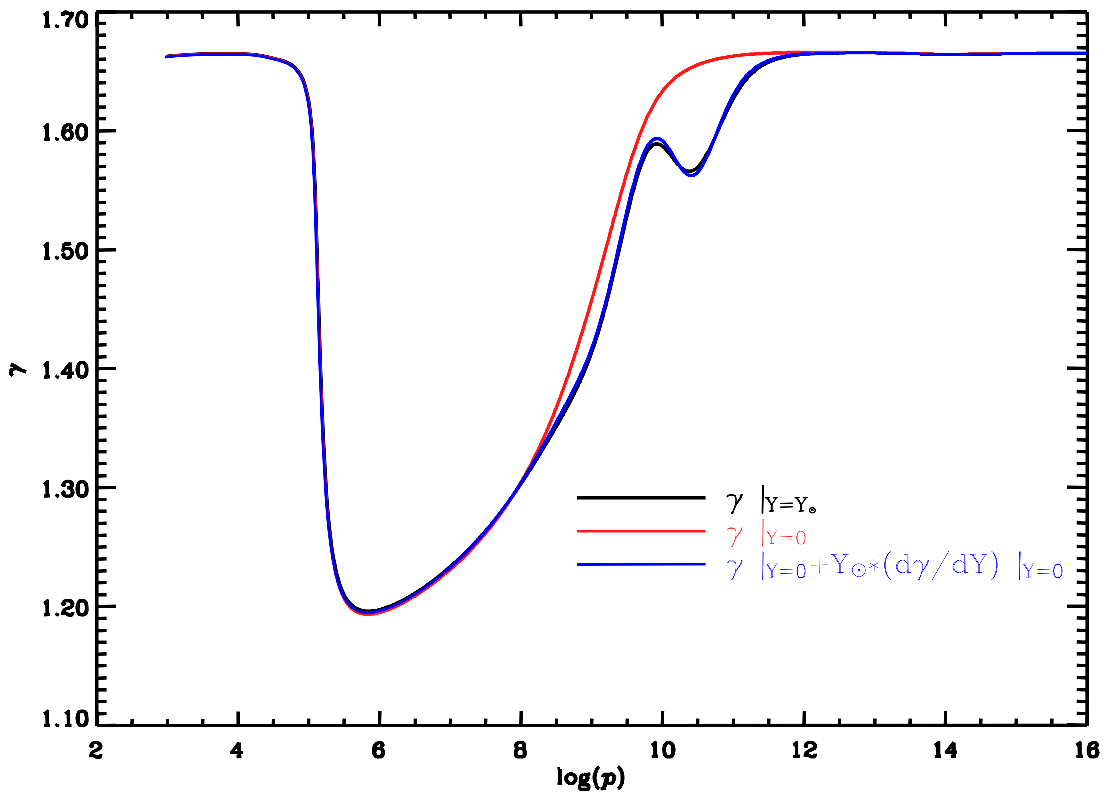
Seismic diagnostics

Expand γ with respect to Y about Y= 0: $\gamma \simeq (\gamma)_{Y=0} + (\partial\gamma/\partial Y)_{Y=0}\, Y$.

Glitches may then be written: $\delta\gamma \simeq (\partial\gamma/\partial Y)_{Y=0}\, Y$

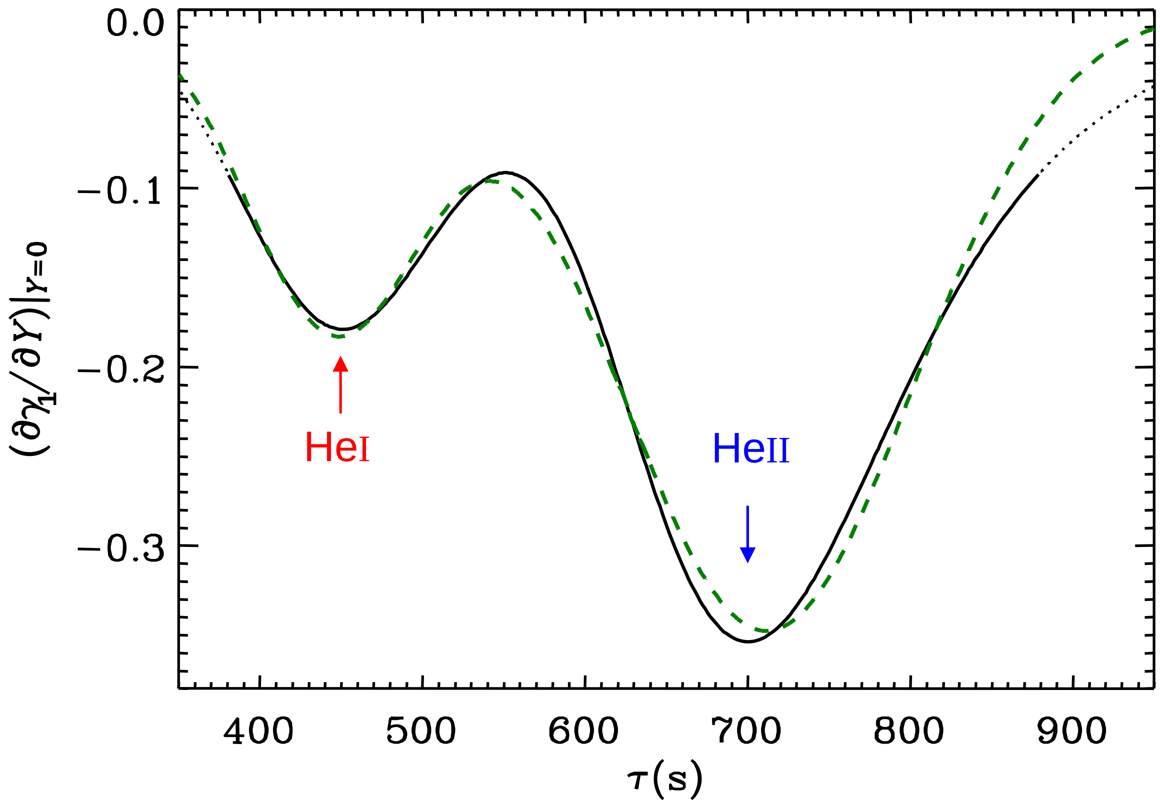
Y …. helium abundance

Adiabatic exponent γ_1



Seismic diagnostics

Glitches may be written as: $\delta\gamma \simeq (\partial\gamma/\partial Y)_{Y=0}\, Y$



Y …. helium abundance

$$\frac{\partial\gamma}{\partial Y} \simeq -\frac{\gamma}{\sqrt{2\pi}Y}\left[\frac{\Gamma_{\rm I}}{\Delta_{\rm I}} e^{-(\tau-\tau_{\rm I})^2/2\Delta_{\rm I}^2} + \frac{\Gamma_{\rm II}}{\Delta_{\rm II}} e^{-(\tau-\tau_{\rm II})^2/2\Delta_{\rm II}^2}\right].$$

Seismic diagnostic

$$\delta\nu \simeq \boxed{\delta_\gamma \nu} + \boxed{\delta_c \nu} + \boxed{\delta_s \nu}$$

He BCZ surface term

$$\boxed{\delta_\gamma \nu} = -\sqrt{2\pi} A_{\rm II} \Delta_{\rm II}^{-1} \left[\nu + \tfrac{1}{2}(m+1)\nu_0\right]$$
$$\times \left[\mu\beta \int_0^T \kappa_{\rm I}^{-1} e^{-(\tau - \eta\tau_{\rm II})^2/2\mu^2\Delta_{\rm II}^2} |x|^{1/2} |\mathrm{Ai}(-x)|^2 \, d\tau \right.$$ ..... HeI
$$\left. + \int_0^T \kappa_{\rm II}^{-1} e^{-(\tau - \tau_{\rm II})^2/2\Delta_{\rm II}^2} |x|^{1/2} |\mathrm{Ai}(-x)|^2 \, d\tau \right]$$ ..... HeII

$$\boxed{\delta_c \nu} \simeq A_c \nu_0^3 \nu^{-2} \left(1 + 1/16\pi^2\tau_0^2\nu^2\right)^{-1/2}$$
$$\times \left\{\cos[2\psi_c + \tan^{-1}(4\pi\tau_0\nu)] - (16\pi^2\tilde{\tau}_c^2\nu^2 + 1)^{1/2}\right\}$$ ..... BCZ

$$\boxed{\delta_s \nu} \simeq \widehat{A} + \widehat{B}\nu + \left[a_0\nu^2/2 + a_1\nu(\ln\nu - 1) - a_2 \ln\nu + a_3/2\nu\right]/h^2$$ ..... Surface (H, ∇_{ad})

$x = f(\omega\tau, \epsilon, m)$ $\psi_c = f(\omega\tau_c, \epsilon_c)$ $h = (\nu_{n+1} - \nu_{n-1})/2$

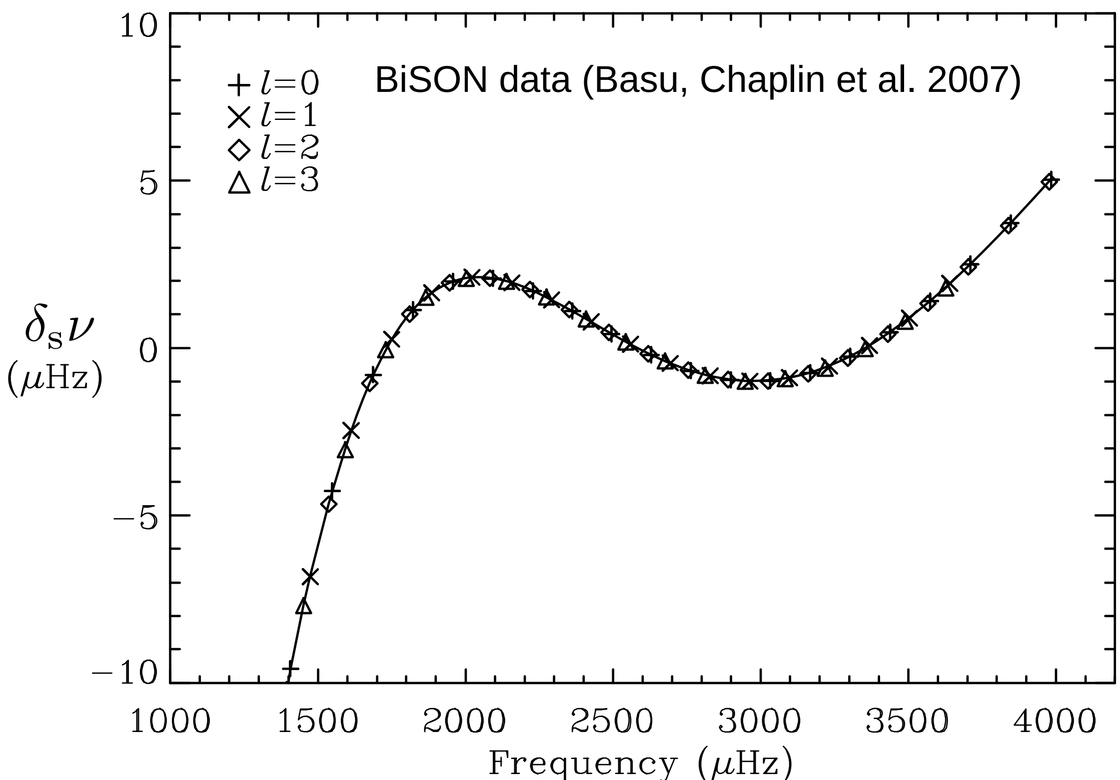
Seismic diagnostics: surface term $\delta_{\rm s}\nu$

$$\delta_{\rm s}\nu \equiv \Delta_2^{-1} \sum_{k=0}^{3} a_k \nu^{-k}$$

$$\simeq \tilde{A} + \tilde{B}\nu + \left[a_0\nu^2/2 + a_1\nu(\ln\nu - 1) - a_2 \ln\nu + a_3/2\nu\right]/h^2$$

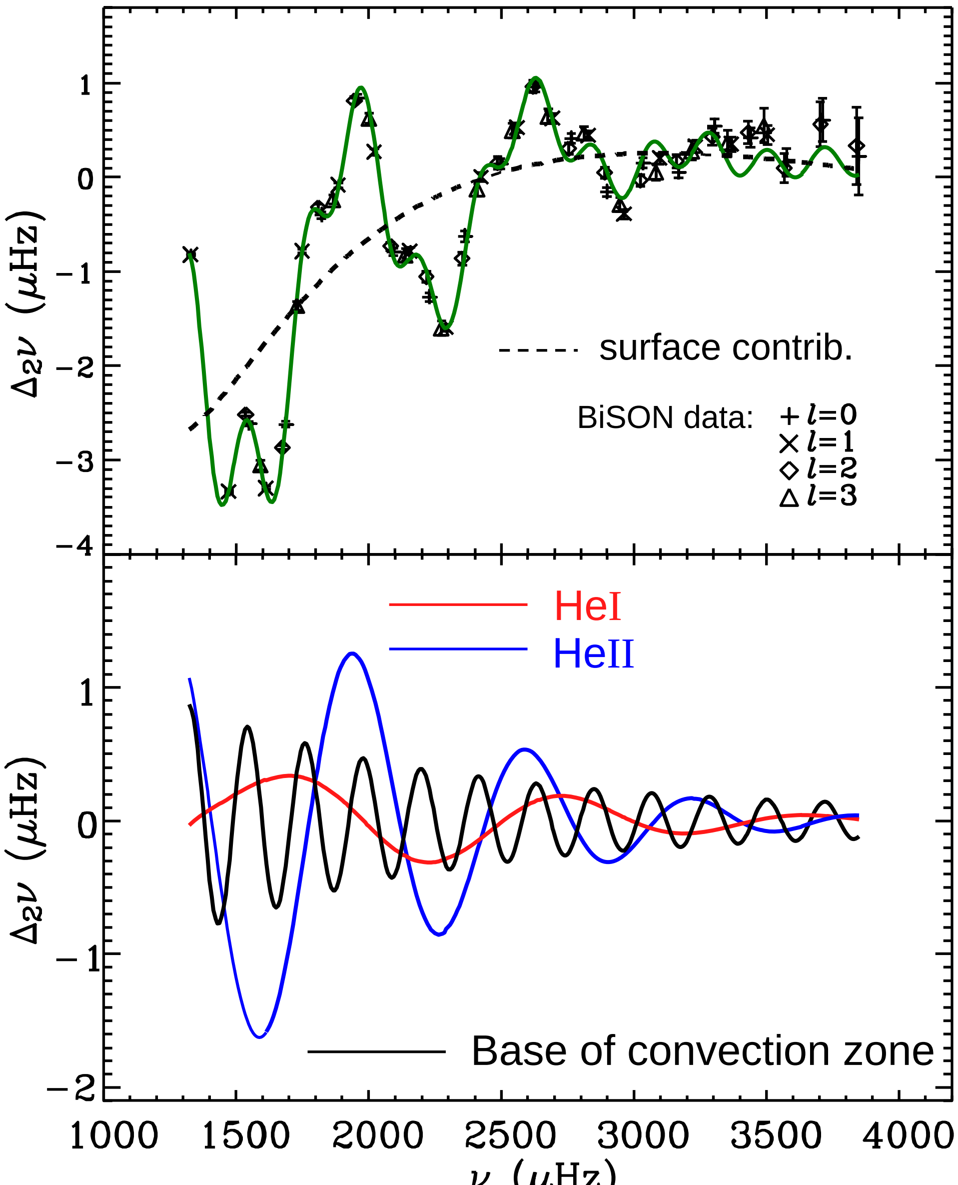
$\tilde{A}$ and $\tilde{B}$ are two undetermined constants of summation of 3rd - order polynomial

Choose $\tilde{A}$ and $\tilde{B}$ by minimizing: $E \equiv ||\delta_{\rm s}\nu||_2 = \sum_n (\tilde{A} + \tilde{B}\nu + F_{\rm s})^2$



Applying the seismic diagnostic to the Sun and simulated data (SONG)

Applying the seismic diagnostic to low-degree p modes: Sun



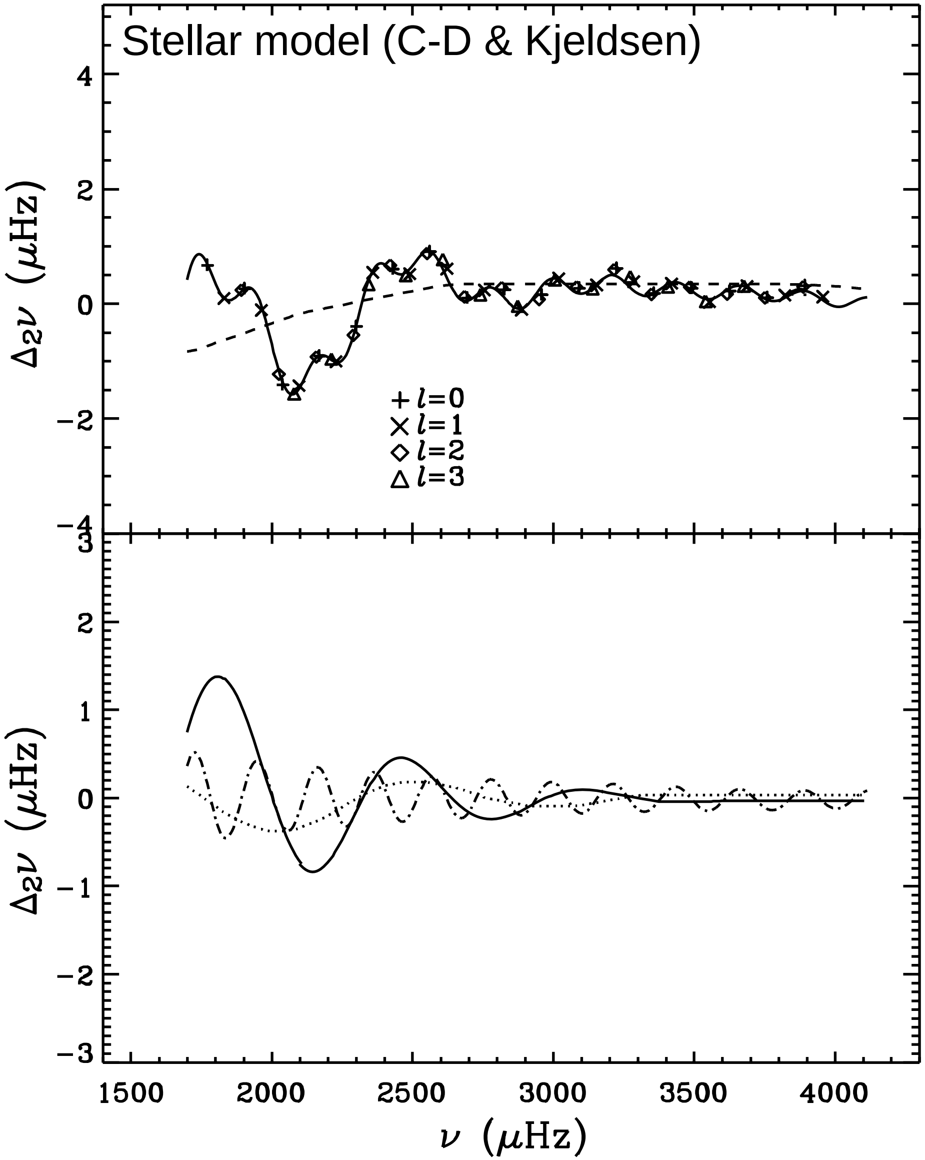
Seismic diagnostic

$$\delta\nu = \underbrace{\delta_{\rm I}\nu + \delta_{\rm II}\nu + \delta_{\rm c}\nu}_{\text{glitch contribution}} + \underbrace{\delta_{\rm s}\nu}_{\text{surface contribution}}$$

$$-\frac{\delta\gamma_1}{\gamma_1} \propto Y$$ …helium abundance

For BiSON data:	For Model S:
$-\delta\gamma_1/\gamma_1\|_{\tau_{\rm II}} = 0.043$	0.045
$\tau_{\rm II} = 819\,{\rm s}$	$815\,{\rm s}$
$\tau_{\rm c} = 2310\,{\rm s}$	$2270\,{\rm s}$

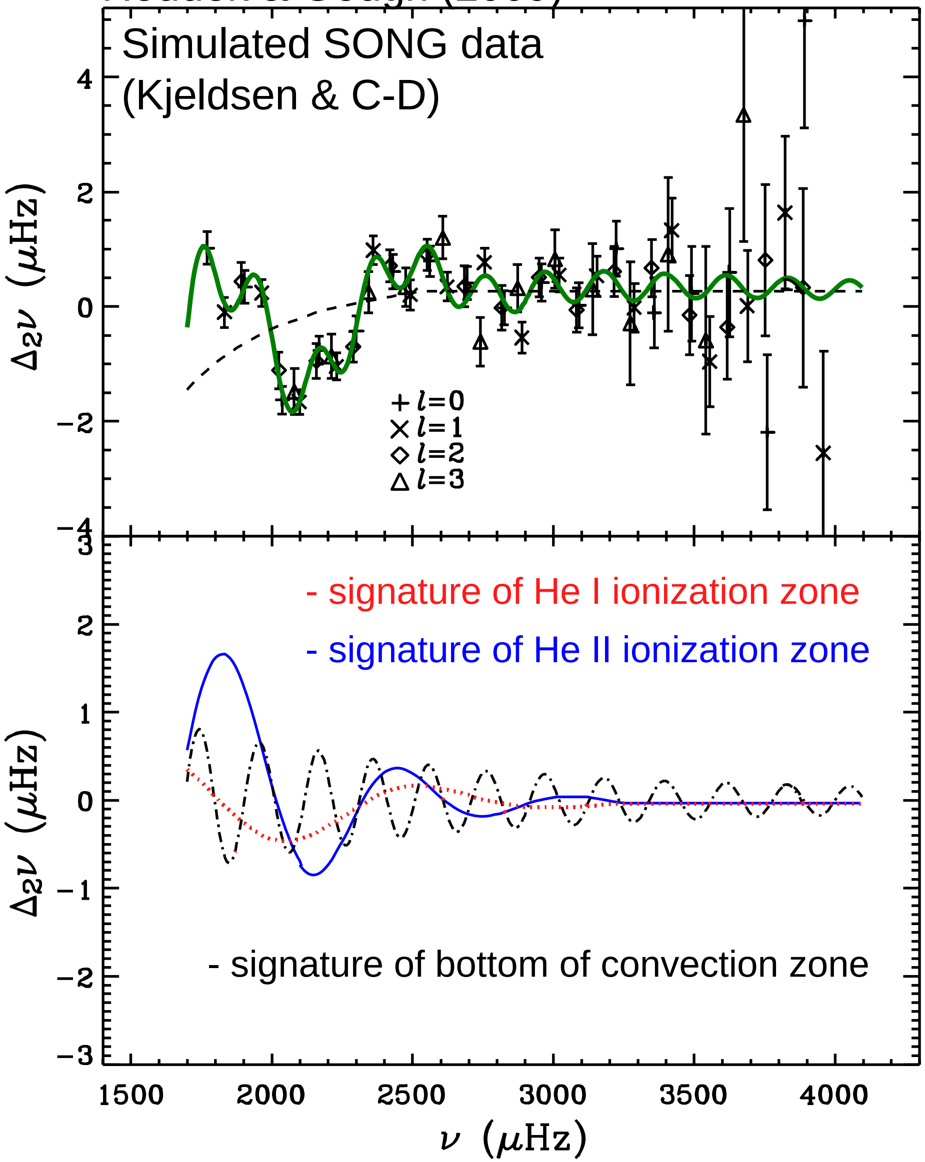
Applying the seismic diagnostic to a solar-like star



Results	for model:
$-\delta\gamma_1/\gamma_1\|_{\tau_{\rm II}} =$	0.062
$\tau_{\rm II} =$	$792\,{\rm s}$
$\tau_{\rm c} =$	$2330\,{\rm s}$

Applying the seismic diagnostic to a solar-like star

Houdek & Gough (2009)



Results for SONG **data:**	for **model**:
$-\delta\gamma_1/\gamma_1\|_{\tau_{\rm II}} = 0.086$	0.062
$\tau_{\rm II} = 818\,{\rm s}$	$792\,{\rm s}$
$\tau_{\rm c} = 2402\,{\rm s}$	$2330\,{\rm s}$

$$\gamma_1 = (\partial \ln p / \partial \ln \rho)_s$$

$-\delta\gamma_1/\gamma_1|_{\tau_{\rm II}}$ … rel. depression of γ_1 in He II ionization zone

$\tau_{\rm II}$ … acoustic depth of HeII ionization zone

$\tau_{\rm c}$ … acoustic depth of bottom of convection zone

Solar/stellar age calibration

Solar/stellar age calibration

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_{\rm s} \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu_{\rm s}}\,\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{{\nu_{\rm s}}^3}\,\nu_0^4 - \frac{F L^6 - G L^4 + H L^2 - I}{{\nu_{\rm s}}^5}\,\nu_0^6$$

- calibration using combinations of the seismically determined parameters

$$\xi_\alpha = (\underbrace{\widehat{A}, \widehat{C}, \widehat{F}}_{\text{age}}, \underbrace{-\delta\gamma_1/\gamma_1}_{Y}), \qquad \alpha = 1,2,3,4\,, \qquad -\delta\gamma_1/\gamma_1 \propto Y$$

$$\widehat{A} = \nu_0 A\,, \quad \widehat{C} = \nu_0^3 C\,, \quad \widehat{F} = \nu_0^5 F$$

- approximate solar value $\xi_\alpha^\odot$ by a two-term expansion about reference value $\xi_\alpha^{\rm r}$

$$\xi_\alpha^\odot = \xi_\alpha^{\rm r} + \left(\frac{\partial \xi_\alpha}{\partial t_\odot}\right)_Z \Delta t + \left(\frac{\partial \xi_\alpha}{\partial Z}\right)_{t_\odot} \Delta Z - \epsilon_{\xi\alpha}\,.$$

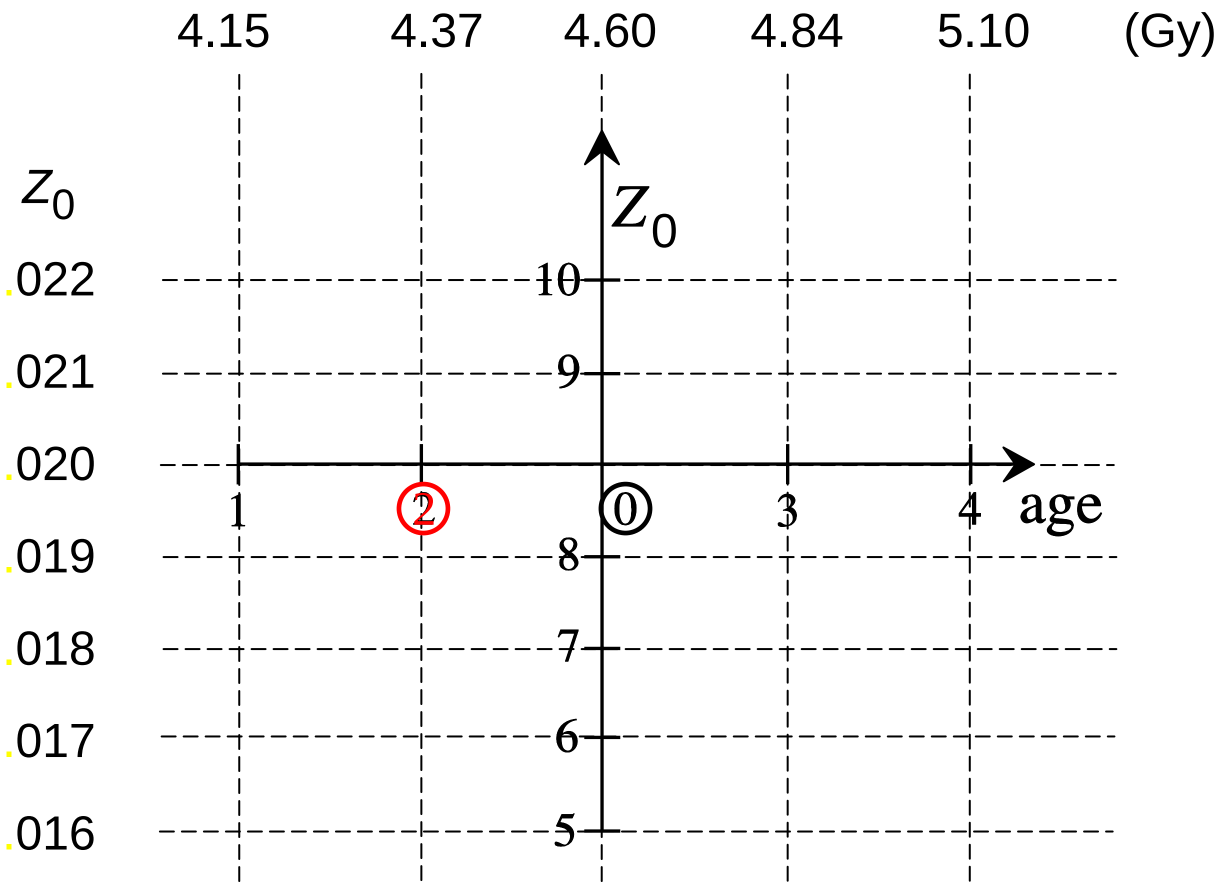
- and the **solution is**:

$$t_\odot = t_{\rm ref} + \Delta t$$
$$Z_\odot = Z_{\rm ref} + \Delta Z$$

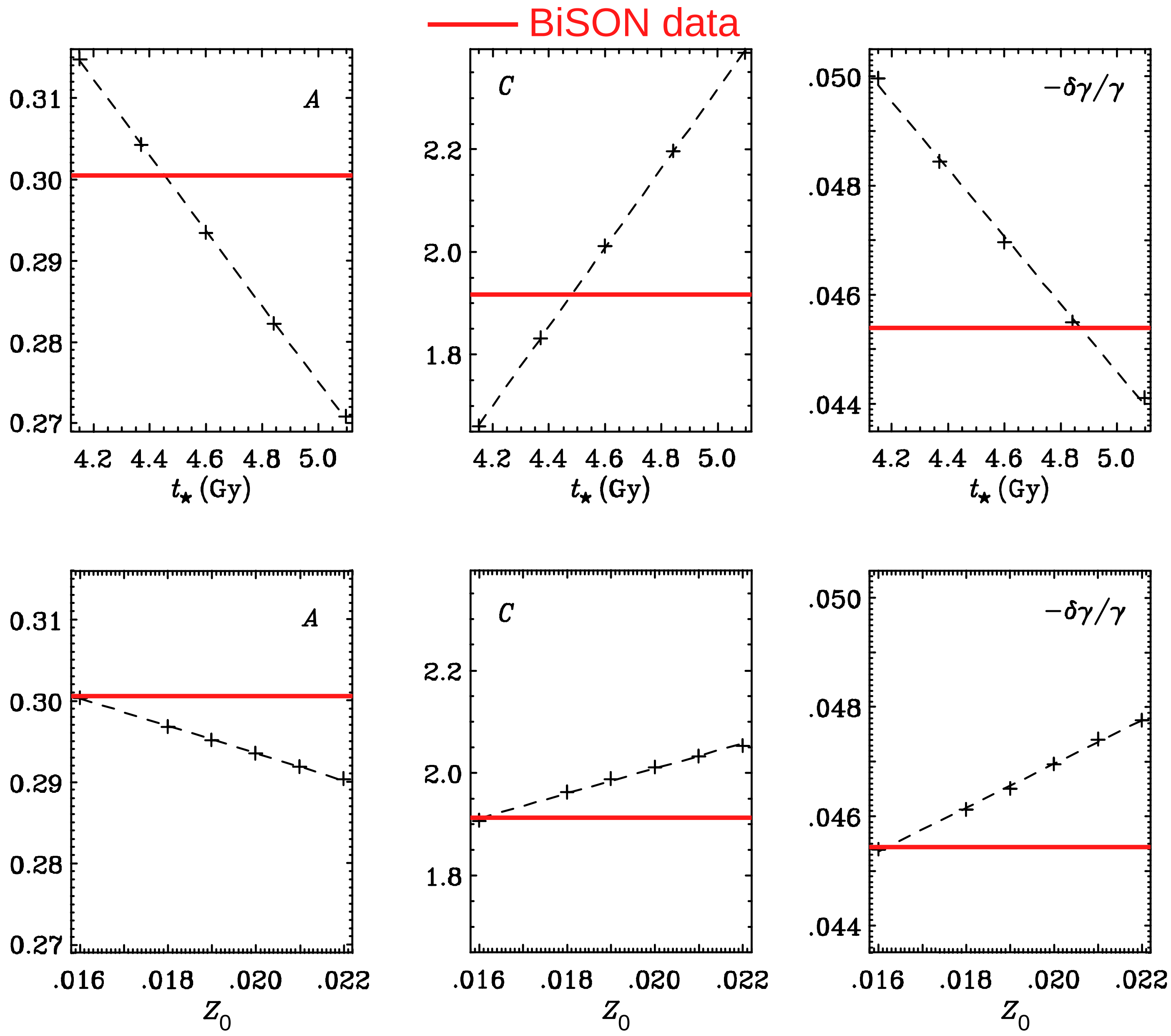
from reference model

Solar/stellar age calibration

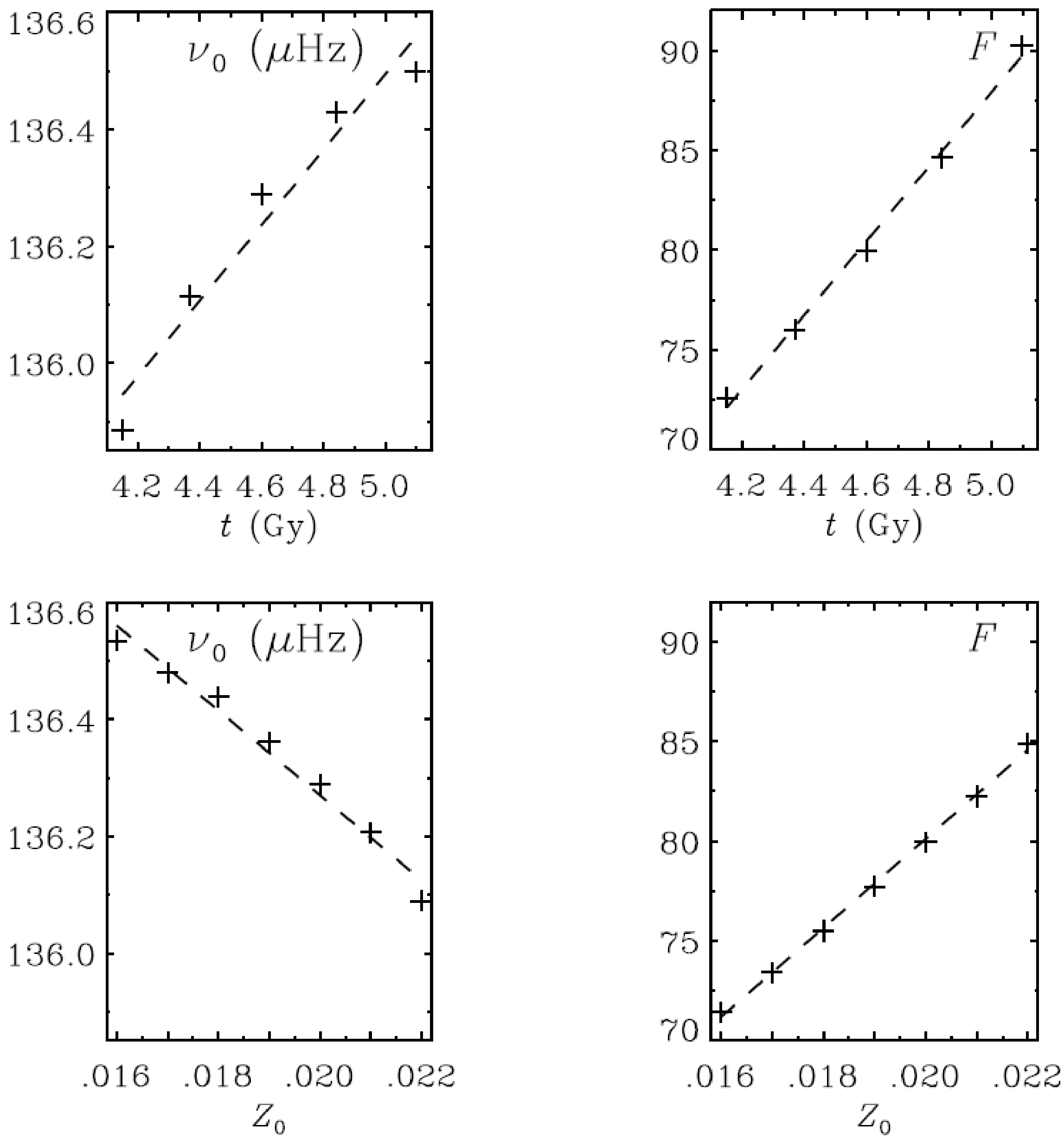
Eleven models calibrated to solar luminosity and radius



Solar/stellar age calibration



Solar/stellar age calibration



Solar/stellar age calibration

partial derivatives

$(\partial A/\partial t_\odot)_Z$	$(\partial A/\partial Z)_{t_\odot}$	$(\partial C/\partial t_\odot)_Z$	$(\partial C/\partial Z)_{t_\odot}$	$[\partial(-\delta\gamma_1/\gamma_1)/\partial t_\odot]_Z$	$[\partial(-\delta\gamma_1/\gamma_1)/\partial Z]_{t_\odot}$
-0.0465	-1.671	0.770	24.2	-0.00618	0.404
-0.7327	-0.1067	1.771	0.231	-0.6074	0.1634

$\left(\frac{\partial\nu_0}{\partial t_\odot}\right)_{Z_0}$	$\left(\frac{\partial\nu_0}{\partial Z}\right)_{t_\odot}$	$\left(\frac{\partial F}{\partial t_\odot}\right)_{Z_0}$	$\left(\frac{\partial F}{\partial Z}\right)_{t_\odot}$
0.6469	-72.66	18.6	2231
0.02203	-0.009973	1.057	0.5391

$\partial \ln\alpha / \partial \ln\beta$

Partial derivatives $H_{\alpha j}$ obtained from two sets of calibrated evolutionary models for the Sun. Values with respect to age $t_{\_}$ are in units of Gy^{-1} ,and frequencies are in μHz.

Solar/stellar age calibration

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_s \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu_s}\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{{\nu_s}^3}\nu_0^4 - \frac{FL^6 - GL^4 + HL^2 - I}{{\nu_s}^5}\nu_0^6$$

- calibration using combinations of the seismically determined parameters

$$\xi_\alpha = (\widehat{A}, \widehat{C}, \widehat{F}, -\delta\gamma_1/\gamma_1), \qquad \alpha = 1,2,3,4\,, \qquad -\delta\gamma_1/\gamma_1 \propto Y$$

ξ_α	$t_\odot$ (Gy)	Z_0	Y_0	$t_\odot$ (Gy)	Z_0	Y_0
$\hat{A}, \hat{C}, \hat{F}, -\delta\gamma_1/\gamma_1$	4.592	0.0156	0.252	4.597	0.0155	0.251
$\hat{A}, \hat{F}, -\delta\gamma_1/\gamma_1$	4.580	0.0157	0.252	4.582	0.0156	0.251
$\hat{C}, \hat{F}, -\delta\gamma_1/\gamma_1$	4.591	0.0157	0.252	4.595	0.0155	0.251
$\hat{A}, \hat{C}, -\delta\gamma_1/\gamma_1$	4.597	0.0160	0.254	4.603	0.0160	0.253
$\hat{A}, \hat{C}, \hat{F}$	4.619	0.0153	0.252	4.632	0.0151	0.248
$\hat{A}, \hat{C}$	4.638	0.0147	0.246	4.654	0.0143	0.245

Referenz model 0:
4.60 Gy, Z_0=0.02

Referenz model 2:
4.37 Gy, Z_0=0.02

Solar/stellar age calibration

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_s \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu_s}\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{{\nu_s}^3}\nu_0^4 - \frac{F L^6 - G L^4 + H L^2 - I}{{\nu_s}^5}\nu_0^6$$

- calibration using combinations of the seismically determined parameters

$$\xi_\alpha = (\widehat{A}, \widehat{C}, \widehat{F}, -\delta\gamma_1/\gamma_1), \qquad \alpha = 1, 2, 3, 4\,, \qquad -\delta\gamma_1/\gamma_1 \propto Y$$

ξ_α	$t_\odot$ (Gy)	Z_0	$C^{1/2}_{\Theta 11}$	$-(-C_{\Theta 12})^{1/2}$	$C^{1/2}_{\Theta 22}$
$\hat{A}, \hat{C}, \hat{F}, -\delta\gamma_1/\gamma_1$	4.592	0.0156	0.039	0.0013	0.0005
$\hat{A}, \hat{F}, -\delta\gamma_1/\gamma_1$	4.580	0.0157	0.045	0.0016	0.0006
$\hat{C}, \hat{F}, -\delta\gamma_1/\gamma_1$	4.591	0.0157	0.044	0.0004	0.0005
$\hat{A}, \hat{C}, -\delta\gamma_1/\gamma_1$	4.597	0.0160	0.045	0.0036	0.0008
$\hat{A}, \hat{C}, \hat{F}$	4.619	0.0153	0.095	0.0104	0.0013
$\hat{A}, \hat{C}$	4.638	0.0147	1.049	0.1791	0.0306

Referenz model 0:
4.60 Gy, Z_0=0.02

error co-variance matrix

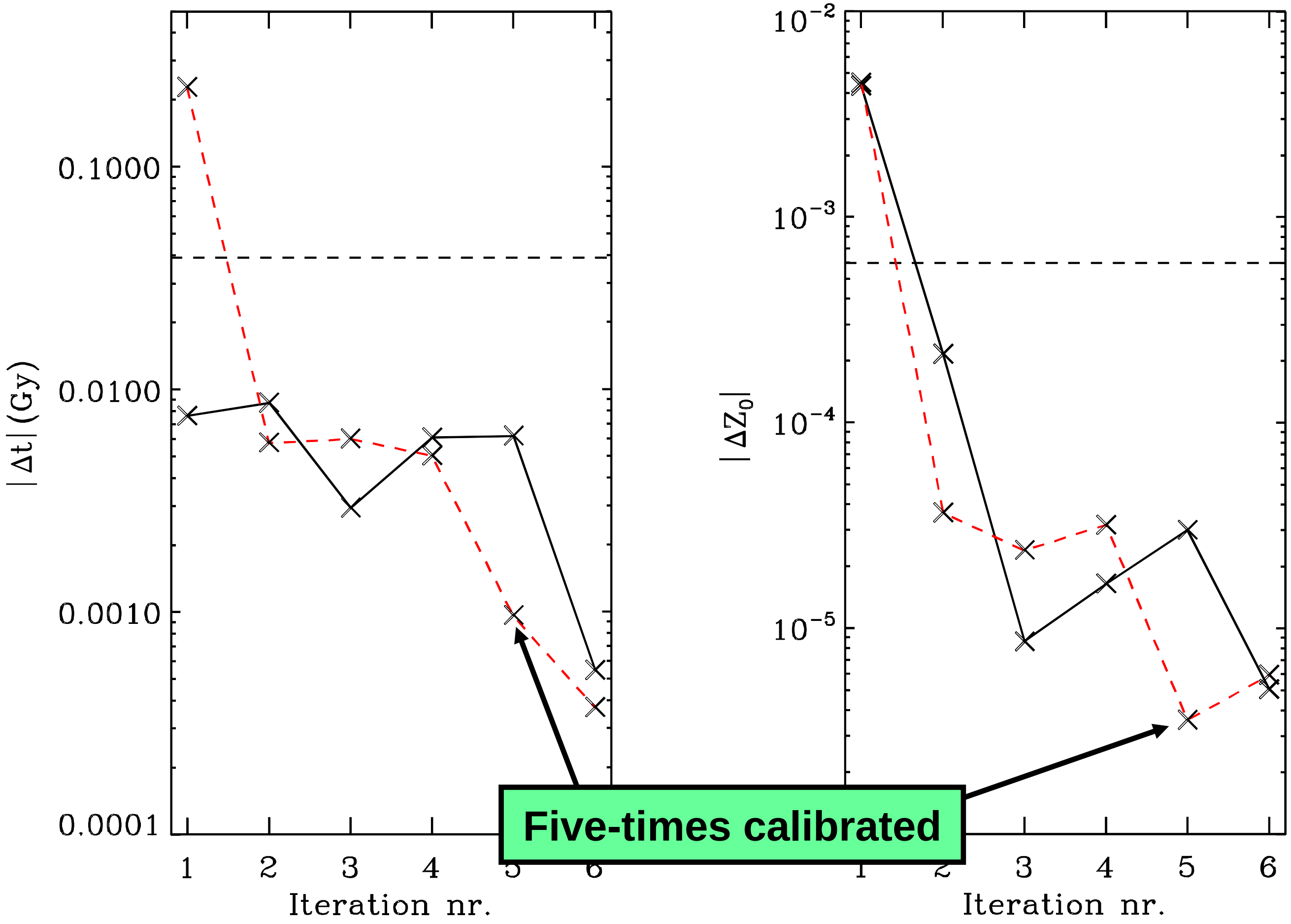
Note: Model S: age=4.60 Gy, Z_0=0.0196

Solar/stellar age calibration

Referenz model⓪: 4.60 Gy, Z=0.02

Referenz model②: 4.37 Gy, Z=0.02

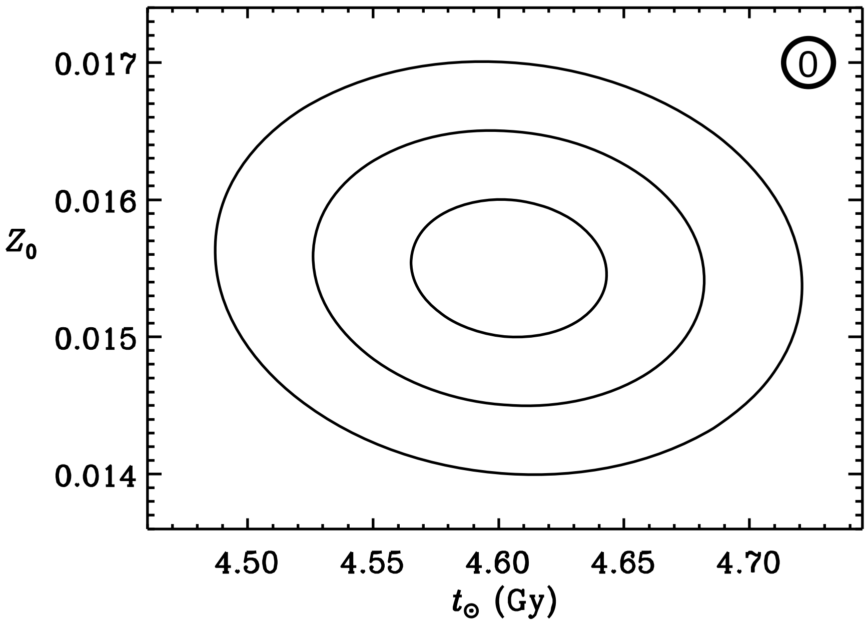
$$(\widehat{A}, \widehat{C}, \widehat{F}, -\delta\gamma_1/\gamma_1)$$



Solar/stellar age calibration

Results for
<u>five-times calibrated</u> reference models
using BiSON data & $(\widehat{A}, \widehat{C}, \widehat{F}, -\delta\gamma_1/\gamma_1)$

	Reference model	$t_\odot$ (Gy)	Z_0	Y_0
0	4.60 Gy/Z_0=0.02	4.604	0.0155	0.250
2	4.37 Gy/Z_0=0.02	4.603	0.0155	0.250



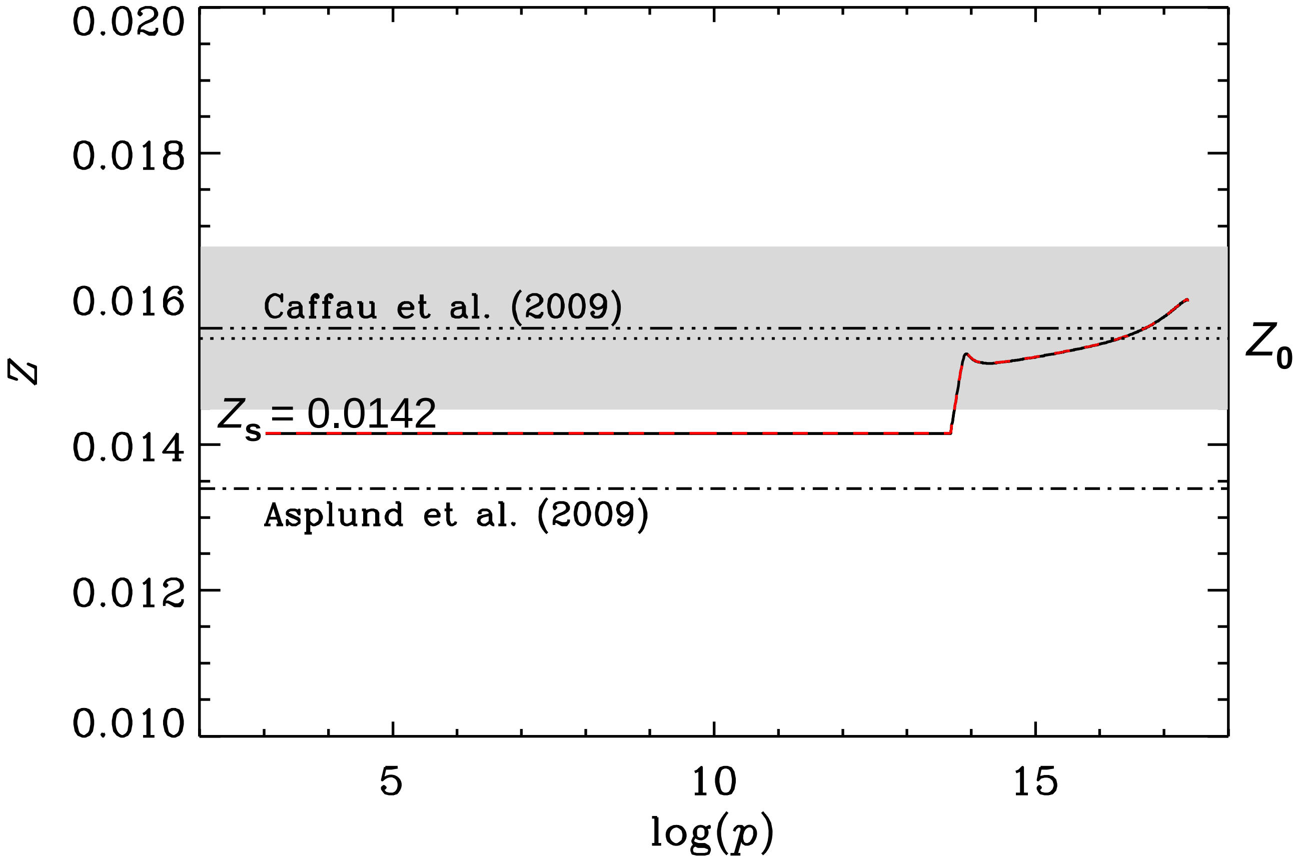
Note: Model S: age = 4.60 Gy, Z_0 = 0.0196

Solar/stellar age calibration

Five-times calibrated

Referenz model⓪: 4.60 Gy, Z_0=0.02

Referenz model②: 4.37 Gy, Z_0=0.02



Solar/stellar age calibration

	Z_s	Z_s/X_s
Caffau et al. (2009)	0.0156±0.0011	0.0213
Houdek & Gough (2011)	0.0142±0.0005	0.0186
Asplund et al. (2009)	0.0134	0.0181

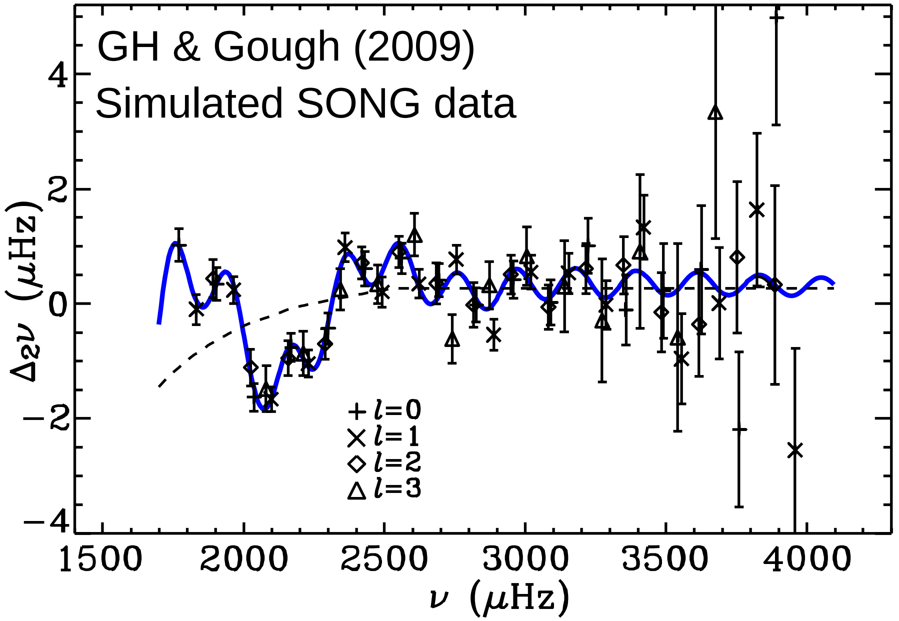
Five-times calibrated reference model

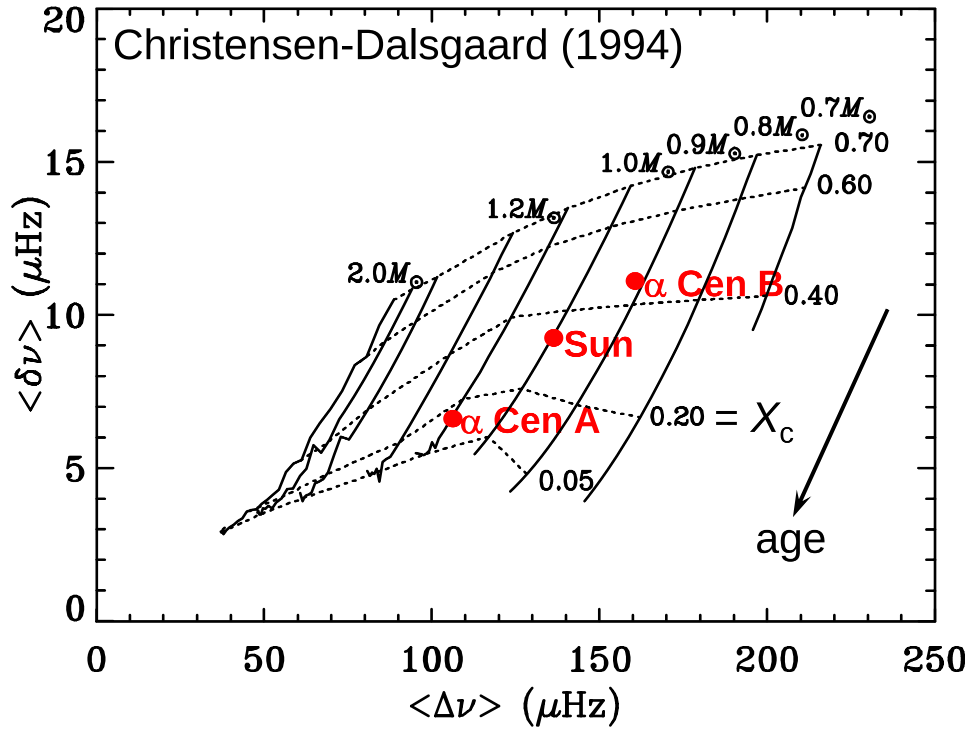
X_s = 0.7618

Y_s = 0.2240

Z_s = 0.0142

Solar/stellar age calibration





$$\widehat{A}, \widehat{C}, \widehat{F} \,, -\delta\gamma_1/\gamma_1$$

$$t_\star(\pm 5\%)$$

<u>Kjeldsen et al. (2008) :</u>

$$\langle\Delta\rangle(\pm 0.5\%) \,, \langle\delta\nu\rangle(\pm 10\%) \,, T_{\rm eff}(\pm 2\%)$$

$$t_\star(\pm 10\%)$$

Summary/conclusion

- Seismic diagnostics (2nd frequency differences) of low-degree modes can be used to estimate gross properties (*Y* , DCZ) of solar-type stars.

- Removing the seismic signature of rapid variations in the background state from the frequencies could substantially improve the calibration of stellar ages and abundances.

- This seismic calibration procedure can be applied to data from CoRoT, Kepler and planned observing campaigns (PLATO, SONG).

- The values of *Z* should not be regarded strictly as statements for initial heavy-element abundance, but rather as a measure of the opacity in the radiative interior.

Happy anniversary Douglas

and

Thank you for the delightful collaboration over the last years, for your contagious joy of thinking, your advice and help and above all for your friendship.