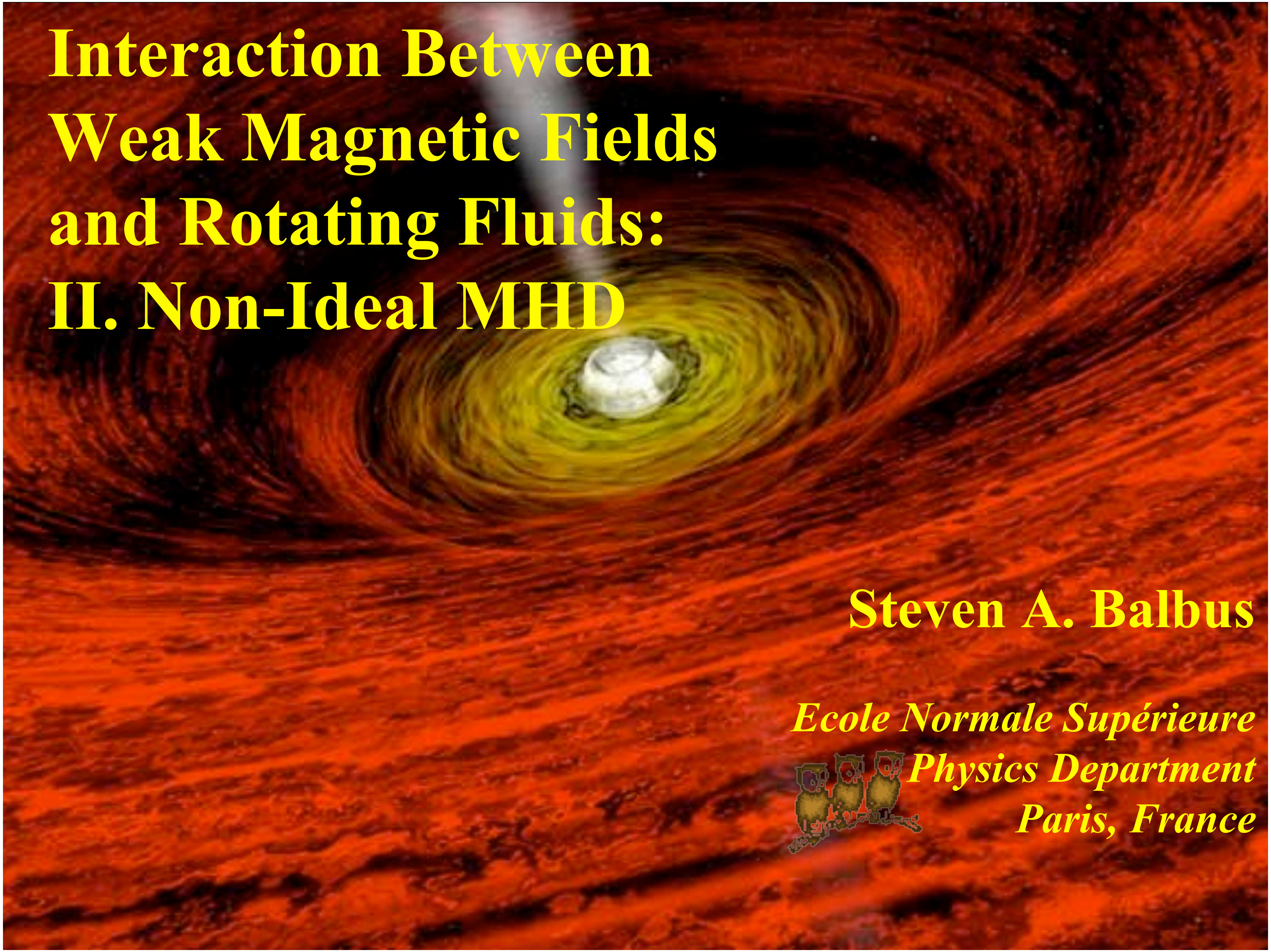

Interaction Between Weak Magnetic Fields and Rotating Fluids: II. Non-Ideal MHD

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Protostellar Disks Do Not Behave Like an Ideal MHD fluid.

$\mathbf{J} = \sigma\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ (Ohm's Law)

$\Rightarrow \quad \mathbf{E} = -(\mathbf{v} \times \mathbf{B}) + \mathbf{J}/\sigma = -(\mathbf{v} \times \mathbf{B}) + \nabla \times \mathbf{B}/(\mu_0\,\sigma)$

Faraday's Law:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{v_e} \times \mathbf{B} - \eta \nabla \times \mathbf{B}]$$

Where $\eta = 1/(\mu_0\,\sigma)$ is the "ohmic resistivity."
There are important deviations from ideal behavior.

Deviations From Ideal MHD.

1. The electron velocity $\mathbf{v}_e$ is not the same as the average fluid velocity, which is almost completely determined by neutral molecules in a protostellar disk:

$$\mathbf{v}_e = \mathbf{v} + (\mathbf{v}_i - \mathbf{v}) + (\mathbf{v}_e - \mathbf{v}_i)$$

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This ion-neutral drift velocity is very small at the densities of interest, but important in the ISM, where it gives rise to "ambipolar diffusion."

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But the ion-electron velocity difference can be large in a low-ionization protostellar disk, because it is proportional to the current…

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1. The electron velocity $\mathbf{v}_e$ is not the same as the average fluid velocity, which is determined by neutral molecules in a protostellar disk:

$$\mathbf{v}_e = \mathbf{v} + (\mathbf{v}_e - \mathbf{v}_i)$$

$$\mathbf{v}_e - \mathbf{v}_i = \mathbf{J}/(n_e\, e)$$

$$\mathbf{v}_e = \mathbf{v} + \mathbf{J}/(n_e\, e)$$

where n_e is the electron number density.

The **J** component of the electron velocity is known as the HALL TERM. (Wardle 99, Balbus &Terquem 01 Salmeron & Wardle 03…)

Ohmic Resistivity…

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{v_e} \times \mathbf{B} - \eta \nabla \times \mathbf{B}]$$

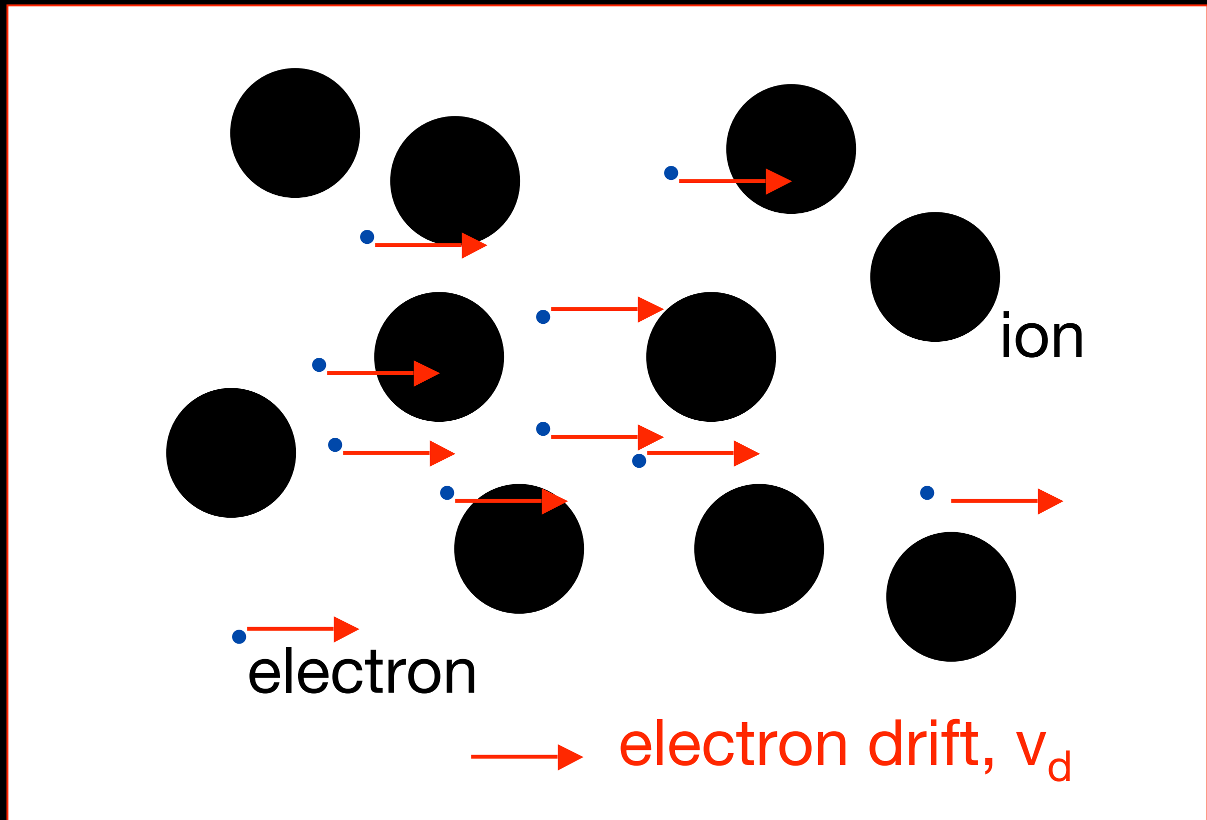
The resisitivity $\eta = 1/(\mu_0 \sigma)$ is comparable in importance to The Hall term in a protostellar disk. In contrast to HALL, OHM is dissipative, and heats the disk.

Resistance is a scattering process, increases entropy.
Conductivity $\sim \lambda$, Resistivity $\sim \lambda^{-1}$

Although viscosity ν and resistivity η are both dissipative diffusivities $[L^2 T^{-1}]$, they behave very differently,

$$\nu \sim \lambda, \quad \eta \sim \lambda^{-1} \ldots$$

Origin of resistivity:



Resistivity is inversely proportional to n_e:

$$m_e v_d / \tau = eE$$

$$v_d = \frac{eE\tau}{m_e}$$

$$J = \sigma E = n_e v_d e = \frac{n_e e^2 E\tau}{m_e}$$

$$\sigma = \frac{n_e e^2 \tau}{m_e} \rightarrow \eta = \frac{m_e c^2}{4\pi n_e e^2 \tau} \rightarrow \mathrm{SI} \rightarrow \frac{m_e}{\mu_0 n_e e^2 \tau}$$

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$\eta = 234\ \ T^{1/2}\ (n/n_e)\ \ cm^2\ s^{-1}$

The Full Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{v_e} \times \mathbf{B} - \eta \nabla \times \mathbf{B}]$$

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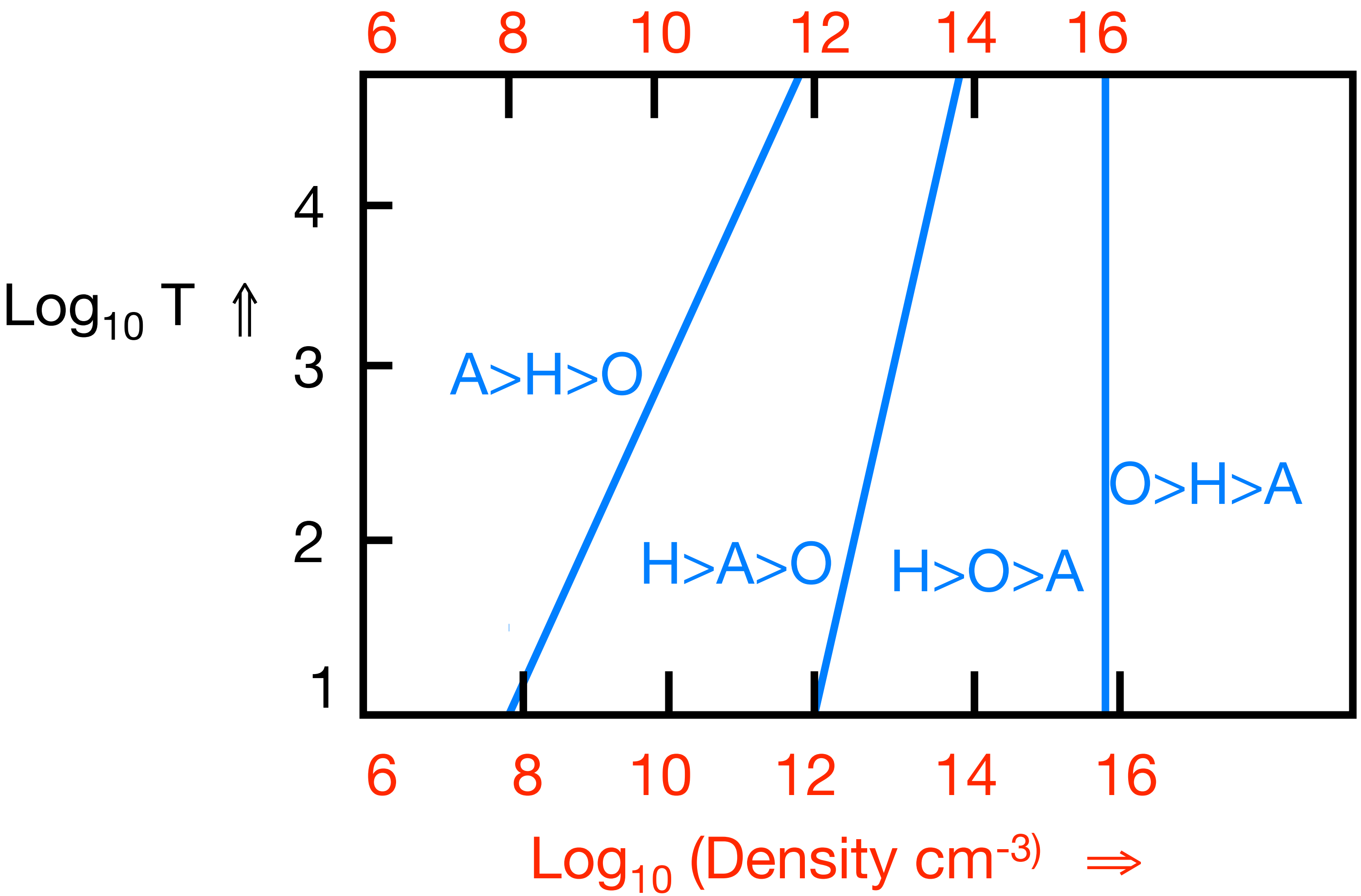
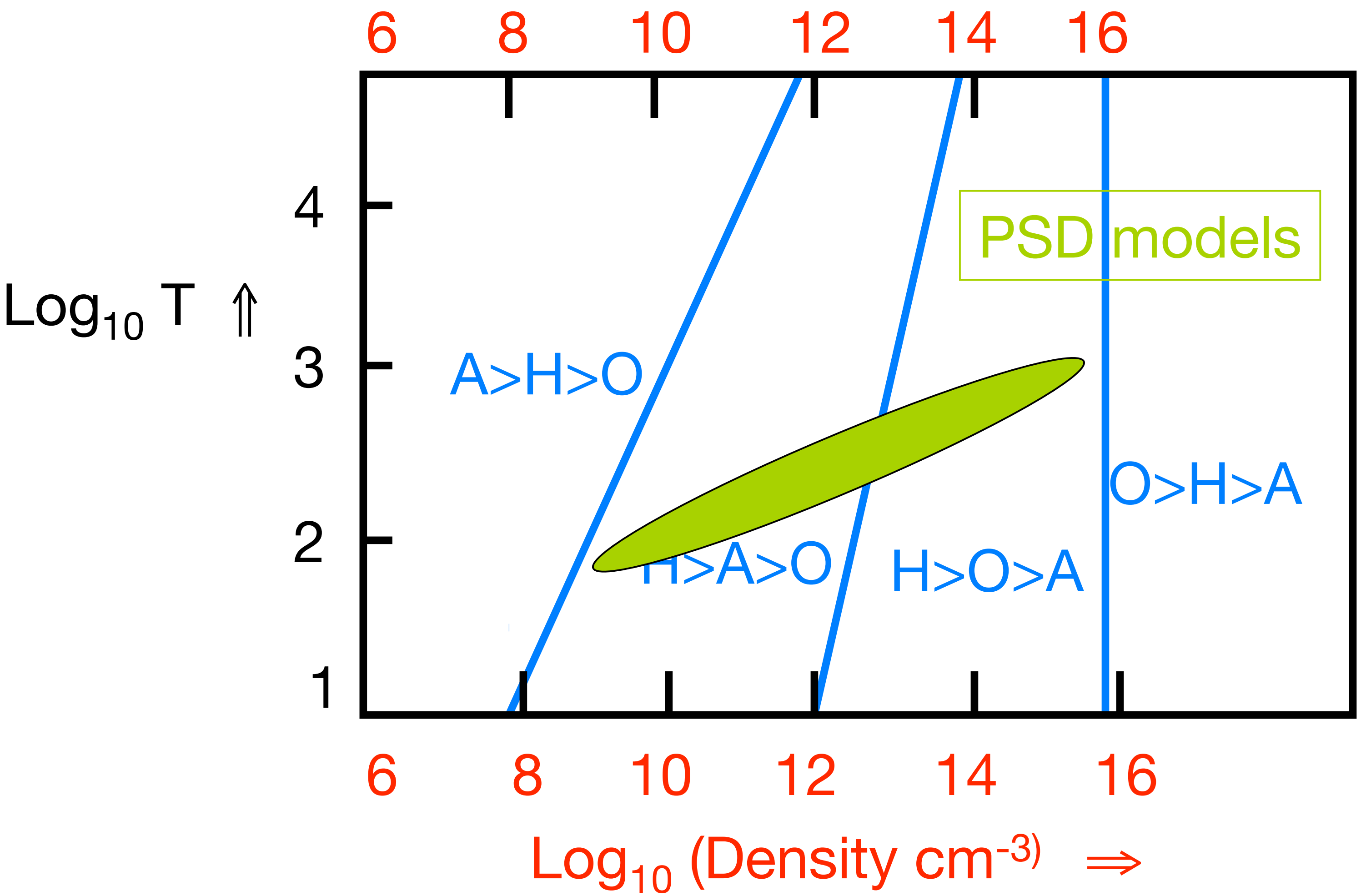
Hall Electromotive Force: Field lines not ‘frozen-in’ to bulk flow.



PARAMETER SPACE FOR NONIDEAL MHD
(Kunz & Balbus 2005)



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Ratio is $v_A L / \eta \equiv Re_M$, the magnetic Reynolds number,
In this form often called the Lundquist number.
Re_M is the key to determining the onset of instability.

ONSET OF INSTABILITY: Ideal, Ohm, Hall

Ideal MRI:

$$k^2 \; < \; | \, d\Omega^2/d \ln R \, | \; [v_A{}^2]^{-1}$$

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Hallic-Ohmic MRI:

$$k^2 < | d\Omega^2/d \ln R | [v_A^2 + v_H^2 + \kappa^2\eta^2/(v_A^2 + f v_H^2)]^{-1} ,$$

$f=\kappa^2/4\Omega^2 = .25$ for Kepler.

ONSET OF INSTABILITY: Ideal, Ohm, Hall

Ideal MRI:

$$(kv_A)^2 \quad [\, (kv_A)^2 + d\Omega^2/d \ln R \,] \; < \; 0$$

Ohmic MRI:

$$(kv_A)^2 \quad [\, (kv_A)^2 + d\Omega^2/d \ln R \,] \; + \kappa^2\eta^2 k^4 < \; 0$$

Hall-Ohm MRI:

$$[\, (kv_A)^2 + \; (kv_H)^2 \, \kappa^2/4\Omega^2 \,] \; \times [\, k^2 \,(v_A{}^2 \; + v_H{}^2) + d\Omega^2/d \ln R \,]$$
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$$+ \kappa^2\eta^2k^4 < \; 0$$

$v_H{}^2 = 2\Omega B/\mu_0 n_e e$, note > 0 or <0

Quantitative Estimate of Hall Effect

$$v_H^2/v_A^2 = (n/n_e)\ 2\Omega m/(eB) = 2\,(n/n_e)\ (\Omega/\omega_{cyc})$$

$$= 7 \times 10^{-12}\,(n/n_e)\ n_{14}^{-1/2}\ T_3^{-1/2}\ (c_S/v_A)$$

In other words, Hall physics is important when:

$$n_e/n < 7 \times 10^{-12}\ n_{14}^{-1/2}\ T_3^{-1/2}\ (c_S/v_A)$$

Resistive Damping

$n_e/n < 7 \times 10^{-12}\ n_{14}^{-1/2}\ T_3^{-1/2}\ (c_S/v_A)$ (HALL)

Resistivity becomes important when $\eta\Omega > v_A^2$, but…

Resisitivity kills MRI when $\eta^2 k^2 > 1.5\ v_A^2$ (from DR).
Smallest possible k is π / H , H = scale height.

$\eta\,\pi / H > 1.22\ v_A \Rightarrow \eta\Omega > 0.38\ v_A c_S$. This translates to an ionization fraction

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SUMMARY:

1. Protostellar disks depart from ideal MHD because their ionization fraction is very low.

2. Both Ohmic dissipation and Hall EMFs are present.

3. Ohmic dissipation surpresses the MRI by forcing instability at longer wavelengths.

4. Hall EMFs can be either stabilizing or destabilizing, depending upon the sign of $\Omega \cdot \mathbf{B}$: + destabilizes, - stabilizes.

5. Preliminary numerics indicate that *level* of turbulence affected more than Re_M at onset (Sano & Stone '02).

MODELING DISKS:

The most important property for determining the dynamical behavior for a protostellar disk is the ionization fraction, f=n_e/n. From this follows Re_M, and in principle everything else.

Unfortunately, determining f is all but impossible. depends on radiation, nebular chemistry, dust grain physics, and (self-consistently!) degree of turbulence.

A small taste of some of the basic ideas follows…

MODELING DISKS

Thermal ionization ($r<0.3$ AU) : the Saha equation.

$$n_e \, n[X^{r+1}] \,/\, n[X^r] = (2g_{r+1}/g_r) \; (2\pi m_e kT/h^2)^{3/2} \; e^{-\Phi/kT}$$

If element X^r is barely ionized, ionization fraction x is:

$$(n_e/n)^2 = x^2 = a \; (2.41 +15) \,/\, n_{H2} \; T^{3/2} \; e^{-\Phi/kT}$$

Where a is the abundance of the element relative to H_2. For K (Na also relevant):

$$x \sim 10^{-12} \; a_{-7}^{1/2} \, T_3^{3/4} \; (e^{-25,188/T} \,/\, 1.15{\times}10^{-11})$$

MODELING DISKS

Ionization: $x \sim 10^{-12}\ a_{-7}^{1/2}\ T_3^{3/4}\quad e^{-[25,188/T\ -\ 25.188]}$

Since resistivity $\eta = 234\ T^{1/2} / x \quad cm^2\ s^{-1}$,

$$\eta \sim T^{-1/4} \exp(25,188/T) \sim 1/Re_M,$$

extremely steep for $T \sim 1000$ K.

A MODEL DISK PROBLEM:

Nonthermal ionization (e.g. Glassgold et al . 2000; Sano et al. 2000; Fromang et al. 2002):

YSOs are active X-ray sources.
L_X 10^{29} - 10^{32} erg s^{-1}, 1 keV $< E_\gamma <$ 5 keV.

H_2 ionization rate ζ can be determined as a function of X-ray spectrum and disk model.

A MODEL DISK PROBLEM:

Recombination (Sano et al. 2000; Fromang et al. 2002):

Recombination processes are complex. Important species: H_2, e^-, H_2^+, metals, dust grains.

Importance of metals is rapid charge exchange with H_2^+ ; e^- recombination much slower compared with molecular dissociative recombination.

Importance of dust is e^- capture and sputtering source of metal atoms.

$T < 1500$ K for dust grains to survive.

A MODEL DISK PROBLEM:

Recombination (Sano et al. 2000; Fromang et al. 2002, Ilger & Nelson 2006):

Simple gas phase model of Fromang et al.:

$$dn_e/dt = \zeta n - \beta_{dis}\; n_e\, n_{mo}^{+} - \beta_{rad}\, n_e\, n_{Me}^{+}$$

$$dn_{mo}^{+}/dt = \zeta n - \beta_{dis}\; n_e\, n_{mo}^{+} - \beta_{trnsfr}\; n_{Me}\, n_{mo}^{+}$$

n_{mo}^{+}, n_{Me}, n_{Me}^{+} = density of molecular ions, metal atoms, metal ions

β_{dis} (>>) β_{rad} , β_{trnsfr} = dissociative recombination, radiative recombination, and charge transfer rates

A MODEL DISK PROBLEM:

Recombination (Sano et al. 2000; Fromang et al. 2002):

Simple limits:

$$x_e = n_e/n = [\zeta/n\beta_{dis}]^{1/2} \quad \text{(no metals!)}$$

$$x_e = n_e/n = [\zeta/n\beta_{rad}]^{1/2} \gg [\zeta/n\beta_{dis}]^{1/2}$$

(metal regulated).

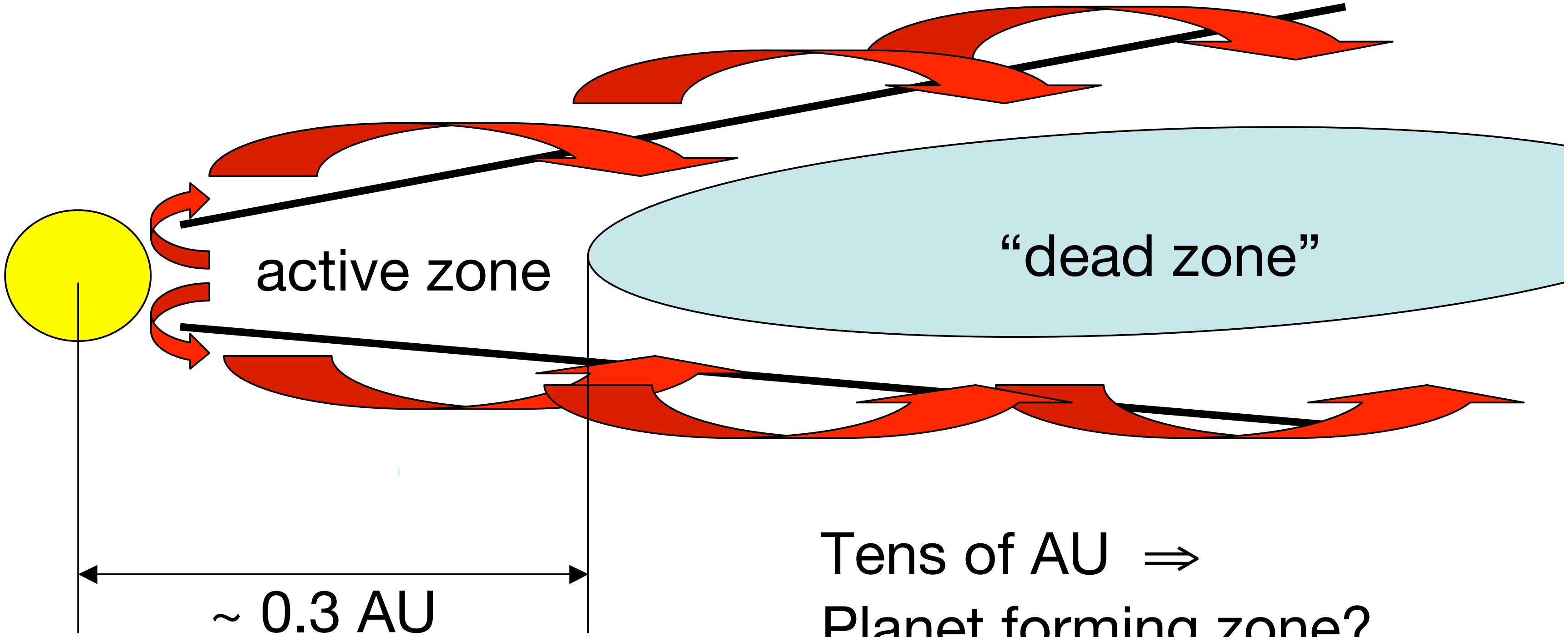
The ionization fraction is an extremely sensitive function of x_M in such models.

A MODEL DISK PROBLEM:

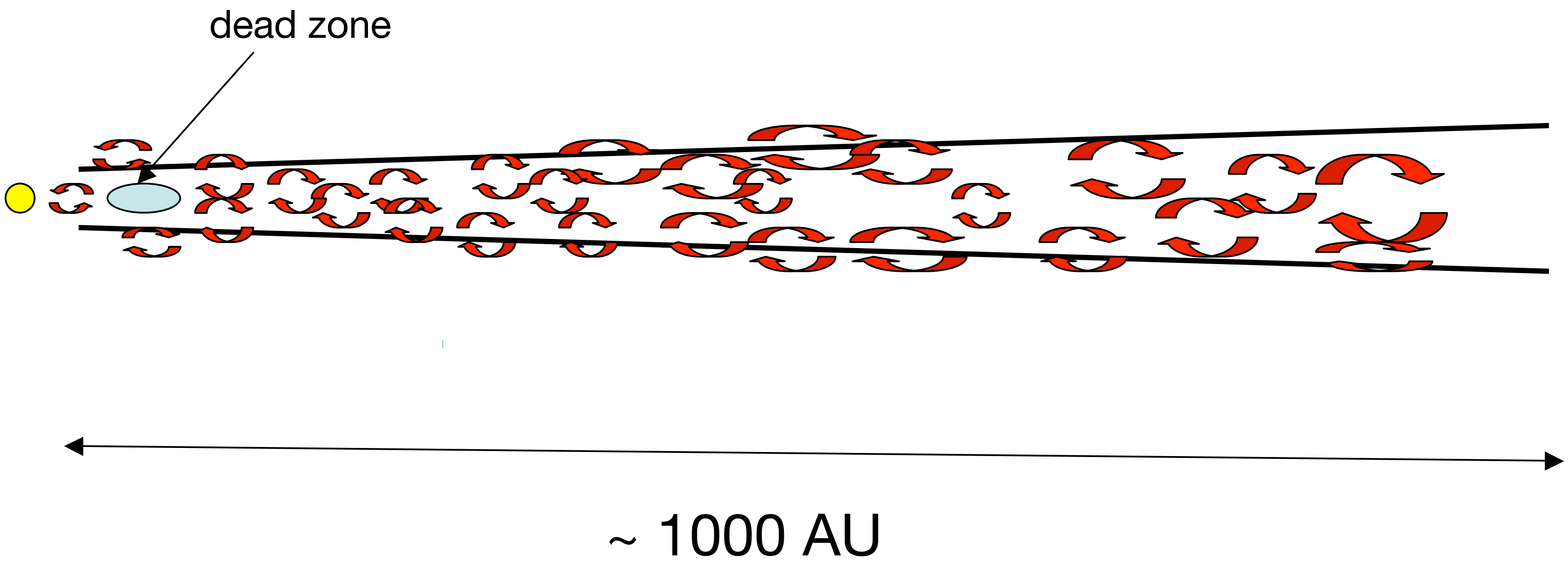
Outer regions (> 30? AU):

Low densities, longer recombination time scales. external ioinzation sources (X-rays, cosmic rays) seem likely to maintain minimal levels of ionization needed for for magnetic coupling.

Ambipolar diffusion regime.



INNER REGIONS OF SOLAR NEBULA



GLOBAL PERSPECTIVE OF SOLAR NEBULA

SUMMARY

1. Protostellar disk structure is sensitive to the ionization fraction which is very difficult to determine.

2. It is likely, however, that a three-zone structure: i) a thermally ionized inner zone [0.3 AU] , ii) a magnetically inactive `dead zone' [30 AU], iii) an active outer zone [>30 AU].

3. Because protostellar disk structure is sensitive to metallicity, we should expect a wide range of behavior.

4. But once this is understood, it is a link with observations---e.g. enhanced magnetic activity should be correlated with higher metallicity.