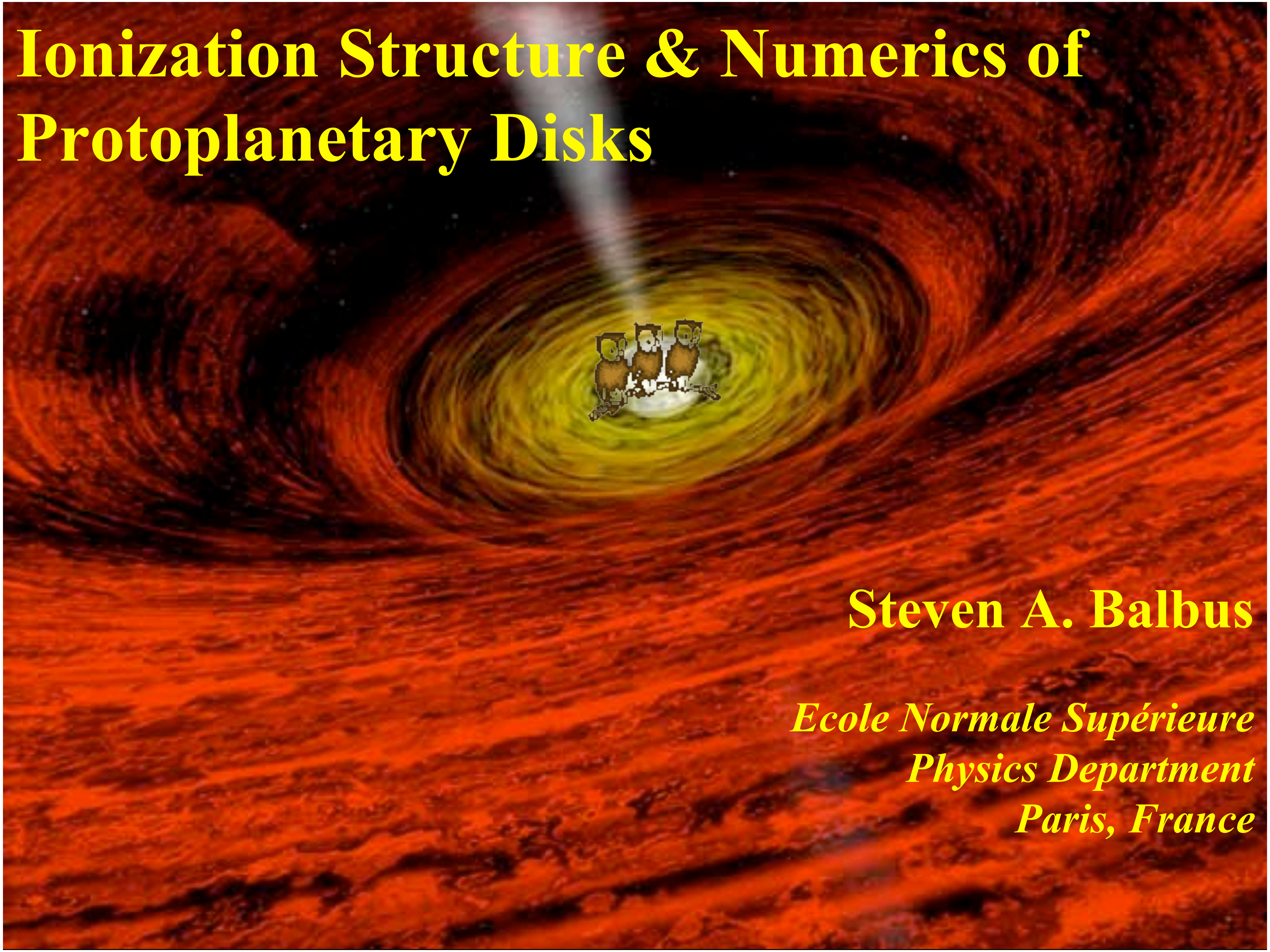
Ionization Structure & Numerics of Protoplanetary Disks



Steven A. Balbus

Ecole Normale Supérieure
Physics Department
Paris, France

FORMATION and EVOLUTION OF PROTOSTELLAR DISKS

Problem is central to understanding star and planet formation.

Initial collapse of a rotating molecular cloud is likely to be planar, not spherical, with a condensed pressurized core and an extended rotating repository of angular momentum. Identify with protostar and protostellar disk.

Early infall stages will be strongly self-gravitating. Deep spiral structure in disk effective at angular momentum transport. Disk turbulence by infall will *not* be effective, for reasons we shall see.

FUNDAMENTALS OF ACCRETION DISKS

- Modern accretion disk theory began in the 1960's from the attempts of Lynden-Bell, Rees, Thorne and others to establish observational signatures from hot gas in orbit around black holes.

- Applications of this formalism to the solar nebula (and protostellar disks more generally), pioneered by Lin and Papaloizou, began in earnest around 1980.

- A quarter of a century later, we still lack a "vanilla" model!

DISK MODEL, version 0.1

- The equation of hydrostatic equilibrium is:

$$\frac{\nabla P}{\rho} = -\nabla\Phi + R\Omega^2 \mathbf{e_R}$$

If $\Omega = \Omega(R)$, right side is exact gradient.

Constant density and constant pressure surfaces coincide.

We may thus define an enthalpy function $H(\rho) = \int dP/\rho$.

$$H(\rho) + \Phi - \int R'\Omega^2(R')dR' = \text{Constant}$$

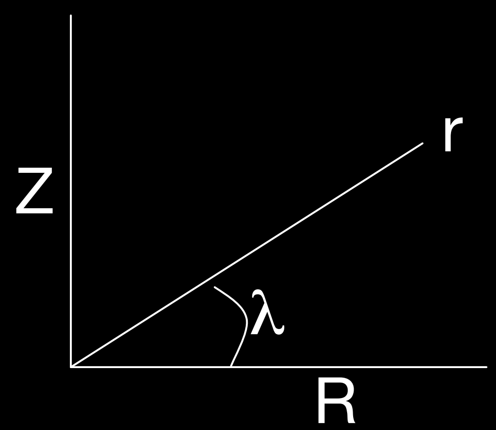
TO FIX IDEAS,

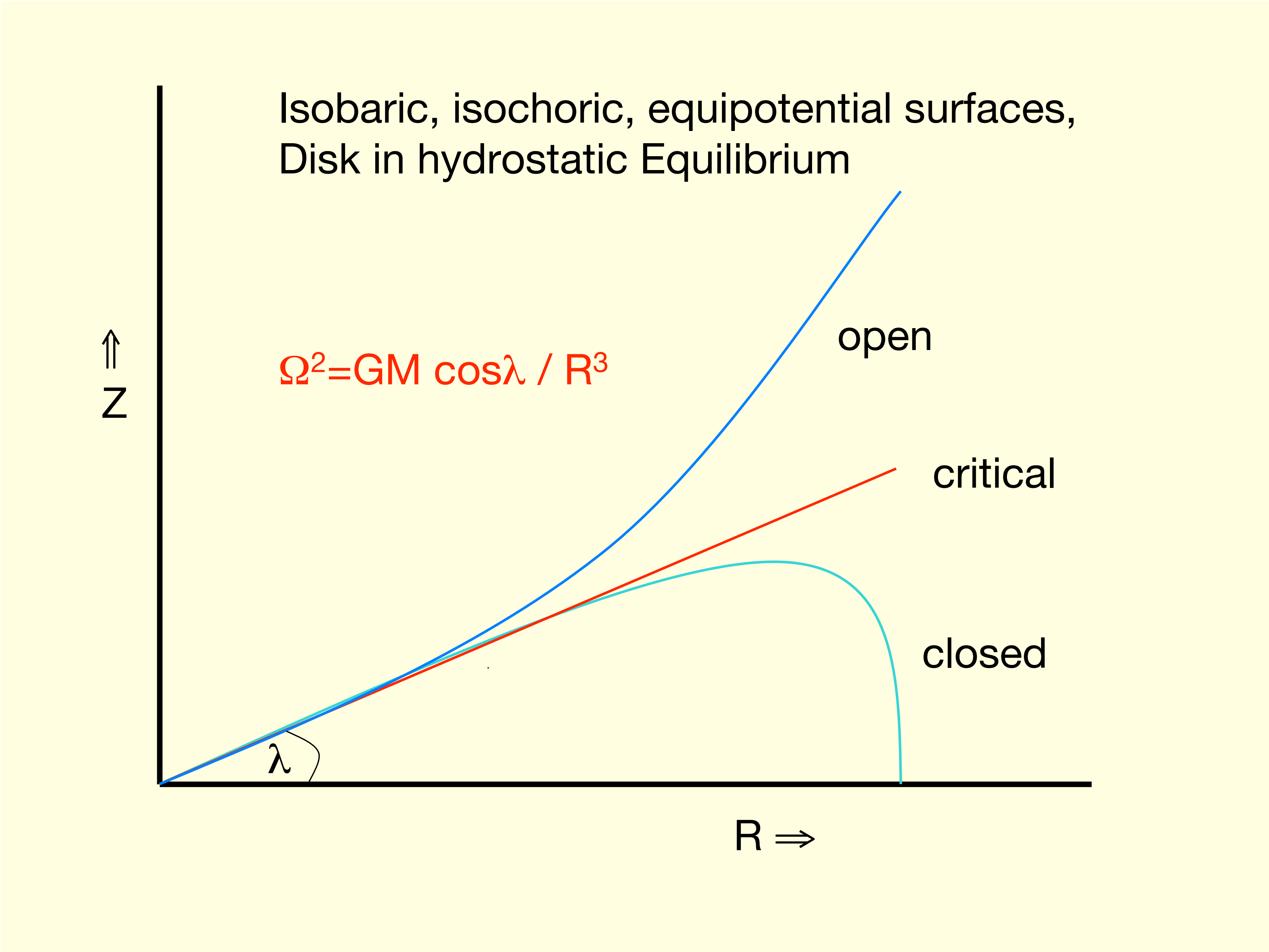
take v^2_{rot} = GM $\cos\alpha/R$, where α is a parameter. This is sub-Keplerian. Hydrostatic equation becomes:

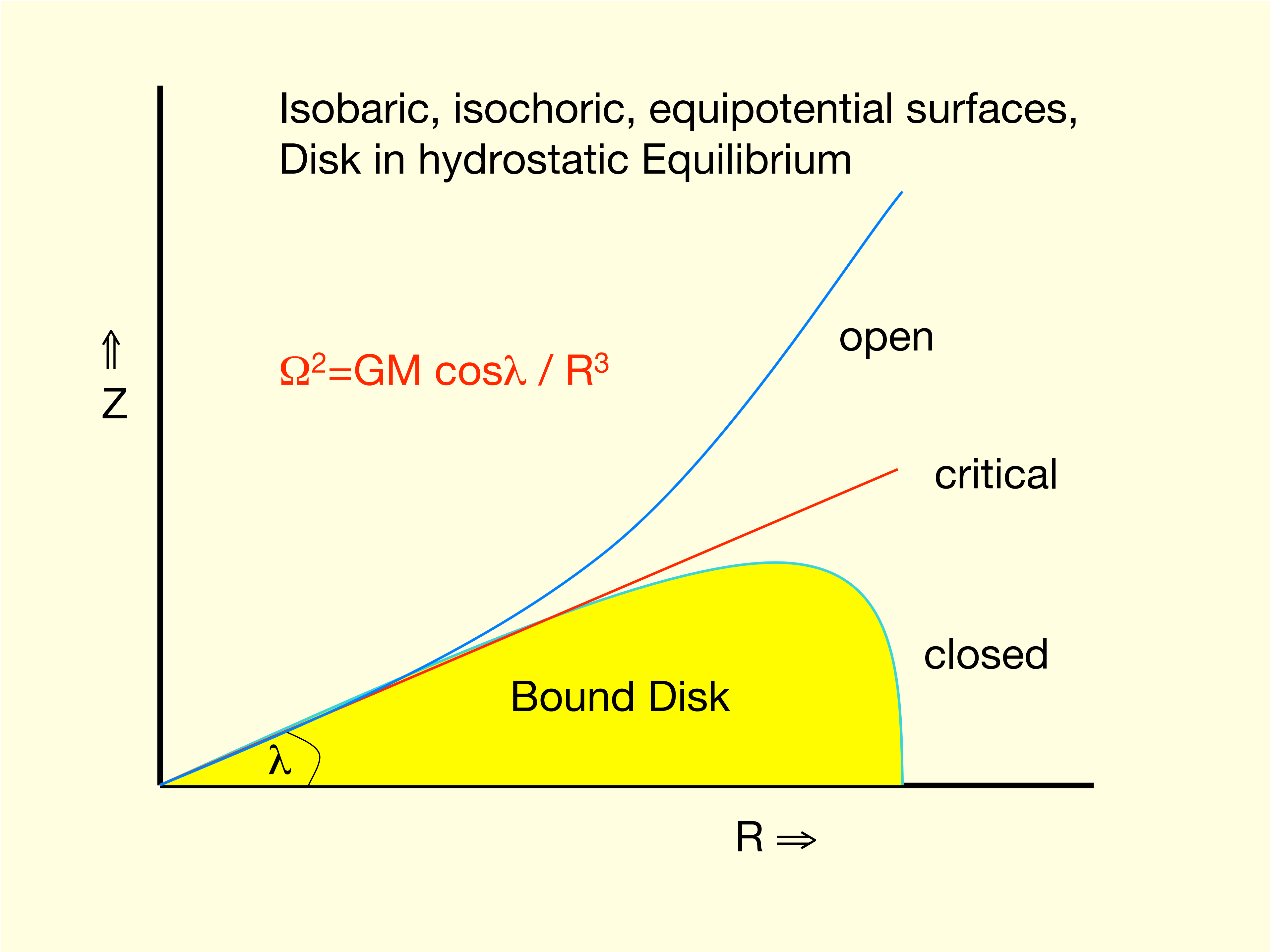
$$H(\rho) - GM/r + GM \cos\alpha/R =$$

$$H(\rho) + GM/r \; [(\cos\alpha/\cos\lambda) - 1] = \text{Constant}$$

(Note that accretion velocity does not enter into HSE.)




Isobaric, isochoric, equipotential surfaces,
Disk in hydrostatic Equilibrium
Ω^2=GM cosλ / R^3
open
critical
closed
$\Uparrow$ Z
R $\Rightarrow$
λ


Isobaric, isochoric, equipotential surfaces,
Disk in hydrostatic Equilibrium
Ω^2=GM cosλ / R^3
⇑
Z
open
critical
closed
Bound Disk
λ
R ⇒

VERSION 0.2: VERTICAL STRUCTURE

If the free energy of differential rotation is extracted, dissipated, and radiated locally, the fundamental equation is:

$$\nabla \bullet \boldsymbol{F}_{rad} = - T_{R\phi} \quad d\Omega/d \ln R \quad ,$$

Where $T_{R\phi} = \langle \rho(u_R \, u_\phi - u_{AR} \, u_{A\phi}) \rangle$ is a mean averaged correlation. In alpha models, $T_{R\phi} = \alpha$ P, with α constant. The equations states that:

$$\int \boldsymbol{F}_{rad} \bullet \boldsymbol{dA} = - \int T_{R\phi} \quad d\Omega/d \ln R \; dV$$

VERSION 0.2: VERTICAL STRUCTURE

Recall:

$$\boldsymbol{F}_{\boldsymbol{rad}} = - \;\; [16\sigma T^3/3\kappa_r\rho\,] \;\; \partial T/\partial z \;\; \boldsymbol{e}_{\boldsymbol{z}}$$

where κ_r is the opacity ($\kappa_r \sim 1 \;\; \mathrm{cm}^2\ \mathrm{g}^{-1}$). Hence,

$$\partial/\partial z \;\; [16\sigma T^3/3\kappa_r\rho \quad \partial/\partial z\,]\, T = \; T_{R\phi} \;\; d\Omega/d\ln R$$

$$= \; \alpha\, P \;\; d\Omega/d\ln R$$

Consider the case of constant opacity κ_r . . .

VERSION 0.2: VERTICAL STRUCTURE

With optical depth $d\tau = - \kappa_r \rho \, dz$, equation takes the form :

$$\boxed{d^2y/ d\tau^2 = - A \, y^{1/4},}$$

$y=T^4$, $\quad dy/d\tau = (3/4\sigma) \, \boldsymbol{F_{rad}}$, $\quad A=(3\alpha k/4\sigma\kappa_r m) \; |d\Omega/d \ln R|$

Integrating, $\quad (dy/d\tau)^2 = - (8A \, T^5/5) + \mathrm{CONST}.$

At the disk surface $(dy/d\tau)^2 = (3T_{eff}^4 \; /4)^2$, hence :

$\mathrm{CONST} = (8A \, T_{eff}^5 \; /5) + (3T_{eff}^{\,4} /4)^2$, and

$$(dy/d\tau)^2 = (8A/5) \; (T_{eff}^5 - T^5) + (3T_{eff}^4 \; /4)^2$$

VERSION 0.2: VERTICAL STRUCTURE

$$(dy/d\tau)^2 = (8A/5)\ (T_{eff}^{5} - T^5) + (3T_{eff}^{4}\ /4)^2$$

T gradient $\Downarrow$ as T $\Uparrow$; T maximum when

$$T_{MAX}^{5} = T_{eff}^{5} + (128A/45)^{-1}\ T_{eff}^{8}$$

At R =1 AU, M = 1 $M_\odot$, this gives:

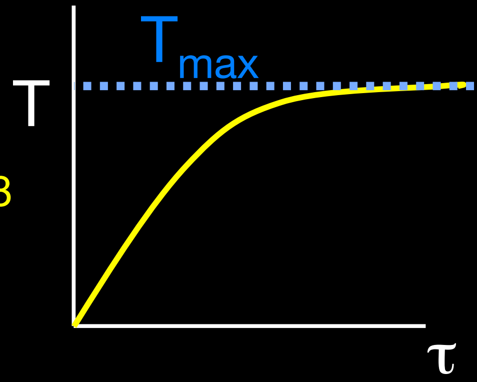
$$T_{MAX} \sim 600\ (\kappa_r / \alpha_{-2})^{1/5}\ (T_{eff} / 152)^{8/5}\ K$$

Interesting, because T > 1000K required for self-consistent MRI . . .

VERSION 0.2: VERTICAL STRUCTURE

$$(dy/d\tau)^2 = (8A/5)\ (T_{eff}^5 - T^5) + (3T_{eff}^4\ /4)^2$$

T gradient $\Downarrow$ as T $\Uparrow$; T maximum when



$$T_{MAX}^5 = T_{eff}^5 + (128A/45)^{-1}\ T_{eff}^8$$

At R =1 AU, M = 1 $M_\odot$, this gives:

$$T_{MAX} \sim 600\ (\kappa_r / \alpha_{-2})^{1/5}\ (T_{eff} / 152)^{8/5}\ \ K$$

Interesting, because T > 1000K required for self-consistent MRI . . .

OTHER ASSUMPTIONS ARE POSSIBLE:

α models are not the only possibility---nor are they are necessarily the most promising.

«β models» (Zahn & coworkers) are based on the prescription

$$T_{R\phi} = \beta\,(R\, d\Omega/\, d \ln R)^2 \ , \ \beta = \text{const.}$$

Invoked for hydrodynamic turbulence, scaling might just apply to MRI saturation as well.

Numerics suggest $v_A \approx$ constant in space. (Galaxy as well…)

Then $T_{R\phi} = \gamma\, \rho, \quad \gamma =$ a ***dimensional*** const.

A (SAMPLE) COMPLETE MODEL:

$$\sigma\, T_{eff}^4 = 3\, GM\, M_{DOT} / 8\pi R^3 = \alpha\, c_S^2\, (3/4)\, \Omega\Sigma$$

$$T_{MAX}^5 = T_{eff}^5 + (128A/45)^{-1}\quad T_{eff}^8$$

$$A \equiv (3\alpha k / 4\sigma\kappa_r m)\ \ |d\Omega/d\ \ln R|$$

Three equations to solve for T_{eff} , T_{MAX} , and Σ, given α and M_{DOT}.

A (SAMPLE) COMPLETE MODEL:

$\sigma T_{eff}^4 = 3\,GM\,M_{DOT} / 8\pi R^3 = \alpha\, c_S^2\,(3/4)\,\Omega\Sigma$

$T_{MAX}^5 = T_{eff}^5 + (128A/45)^{-1}\;\; T_{eff}^8$

RESULTS: $T_c = 589\text{ K}\;(\alpha_{-2})^{-1/5}\;\;\kappa_r^{1/5}\;\;M_{DOT-7}^{2/5}\;\;R_{AU}^{-9/10}$

$T_{eff} = 152\text{ K}\;\;M_{DOT-7}^{1/4}\;\;R_{AU}^{-3/4}$

$\Sigma = 965\text{ g cm}^{-2}\;(\alpha_{-2})^{-4/5}\;\;\kappa_r^{-1/5}\;\;M_{DOT-7}^{3/5}\;\;R_{AU}^{-3/5}$

$\rho = 6.68\times 10^{-10}\text{ g cm}^{-3}\;(\alpha_{-2})^{-9/10}\;\;\kappa_r^{-3/10}\;\;M_{DOT-7}^{2/5}\;\;R_{AU}^{-33/20}$

POSSIBLE IONIZATION SOURCES

1. Cosmic Rays (Gammie 1996, Sano et al. 2000)

2. Radioactive Isotopes (many studies)

3. Chromospheric X-rays (Igea & Glassgold 1999, Fromang et al. 2002, Ilger & Nelson 2006abc)

Cosmic rays uncertain, excluded by stellar winds and possibly by plasma instabilities. Radioactivity ultimately too small. X-rays are both a potent source *and* observed.

IONIZATION STRUCTURE OF α-MODELS

(Fromang et al. 2002, Ilger & Nelson 2006a)

1. Ionization depends upon details of chemistry (nature of atomic and molecular species) as well as presence of dust grains.

2. Simple models tend to be to "optimistic": predict higher levels and more extended active zone.

3. Submicron dust grains are MRI-killers, chemistry less important.   But:

4. Suppresing the MRI leads to quiesence and dust settling which turns the MRI back on, which mixes the dust back in…!

Table A.1. List of gaseous species including free electrons and grain particles with different electric excess charges.

H^+	H	H_2	H_2^+	H_3^+
He	He^+	C	C^+	CH
CH^+	CH_2	CH_2^+	N^+	N
CH_3^+	NH^+	CH_3	NH	CH_4^+
NH_2^+	O	NH_2	O^+	CH_4
OH	NH_3	NH_3^+	OH^+	CH_5^+
H_2O	NH_4^+	H_2O^+	H_3O^+	Mg
C_2	C_2^+	Mg^+	C_2H^+	C_2H
CN	C_2H_2	CN^+	$C_2H_2^+$	HCN
$C_2H_3^+$	HNC	C_2H_3	HCN^+	H_2NC^+
Si^+	$HCNH^+$	CO^+	CO	Si
N_2^+	N_2	$C_2H_4^+$	SiH^+	HCO^+
HCO	SiH	HN_2^+	NO^+	H_2CO^+
SiH_2	NO	SiH_2^+	H_2CO	H_3CO^+
SiH_3	SiH_3^+	S^+	CH_3OH^+	SiH_4
CH_3OH	O_2^+	S	SiH_4^+	O_2
HS^+	$CH_3OH_2^+$	HS	SiH_5^+	H_2S
H_2S^+	H_3S^+	C_3	C_3^+	C_3H
C_3H^+	C_3H_2	$C_3H_2^+$	C_2N^+	CNC^+
C_3H_3	$C_3H_3^+$	C_3H_4	SiC^+	$C_3H_4^+$
SiC	C_2O^+	$HCSi$	HC_2O^+	$HCSi^+$
CH_3CN^+	$C_3H_5^+$	CH_3CN	CH_2CO	SiN^+
SiN	CH_2CO^+	$H_4C_2N^+$	CH_3CO^+	$HNSi$
$HNSi^+$	CO_2^+	SiO^+	CS	SiO
CO_2	CS^+	HCS	HCO_2^+	$SiOH^+$
HCS^+	NS^+	NS	H_2CS	H_2CS^+
H_3CS^+	HNS^+	C_4^+	SO^+	SO
C_4	HSO^+	C_4H^+	C_4H	C_3N
$C_4H_2^+$	C_3N^+	C_4H_2	HC_3N^+	HC_3N
C_3O^+	C_3O	$H_2C_3N^+$	HC_3O^+	$C_3H_2O^+$
$H_3C_3O^+$	C_2S^+	C_2S	Fe^+	Fe
HC_2S^+	SiS	OCS^+	SiO_2	SiS^+
OCS	$HSiS^+$	$HOCS^+$	SO_2^+	SO_2
S_2	HSO_2^+	$H_4C_4N^+$	$H_2S_2^+$	C_3S
C_3S^+	HC_3S^+	C_7^+	e^-	gr
gr^{2-}	gr^-	gr^+	gr^{2+}	

List of gaseous species from Ilger & Nelson 2006a

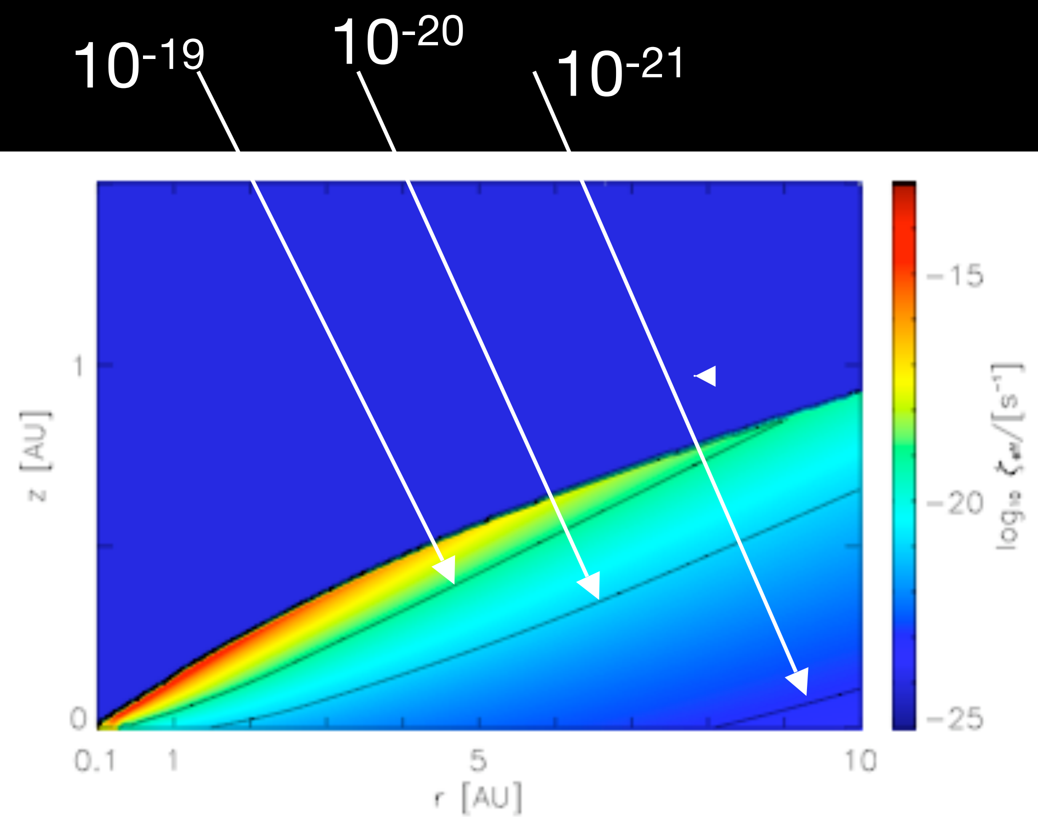


Fig. 6. The effective X-ray ionisation rate $\zeta_{\rm eff}$ per hydrogen nucleus (including contributions due to thermal ionisation of potassium). The disk parameters are $\alpha = 10^{-2}$ and $\dot{M} = 10^{-7}\ M_\odot\,{\rm yr}^{-1}$. The contour lines refer to values of $\zeta_{\rm eff}$: 10^{-19}, 10^{-21}, and $10^{-23}\ {\rm s}^{-1}$.

Ilger and Nelson standard model of ionization structure

ACTIVE--DEAD ZONE INTERFACE

The boundary between the active and dead zones is unstable.

In a turbulent system, a classical method of modeling the fluctuation amplitude y is to use a "Landau Equation:"

$dy/dt = [\gamma - \eta(T)]\, y - Ay^3$

Where γ is a linear growth rate, η a damping rate --- here T dependent! --- and A is a nonlinear saturation term, possibly T dependent.

Cooling: $dT/dt = Wy^2 - C(T)$

ACTIVE--DEAD ZONE INTERFACE

The boundary between the active and dead zone is unstable.

In a turbulent system, a classical method of modeling the fluctuation amplitude y is to use a "Landau Equation:"

$$dy/dt = [\, \gamma - \eta(T) \,]\, y - Ay^3$$

Where γ is a linear growth rate, η a damping rate --- here T dependent! --- and A is a nonlinear saturation term, possibly T dependent.

Cooling: $dT/dt = Wy^2 - C(T)$

$-T_{R\phi}\, d\Omega/d\ln R$

ACTIVE--DEAD ZONE INTERFACE

The boundary between the active and dead zone is unstable.

In a turbulent system, a classical method of modeling the fluctuation amplitude y is to use a "Landau Equation:"

$$dy/dt = [\gamma - \eta(T)]\, y - Ay^3$$

Where γ is a linear growth rate, η a damping rate --- here T dependent! --- and A is a nonlinear saturation term, possibly T dependent.

Cooling: $dT/dt = Wy^2 - C(T)$

T-dependent cooling function

ACTIVE--DEAD ZONE INTERFACE

The boundary between the active and dead zone is unstable.

For protostellar disks, $\eta(T) = 234\ T^{1/2}\ (n_n / n_e)\ cm^2\ s^{-1}$, where n_n/n_e is *very* T-sensitive.

Result: $\eta(T) \sim T^{-1/4} \exp(T_0 / T)$.   Take A=constant.

The steady state $\gamma = \eta(T_0)$, $W\ y_0^2 = C(T_0)$ is ***unstable*** if either

$\partial C/\partial T < 0$, or $\partial \eta/\partial T < 0$, (Balbus & Lesaffre 2006).

Active-inactive interface is therefore a region of instability!

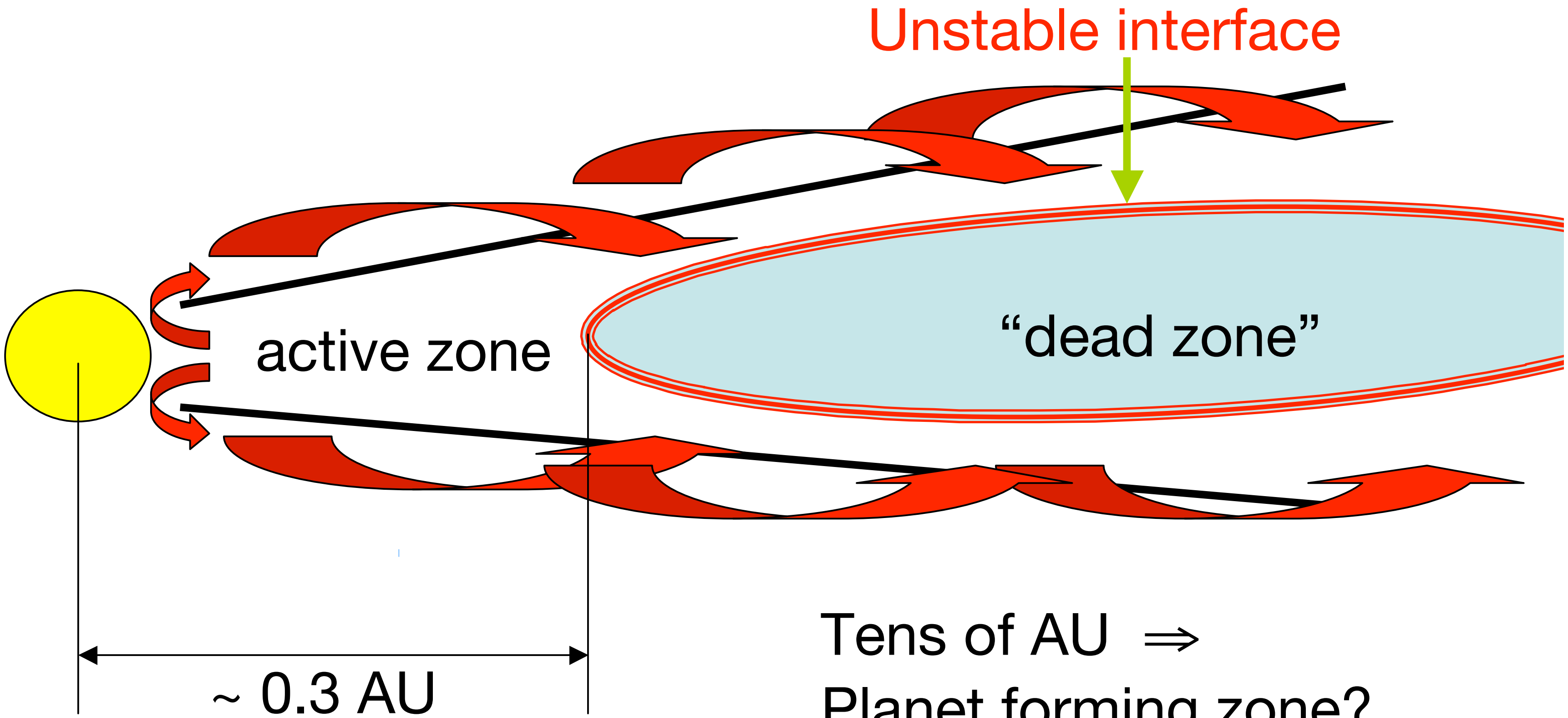
ACTIVE--DEAD ZONE INTERFACE

The physical reasoning is simple:

If $\partial C/\partial T < 0$, the cooling increases as the temperature decreases, T decreases more, a runaway ensues (Field 1965.)

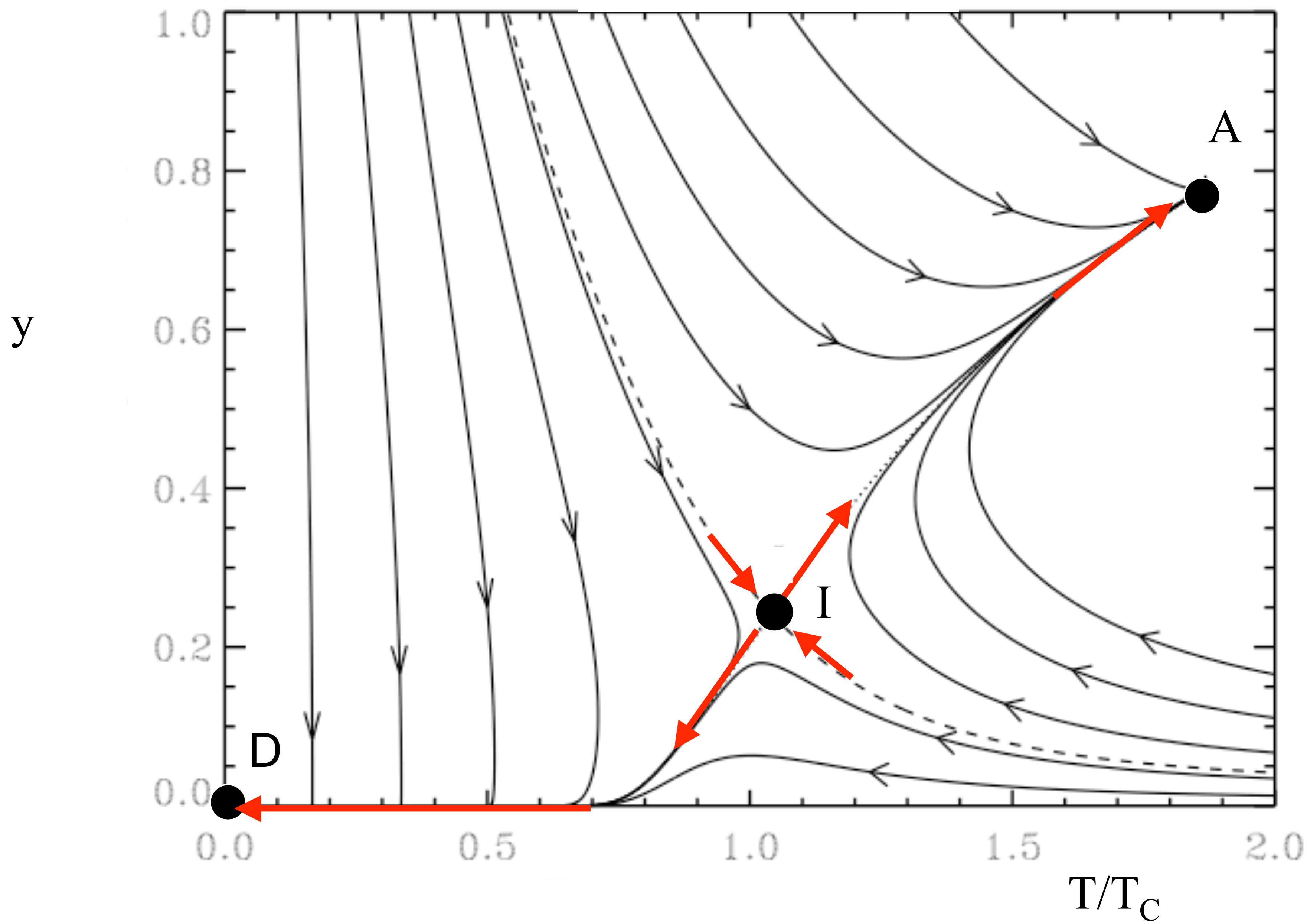
If $\partial \eta/\partial T < 0$, increasing the temperature decreases the resistivity and ***raises*** the fluctuation amplitude y. The turbulent heating then goes up, which raises the temperature even more.

This behavior is not limited to protostellar disks:
Dwarf Novae may also have dead zones. Both DN and PSD show outbursts.



INNER REGIONS OF SOLAR NEBULA

Nonlinear Calculation by P. Lesaffre Phase space



SUMMARY

1. If accretion velocity is small, v_{rot} (R), gross disk structure is relatively simple.

2. Accretion mass flux may exhibit complicated structure.

3. Ionization is *certainly* complex. Sensitive to molecular chemistry and grain physics.

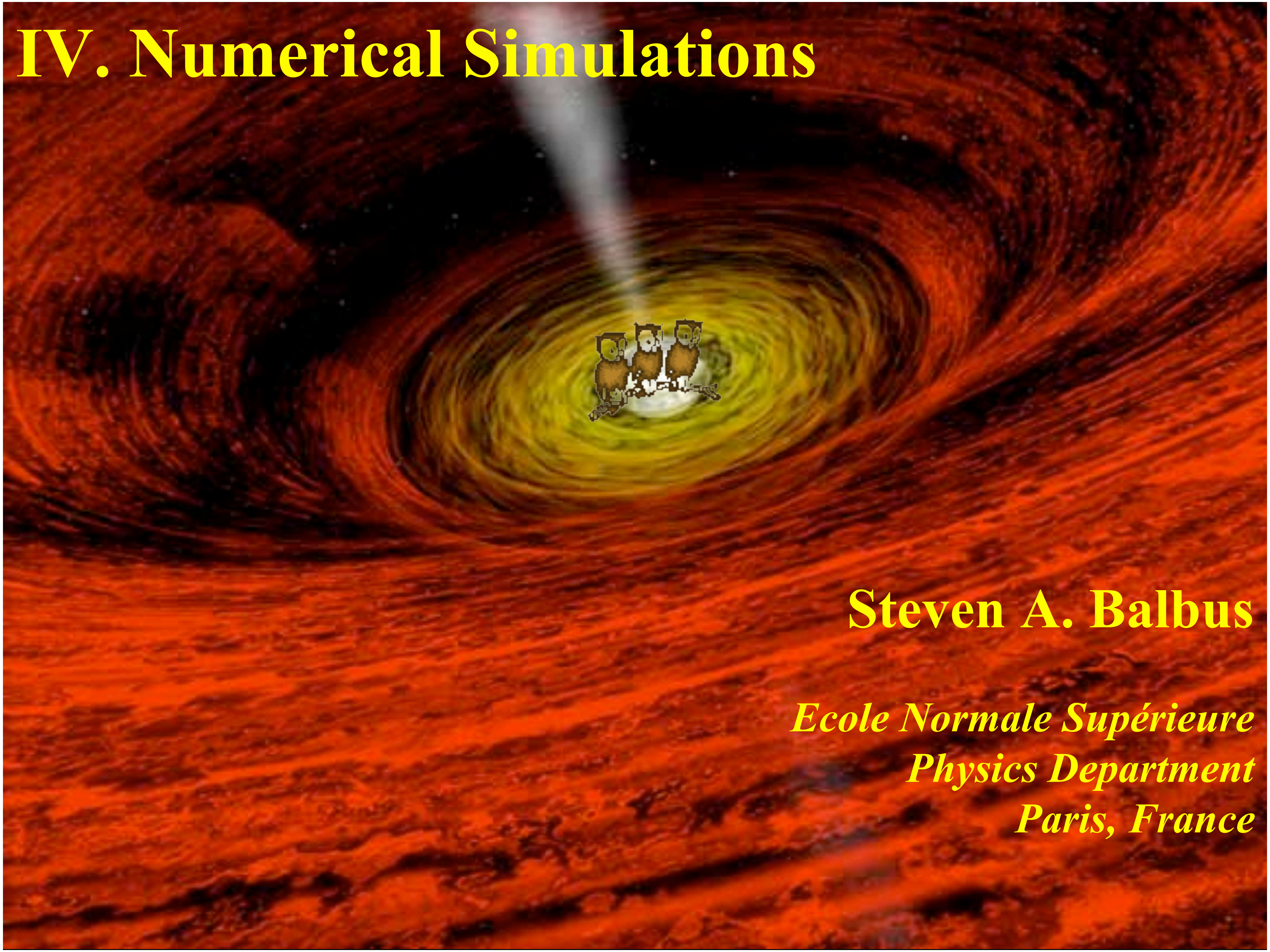
4. Grains kill MRI, but killing MRI settles grains. Self-consistent solution needed.

5. Despite uncertainty, magnetic decoupling is likely on scales of 10 AU. Vertical, temporal extent uncertain.

SUMMARY

6. Active-decoupled boundary is unstable, and may be the site of eruptive behavior, and different disk states.

IV. Numerical Simulations



Steven A. Balbus

Ecole Normale Supérieure
Physics Department
Paris, France

EFFECTS OF MHD TURBULENCE

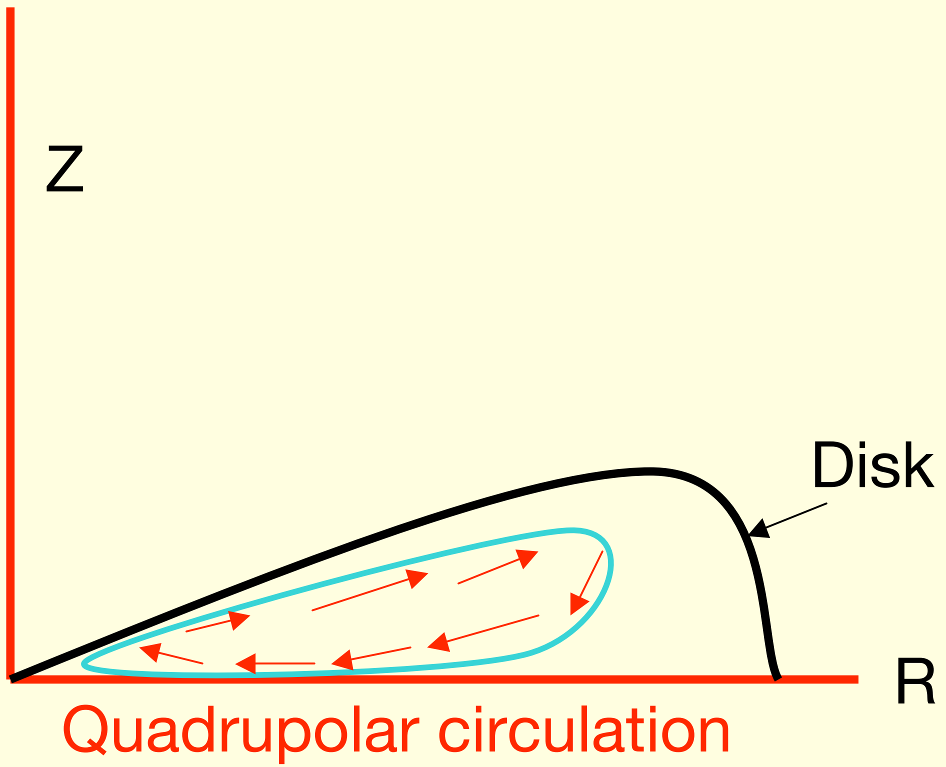
1. Self-consistent models involve HSE and a decoupled mass conservation equation, $\nabla \bullet (\rho \mathbf{v}) = 0.$

2. Simplest models take a radial mass flux, $\rho r^2\ v_R = \mathrm{Const.}$

3. An obvious generalization is to try $\rho \mathbf{v} = \nabla\Phi,\ \nabla^2\Phi=0$ with
$$\Phi \sim\ 1/r + P_2(\cos\theta)\ /\ r^3\ + \ldots$$

4. Quadrupolar structure is natural because turbulent forcing is quadrupolar, $\langle v_i\ v_j - v_{Ai}\ v_{Aj}\rangle\ \partial v_i/\partial v_j$. Non-alpha.

5. Seen in simulations. Potential importance for *mixing.*


Z
Disk
R
Quadrupolar circulation

NUMERICAL SIMULATIONS OF PROTOSTELLAR DISKS

NUMERICAL SIMULATIONS OF PROTOSTELLAR DISKS

Only the tiniest pieces have been attempted: Ohmic dissipation, Hall MHD (Stone, Sano and coworkers).

WHY NOT MORE?

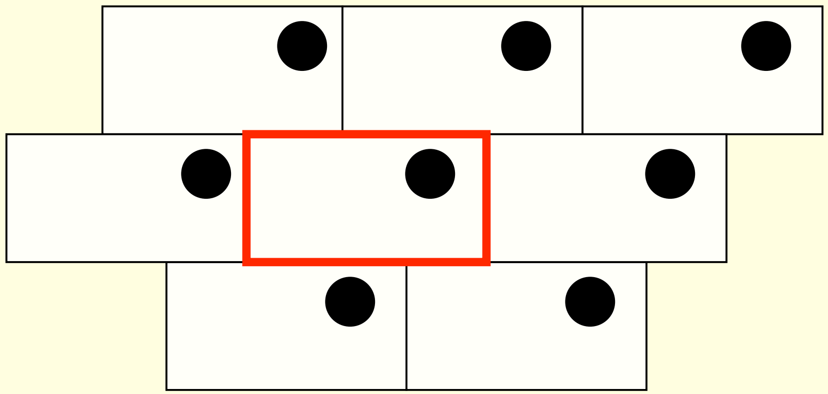
1. Dynamic range requirements.

2. Chemistry coupled to dynamics is not yet feasible, even in very local models.

3. Radiation dynamics is in its infancy (Hirose, Krolik & Stone 2006).

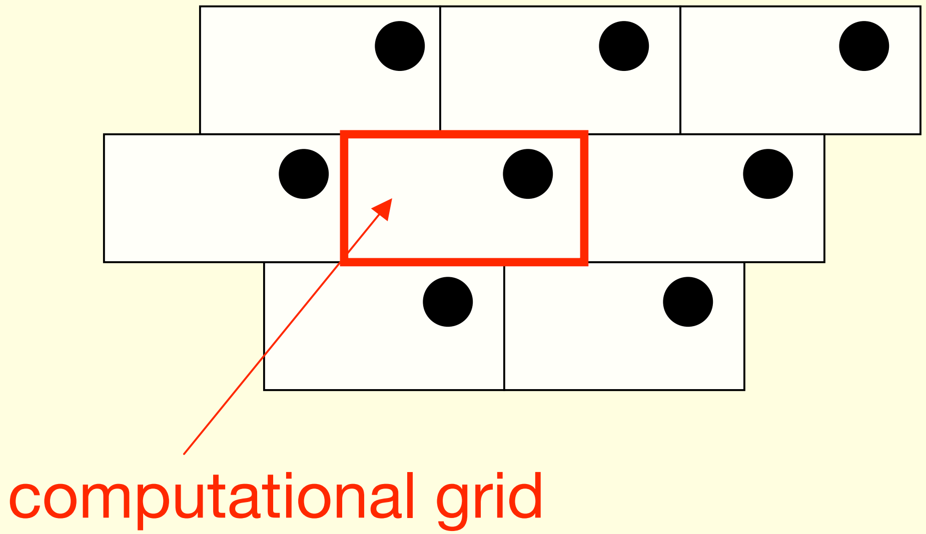
Long history of MHD polytropic simulations.
Let us start with the “Shearing Box:”

LOCAL SHEARING BOX SIMULATION:



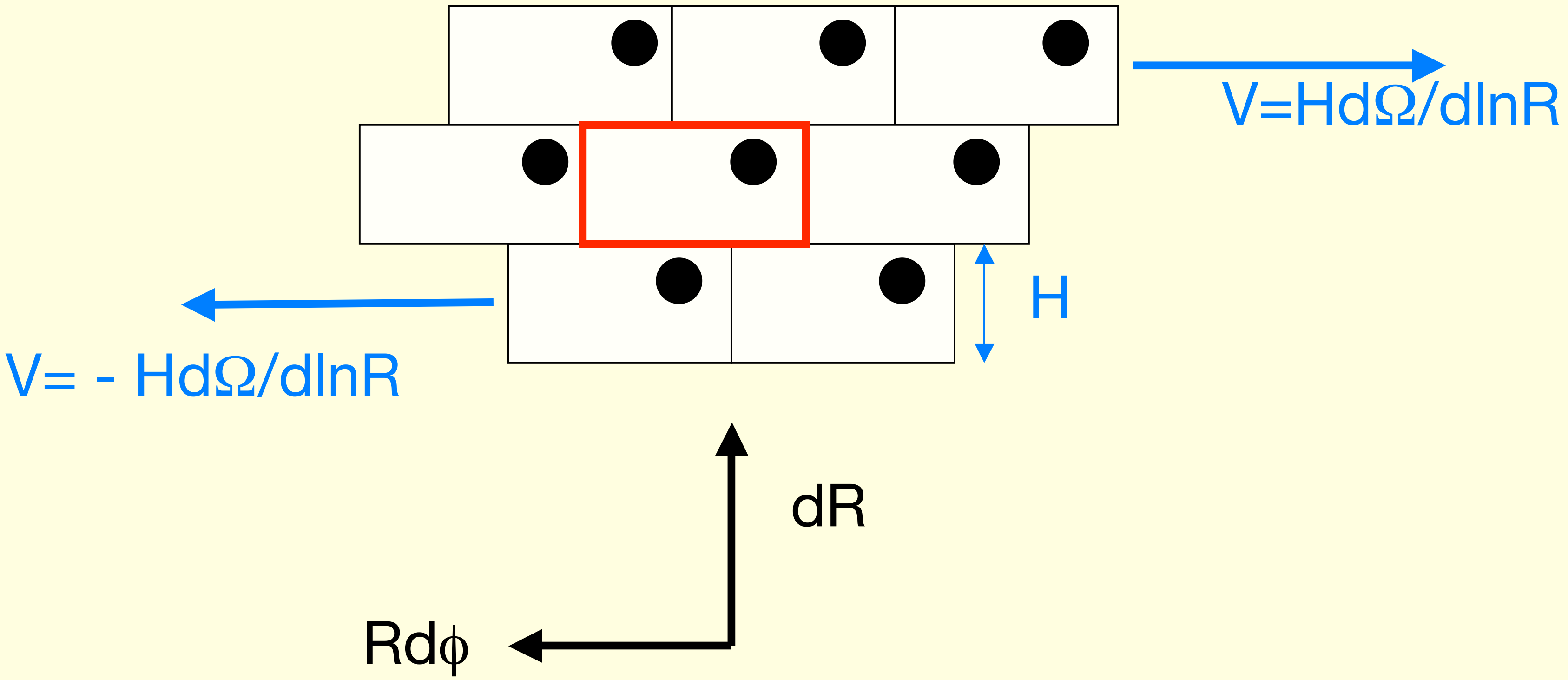
Ref: Hawley, Gammie, & Balbus 1995

LOCAL SHEARING BOX SIMULATION:



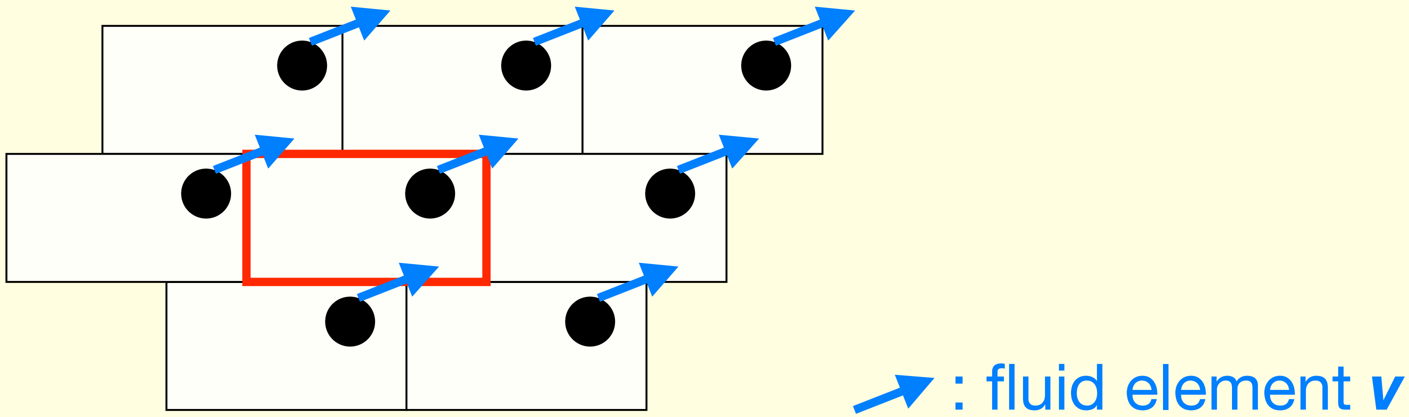
Ref: Hawley, Gammie, & Balbus 1995

LOCAL SHEARING BOX SIMULATION:



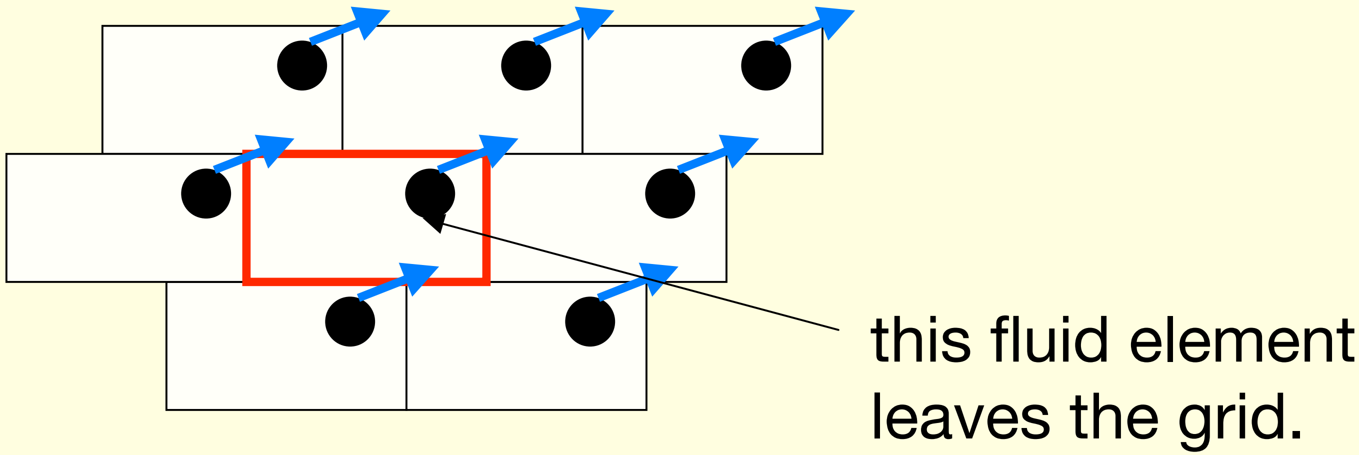
Ref: Hawley, Gammie, & Balbus 1995

LOCAL SHEARING BOX SIMULATION:



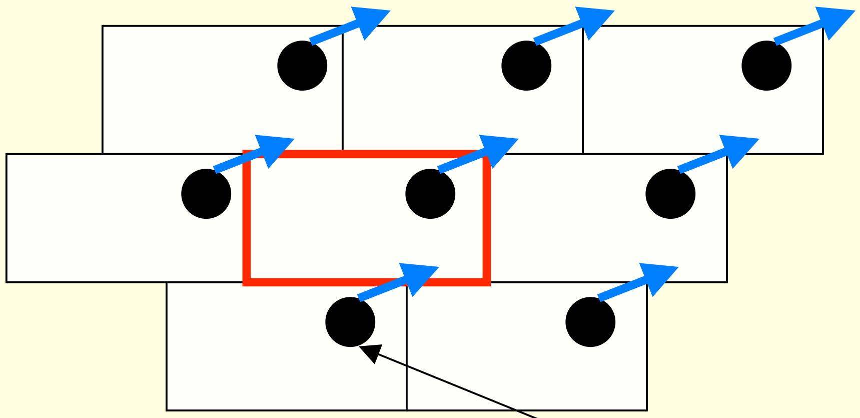
Ref: Hawley, Gammie, & Balbus 1995

LOCAL SHEARING BOX SIMULATION:



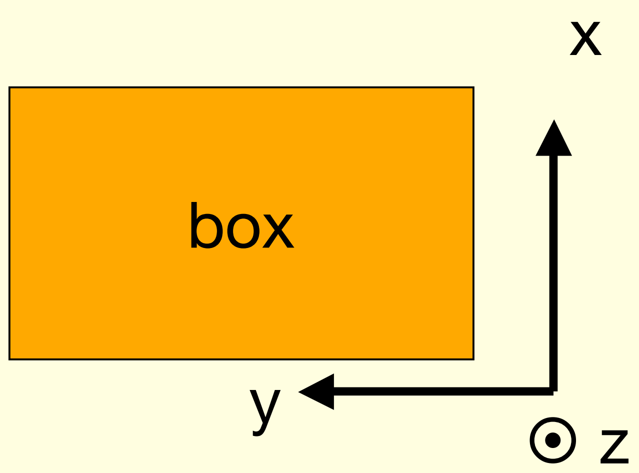
Ref: Hawley, Gammie, & Balbus 1995

LOCAL SHEARING BOX SIMULATION:



and is replaced by this one.

Ref: Hawley, Gammie, & Balbus 1995

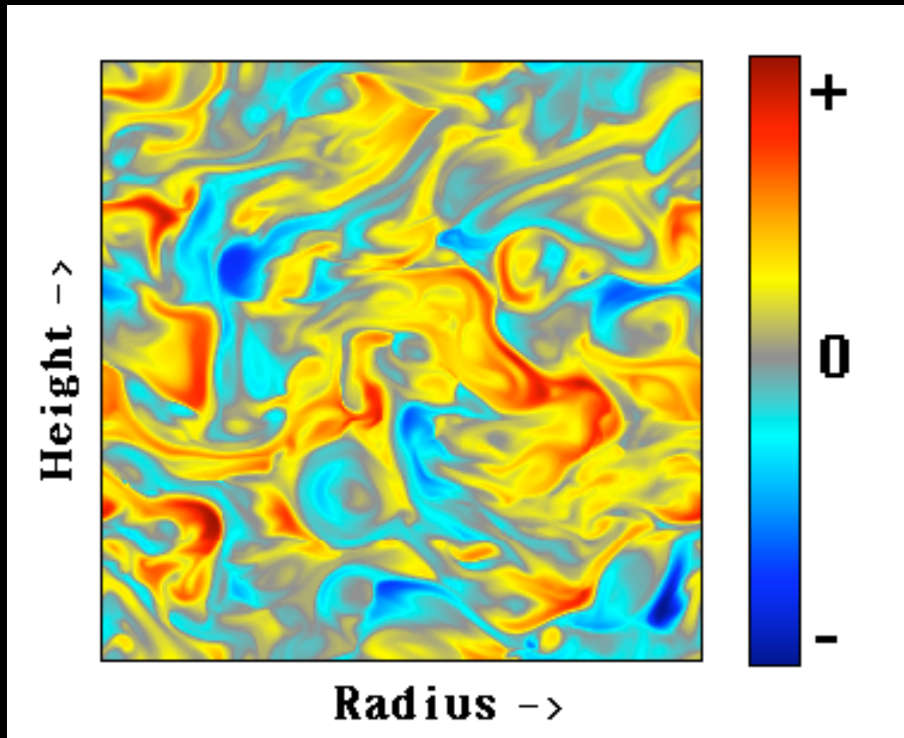


Equations of motion for shearing box:

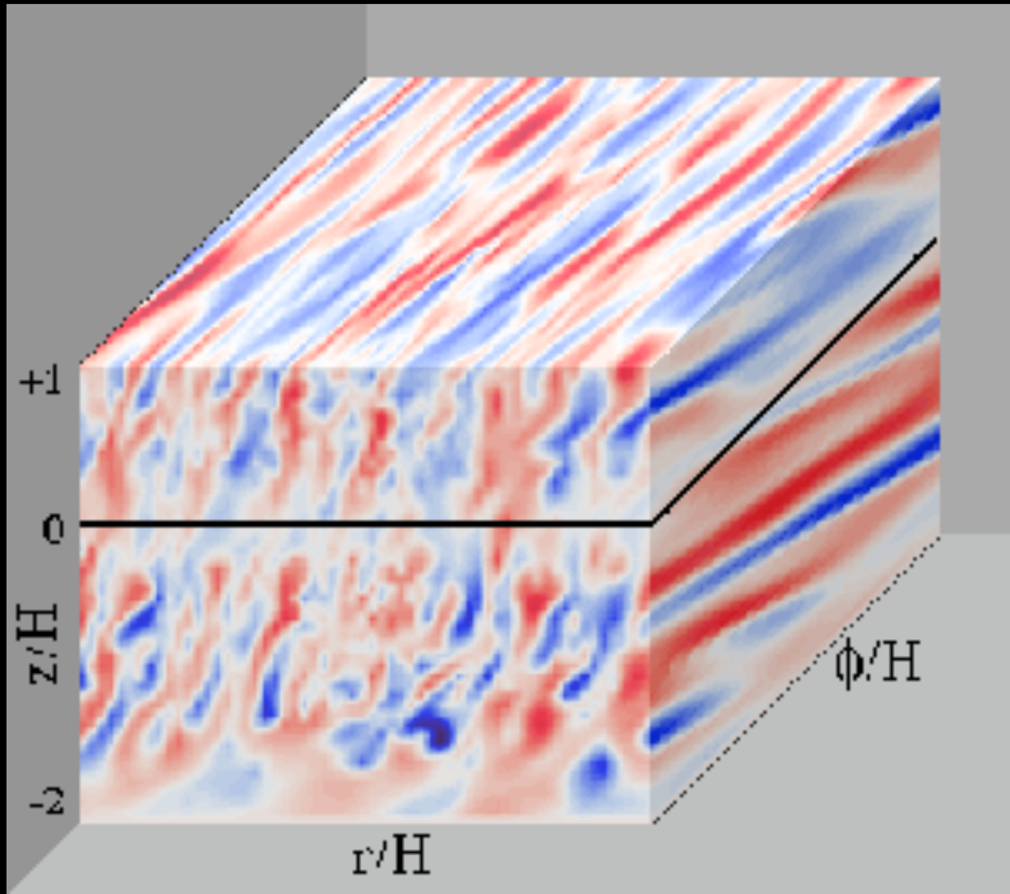
$$\frac{Dv_{\mathbf{x}}}{Dt} - 2\Omega v_y = -x\frac{d\Omega^2}{d\ln R} - \frac{\partial P}{\partial x} + (\mathbf{J}\times\mathbf{B})_x \qquad \text{(radial)}$$

$$\frac{Dv_y}{Dt} + 2\Omega v_x = -\frac{\partial P}{\partial y} + (\mathbf{J}\times\mathbf{B})_y \qquad \text{(azimuthal)}$$

$$\frac{Dv_z}{Dt} = -\frac{\partial P}{\partial z} + (\mathbf{J}\times\mathbf{B})_z \qquad \text{(vertical)}$$



2D shearing box simulation from Hawley & Balbus 1992: Angular momentum contours.



Simulation of convection in shearing box (Stone & Balbus 96)

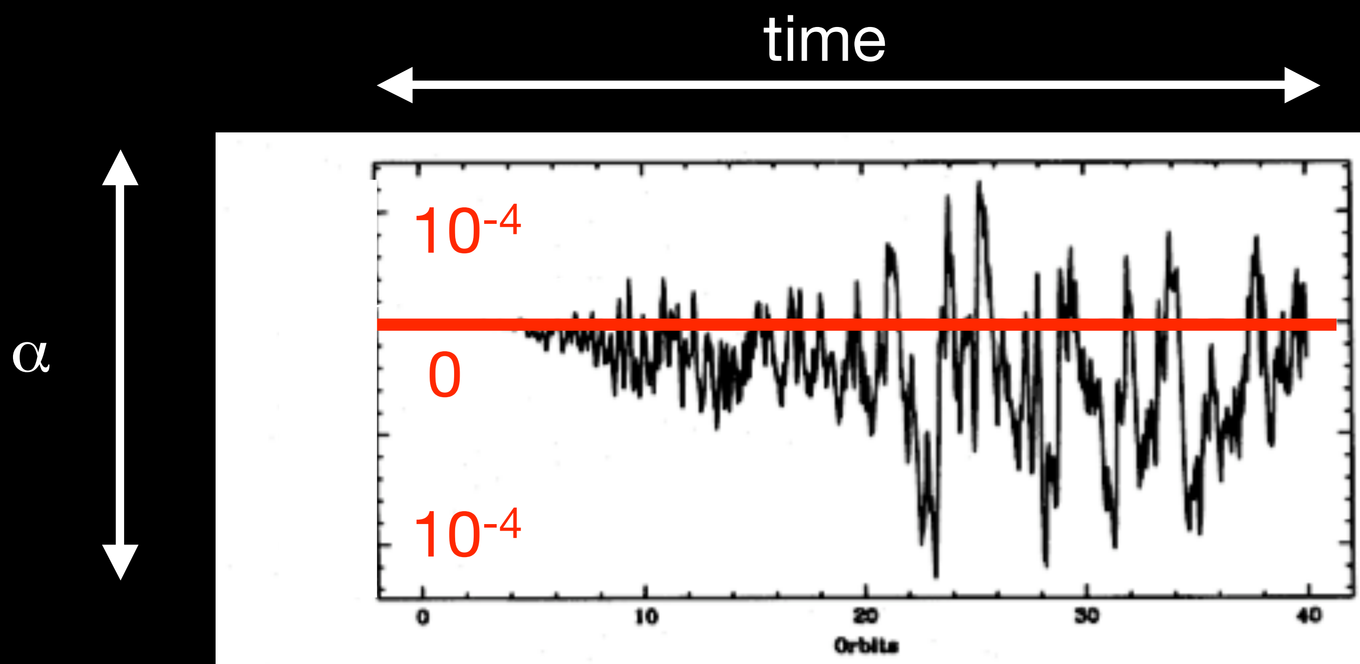


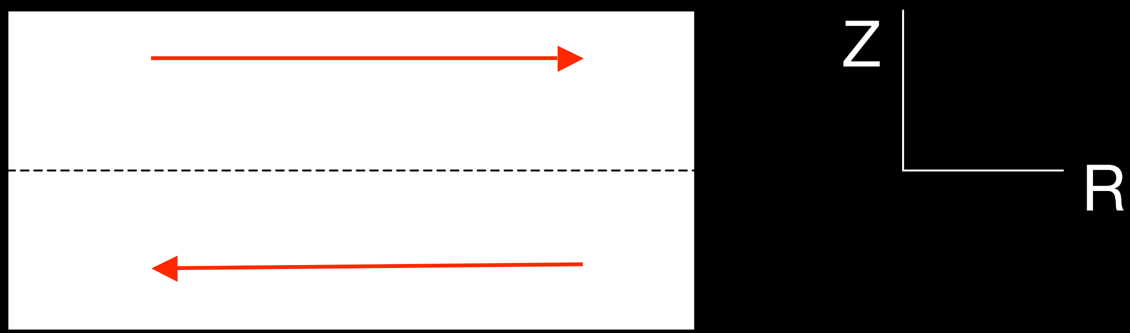
FIG. 2.—*Top*: Time history of the vertical kinetic energy during the three-dimensional evolution of a convectively unstable accretion disk with a vertical structure given in Fig. 1 and with an ad hoc heating source added at the disk midplane. *Bottom*: Time history of the Reynolds stress normalized by the pressure at the disk midplane.

The stress tensor of a convectively driven shearing box is negative (Stone & Balbus 1996)!

SHEARING BOX SUMMARY:

1. Very different behaviors depending on whether mean B_z vanishes or not. Turbulence cannot destroy mean ***B*** in SBS.

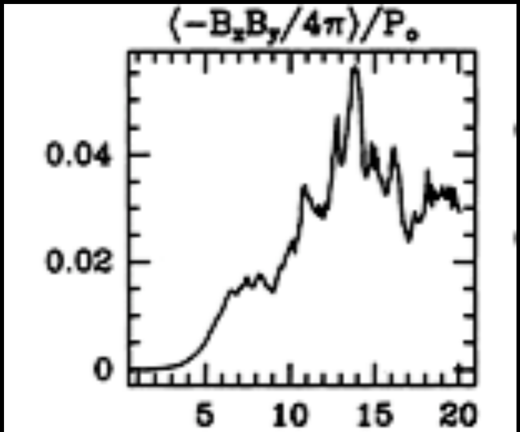
2. Early 2D simulations with $\langle B_z \rangle$ did not saturate! Instead, Exponentially growing streams formed:



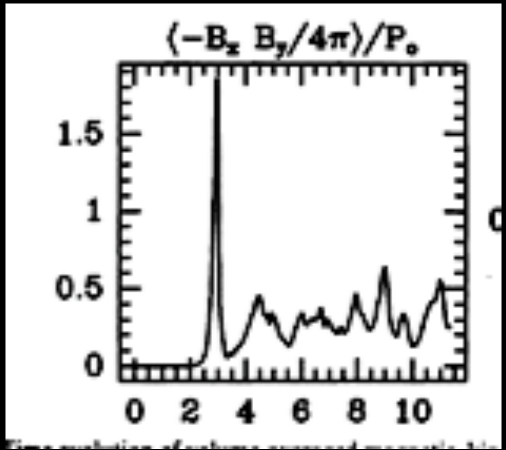
SHEARING BOX SUMMARY:

3. In 3D, "channel solution" is K-H unstable (Goodman and Xu 1994), breaks down. K-H is critical for turbulent cascade.

4. Finite <B_z> simulations in 3D show higher $T_{R\phi}$ than finite <B_R> or <B_ϕ>. If zero initial mean field, no evidence of convergence. From Hawley, Gammie, Balbus 1995:



Mean B_ϕ Field



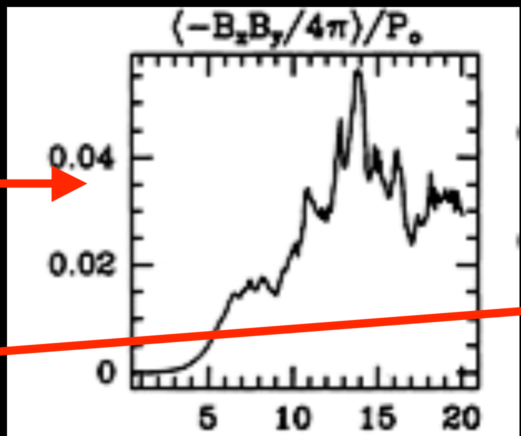
Mean B_Z Field

SHEARING BOX SUMMARY:

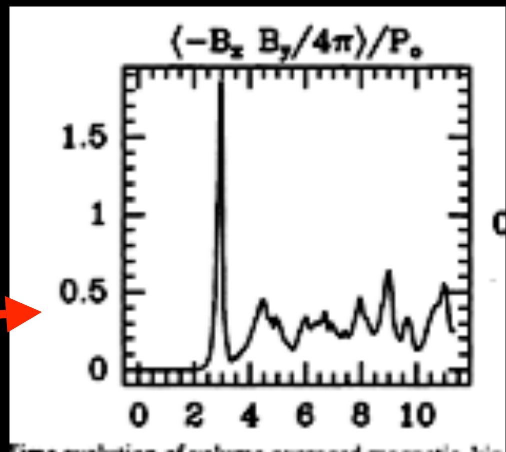
3. In 3D, “channel solution” is K-H unstable (Goodman and Xu 1994). Breaks down. K-H critical for turbulent cascade.

4. Finite $\langle B_z \rangle$ simulations in 3D show higher $T_{R\phi}$ than finite $\langle B_R \rangle$ or $\langle B_\phi \rangle$. If zero initial mean field, no evidence of convergence. From Hawley, Gammie, Balbus 1995:

Note Scale!



Mean B_ϕ Field



Mean B_Z Field

SHEARING BOX SUMMARY:

5. Restricted dynamical range of SB allows more complex MHD. For PSDs, Ohmic and Hall effects most interesting.

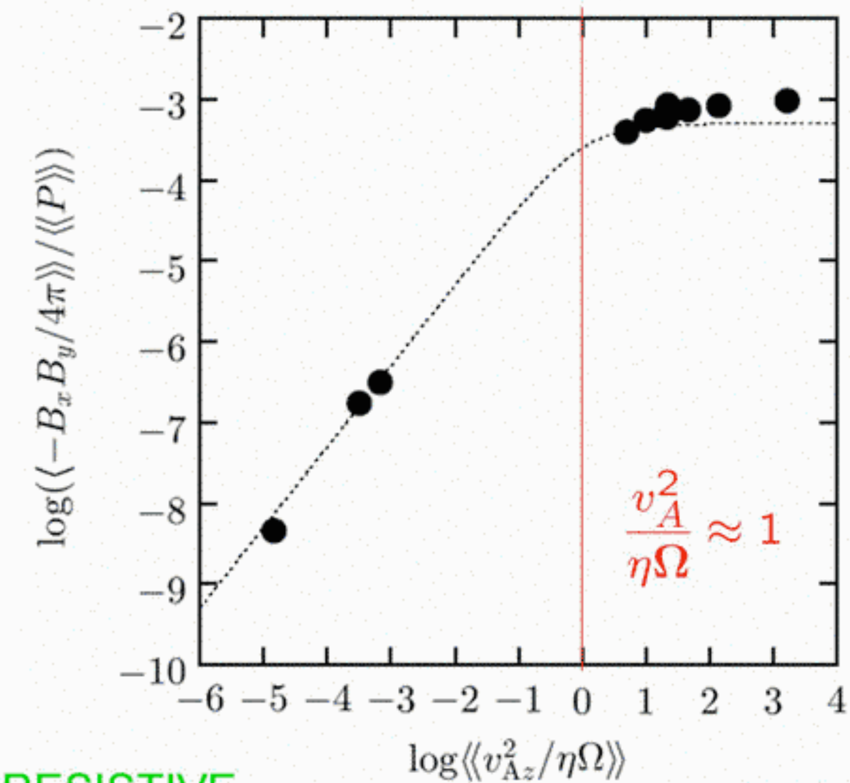
6. Most detailed studies of dissipation by T. Sano and collaborators. Key parameter is Lundquist number:

$$Re_M = v_A{}^2/\eta\Omega$$

In B_z simulations, this must exceed unity to avoid Ohmic dissipation.

Critical Value

α



← RESISTIVE

IDEAL MHD →

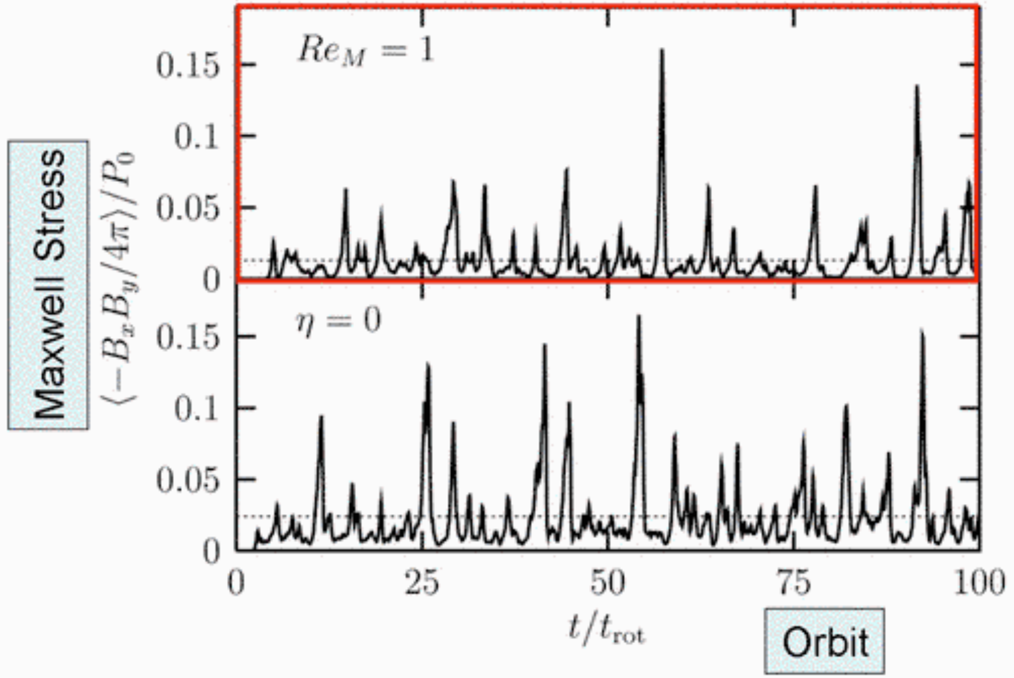
From Sano et al. 2001

Time Variability

(Sano & Inutsuka 2001)

- Maxwell Stress in the Saturated State:
  - Highly Fluctuating
  - Spike-Shaped Excursions

Uniform Bz Cases



Over longer time scales, B_z runs show highly fluctuating time signals, reminiscent of Shot Noise.

SHEARING BOX STATE OF THE ART:

Recently, the first “Shakura-Sunyaev model” has been constructed from first principles (Hirose, Krolik, Stone 2006; Blaes et al. 2006 for spectra).

Full energy equation, including radiation transport. Vertical stratification. Energy “dissipated” goes into heating gas, radiatively diffused and lost through surface. Kramers op.

No explicit resistivity or viscosity, so energy not truly dissipated. Instead, lost at grid scale and recycled as heat.

Photosphere occurs at z= 7 (c_S/Ω), very high [cf. 3 (c_S/Ω) in Analytic models]. Significant departures from Planckian. Magnetic pressure dominates at high latitude.

GLOBAL MODELS

Global Models of accretion disks began in 1998 (Armitage), and were developed by Hawley (2000).

Pacynski-Wiita potential $1/|r - r_g|$, developed for black holes, used almost universally (infalling boundary condition at finite r).

Extremely demanding of computational resources, simple polytropes so far, a few runs with resistive heating. Used for PSDs, BHDs alike!

GLOBAL MODELS

Typically, hard EOS accrete with great difficulty, isothermal less so. Bound and unbound equipotential surfaces all converge:

This means it is easy for blackflow to move from bound to unbound surfaces, simulations often reveal jets from this region; start thermal, become MHD.

GLOBAL MODELS

A quasi-Keplerian disk & jet structure readily emerge from pressure supported tori as initial conditions.

Beyond this, there is no regularity: *highly fluctuating* both spatially and temporally. Underlying average structure is very difficult to extract. $T_{R\phi}$, a correlated average of fluctuations, is impossible to describe in a simple way, certainly is NOT αP.

Radial mixing is efficient in simulations. If representative of PSDs, a result of considerable practical importance.

Fluctuation Equation:

$$\boxed{\nabla\cdot\left(\frac{\gamma\rho}{\gamma-1}\langle\delta\boldsymbol{v}\,\delta\tau\rangle\right)+\frac{\tau}{\gamma-1}\langle\rho\boldsymbol{v}\rangle\cdot\nabla\ln P\rho^{-\gamma}=-Q^{-}-T_{R\phi}\frac{d\Omega}{d\ln R}}$$

$$\tau=kT/\mu,\ T_{R\phi}=\rho\langle\delta\boldsymbol{v}\delta v_{\phi}-\boldsymbol{v_A}v_{A\phi}\rangle$$

$$\begin{aligned}\frac{\gamma\rho}{\gamma-1}\langle\delta\boldsymbol{v}\,\delta\tau\rangle &= \langle\delta P\,\delta\boldsymbol{v}\rangle \quad \text{(acoustic)}\\ &= C_P\langle\delta\boldsymbol{v}\delta T\rangle \ \text{(isobaric.)}\end{aligned}$$

Dominant balances (4 out of $4!/2!2!=6$):

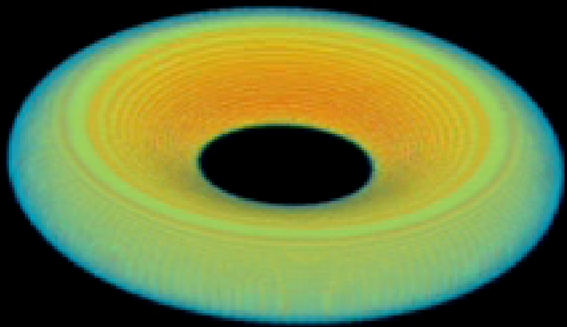
$$-Q^{-}=T_{R\phi}\frac{d\Omega}{d\ln R}\ \text{(classical disk theory)}$$

$$\nabla\cdot\left(\frac{\gamma\rho}{\gamma-1}\langle\delta\boldsymbol{v}\,\delta\tau\rangle\right)=-T_{R\phi}\frac{d\Omega}{d\ln R}\ \text{(wave action)}$$

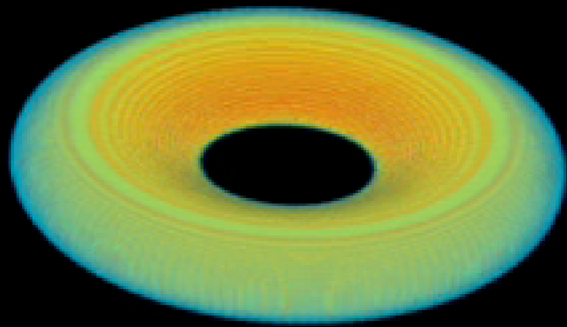
$$\frac{\tau}{\gamma-1}\langle\rho\boldsymbol{v}\rangle\cdot\nabla\ln P\rho^{-\gamma}=-Q^{-}\ \text{(classical cooling flow.)}$$

$$\frac{\tau}{\gamma-1}\langle\rho\boldsymbol{v}\rangle\cdot\nabla\ln P\rho^{-\gamma}=-T_{R\phi}\frac{d\Omega}{d\ln R}\ \text{(RIAF's)}$$

Global Simulations of the MRI, *Hawley 2000*



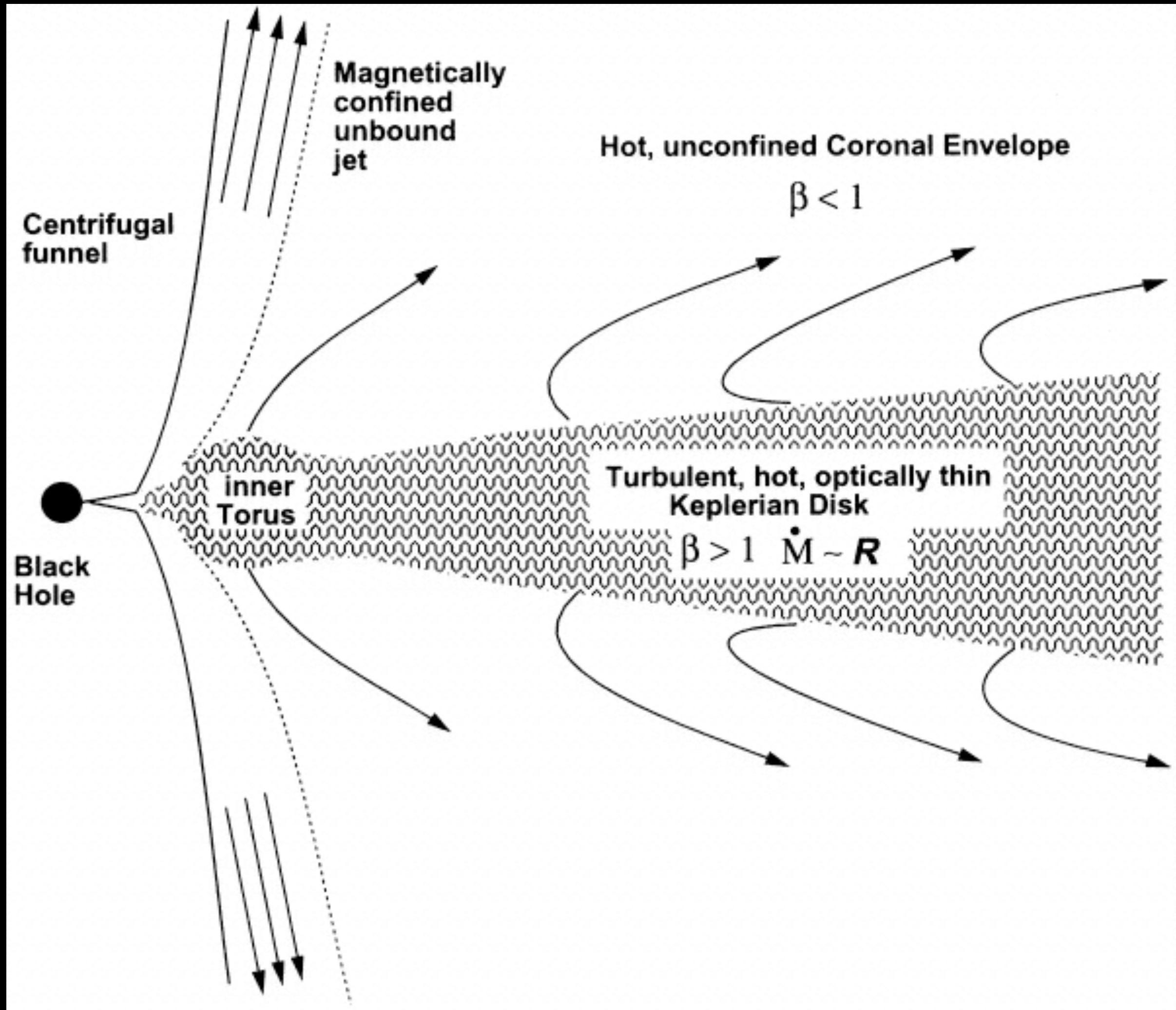
Meridional Plane



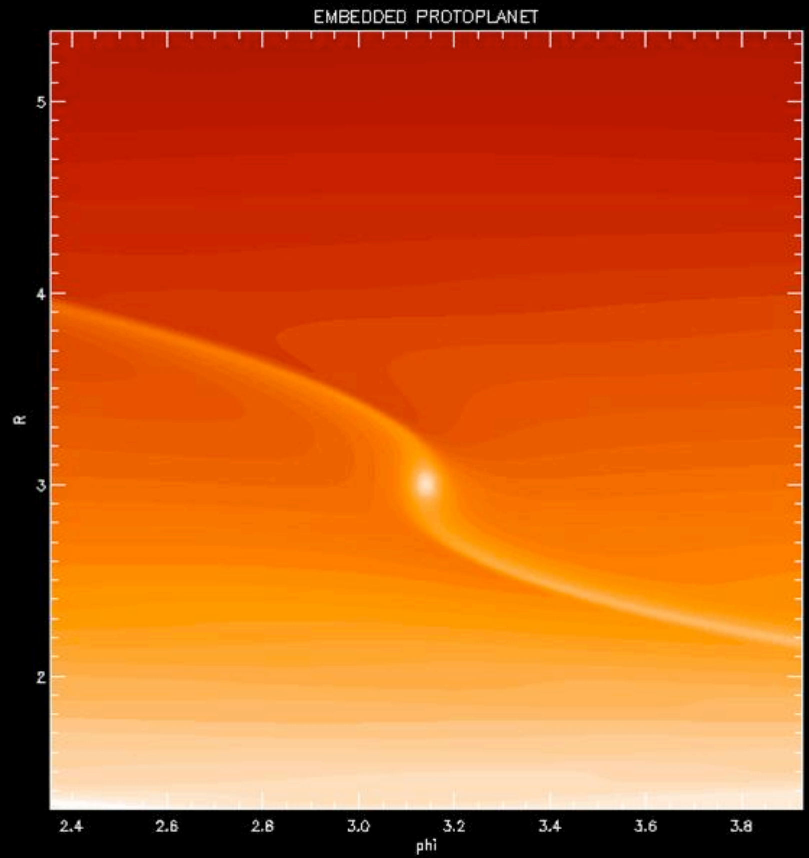
Equatorial Plane

disk and jet

by *John Hawley*

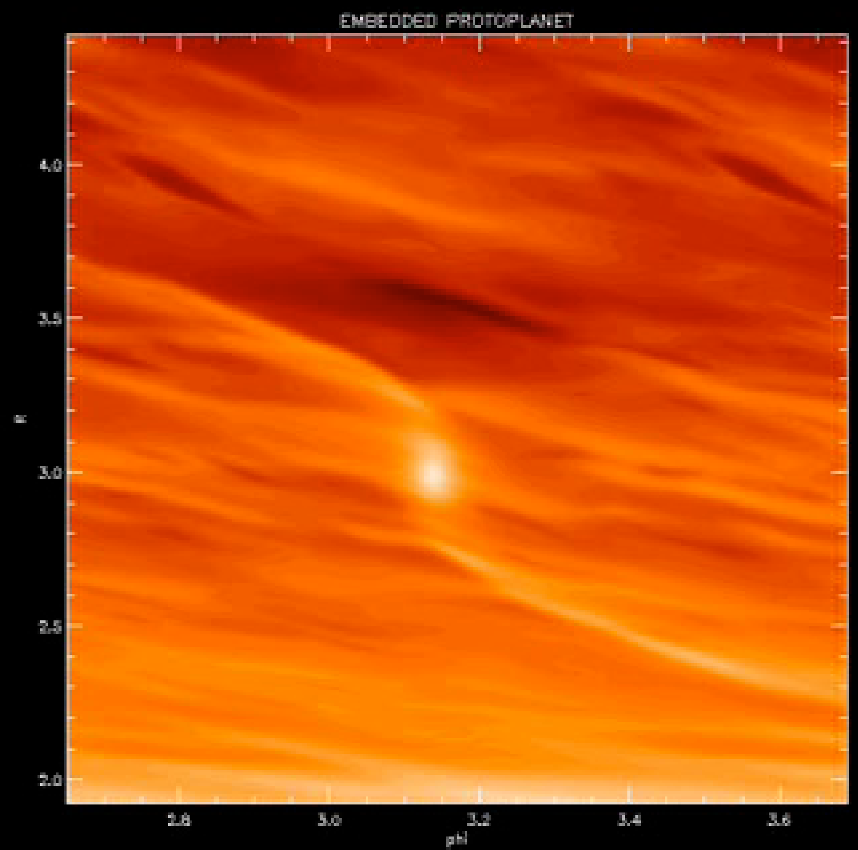

Magnetically
confined
unbound
jet
Hot, unconfined Coronal Envelope
$\beta < 1$
Centrifugal
funnel
inner
Torus
Turbulent, hot, optically thin
Keplerian Disk
$\beta > 1$ $\dot{M} \sim R$
Black
Hole

PLANET IN A DISK (NELSON & PAPALOIZOU 2005):



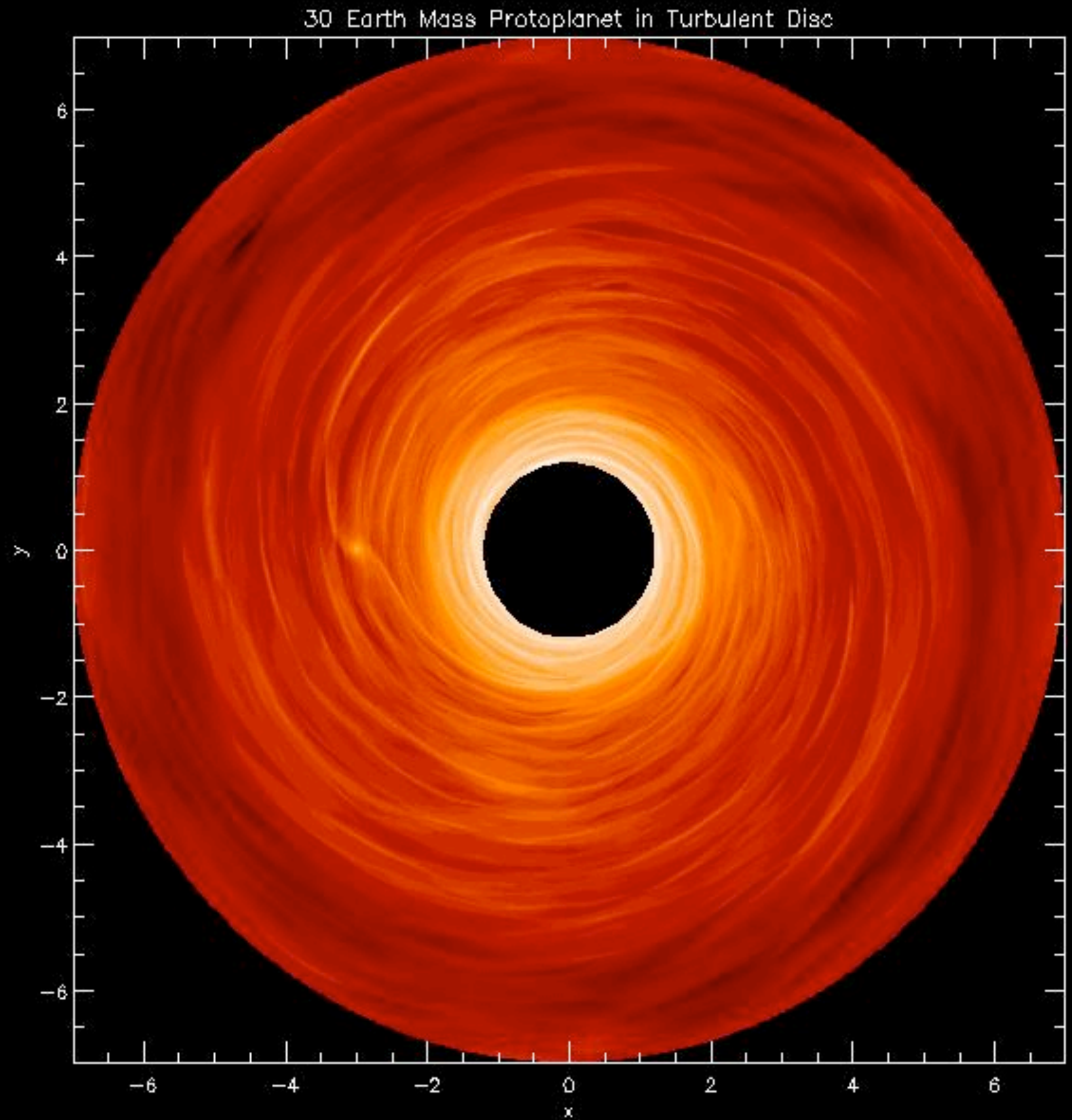
30 $M_\oplus$ planet in laminar disk.

PLANET IN A DISK (NELSON & PAPALOIZOU 2005):



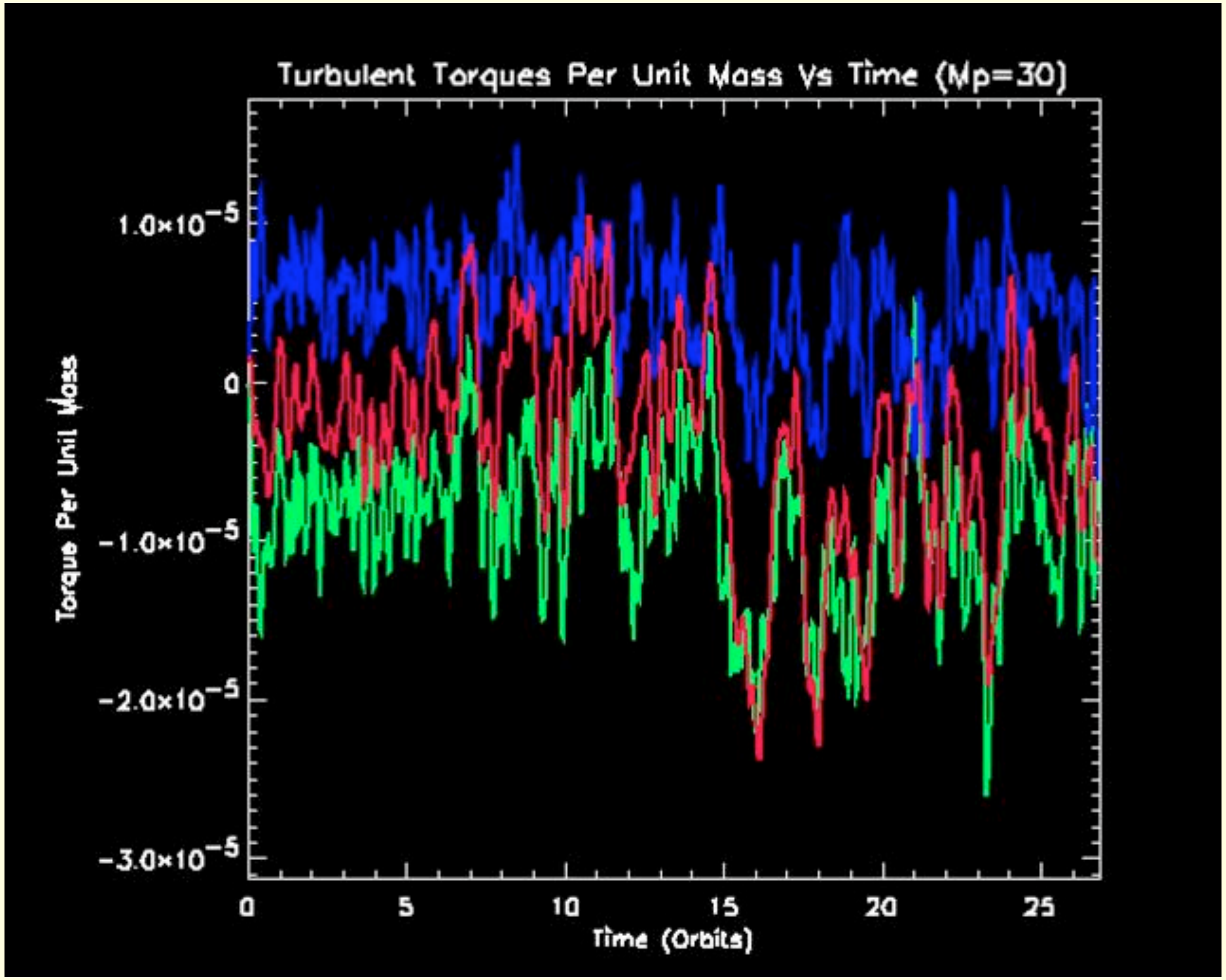
30 $M_{\oplus}$ planet in *turbulent* disk.

(NP05)



30 $M_{\oplus}$ planet in *turbulent* disk: midplane.

(NP05) ↑



Torques on $M_{\oplus}$ planet. Blue: outer Green: inner Red: net.
Highly stochastic, long term behavior unresolved.

SUMMARY

1. Realistic PSD simulations not yet feasible.

2. Numerics indicate a highly fluctuating flow. Analytic prescriptions unimpressive, but not obvious how to improve them.

3. Two broad approaches: shearing box and global disks. (There is nothing in between, despite occasional attempts to "improve" shearing box.)

4. Local simulations: small dynamic range, more complex physics, e.g. Ohmic, Hall MHD; radiative losses.

5. Global simulations: extended dynamic range, simple EOS.

SUMMARY

6. Gross morphology of global simulations *do* show some regularity: Keplerian disk, mixing, “corona,” jet.

7. More detailed structure (e.g. transport via $T_{R\phi}$) is *not* simple: no obvious α prescription from simulations.

8. Planet migration is dominated by MHD turbulence, *when present.*

9. The PSD problem has yet to be addressed. Mixing of active and decoupled zones, temperature sensitive resistivity, radiation are the heart of PSDs---even a restricted combination would be of interest.