COSMOLOGY WITH NEGATIVE ENERGY

Tomislav Prokopec, ITP and Spinoza Institute, Utrecht U.

CONTENTS

1) MOTIVATION

2) ANTI-DE SITTER SPACE: CLASSICAL BOUNCE

3) QUANTUM BACKREACTION: MODEL I
 - early bouncing phase
 - intermediate inflationary epoch
 - late matter era

4) QUANTUM BACKREACTION: MODEL II
 - early stage
 - late radiation epoch

5) SUMMARY AND DISCUSSION

6) OUTLOOK:
TOWARDS REALISTIC QUANTUM BACKREACTION IN COSMOLOGY

MOTIVATION

- STRING THEORY PRESERVES SUPERSYMMETRY WHEN COMPACTIFIED (ON CALABI-YAU SPACES) ON ANTI-DE SITTER OR MINKOWSKI BACKGROUNDS.

- An empty anti-de Sitter space (AdS) is inconsistent with homogeneous cosmology: AdS represents an inhomogeneous static space.

- So far, meta-stable string-inspired cosmologies have been constructed/considered.

Kachru, Kalosh, Linde, Trivedi (KKLT, 2003)

Kachru, Kalosh, Linde, Maldacena, McAllister, Trivedi (KKLMMT, 2003)

- These cosmologies involve an uncomfortably large amount of fine tuning, and are hence not universally accepted as the solution to the above problem.

THE QUESTION WE POSE HERE IS: Are there (good) alternatives?

ANTI-DE SITTER: CLASSICAL BOUNCE

CLASSICAL FRIEDMANN EQUATIONS:

$$H^2 = \frac{8\pi G_N}{3c^2}\rho + \frac{\Lambda}{3}, \quad \dot{H} = -\frac{4\pi G_N}{c^2}(\rho + p), \quad \rho = \frac{\rho_0}{a^{2\varepsilon_0}}, \quad \varepsilon_0 = \text{const.}$$

FOR: $\Lambda<0$

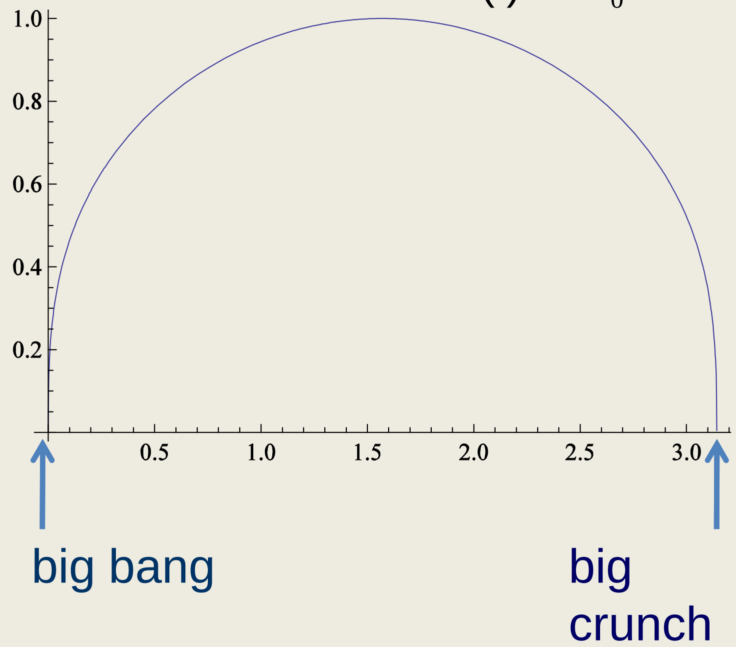
$$a(t) = a_0\left[\sin\left(\sqrt{\frac{-\Lambda}{3}}\varepsilon_0 t\right)\right]^{\frac{1}{\varepsilon_0}}, \quad H(t) = \sqrt{-\frac{\Lambda}{3}}\cot\left(\sqrt{-\frac{\Lambda}{3}}\varepsilon_0 t\right)$$

- The Universe exhibits big bang [$t_{bb}=0$] and big crunch [$t_{bc}=(\pi/\varepsilon_0)\sqrt{(-\Lambda/3)}$] singularities.

RICCI SCALAR:

$$R(t) = (-2\Lambda)\frac{2\cos^2\left(\sqrt{-\frac{\Lambda}{3}}\varepsilon_0 t\right) - \varepsilon_0}{\sin^2\left(\sqrt{-\frac{\Lambda}{3}}\varepsilon_0 t\right)}$$

SCALE FACTOR a(t) for $\varepsilon_0 = 3$



$\text{NB}: R(t) \to \infty$ at b.b. and b.c.

MODEL I: EARLY OSCILLATING PHASE

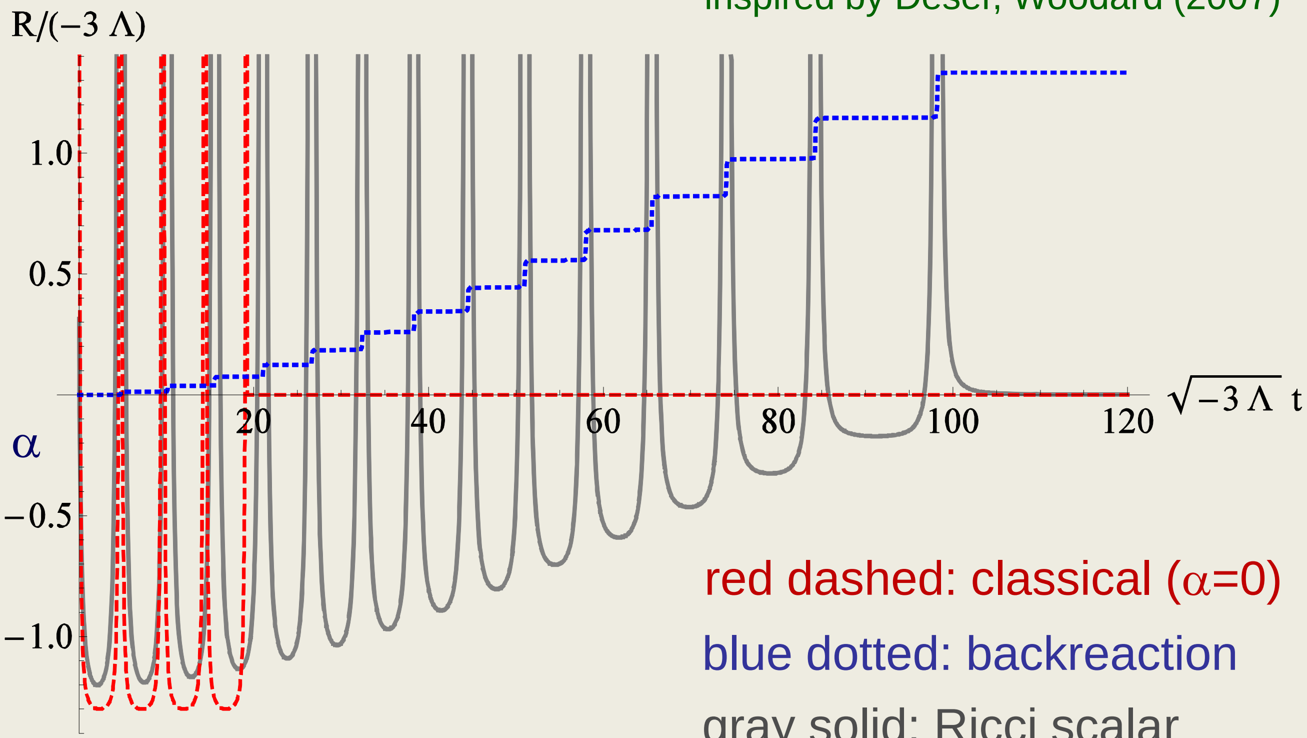
MODELING QUANTUM BACKREACTION WITH $-\alpha\int_{t_0}^{t} dt' G_{\rm ret}(t;t')R(t')$ in H² EQ.

$$G_{\rm ret}(t;t') \equiv \left[\sqrt{-g}\, g^{\mu\nu}\nabla_\mu\nabla_\nu\delta(t-t')\right]^{-1}_{\rm ret} = \left(\int_{t_0}^{t}\frac{d\tilde{t}}{a^3(\tilde{t})} - \int_{t_0}^{t'}\frac{d\tilde{t}}{a^3(\tilde{t})}\right)\Theta(t-t')$$

inspired by Deser, Woodard (2007)

● EARLY EVOLUTION:
CLASSICAL BOUNCE
⇒ OSCILLATING PHASE

of `bounces' depends on α

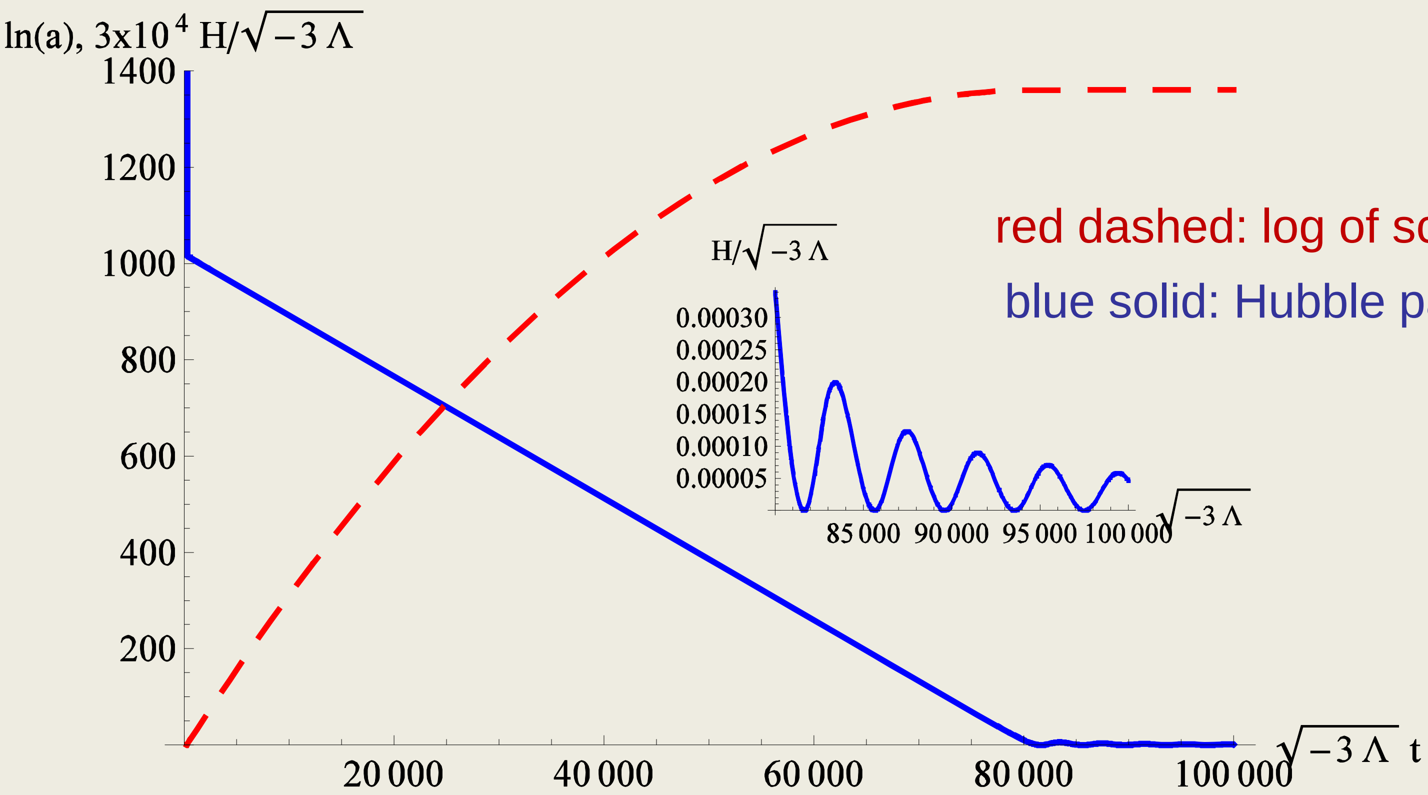


red dashed: classical (α=0)

blue dotted: backreaction

gray solid: Ricci scalar

MODEL I: INFLATION

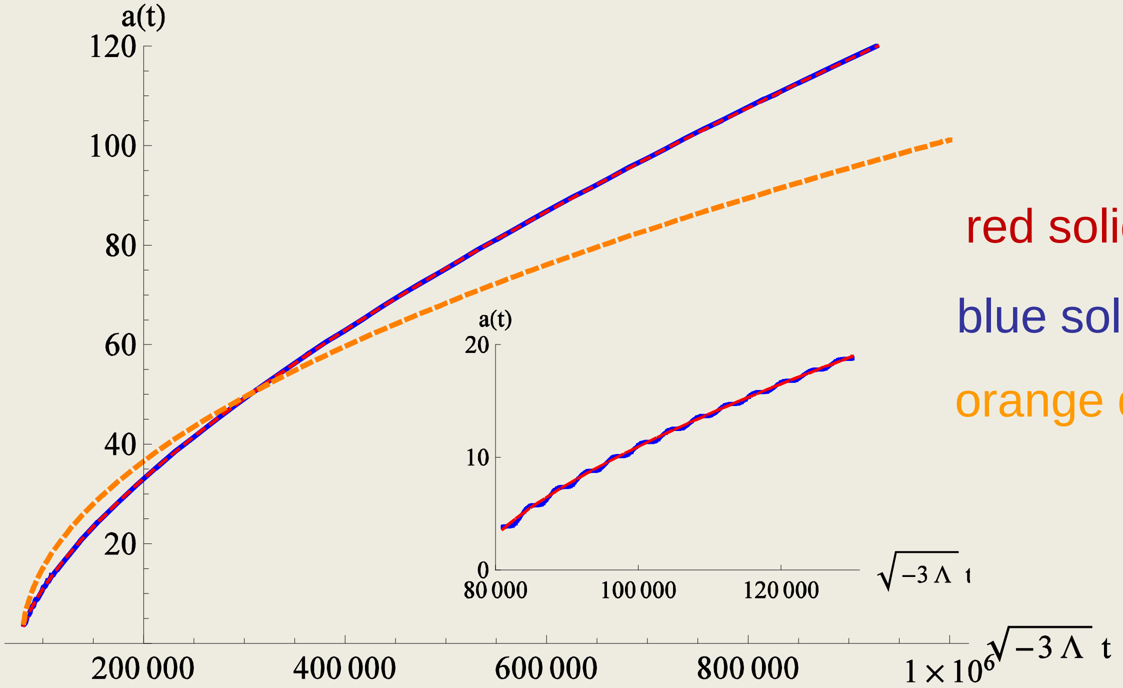


red dashed: log of scale factor

blue solid: Hubble parameter

- the duration of inflation (dH/dt) depends on α: smaller $\alpha \Rightarrow$ longer inflation.

MODEL I: LATE TIME MATTER



red solid: matt. era: a∝ t^(2/3)

blue solid: numerical solution a(t)

orange dashed: rad. era, a∝ t^(1/2)

NB1: Late time matter era scaling (ε→3/2) does not depend on the matter EOS (that dominates @ early times):

$$\varepsilon_0 = \frac{3}{2}(1+w), \quad w = \frac{p}{\rho}$$

NB2: There are small (potentially observable?) oscillations on top of the matter solution.

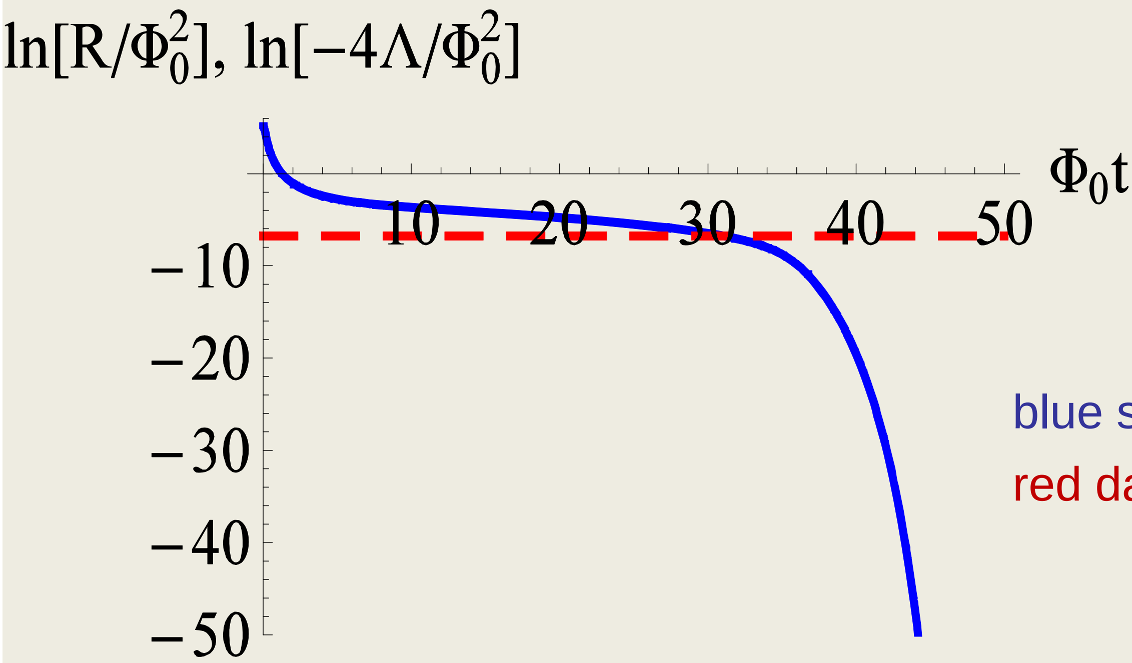
MODEL II: RICCI CURVATURE

ADD THE NONLOCAL POTENTIAL: $V(\phi) = \frac{\lambda_0}{4!}\phi^4 + \frac{\lambda_1}{4!}\phi^4 \ln\left(\frac{\phi^4}{R^2}\right)$

- inspired by the 1 loop effective potential of Yukawa theory

Prokopec (2006)

RESULT: RICCI SCALAR R(t) DOES NOT BLOW UP. INSTEAD: R(t) $\rightarrow$ 0.

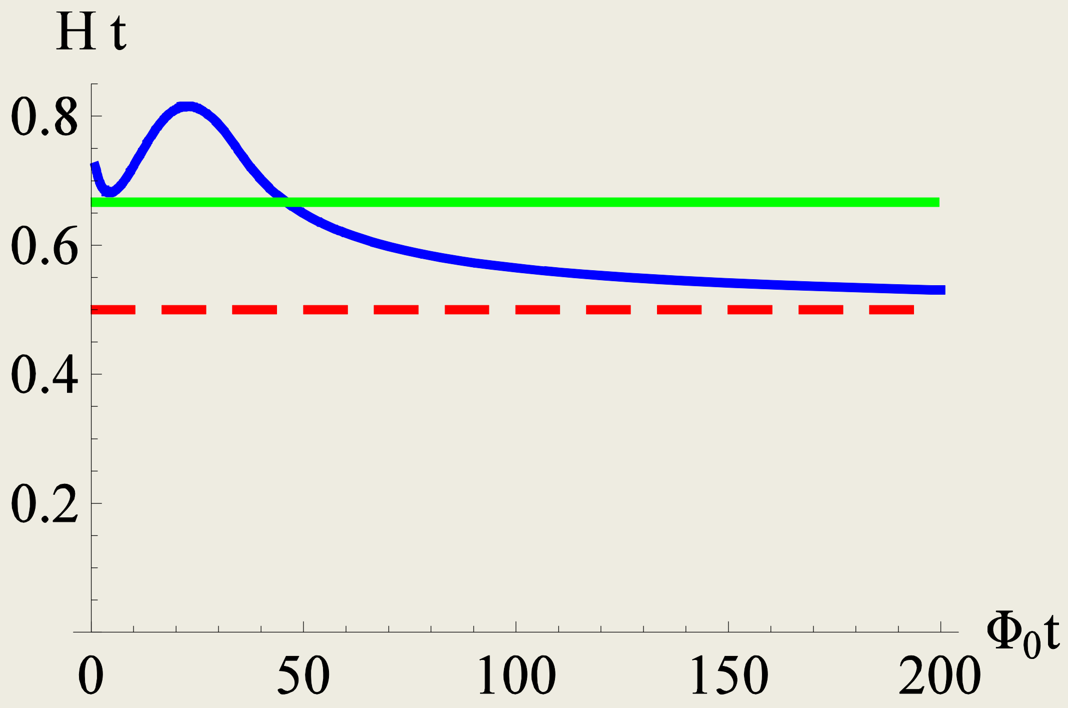


blue solid: log of Ricci scalar

red dashed: log of min cosmological term

MODEL II: LATE TIME RADIATION

UNLIKE IN MODEL I, IN MODEL II LATE TIME UNIVERSE ASYMPTOTES TO A RADIATION ERA (here there are no small oscillations).



blue solid: tH (numerical solution)

green solid: tH=2/3 (matter era)

red dashed: tH=1/2 (radiation era)

NB: Neither Model I nor Model II can model dark energy
(but beware of the late time oscillations in Model I).

SUMMARY AND DISCUSSION

- We have considered two nonlocal cosmologies which – starting with a negative cosmological term and homogeneous matter -- yield consistent late time cosmology (radiation or matter era)

- MODELS I and II discussed here are TOY MODELS for quantum backreaction, however they do share some of the features of realistic quantum backreaction.

Deser, Woodard,Tsamis (2007-2010)
Prokopec, gr-qc/0603088 (2006)

- MODELS I and II illustrate that quantum backreaction can qualitatively change the evolution of the Universe.

OUTLOOK: TOWARDS REALISTIC QUANTUM BACKREACTION IN COSMOLOGY

- Step 1 to study realistic quantum backreaction in cosmology is to renormalise the relevant correlators (stress energy) in general FLRW spaces. This has been Recently done by Weinberg (Pauli-Vilars) and by myself and Glavan (dim reg).

S. Weinberg, arXiv: 1011.1630 (2010)

Prokopec. Glavan, in preparation (2011)

- Step 2 is to consistently (numerically) solve the quantum corrected Friedmann equation, as well as the field equations.

- A lot of work needs to be done, before we will understand are there realistic conditions in which the quantum backreaction is large, s.t. it dominates/determines the late time evolution of the Univere.
- At the moment, there are studies of quantum backreaction in fixed bacgrounds, primarily de Sitter and quasi-de Sitter spaces.

Tsamis Woodard (1995-96)
Janssen, Prokopec (2008-09)
Janssen, Miao, Prokopec, Woodard (2009)
Koivisto, Prokopec (2010)

- large quantum backreaction could not only shed light on the very early universe, but also explain the Universe's dark energy.