Cosmic Microwave Background

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Plan of the lectures

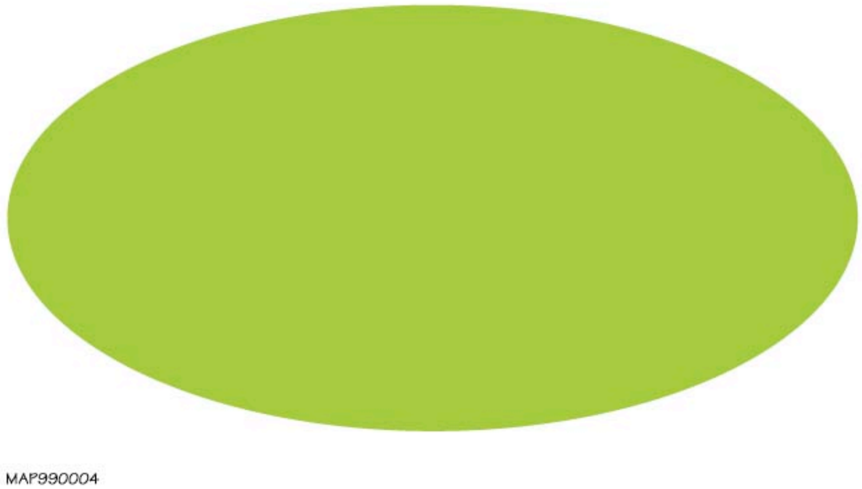
Lecture 1
Physics of the CMB: Thermal history & recombination
Acoustic Oscillations
Boltzmann equation
Harmonic expansion

Lecture 2
CMB Spectra
Polarization and CMB lensing
Status of observations

References:
Cosmological perturbations: Kodama & Sasaki 1984
CMB physics and anisotropies: Hu & Dodelson 2002
Normal modes and Boltzmann equation: Hu & White 1997
Experiments: lambda.gfsc.nasa.gov

Clues of the Universe

CMB: relic light from the Big Bang



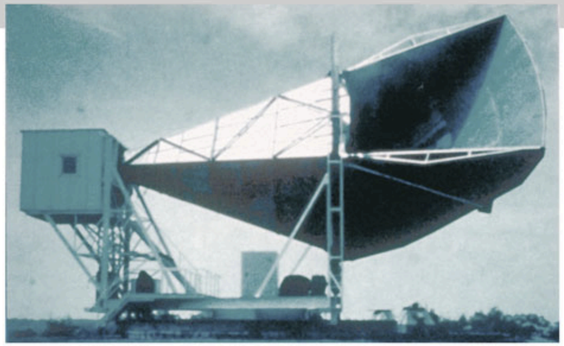
Light emitted 380.000 yrs (z = 1090) after the Big Bang, now in the microwave

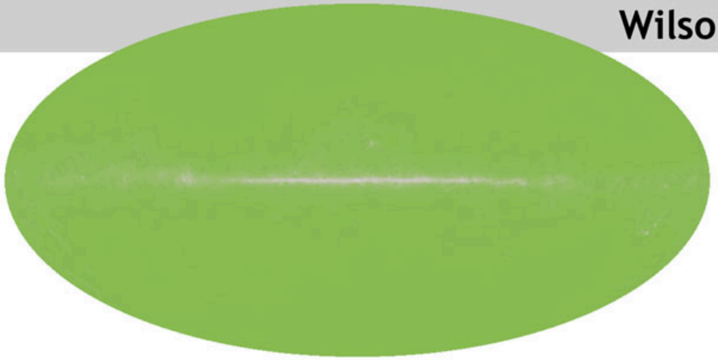
It looks (almost) the same in every direction

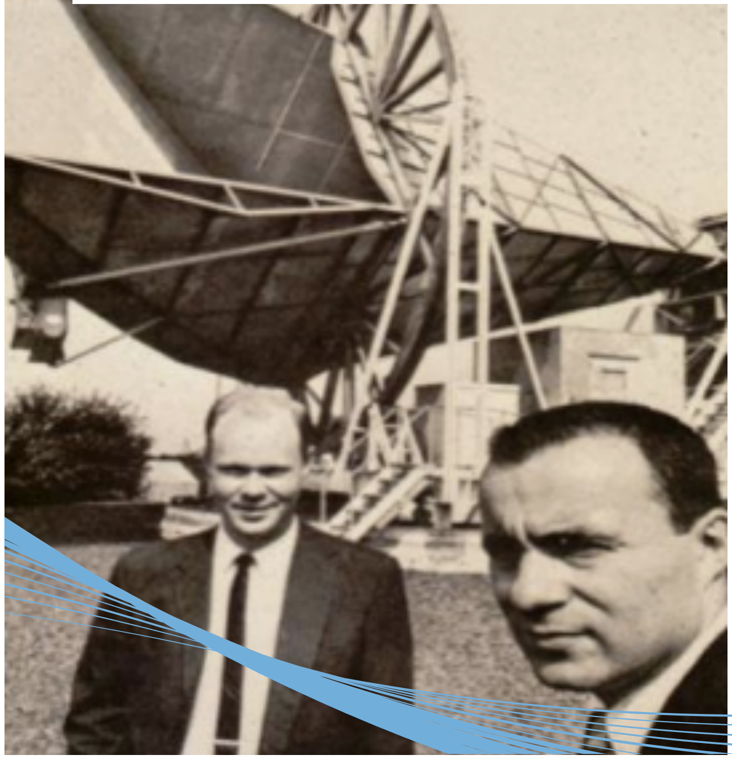
History

1965

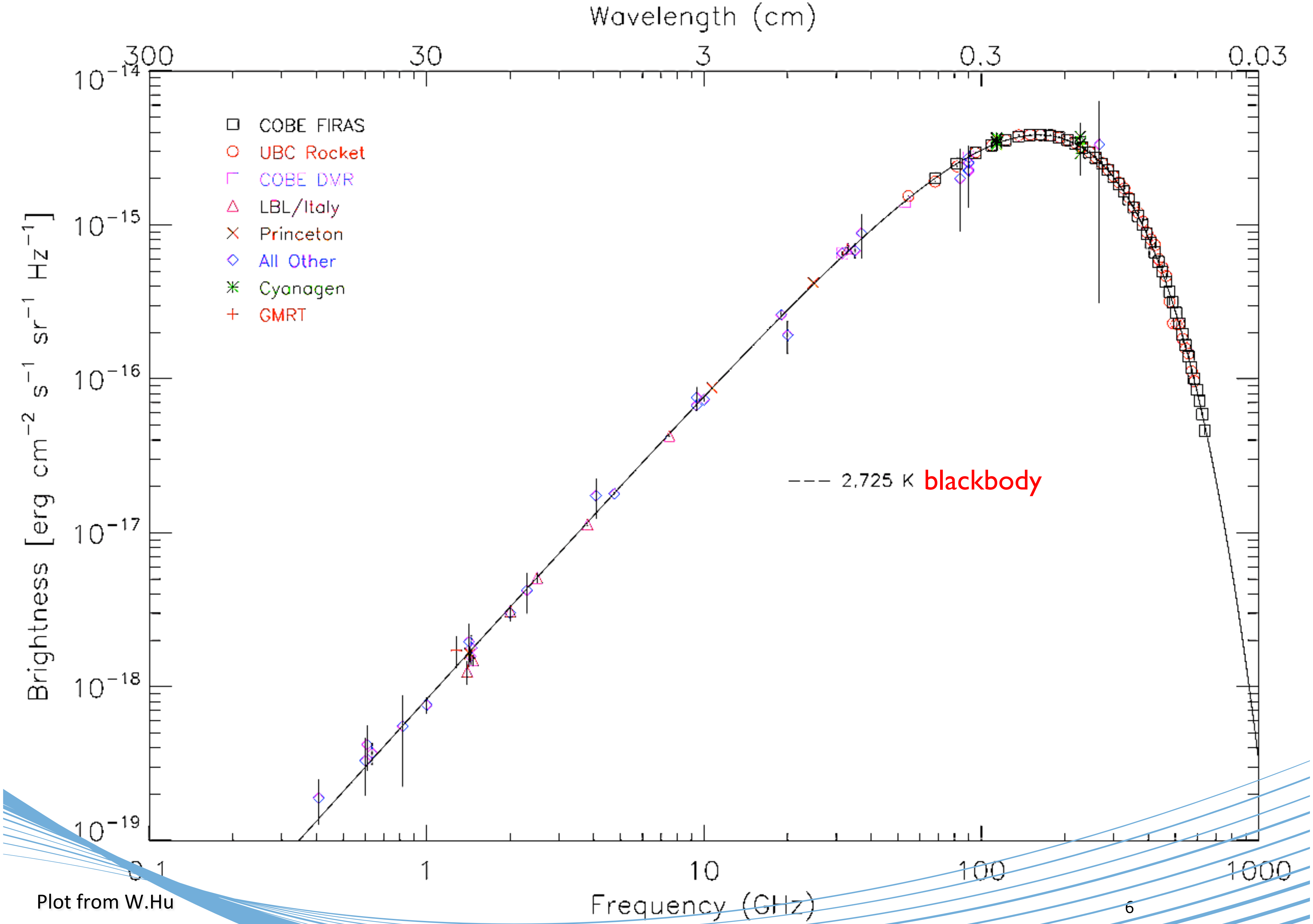
Penzias and Wilson







- Predicted in 1948 (Ralph lpher, Robert Herman, G. Gamow)
- First observed in 1965 by Penzias & Wilson at the Bell Telephone Laboratories in New Jersey. The radiation was acting as a source of excess noise in a radio receiver they were building.
- Researchers (Robert Dicke, Dave Wilkinson, Peebles, Roll) realised it was CMB
- Nobel Prize in 1978 to Penzias & Wilson for the discovery

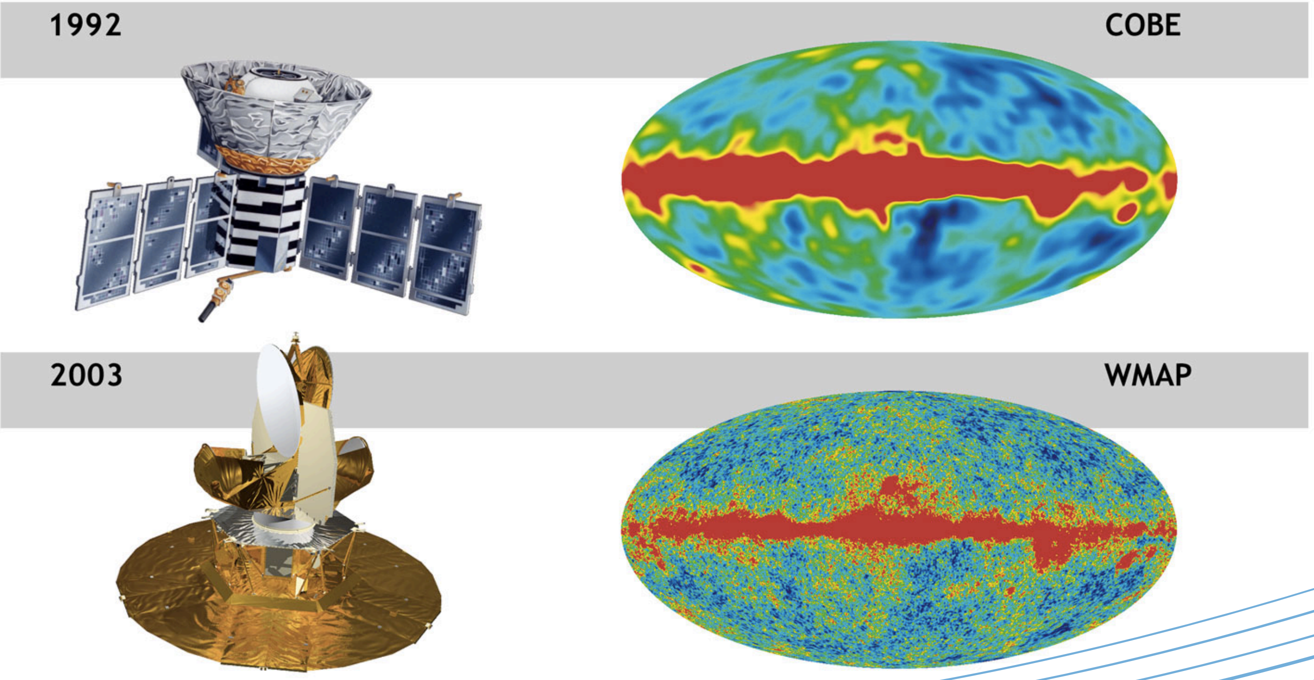


Plot from W.Hu

CMB anisotropies

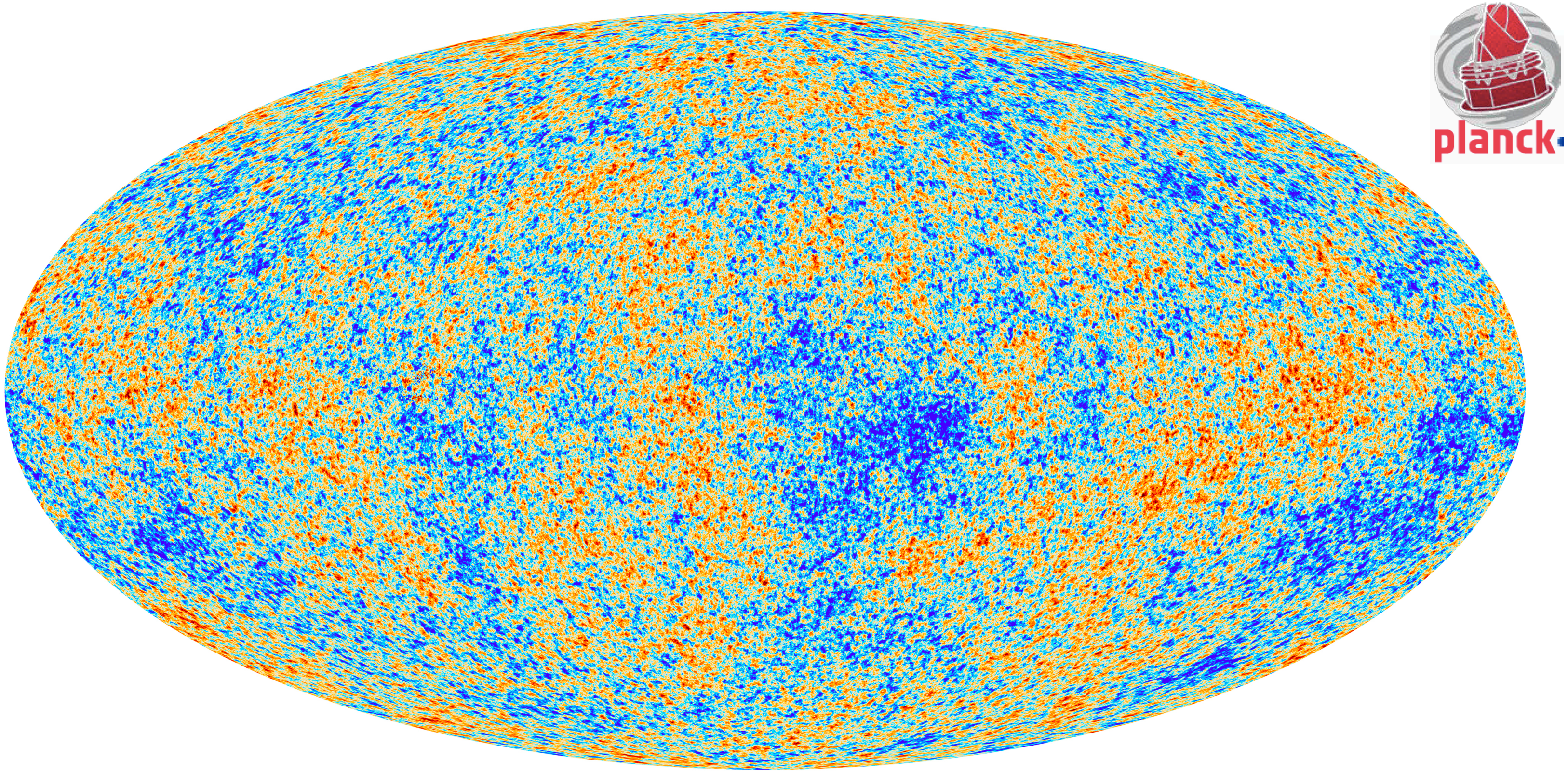
If you look at angles of about 1 degree or smaller you see anisotropies

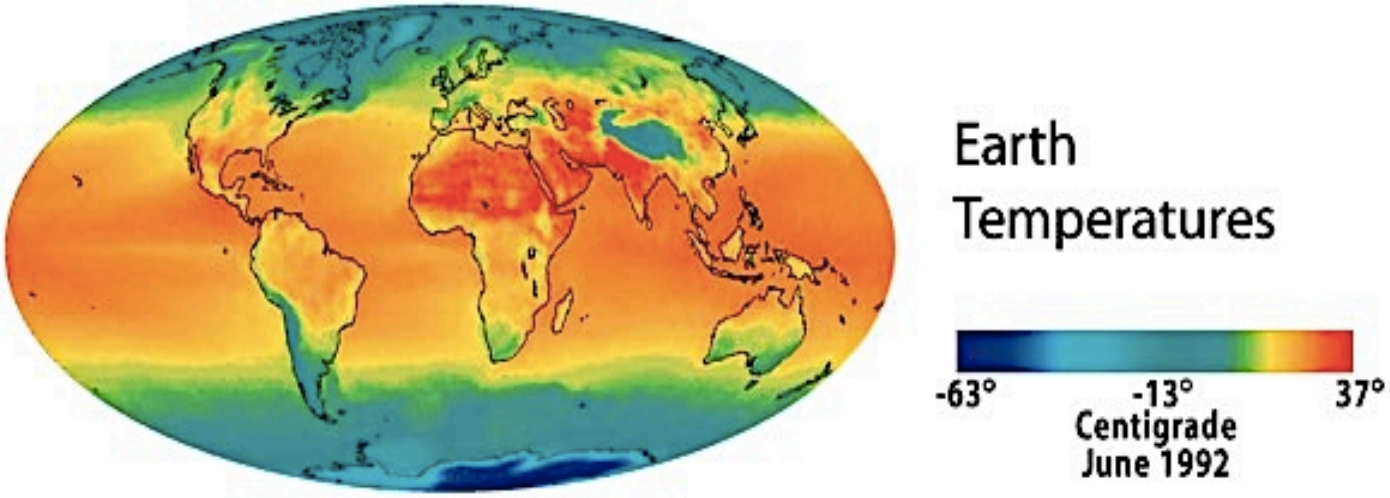
Anisotropies predicted in 1970s-1980s and detected in 1990 with COBE

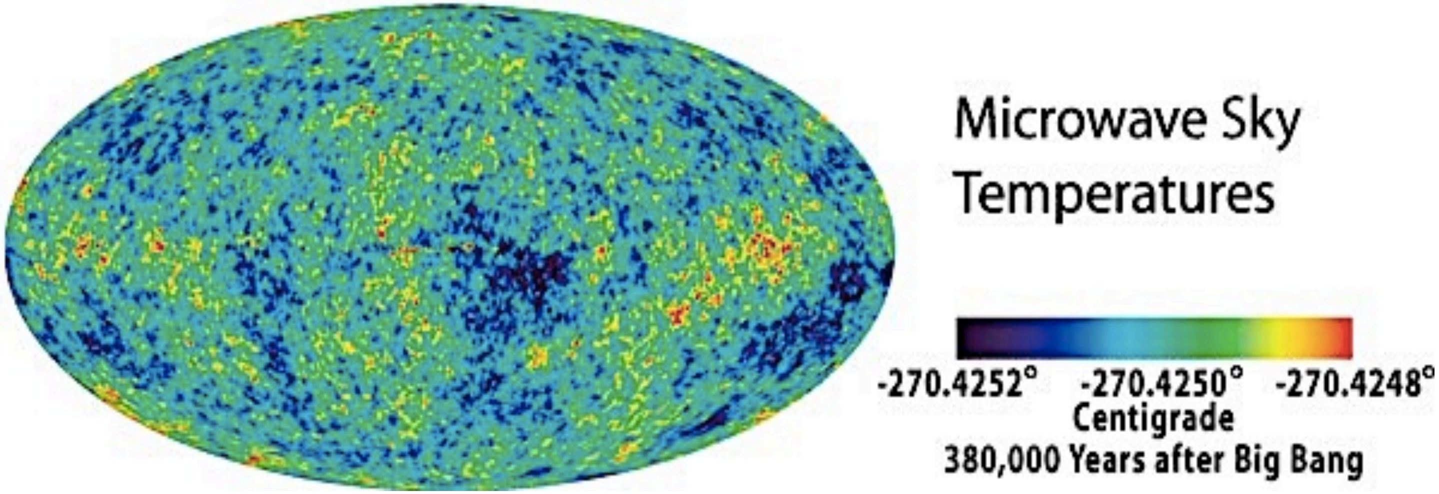


CMB anisotropies

Colors indicate differences in temperature







Deviations from black body are of order 10^{-5}

Thermal history

Thomson scattering

Black body: the radiation is thermalized thanks to efficient interactions

Hydrogen is ionized, photons can scatter on free electrons: Thomson scattering

$$\gamma + e^- \rightarrow \gamma + e^-$$

Mean free path:

$$\lambda_T = \frac{1}{a n_e \sigma_T} \sim 4Mpc \frac{1}{X_e} \left(\frac{0.0125}{\Omega_b h^2}\right) \left(\frac{0.9}{1 - Y_P/2}\right) \left(\frac{1000}{1+z}\right)^2$$

Density of free electron

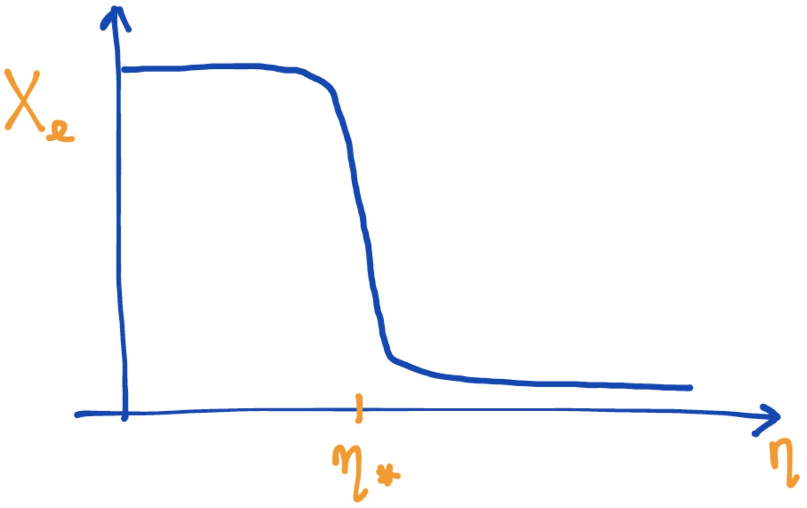
Cross section:

$$\sigma_T \sim 6 \times 10^{-25} cm^2$$

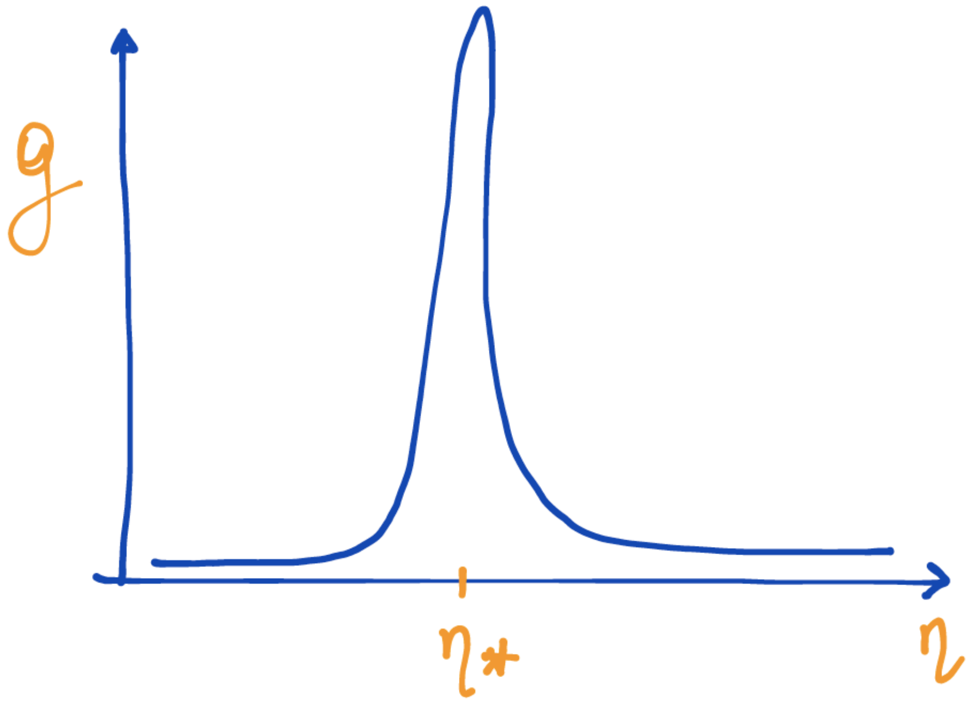
Decoupling occurs when $\lambda_T \sim \mathcal{H}^{-1}(\eta_*)$

z = 1100, T = 3000 K

Ionization fraction and visibility function



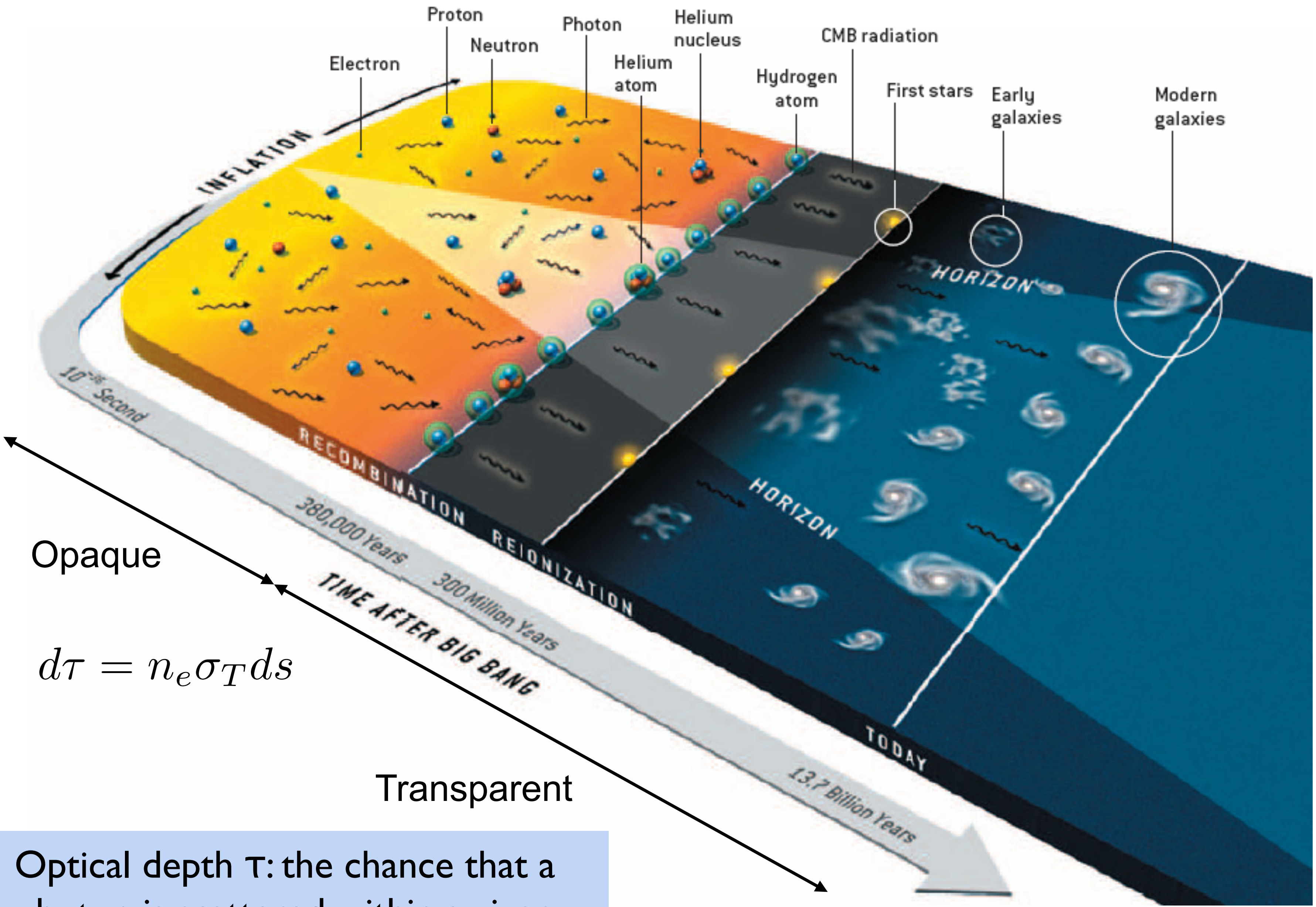
Fraction of free electrons



Probability that a photon we observe last scattered at some position along the line of sight

Evolution of the universe

Hu & White, Sci. Am., 290 44 (200

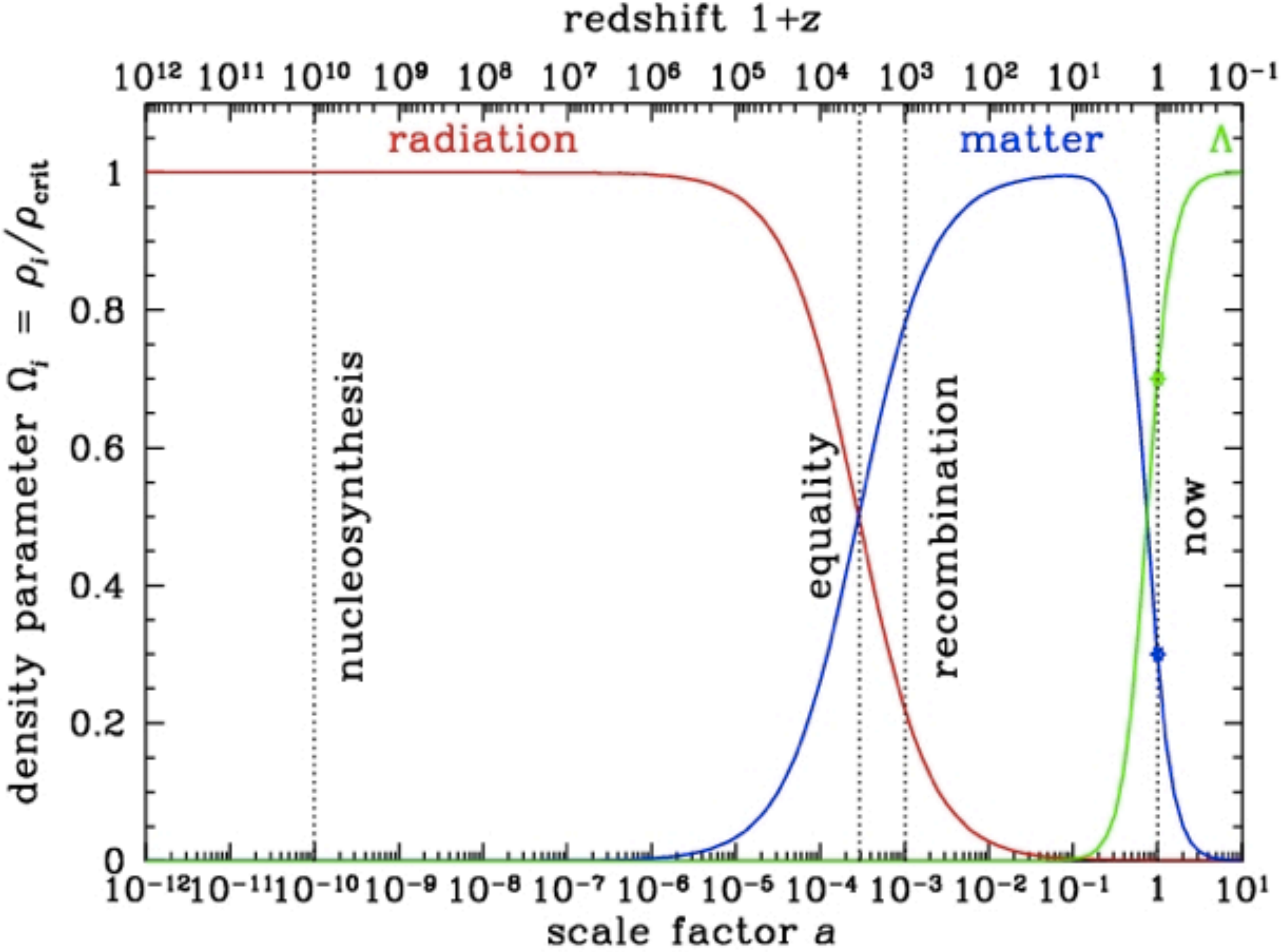


Opaque

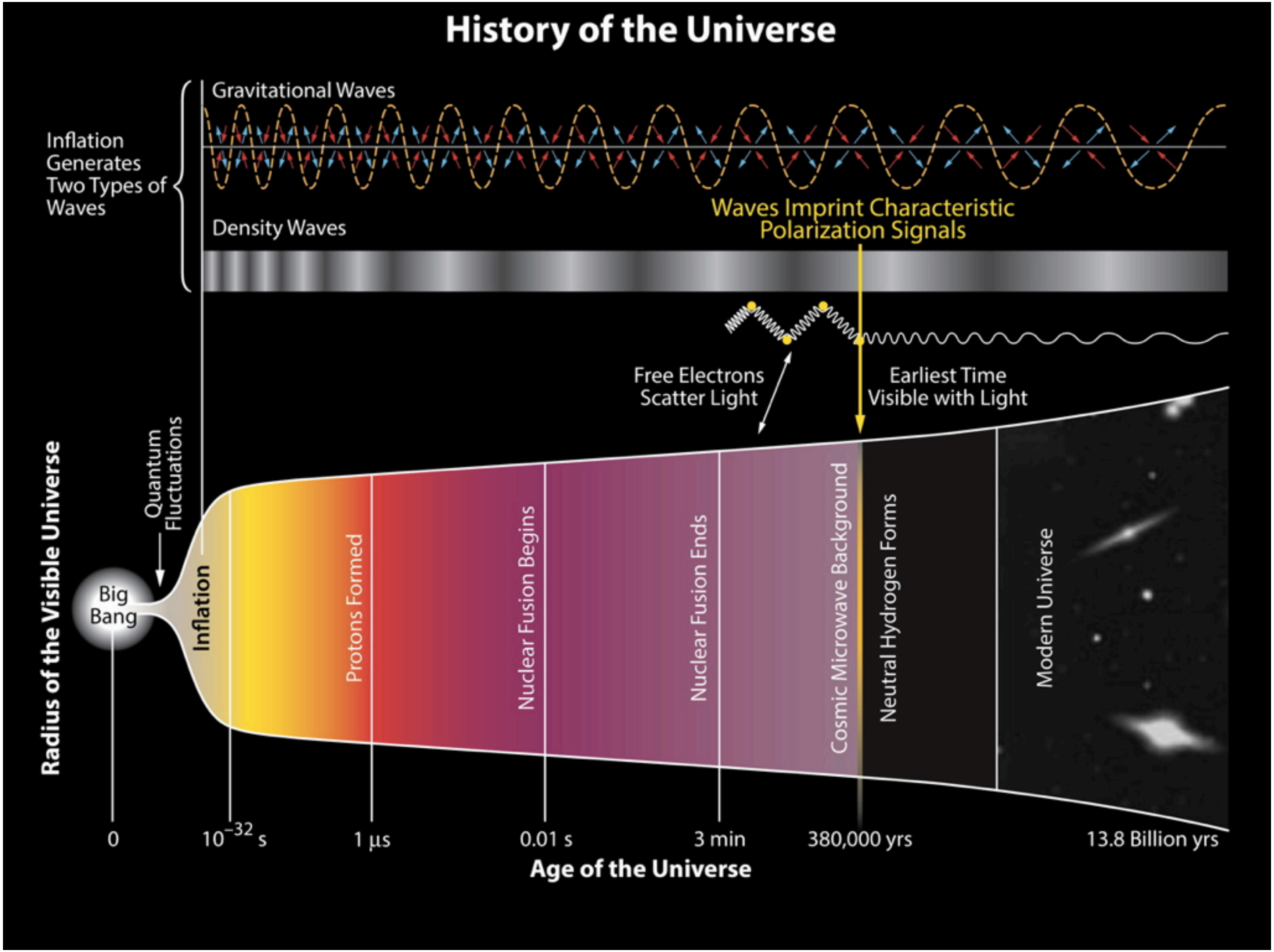
$$d\tau = n_e \sigma_T ds$$

Transparent

Optical depth τ: the chance that a photon is scattered within a given interval of length ds



Plot by B. Fields


History of the Universe
Inflation Generates Two Types of Waves
Gravitational Waves
Density Waves
Waves Imprint Characteristic Polarization Signals
Free Electrons Scatter Light
Earliest Time Visible with Light
Radius of the Visible Universe
Quantum Fluctuations
Big Bang
Inflation
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Neutral Hydrogen Forms
Modern Universe
0
10^-32 s
1 µs
0.01 s
3 min
380,000 yrs
13.8 Billion yrs
Age of the Universe

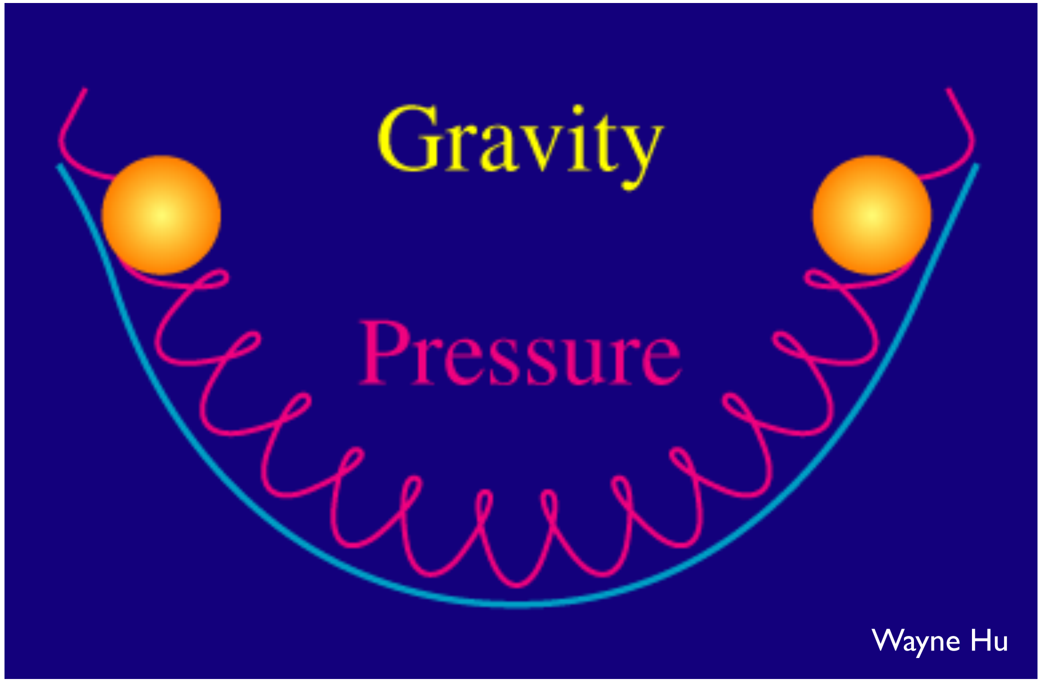
Acoustic oscillations

Sound waves

Before last scattering, photons, electrons and protons behave as a single fluid.

The tight coupling between baryons and photons produce oscillations in the plasma.

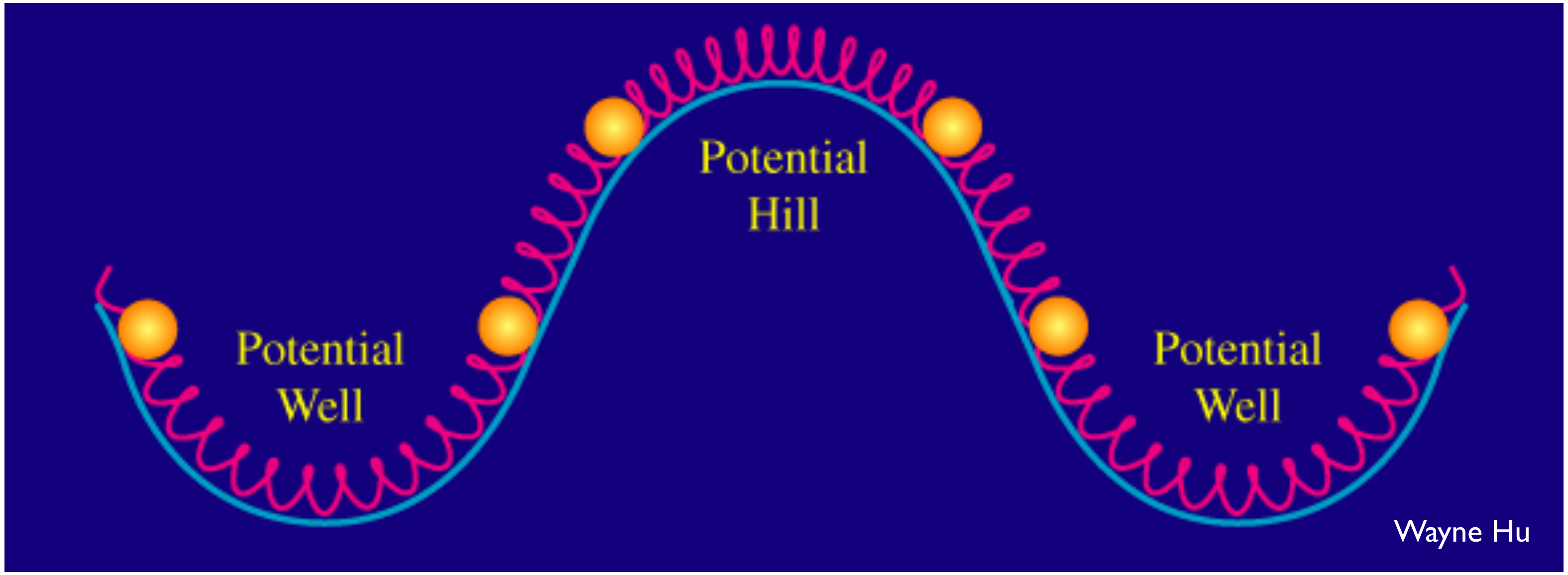
Sound waves of the baryon-photon fluid, gravity/pressure, compressions/ rarefactions.



Compressions and rarefactions

Regions of high density generate potential wells (compression).

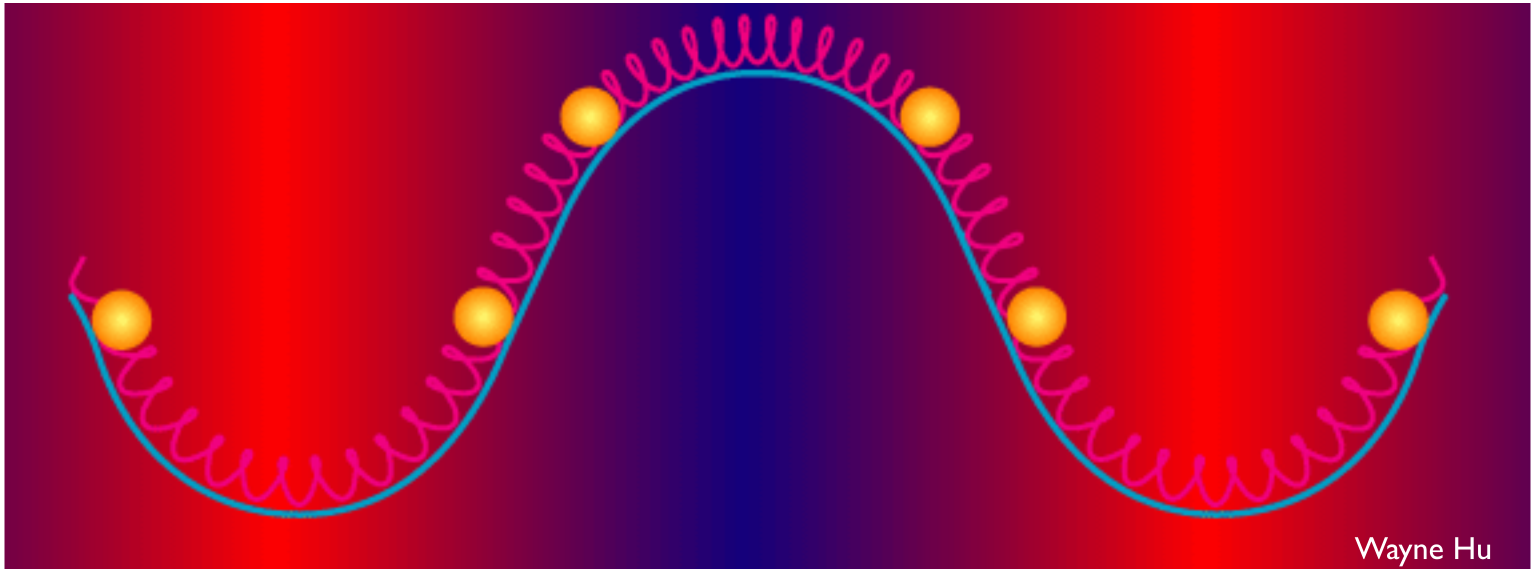
Regions of low density generate potential hills (rarefaction).



Hot and cold

Wherever the density of p and e is higher, also the photon density is higher.

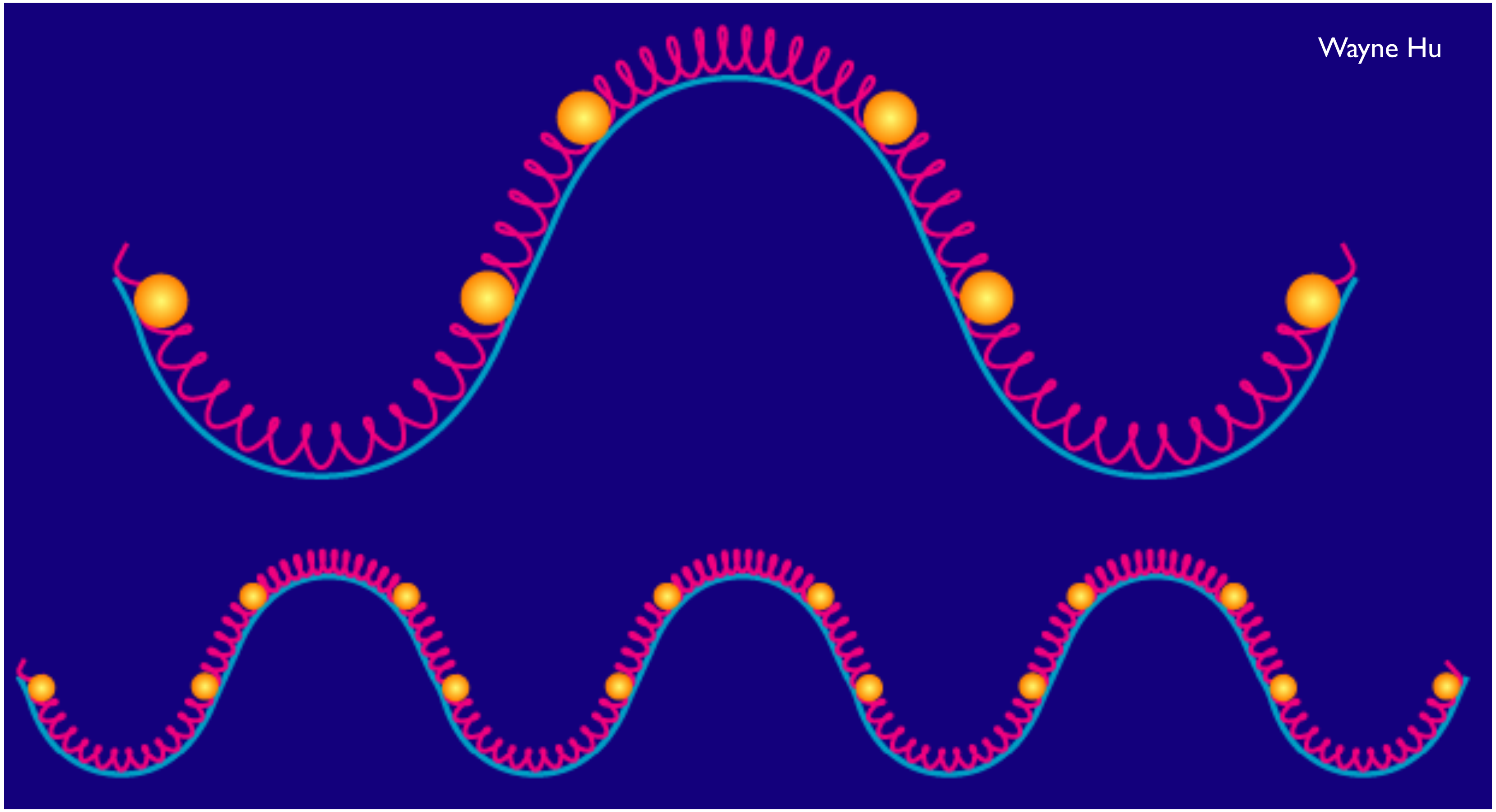
CMB is locally hotter in regions where the acoustic wave causes compression and cooler where it causes rarefaction. (in the picture, blue is hotter, red is colder)



Wayne Hu

Modes

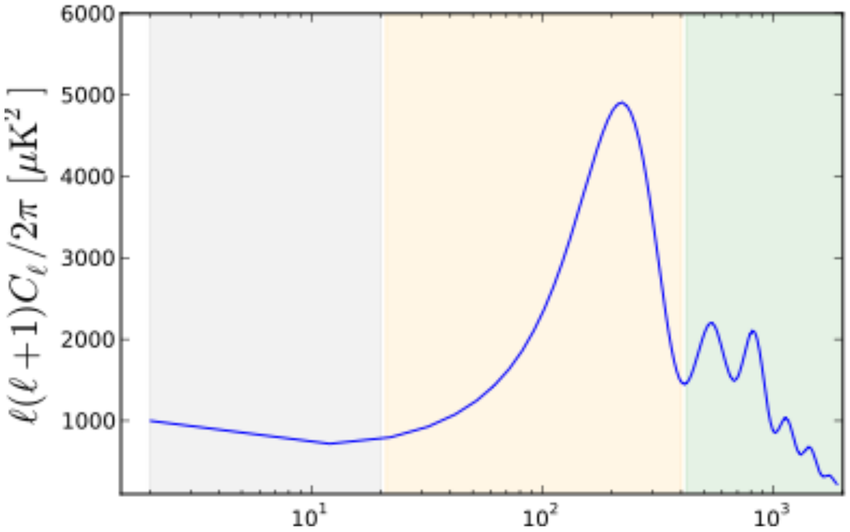
Inflation leads to fluctuations at all scales.
Mathematically, we decompose the fluctuation in Fourier space into plane waves of various wavelenghts. Each mode behaves independently (in linear theory).


Wayne Hu

Freeze

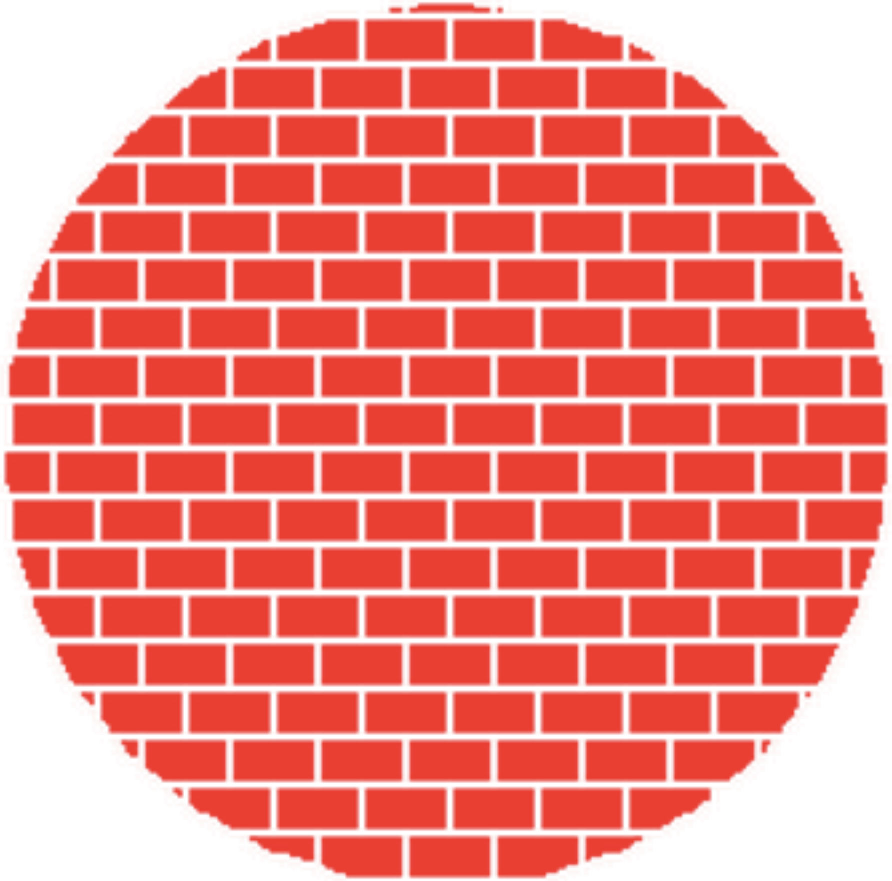
At recombination tight coupling breaks and oscillations freeze.
When this happens, modes will be caught at some point of their oscillations: extrema of their oscillations represent peaks in the CMB.

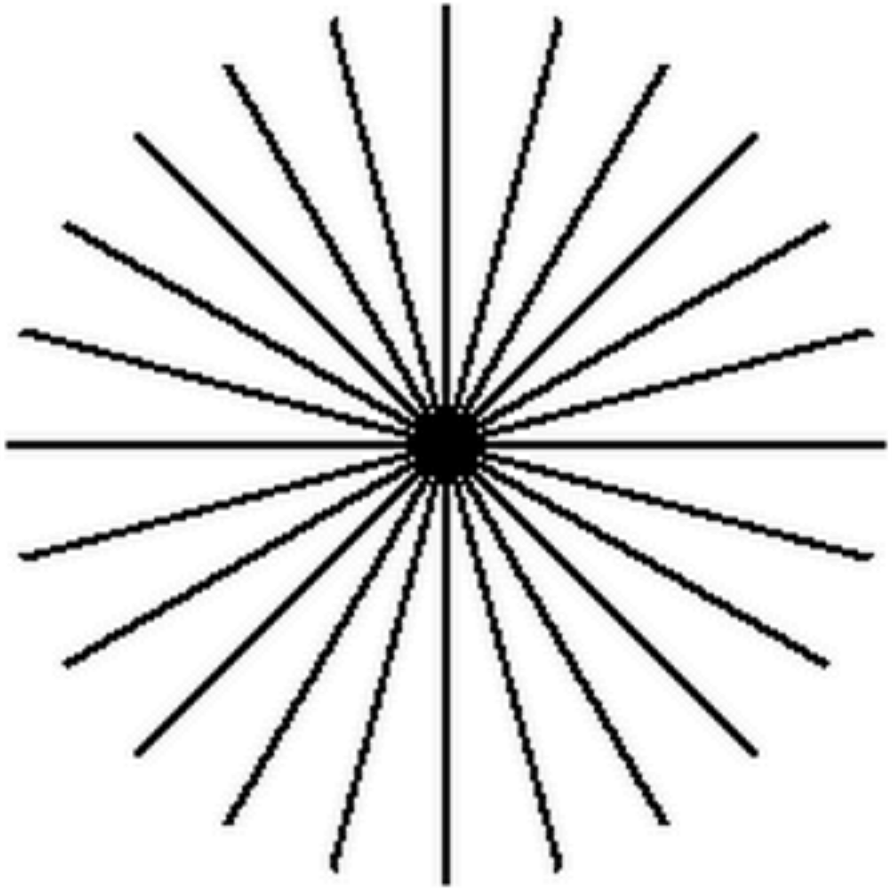
First peak: sound waves that were just starting their first period of compression when the last scattering occurred.

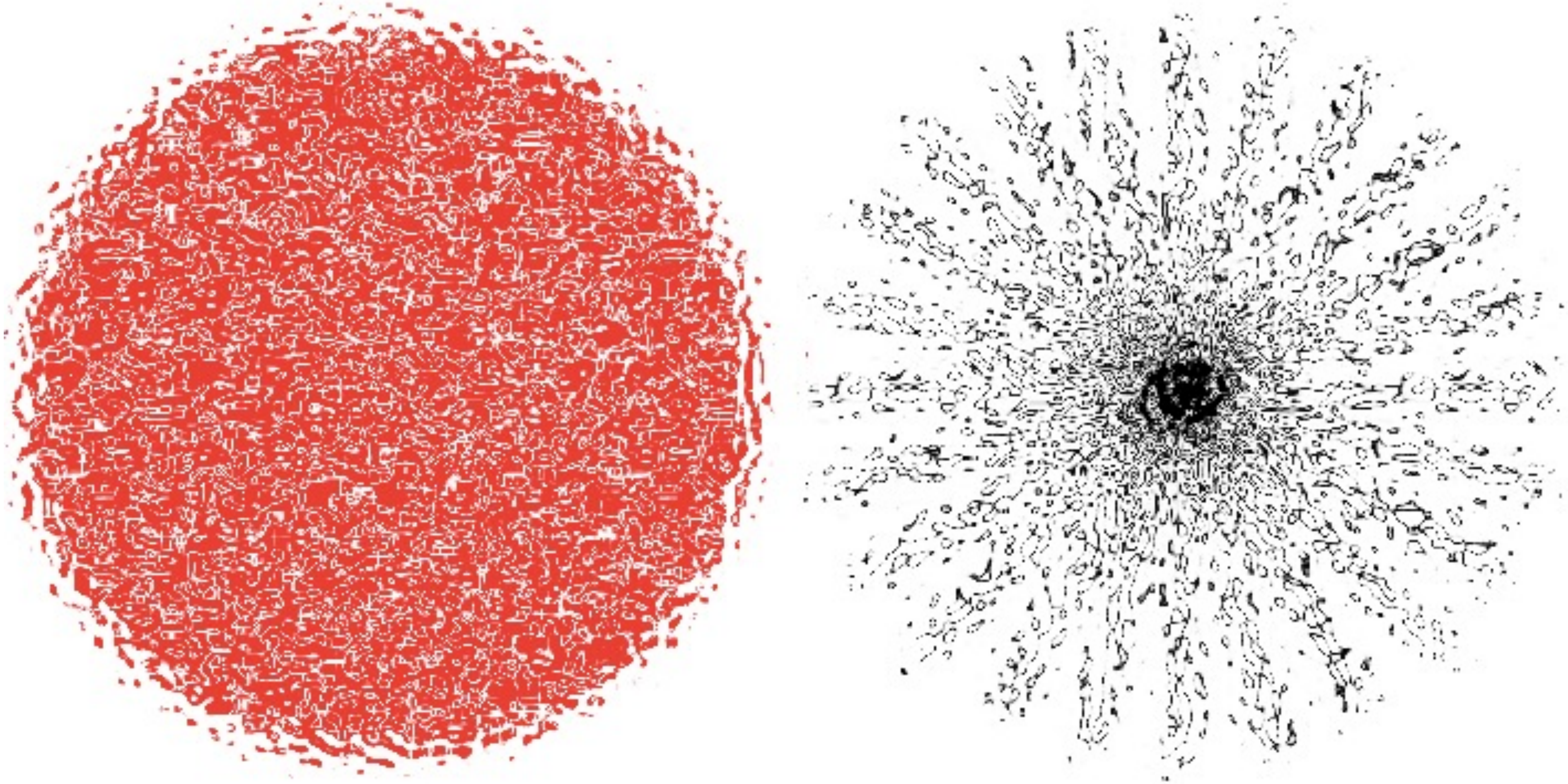


$$l = \pi/\theta$$

Snapshot of primordial perturbations







Photon-baryon fluid: basic description

If we only had photons-baryon fluid perturbations (neglecting gravity) could be described by continuity and Euler equations

$$\dot{\delta}_\gamma = -(1+w_\gamma)kv_\gamma$$

$$\dot{v}_\gamma = kc_s^2\frac{3}{4}\delta_\gamma$$

$$\delta_\gamma = 4\frac{\delta T}{T} \equiv 4\Theta$$

Temperature fluctuations

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0$$

$$c_s^2 = \frac{\delta p_\gamma}{\delta \rho_\gamma} = \frac{\dot{p}_\gamma}{\dot{\rho}_\gamma}$$

(Adiabatic IC)

Harmonic oscillator $\Theta(\eta) = \Theta(0)cos(ks)$

Temperature oscillations represent the heating and cooling of a fluid that is compressed and rarefied by a sound wave.

For scales larger than the sound horizon (ks <<1) perturbations are frozen to the IC; on small scales the amplitude of the Fourier modes will have oscillations.

Fundamental scale

Modes that are caught at max or minima in their oscillations at recombination correspond to peaks in the power of anisotropies:

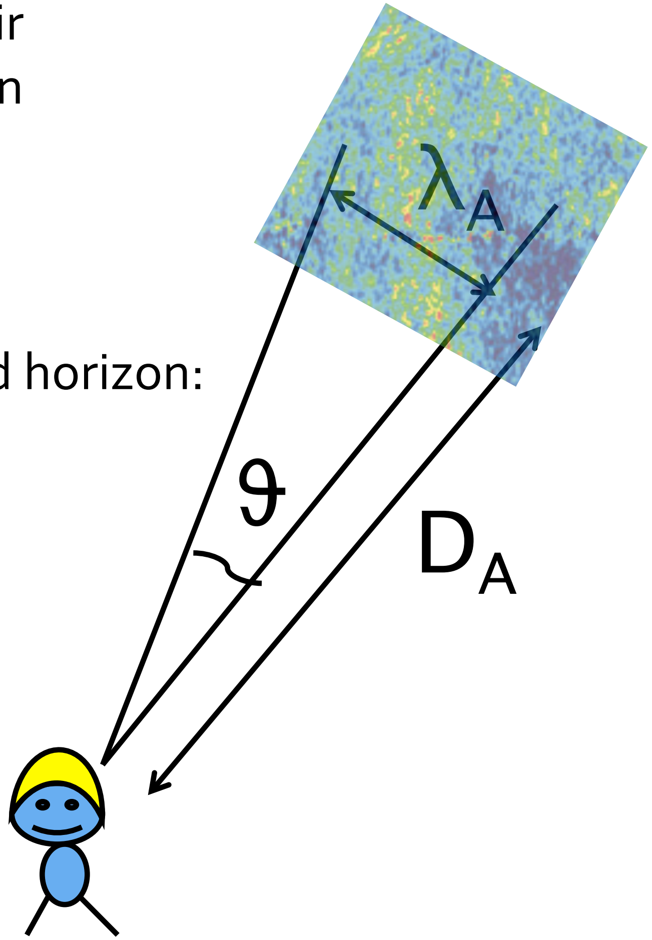
$$k_n s_* = n\pi$$

Fundamental scale related to the inverse of the sound horizon:

$$k_A = \pi / s_*$$

This scale translates into a fundamental angular scale, related to the angular diameter distance:

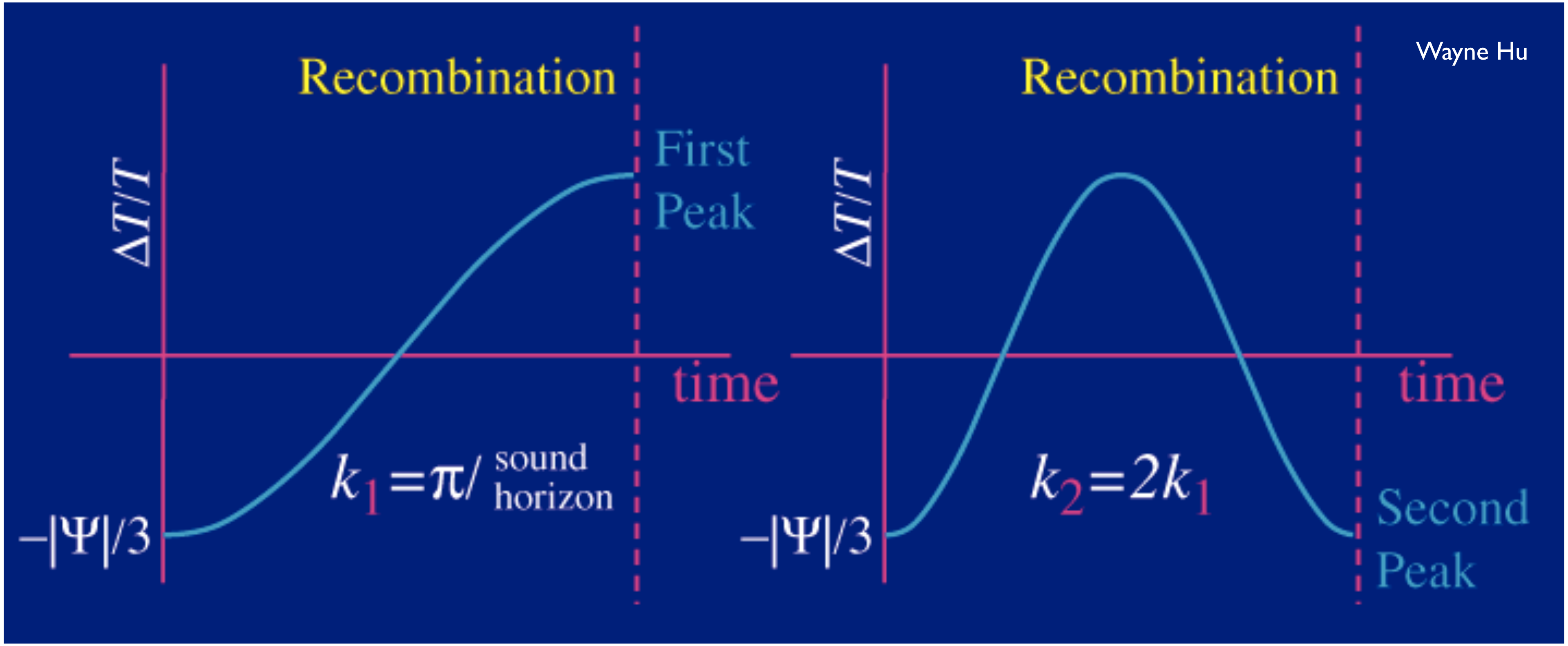
$$\theta_A = \lambda_A / D_A$$



which in a MD universe is about 1 degree and corresponds to first peak.

Sound waves and CMB peaks

The mode of the peaks are related to a fundamental scale: the distance sound can travel until recombination (i.e. the sound horizon).



$$ks = n\pi$$

Adding gravity

Photons climb out of potential wells at recombination and loose energy. Gravity changes the continuity and Euler equations and therefore also the temperature fluctuation:

$$\ddot{\Theta} + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - \ddot{\Phi}$$

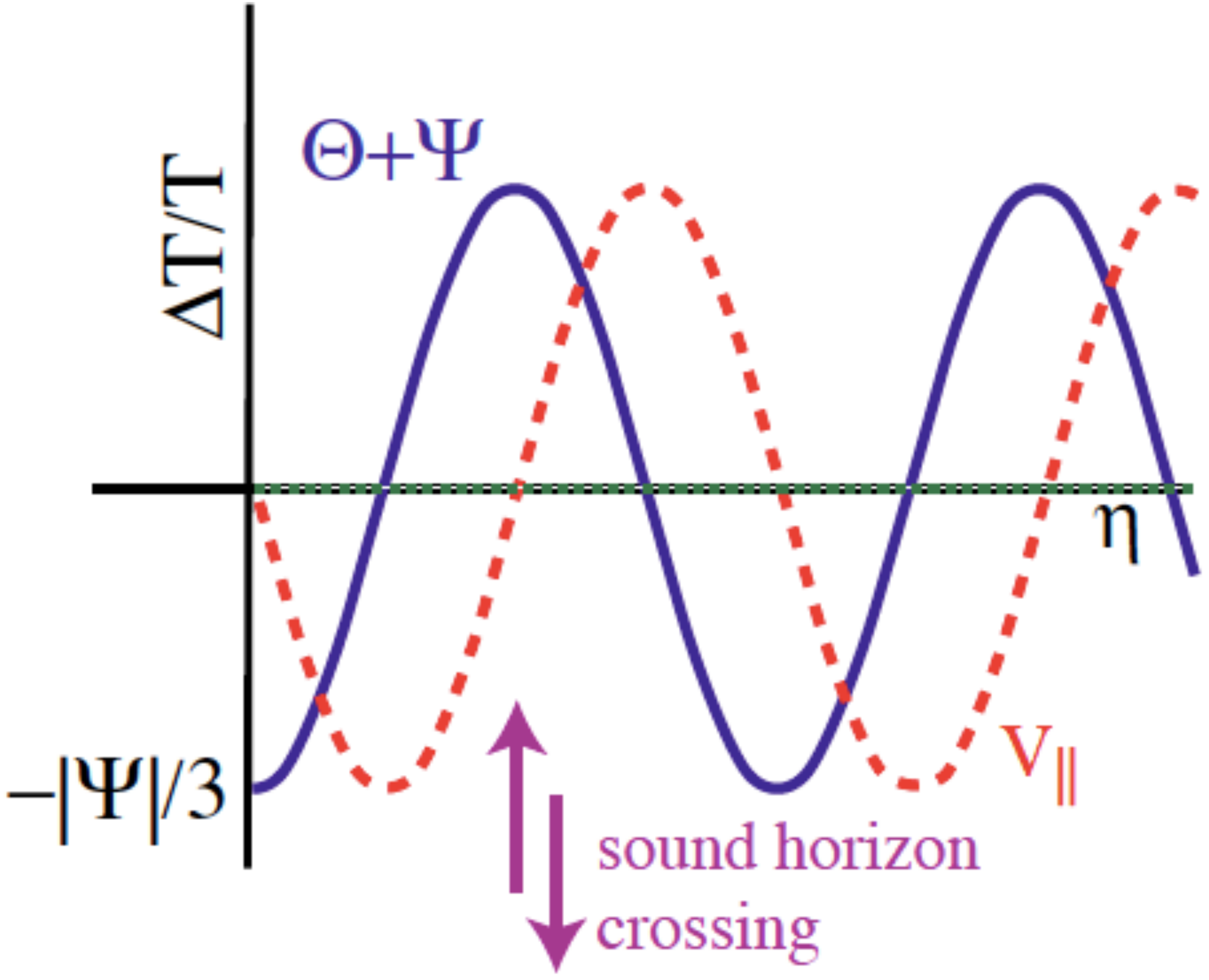
In a flat universe and in the absence of pressure, the potentials are constant. Same equation as before but for the 'effective temperature':

$$[\Theta + \Psi](\eta) = [\Theta + \Psi](0)\ \cos(ks)$$

solution is just an offset

It oscillates around zero with an amplitude given by the IC (IC are set when the perturbation is outside horizon).
The large scale limit is the 'Sachs Wolfe'.

Effective temperature



Plot by Wayne Hu

Baryon dragging

We have neglected baryons in the dynamics of oscillations

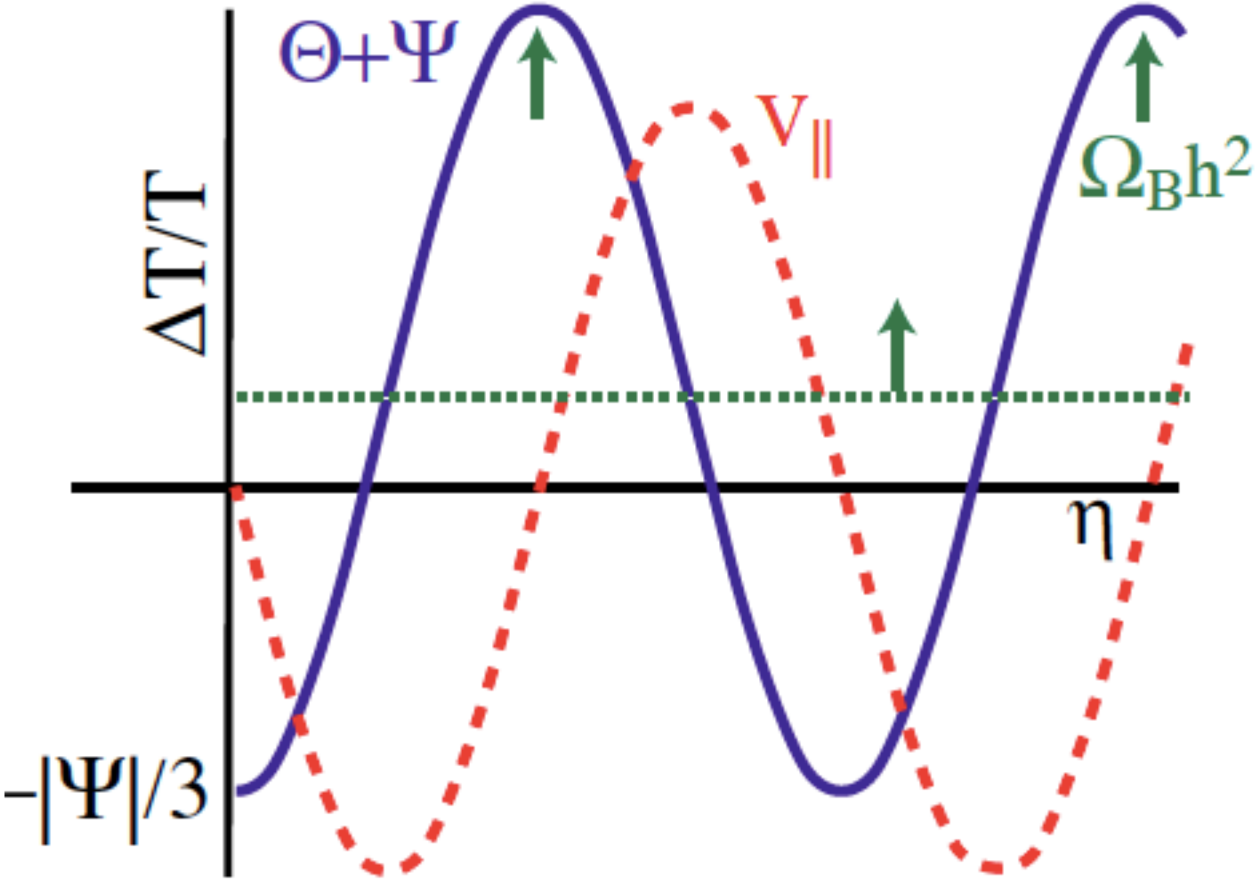
$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Phi})$$

where now the sound speed $c_s^2 = \frac{1}{3}\frac{1}{1+R}$ $R \equiv \frac{p_b + \rho_b}{p_\gamma + \rho_\gamma}$

Baryons lower the sound speed (so decrease the sound horizon). They enhance the compressions, i.e. change odd/even relative amplitude of the oscillations

$$[\Theta + (1+R)\Psi](\eta) = [\Theta + (1+R)\Psi](0)\cos(ks)$$

Baryon dragging



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Doppler effect

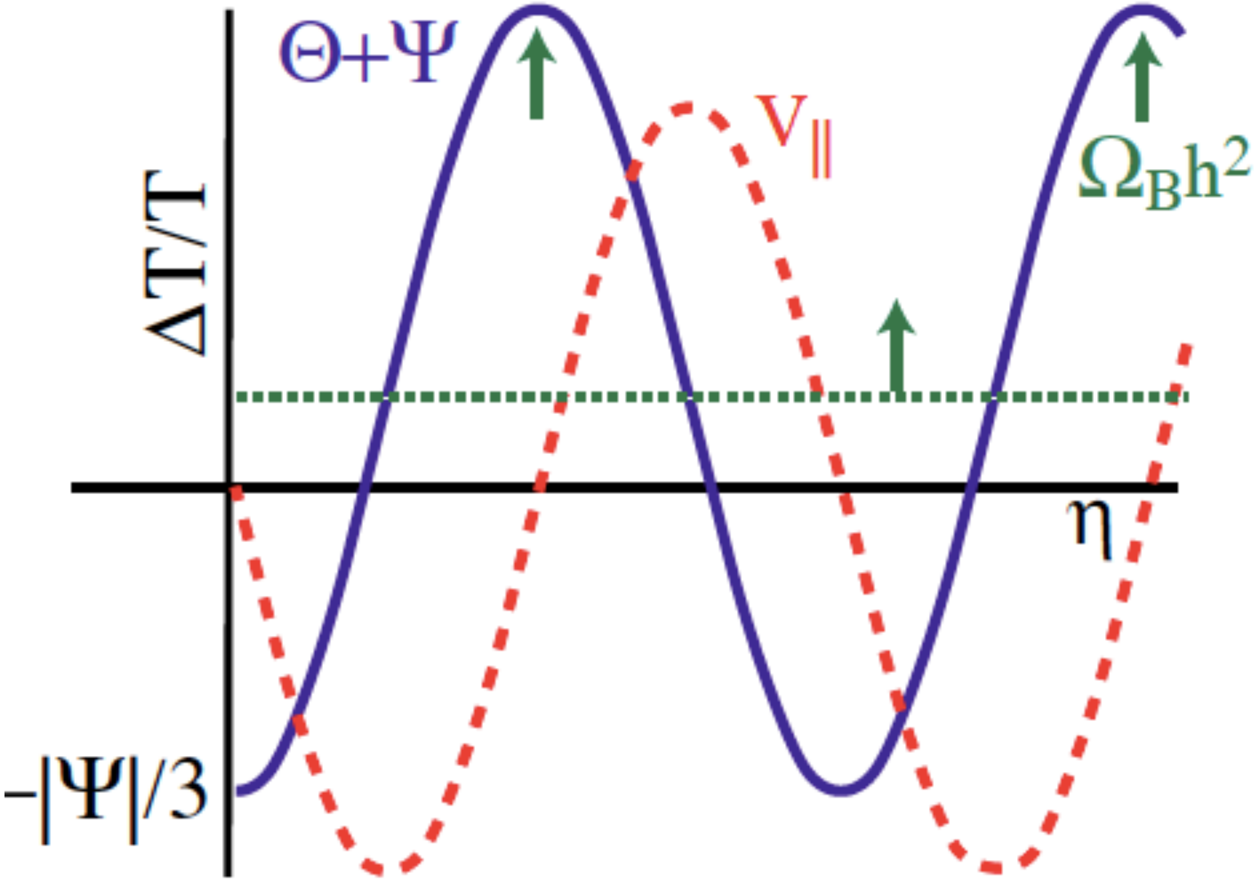
Fluid velocity also changes the observed temperature via the Doppler shift, due to the velocity of the fluid with respect to our reference system:

$$\left(\frac{\Delta T}{T}\right)_{dop} = \hat{n} \cdot v_\gamma$$

At extrema, velocity is zero, has nodes where temperature oscillations have peaks and deeps.

This term also oscillates but with phase shift of π/2 (so as a sin(ks)).

Doppler effect



Velocity oscillations are symmetric around zero. Equal amplitude.

Radiation driving effect

Radiation contributes to the expansion and to the Poisson equation, so to the gravitational potential itself. If radiation dominates, potentials are not constant but decay.

The more radiation there is, the more it decays, the less energy the photon looses to fight against compression (and also less redshift to climb out the well).

We expect a difference in amplitude between modes that start oscillating in RDE and in MDE.

Small scales cross horizon earlier, so they start to oscillate earlier.

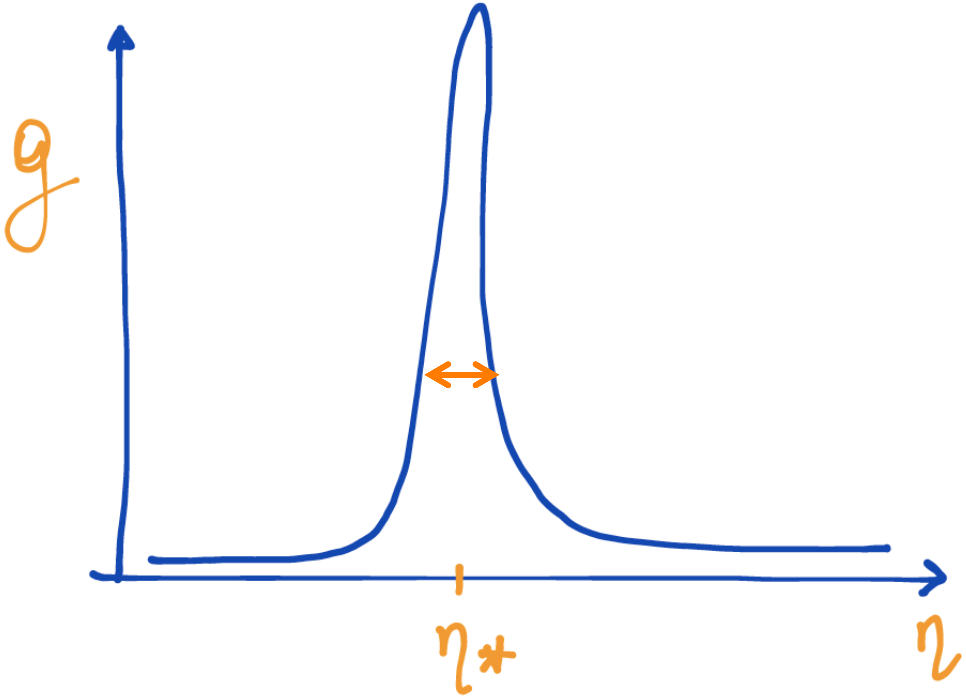
Enhancement in amplitude of peaks (higher l). Can be used to infer matter radiation equality.

Diffusion Damping

Recombination is not instantaneous.

Some photons still scatter: damping at scales smaller than the thickness of last scattering 'surface'. The coupling is not perfectly tight, there is diffusion. Anisotropic stress contributes to photon Euler equation.

Suppression beyond the 3rd peak



Final oscillator equation

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + \frac{k^2 c_s^2}{\dot{\tau}}[A_v + A_h]\dot{\Theta} + k^2 c_s^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Phi})$$

Baryons in sound speed

Damping term

Fundamental scale

Gravitational driving

(just from fluid dynamics, no transfer)

Boltzmann equation

Radiative transfer

We need to transport the radiation from the initial conditions to the observer. Photons decouple so we cannot describe them with fluid equations.

This is done using the Boltzmann equation (radiative transfer equation).

Collisionless part: gravity and photon distribution

$$\frac{df}{d\eta} = C[f]$$

Collision term due to Thomson scattering

Where the distribution function is:

$$f = \frac{g_\gamma}{e^{\frac{E}{k_B T}(1-\Theta)} - 1}$$

$$\frac{df}{d\eta} = \dot{f} + \frac{\partial f}{\partial x^i}\frac{dx^i}{d\eta} + \frac{\partial f}{\partial E}\frac{dE}{d\eta} + \frac{\partial f}{\partial n^i}\frac{dn^i}{d\eta} = C[f]$$

Background Boltzmann equation

$$\frac{df}{d\eta} = \dot{f} + \frac{\partial f}{\partial x^i}\frac{dx^i}{d\eta} + \frac{\partial f}{\partial E}\frac{dE}{d\eta} + \frac{\partial f}{\partial n^i}\frac{dn^i}{d\eta} = C[f]$$

= 0

1st order

Both 1st order,
the product is
2nd order

C = 0

Only bkg
contribution

Perturbed Boltzmann equation

$$\frac{df}{d\eta} = \underset{\dot{\Theta}}{\underset{\downarrow}{\dot{f}}} + \underset{\vec{\nabla}\Theta}{\underset{\downarrow}{\frac{\partial f}{\partial x^i}}}\,\underset{\hat{n}}{\underset{\downarrow}{\frac{dx^i}{d\eta}}} + \underset{\delta G}{\underset{\downarrow}{\frac{\partial f}{\partial E}\,\frac{dE}{d\eta}}} + \underset{\downarrow}{\cancel{\frac{\partial f}{\partial n^i}\,\frac{dn^i}{d\eta}}} = \underset{\delta c}{\underset{\downarrow}{C[f]}}$$

Both 1st order,
the product is
2nd order

Obtained from the
geodesic equation for the
photons; written in terms
of metric perturbations

$$\dot{\Theta} + \hat{n} \cdot \vec{\nabla}\Theta - \dot{\Phi} + \hat{n} \cdot \vec{\nabla}\Psi = \delta C$$

Thomson scattering and tight coupling

$$\dot{\Theta} + \hat{n} \cdot \vec{\nabla}\Theta - \dot{\Phi} + \hat{n} \cdot \vec{\nabla}\Psi = \boxed{-\Gamma(\Theta - \Theta_0 - \hat{n} \cdot \vec{v}_e)}$$

Monopole

Dipole

Thomson scattering rate
(inverse of mean free path)

When Γ is large (before decoupling) only monopole and dipole contributions.

In tight coupling regime all higher multipoles vanish. Photon perturbations can be described in terms of two independent variables, like for a fluid

$$\Theta_0 \sim \delta_\gamma \qquad\qquad \Theta_1 \sim v_\gamma$$

The interaction imposes a common bulk velocity to e, photons, b.

Decomposition in normal modes (k frame)

We expand the direction dependence in spherical harmonics and the space dependence in Fourier space:

$$\Theta(\eta, \vec{x}, \vec{n}) = \frac{\delta T}{T} = \int \frac{d^3k}{(2\pi)^3} \sum_l \sum_{m=-2}^{2} \Theta_{lm}(\eta, \vec{k}) G_{lm}(\vec{k}, \vec{x}, \hat{n})$$

Normal bases, product of plane waves and spherical harmonics

$$G_{lm}(\vec{k}, \vec{x}, \hat{n}) \sim Y(\vec{k}, \vec{x}) Y_{lm}(\hat{n}_k)$$

Expansion done keeping the k vector as the polar axis.

m = 0 scalars
m = 1 vectors
m = 2 tensors

Decomposition in normal modes (k frame)

Boltzmann equation for the expansion coefficients:

$$\dot{\Theta}_l^{(m)} = k\left[\frac{\kappa_l^m}{2l+1}\Theta_{l-1}^{(m)} - \frac{\kappa_{l+1}^m}{2l+3}\Theta_{l+1}^{(m)}\right] - \dot{\tau}\Theta_l^{(m)} + S_l^{(m)}$$

Free streaming terms:
the power is transferred dynamically from l to l +1
The anisotropy power at high l is populated

Friction term (due to Thomson scattering)

Gravitational source, monopole, dipole

Scalar perturbations select m = 0

Originally CMB codes were solving the whole hierarchy up to the multipole l of interest.

Solving the Boltzmann equation

Integrating in time along the line of sight simplifies things.

$$\Theta_l(\eta_0, k) = \int_{\eta_{ini}}^{\eta_0} \underbrace{S(\eta, k)}_{\text{Physical Sources}} \underbrace{j_\ell(k(\eta_0 - \eta))}_{\text{Geometrical part}} d\eta$$

$$S(\eta, k) \equiv g(\overbrace{\Theta_0 + \Psi}^{\text{SW}}) + \overbrace{\left(g\frac{v_b}{k}\right)'}^{\text{Doppler}} + e^{-\tau}(\overbrace{\Phi' + \Psi'}^{\text{ISW}}) + g\,P$$

Peaked around recombination
Encodes the initial anisotropy power at recombination

Peaked around recombination
Coupling of photons and electrons

Still need hierarchy to calculate first multipoles, P, neutrinos

ISW effect

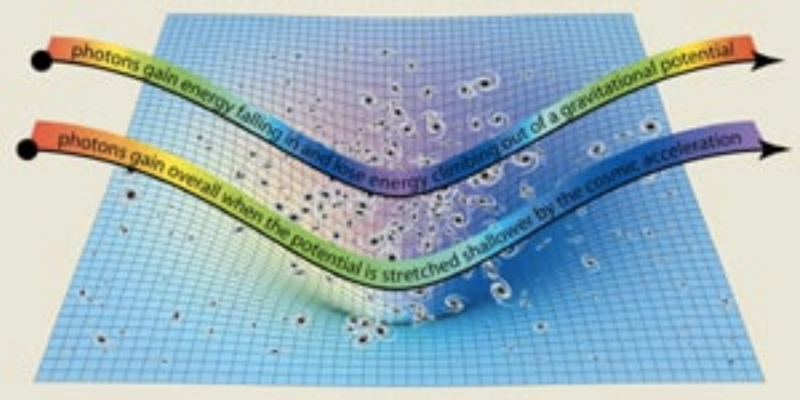
After last scattering, photons free stream. Only gravitational effects (and reionization) can alter the temperature.

The changes in time of gravitational potentials must be integrated along the line of sight.

Early (RDE to MDE) and Late ISW (RDE to DE)

Contributions of LISW arise from a distance closer to us, that subtend larger angles on the sky.

Enhancement of the anisotropic power at small multipoles.



CMB spectra

Temperature power spectrum

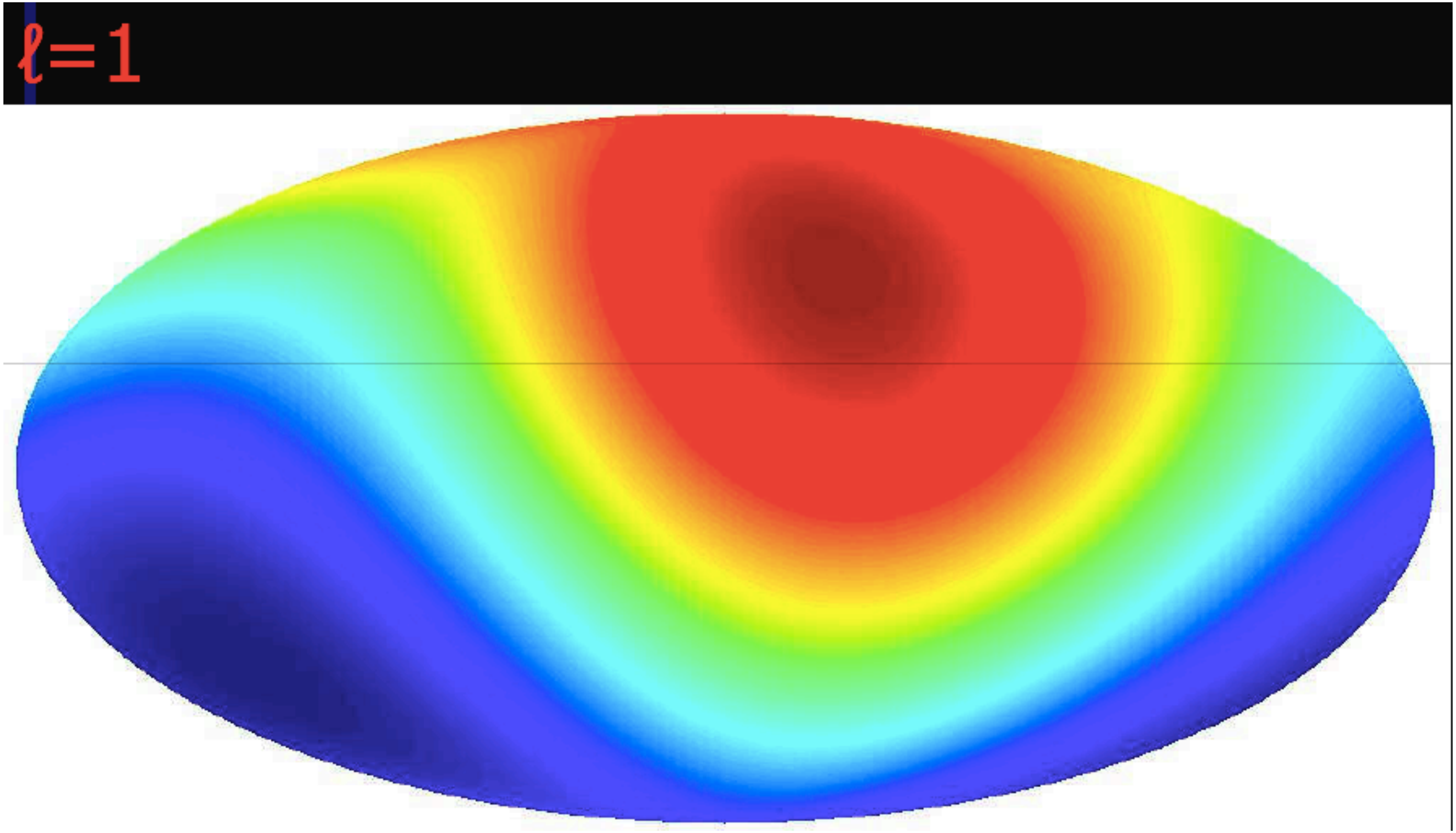
In the laboratory frame:

$$\frac{\delta T}{T}(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

a_{lm} can be written in terms of the primordial curvature power spectrum:

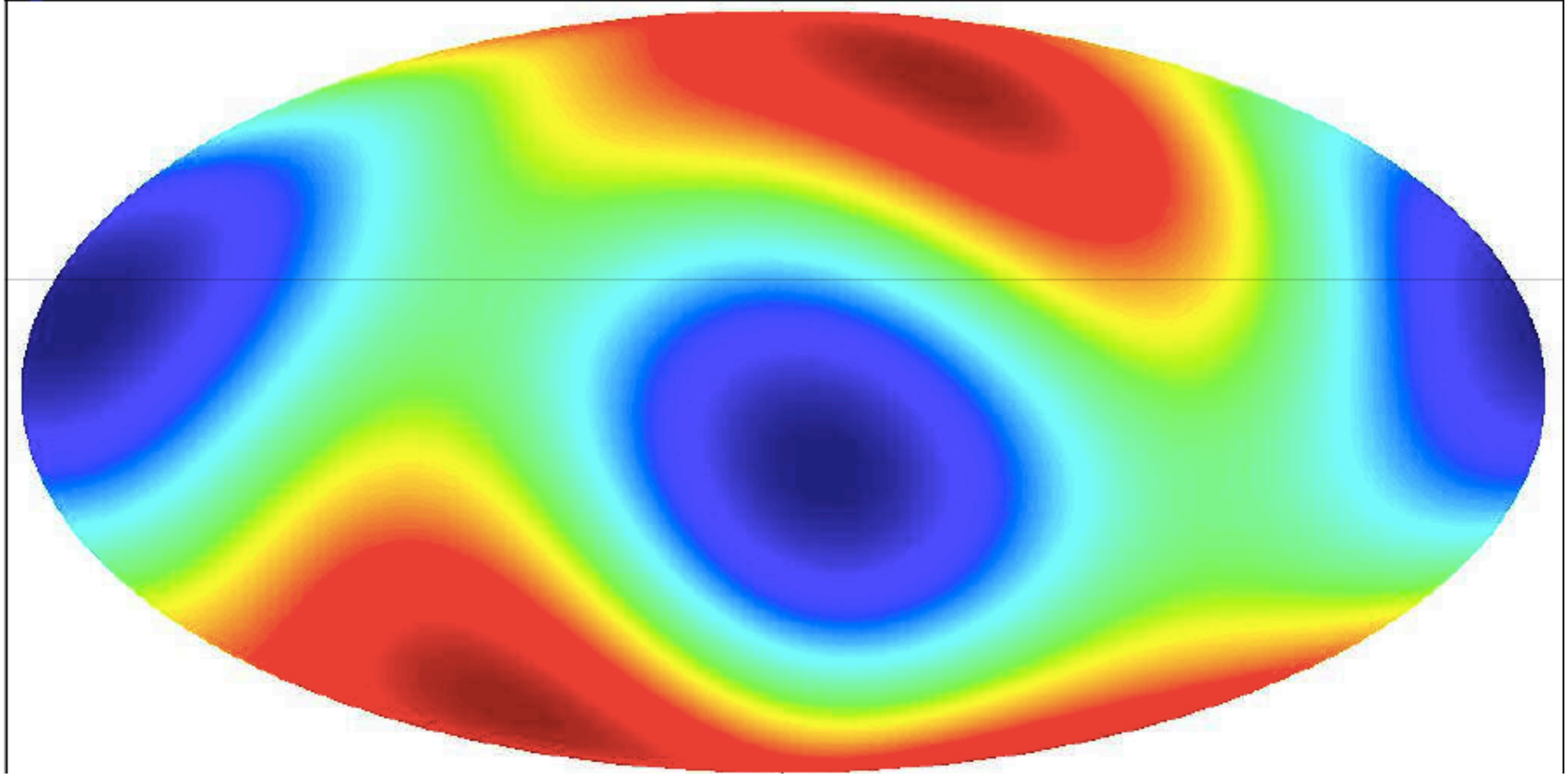
For Gaussian fluctuations, the statistical content of the maps is encapsulated in the two point correlation function

$$\boxed{< a_{lm} a^{*}_{l'm'} >} = \delta_{ll'}\delta_{mm'} \underbrace{\left[\frac{1}{2\pi^2}\int \frac{dk}{k}\Theta_l^2(\eta_0, k)\mathcal{P}_{\mathcal{R}}(k)\right]}_{\boxed{\equiv C_l}}$$

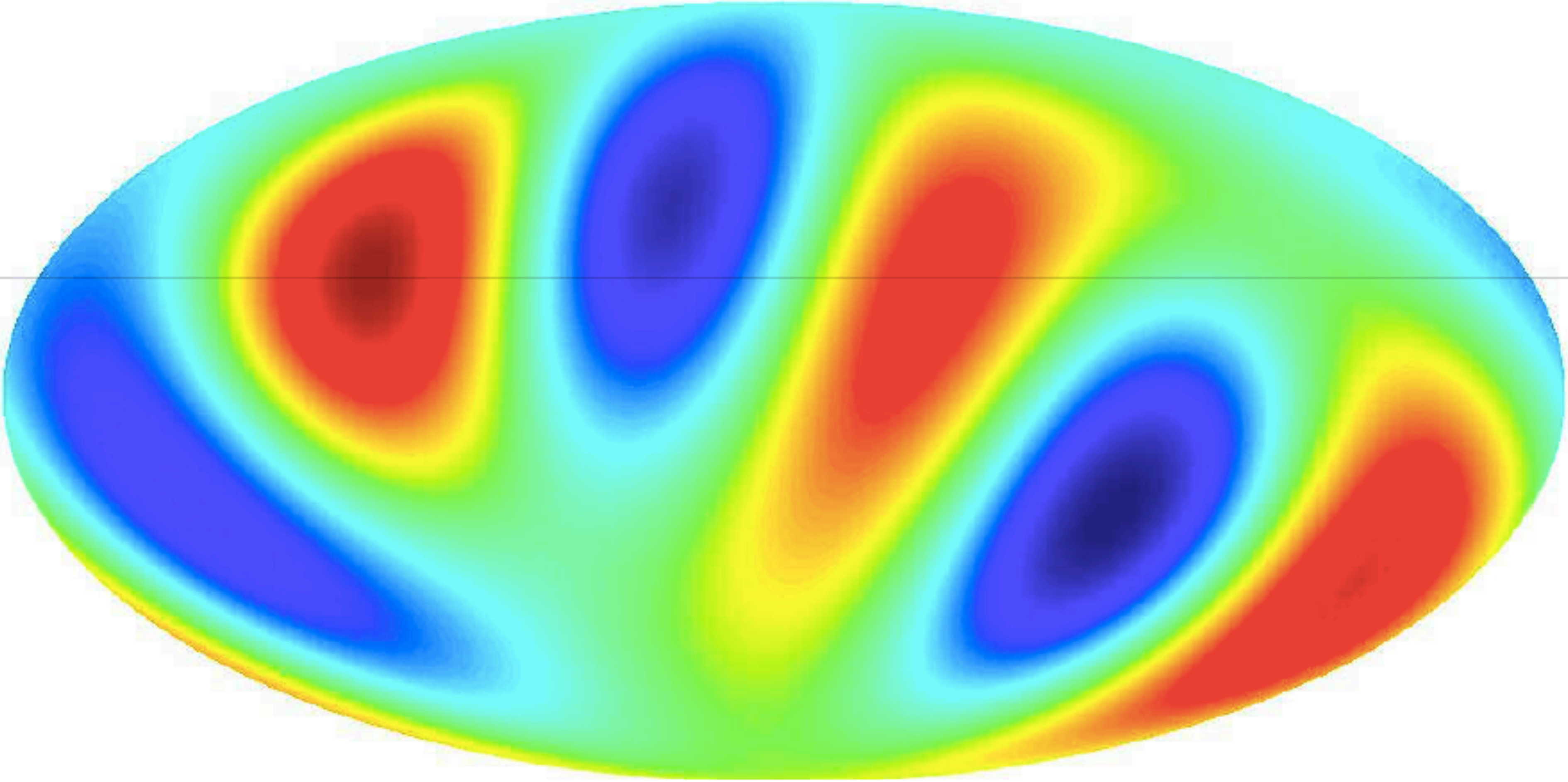


M. Bartelmann, F. Abdalla

ℓ=2



$\ell=3$



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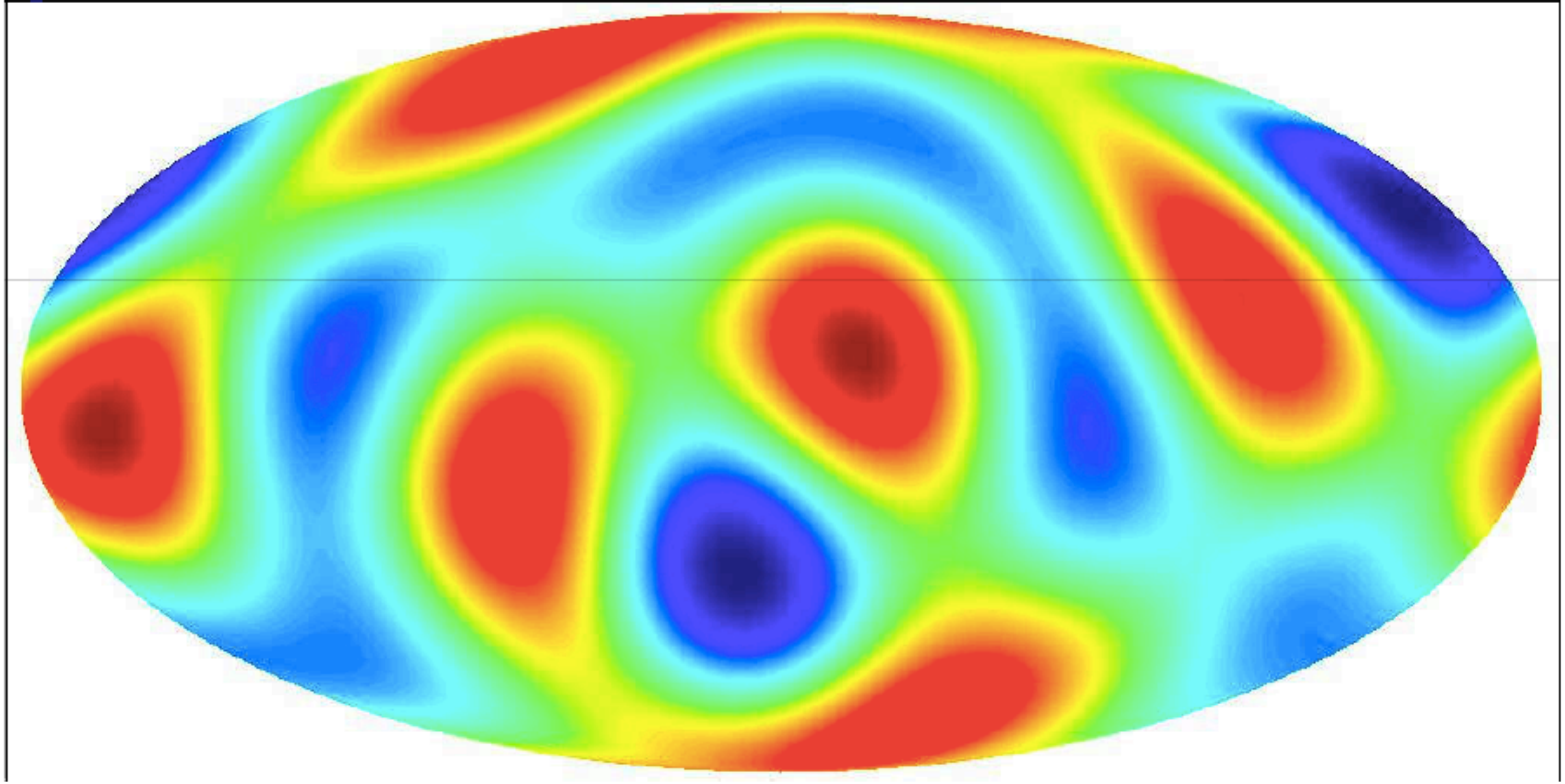
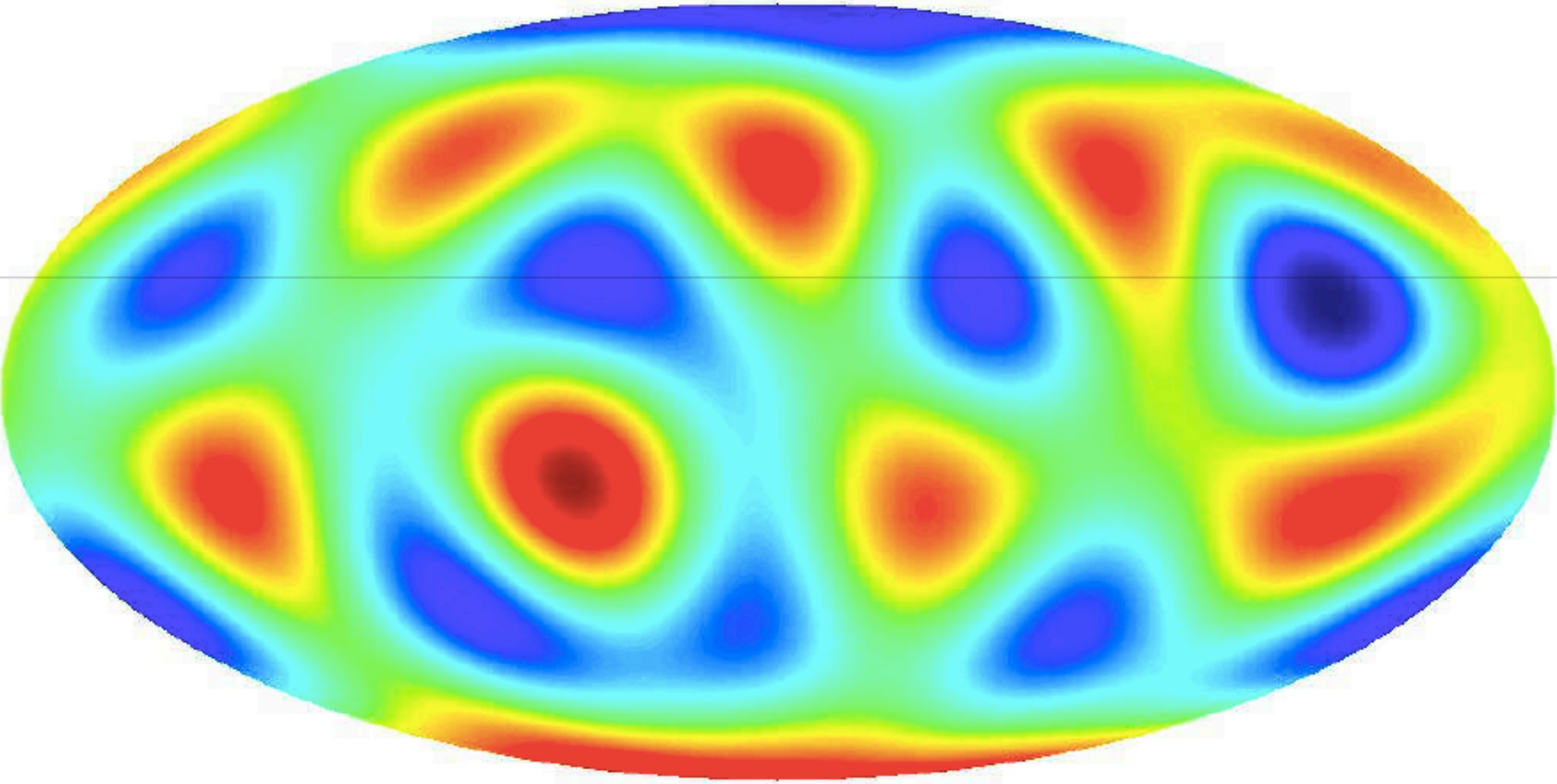
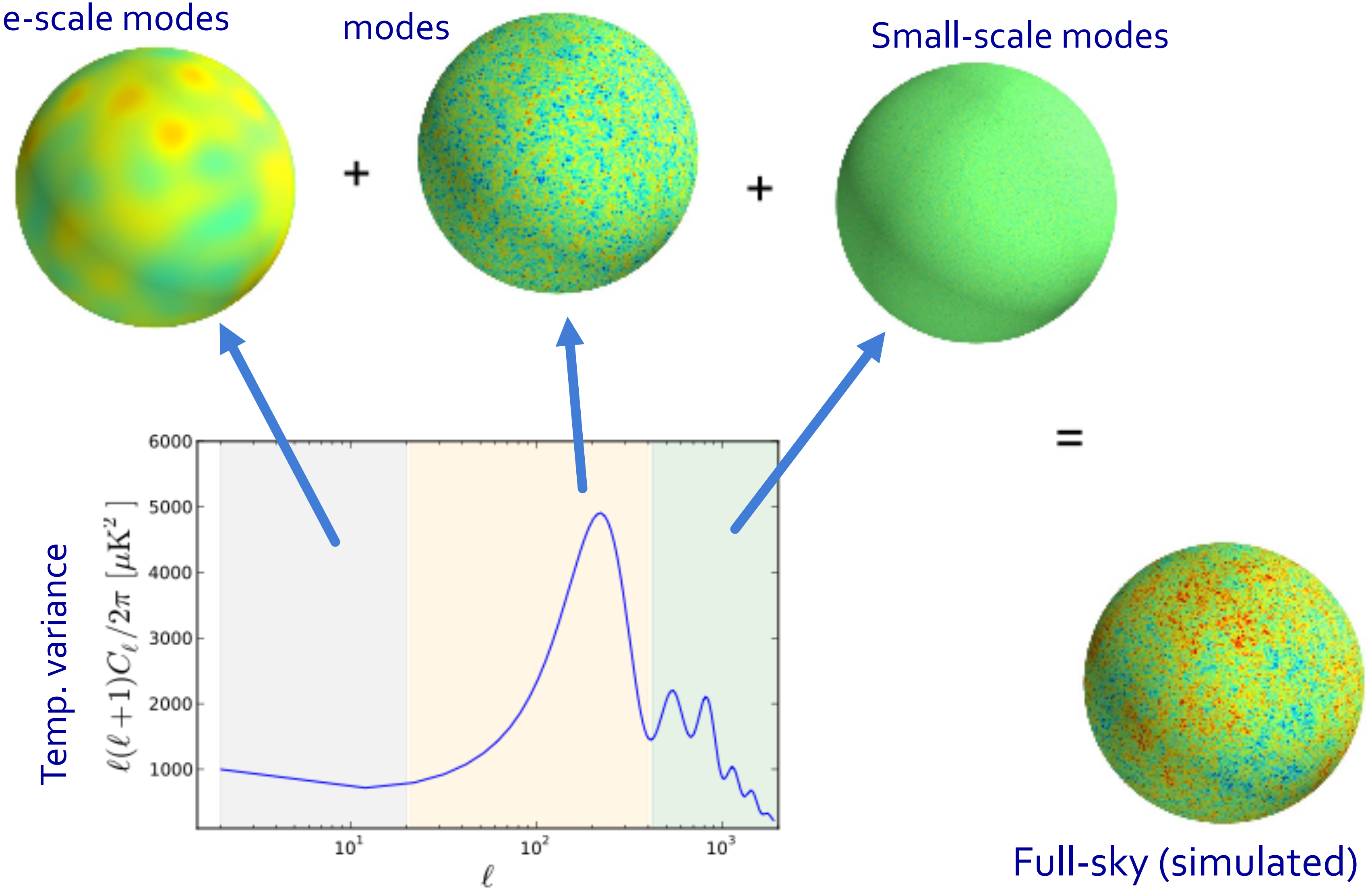
$\ell=4$



$\ell=5$



M. Bartelmann, F. Abdalla



Large-scale modes

Intermediate-scale modes

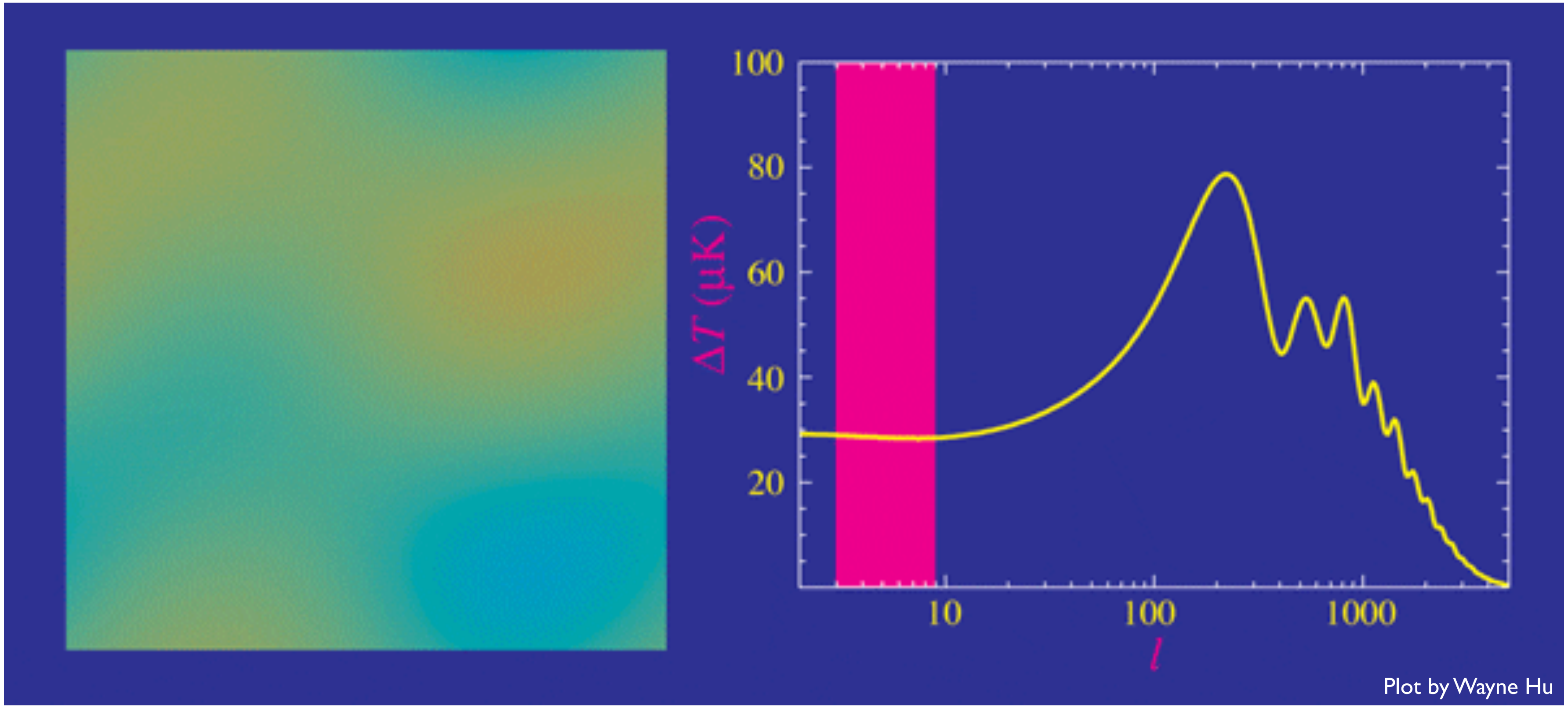
Small-scale modes

Temp. variance

Full-sky (simulated) map of the CMB

l = oscillations per ~180 degrees

Courtesy of Douglas Scott



Plot by Wayne Hu

Cosmic variance

The average $< a_{lm} a^*_{l'm'} >$ is over many realizations, i.e. many 'given universes' obeying the same cosmological model.

We only have access to our sky, one realization.
The squares $|a_{lm}|^2$ will not be equal to C_l
There will be some scattering around its value.

Because of isotropy, we know that the distribution of $|a_{lm}|^2$

Is independent of m. We can then average over m, to reduce the scattering.

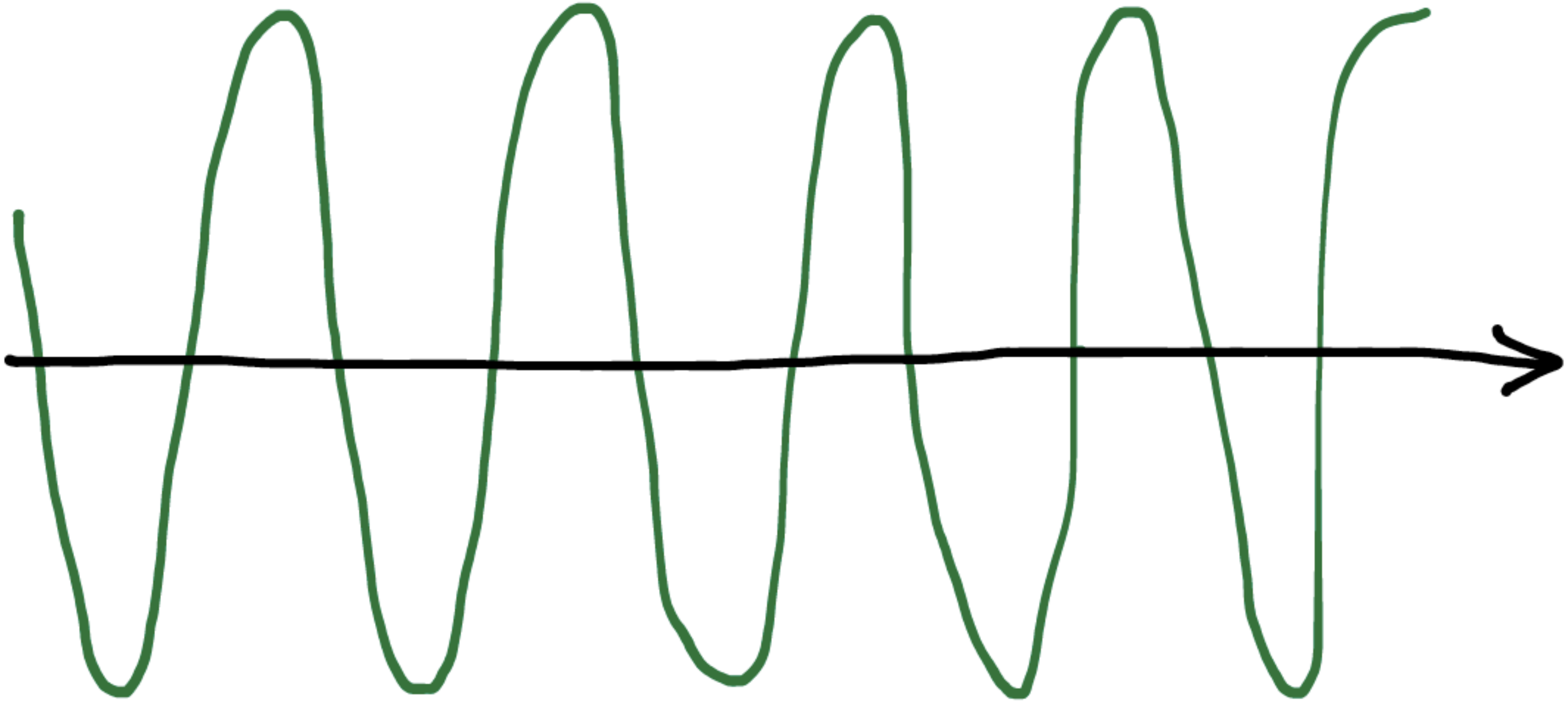
The best estimator of the C_l power spectrum is then:

$$\boxed{\hat{C}_l \equiv \frac{1}{2l+1} \sum_{-l \leq m \leq l} |a_{lm}|^2}$$

$$\boxed{< (\hat{C}_l - C_l)^2 >= \frac{2}{2l+1} C_l^2}$$

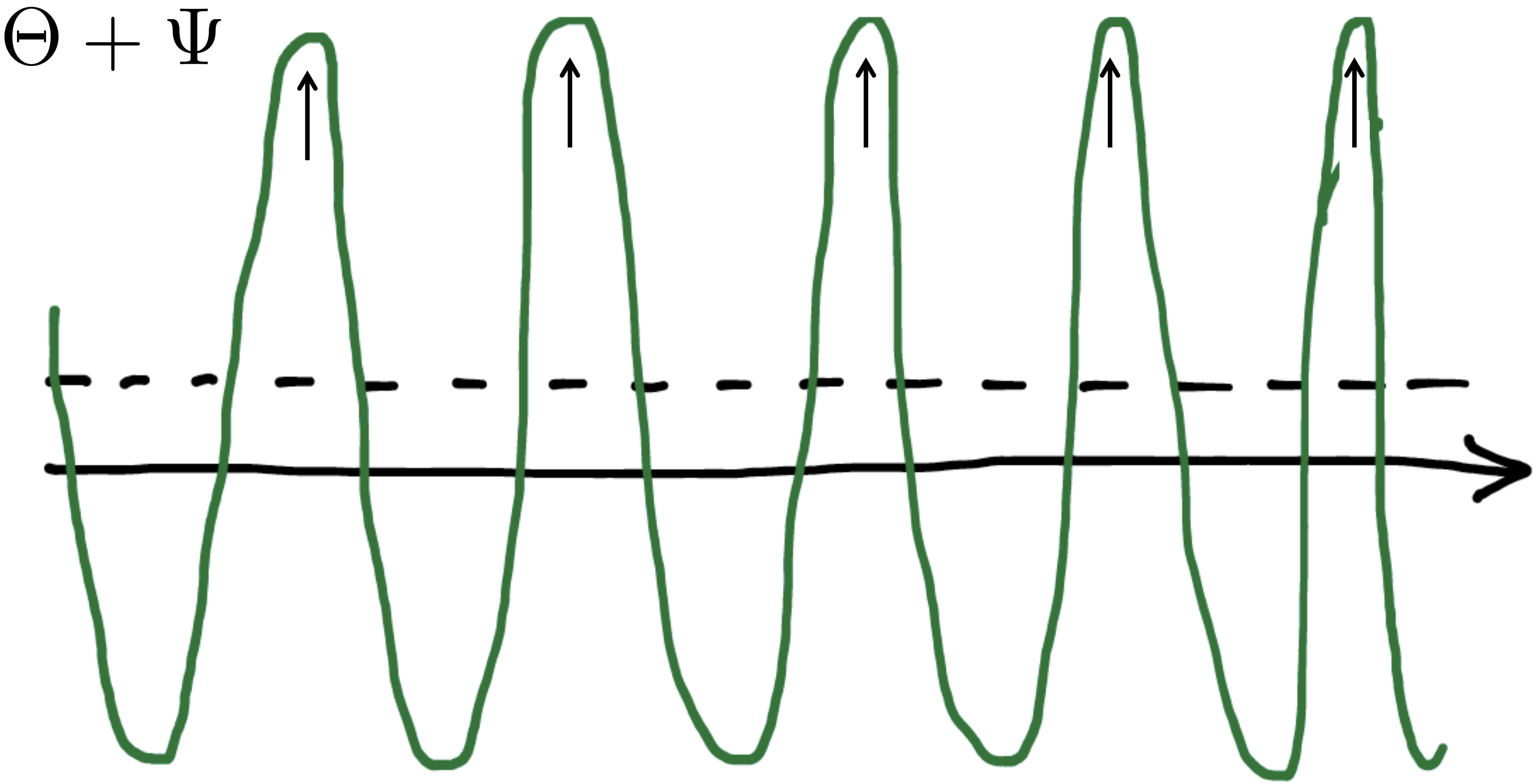
Oscillations of a tight fluid, equal amplitude

$\Theta + \Psi$



Baryon dragging

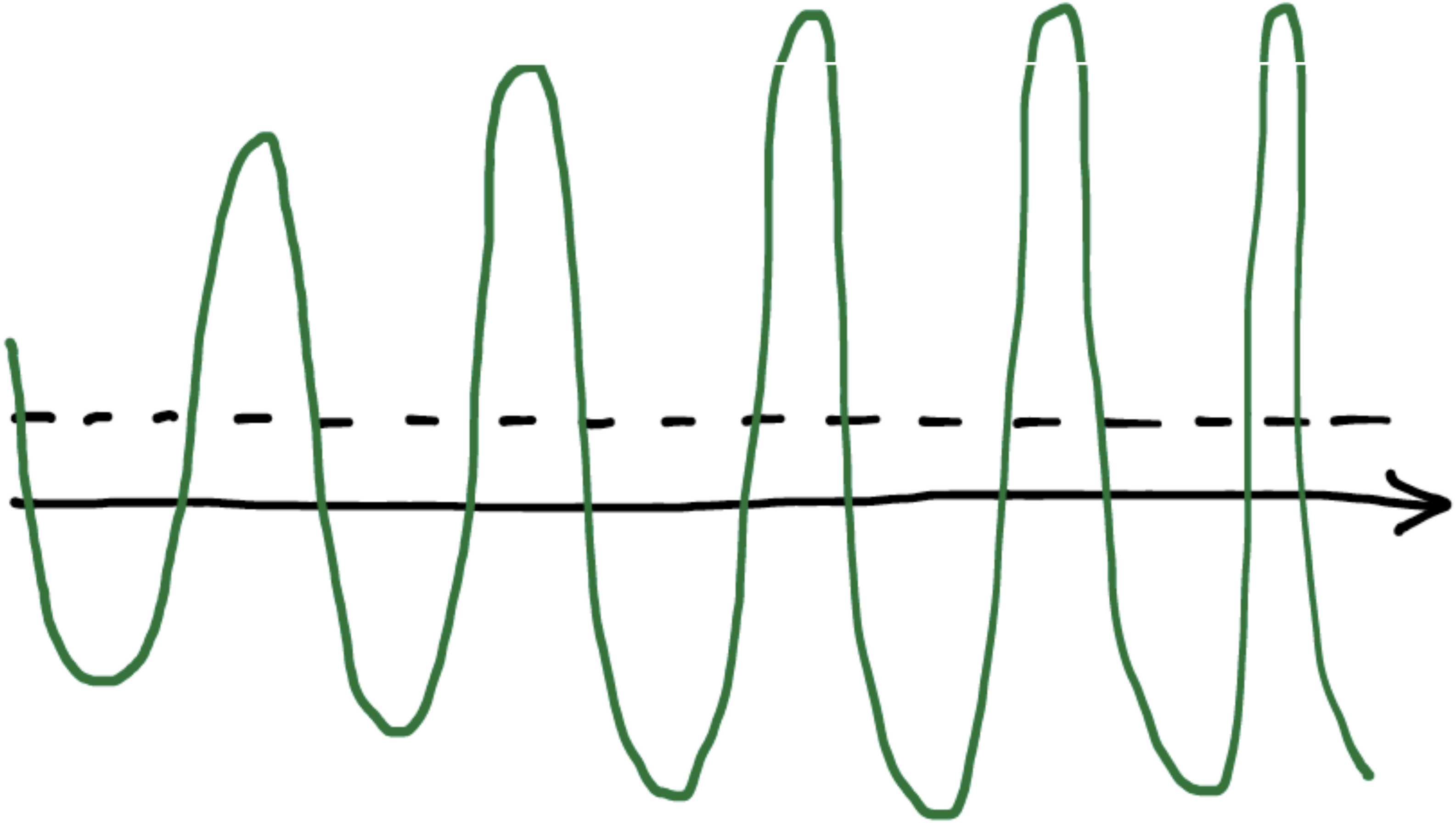
enhances compressions, shifts equilibrium point



Gravitational driving

Enhances small scales with respect to large ones

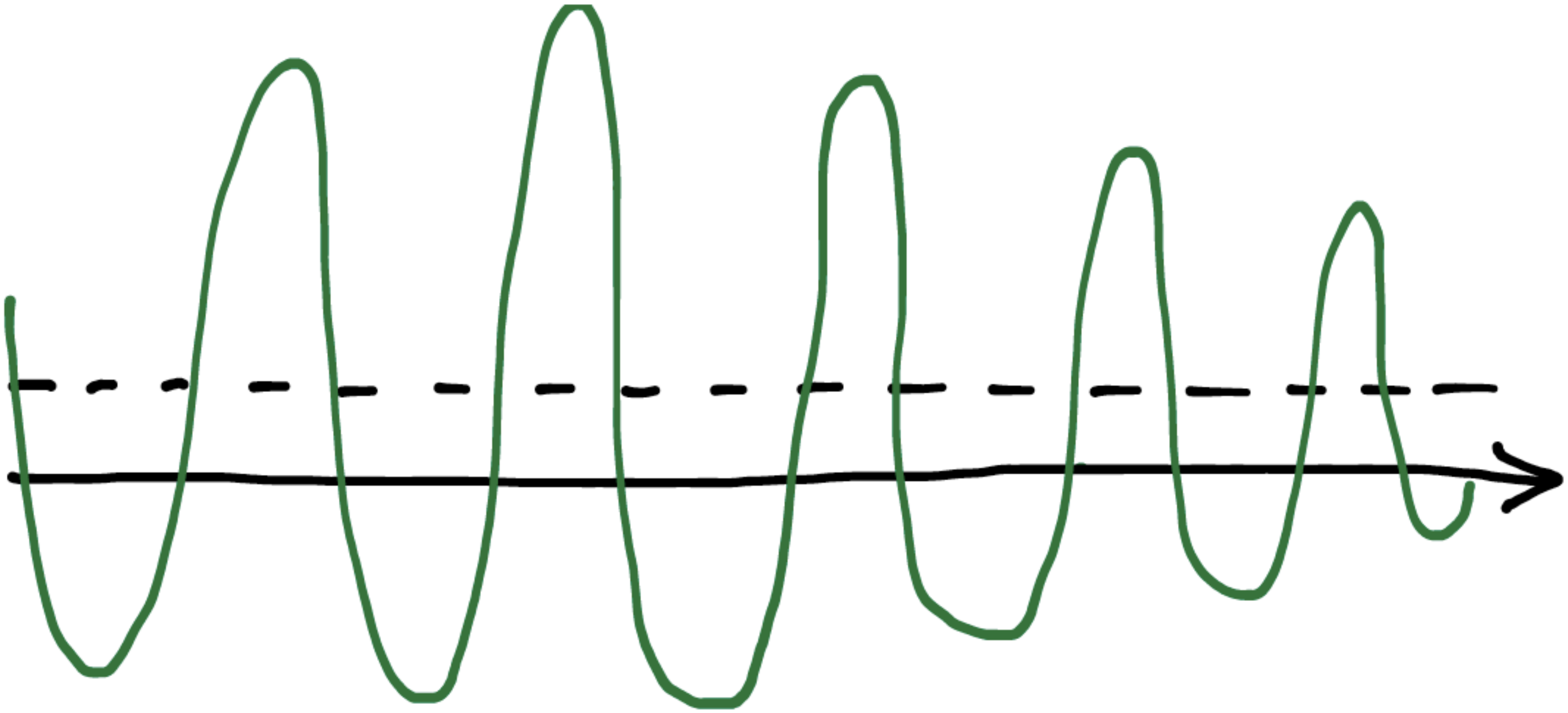
$\Theta + \Psi$



Diffusion damping

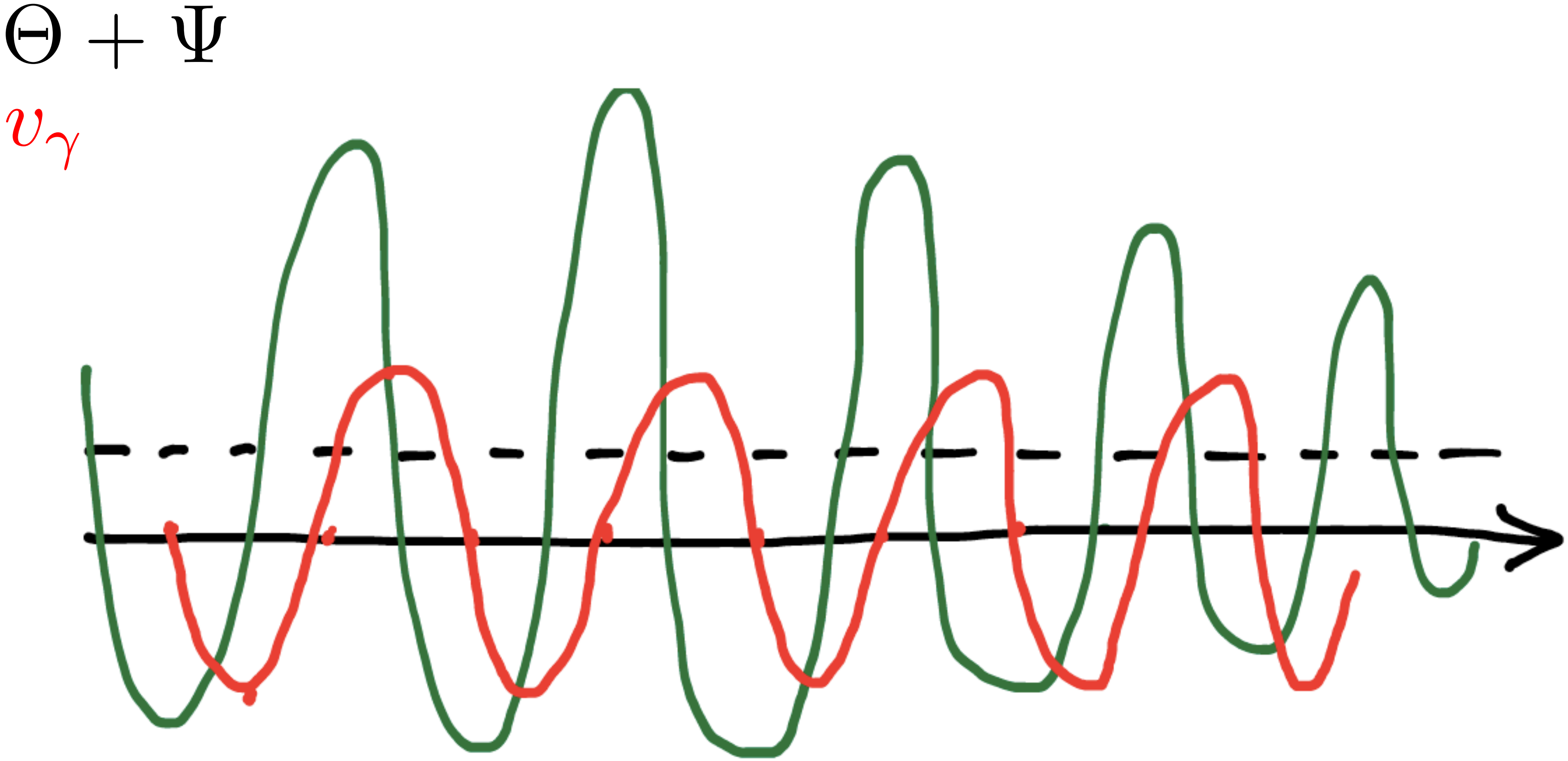
Suppresses small scales

$\Theta + \Psi$

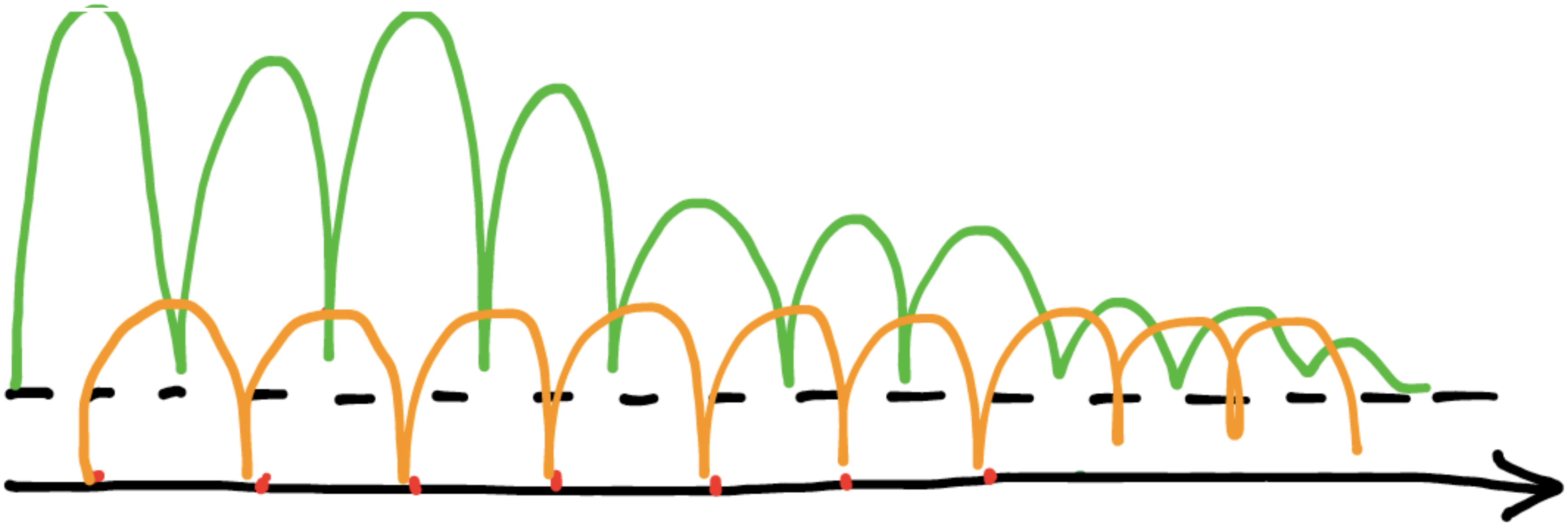


Doppler effect

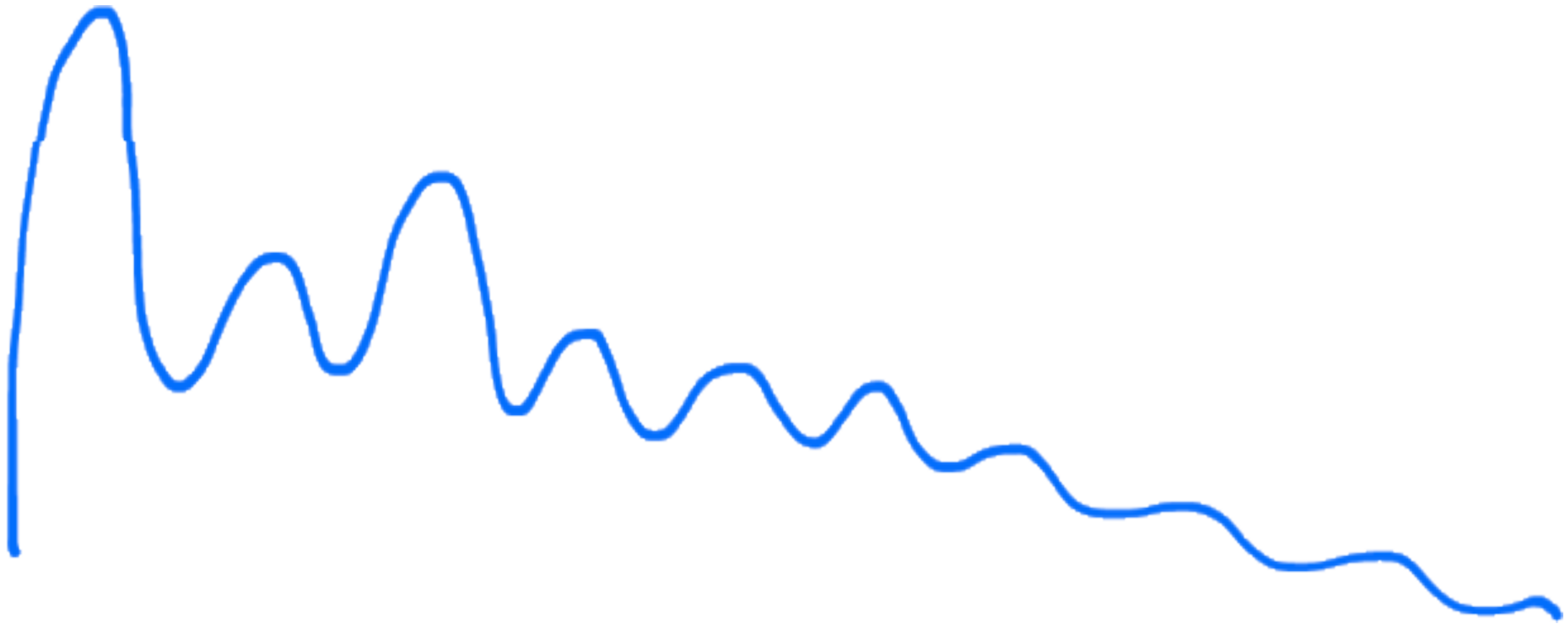
Out of phase of 90 degrees, equal amplitude



Square both

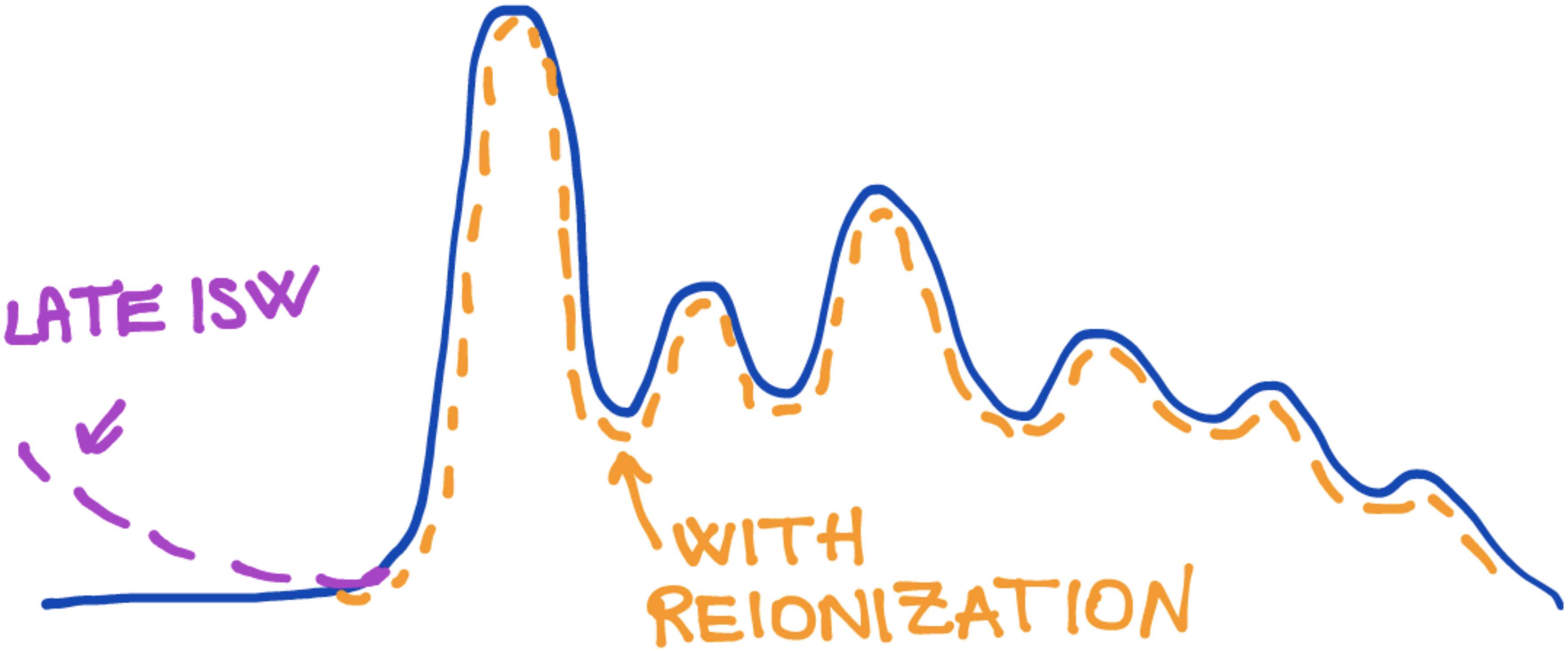


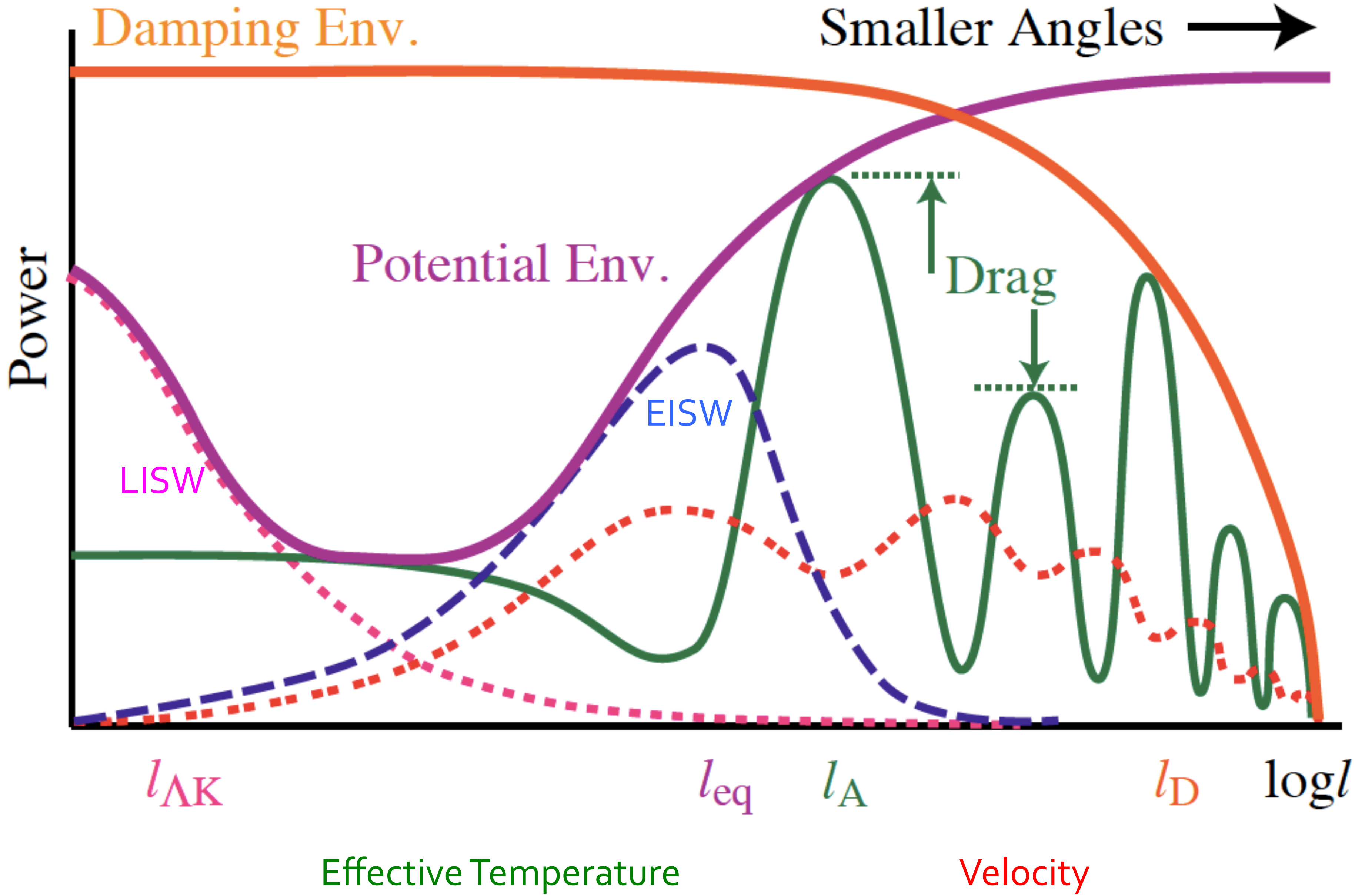
Sum



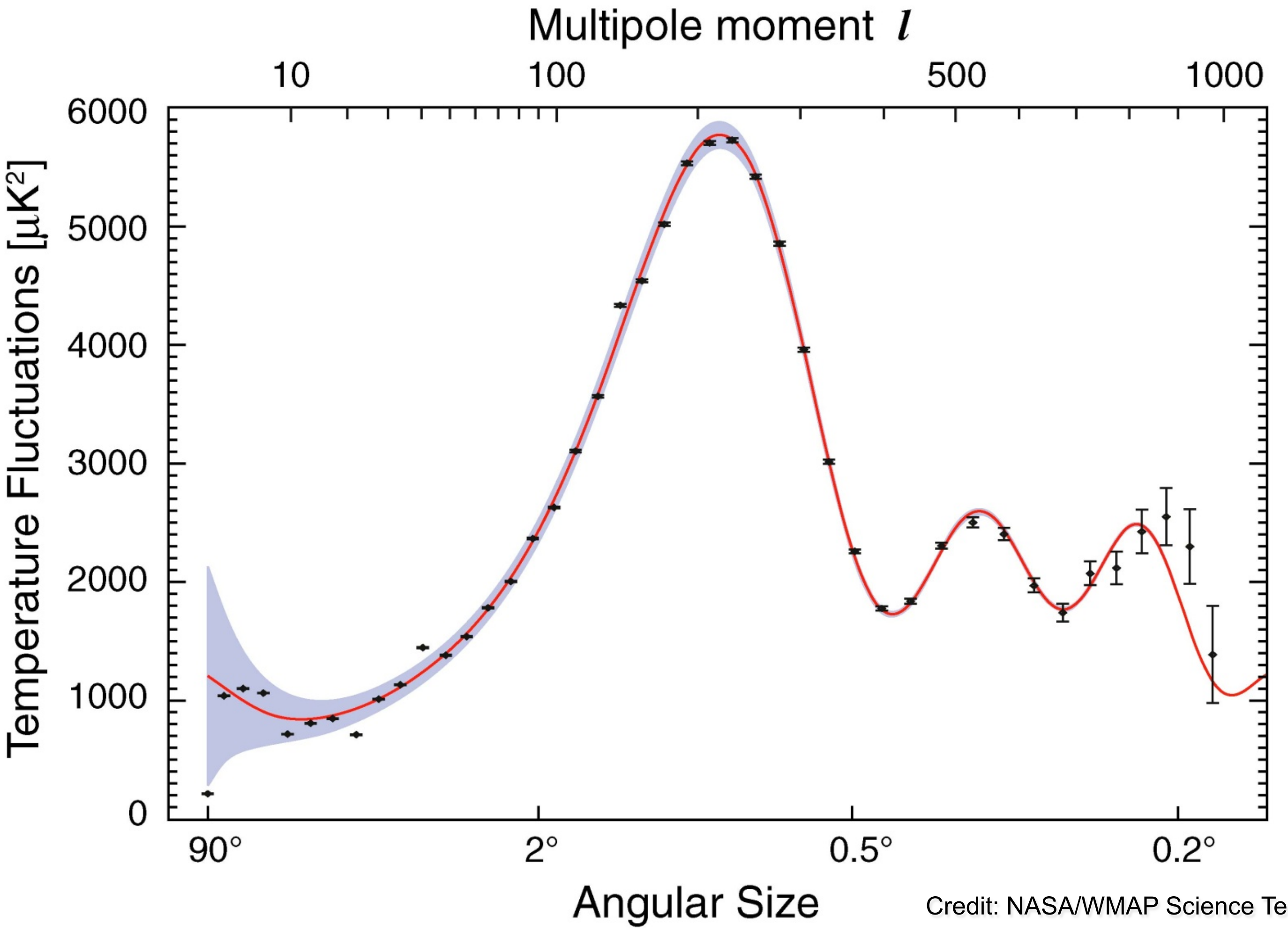
$|a_{lm}|^2$

Modified along the line of sight




Damping Env.
Smaller Angles →
Potential Env.
Drag
Power
EISW
LISW
$l_{\Lambda K}$
l_{eq}
l_A
l_D
$\log l$
Effective Temperature
Velocity

The detailed structure tells us about the cosmological parameters



Credit: NASA/WMAP Science Team

You can check here how a cosmological constant influences CMB peaks:

http://map.gsfc.nasa.gov/resources/camb_tool/index.html