

Vidago Workshop 2006

Physical Processes in Circumstellar Disks Around Young Stars

Disk Hydrodynamics

Talk #2: A Wonderland of Instabilities

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BIG QUESTIONS

What Hydrodynamical ($B=0$) Processes Really Affect Disk Evolution?

- Which can and do alter the structure of disks?
- Which, if any, can and do affect planet formation?

Which Can Transport Mass & Ang. Mom.?

- How efficient are they?
- Is the transport local or global?

Outline of Talk #2

A Wonderland of Instabilities

- Classic results
 - Hydrodynamic instability
 - Convective instability
 - Baroclinic instability
- Application to disks
 - Instabilities
 - hydrodynamic
 - convective
 - baroclinic and others
 - Large-scale structures
 - Tapping the energy in Keplerian shear
- Gravitational instabilities
- Conclusions

Classic Results

Classic Results

Tassoul 1978
Shu 1992
Paploizou & Lin 1995
Balbus & Hawley 1998
Balbus 2003
Gammie & Johnson 2005

Hydrodynamic Instability

- Why should shear flows be unstable?
 - Consider a simple linear planar shear
 - $dv_x/dz = v_x' = \text{constant}$
 - There's energy to be had
 - redistribute momentum so $v_x = \text{constant}$ over $z=z_0 \pm \varepsilon$
 - all else equal, the kinetic energy decreases by a fraction of order $(\varepsilon v_x' / v_x)^2$
 - this energy can feed full-blown turbulence
 - What can keep the shear flow stable?
 - high viscosity
 - stratification ($s=\text{entropy}$ increases "upward")

Classic Results

Tassoul 1978
Shu 1992
Paploizou & Lin 1995
Balbus & Hawley 1998
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Hydrodynamic Instability (cont'd)

- Conditions for instability of shear flow
 - Shear flows are *usually* unstable for
 - $Re = \text{Reynolds number} = vL/\nu > 10^3 - 10^4$
 - for low Re , viscosity can keep the flow laminar
 - at high Re , turbulence ensues
 - Stratified shears are stable when
 - $Ri = \text{Richardson number} = (g_z s' / c_p) / (v_x')^2 > 1/4$
 - Ri measures the relative importance of buoyancy and shear
 - Naïve application to astrophysical disks
 - molecular $\nu \sim \nu_{\text{th}} \sim 10^4 - 10^5 \text{ cm}^2/\text{s}$
 - $Re \sim 10^{15}$ and $Ri \sim 10^{-2} \Rightarrow$ **unstable?**

Classic Results

Tassoul 1978
Shu 1992
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Convective Instability

- Condition for convective instability
 - Consider a plane parallel H.E. layer in R.E.
 - R.E. = radiative equilibrium = radiation carries all the energy flux
 - Convective instability
 - Schwarzschild criterion
 - $ds/dz_{R.E.} = s'(R.E.) < 0$
 - for high Re, convective flows are turbulent
 - Naïve application to disks
 - young stellar disks are convectively unstable in the z-direction for a wide range of conditions
 - a source of turbulent a.m. transport?

Classic Results

Tassoul 1978
Shu 1992
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Baroclinic Instability

- Condition for baroclinic instability
 - Consider an axisymmetric rotating flow in a gravitational field (e.g., planet atmosphere)
 - barocline: constant entropy surfaces not the same as constant effective potential surfaces
 - Baroclinic instability
 - Høiland criterion: instability occurs when
 - $j = r^2\Omega$ decreases outward on an $s=\text{const.}$ surface
 - inertial waves and vortices can result
 - Naïve application to disks
 - such conditions can occur in disks
 - what happen then?

Applications to Disks

Application to Disks

Tassoul 1978
Shu 1992
Paploizou & Lin 1995
Balbus & Hawley 1998
Hawley et al. 1999
Balbus 2003
Gammie & Johnson 2005

Instabilities



What is unique here?



New dynamical elements



angular momentum



centrifugal and Coriolis forces



Axisymmetric stability



imagine exchanging rings of fluid between r_1 and $r_2 > r_1$ while conserving specific ang. mom.



if $j_1 > j_2$, the new state has less rotational K.E. and nonrestorative excess centrifugal forces



Rayleigh criterion: $\partial j / \partial r < 0 \Leftrightarrow$ instability!



$\partial^2 j / \partial r = \partial(r^2 \Omega)^2 / \partial r = r^3 \kappa^2$

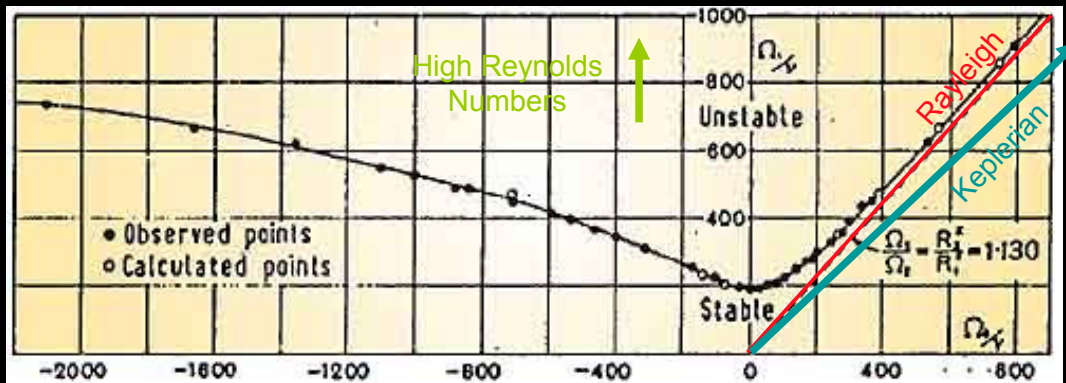


Taylor-Couette flow between rotating cylinders

Application to Disks

Taylor-Couette Flow

Taylor 1923
Shu 1992
Brenner & Stone 2000
Barnes 2005



Keplerian disks are stable
by the Rayleigh criterion!
Taylor-Couette flow suggests
stability even though $Re \gg 1$.
Caution: Incompressible & cylindrical

Application to Disks

Papaloizou & Pringle 1984, 1985
Goldreich et al 1986
Narayan & Goldreich 1989
Papaloizou & Lin 1995
Hawley et al. 1999
Balbus 2003
Johnson & Gammie 2006

Instabilities (cont'd)



Hydrodynamic



Precision local simulations

- no linear or nonlinear instabilities detected for Keplerian shear
- Re large does not cause local instability **but**
- it is hard to exclude truly global effects



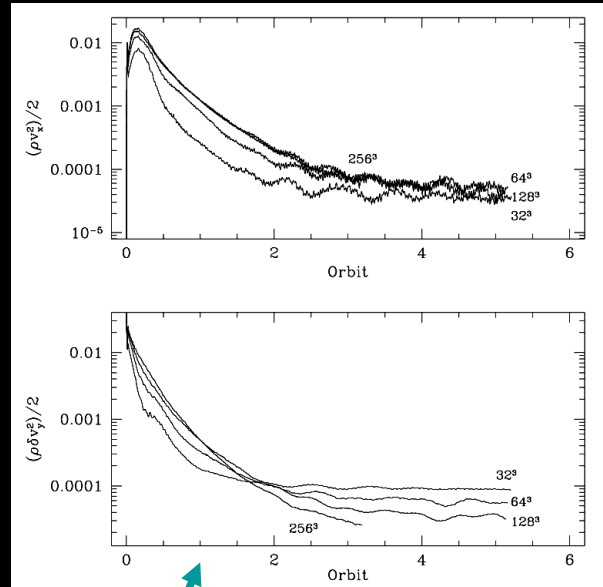
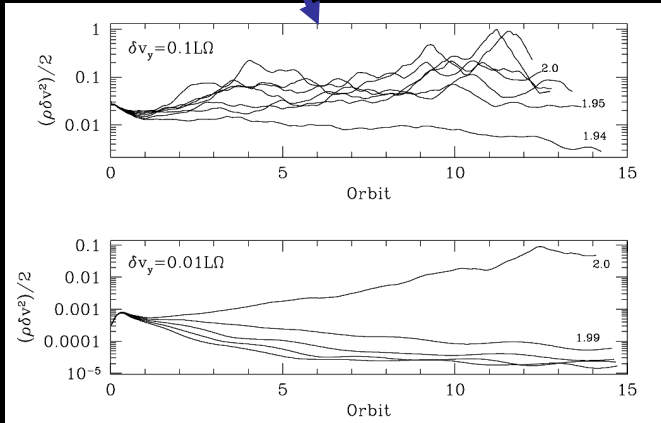
Interesting known global exception

- Papaloizou & Pringle:** narrow fluid annuli can be unstable due to interacting edge waves
- instability for $q > 1.732$ with $\Omega \sim r^{-q}$ **but**
- Keplerian annuli ($q = 1.5$) are stable

Application to Disks

Hawley et al. 1999: Local Simulations

Simulations for $\Omega \sim r^q$ near the Rayleigh stability regime ($q = 2$).
 Shear acts like a planar shear because the epicyclic frequency $\kappa = 0$.
 No growth for $q < 1.95$.
 3D Shearing Box, Isentropic Equilibrium



The Keplerian case ($q = 1.5$) is strongly damped and is stable regardless of code or resolution used.







Application to Disks

Lin & Papaloizou 1980
 Stone & Balbus 1996
 Cabot 1996
 Balbus 2000
 Klahr & Bodenheimer 2000, 2003
 Johnson & Gammie 2006

Instabilities (cont'd)



Convection (z-direction)

-  Precision local simulations and analytic work
 -  convection in disks causes r, ϕ Reynolds stresses that have the wrong sign
 -  convection transports a.m. inward
-  Some residual uncertainty from global 3D radiative hydro simulations but
 -  effective $|\alpha|$'s probably not large in any case
 -  no more than a few $\times 10^{-3}$ and probably much less

Application to Disks

Lin & Papaloizou 1995
Lovelace et al. 1999
Li et al. 2000
Klahr & Bodenheimer 2003
Johnson & Gammie 2006
Klahr & Bodenheimer 2006

Instabilities (cont'd)



Instabilities involving vorticity & entropy



Analogs of Papaloizou-Pringle and baroclinic instabilities



disks are broad, but they can have localized features which can act like edges



entropy gradients can also drive instabilities



Bumps, jumps, & extrema



vortensity $\nabla_{\mathbf{x}}\mathbf{v}/\Sigma$



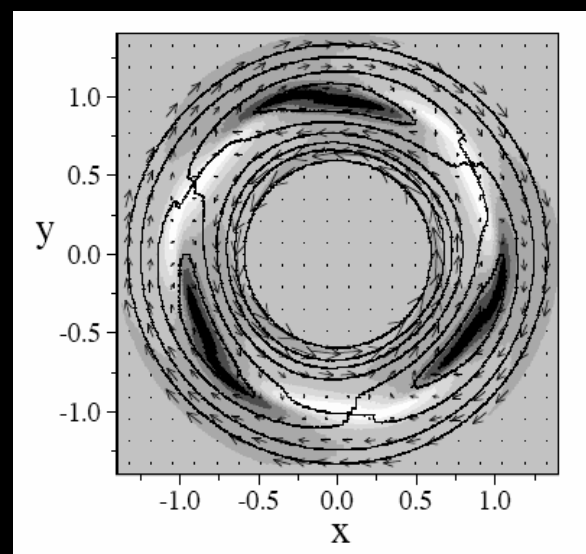
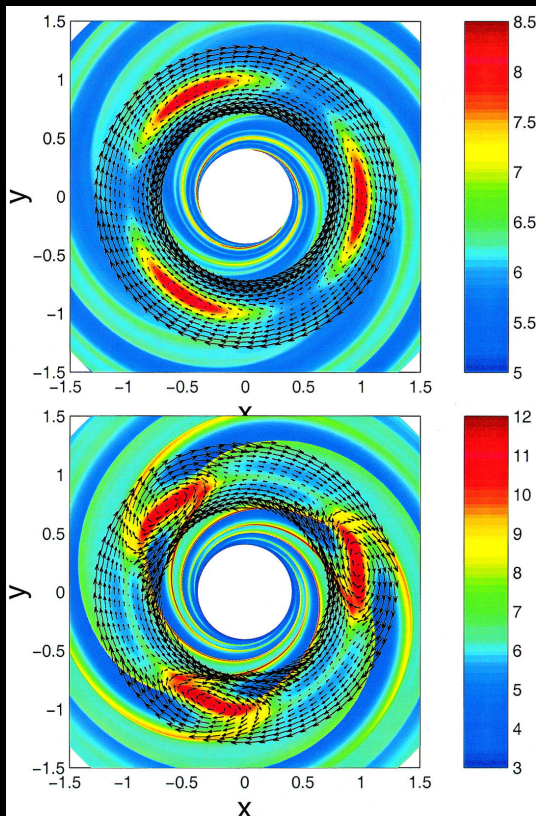
“special” function $(\Sigma\Omega/\kappa^2)s^{2\Gamma}$



extrema or sharp jumps in the r-distributions of these quantities can induce instability

Application to Disks

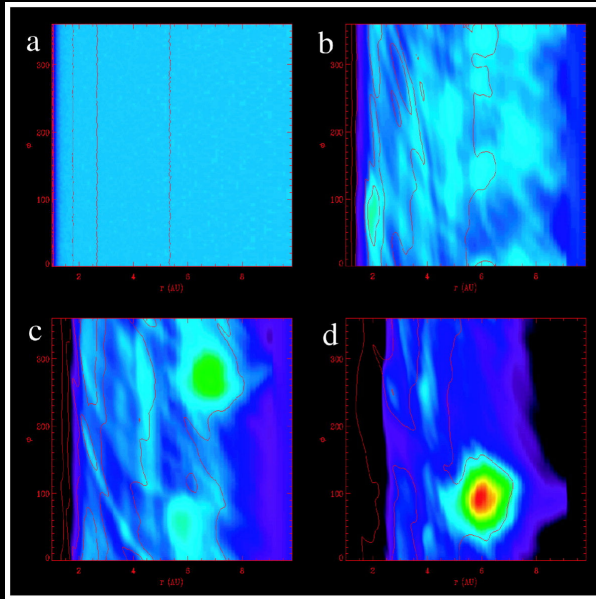
Li et al. 2000, 2001: Bumps and Jumps



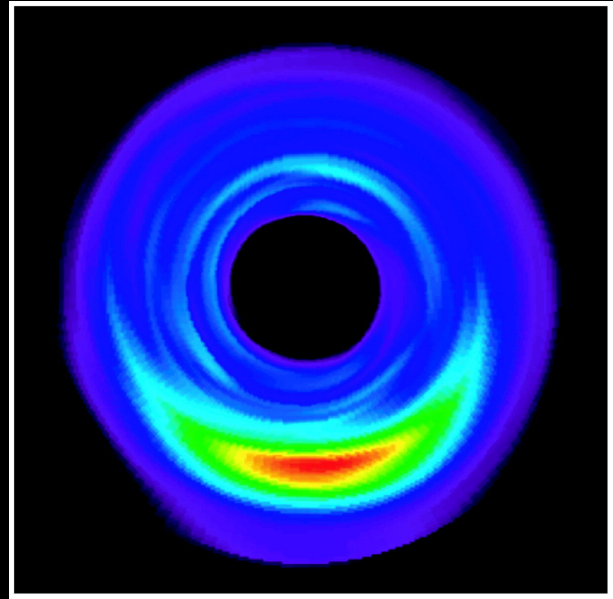
Instability results in anticyclonic vortices with large effective α but variable and global.
2D Nonlinear Simulation

Application to Disks

Klahr & Bodenheimer 2003: $s'(r) < 0$



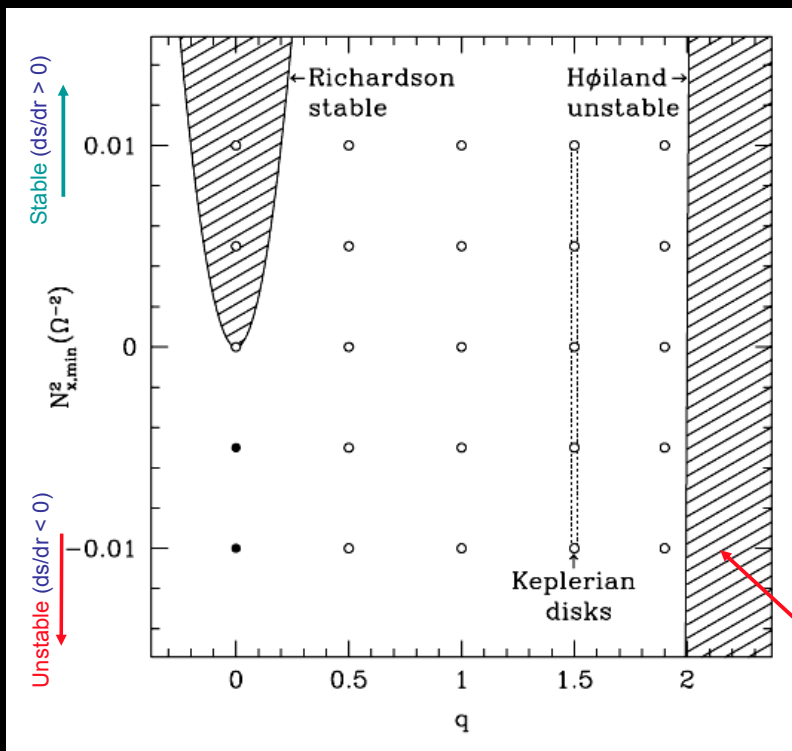
Large-scale dense anticyclonic vortex grows when $s(r) < 0$.
2D Annular Box, Radiative Hydro
No self-gravity



Same result in 3D.
3D Toroidal Grid, Radiative Hydro
No self-gravity

Application to Disks

Johnson & Gammie 2006: $s(r)$



Simulations for $\Omega \sim r^{-q}$ with radial entropy s gradients.
Only pure convective instability is detected.
Keplerian disks seem locally stable.
Cannot rule out global instability.
2D Shearing Box
Nonisentropic Equilibrium









$dj/dr < 0$

Application to Disks

Admas & Watkins 1995
Li et al. 2001
Godon & Livio 1999, 2000
Klahr & Bodenheimer 2003
Umurhu & Regev 2004
Barranco & Marcus 2005
Johnson & Gammie 2005, 2006

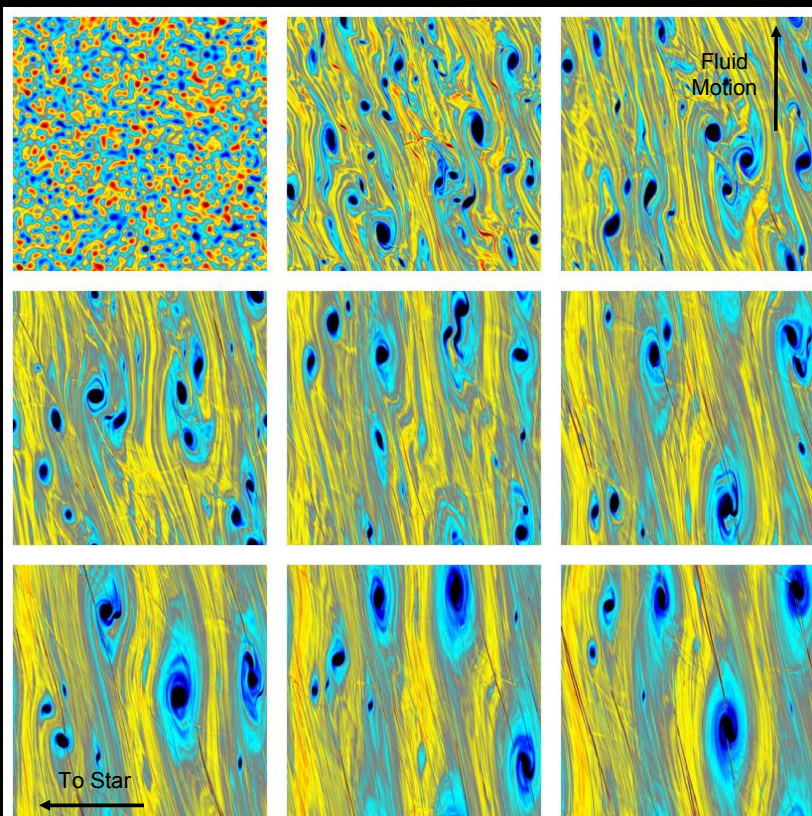
Large-Scale Structures

Vortices

-  Negative eddy viscosity
 -  large-scale vortices in shallow flows preferentially absorb smaller eddies with the same vorticity
 -  example: **Jupiter's Red Spot**
 -  thin disks might exhibit a similar phenomenon
-  Stresses
 -  the preferential sign for large-scale vorticity (anticyclonic) causes net Reynolds stresses of the right sign
-  Various instabilities just discussed
 -  could act as a source for vortical motions

Application to Disks

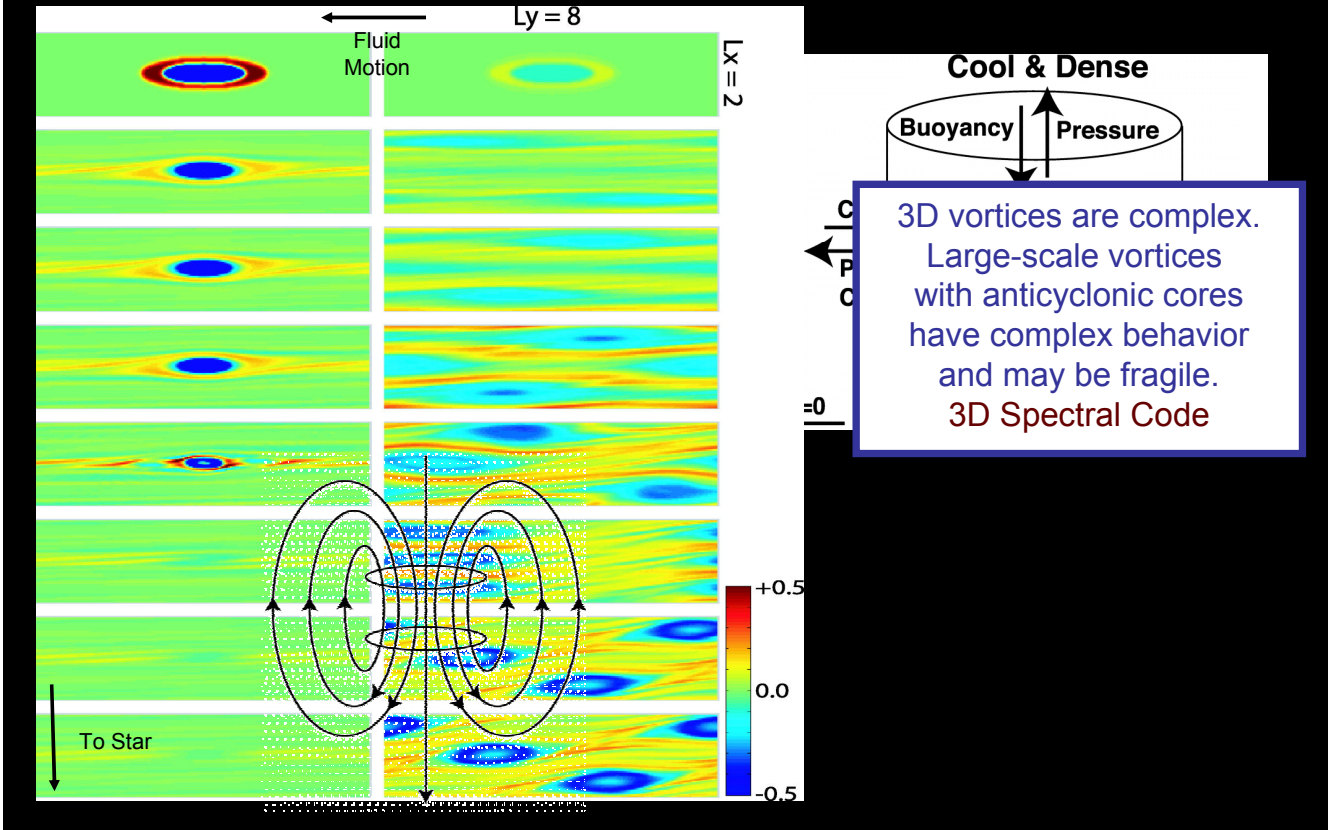
Johnson & Gammie 2005: 2D Vortex Growth



Large-scale anticyclonic vortices grow and are sustained in 2D Keplerian shear.
2D Shearing Box
Isothermal EOS
No self-gravity

Application to Disks

Barranco & Marcus 2005: 3D Vortices

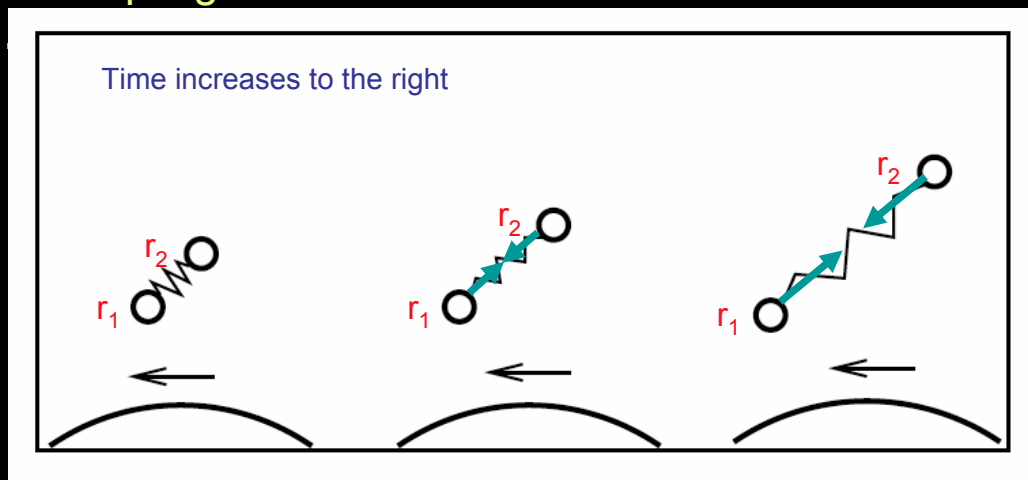


Application to Disks

Tapping the Energy of Keplerian shear



Coupling the fluid elements



what kind of 'weak springs' are available?

- ☉ B-fields \Rightarrow Magnetorotational Instability (MRI)
- ☉ disk self-gravity \Rightarrow Gravitational Instability (GI)

Conclusions

Conclusions

Summary

- ☉ **Instabilities that probably do not work**
 - ☉ Hydrodynamic instability due to shear
 - ☉ Keplerian disks are locally **stable**
 - ☉ global stability difficult to establish
 - ☉ **Convective instability in the z-direction**
 - ☉ convection probably produces only weak **inward** a.m. transport
 - ☉ lingering uncertainty about the global regime

Conclusions

Summary (cont'd)

- ☉ **Instabilities that probably do occur**
 - ☉ **Hydrodynamic instabilities due to edges**
 - ☉ Keplerian shears are **stable**
 - ☉ but localized deviations from Keplerian could occur
 - ☉ **Instabilities due to extrema**
 - ☉ extrema of some properties in bumps or edges can induce waves to grow
 - ☉ transport can be substantial
 - ☉ **Vortices**
 - ☉ large-scale anticyclonic vortices grow from vorticity noise in 2D
 - ☉ their fate in 3D is uncertain

Conclusions

Summary (cont'd)

- ☉ **Instabilities that **definitely** work**
 - ☉ **Magnetorotational instability**
 - ☉ requires a sufficiently conducting fluid
 - ☉ weak magnetic field
 - ☉ others will discuss this at length
 - ☉ **Gravitational instabilities**
 - ☉ require a disk that is sufficiently massive or cold
- ☉ **Rest of my talks**
 - ☉ The focus will be on GIs
 - ☉ But there is clearly room for further work on other instability mechanisms

