

**Vidago Workshop 2006**  
Physical Processes in Circumstellar Disks Around Young Stars

# Disk Hydrodynamics

## Talk #1: Introduction

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### PERSONAL INTRODUCTION:

What Kind of Fool Am I?

What Kind of Astrophysicist Am I?

- Numerical simulator
- Using mostly grid-based 3D hydro



How Did I End Up Studying Disks?

- I came to disk dynamics by extrapolation from global studies of rotating stars & now I have to think about local disk physics
- Relevant T-shirt slogan? “Think globally, drink locally!”

Notice: **3D, Global, Rotating, Self-Gravitating**

## BIG QUESTIONS

What Purely Hydrodynamical ( $B=0$ ) Processes Affect Disk Evolution?



-  Which can and do alter the structure of disks?
-  Which, if any, can and do affect planet formation?

Which Can Transport Mass & Ang. Mom.?

-  How efficient are they?
-  Is the transport local or global?

## STRONG PERSONAL BIAS






What Purely Hydrodynamical ( $B=0$ ) Processes Affect Disk Evolution?

-  Which can and do alter the structure of disks? **Gravitational Instabilities**
-  Which, if any, can and do affect planet formation? **GIs**













Which Can Transport Mass & Ang. Mom.? **GIs**

-  How efficient are they? **Very**
-  Is the transport local or **global**?

## Outline of Talks

-  Talk #1: Introduction
-  Talk #2: A Wonderland of Instabilities
-  Talk #3: Gravitational Instabilities I
-  Talk #4: Gravitational Instabilities II
-  Talk #5: Special Effects and the Future

## Outline of Talk #1

-  What is a disk?
  -  Thin sheet vs a flat star
-  Important processes
  -  Internal
  -  External
-  Origins of disks
  -  Rotating Cloud Collapse
  -  Turbulent Cloud Collapse
-  Evolution due to stresses
  -  Turbulent accretion disks
  -  The big three stressors
  -  Overstressed?

# What is a Disk?

Hartmann 1998

## What is a Disk?

### Disks as Thin

- ④ Zero-order axisymmetric equilibrium
  - ④ Gas orbiting a star in circular coplanar orbits
- ④ Radially smooth and dominated by
  - ④ Centrifugal balance in the r-direction
    - ④  $\partial P / \partial r \sim \partial \rho c_s^2 / \partial r \ll \rho r \Omega^2 \Rightarrow c_s \ll r \Omega$
    - ④  $M_d \ll M_s \Rightarrow$  nearly Keplerian  $\Omega = (GM_s / r^3)^{1/2}$
    - ④  $M_d < M_s \Rightarrow \Omega \approx (GM_r / r^3)^{1/2}$
  - ④ Pressure balance in the z-direction
    - ④  $\partial P / \partial z = \rho g_z \Rightarrow c_s \sim \Omega H$  where  $\Sigma \sim 2\rho H$
    - ④ and so  $c_s^2 / (r \Omega)^2 \sim (H/r)^2 \ll 1$
    - ④  $H/r < 0.1 \Rightarrow e_{\text{therm}} / e_{\text{rot}} < 0.01$



# What is a Disk?

Shakura & Sunyaev 1973  
Lynden-Bell & Pringle 1974  
Hartmann 1998  
Gammie 2001  
Balbus 2003  
Gammie & Johnson 2005

## Disks as Thin (cont'd)



### Vertically integrated equations

- Time-dependent Keplerian viscous accretion disk theory for  $\nu (= \alpha c_s H) \ll r^2 \Omega$

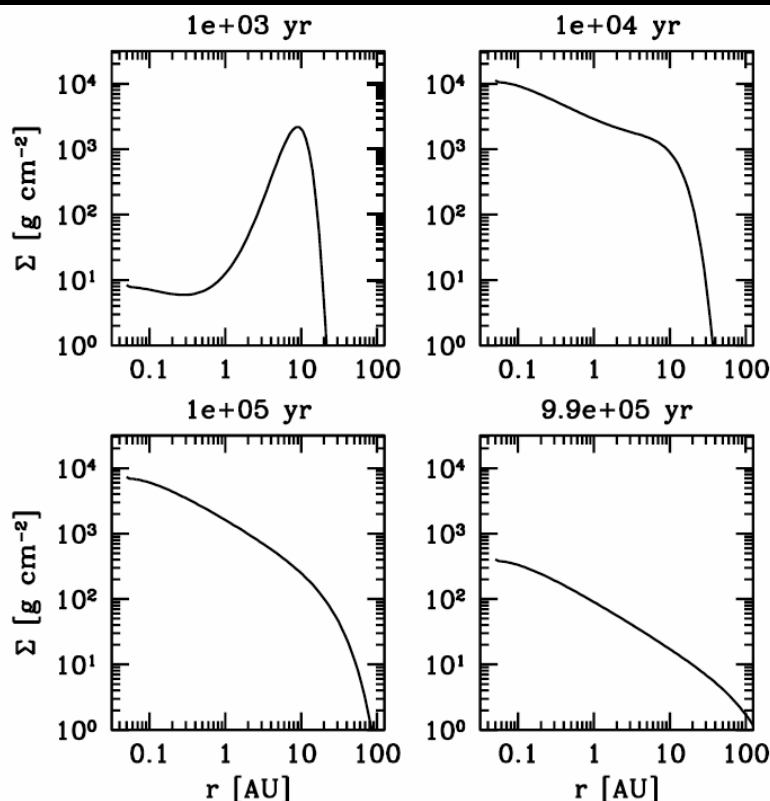
$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[ R^{1/2} \frac{\partial}{\partial R} (\nu R^{1/2} \Sigma) \right]$$

- Equation of state (low frequency)
  - $U = \int u dz$  and  $P = \int P dz$
  - $P = (\gamma - 1)u \Rightarrow P = (\Gamma - 1)U$
  - $g_z$  dominated by star:  $\Gamma = (3\gamma - 1)/(\gamma + 1)$
  - $g_z$  dominated by disk:  $\Gamma = 3 - 2/\gamma$
- Instantaneous adjustment of equilibrium in the z-direction assumed

# What is a Disk

## Simple $\alpha$ -Disk Evolution

Gammie & Johnson 2005



### Simple "Solar Nebula"

$$M_d/M_s = 0.1$$

$$M_s = 1.0 M_\odot$$











$$\alpha = 10^{-2}$$

$$\text{fixed } T \sim r^{-3/4}$$

The point here is that even this simplistic evolution is not well described as a "steady state".

# What is a Disk?












## Disks as Thick

-  **Zero-order axisymmetric equilibrium**
  -  **Centrifugal balance in the r-direction**
    -   $P_{\text{rot}} = \text{rotation period} = 2\pi/\Omega$
    -  sets the “dynamic” time scale
  -  **Pressure balance in the z-direction**
    -   $c_s/H = \text{the vertical sound crossing time} = t_z$
    -   $c_s/H \sim \Omega \Rightarrow t_z \sim P_{\text{rot}}/2\pi$
-  **Dynamic time scales are comparable in r and z regardless of H/r!**
  -  **r and z dynamics are coupled**
    -  3D disks have richer dynamics than 2D disks

# What is a Disk?

Lin & Papaloizou 1980  
 Hartmann 1998  
 Pickett et al. 1996, 2000  
 Lubow & Ogilvie 1998  
 Dullemond et al. 2006  
 Boley & Durisen 2006

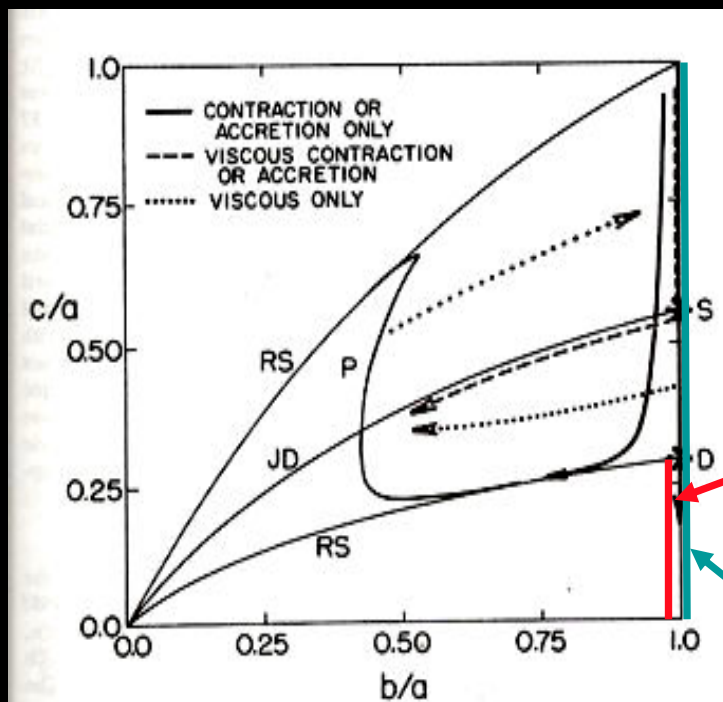
## Disks as Thick (cont'd)

-  **Critical physics**
  -  **Energy transport couples in 3D**
    -  radiation flows in both r and z
    -  convection cells & vortices  $\Delta r \sim 2H$
  -  **External effects often set a vertical B.C.**
    -  accretion
    -  irradiation
-  **Global dynamics resembles flattened star**
  -  “Density” waves are also surface distortions
-  **Vertical dynamics**
  -  Hydraulic jumps (“shock bores”)

# What is a Disk?

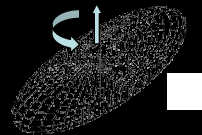
## Disks as Thick

Chandrasekhar 1969  
Tassoul 1978  
Hachisu & Eriguchi 1984  
Durisen & Tohline 1985, 2001



### Ellipsoidal Equilibrium States:

Uniformly Rotating  
Uniform Density  
Incompressible Fluids  
Semi-axes  $c \leq b \leq a$

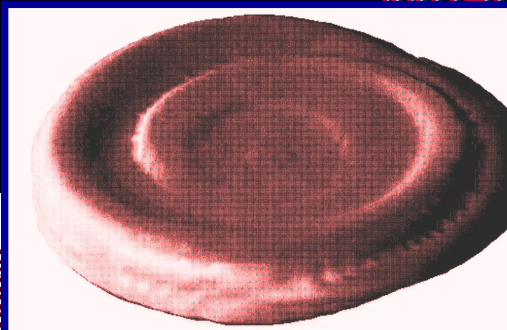


Dynamic Bar Instability  
(Kelvin or f-mode)  
 $T/|W| \geq 0.27$

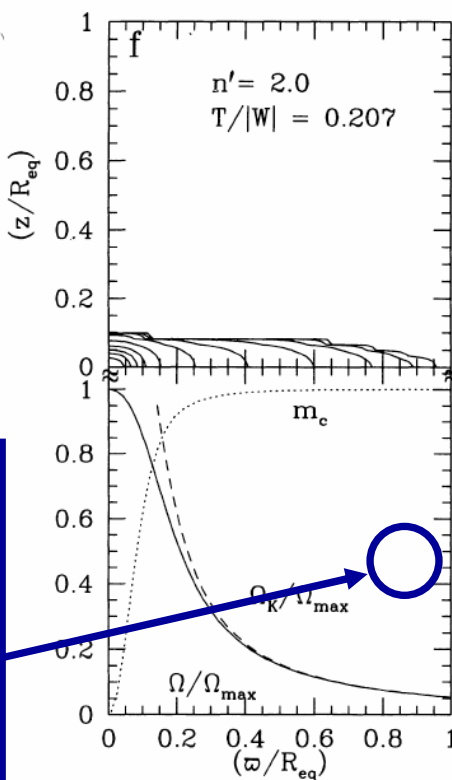
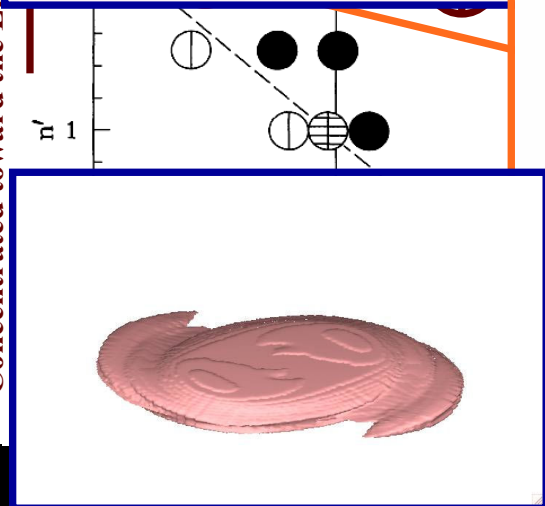
Maclaurin Spheroids  
(axisymmetric  $\Rightarrow$   
"thick" disks)

# What is a Disk?

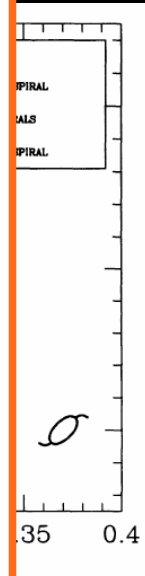
Pickett et al. 1996



Angular Momentum More  
Concentrated toward the Equator



States:  
Rotating  
 $\rho = K\rho^{5/3}$

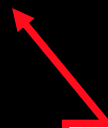


# What is a Disk?

Shock Bore

Boley & Durisen 2006

**Driven Spiral Shock  
in a High Mass  
Solar Nebula Model  
Meridional Slice**



Shock Heating  
by artificial viscosity

Animation courtesy of  
Aaron C. Boley

## Important Processes

# Important Processes

## Internal Processes



### Gravitational

- ④ Disk self-gravity
- ④ Gravity of embedded objects
  - ④ planets & planetesimals
  - ④ gravitationally bound protoplanets
  - ④ transient clumps & other structures



### Material

- ④ Growth and settling of solids
- ④ Chemistry
  - ④ gas phase & on grains
- ④ Dust
  - ④ major opacity source

# Important Processes

## Internal Processes (cont'd)



### Radiation (& radioactivity)

- ④ Energy
  - ④ deposition
  - ④ transport
- ④ Nonlocal effects
  - ④ irradiation
  - ④ shadowing
- ④ Ionization equilibrium
  - ④ thermal & nonthermal



### Thermodynamics

- ④ Equation of state
  - ④ molecular hydrogen
  - ④ effective  $\Gamma_1$

# Important Processes

## Internal Processes (cont'd)

- **Hydrodynamics & MHD**
  - **Instabilities**
    - hydrodynamical
    - gravitational
    - magnetohydrodynamical
    - turbulence due to instabilities
  - **Gas/solid interactions**
    - gas drag
    - gravity
    - differential drift & mixing of solids
  - **Shocks & shock bores**
  - **Magnetic fields**

# Important Processes

## External Processes

- **Gravitational**
  - Binary or multiple components
  - Tidal encounters
- **Material**
  - Accretion or wind outflow
  - Disk collisions
- **Radiation & energetic particles**
  - Cosmic rays
  - Central or nearby star(s)
  - Warm envelope around the star & disk
- **Global magnetic fields**

# Formation

Cassen 1994  
Hartmann 1998

## Formation

### Rotating Cloud Collapse



#### Initial cloud state



No universal agreement



Probably: centrally concentrated clumps in a turbulent MHD environment



#### General



Collapse time scale



$$t_{\text{ff}} = (3\pi/32G\rho_{\text{cl}})^{1/2} \sim 10^5 \text{ yrs}$$



Disk evolution time due to transport



$$t_{\text{ev}} \approx r^2/\nu = r^2/\alpha c_s H = (r/H)^2 P_{\text{rot}}/2\pi\alpha$$



$$t_{\text{ev}} \approx 10^4 P_{\text{rot}} \text{ for } H/r = 0.1 \text{ \& } \alpha = 10^{-2}$$



Accumulation disk where or when  $t_{\text{ff}} < t_{\text{ev}}$



Transition to accretion disk after a few  $t_{\text{ff}}$

# Formation

Shu 1977  
Cassen & Moosman 1981  
Cassen & Summers 1983  
Tereby et al. 1984

## Rotating Cloud Collapse (cont'd)



### Singular isothermal cloud in slow UR



#### Analytic solutions



$dM/dt(r,t)$ ,  $M \sim t$ , and specific a.m.  $j_d \sim t^2$



once  $r_d > r_s \Rightarrow r_d \sim j_d^2/M \sim t^3$



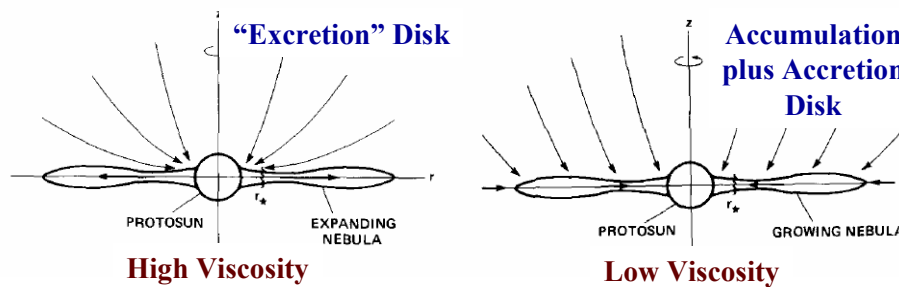
the disk grows rapidly in radius



#### High vs low viscosity



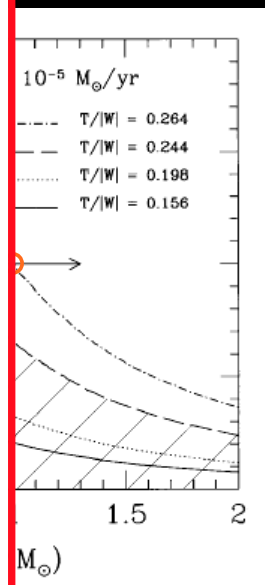
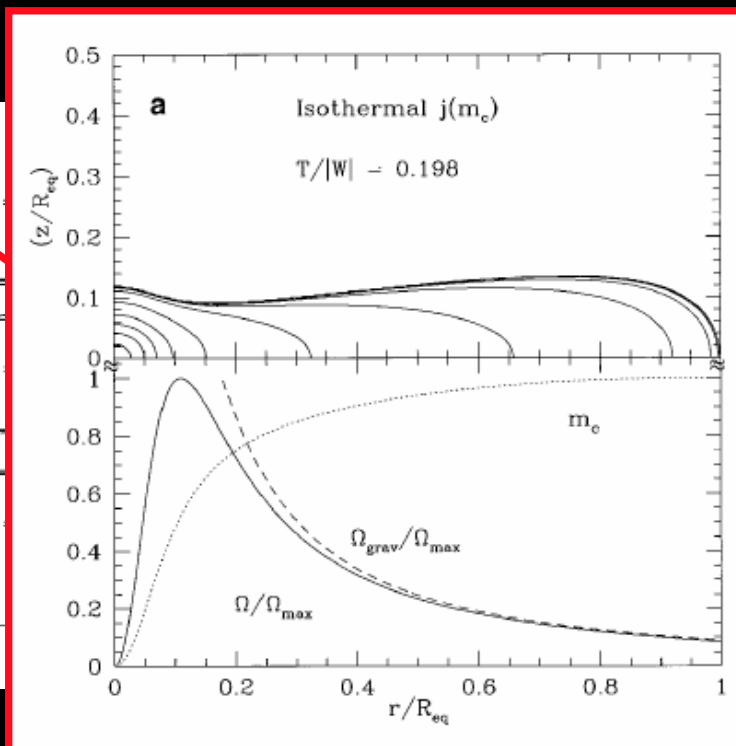
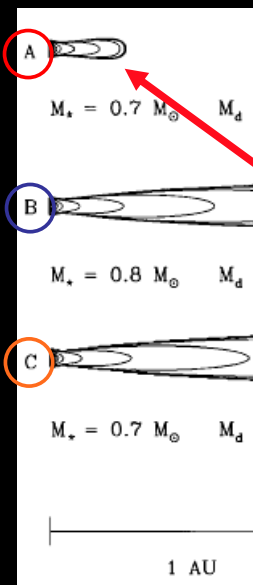
disk radius grows even faster when  $t_{ev}(r_d) < t$



# Formation

## Rotating Cloud Collapse

Stahler et al. 1984  
Yang et al. 1991  
Pickett et al. 1997



Pure Accumulation Star/Disk Systems



# Formation

Cassen 1994  
Laughlin & Bodenheimer 1994  
Bate 1998  
Yorke & Bodenheimer 1999  
Matzner & Levin 2005  
Vorobyov & Basu 2006

## Rotating Cloud Collapse (cont'd)



### More general collapses



#### Quasi-analytic

- evolutions differ depending on  $dM/dt(r,t)$ ,  $J(r,t)$ , and  $t_{ev}(r,t)$
- depend critically on assumed physics of  $t_{ev}$  if and when a disk forms



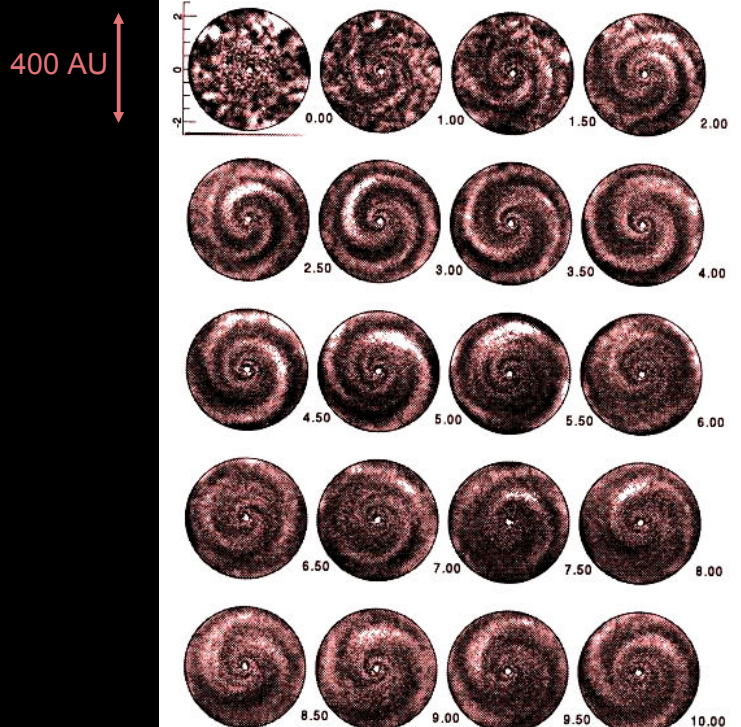
#### Numerical solutions in 3D

- sensitive to initial conditions
- sensitive to assumed physics of  $t_{ev}$
- eruption of GIs in disk may control  $t_{ev}$  in early stages of evolution
- physics is “calculated” in some recent cases

# Formation

Laughlin & Bodenheimer 1994

## Rotating Cloud Collapse



**Unstable  
Accumulated Disk:  
Collapse from  
Small UR Singular  
Isothermal Cloud**

**SPH 25,000  
particles**

One time unit  
= 477 yrs

# Formation

## Rotating Cloud Collapse

Bate 1998

100 AU



# Formation

## Turbulent Cloud Collapse

Bate & Bonnell 2005

**50 M<sub>⊙</sub> Cloud**  
**R = 0.6 pc**  
**T = 10 K**  
**M<sub>J</sub> = M<sub>⊙</sub>**  
**P(k) ~ k<sup>-4</sup>**

**SPH 3.5x10<sup>6</sup>**  
**particles**

# Disk Evolution

## Disk Evolution

### General



#### External processes



Already mentioned



#### Internal processes



##### Evolution of disk constituents



particle growth and settling



planet formation and migration



chemistry



##### Internal stresses that create torques



molecular viscosity too small



$t_{\text{ev}} \sim r_d^2/\nu > \text{Hubble time}$



turbulent stresses due to instabilities

# Disk Evolution

Shakura & Sunyaev 1973  
Lynden-Bell & Pringle 1974  
Hartmann 1998  
Gammie 2001  
Balbus 2003

## Turbulent Evolution

- **Axisymmetric equilibrium**
  - Simple  $\phi$ -independent forces
    - gravity, rotation, isotropic pressure
    - specific ang. mom.  $\Rightarrow$  orbit radius
- **Angular momentum transport**
  - Energy release by mass $\downarrow$  & ang mom $\uparrow$
  - Consider transport as a perturbation
    - deviations from equilibrium small
    - ang. mom. changes lead to orbit changes
  - Net z-components of the torque
    - requires off-diagonal stress tensor terms
    - generally requires nonaxisymmetry

# Disk Evolution

Shakura & Sunyaev 1973  
Lynden-Bell & Pringle 1974  
Hartmann 1998  
Gammie 2001  
Balbus 2003

## Turbulent Evolution (cont'd)

- **Turbulent accretion disk**
  - Consider physical quantities fluctuating about the axisymmetric equilibrium
    - $\langle f \rangle =$  "local" average in space and time
  - Thin-disk evolution equation

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left[ \frac{1}{(R^2 \Omega)'} \frac{\partial}{\partial R} (\Sigma R^2 W_{R\phi}) \right]$$

- **Keplerian viscous evol. eq. recovered by**
  - $W_{r\phi} = \alpha c_s^2 \Rightarrow \alpha$  compares  $r\phi$  stresses to gas pressure
  - $\nu = \alpha c_s^2 / \Omega = \alpha c_s H$  with Keplerian  $\Omega$

# Disk Evolution








Lynden-Bell & Kalnajs 1972  
Balbus & Papaloizou 1999  
Gammie 2001  
Balbus 2003

## Turbulent Evolution (cont'd)

### Evolution equation

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left[ \frac{1}{(R^2 \Omega)'} \frac{\partial}{\partial R} (\Sigma R^2 W_{R\phi}) \right]$$

### The Big Three Stressors











-  What stresses change z-comp of ang mom?
-  Reynolds (or Hydrodynamic) stress
  -   $W_{r\phi}^R = \langle \int dz v_r \delta v_\phi \rangle$
-  Gravitational (or Newtonian) stress
  -   $W_{r\phi}^G = \langle \int dz g_r g_\phi / 4\pi G \rho \rangle$
-  Maxwell (or Magnetic) stress
  -   $W_{r\phi}^M = - \langle \int dz B_r B_\phi / 4\pi \rho \rangle$

# Disk Evolution

Balbus & Papaloizou 1999  
Gammie 2001  
Balbus 2003

## Turbulent Evolution (cont'd)

### Local vs Global

-  Two ways to be intrinsically global
  -  #1: local treatment cannot determine  $W_{r\phi}$
  -  #2: energy dissipation is not local
-  Thin disk, local transport, in thermal equil.
  -   $\alpha = 4/[9\Gamma(\Gamma-1)t_{\text{cool}}\Omega]$
  -   $t_{\text{cool}} = U/F = \text{cooling time}$
  -  note that  $t_{\text{cool}}\Omega = \text{constant}$  is necessary for a steady state (i.e., constant accretion rate)
-  Global
  -  for either #1 or #2, alpha-disk breaks down
  -  in case #2, waves transport energy

# Disk Evolution

## Turbulent Evolution (cont'd)

- ④ Is there a maximum stress?
  - ④ Can  $W_{r\phi}$  (or  $\alpha$ ) get too large?
  - ④ What then?
    - ④ disk fragmentation?
  - ④ For gravitational stresses
    - ④  $\alpha > 0.06$  in  $t_{\text{cool}}\Omega = \text{constant}$  simulations of self-gravitating disks
    - ④ stress levels  $\alpha > 0.1$  are sustained without fragmentation for  $t_{\text{cool}} = \text{constant}$  (& are also large in global bursts)
    - ④ cooling controls gravitational fragmentation
  - ④ “She can’t take no more, Captain!”

# Disk Evolution

## Fragmenting Disk

80 AU





One time unit  
is an orp or  
Outer Rotation  
Period =  
180 years

$M_s = 1.0M_\odot$   
 $M_d/M_s = 0.14$   
 $\gamma = 7/5 \text{ gas}$   
 $t_{\text{cool}} = 1 \text{ orp}$

## Big Questions Revisited

### BIG QUESTIONS

What Hydrodynamical ( $\mathbf{B}=0$ ) Processes Really Affect Disk Evolution?

-  Which can and do alter the structure of disks?
-  Which, if any, can and do affect planet formation?

Which Can Transport Mass & Ang. Mom.?

-  How efficient are they?
-  Is the transport local or global?