Hydromagnetic Disk Outflows

The equations of a steady-state, ideal-MHD flow are:

mass conservation (continuity equation)

$$\nabla\cdot(\rho\mathbf{V}) = 0 \ , \tag{1}$$

momentum conservation (force equation)

$$\rho\mathbf{V}\cdot\nabla\mathbf{V} = -\nabla P - \rho\nabla\Phi + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} \tag{2}$$

(with $P = \rho k_{\rm B}T/\mu m_{\rm H}$),

induction equation (magnetic field evolution)

$$\nabla\times(\mathbf{V}\times\mathbf{B}) = 0 \ , \tag{3}$$

solenoidal condition on $\mathbf{B}$ (no magnetic monopole)

$$\nabla\cdot\mathbf{B} = 0 \ . \tag{4}$$

In general, one also needs to specify an entropy conservation equation (balance of heating and cooling). For simplicity we will, however, specialize initially to isothermal flows (constant sound speed C).

Some general properties of axisymmetric outflows (use cylindrical coordinates $\{r,\ \phi\ z\}$; label poloidal components by a subscript p):

♣ One can write $\mathbf{B} = \mathbf{B}_\mathrm{p} + \mathbf{B}_\phi$, with $\mathbf{B}_\mathrm{p} = (\nabla A \times \hat{\phi}/r$, where the poloidal magnetic flux function $A(r\,, z) = \Psi/2\pi$, and $\Psi = \iint \mathbf{B}_\mathrm{p} \cdot d\mathbf{S}$ is the poloidal flux. The function A can be used to label the field lines.

♣ The solution of the induction equation (3),

$$\mathbf{V} \times \mathbf{B} = \nabla \chi, \tag{5}$$

shows that the electric field $\mathbf{E} = -(\mathbf{V} \times \mathbf{B})/c$ is derivable from an electrostatic potential (χ). Since $\partial\chi/\partial\phi = 0$, this implies $\mathbf{V}_\mathrm{p} \parallel \mathbf{B}_\mathrm{p}$. Equivalently,

$$\rho \mathbf{V}_\mathrm{p} = k \mathbf{B}_\mathrm{p}\ , \tag{6}$$

where k is a flux-surface constant ($\mathbf{B} \cdot \nabla k = 0$, using $\nabla \cdot (\rho \mathbf{V}) = 0$ and $\nabla \cdot \mathbf{B} = 0$). Physically, k is the wind mass load function,

$$k = \frac{\rho V_\mathrm{p}}{B_\mathrm{p}} = \frac{d\dot{M}_\mathrm{w}}{d\Psi}\ , \tag{7}$$

whose value is determined by the physical conditions near the top of the disk (more precisely, at the sonic, or slow-magnetosonic, critical surface).

♣ From $\mathbf{B}_{\rm p}\cdot$ (eq. [5]) and eq. (6), it follows that the field-line angular velocity $\omega = \Omega - (kB_\phi/\rho r)$ is also a flux-surface constant. By writing this relation as

$$B_\phi = \frac{\rho r}{k}(\Omega - \omega)\ , \tag{8}$$

it is seen that ω can be identified with the angular velocity of the matter at the point where $B_\phi = 0$ [the disk midplane (subscript 0), assuming reflection symmetry].

♣ Using the ϕ component of the momentum-conservation equation (2) and applying eq. (6) and the field-line constancy of k, one finds that

$$l = rV_\phi - \frac{rB_\phi}{4\pi k} \tag{9}$$

is a flux-surface constant as well, representing the conserved total specific angular momentum (matter + electromagnetic contributions) along a poloidal field line (or streamline).

♣ In a centrifugally driven wind (CDW), the magnetic component of the total specific angular momentum dominates over the matter component near the disk surface,

$$\frac{1}{\rho V_{\rm p}}\frac{r B_{\rm p} B_\phi}{4\pi} \gg r V_\phi \ ,$$

whereas at large distances this inequality is reversed. The transfer of angular momentum from the field to the matter is the essence of the "centrifugal" acceleration process. This transfer also embodies the capacity of such a wind to act as an efficient angular momentum transport mechanism. In fact, $r B_{\rm p,s} B_{\phi,\rm s}/4\pi$ represents the magnetic torque per unit area that is exerted on the surface of the disk (subscript $\rm s$).

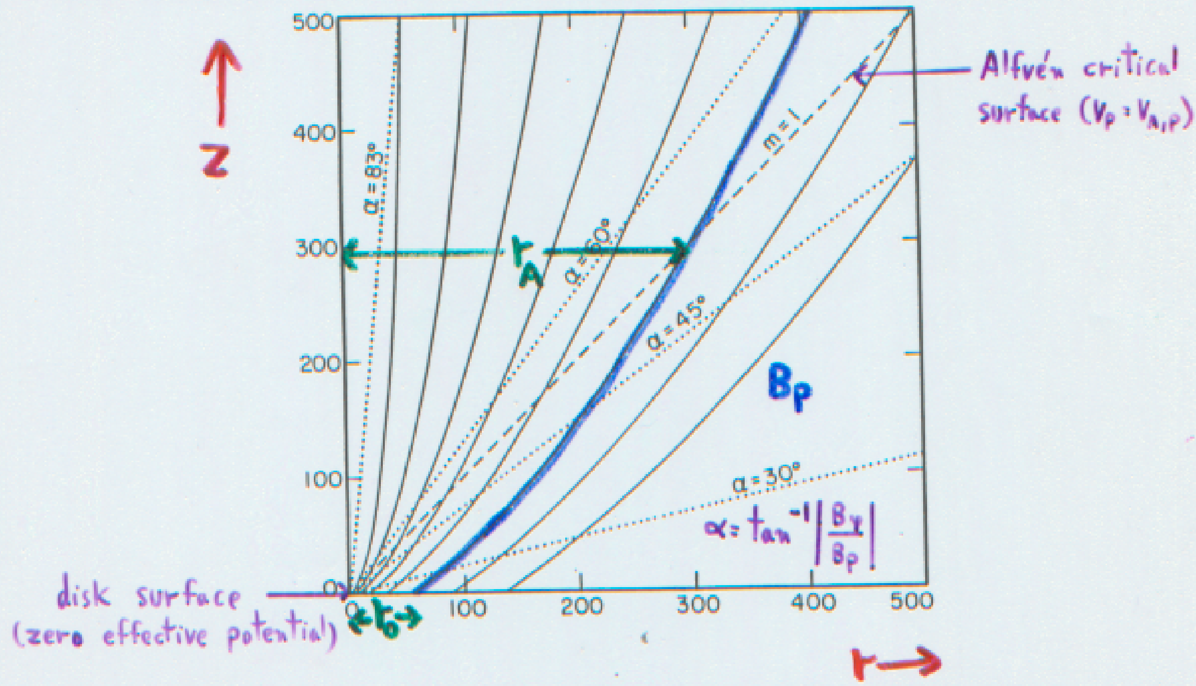
The value of $B_{\phi,\rm s}$ is determined by the conditions outside the disk, essentially by the inertia of the matter that absorbs the transported angular momentum and exerts a back torque on the disk. In the case of a CDW, $B_{\phi,\rm s}$ is effectively fixed by the regularity condition at the Alfvén critical surface. This condition yields the value of the "Alfvén lever arm" $r_{\rm A}$, which satisfies

$$l = \omega r_{\rm A}^2 \ . \qquad (10)$$

The rate of angular momentum transport by the wind is thus $\sim \dot{M}_{\rm w} \Omega_0 r_{\rm A}^2$ (where we set $\omega = \Omega_0$), whereas the rate at which angular momentum is advected inward by the accretion disk is $\sim \dot{M}_{\rm in} \Omega_0 r_0^2$.

Centrifugally Driven Outflows

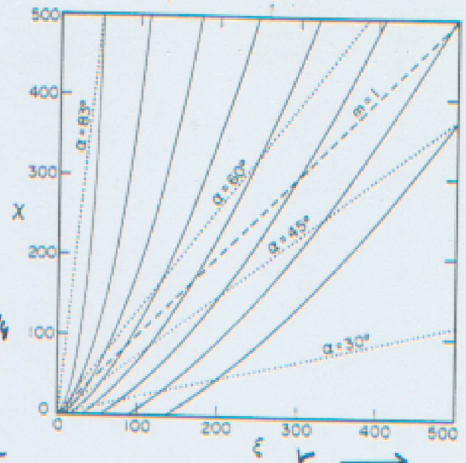
"bead on wire" analogy: in a Keplerian disk, matter is flung out if the wire makes an angle $\leq 60°$ to the surface



- Blandford & Payne (1982) wind solution (cold gas, radial self-similarity)
  - flow is magnetically dominated above the disk surface
  - flow becomes super-Alfvénic and magnetically collimated farther out
  - $(\dot{M}_{wind}/\dot{M}_{in}) \approx (r_0/r_A)^2$ can be $\ll 1$, $v_{p\infty} \approx (r_A/r_0)\, v_K(r_0)$

disk field-driven wind models (Keplerian disks)

Blandford & Payne '82



(spherical) radial self-similarity

$B/\sqrt{\rho} \propto V \propto R^{-1/2}$

$\dot{M}_w(r_0) = \text{const} \propto \rho_0 V_0 r_0^2 \Rightarrow \rho_0 \propto r_0^{-3/2}, B_0 \propto r_0^{-5/4}$

(dimensionless) specific ang. mom. λ, mass-to-flux ratio κ, and $\tan^{-1}(B_r/B_z)_0$ const. throughout the flow

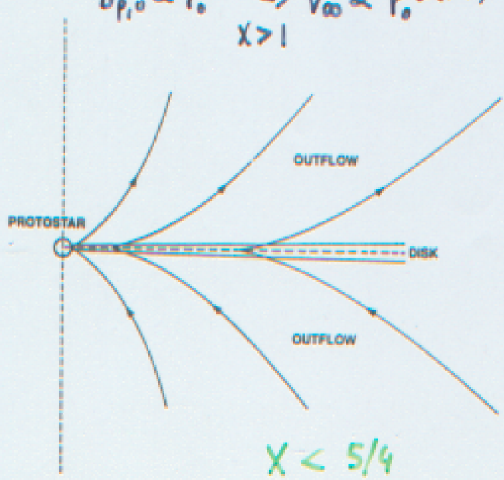
Generalizations:

(i) self-similar flows with $\rho \propto R^{-\alpha}$: $B \propto \rho^{1/2} V \propto R^{-\frac{\alpha+1}{2}}$, $\dot{M}_w(r_0) \propto r_0^{(\frac{3}{2}-\alpha)}$ Contopoulos & Lovelace '94

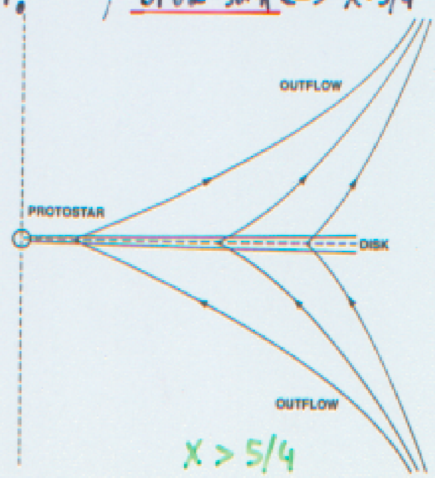
favored "minimum energy" model: $I = 2\pi c r B_\varphi = \text{const.}$ across jet $\Rightarrow \alpha = 1$

(ii) non-self-similar - $\frac{\Omega^2 r_A^2}{V_A^2}(\nabla r_A)_A^2 = \text{const.}$ throughout the flow (Pelletier & Pudritz '92)

$B_{p,0} \propto r_0^{-x} \Rightarrow V_\infty \propto r_0^{2(1-x)}$, $\dot{M}_w \propto r_0^{(4x-5)}$; BP82 sol'n $\Leftrightarrow x = 5/4$
$x > 1$



$X < 5/4$



$X > 5/4$

$I = \text{const.} \Rightarrow x = 3/2$ ($V_\infty \propto r_0^{-1}$, $\dot{M}_w \propto r_0$)

(n.b., $V_\infty \propto r_0^{-1/2}$, $\dot{M}_w \propto r_0^{1/2}$ for $I=$const. sol'n of CL94)

Hence, wind transport can enable accretion at a rate

$$\dot{M}_{\rm in} \simeq (r_{\rm A}/r_0)^2\, \dot{M}_{\rm w}\ . \tag{11}$$

CDW solutions can have $r_{\rm A}/r_0 \sim 3$ for reasonable parameters, indicating that it is in principle possible for such outflows to transport the bulk of the excess angular momentum in protostellar disks (yielding $\dot{M}_{\rm w}/\dot{M}_{\rm in} \simeq 0.1$).

As we will discuss shortly, the wind-launching condition imposes a minimum value on the magnitude of B_r/B_z at the disk surface. If this condition is not satisfied, a steady-state wind will not form. However, the disk can still lose angular momentum vertically by launching torsional Alfvén waves that propagate along the large-scale magnetic field and set up the ambient medium into rotation. In this magnetic braking process, the inertia of the external gas responsible for the back torque on the disk enters through the value of the Alfvén speed $V_{\rm A} = B/\sqrt{4\pi\rho}$ in the ambient medium, and the azimuthal surface magnetic field is given by

$$B_{\phi,{\rm s}} = -\frac{\Psi}{\pi r^2}\frac{V_\phi}{V_{\rm A,ext}} \tag{12}$$

(e.g., Basu & Mouschovias 1994).

♣ By taking the dot product of the force equation (2) with $\mathbf{B}_\mathrm{p}$, one finds that the specific energy

$$E = \frac{1}{2}V^2 - \frac{B_\phi B_\mathrm{p}}{4\pi\rho V_\mathrm{p}} + h + \Phi \ , \qquad (13)$$

where h $[= C^2 \ln(\rho/\rho_\mathrm{A})$, with $\rho_\mathrm{A} = 4\pi k^2]$ is the specific enthalpy and Φ is the gravitational potential, is constant along flux surfaces. This is the generalized Bernoulli equation, in which the magnetic term arises from the poloidal component of the Poynting flux $c\,\mathbf{E}\times\mathbf{B}/4\pi$. Using eqs. (9) and (10), this term can be rewritten as $\Omega_0(\Omega_0 r_\mathrm{A}^2 - \Omega r^2)$, and by approximating $E \approx V_\infty^2/2$ as $r \to \infty$ and assuming that $(r_\mathrm{A}/r_0)^2 \gg 1$, one can estimate the value of the asymptotic poloidal speed as

$$V_\infty \simeq 2^{1/2}\Omega_0 r_\mathrm{A} \ , \qquad (14)$$

or $V_\infty/V_\mathrm{K} \approx 2^{1/2} r_\mathrm{A}/r_0$. This shows that such outflows can attain speeds that exceed (by a factor of up to a few) the Keplerian speed $V_\mathrm{K} = r_0\Omega_0$ at the base of the flow and thus successfully account for the measured velocities of YSO jets.

By combining E, ω, and l, one can form the field-line constant

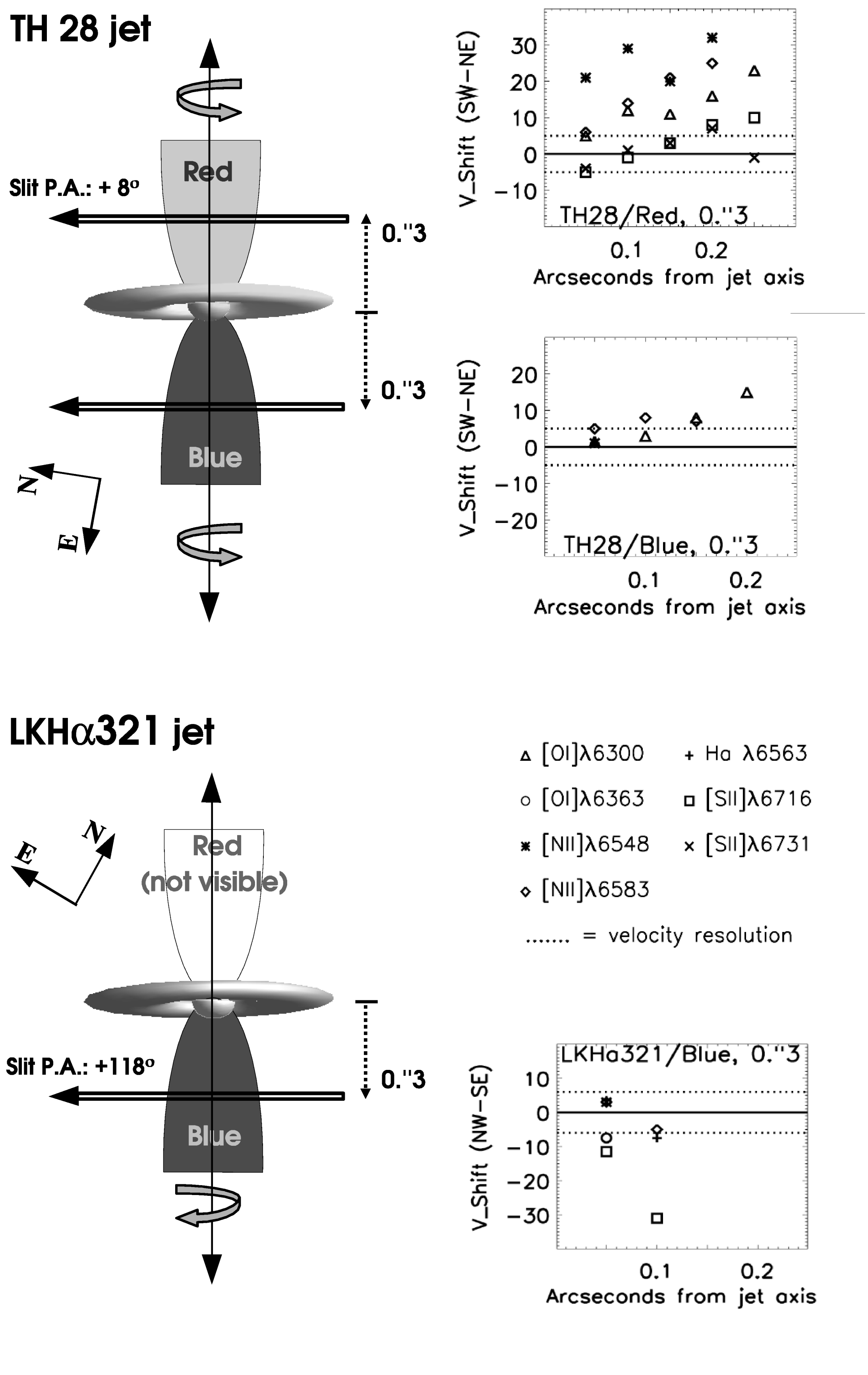
$$H \equiv E - \omega l = \frac{1}{2}V^2 - r^2\omega\Omega + \Phi \ , \qquad (15)$$

where we omitted h from the explicit expression. Evaluating at r_0 gives $H = -\frac{3}{2}V_{\rm K}^2 = -\frac{3}{2}(GM_*\Omega_0)^{2/3}$, and when evaluating also at large distances (r_∞) one gets

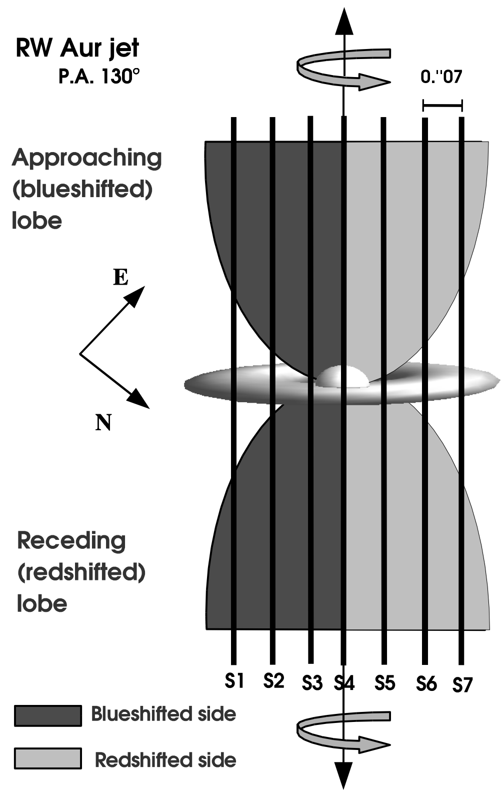
$$r_\infty V_{\phi,\infty}\Omega_0 - \frac{3}{2}(GM_*)^{2/3}\Omega_0^{2/3} - \frac{1}{2}(V_{{\rm p},\infty}^2 + V_{\phi,\infty}^2) \approx 0 \ . \qquad (16)$$

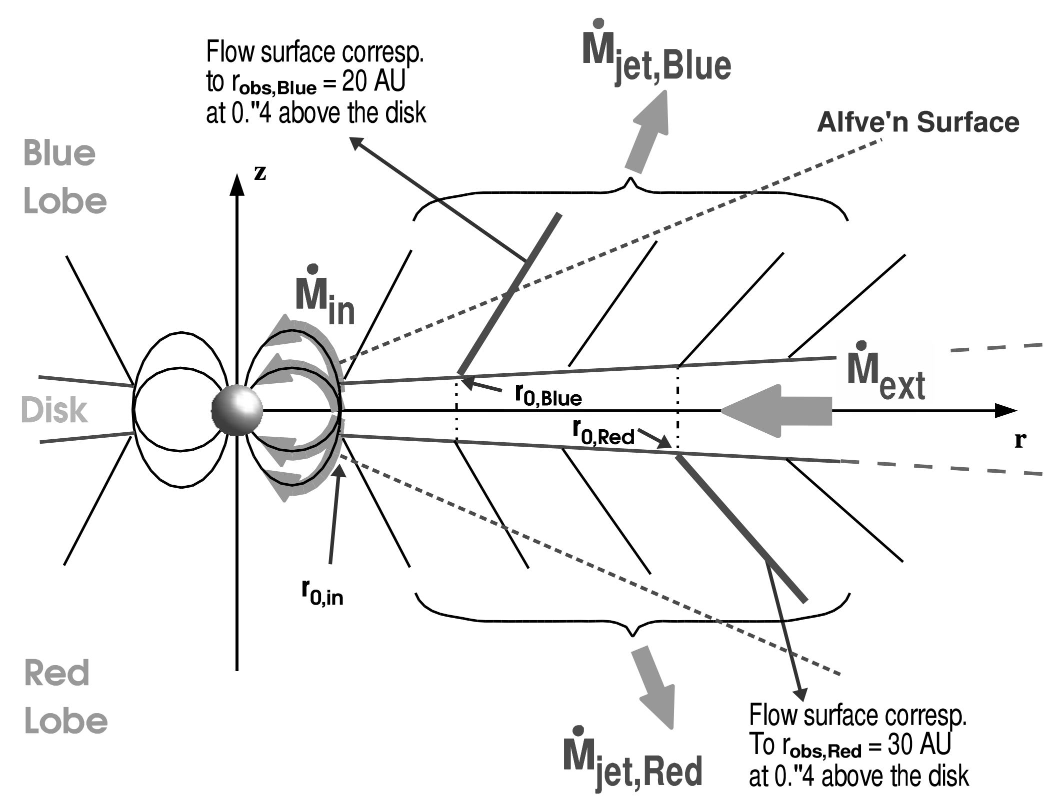
If one can measure the poloidal and azimuthal speeds at a location r_∞ in a YSO jet and estimate the YSO mass M_*, one can use eq. (15) (regarded as a cubic in $\Omega_0^{1/3}$) to infer the jet launching radius $r_0 = (GM_*/\Omega^2)^{1/3}$ (Anderson et al. 2003).

Recent high-resolution spectroscopic observations (using, e.g., STIS on board *HST*) have reavealed transverse velocity gradients that, when interpreted as jet rotation, imply (using eq. [16]) launching radii $r_0 \gtrsim 1$ AU in several YSOs, indicating that the jets originate in a disk.



Coffey et al. (2004)





Woitas et al. (2005)

It is further inferred that the jets carry a large fraction of the disk angular momentum in the launching region. Eqs. (8) and (10) can be used to also estimate $|B_{\phi,\infty}/B_{\rm p,\infty}|$ and $r_{\rm A}/r_0$, respectively, and verify that they are significantly greater than 1, as required for self-consistency (Woitas et al. 2005).

The interpretation of the transverse velocity gradients is, however, still controversial (e.g., Soker 2005; Cerqueira et al. 2006). Furthermore, in at least one case (RW Aur; Cabrit et al. 2006) it was found that the sense of rotation of the underlying disk is opposite to that inferred in the jets. It thus appears that, while the evidence presented so far in favor of significant angular-momentum transport by disk-driven jets is tantalizing, this issue is not yet fully settled.

Critical points of the outflow

If one regards the magnetic field configuration as given and considers the poloidal flow along $\mathbf{B}_{\rm p}$, one can derive the location of the critical points and the values of the critical speeds ($= V_{\rm p}$ at the critical points) by regarding the Bernoulli integral as a function of a spatial coordinate along the field line and the density [e.g., $E = E(s, \rho)$, where s is the arc-length of the streamline] and deriving the extrema of E by setting $\partial E/\partial \rho = 0$ (which yields the critical speeds) and $\partial E/\partial s = 0$ (which, together with the other relation yields the critical point locations).

- The critical points occur in stationary flows at the locations where the fluid velocity equals the speed of a backward-propagating disturbance, and as such they represent relics of the initial conditions in the time-dependent equations (Blandford & Payne 1982).

- The critical points obtained from the generalized Bernoulli equation correspond to $V_{\rm p}$ becoming equal to either the slow or the fast magnetosonic wave speeds (e.g., Sakurai 1985).

- In the full wind problem, the shape of the field lines must be determined as part of the solution by solving also the transfield (or Grad-Shafranov) equation, which involves the force balance *across* the flux surfaces. This equation introduces a critical point corresponding to $V_{\rm p} = V_{\rm A,p}$ (Okamoto 1975). However, the critical points of the combined Bernoulli and transfield equations are in general different from those obtained when these two equations are solved simultaneously. The modified slow, Alfvén, and fast points occur on surfaces that correspond to the so-called limiting characteristics (or *separatrices*; e.g., Tsinganos et al. 1996).

Example: the slow-magnetosonic critical point

In the magnetically dominated region above the disk surface the shape of the field lines changes on the scale of the spherical radius R. Anticipating that the height $z_{\rm sms}$ of the s.m.s. point is $\ll R$, we approximate the shape of the field line just above the point $\{r_{\rm s},\, z_{\rm s}\}$ on the disk surface by a straight line

$$r = r_{\rm s} + s \sin\theta_{\rm s} \,, \quad z = z_{\rm s} + s\cos\theta_{\rm s} \,, \tag{17}$$

where the angle $\theta_{\rm s}$ gives the field-line inclination at the disk surface ($\sin\theta_{\rm s} = B_{r,{\rm s}}/B_{{\rm p},{\rm s}}$; $\tan\theta_{\rm s} = B_{r,{\rm s}}/B_z$). The Bernoulli equation (in the form $H = E - \omega l$) then becomes, after substituting $\Phi = -GM_*/(r^2+z^2)^{1/2}$,

$$\begin{aligned} H(s,\rho) \quad = \quad & \frac{k^2 B_{\rm p}^2}{2\rho^2} + \frac{\omega^2}{2}(r_{\rm s} + s\sin\theta_{\rm s})^2 \left(\frac{\Omega}{\omega}\right)\left(\frac{\Omega}{\omega} - 2\right) \\ & - \frac{GM_*}{[(r_{\rm s} + s\sin\theta_{\rm s})^2 + (z_{\rm s} + s\cos\theta_{\rm s})^2]^{1/2}} \\ & + C^2 \ln\left(\frac{\rho}{\rho_{\rm A}}\right) \,, \end{aligned} \tag{18}$$

where $\Omega(s,\rho)$ is given by combining

$$\Omega = \frac{1 - \frac{r_{\rm A}^2 \rho_{\rm A}}{r^2 \rho}}{1 - \frac{\rho_{\rm A}}{\rho}}\, \omega \tag{19}$$

(obtained from eqs. [8]-[10]) and eq. (17).

We fix $\omega \approx \Omega(r_\mathrm{s}) = V_\mathrm{K}(r_\mathrm{s})/r_\mathrm{s}$ (see below). Setting $\partial H/\partial\rho = 0$ yields the s.m.s. speed:

$$V_\mathrm{sms} = \frac{B_\mathrm{p,s}}{B_\mathrm{s}}\, C \ , \tag{20}$$

where $B_\mathrm{s} = (B_\mathrm{p,s}^2 + B_{\phi,\mathrm{s}}^2)^{1/2}$. Setting also $\partial H/\partial r = 0$ and approximating $r_\mathrm{sms} \approx r_\mathrm{s}$ then gives the height of this point:

$$\frac{z_\mathrm{sms}}{z_\mathrm{s}} = \frac{3\tan^2\theta_\mathrm{s}}{3\tan^2\theta_\mathrm{s} - 1} \ . \tag{21}$$

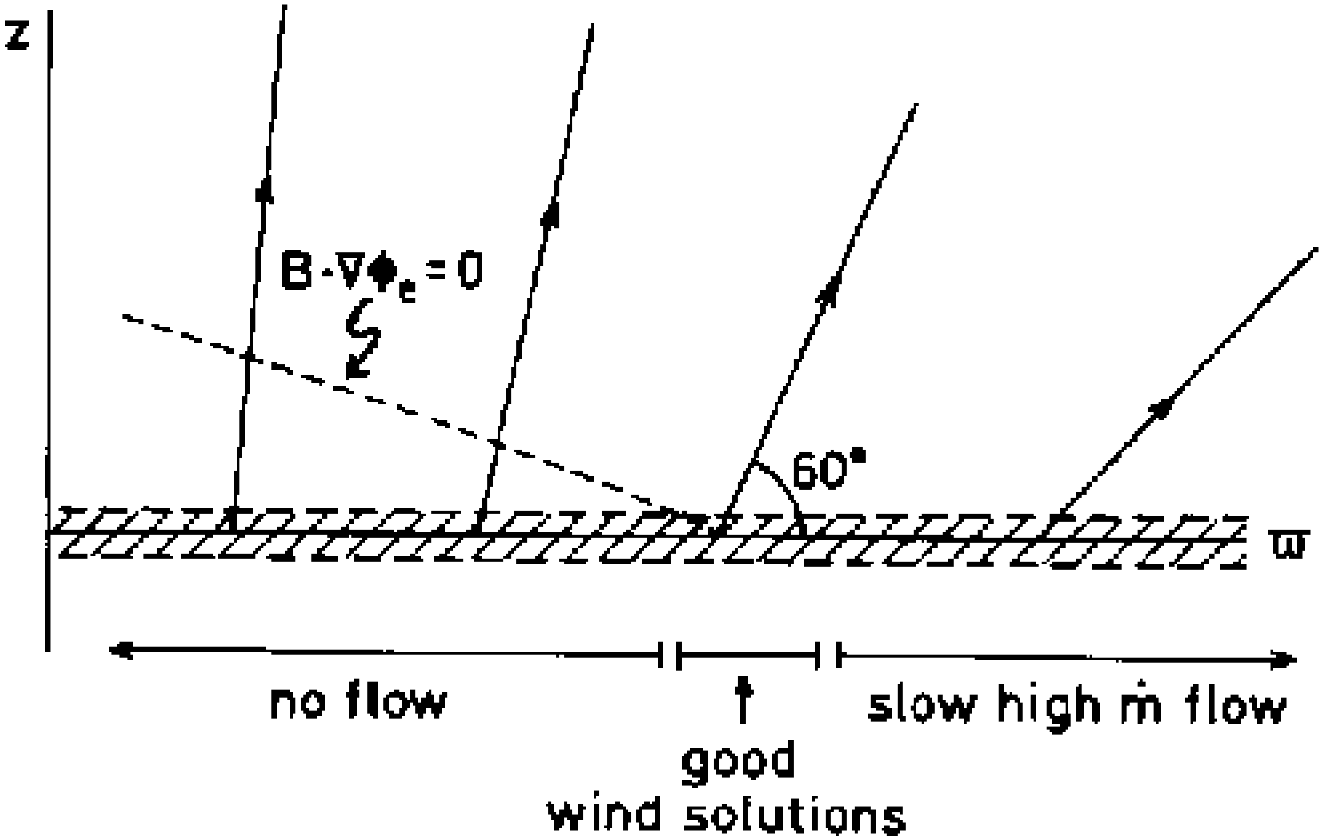
Eq. (21) gives a meaningful result only if $\tan\theta_\mathrm{s} > 1/\sqrt{3}$, i.e., if the field line is inclined at an angle $> 30°$ to the z axis. This is the launching condition of a centrifugally driven wind from a Keplerian accretion disk that Blandford & Payne (1982) showed could be derived by using the mechanical analogy of a "bead on a rigid wire" and considering the stability of equilibria in the effective (gravitational + centrifugal) potential. The relationship of this to the above derivation becomes clear when one notes that, in the limit $\omega \approx \Omega$, the 2nd term on the r.h.s. of eq. (18) becomes equal to the centrifugal potential $-\omega^2 r^2/2$, so the 2nd and 3rd terms (which together dominate the r.h.s.) are just Φ_eff.

The condition $\partial H/\partial \rho = 0$ is, in fact, equivalent to $\mathbf{B}_{\rm p}\cdot\nabla\Phi_{\rm eff} = 0$, which can be alternatively used to obtain the s.m.s. point (e.g., Ogilvie 1997).

The physical interpretation is that, above the disk, $\Phi_{\rm eff}$ continues to increases along the field line until it reaches a maximum at $z_{\rm sms}$. The energy to overcome the potential difference between $z_{\rm s}$ and $z_{\rm sms}$ derives from the gas enthalpy at the disk surface and the location of the sms point determines the gas density there and, correspondingly, the mass flux injected into the wind.

When $\theta_{\rm s}$ approaches (and decreases below) 30° the field-line curvature needs to be taken into account in the analysis. The height of the sonic point rapidly increases to $\sim R$, with the potential difference growing to $\sim GM_*/R$: the launcing problem becomes essentially that of a thermally driven spherical wind (e.g., Levinson 2006).

♣ It has been argued (Spruit 1996) that the field-line inclination varies along the disk surface and only in a narrow radial range the conditions are favorable for launching a wind that is not "overloaded" (and thus potentially super-Alfvénic from the start; Cao & Spruit 1994).



This is an intriguing suggestion, especially in view of the fact that so far there is no observational evidence for an extended wind-driving region in protostellar disks. However, in principle it may be possible to launch outflows over a large radial range. In particular:

* If the magnetic flux is advected inward by the accretion flow, the field-line inclination through much of the disk could be $> 30^\circ$ (lecture 4).

* As we will discuss shortly, the field lines in wind-launcing disks are likely to compress the disk and reduce the density scale height. In this case, even though $z_{\rm sms}$ decreases as $\theta_{\rm s}$ is increased, $z_{\rm sms}/z_{\rm s}$ may "bottom out" (e.g., Ogilvie & Livio 1998), so the wind need not necessarily "overload."

Centrifugally driven disk winds possess several generic properties that could have potentially significant observational ramifications.

* The strong acceleration above the disk surface leads to a strong vertical density stratification of the outflows (Safier 1993a,b).

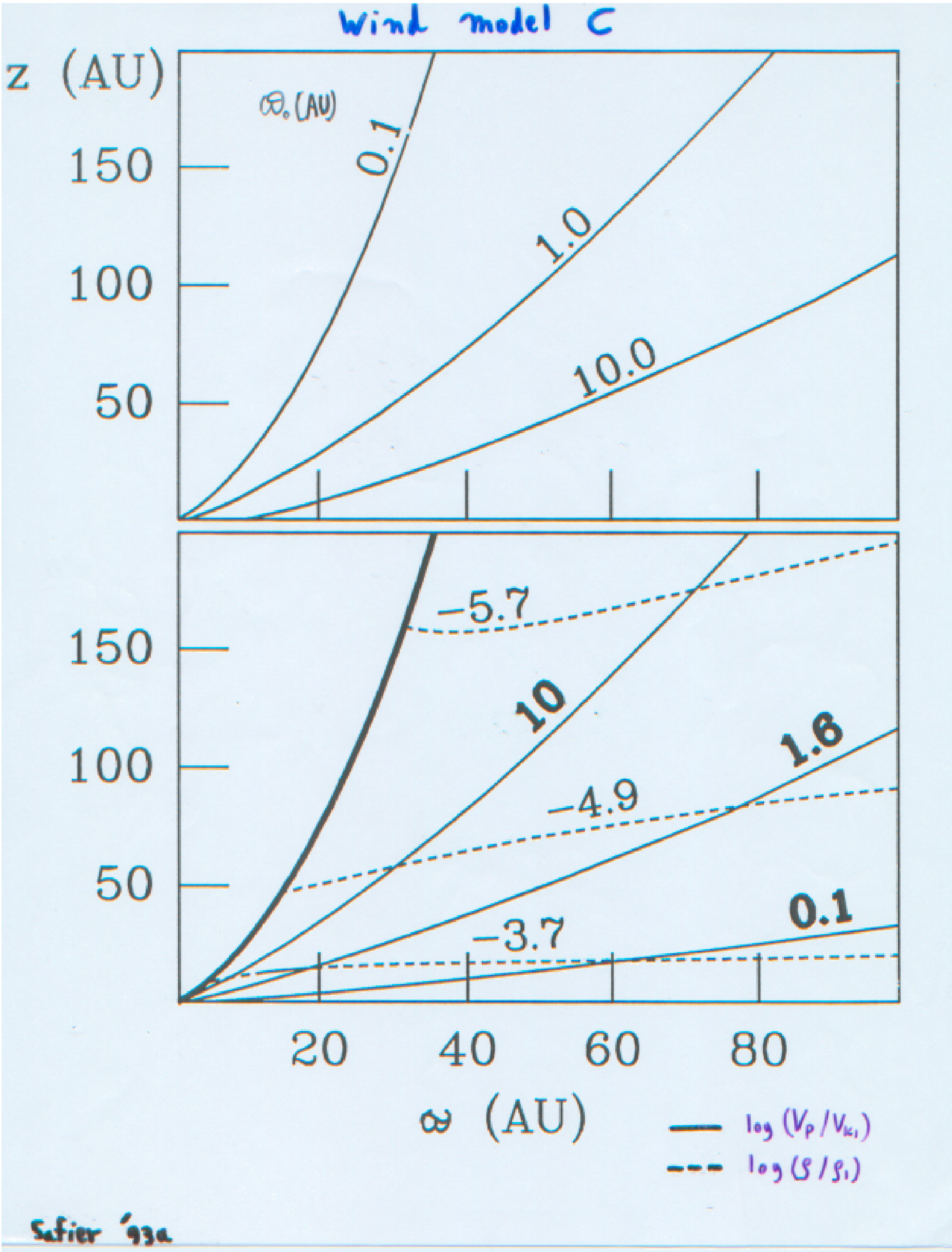
* The strong momentum flux in the flows implies that, if winds are launched from beyond the *dust sublimation radius*, they will readily uplift dust from the disk (up to at least μm-size grains at $1\ \mathrm{AU}$; see poster by S. Teitler). There is observational support for this from mid-IR observations of the Orion Nebula (Smith et al. 2005).

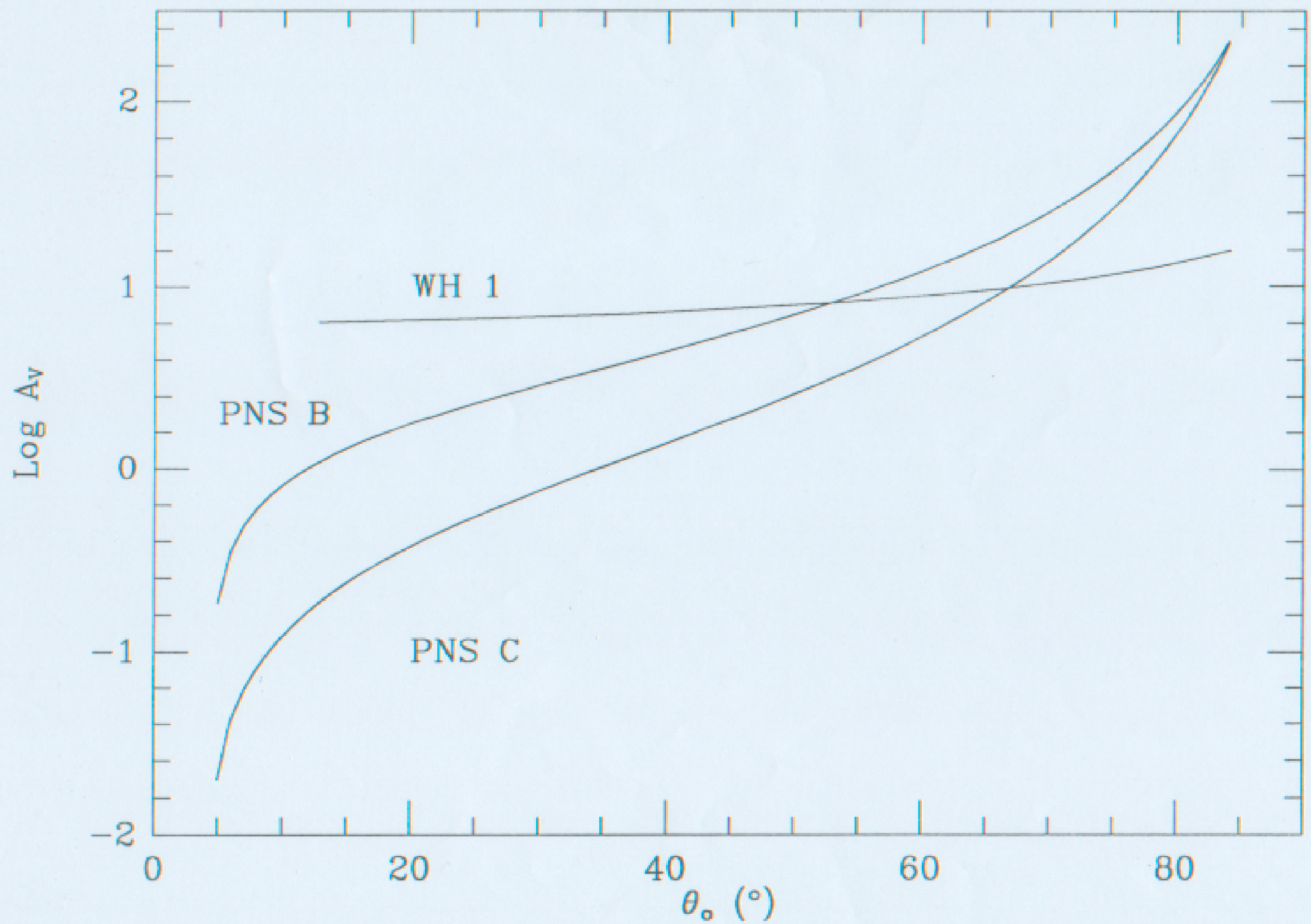
This could lead to:

- Angle-dependent obscuration and shielding of the central continuum – distinct from an infalling cloud envelope. (In an application to AGNs, this effect was invoked to account for the Seyfert 1/Seyfert 2 dichotomy; Königl & Kartje 1994.)

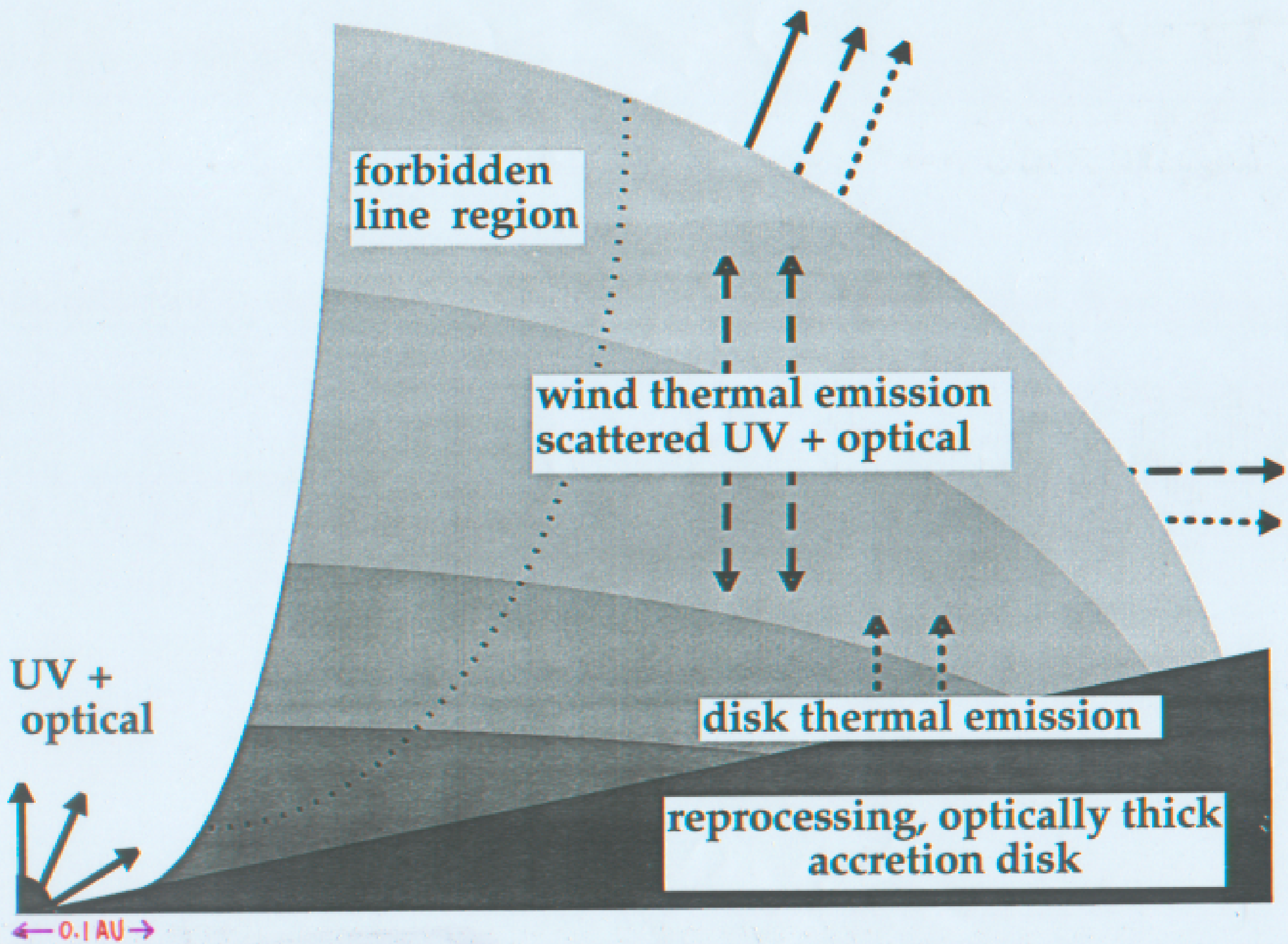
- Unique polarization and scattering characteristics. (For the AGN application, see Kartje 1995.)

- Efficient reprocessing of the central continuum to the IR.

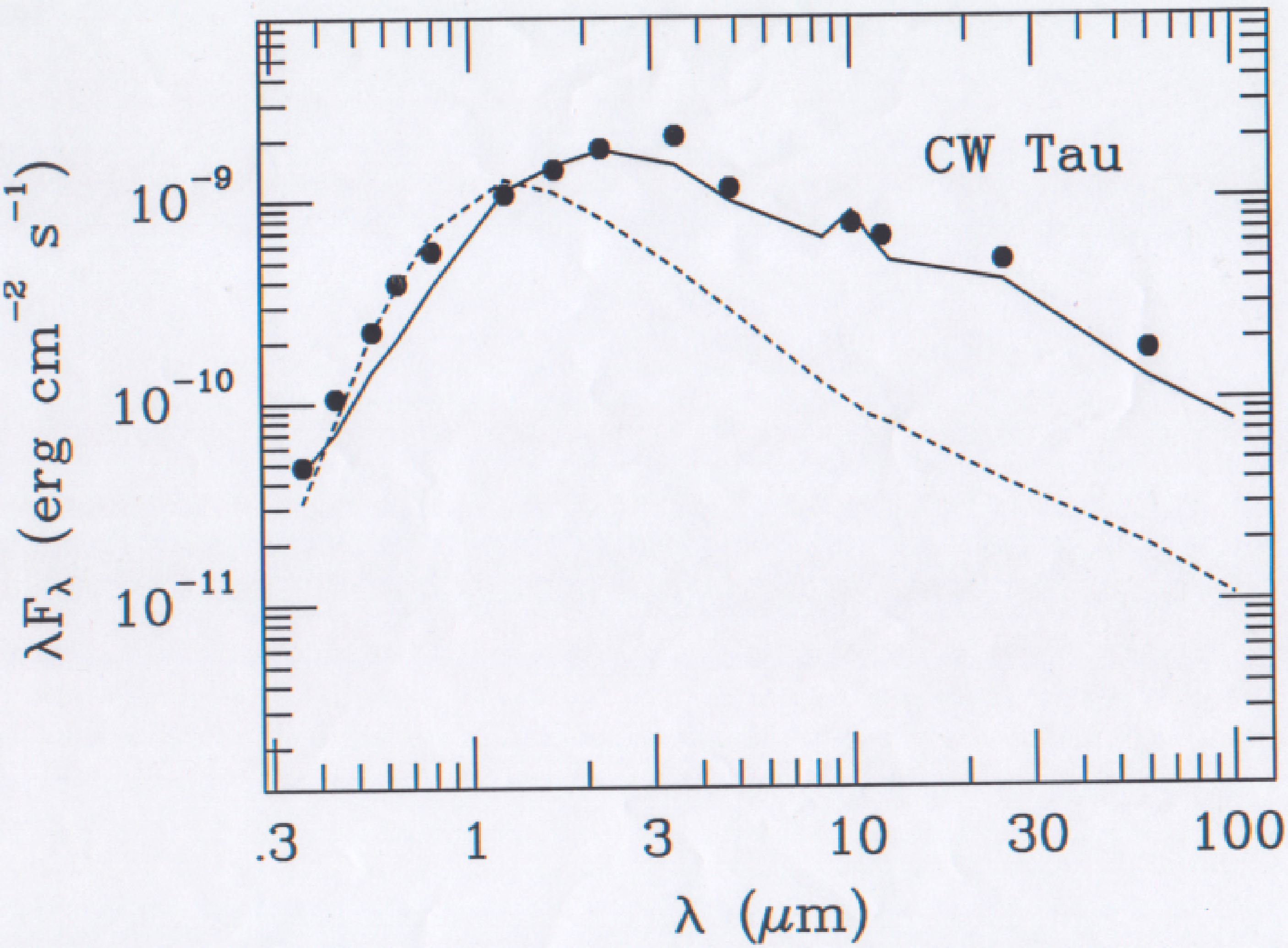

Wind model C
z (AU)
ϖ₀ (AU)
0.1
1.0
10.0
150
100
50
−5.7
10
1.6
−4.9
0.1
−3.7
20
40
60
80
ϖ (AU)
log (Vp / Vk,1)
log (ρ / ρ1)
Safier '93a


Log A_V
2
1
0
−1
−2
0
20
40
60
80
θ_o (°)
WH 1
PNS B
PNS C

YSO



forbidden
line region
wind thermal emission
scattered UV + optical
UV +
optical
disk thermal emission
reprocessing, optically thick
accretion disk
0.1 AU



CW Tau
λF_λ (erg cm^-2 s^-1)
10^-9
10^-10
10^-11
.3
1
3
10
30
100
λ (μm)

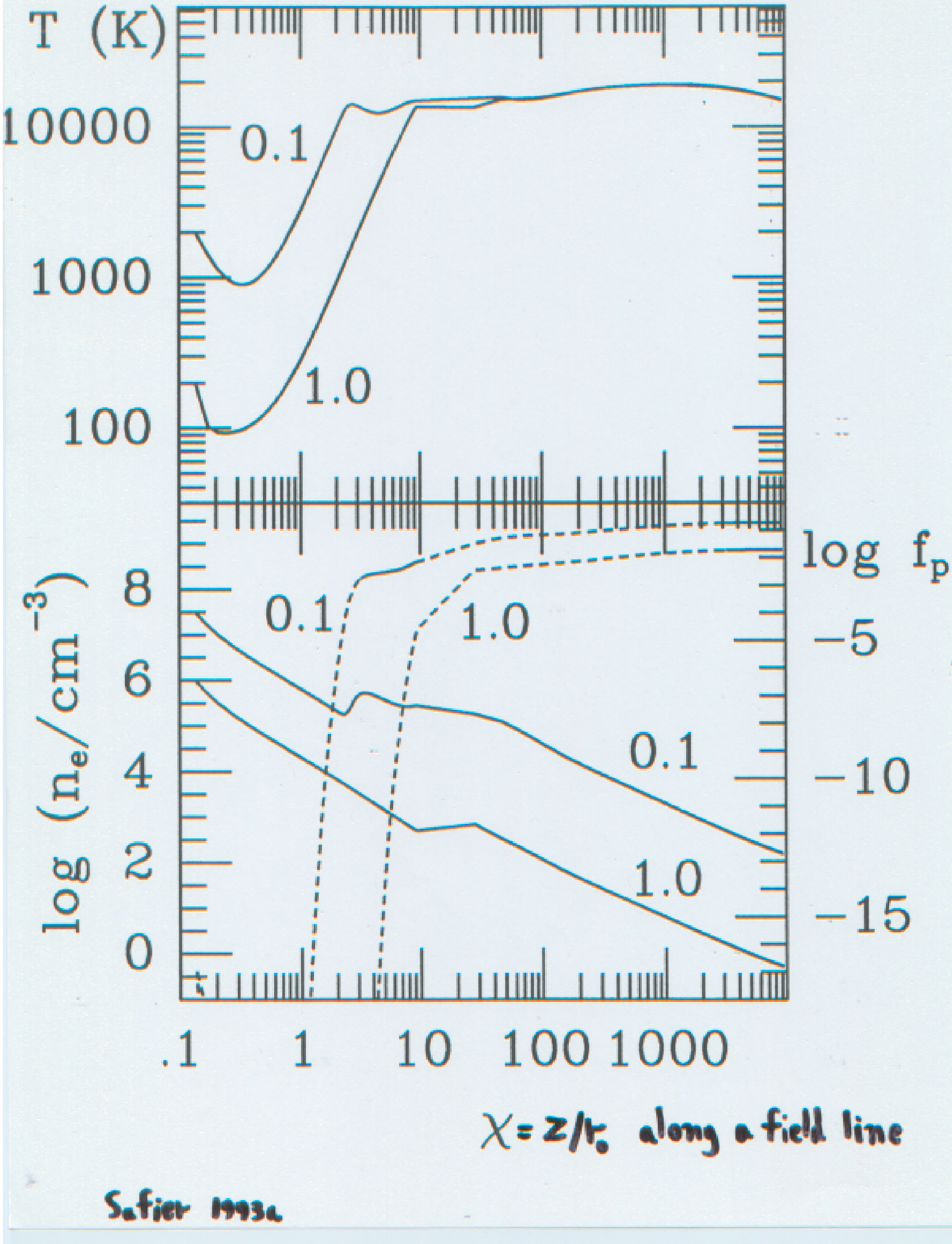
$\ast$ A wind emerging from a weakly ionized disk will undergo heating by ion-neutral drag (ambipolar diffusion) even though its dynamics is governed by ideal MHD. Since adiabatic cooling is relatively inefficent in collimated disk outflows, the temperature could rapidly rise to $\sim 10^4$ K and continue to be kept at that value by an inherent heating/ionization feedback mechanism (safier 1993a; see, however, Shang et al. 2002).

This could contribute to:

- Forbidden line emission from CTTs winds (Safier 1993b; Cabrit et al. 1999; Garcia et al. 2001a,b).

- Thermal radio emission from CTTs jets (Martin 1996).

  In both of these cases, an additional source of heating may be needed (possibly associated with dissipation of weak shocks and turbulence – O'Brien et al. 2003; Shang et al. 2004).



T (K)
10000
1000
100
0.1
1.0
log (n$_e$/cm^{-3})
8
6
4
2
0
log f$_p$
−5
−10
−15
0.1
1.0
0.1
1.0
.1
1
10
100
1000
χ = z/r$_0$ along a field line
Safier 1993a

* On large scales, centrifugally driven outflows assume a structure of a collimated jet (most noticeable in the density contours) and a surrounding wide-angle wind (Shu et al. 1995; Li 1996).

- This bears directly on the general morphology of YSO sources, and in particular on the shapes of the (wind/jet)-driven molecular outflow lobes.