Magnetic Diffusivity in Protostellar Disks

Faraday's Law

$$\frac{\partial \mathbf{B}}{\partial t} = -c\,\nabla \times \mathbf{E}\ , \tag{22}$$

where the lab-frame electric field $\mathbf{E}$ is related to the comoving electric field $\mathbf{E}'$ by

$$\mathbf{E} = \mathbf{E}' - \frac{\mathbf{V}}{c}\times\mathbf{B}\ . \tag{23}$$

Ampère's Law

$$\mathbf{j} = \frac{c}{4\pi}\nabla \times \mathbf{B}\ . \tag{24}$$

Ohm's Law

$$\mathbf{j} = \sigma \cdot \mathbf{E}' = \sigma_{\|}\mathbf{E}'_{\|} + \sigma_{\mathrm{H}}\hat{\mathbf{b}} \times \mathbf{E}'_{\perp} + \sigma_{\mathrm{P}}\mathbf{E}'_{\perp}\ , \tag{25}$$

where $\sigma_{\|}$, σ_{H}, and σ_{P} are the magnetic field-parallel, Hall, and Pedersen conductivities, respectively, and $\hat{\mathbf{b}} = \mathbf{B}/B$.

In the limit $\sigma \to \infty$, $\mathbf{E}' \to 0$ and $\mathbf{E} \to -\mathbf{V} \times \mathbf{B}/c$, the ideal-MHD result.

Thus

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times\left[\mathbf{V}\times\mathbf{B} - \frac{\mathbf{j}}{\sigma_\parallel} - \frac{\sigma_\mathrm{H}}{\sigma_\perp^2}\mathbf{j}\times\mathbf{b} - \left(\frac{\sigma_\mathrm{P}}{\sigma_\perp^2} - \frac{1}{\sigma_\parallel}\right)(\mathbf{j}\times\mathbf{b})\times\mathbf{b}\right], \quad (26)$$

where $\sigma_\perp^2 = \sigma_\mathrm{H}^2 + \sigma_\mathrm{P}^2$. The magnetic field evolution is thus seen to be determined by the balance of the inductive, Ohmic, Hall, and ambipolar terms.

The conductivities can be expressed in terms of the current carriers' charges, number densities, and Hall parameters

$$\beta_\mathrm{j} = \frac{\omega_\mathrm{c,j}}{\nu_\mathrm{jn}}, \quad (27)$$

where $\omega_\mathrm{c,j} = eZ_\mathrm{j}B/m_\mathrm{j}c$ and $\nu_\mathrm{jn} = \gamma_\mathrm{j}\rho$ are, resepctively, particle j's cyclotron frequency and collision frequency (with the dominant neutral component).

Example: neutral-ion-electron (n-i-e) gas

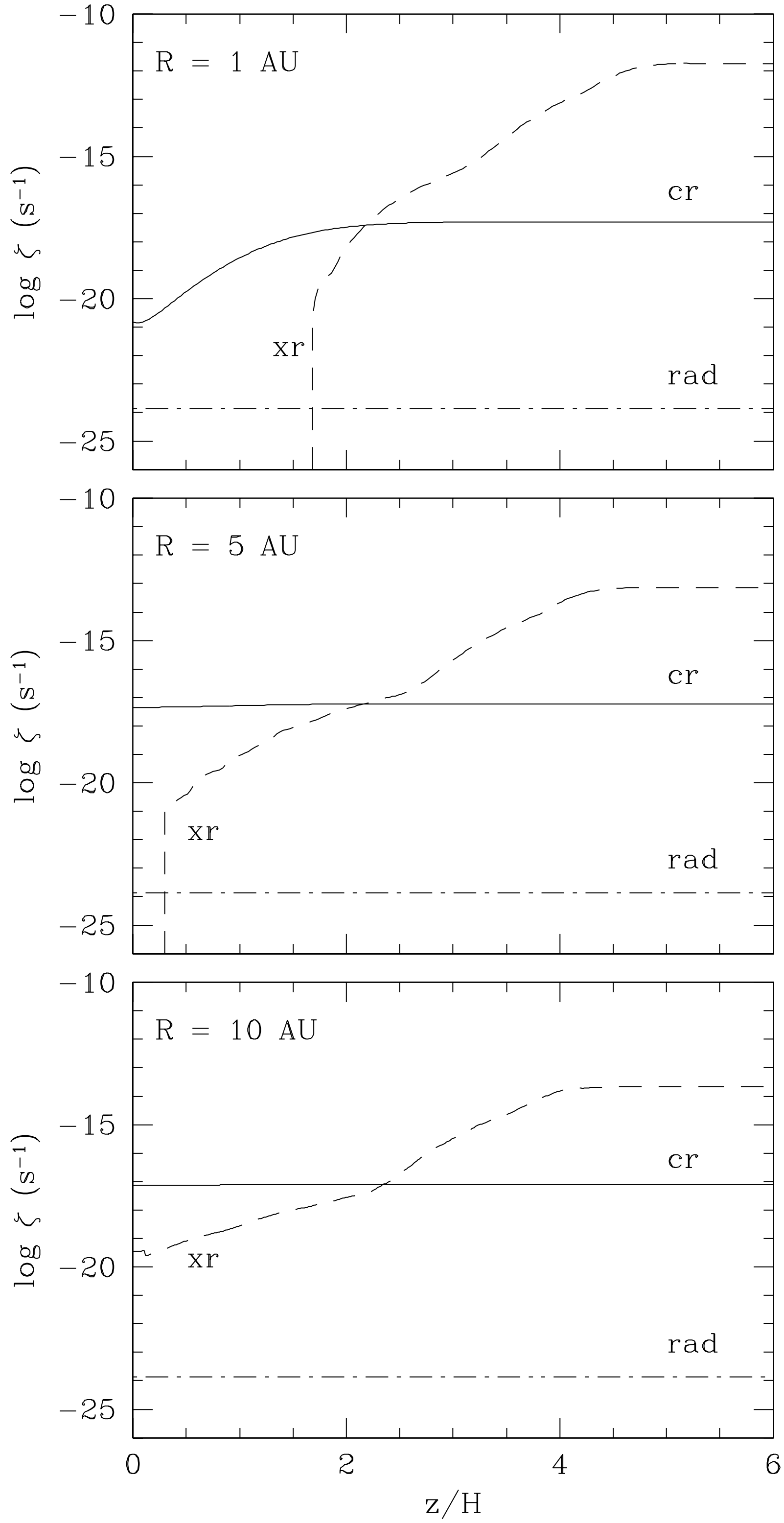
$$\mathbf{E} = -\frac{\mathbf{V}\times\mathbf{B}}{c} + \frac{\mathbf{j} + \beta_{\rm e}\,\mathbf{j}\times\mathbf{b} + \beta_{\rm e}\,\beta_{\rm i}\,\mathbf{b}\times(\mathbf{j}\times\mathbf{b})}{\sigma_{\rm e}}\,, \tag{28}$$

where $\beta_{\rm i} \approx 10^{-4}T^{1/2}\beta_{\rm e}$ and $\sigma_{\rm e} = e^2 n_{\rm e}/m_{\rm e}\nu_{\rm en}$.

In the limit of negligible Ohmic resistivity ($\beta_{\rm e} \gg 1$), eq. (28) reduces to

$$\mathbf{E} = -[\mathbf{V}_{\rm n} + (\mathbf{V}_{\rm e} - \mathbf{V}_{\rm i}) + (\mathbf{V}_{\rm i} - \mathbf{V}_{\rm n})]\times\mathbf{B}/c\,, \tag{29}$$

which shows that the field is "frozen" into the electrons (the particles with the highest mobility $eZ_{\rm j}/m_{\rm j}$). The gas is in the ambipolar diffusion-dominated regime when $\beta_{\rm i} > 1$, in which case the ions and electrons effectively move together and drift relative to the neutrals, whereas when $\beta_{\rm i} < 1$ the gas is in the Hall-dominated regime, in which case the ions effectively move with the neutrals and drift relative to the electrons.



Salmeron & Wardle (2005)

Collisional (thermal) ionization becomes important at $R \lesssim 0.1$ AU (for $T \gtrsim 10^3$ K — Gammie '96; Li '96).

YSO disk models

"gate" ↓ (between log r = −1 and 0)

$\log(r/AU)$:	-1	0	1	2
composition:	i-e **	n-g_+-g_- (or n-i-e?)	n-i-g_-	n-i-e-g_-
T (°K):	10^4 **	300	100	30
$\log(n_c/cm^{-3})$:	17	14.5 (13)*		10
$\log(B_c/G)$:	1	0		-2
$\log(B_{eq}/G)$: $= (8\pi n_c kT)^{1/2}$	3 **	1 (0.5)*		-1
diffusion mechanism:	anomalous Ohmic diffusivity **	ambipolar diffusion (in outer layers)		ambipolar diffusion (+ e-i & g_--n drifts)

* at base of accreting outer layers

** during the "high" phase of thermal ionization instability

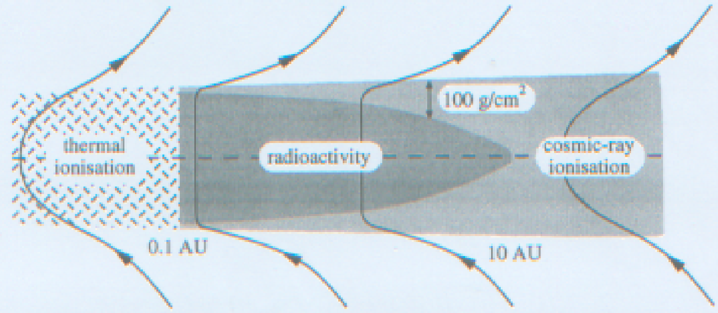


Figure 1. A sketch of the ionisation structure and magnetic field configuration expected for a protostellar disk. Cosmic rays can penetrate the first 100 g/cm^2 of the disk, so cannot maintain significant ionisation deep into the disk inside about 10 AU. Thermal ionisation of metals becomes significant within 0.1 AU where the disk temperature exceeds 1000 K. The magnetic field does not interact with the disk material in the region where radioactivity is the dominant form of ionisation because the conductivity there is negligible.

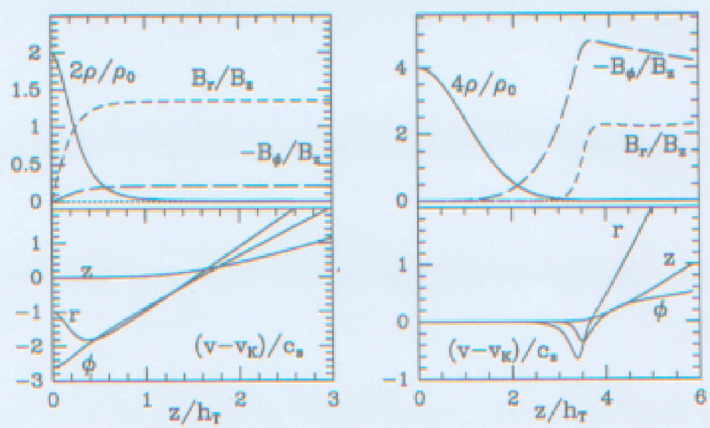
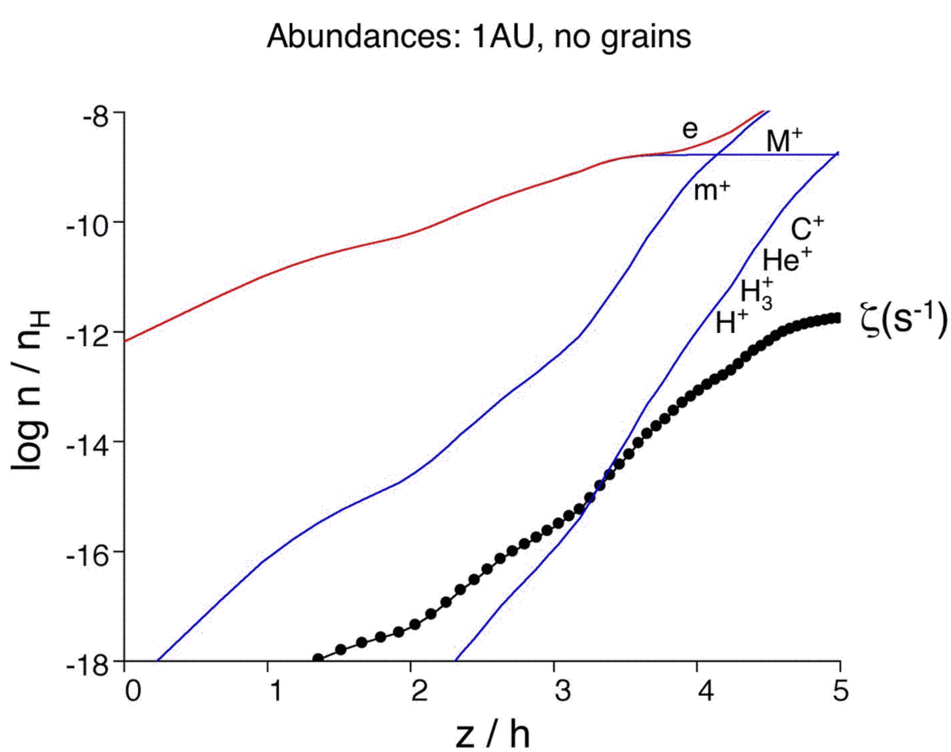


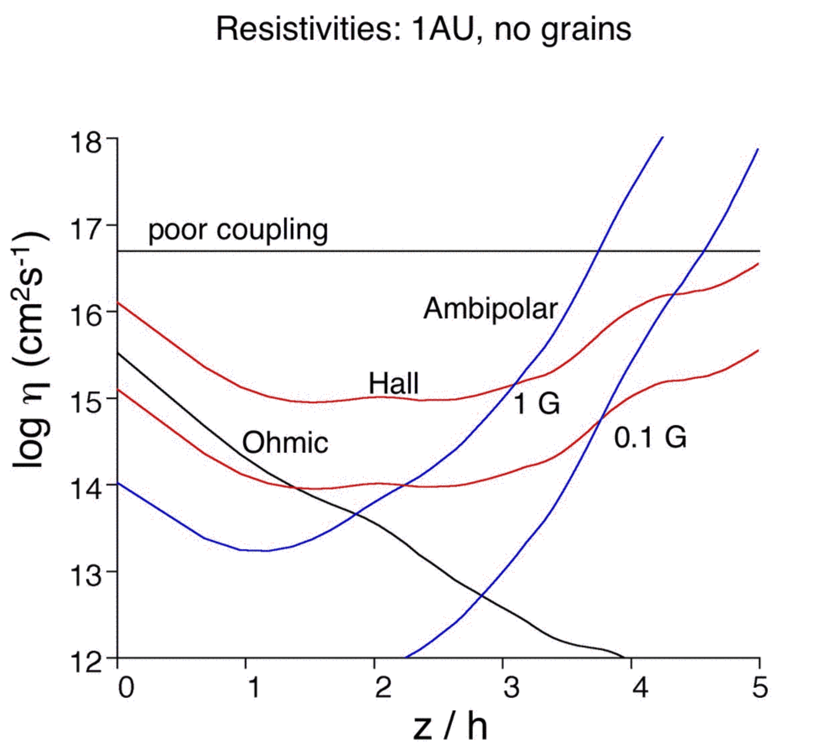
Figure 2. Vertical structure of a protostellar disk outside of 10 AU (left) and at 1 AU (right). The top panel of each figure plots the run of density and of the magnetic field components with height above the midplane in units of the isothermal scale height. The velocity components (relative to Keplerian and normalised by the sound speed) are plotted in the lower panels.

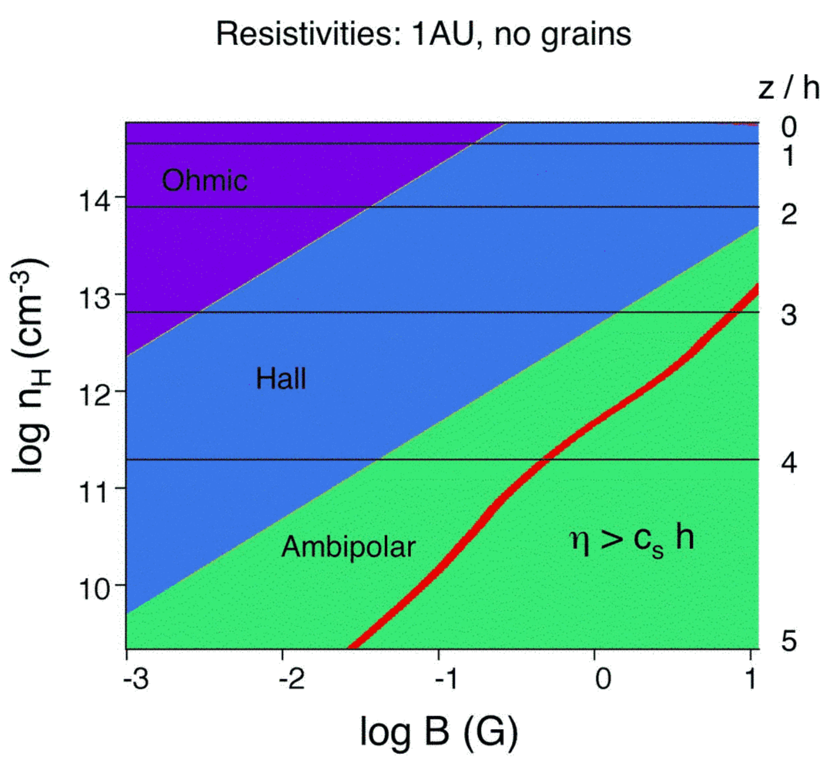
Wardle '97

Resistivity calculations

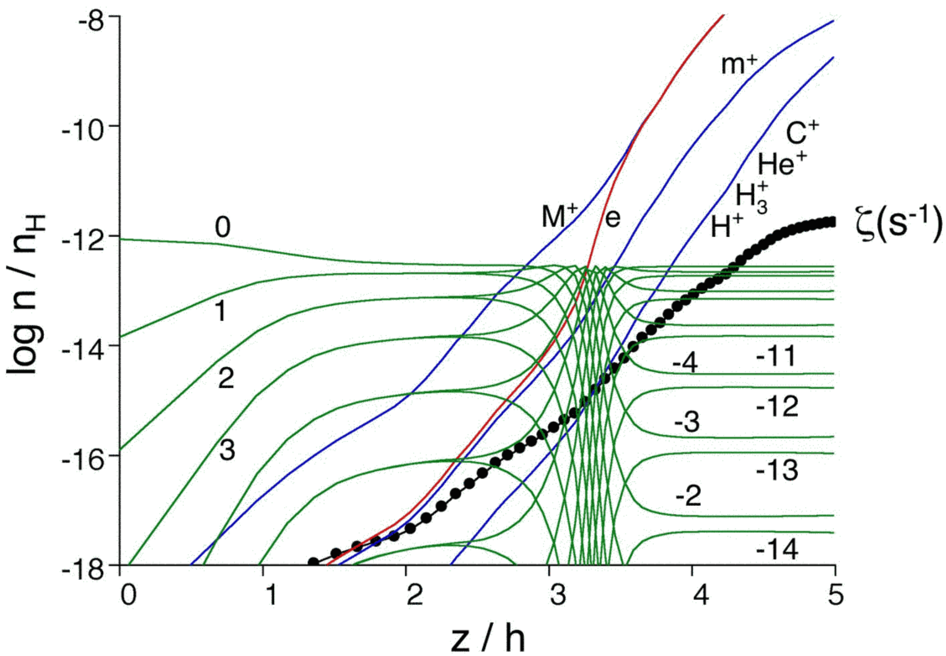
- minimum solar nebula
  - assume isothermal in z-direction
- ionisation by cosmic rays and x-rays from central star
- simple reaction scheme following Nishi, Nakano & Umebayashi (1993)
  - H^+,H_3^+,He^+,C^+,molecular (M^+) and metal ions (M^+), e^-, and charged grains
  - extended to allow high grain charge (T larger than in molecular clouds)
- adopt model for grains
  - none, single size grains, MRN size distribution, MRN + ice mantles, etc
  - results for "no grains" or 0.1 μm grains presented here
- evaluate resistivity components
  - when can the field couple to the shear in the disc?
  - which form of diffusion is dominant?



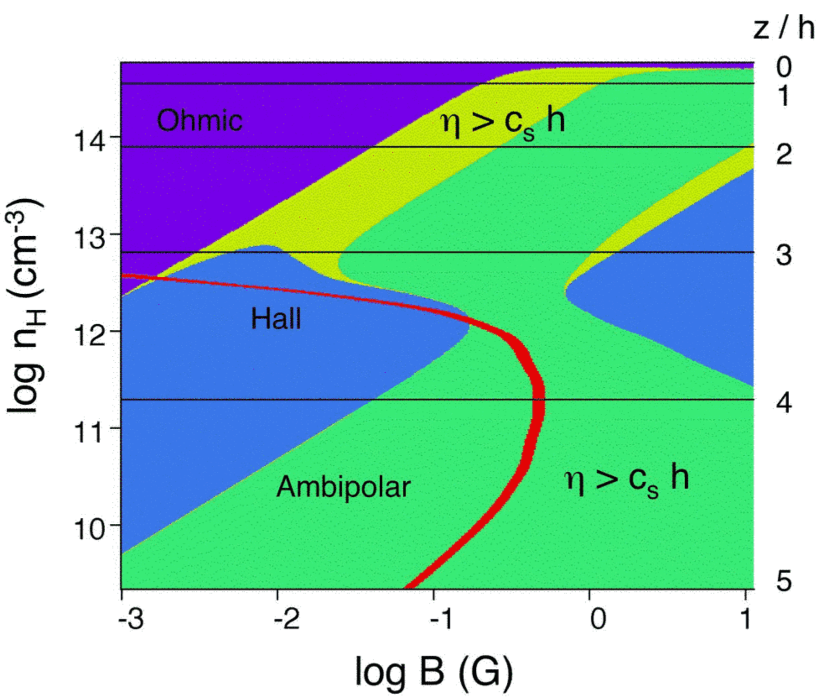

Resistivities: 1AU, no grains
log η (cm²s⁻¹)
z / h
poor coupling
Ambipolar
Hall
1 G
0.1 G
Ohmic
18
17
16
15
14
13
12
0
1
2
3
4
5

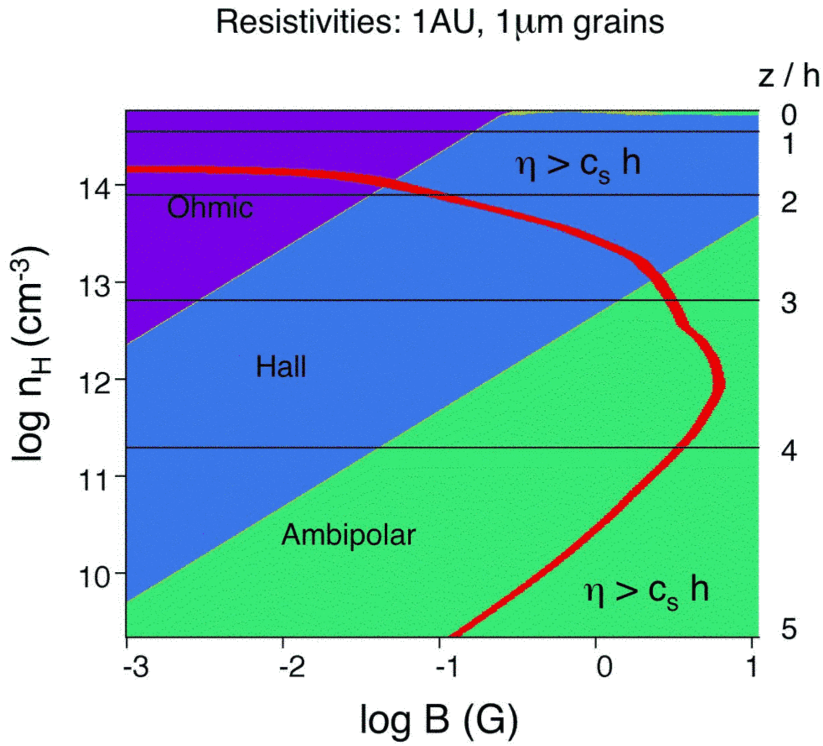

Resistivities: 1AU, no grains
z / h
Ohmic
Hall
Ambipolar
η > cs h
log nH (cm⁻³)
log B (G)
14
13
12
11
10
-3
-2
-1
0
1
0
1
2
3
4
5

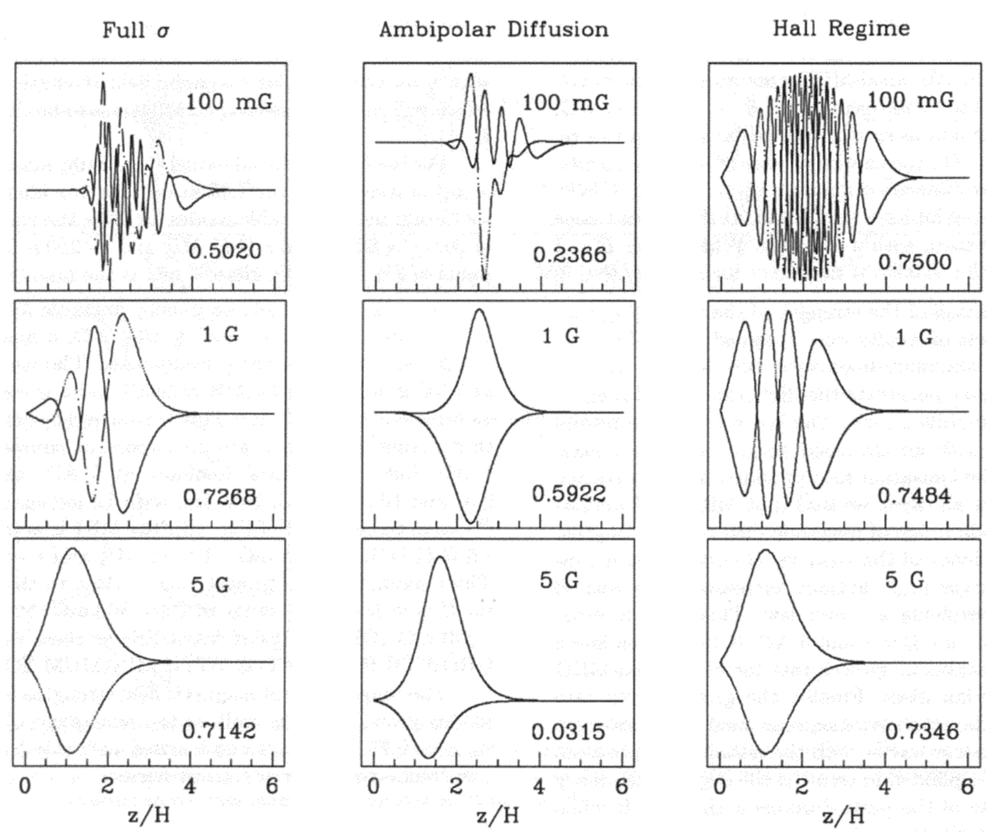
Abundances: 1AU, 0.1μm grains



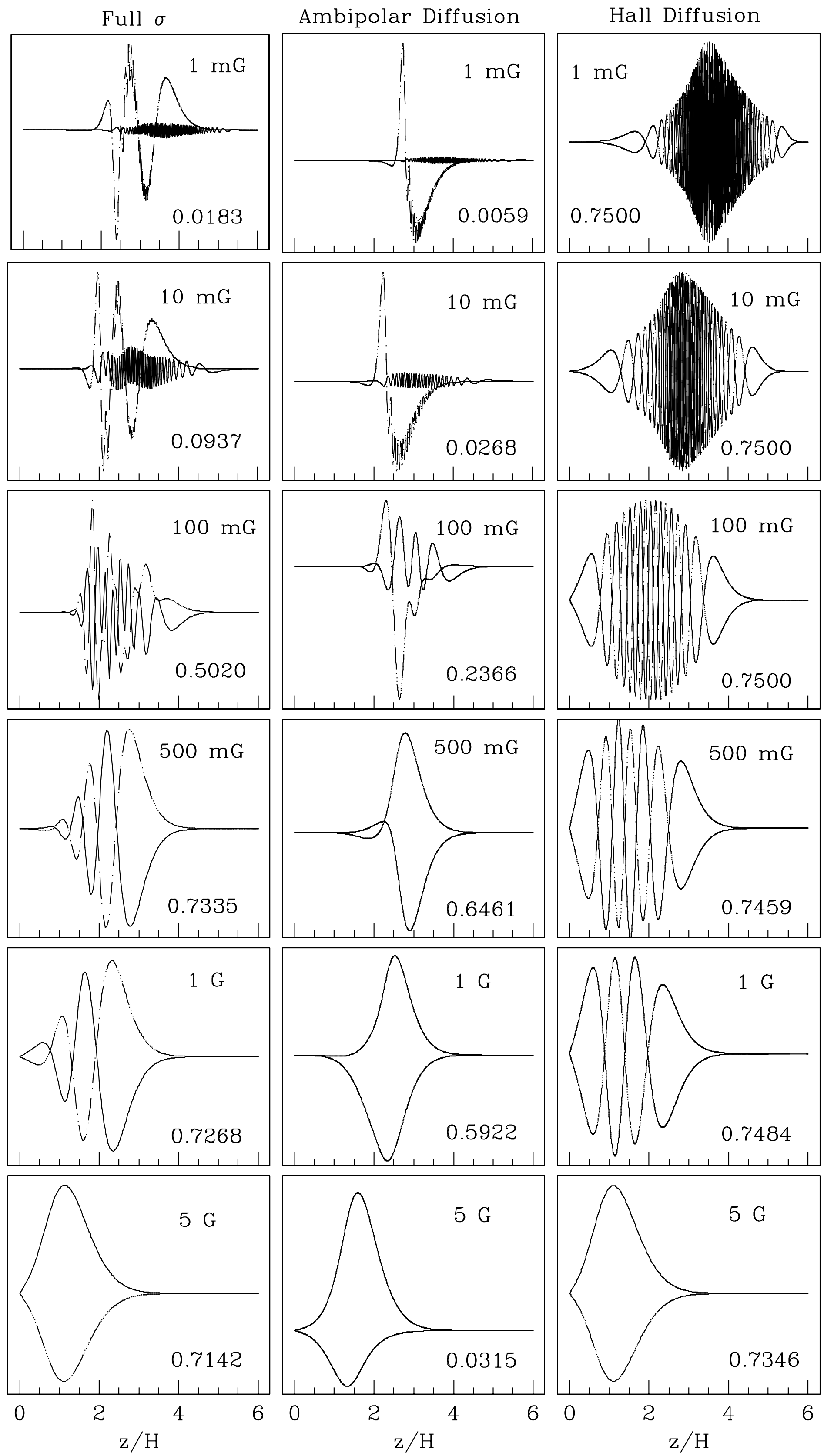
Resistivities: 1AU, 0.1μm grains







Salmeron & Wardle MNRAS in press



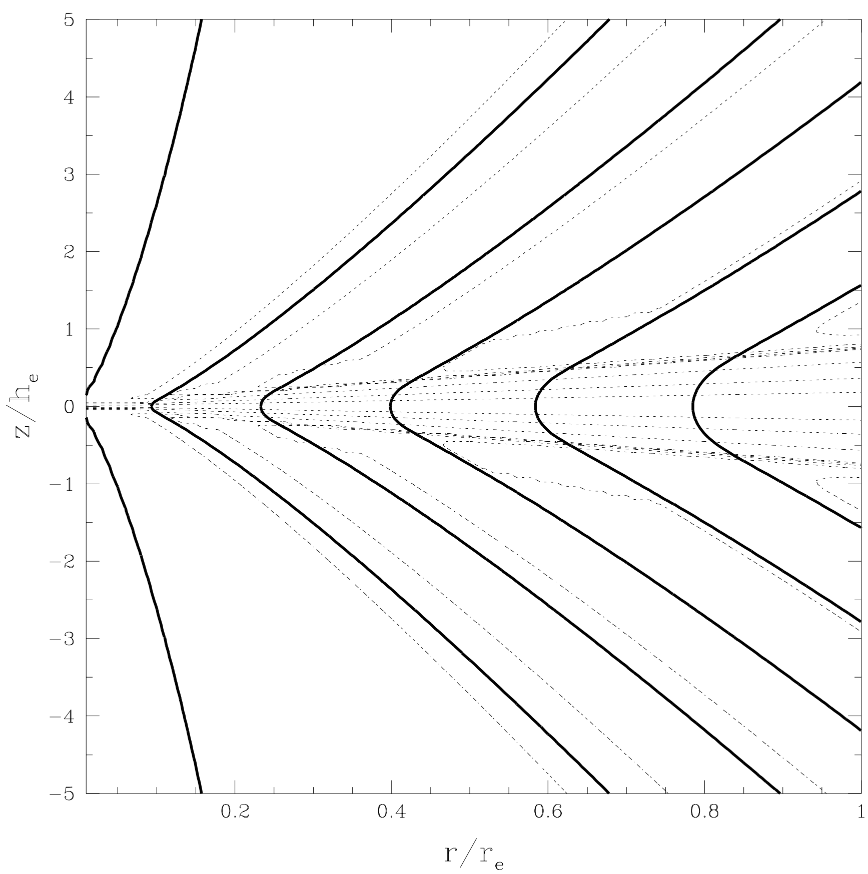
Salmeron & Wardle (2005)

Equilibrium Disk/Wind Models

Pure vertical transport model in the ambipolar-diffusion/Hall regime (Wardle & Königl 1993):

- Isothermal, geometrically thin, Keplerian rotation law, even field symmetry;
- Radially localized disk solution matched onto radially self-similar, ideal-MHD wind solution (Blandford & Payne 1982).

Results confirmed by global self-similar disk/wind solution (Li 1996).



Ferreira (1997)

Joule dissipation

- In the ambipolar diffusion regime, even though the magnetic field is "frozen" into the electrons (as in ideal MHD), there is electromagnetic energy dissipation (associated with the ion–neutral collisional drag) since $\mathbf{E}' \neq 0$.
- Hall diffusion does not lead to Joule dissipation since $\mathbf{j} \cdot \mathbf{E}' = 0$.

In a weakly ionized $\mathrm{n\text{-}i\text{-}e}$ gas, the equation of motion of the charge carriers (subscript $\mathrm{j = i \text{ or } e}$) can be approximated by

$$eZ_{\mathrm{j}}n_{\mathrm{j}}\left(\mathbf{E} + \frac{\mathbf{V}_{\mathrm{j}}}{c}\times\mathbf{B}\right) = -\mathbf{F}_{\mathrm{nj}} = \gamma_{\mathrm{j}}\rho_{\mathrm{j}}\rho_{\mathrm{n}}(\mathbf{V}_{\mathrm{j}} - \mathbf{V}_{\mathrm{n}})\ , \tag{30}$$

representing the near balance of the Lorentz and collisional drag forces. The collisional drag terms also appear (with opposite signs) in the equation of motion of the dominant neutral component ($\rho_n \simeq \rho$), and this is how the Lorentz force is transmitted to the neutrals. This result is obtained formally by eliminating $\mathbf{E}$ from the ion & electron equations of motion, which gives

$$\mathbf{F}_{\mathrm{in}} + \mathbf{F}_{\mathrm{en}} = \frac{\mathbf{j}\times\mathbf{B}}{c}\ . \tag{31}$$

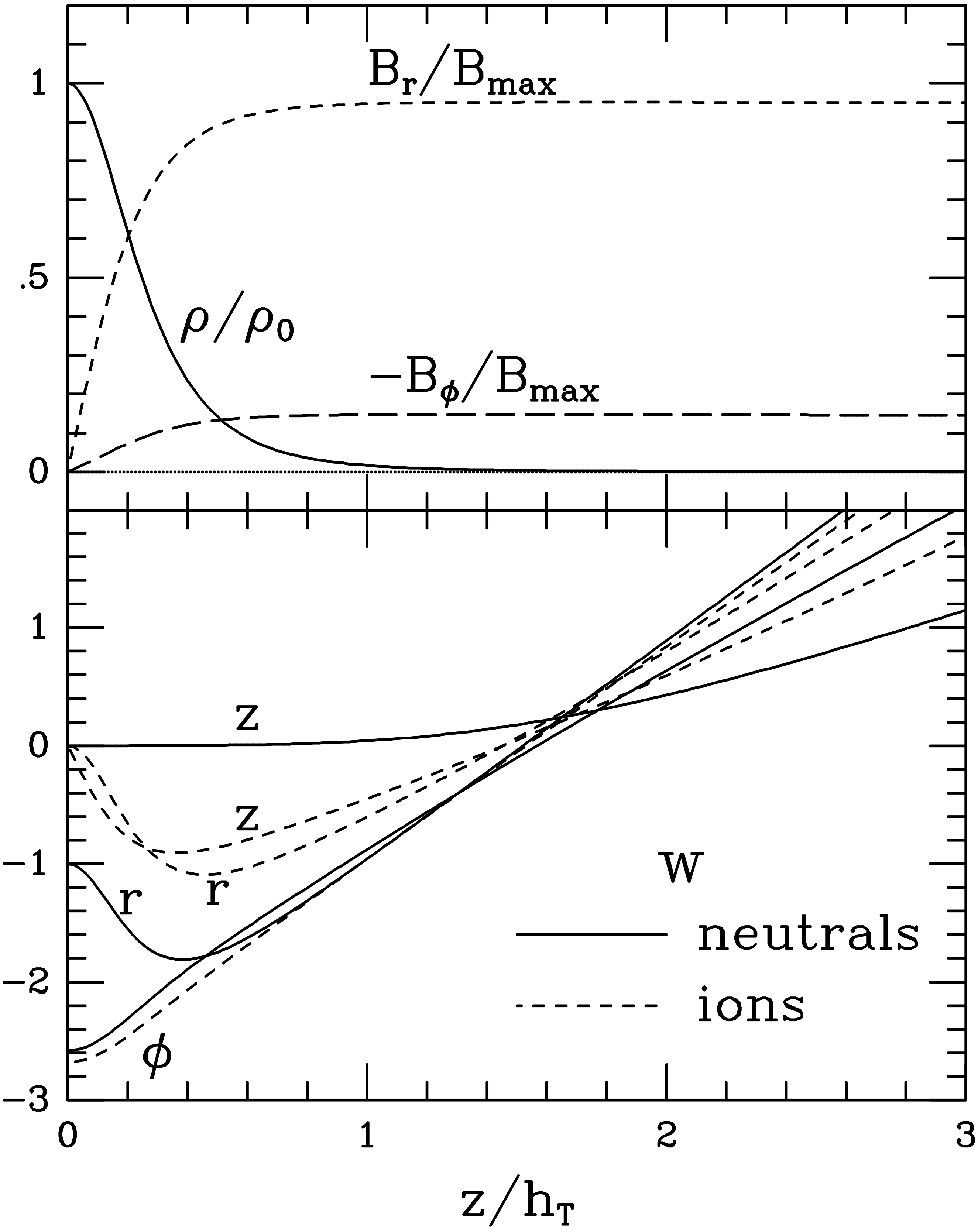
In the thin-disk approximation, the magnetic terms introduced in this way into the r.h.s. of the neutrals' equation of motion (eq. [2]) are

radial component: $(B_z/4\pi)dB_r/dz$
representing the magnetic tension force that typically acts in opposition to central gravity;

azimuthal component: $(B_z/4\pi)dB_\phi/dz$
representing the magnetic torque that, inside the disk, typically transfers angular momentum from the matter to the field;

vertical component: $-\frac{d}{dz}\,\frac{B_r^2+B_\phi^2}{8\pi}$
representing the magnetic squeezing of the disk (which acts in the same direction as the gravitational tidal force and in opposition to the thermal pressure-gradient force).

The "local" (narrow radial slice) approximation consists of neglecting essentially all radial derivatives except that of V_ϕ and setting $\rho V_z = \mathrm{const}$. In addition, ρ_i was taken to be constant for simplicity.



$$\mathbf{W} = \frac{\mathbf{V} - V_K \hat{\phi}}{C}\,, \quad h_T = \frac{C}{\Omega_K}$$

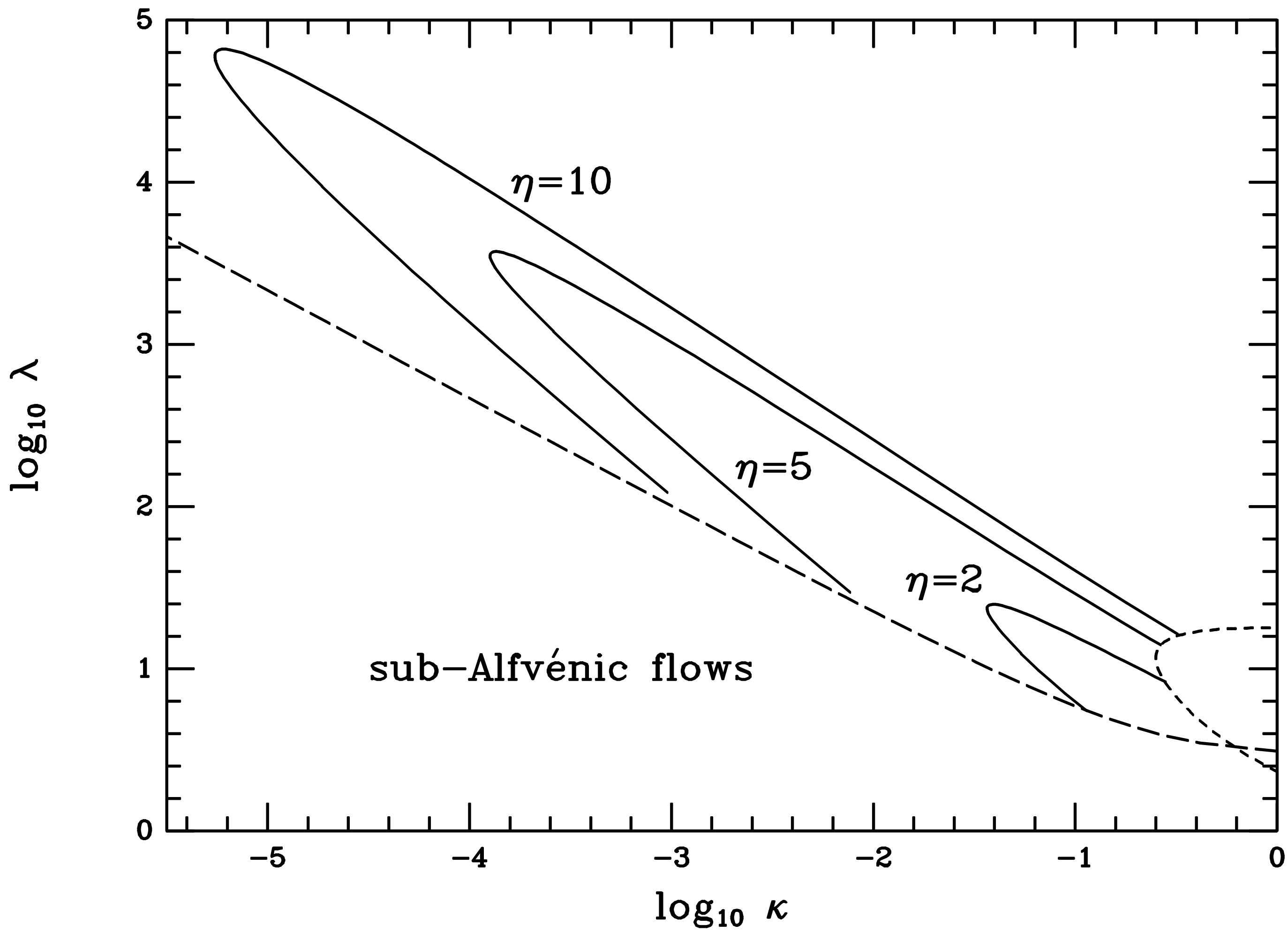
wind parameters:

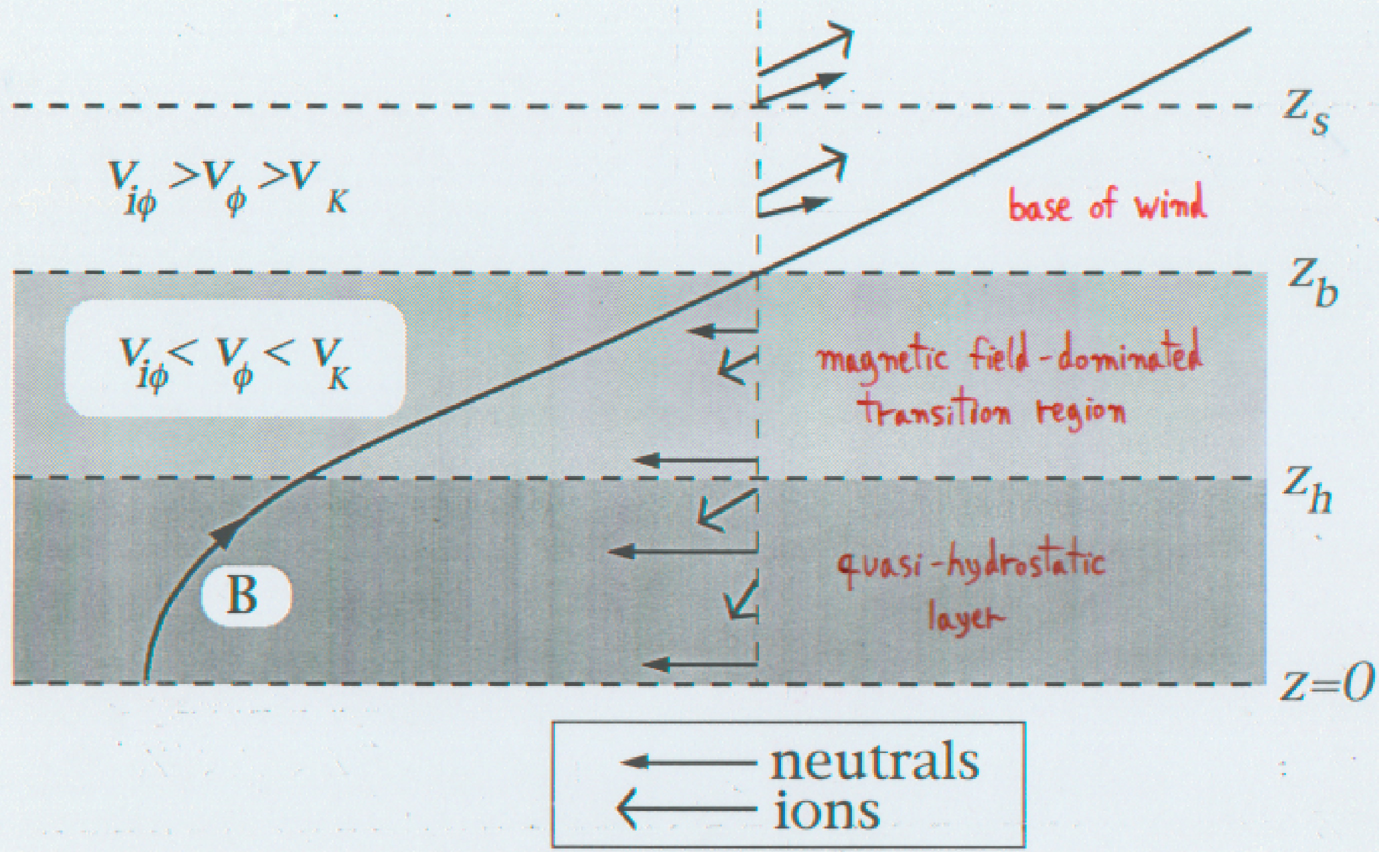
$\lambda \equiv l/(V_{\rm K} r_0)$: normalized specific angular momentum
$\kappa \equiv k(V_{\rm K}/B_0)$: normalized mass/magnetic flux ratio

disk parameters:

$\epsilon_B \equiv -V_{\rm i,0}/C$: normalized drift speed of $\mathbf{B}$ lines
$\eta \equiv \nu_{\rm ni}/\Omega_{\rm K}$: neutral–ion coupling strength $[\nu_{\rm ni} = (\rho_{\rm i}/\rho_{\rm n})\nu_{\rm in}]$
$a \equiv V_{\rm A,0}/C$: magnetic field strength

$\eta > 1$ and $a \lesssim 1$ required to drive a wind ($\eta > 1$ everywhere in strongly coupled disks)





$V_{i\phi} > V_\phi > V_K$
base of wind
$V_{i\phi} < V_\phi < V_K$
magnetic field-dominated transition region
B
quasi-hydrostatic layer
z_s
z_b
z_h
$z=0$
neutrals
ions

Further details

The disk solution can be used to obtain $\tan\theta_{\rm s}$ and the mass-loading parameter κ. By matching to the wind solution one deduces λ and hence $B_{\phi,{\rm s}}/B_z = \kappa(\lambda-1)$. One then adjusts $\epsilon \equiv -V_{r,0}/C$ until a self-consistent disk/wind solution is found.

Wardle & Königl (1993) treated $\epsilon_{\rm B}$ $(= -cE_\phi/CB_z)$ as a free parameter and showed that the results (except for the values of $B_{\phi,{\rm s}}$ and $V_{r,0}$) were insensitive to its value (as expected given that the only change in the equations involves changing the radial velocity of the reference frame in which the field lines are stationary). In a more complete treatment, the value of $\epsilon_{\rm B}$ will be determined by the global field distribution outside the disk (Ogilvie & Livio 2001; lecture 4).

By employing the hydrostatic approximation $(V_z \to 0)$, one obtains a set of algebraic relations that can be used to derive useful constraints on the disk solutions.

In the case of a strongly coupled, ambipolar diffusion-dominated disk with a constant η $(\propto \rho_{\rm i})$, one finds:

$$(2\eta)^{-1/2} \lesssim a \lesssim \sqrt{3} \lesssim \epsilon\eta \lesssim V_{\rm K}/2C \ . \qquad (32)$$

Inequality

1 $\Leftrightarrow$ disk remains sub-Keplerian throughout.
2 $\Leftrightarrow$ wind launching condition ($\tan\theta_{\rm s} > 1/\sqrt{3}$).
3 $\Leftrightarrow$ top of disk ($z_{\rm s}$) $>$ density scale height (h).
4 $\Leftrightarrow$ Joule heating $<$ gravitational energy release.

1st & 2nd inequalities $\Rightarrow$ the coupling parameter η has a lower bound ($= 1$, from a more detailed analysis).

2nd & 3rd inequalities $\Rightarrow$ magnetic squeezing dominates the vertical confinement of the disk ($h/h_{\rm T} \approx a/\epsilon\eta < 1$).

♣ Eq. (32) can also be used to:

- Demonstrate that the minimum wavelength of the most unstable linear MRI mode exceeds the disk scale height.

- Identify the region in the disk that is susceptible to MRI-induced turbulence. One can use this result to construct "hybrid" disk models in which both radial and vertical angular momentum transport mechanisms operate (possibly dominating in different vertical segments at the same radial location; see poster by R. Salmeron).

Stability considerations

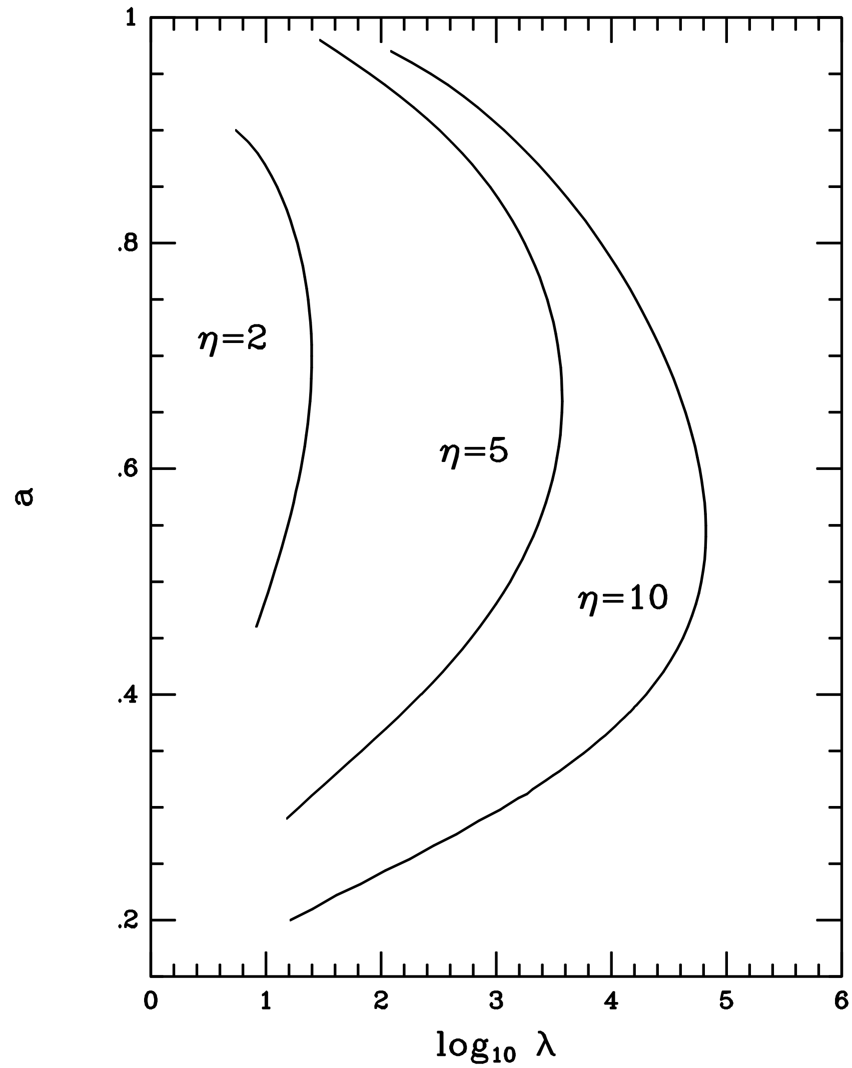
Strongly coupled, wind-driving disks naturally lie in a stability "window," in which the magnetic field is strong enough to largely suppress the MRI but not so strong as to be subject to the *radial interchange* instability (Königl & Wardle 1996).

However, based on approximate equilibrium models, Lubow et al. (1994b) and Cao & Spruit (2002) suggested that such disks might be inherently unstable.

They attributed this behavior to the sensitivity of the outflowing mass flux to changes in the field-line inclination at the disk surface:

$$|V_r| \ \text{increases} \quad \Rightarrow \quad \tan\theta_{\rm s} \ \text{increases}$$

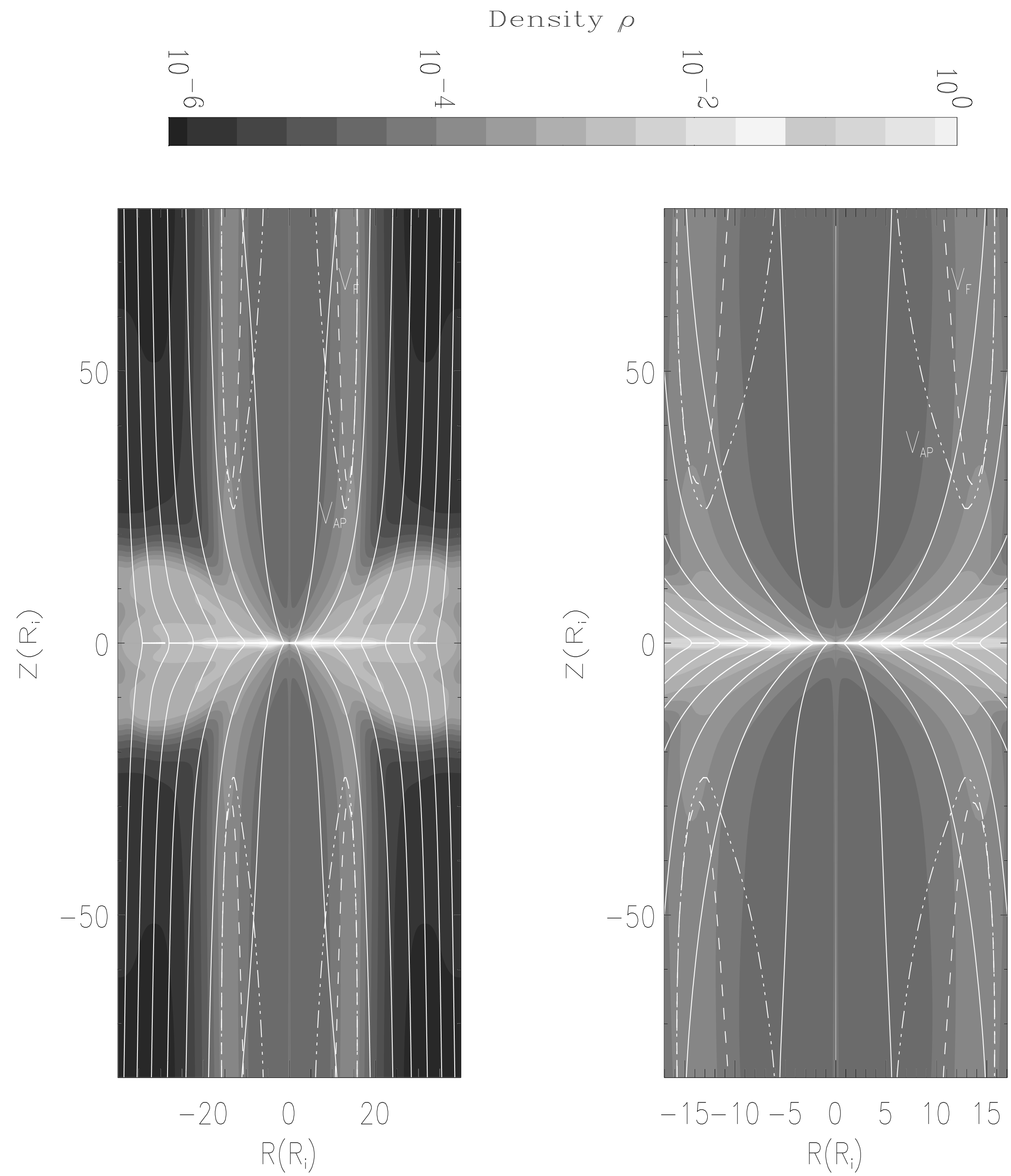
$$\Rightarrow \quad \dot{M}_{\rm w} \ \text{(and hence } \dot{M}_{\rm w}\, l) \ \text{increase}$$
$$\Rightarrow \quad |V_r| \ \text{increases even more....}$$



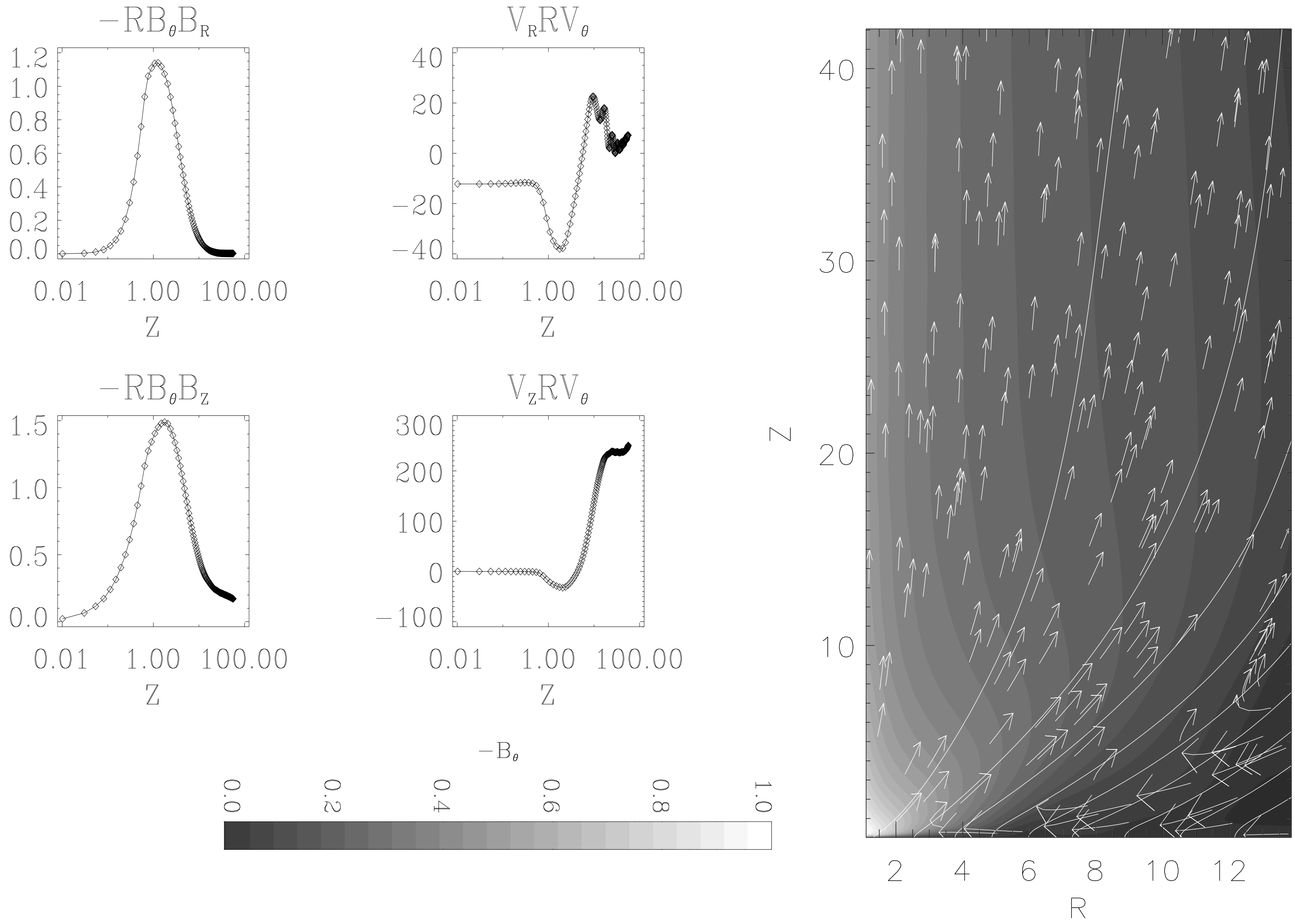
The issue was reexamined by Königl (2004), who used the diffusive disk models of Wardle & Königl (1993; see also Li 1996) and appealed to the fact that the stability properties generally change at a turning point of the equilibrium curve. He identified the branch along which the increase in the field-line tension force ($\propto B_{rs}$) with decreasing field-line inclination evidently leads to stability, and argued that real YSO systems likely correspond to that branch.

A full resolution of this question will be provided by global numerical simulations.

simulations of jet-driving diffusive disks



Casse & Keppens (2002)

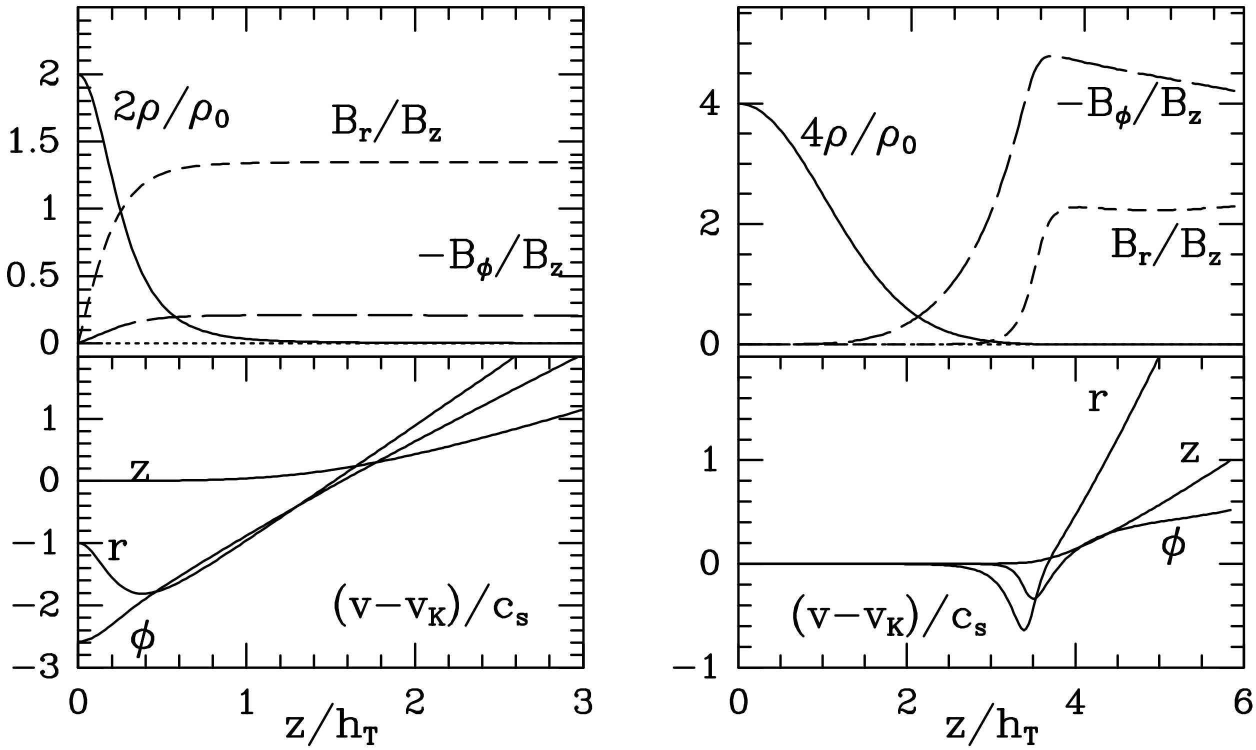


$-RB_\theta B_R$
$V_R R V_\theta$
$-RB_\theta B_Z$
$V_Z R V_\theta$
Z
R
$-B_\theta$

weakly coupled disks

$\eta < 1$ near the midplane and increases to $\gtrsim 1$ near the surface (Li 1996; Wardle 1997).

strong coupling (left) vs. weak coupling (right)



Wardle (1997)

In strongly coupled disks: $V_{\rm A,0} \lesssim C$, $| < V_r > | \sim C$, $B_{r,\rm s} > |B_{\phi,\rm s}|$ (with B_r increasing already at $z = 0$).

In weakly coupled disks: $V_{\rm A,0} \ll C$, $| < V_r > | \ll C$, $B_{r,\rm s} < |B_{\phi,\rm s}|$ [with B_r taking off only when η increases above 1 (in AD regime); $(dB_r/dB_\phi)_0 = -2\eta$].

Note that angular momentum is transported vertically even in weakly coupled regions where $B_r \approx 0$ but $|B_\phi| \gg B_r$, since the torque is $\propto B_z dB_\phi/dz$. This could have implications to the question of "dead zones" in the disk.