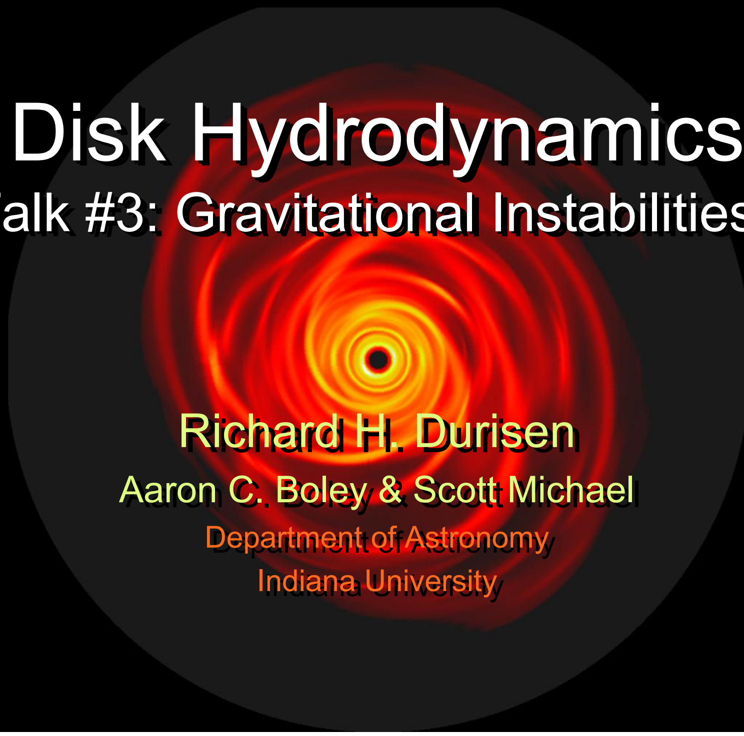
Vidago Workshop 2006

Physical Processes in Circumstellar Disks Around Young Stars



Disk Hydrodynamics

Talk #3: Gravitational Instabilities I

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Outline of Talk #3

Gravitational Instabilities I

- GI basics to 1998
- Simple equations of state
  - Comparisons
  - Isothermal limit
- Idealized cooling
  - Constant $t_{cool}\Omega$
  - Constant t_{cool}
  - Local vs Global?
  - Effect of γ and $\Sigma(r)$
- Conclusions

GI Basics to 1998

Basics to 1998

Toomre 1964, 1981
Goldreich & Lynden-Bell 1965
Lynden-Bell & Kalnajs 1972
Binney & Tremaine 1987
Papaloizou & Savonije 1991
Paploizou &Lin 1995

Gravitational Instabilities (GIs) in Disks

- Toomre's stability parameter
  - For $Q = c_s\kappa/\pi G\Sigma < 1 \Rightarrow$ linear ring instability
    - Self-gravitating perturbations grow that are
      - too large for pressure to stabilize
      - too small to be stabilized by shear
    - $Q \sim t_{ff}^2/t_z P_{rot}$ with t_{ff} due local disk self-gravity
  - For $Q < 1.5 - 1.7 \Rightarrow$ linear spiral instability
- General Features
  - Growth from noise on a dynamic time scale
  - Predominance of trailing spirals
  - Lindblad resonances for m-armed spiral
    - $\Omega = \Omega_{pat} \pm \kappa/m$ (+ inner and - outer)

Basics to 1998

Goldreich & Lynden-Bell 1965
Toomre 1981
Pringle 1981
Larson 1984
Boss 1995
Durisen et al. 1986
Tomley et al. 1991,1994
Laughlin & Rozyczka 1996
Laughlin et al. 1997, 1998

GI's in Disks (cont'd)

- Mass & angular momentum transport
  - Gravitational torques due to a trailing spiral
    - $r < r_{CR} \Rightarrow$ mass inflow; $r > r_{CR} \Rightarrow$ mass outflow
- What determines the nonlinear amplitude?
  - Balance of heating versus cooling
    - shock heating versus radiative cooling
    - disks hover near Q ~ 1 to 1.5
    - mass inflow is the ultimate source of energy
  - Saturation through mode coupling
  - Fragmentation
    - if cooling is fast enough

Basics to 1998

Toomre 1964, 1981
Goldreich & Lynden-Bell 1965
Binney & Tremaine 1987
Adams et al . 1989
Shu et al. 1990

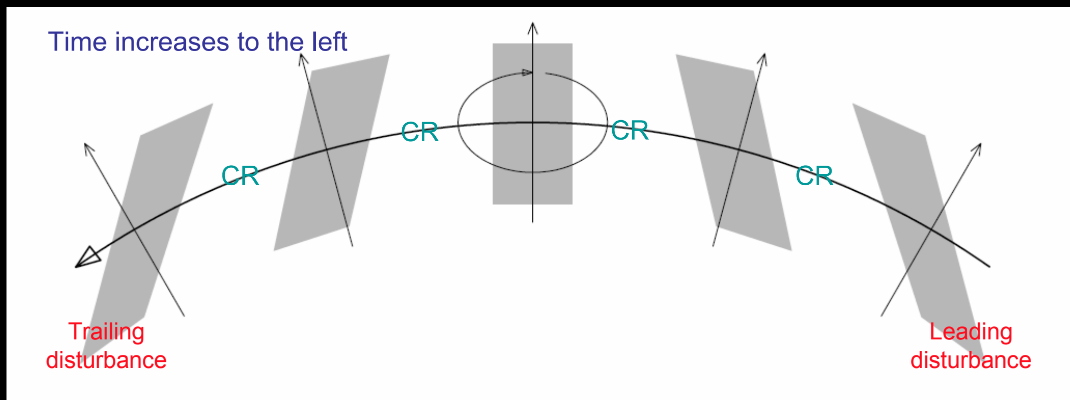
GI's in Disks (cont'd)

- Growth mechanism
  - Swing
  - SLING
- Methodologies
  - Linear
    - WKB analysis
  - Nonlinear simulations
    - local shearing box
    - grid-based hydro
    - SPH hydro

Basics to 1998

Swing Amplification

Toomre 1981
Binney & Tremaine 1987
Barnes 2005

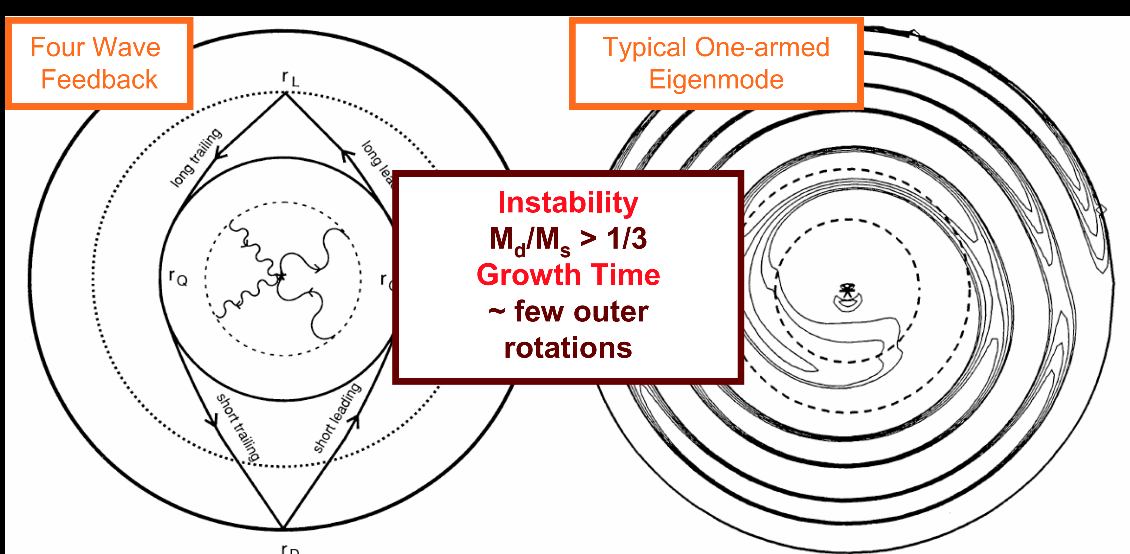


Retrograde epicyclic motion keeps a fluid element at a fixed position in a long wavelength disturbance at corotation as it "swings" from leading to trailing. Self-gravity is thus given time to amplify it.

Basics to 1998

SLING: Stimulation by Long-range Interaction of Newtonian Gravity

Adams et al. 1989
Shu et al. 1990

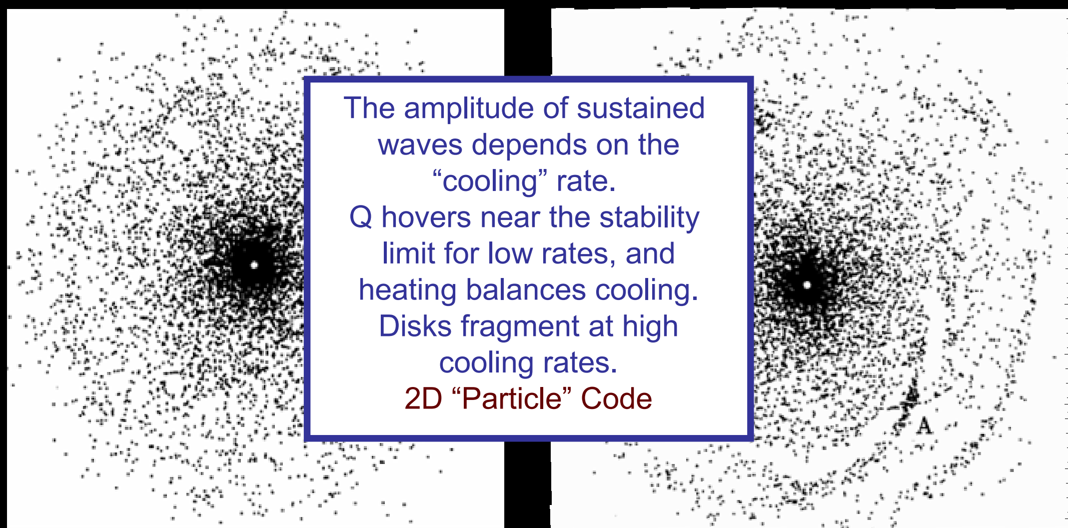


Growth occurs if the returning wave is in phase with the perturbation due to the star's displacement.

Goldreich & Lynden-Bell 1965
Tomley et al. 1991,1994

Basics to 1998
Heating & Cooling

Tomley et al. 1991



The amplitude of sustained waves depends on the “cooling” rate.
Q hovers near the stability limit for low rates, and heating balances cooling.
Disks fragment at high cooling rates.
2D “Particle” Code

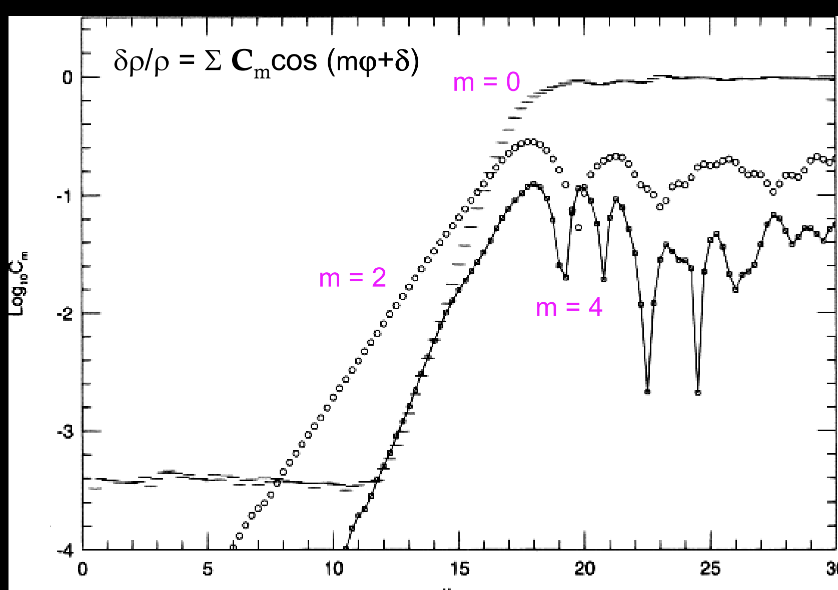
Low Cooling Rate

High Cooling Rate

Laughlin & Rozyczka 1996
Laughlin et al. 1997, 1998

Basics to 1998
Mode Coupling

Global spiral amplitudes saturate through mode coupling.
Mass transport is not well approximated by an α disk.
3D Governing Equations of second and third order plus 3D Hydro Simulations
Polytropic EOS



Laughlin et al. 1997

Reminders

Questions Left in 1998

- Which mechanism(s) causes GIs to grow?
- What mechanism(s) limits their growth?
- Exactly how do disks evolve under GIs?
  - Mass and angular momentum transport?
  - Fragmentation?
- Do they act like a local or global mechanism?

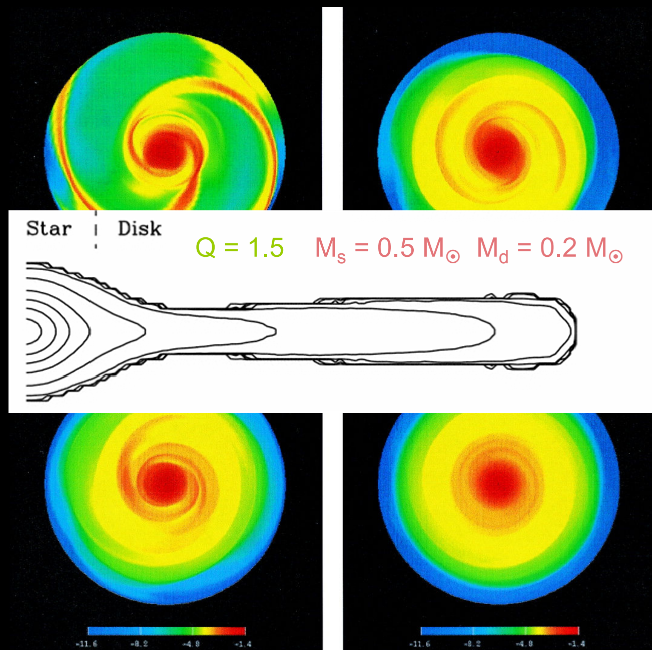
Simple EOS's

Simple EOS's

Pickett et al. 1998 & 2000

LOCALLY ISOTHERMAL

LOCALLY ISENTROPIC

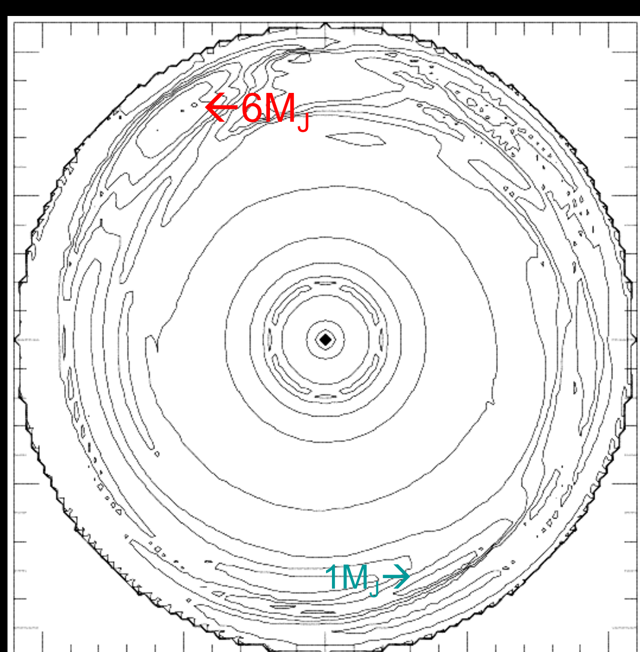


ADIABATIC

SHOCK HEATING BY ARTIFICIAL VISCOSITY

Simple EOS's

Boss 1998



Solar Nebula Model

$Q_{min} < 1$

$M_* = 1\ M_\odot$

$M_d/M_* = 0.1$

R = 10 AU

Disk expansion not allowed

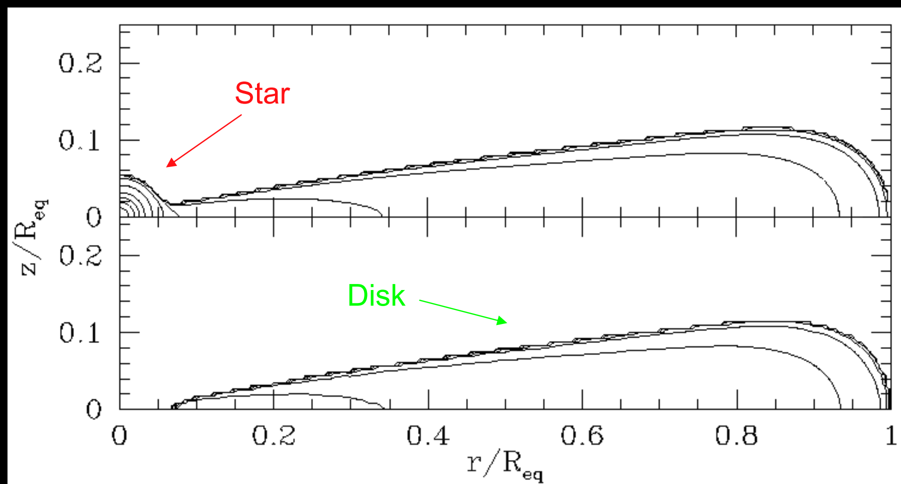
Locally isothermal

'PLANETS' FORM!

Simple EOS's

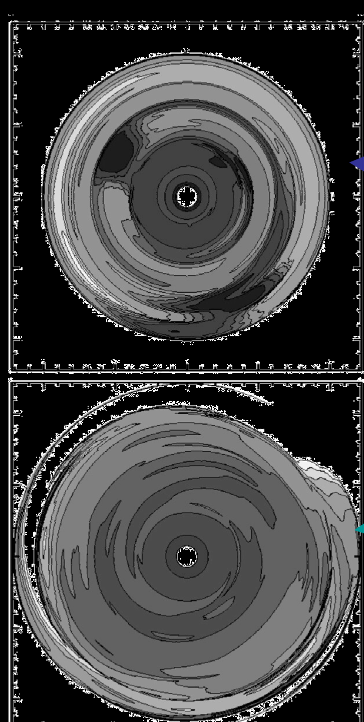
Mejía Star/Disk Models

$M_d/M_s=1/7$, $R_d/R_s=20$, $\Sigma(r) \sim r^{-1/2}$



Simple EOS's

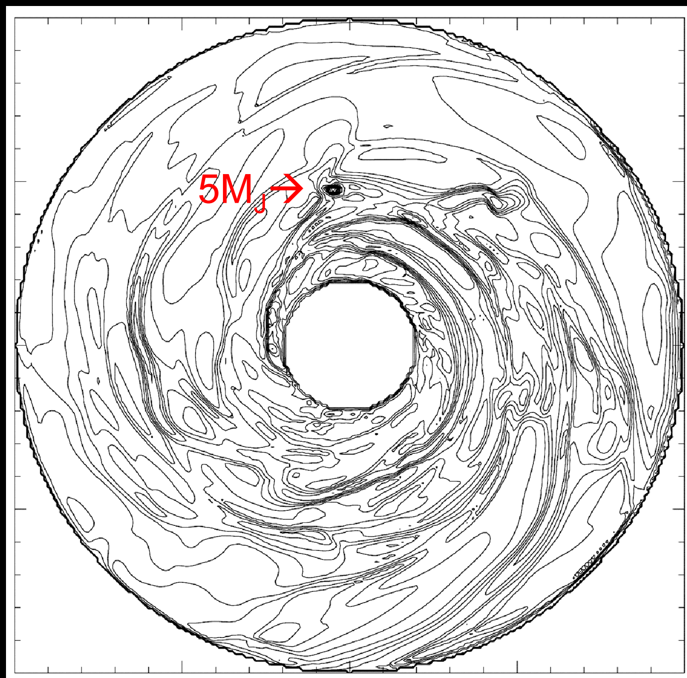
Pickett et al. 2000



Isothermal Evolution
$r = 0.76$ to 10 AU
$M_s = 1\ M_\odot$
$M_d/M_s = 0.13$
$Q \sim 1.1$
Mimics Boss radial velocity damping in the outer disk

Same case with free disk expansion.
Planets go away

Simple EOS's
Isothermal Disk Fragmentation



High Resolution Simulation

$Q_{min} = 1.3$
$M_* = 1\ M_\odot$
$M_d = 0.09\ M_\odot$
$R_d = 20$ AU

Persistent Dense Clump Forms!

Boss 2000, 2005

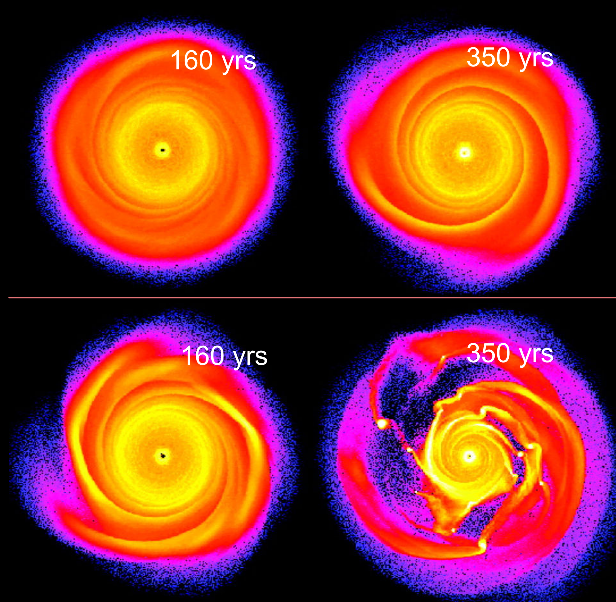
Simple EOS's
Mayer et al. 2002 & 2004

High Resolution Isothermal SPH Simulations

$Q_{min} = 1.75$
$M_d = 0.08\ M_\odot$

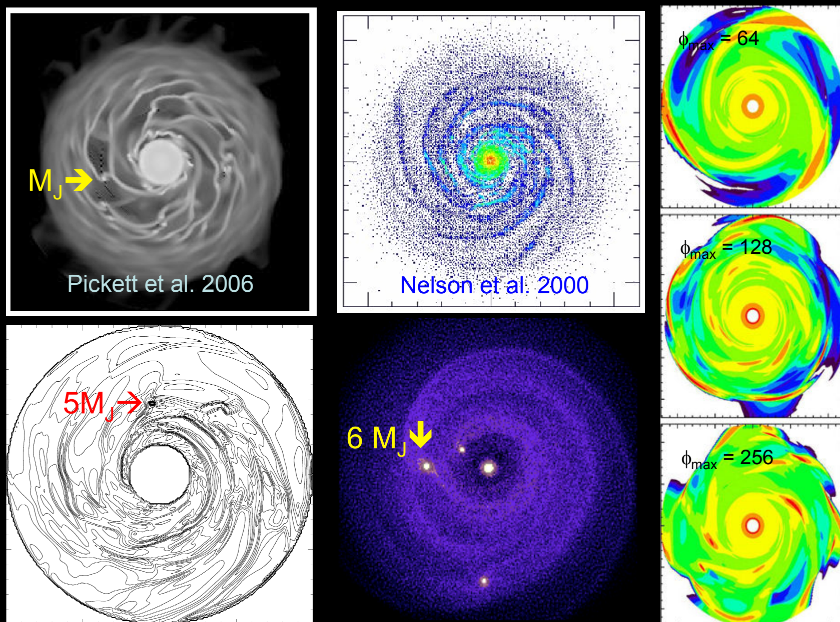
$M_* = 1\ M_\odot$
$R_d = 20$ AU

$Q_{min} = 1.40$
$M_d = 0.10\ M_\odot$



Simple EOS's

Isothermal Disk Fragmentation



Boss 2000 Mayer et al. 2002 & 2004 Pickett et al. 2003

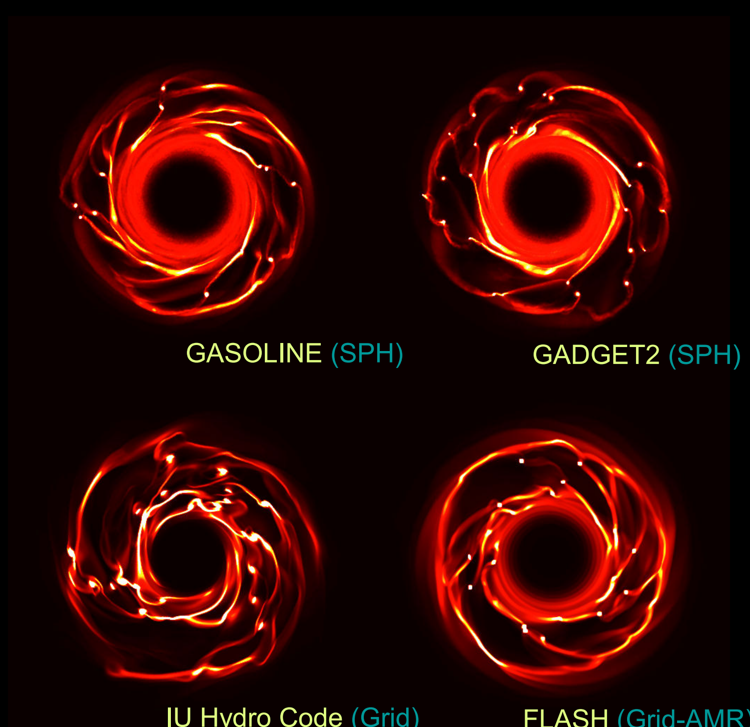
Local regions of thin disks fragment for $Q < \sim 1.4$

Johnson & Gammie 2003

Agree that dense clumps can form but not about how long they live

Idealized Cooling

Mayer et al. 2006



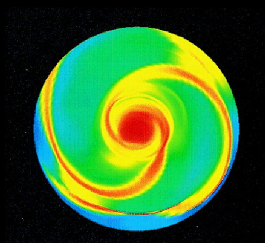
Multi-Code Comparison:
Isothermal Disk
High Resolution
$r_{outer} = 20AU$
$M_d/M_s = 0.055$
$Q < 1.4$

Simple EOS's

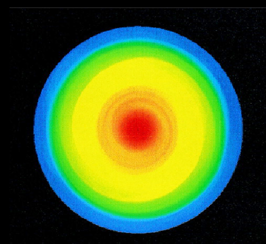
Heating/Cooling & Fragmentation

Pickett et al. 1998 & 2000

Isothermal

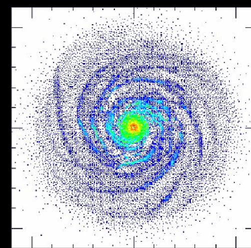


Unbalanced Heating

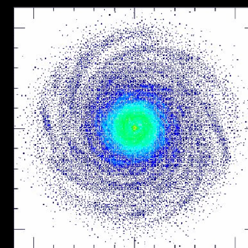


Nelson et al. 1998 & 2000

Isothermal

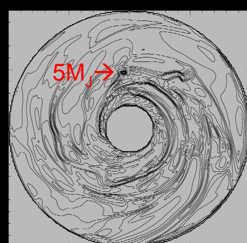


Heating & Cooling

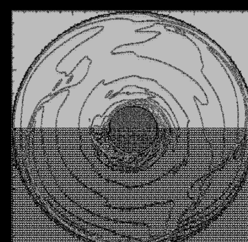


Boss 2000, 2001, 2005

Isothermal



Strong Heating



Simple EOS's

Consensus Conclusions:

- Mechanism
  - Swing not SLING at least for 10's orbits
- Thermal physics sets wave amplitude
  - Mode interactions do occur
  - Heating ↑ ⇒ amplitude of spirals ↓
  - Cooling ↑ ⇒ amplitude of spirals ↑
  - Amplitude set by balance of heating & cooling unless fragmentation occurs
- Nonlinear outcome
  - Multiple multi-arm spirals
  - Dense structure (clumps) form with severe cooling (esp., isothermal)

Idealized Cooling

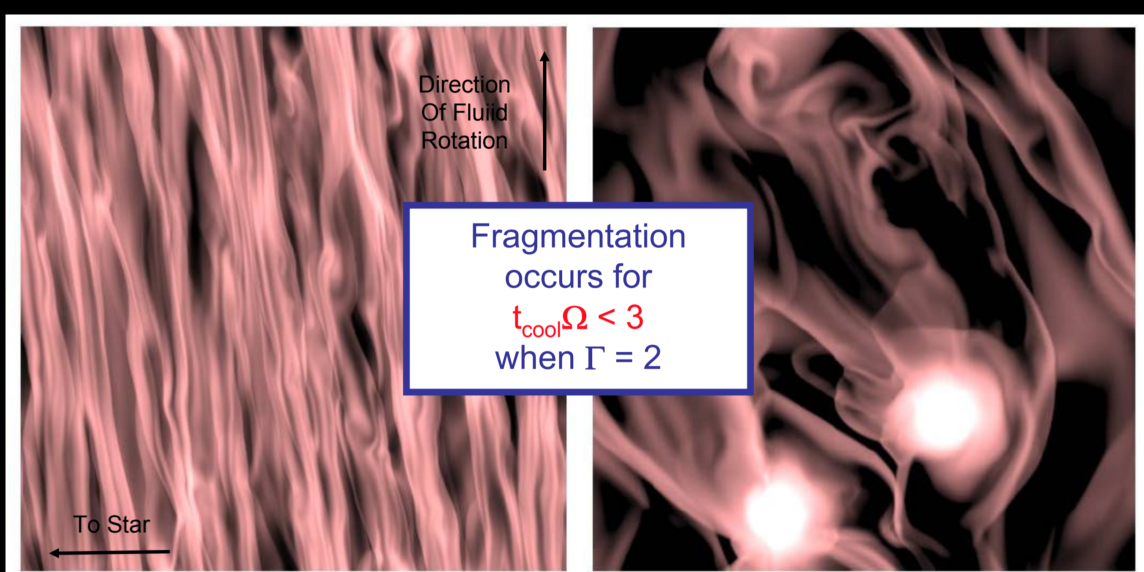
t_{cool} = constant

$t_{cool}\Omega$ = constant

$\Lambda = u_{int}/t_{cool}$

Idealized Cooling

Gammie 2001

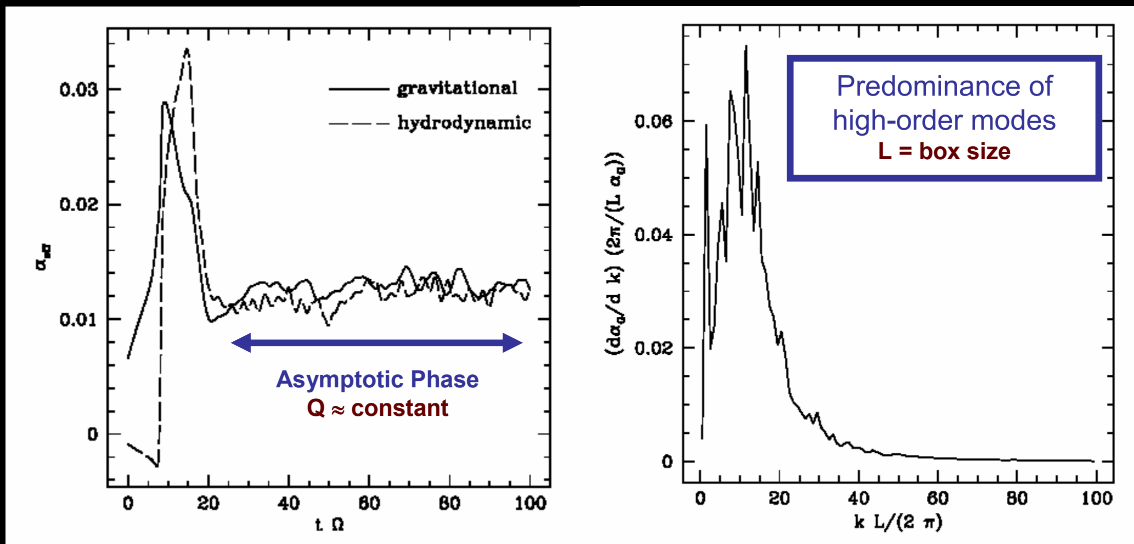


$t_{cool}\Omega = 50$

$t_{cool}\Omega = 2$

Idealized Cooling

Gammie 2001



Idealized Cooling

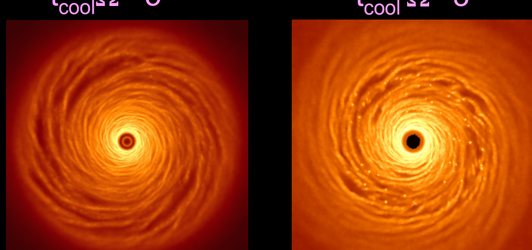
Heating/Cooling & Fragmentation for γ=5/3

Gammie 2001 (2D Shearing Box)

Local regions of thin disks fragment for
$t_{cool}\,\Omega < 3$ to 6
$t_{cool} < 0.5$ to $1.0\ P_{rot}$

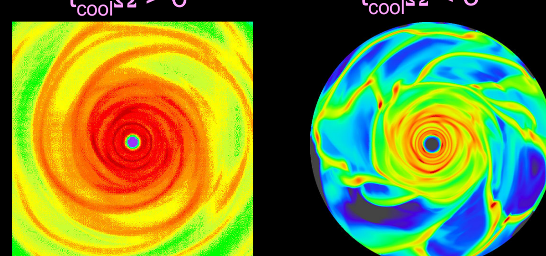
Rice et al. 2003 (3D SPH)

$t_{cool}\Omega = 5$ $t_{cool}\,\Omega = 3$



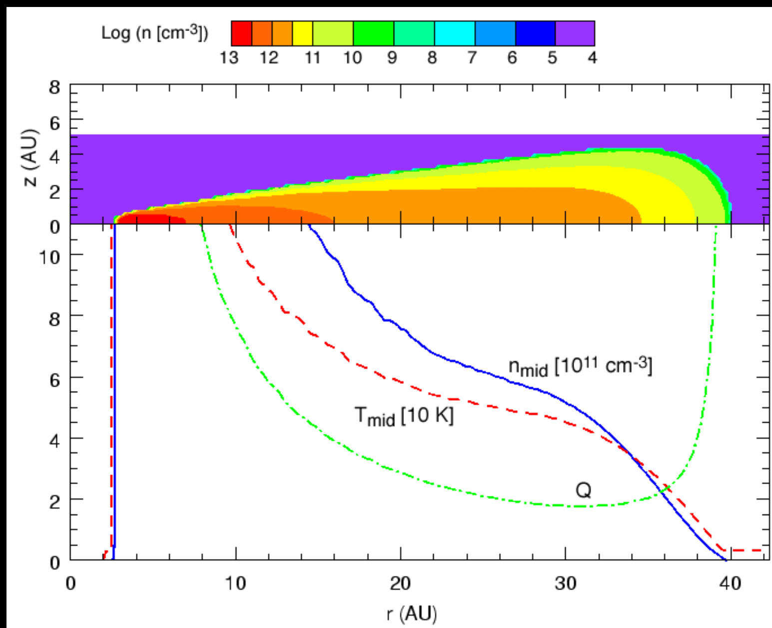
Mejía et al. 2005 (3D Grid-Based)

$t_{cool}\Omega > 6$ $t_{cool}\Omega < 3$



Idealized Cooling

Mejía et al. 2005



Initial model

R = 40 AU

$M_d = 0.07M_\odot$

$M_* = 0.5M_\odot$

$\Sigma(r) \sim r^{-1/2}$

Q_{min}=1.8

ORP = outer rotation period = 250 years

Idealized Cooling

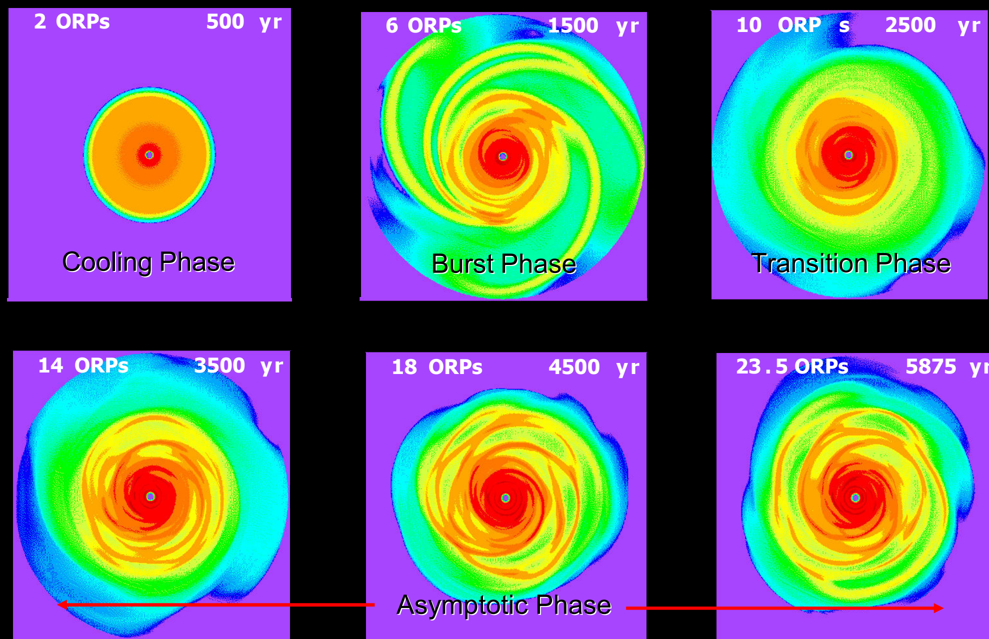
t_{cool} = 2 orps (500 yrs) = constant

Mejía et al. 2005

Animation courtesy of John Rosheck

Idealized Cooling

t_{cool} = 2 orps (500 yrs) = constant



Pickett et al. 2003 & Mejía et al. 2005

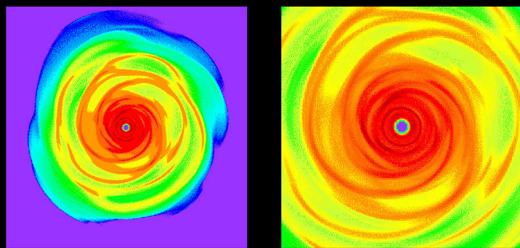
Idealized Cooling

t_{cool} = 1/4 orp (63 yrs) = constant

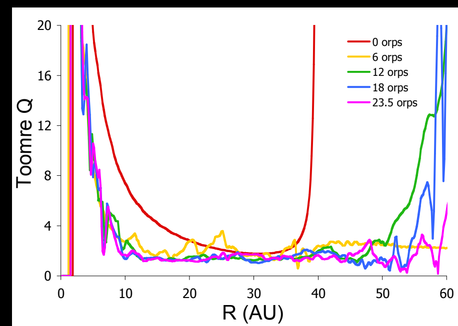
Animation courtesy of
Annie C. Mejía

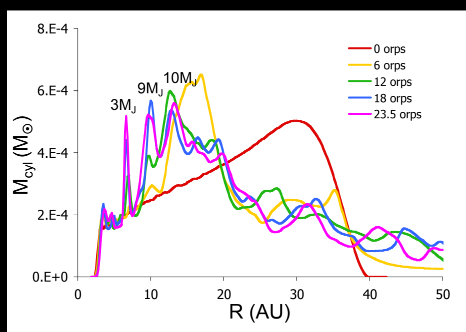
Idealized Cooling

t_{cool} = 2 orps (500 yrs) = constant



23.5 ORPs





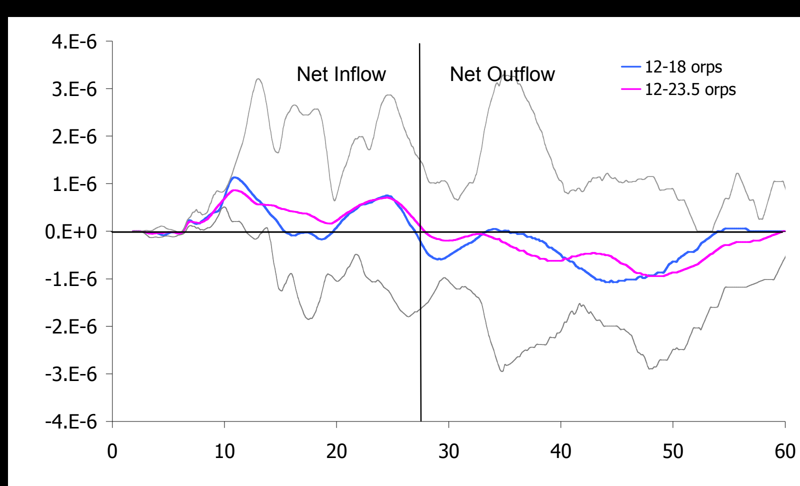
- After 12 orps, azimuthal averages approach asymptotic values
- Q ~ 1.46 between 10 and 50 AU
- Rings grow in the inner disk

Mejía et al. 2005

Idealized Cooling

t_{cool} = 2 orps = constant

Asymptotic Inflow Rate $M_\odot$/yr



r(AU)

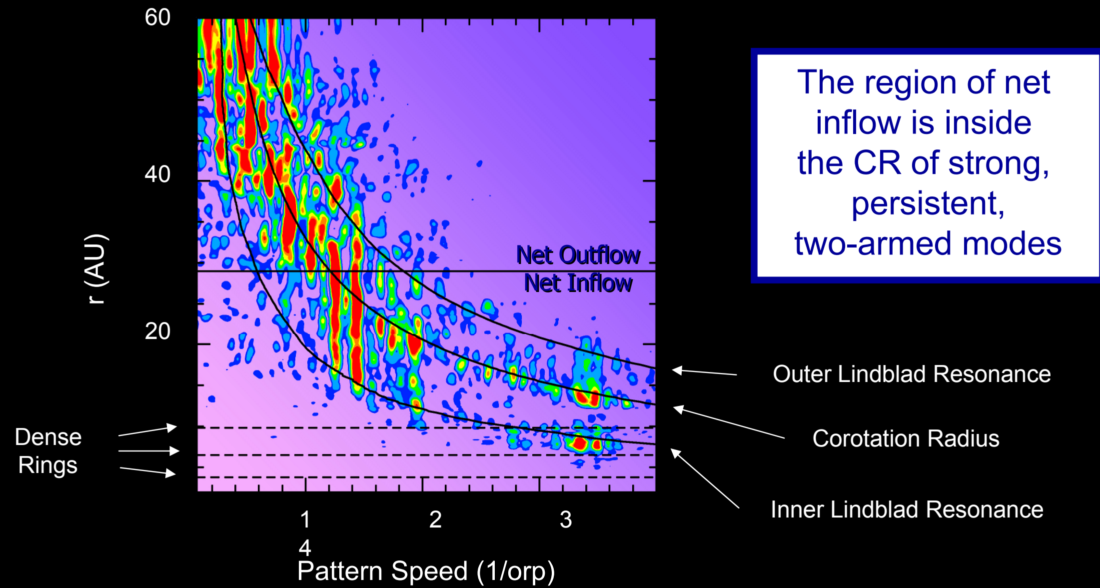
Mass inflow burst ~ 10^{-5} $M_\odot$/yr

Asymptotic ~5×10^{-7} $M_\odot$/yr

Mejía et al. 2005

Idealized Cooling

Global Modes for t_{cool} = 2 orps = constant

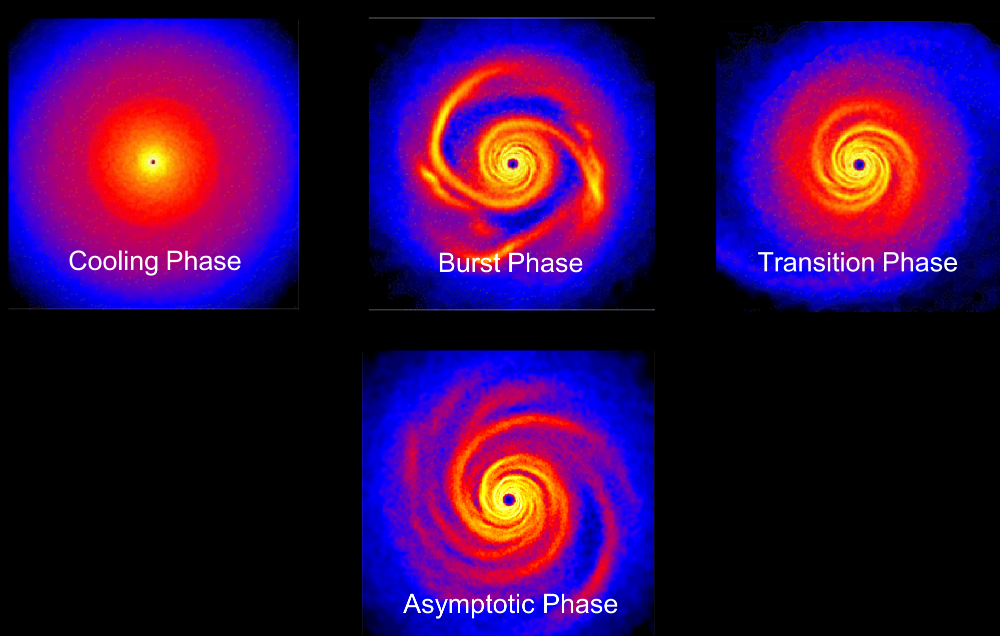


m = 2 Power Spectrum
from Fourier decomposition in φ

Mejía et al. 2005

Idealized Cooling

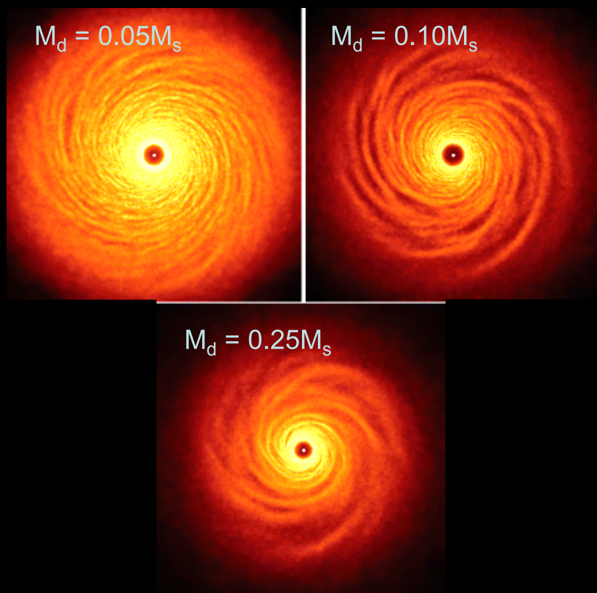
$t_{cool}\Omega$ = 7.5, M_d = 0.5M_s



Lodato & Rice 2005

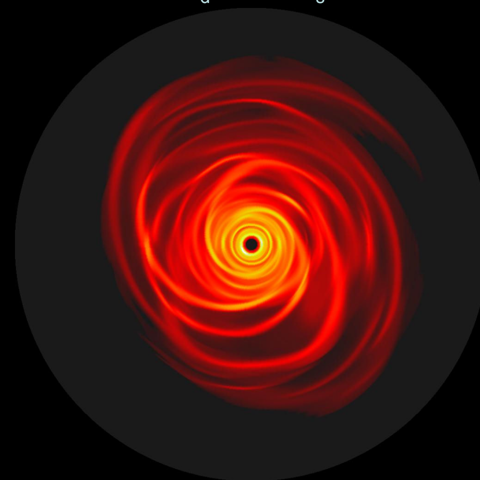
Idealized Cooling

$t_{cool}\Omega$ = 7.5 versus t_{cool} = 2 orps



Lodato & Rice 2004

$M_d = 0.14M_s$



Mejía et al. 2005

Idealized Cooling

Local or Global

Lin & Pringle 1987 & 1990
Gammie 2001
Lodato & Rice 2004, 2005

Local Transport by GIs

- Local behavior occurs when
  - $M_d < 0.5M_s$ ($H < 0.1r$) and
  - t_{cool} is local, e.g., $t_{cool}\Omega$ = constant as in a steady-state accretion disk
- Then
  - Transport is well characterized by a local α-viscosity
  - $\alpha = 4/[9\Gamma(\Gamma-1)t_{cool}\Omega]$
  - High-order tightly wrapped spirals predominate

Idealized Cooling

Local or Global

Laughlin & Rozczyka 1996
Balbus & Paploizou 1999
Fromang et al. 2004
Lodato & Rice 2005
Mejia et al. 2005
Boley et al. 2006
Michael et al. 2006

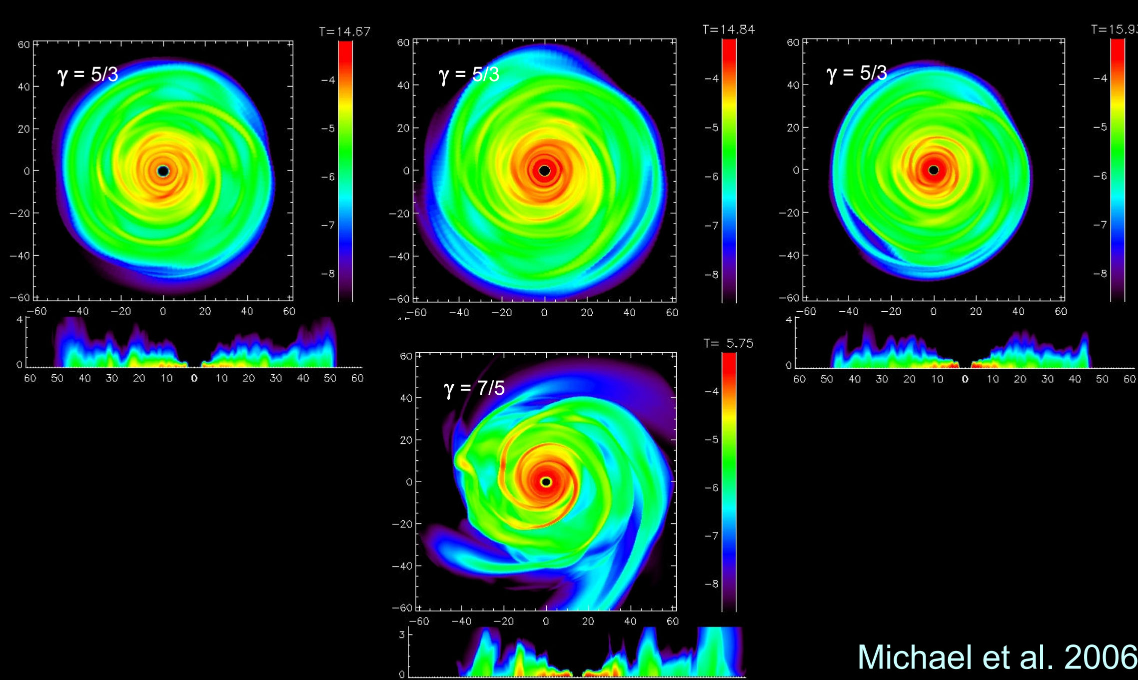
Global Transport by GIs

- Global behavior occurs when
  - For any M_d, when t_{cool} is global
  - For large M_d ($H > 0.1r$), even for local t_{cool}
- Then
  - Transport is not well characterized by a local α-viscosity
  - $\alpha_{eff} = W_{r\varphi}/c_s^2$ is rather nonuniform in space and time
  - $\alpha > 4/[9\Gamma(\Gamma-1)t_{cool}\Omega]$ by factor ~ 10
  - Low-order open spirals dominate
  - Rings can form at the edges of GI-activity

Idealized Cooling

Different $\Sigma(r)$ for t_{cool} = constant = 2 orps

$\Sigma \sim r^{-1/2}$ $\Sigma \sim r^{-1}$ $\Sigma \sim r^{-3/2}$



Michael et al. 2006

Idealized Cooling

Gammie 2001
Johnson & Gammie 2003
Rice et al. 2003, 2005
Mejia et al. 2005
Michael et al. 2006

Combined Effect of t_{cool} and EOS

- $\gamma = 5/3$
  - Fragmentation for $t_{cool}\Omega < 3\text{-}6$ ($t_{cool} < P_{rot}$)
  - Hydrogen not rotationally excited or dissociating
- $\gamma = 7/5$
  - Fragmentation for $t_{cool}\Omega < 12$ ($t_{cool} < 2P_{rot}$)
  - Hydrogen that is rotationally excited
- General $\gamma > 1$
  - Fragmentation for $\alpha > 0.06$ (with local t_{cool})
- Isothermal
  - Fragmentation whenever $Q < 1.4$

Conclusions

Conclusions
Simple EOS's & Idealized Cooling

Conclusions

- Governing physics
  - Thermal physics regulates GIs in disks
  - GIs get stronger as t_{cool} decreases
- Approximate fragmentation criteria
  - Cold disks fragment for fast enough cooling
  - $t_{cool}\Omega <$ const. ↑ as γ ↓
  - $\alpha > \alpha_{crit} \Rightarrow$ maximum stress
    - depends on $t_{cool}(r)$
  - **Q < 1.4** for an isothermal disk

Conclusions
Simple EOS's & Idealized Cooling

Conclusions (cont.)

- GIs in unfragmented disks
  - Can initiate with a strong burst and
  - Achieve an asymptotic state of T.E. with ongoing mass and ang. mom. transport
  - Local vs global depends on $t_{cool}(r)$, H/r, and M_d/M_s
  - Strong bursts may repeat in massive disks
- Areas of disagreement
  - Longevity of clumps in fragmented disk
  - Exactly when GIs behave locally or globally

Conclusions
Questions Partly Answered

- Which mechanism(s) causes GIs to grow?

  Swing

- What mechanism(s) limits their growth?

  Thermal physics controls growth

- How do disks evolve under GIs?
  - Mass and angular momentum transport?

    Effective α increases as t_{cool} decreases
  - Fragmentation?

    A cold disk cooling rapidly fragments
- Do they act like a local or global mechanism?

  Depends on $t_{cool}(r)$ and so $\alpha(r,t)$ generally

Conclusions
The Next Questions

- What do REAL disks do?
  - Realistic radiative cooling
  - Realistic equation of state
  - Realistic conditions and environments
- How do REAL disks evolve under GIs?
  - Mass and angular momentum transport?
  - Fragmentation?
- Are GIs in REAL disks local or global?