Lectures on Modifying Gravity

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Outline

- GR
- Why you shouldn't modify GR
- Why you might want to modify GR
- The landscape of theories
- Observables

What is General Relativity?

Newton

$$\mathbf{g} = -\nabla\Phi$$

$$\nabla^2\Phi = 4\pi G\rho$$

Einstein

$$\frac{d^2x^\beta}{d\tau^2} + \Gamma^\beta{}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$$

$$G^{\alpha\beta} = \frac{8\pi G}{c^4}T^{\alpha\beta} - \Lambda g^{\alpha\beta}$$

What is General Relativity

Curvature

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}R(g)+\int d^4x\sqrt{-g}\mathcal{L}(g,\mathrm{matter})$$

Metric of space time

Classical Tests?

- Precession of the Perihelion of Mercury
- Gravitational Lensing
- Gravitational Redshift
- Shapiro Time Delay

Current Constraints

Parameter	Bound	Effects	Experiment
$\gamma - 1$	2.3×10^{-5}	Time delay, light deflection	Cassini tracking
$\beta - 1$	2.3×10^{-4}	Nordtvedt effect, Perihelion shift	Nordtvedt effect
ξ	0.001	Earth tides	Gravimeter data
α_1	10^{-4}	Orbit polarization	Lunar laser ranging
α_2	4×10^{-7}	Spin precession	Solar alignment with ecliptic
α_3	4×10^{-20}	Self-acceleration	Pulsar spin-down statistics
ζ_1	0.02	-	Combined PPN bounds
ζ_2	4×10^{-5}	Binary pulsar acceleration	PSR 1913+16
ζ_3	10^{-8}	Newton's 3rd law	Lunar acceleration
ζ_4	0.006	-	Usually not independent

Constraints

- Constraints are of order

$$\frac{\dot{G}}{G} \sim 10^{-13} \text{ to } 10^{-11} \quad \mathrm{yr}^{-1}$$

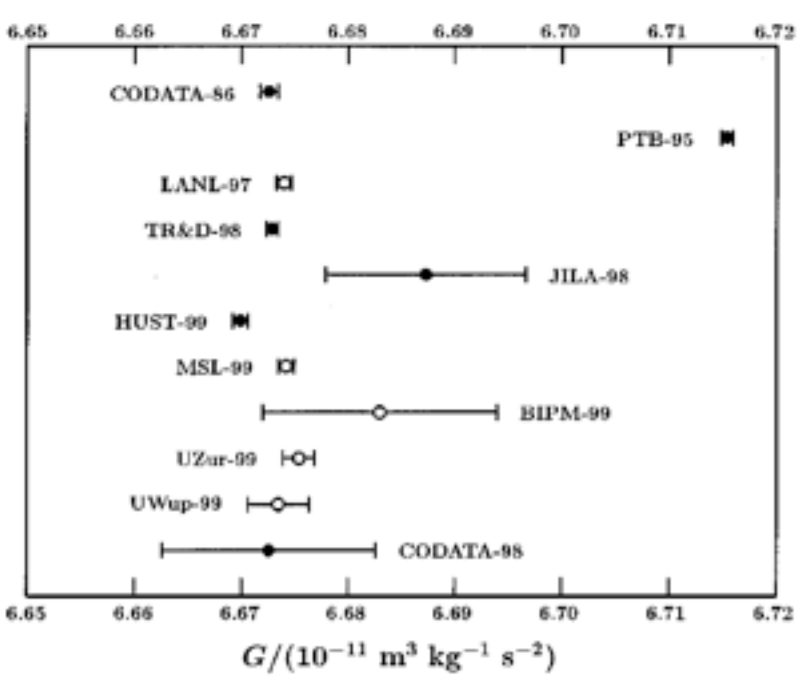
- Discrepancies in measurements of G
- Electromagnetic/Weak/Strong forces are weak on large scales- look there.

J.P. Uzan- Living Rev. Relativity, 14 (2011), 2

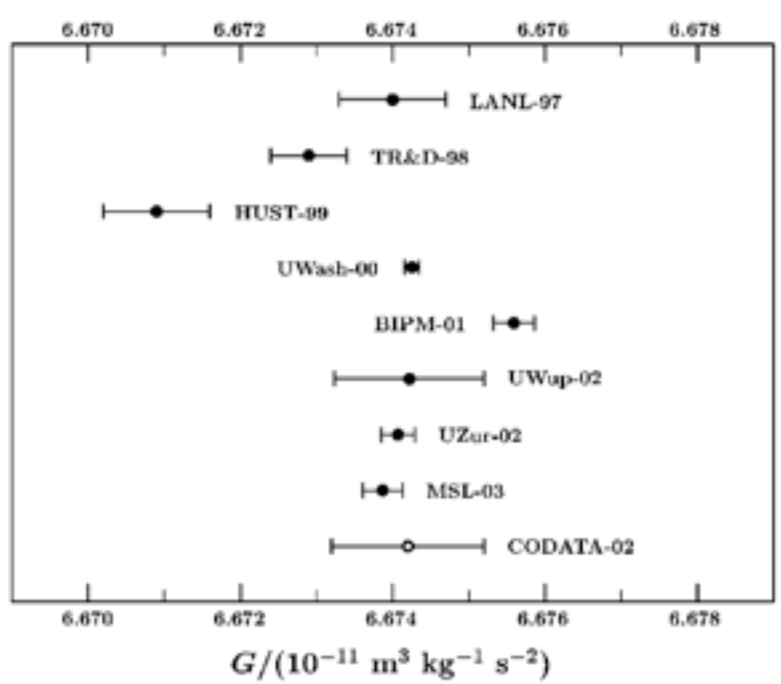
CODATA over time

Cavendish Torsion Balance
(Science Museum)

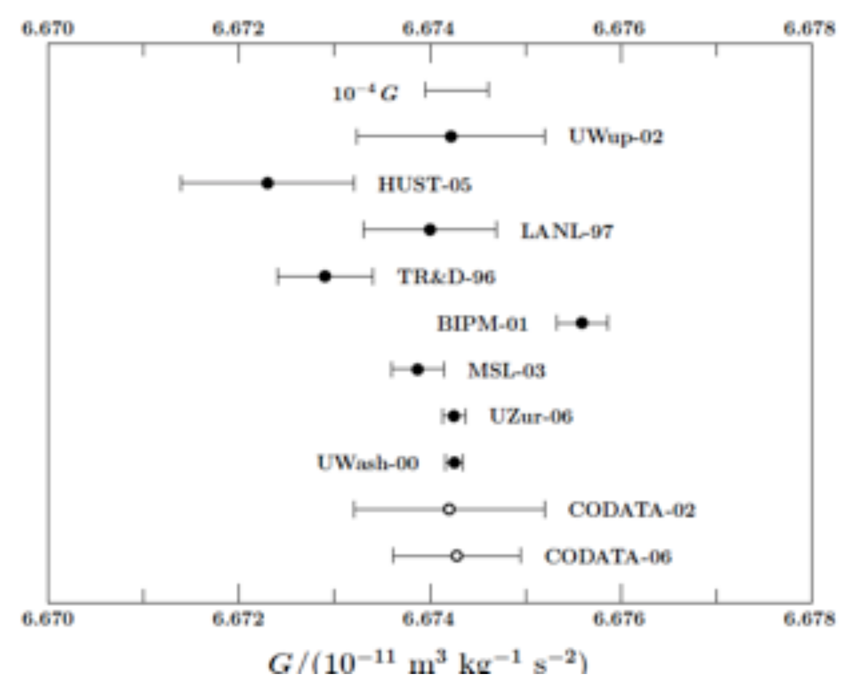
1998



2002



2006



Measurement of G

TABLE XVII. Summary of the results of measurements of the Newtonian constant of gravitation G relevant to the 2010 adjustment.

Source	Identification[a]	Method	Value (10^{-11} m^3 kg^{-1} s^{-2})	Rel. stand. uncert. u_r
Luther and Towler (1982)	NIST-82	Fiber torsion balance, dynamic mode	6.672 48(43)	6.4×10^{-5}
Karagioz and Izmailov (1996)	TR&D-96	Fiber torsion balance, dynamic mode	6.672 9(5)	7.5×10^{-5}
Bagley and Luther (1997)	LANL-97	Fiber torsion balance, dynamic mode	6.673 98(70)	1.0×10^{-4}
Gundlach and Merkowitz (2000, 2002)	UWash-00	Fiber torsion balance, dynamic compensation	6.674 255(92)	1.4×10^{-5}
Quinn *et al.* (2001)	BIPM-01	Strip torsion balance, compensation mode, static deflection	6.675 59(27)	4.0×10^{-5}
Kleinevoß (2002); Kleinvoß *et al.* (2002)	UWup-02	Suspended body, displacement	6.674 22(98)	1.5×10^{-4}
Armstrong and Fitzgerald (2003)	MSL-03	Strip torsion balance, compensation mode	6.673 87(27)	4.0×10^{-5}
Hu, Guo, and Luo (2005)	HUST-05	Fiber torsion balance, dynamic mode	6.672 28(87)	1.3×10^{-4}
Schlamminger *et al.* (2006)	UZur-06	Stationary body, weight change	6.674 25(12)	1.9×10^{-5}
Luo *et al.* (2009); Tu *et al.* (2010)	HUST-09	Fiber torsion balance, dynamic mode	6.673 49(18)	2.7×10^{-5}
Parks and Faller (2010)	JILA-10	Suspended body, displacement	6.672 34(14)	2.1×10^{-5}

[a]NIST: National Institute of Standards and Technology, Gaithersburg, MD, USA; TR&D: Tribotech Research and Development Company, Moscow, Russian Federation; LANL: Los Alamos National Laboratory, Los Alamos, New Mexico, USA; UWash: University of Washington, Seattle, Washington, USA; BIPM: International Bureau of Weights and Measures, Sèvres, France; UWup: University of Wuppertal, Wuppertal, Germany; MSL: Measurement Standards Laboratory, Lower Hutt, New Zealand; HUST: Huazhong University of Science and Technology, Wuhan, PRC; UZur: University of Zurich, Zurich, Switzerland; JILA: a joint institute of the University of Colorado and NIST, Boulder, Colorado, USA.

Rev. Mod. Phys., Vol. 84, No. 4, October–December 2012

CODATA2010

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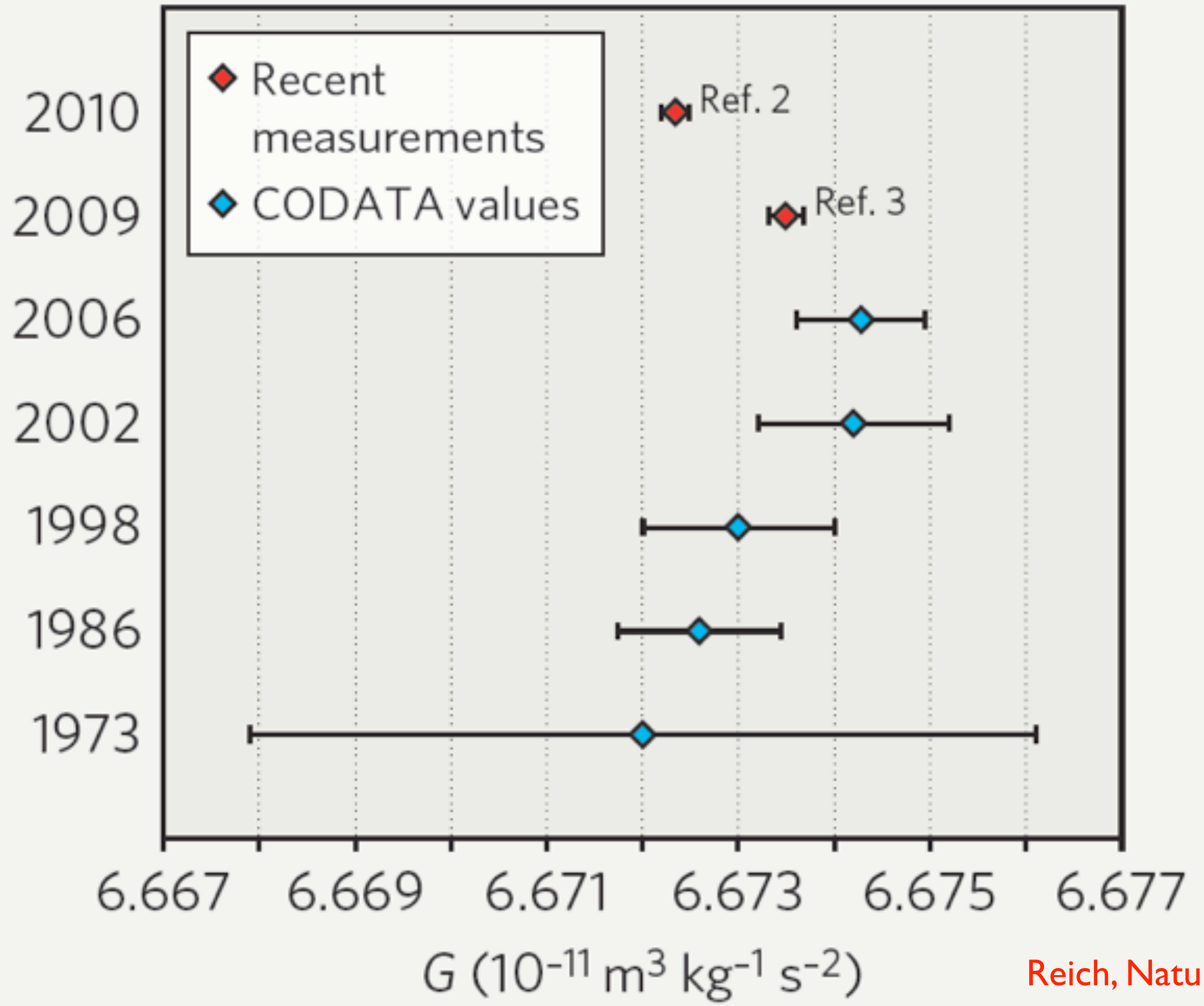
A CHANGING CONSTANT

The recommended value of *G* — the gravitational constant — has risen slightly over the past four decades. But the latest measurements will probably cause a downward revision.



Reich, Nature, 1030, 466, 2010

TABLE XVII. Summ … to the 2010 adjustment.

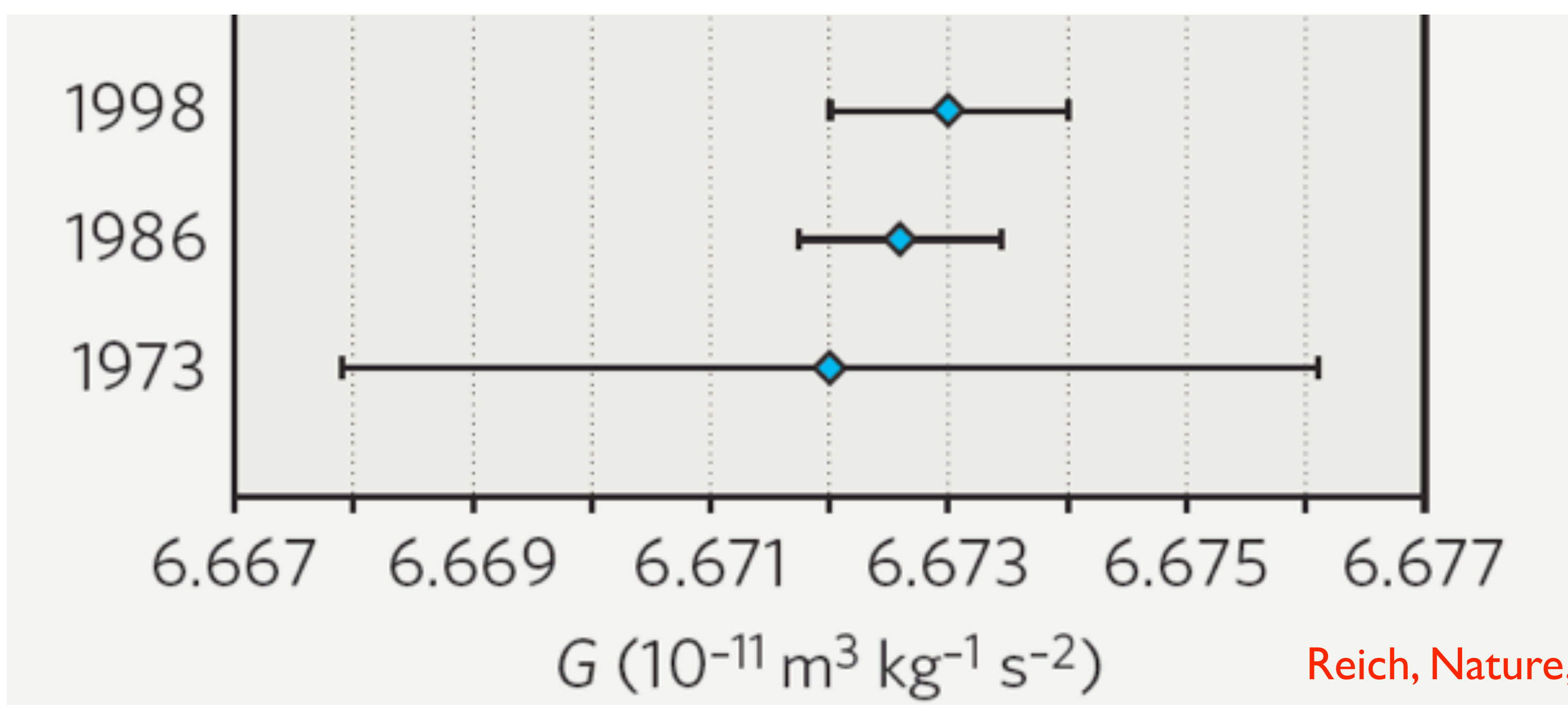
Source	… e (10^{-11} m^3 kg^{-1} s^{-2})	Rel. stand. uncert. u_r
Luther and Towler (19	248(43)	6.4×10^{-5}
Karagioz and Izmailov	29(5)	7.5×10^{-5}
Bagley and Luther (19	398(70)	1.0×10^{-4}
Gundlach and Merkow	4255(92)	1.4×10^{-5}
Quinn *et al.* (2001)	559(27)	4.0×10^{-5}
Kleinevoß (2002); Kle	422(98)	1.5×10^{-4}
Armstrong and Fitzger	387(27)	4.0×10^{-5}
Hu, Guo, and Luo (20	228(87)	1.3×10^{-4}
Schlamminger *et al.* (	425(12)	1.9×10^{-5}
Luo *et al.* (2009); Tu	349(18)	2.7×10^{-5}
Parks and Faller (2010	234(14)	2.1×10^{-5}

[a]NIST: National Inst … Research and Development Company, Moscow, … w Mexico, USA; UWash: University of Washi … Measures, Sèvres, France; UWup: University o … ower Hutt, New Zealand; HUST: Huazhong Ur … rich, Switzerland; JILA: a joint institute of the U …

A CHANGING CONSTANT

The recommended value of *G* — the gravitational constant — has risen slightly over the past four decades. But the latest measurements will probably cause a downward revision.

Only a factor of ~ 10 better than Cavendish's original measurement!



Reich, Nature, 1030, 466, 2010

Why you might not want to modify General Relativity.

- The spin-2 field theoretic argument
- The effective field theory argument
- The geometrodynamical argument
- Lovelock's theorem
- Ghosts and Instabilities
- Ostrogradski's "Theorem"
- The initial condition problem

The Fierz-Pauli Action

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$ Spin-2 field

$$S = \frac{1}{16\pi G}\int d^4x\left[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \partial_\lambda h \partial^\lambda h\right]$$

General covariance
leads to
gauge invariance

$$y^\mu = x^\mu + \xi^\mu$$

$$h_{\alpha\beta} \to h_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha$$

$$g_{\mu\nu}(y) = \frac{\partial x^\alpha}{\partial y^\mu}\frac{\partial x^\beta}{\partial y^\nu} g_{\alpha\beta}(x)$$

The Fierz-Pauli Action

Couple to matter: $S_M = \int d^4x h_{\alpha\beta} T_M^{\alpha\beta}$

Incompatible with: $\Box h_{\mu\nu} = 16\pi G T_{\mu\nu}^M$

Self energy of the graviton:

$$\Box h_{\mu\nu} = 16\pi G(T_{\mu\nu}^M + T_{\mu\nu}^G) \qquad T_{\mu\nu}^M \sim (\partial h)(\partial h)$$

Conjecture: unique non-linear completion is GR...

... but see Padmanabhan (2004)

Feynman (1963)
Weinberg (1965)
Deser (1970)

Effective Field Theory

Renormalisable QED with electron and muon:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \overline{\psi}(\gamma^\mu D_\mu + m)\psi - \overline{\chi}(\gamma^\mu D_\mu + M)\chi$$

Non- renormalisable QED with electron only:

$$\mathcal{L}_{\rm eff} \quad = \quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \overline{\psi}(\gamma^\mu D_\mu + m)\psi + \frac{k_1\,\alpha}{30\pi M^2}F^{\mu\nu}\Box F_{\mu\nu} + \cdots$$

Burgess (2009)

The rules of EFT: write out the most general action consisent with locality, symmetries and degrees of freedom.

Weinberg (1979)

Effective Field Theory

Cut off: m $\qquad a_i \sim \mathcal{O}(1)$

$$
\begin{aligned}
-\frac{\mathcal{L}_{\rm eff}}{\sqrt{-g}} \;=\;& \lambda + \frac{M_p^2}{2} R + a_1 R_{\mu\nu} R^{\mu\nu} \\
& + a_2 R^2 + a_3 R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} + a_4 \Box R \\
& + \frac{b_1}{m^2} R^3 + \frac{b_2}{m^2} R R_{\mu\nu} R^{\mu\nu} + \frac{b_3}{m^2} R_{\mu\nu} R^{\nu\lambda} R_\lambda{}^\mu
\end{aligned}
$$

$$\frac{M_p^2}{2} R + a_2 R^2 \sim \frac{M_p^2}{2} R \left(1 + 2a_2 \frac{R}{M_p}\right)$$ but $$\frac{R}{M_p} \ll 1$$

Geometrodynamics

ADM decomposition

$$ds^2 = \left(-N^2 + g^{mn} N_m N_n\right) dt^2 + N_i dt x^i + g_{ij} dx^i dx^j$$

$$S = \int d^4x \left(\pi^{ij} g_{ij} - N\mathcal{H} - N^i \mathcal{H}_i\right)$$

$\mathcal{H}$ and $\mathcal{H}_i$ are the constraints.

"Einsteinian geometrodynamics is the only (time reversible) canonical representation of the set of generators of deformations of a spacelike hypersurface embedded in a Riemannian spacetime, if the intrinsic metric of the hypersurface and its conjugate momentum are the sole canonical variables."

Hojman, Kuchar & Teitelboim (1976)

Einstein Gravity

Curvature

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}R(g) + \int d^4x\sqrt{-g}\mathcal{L}(g, \mathrm{matter})$$

Metric of space time

Lovelock's theorem (1971) :*"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."*

Ghosts and Instabilities

Ghosts are classical and quantum instabilities that arise from negative kinetic energy of a field.

Modifying general relativity seems to (almost) inevitably lead to extra degrees of freedom.

Extra degrees of freedom are (almost) inevitably ghosts.

Ostragradski's Theorem

2nd order Lagrangian: $L(q, \dot{q})$

Conjugate variables:

$$Q \equiv q$$

$$P \equiv \frac{\partial L}{\partial \dot{q}}$$

Time evolution:

$$\dot{Q} = \frac{\partial H}{\partial P}$$

$$H(Q, P) = P\dot{q} - L$$

$$\dot{P} = -\frac{\partial H}{\partial Q}$$

Ostragradski's Theorem

Higher order Lagrangian: $L(q, \dot{q}, \ddot{q})$

Conjugate variables: $Q_1 \equiv q \quad P_1 \equiv \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}}$

$$Q_2 \equiv \dot{q}$$

$$P_2 \equiv \frac{\partial L}{\partial \ddot{q}}$$

Time evolution:

$$H = P_1 Q_2 + P_2 a(Q_1, Q_2, P_2) - L(Q_1, Q_2, P_2)$$

$$\dot{Q}_i = \frac{\partial H}{\partial P_i}$$

Instability in P_1

$$\dot{P}_i = -\frac{\partial H}{\partial Q_i}$$

Initial Condition Problem

Predictive Theory (well posed):

- Existence
- Uniqueness
- Depend continuously on initial conditions

Desiderata: theory should be *strongly* hyperbolic.

General Relativity is *weakly* hyperbolic- only recently proven to be well posed.

Adding more terms- anything can happen ...

Why Modify?

Equivalence Principle

Geodesic action for a particle ...

$$S_{\rm matter} = -\int mc\sqrt{-g_{\mu\nu}(\mathbf{x})v^\mu v^\nu}\mathrm{d}t$$

... with non-constant constants...

$$S_{\rm matter} = -\int m_A[\alpha_j]c\sqrt{-g_{\mu\nu}(\mathbf{x})v^\mu v^\nu}\mathrm{d}t$$

... leads to non-geodesic motion

$$u^\nu \nabla_\nu u^\mu = -\underline{\left(\sum_i \frac{\partial \ln m_A}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial x^\beta}\right)\left(g^{\beta\mu} + u^\beta u^\mu\right)}$$

Gravity is not a Gauge Theory

- Other three forces are gauge fields.
- Described by Yang-Mills-non-linear but incredibly predictive (see LHC 2012)
- General Covariance might be a gauge symmetry ...
- ... but doesn't work that way.

Non-renormalizability

Coupling constant: $[\alpha] = M^d$

N -leg Feynman amplitude $\int p^{A-Nd} dp$

Gravitational coupling constant: $G \sim M^{-2}$

N -leg Feynman amplitude $\int p^{A+2N} dp$

Diverges! No way to reabsorb

Maybe EFT (but see above)

Weyl, Eddington, Dirac and Large Numbers

$$\frac{e^2}{4\pi G m_e^2} \simeq \sqrt{N} \simeq 10^{42}$$

Eddington

$$G \simeq \left(\frac{c^3}{M}\right) t^3$$

Milne

$$G \sim \frac{M}{R}$$

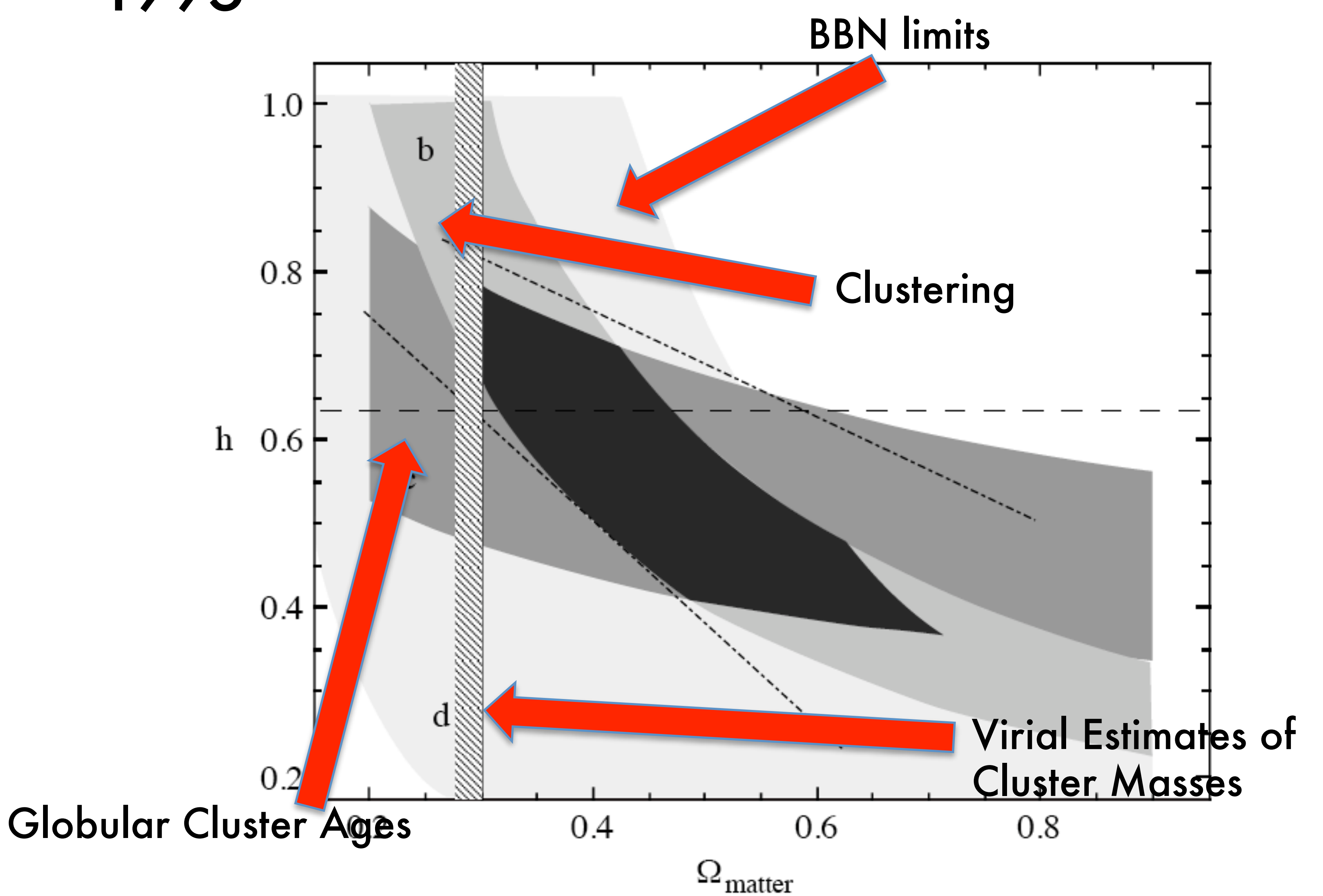
Dirac

$$"\Box" \frac{1}{G} \propto \rho$$

The Dark Universe

1984
1995

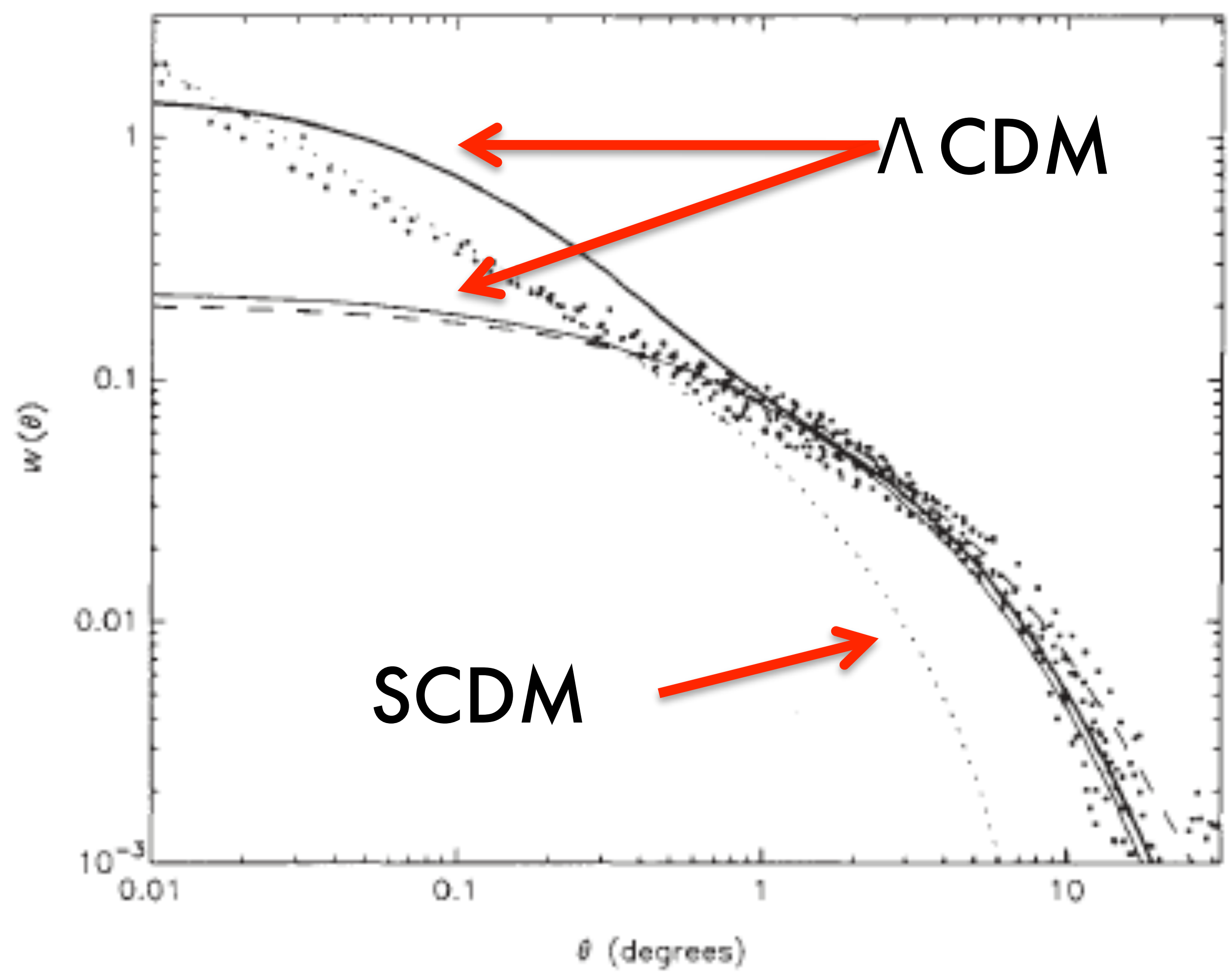
Universe must be Flat: $\Omega_K = 0$



Krauss and Turner 1994

The Dark Universe

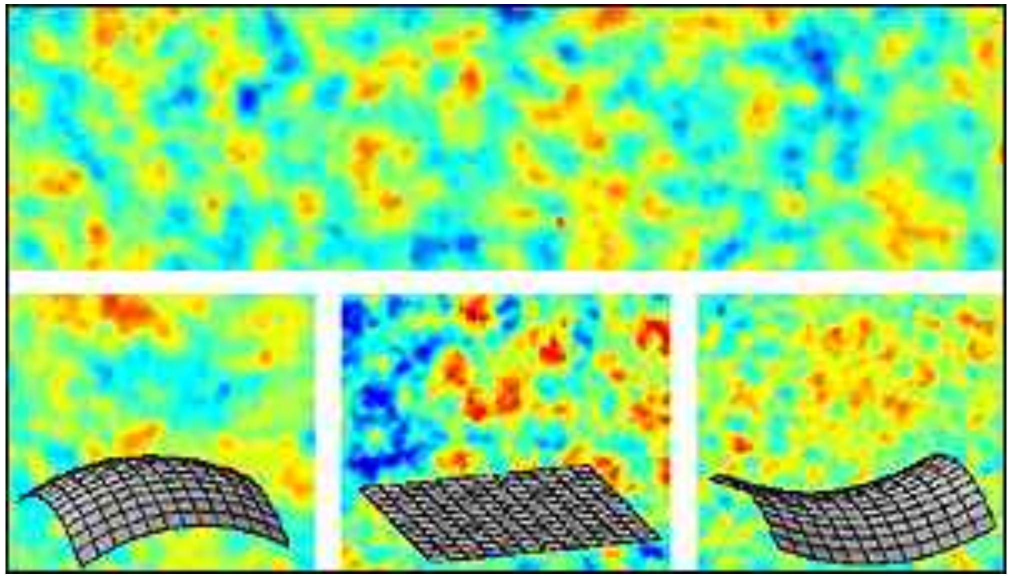
1990 Λ from Large Scale Structure



Efstathiou, Sutherland, Maddox

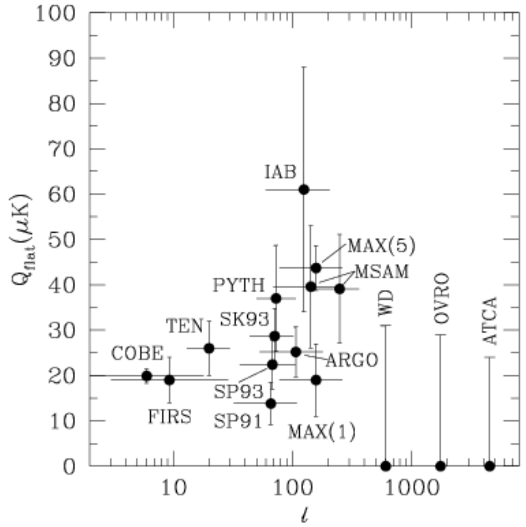
The Dark Universe

Is the Universe Flat ?

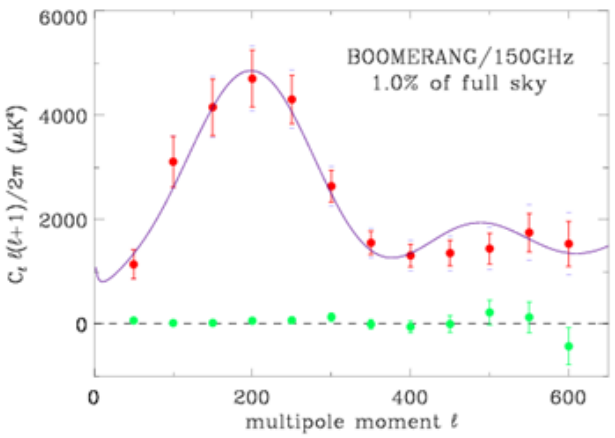


Angular Diameter
Distance from CMB

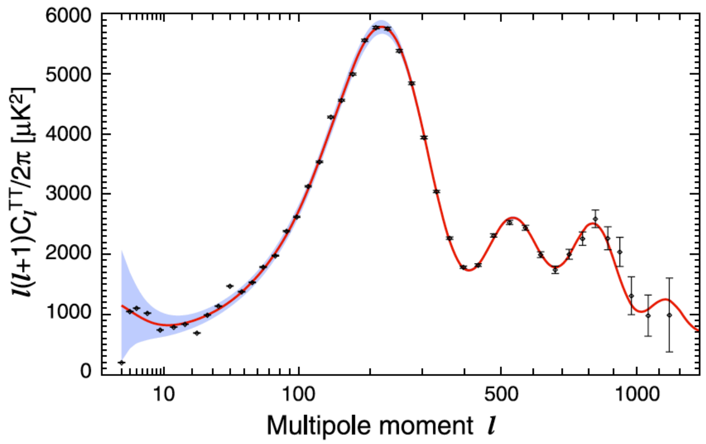
Scott 1995



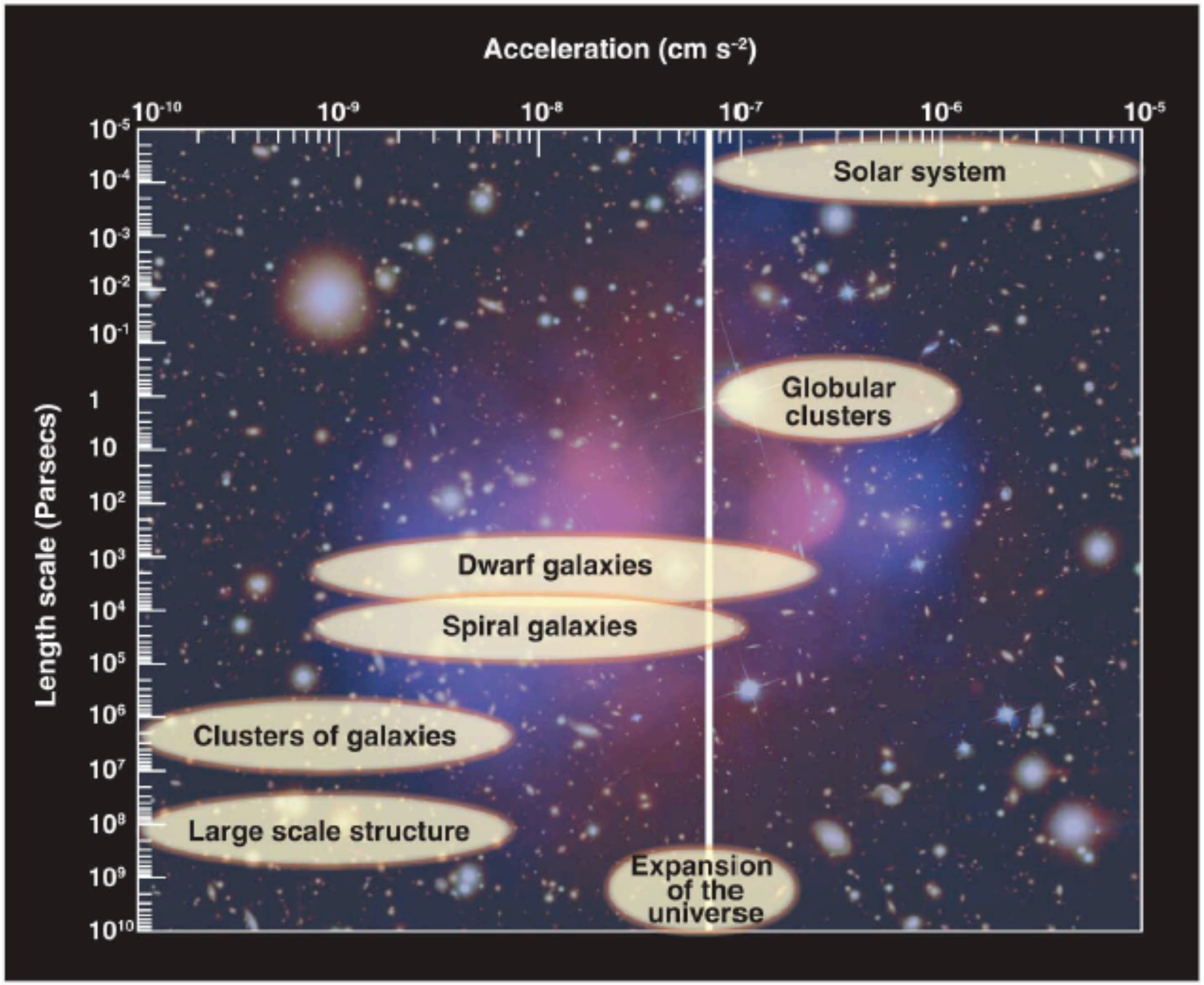
Boomerang 2000



WMAP 2010



The Dark Universe



Ferreira & Starkman, 2009

The landscape of theories

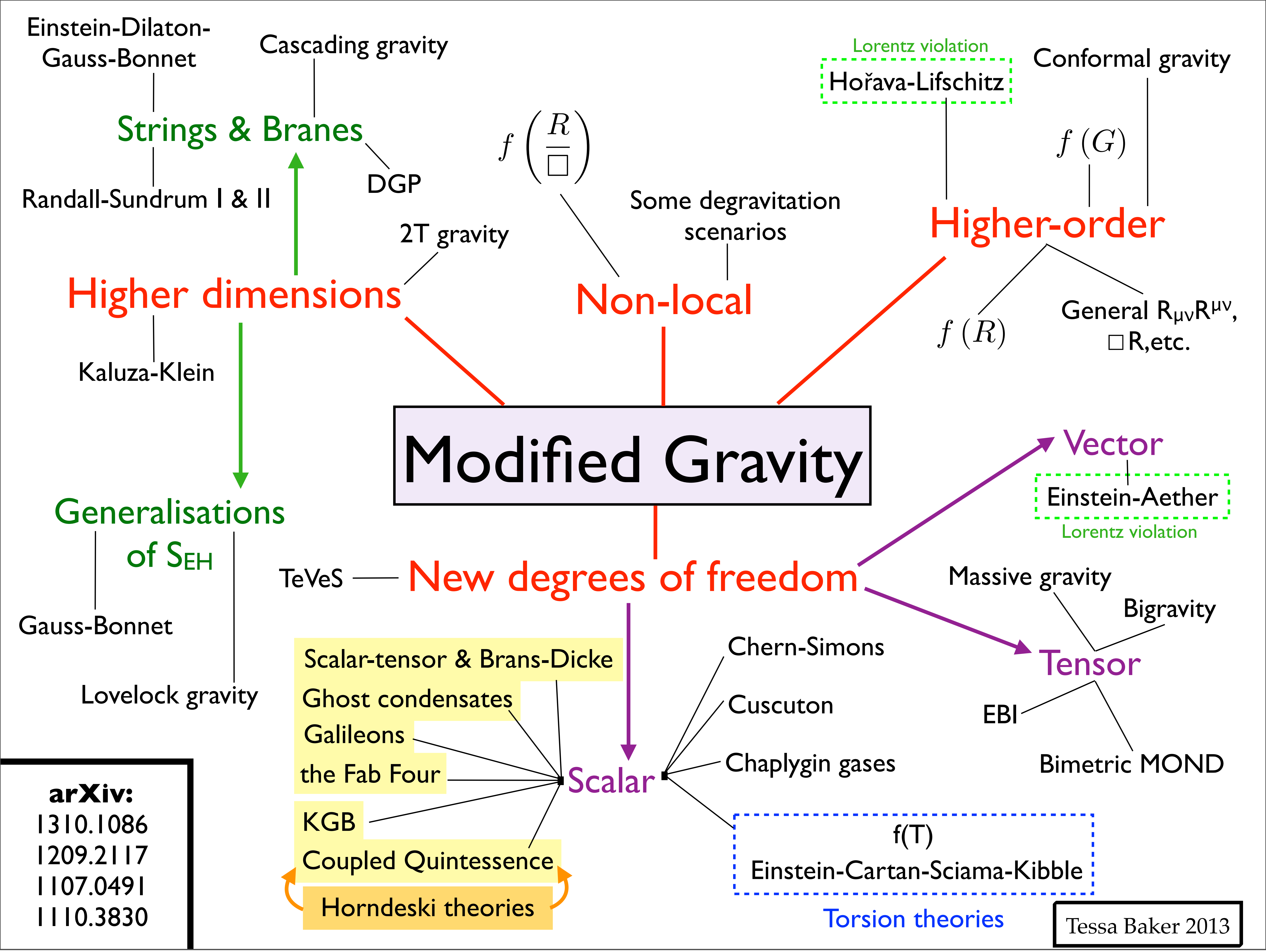
Einstein Gravity

Curvature

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}R(g)+\int d^4x\sqrt{-g}\mathcal{L}(g,\mathrm{matter})$$

Metric of space time

Lovelock's theorem (1971) :*"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."*


Modified Gravity
Higher dimensions
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
Randall-Sundrum I & II
DGP
2T gravity
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
Non-local
$f\left(\frac{R}{\Box}\right)$
Some degravitation scenarios
Higher-order
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.
New degrees of freedom
TeVeS
Vector
Einstein-Aether
Lorentz violation
Tensor
Massive gravity
Bigravity
EBI
Bimetric MOND
Scalar
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Chern-Simons
Cuscuton
Chaplygin gases
f(T)
Einstein-Cartan-Sciama-Kibble
Torsion theories
arXiv:
1310.1086
1209.2117
1107.0491
1110.3830
Tessa Baker 2013

How to Modify

- No action
- Higher dimensions
- Extra degrees of freedom
- Higher derivatives
- Non-locality

No-action

Emergence: space-time as a fluid, crystal- the metric (or any gravitational degree of freedom) as an emergent property. E.g. Navier-Stokes equation, ...

Classicality: all theories are fundamentally quantum. Dynamics is *solely* defined in terms of

$$\langle g_{\mu\nu}(x)\rangle$$
$$\langle g_{\mu\nu}(x)g_{\alpha\beta}(x')\rangle$$
$$\langle g_{\mu\nu}(x)g_{\alpha\beta}(x')g_{\chi\xi}(x'')\rangle$$
$$\cdots$$

Higher Dimensions: Kaluza-Klein

5-dimensional space time

$$X^A = (x^\mu, z)$$

$$\gamma_{AB}$$

$$R_{AB}$$

Compactify: $\gamma_{AB}(X) = \sum_n \gamma_{AB}^{(n)}(x^\mu) e^{\frac{inz}{L}}$

New mass scale: $L \sim 1/M$

current constraints: $M > 1 \text{ TeV}$

Natural scale is Planck mass...

Higher Dimensions: Kaluza-Klein

Expand the metric- new fields:

$$\gamma_{\mu\nu} = e^{\frac{1}{\sqrt{3}}\phi} g_{\mu\nu} + e^{-\frac{2}{\sqrt{3}}\phi} A_\mu A_\nu \qquad \gamma_{\mu z} = e^{-\frac{2}{\sqrt{3}}\phi} A_\mu$$

$$\gamma_{zz} = e^{-\frac{2}{\sqrt{3}}\phi}$$

$$S[\gamma] = \frac{1}{16\pi G_D} \int d^D X \sqrt{-\gamma} \mathcal{R} =$$

$$\frac{L}{16\pi G_D} \int d^4 x \sqrt{-g} \left[R - \frac{1}{2} (\nabla\phi)^2 - e^{-2\sqrt{3}\phi} F^2 \right]$$

Einstein-Maxwell-*Dilaton* gravity.

Problem: how do we stabilize the Dilaton (or “moduli”)?

(Large) Higher Dimensions

Solving the hierarchy problem

$$\boxed{M_{\rm Pl} \gg M_{\rm EW}}$$

where $(M_D)^{D-2} \equiv \dfrac{1}{8\pi G_D}$

Arkanhi-Hamed, Dimopolous, Dvali

$$S[\gamma] = \frac{(M_D)^{D-2}}{2} \int d^D X \sqrt{-\gamma}\mathcal{R}$$

$$M_{\rm Pl}^2 = (M_D)^{2+n}\, L^n$$

compactify (volume V_n)

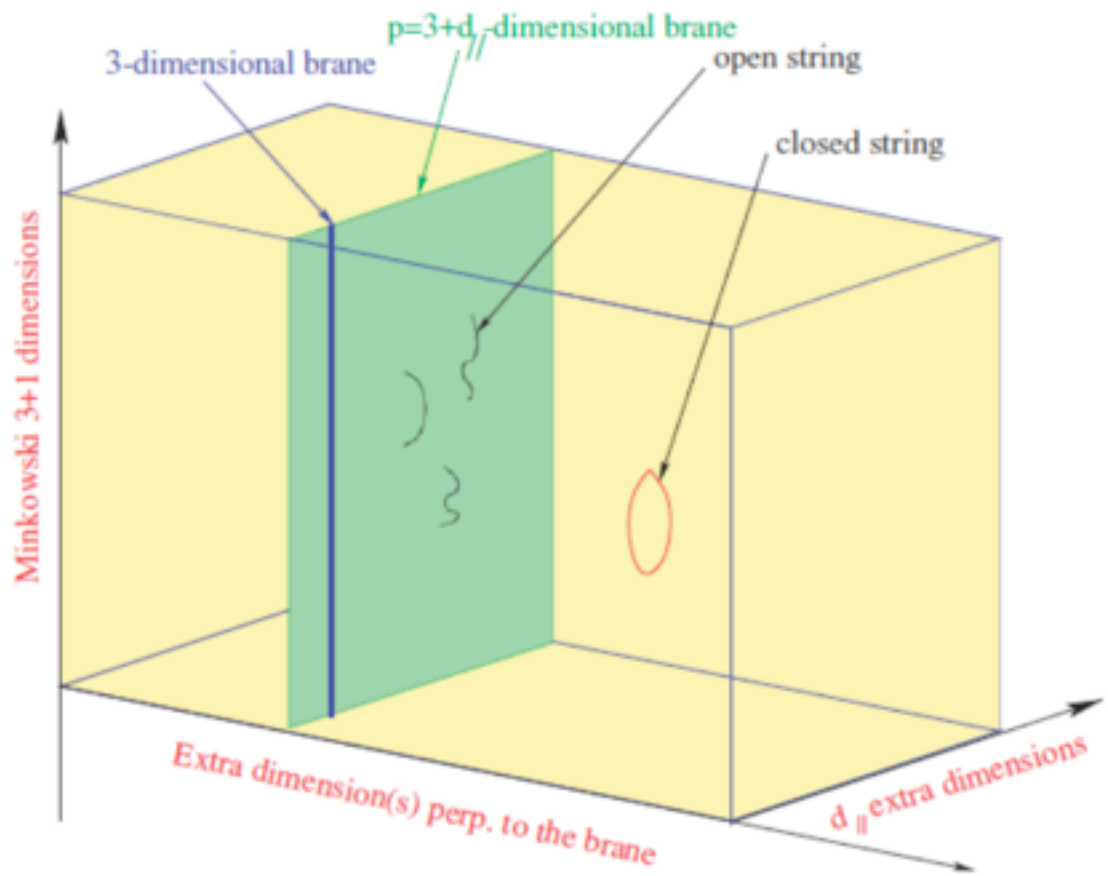
$$M_D \simeq M_{\rm EW}$$

$$L \sim \mathrm{mm}$$

$$S[g] = \frac{(M_D)^{D-2}\, V_n}{2} \int d^4x \sqrt{-g} R$$

(Large) Higher Dimensions

KK states too light so: localize on a brane.



Antoniadis 2007

Large, compact, extra dimensions but small brane thickness.

(Warped) Higher Dimensions

Instead of compactifying, warp the extra dimensions

$$S[\gamma] = \frac{M_5^2}{2} \int d^D X \sqrt{-\gamma}\,(\mathcal{R} - 2\Lambda) - \sigma \int d^4 x \sqrt{-g}$$

$$M_{Pl}^2 = M_5^3 \ell$$

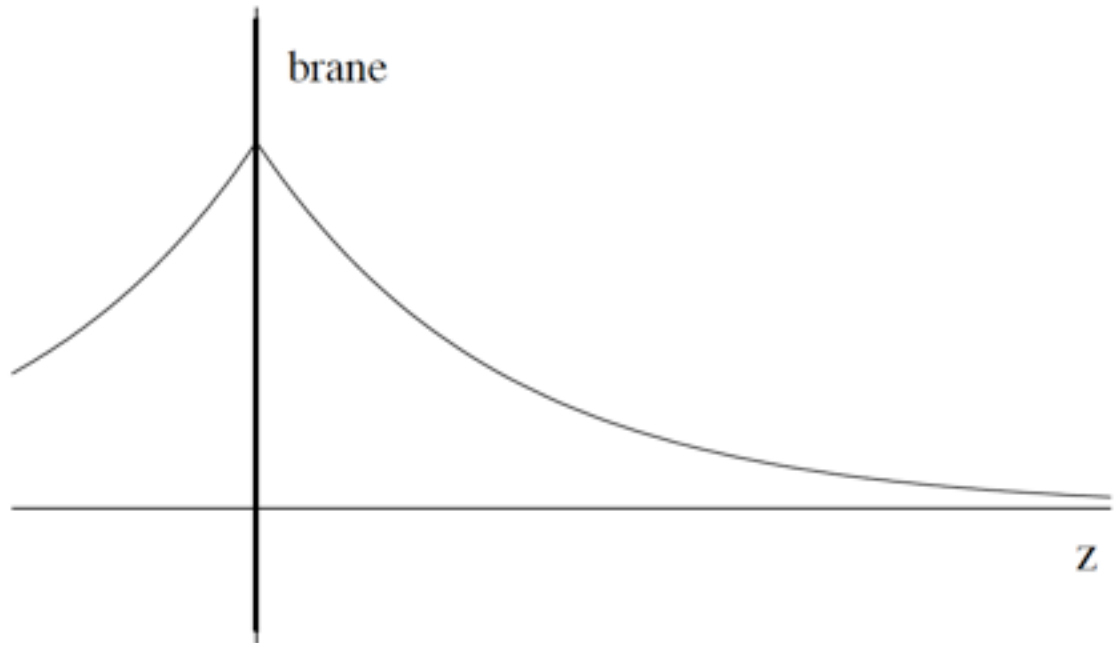
$$\Lambda = -\frac{6}{\ell^2}$$ (negative, i.e. AdS bulk).

$$\sigma = 6\frac{M_5^3}{\ell}$$

(Warped) Higher Dimensions

Randall-Sundrum- infinite, warped extra dimensions

Randall-Sundrum II



Metric: $ds^2 = e^{-2|z|/l}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2$

4D Newtons' constant: $G = \frac{G_5}{\ell}$

Effect at *early* times: $H^2 = \frac{M_{Pl}^2}{3}\rho\left(1 - \frac{\rho}{2\sigma}\right)$

(Warped) Higher Dimensions: DGP

Need IR modification: add geometry to brane

$$S[\gamma] = \frac{M_5^2}{2}\int d^D X\sqrt{-\gamma}\mathcal{R} + \int d^4x\sqrt{-g}\left(-M_5^3 K + \frac{M_4^2}{2}R - \sigma\right)$$

Brane curvatures: extrinsic, K, and intrinsic, R.

De Sitter brane with curvature $\sim \frac{M_5^3}{M_4^2}\left[\epsilon + \sqrt{1 + \frac{M_4^2\sigma}{6M_5^3}}\right]$

$\epsilon = +1$ "self accelerating" (ghosts...)

$\epsilon = -1$ "normal"

crossover scale: $\frac{1}{r_c} \sim \frac{M_5^3}{M_4^2}$

Extra Degrees of Freedom

- Scalars
- Vectors
- Tensors

Scalars: Jordan-Brans-Dicke

$$S = \int \sqrt{-g} d^4x \left[\phi R - \frac{\omega}{\phi} (\nabla\phi)^2 \right]$$

$$\boxed{\Box\phi = \frac{1}{(2\omega+3)} T_{matt}} \qquad \boxed{G = \frac{4+2\omega}{3+2\omega} \frac{1}{\phi_0}}$$

Recall Dirac: $\text{"}\Box\text{"} \frac{1}{G} \quad \propto \quad \rho$ GR: $\omega \to \infty$

Scalars: Jordan-Brans-Dicke

$$\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}(\nabla\phi)^2 + L_{\rm M}\right]$$

conformal transformation $g_{\mu\nu} \to \psi g_{\mu\nu}$

$$\int d^4x\sqrt{-g}\left[\frac{\phi}{\psi}R - \frac{3}{2}\frac{\phi(\nabla\psi)^2}{\psi^3} - \frac{\omega}{\phi\psi}(\nabla\phi)^2 + \frac{L_{\rm M}}{\psi^2}\right]$$

simplify with $\phi = \psi$ and $\phi = e^\alpha$

$$\int d^4x\sqrt{-g}\left[R - \left(\omega + \frac{3}{2}\right)(\nabla\alpha)^2 + e^{-2\alpha}L_{\rm M}\right]$$

non-minimal coupling

Cassini: $\omega > 40,000$

Scalars: Jordan-Brans-Dicke

Jordan Frame

$$\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}(\nabla\phi)^2 + L_{\mathrm{M}}\right]$$

Einstein Frame

$$\int d^4x\sqrt{-g}\left[R - \left(\omega + \frac{3}{2}\right)(\nabla\alpha)^2 + e^{-2\alpha}L_{\mathrm{M}}\right]$$

Scalars: Jordan-Brans-Dicke

Generalize a bit...

$$\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}(\nabla\phi)^2 + L_{\rm M}\right]$$

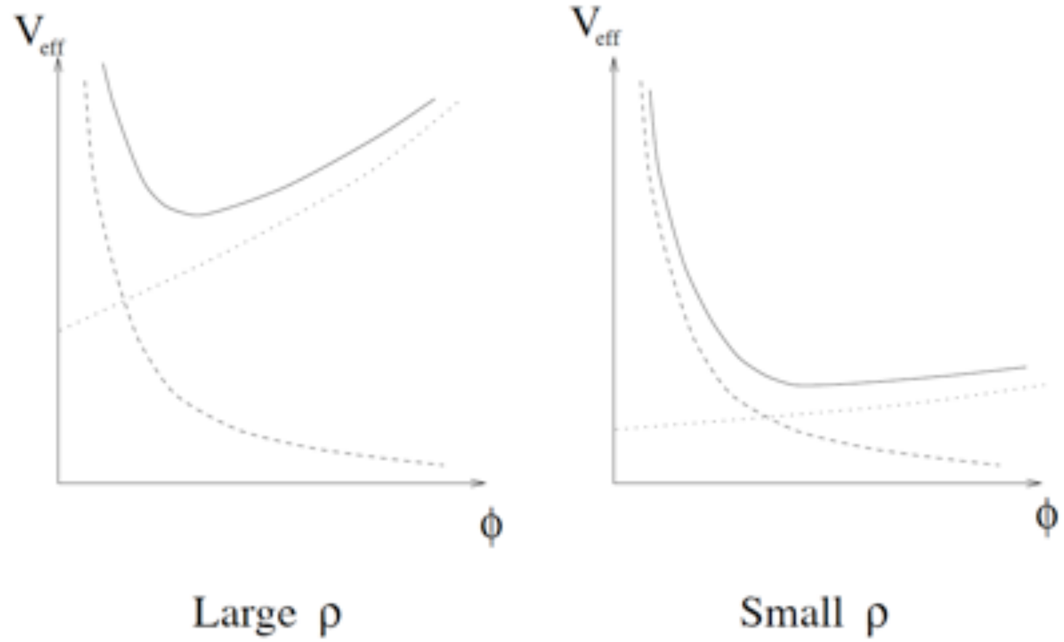
or a lot ...

$$\int d^4x\sqrt{-g}\left[f(\phi)R - \frac{\omega(\phi)}{\phi}(\nabla\phi)^2 + V(\phi) + L_{\rm M}\right]$$

What is the most general scalar tensor theory?

Scalars: Jordan-Brans-Dicke

Screening

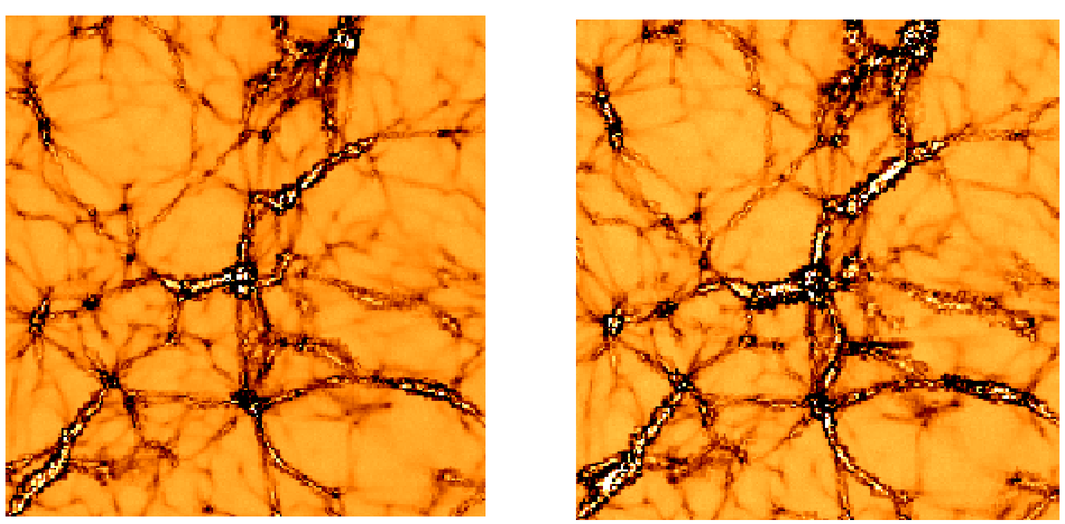


$$V_{eff}(\phi) \equiv V(\phi) + \sum \rho_i e^{\beta_i \phi / M_{Pl}}$$

$$m^2_{min} = V_{,\phi\phi}(\phi_{min}) + \sum_i \frac{\beta_i^2}{M^2_{Pl}} \rho_i e^{\beta_i \phi_{min} / M_{Pl}}$$

Khoury & Weltman, 2003

GR f(R) gravity



Courtesy of Hans Winther

Scalars: Galileons

Limit ourselves to 2nd order equations

Weak field: $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$

Fierz-Pauli + ... $L = L_{GR}(h) + L_\pi(\pi, \partial\pi, \partial^2\pi)$

Shift symmetry $\pi = \pi + b_\mu x^\mu + c$

- historical (see DGP)
- quantum correction protected
- non-linear corrections on small scales

E.g. DGP $L_\pi = M_{Pl}^2 \left[\frac{1}{2}\pi\partial^2\pi - r_c^2(\partial\pi)^2\partial^2\pi\right] + \frac{1}{2}\pi T$

Scalars: Galileons

Complete Galileon action

$$L_{Galileon} = \sum_{i=1}^{5} c_i L_i(\pi, \partial\pi, \partial^2\pi)$$

Constituents: $L_1 = \pi \quad L_2 = -\frac{1}{2}\partial\pi \cdot \partial\pi \quad L_3 = -\frac{1}{2}\Box\pi\partial\pi \cdot \partial\pi$

$$L_4 = -\frac{1}{2}\left[(\Box\pi)^2 - \partial_\mu\partial\nu\partial^\mu\partial^\nu\pi\right]\partial\pi\cdot\partial\pi$$

$$L_5 = -\frac{1}{2}\left[(\Box\pi)^3 - 3(\Box\pi)\partial_\mu\partial_\nu\pi\partial^\mu\partial^\nu\pi\right.$$
$$\left.+\partial_\mu\partial_\nu\pi\partial_\gamma\partial^\mu\pi\partial^\nu\partial^\gamma\pi + \mathrm{perms}\right]\partial\pi\cdot\partial\pi$$

Can make theory covariant: $\partial_\mu \to \nabla_\mu, R_{\mu\nu}$

Scalars: Galileons

Screening (again): Vainshtain mechanism

Consider cubic galileon around an object with mass M

$$c_2 \frac{\pi'}{r} + \frac{2c_3}{M^3}\left(\frac{\pi'}{r}\right)^2 = \frac{M}{4\pi\Lambda r^3} \qquad \Lambda \sim \left(\frac{M_P}{r_c^2}\right)^{1/3}$$

Fifth force is: $\vec{F}_\pi = \frac{1}{\Lambda}\vec{\nabla}\pi$

$$\frac{F_\phi}{F_\mathrm{N}} = \begin{cases} \frac{2}{c_2}\Big(\frac{M_\mathrm{P}}{\Lambda}\Big)^2\Big(\frac{r}{R_\mathrm{v}}\Big)^{3/2} & r \ll R_\mathrm{v} \\ \frac{2}{c_2}\Big(\frac{M_\mathrm{P}}{\Lambda}\Big)^2 & r \gg R_\mathrm{v} \end{cases}$$

i.e. supression on scales below the Vainshtain radius

$$R_\mathrm{V} = \frac{1}{M}\left(\frac{c_3 M}{2\pi c_2^2|\Lambda|}\right)^{1/3}$$

Scalars: Horndeski

Drop shift symmetry, generalize alot!

$$
\begin{aligned}
\mathcal{L}_H \quad = \quad & \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\Big[\kappa_1\nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi \\
& \qquad +\kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} - 4\kappa_{3,X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\Big] \\
& +\delta^{\alpha\beta}_{\mu\nu}\Big[(F+2W)R_{\alpha\beta}{}^{\mu\nu} - 4F_{,X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\Big] \\
& -3[2(F+2W)_{,\phi} + X\kappa_8]\nabla_\mu\nabla^\mu\phi + \kappa_9(\phi, X),
\end{aligned}
$$

Where $X = \nabla_\mu\phi\nabla^\mu\phi$ with $\kappa_i = \kappa_i(\phi, X)$ and $F = F(\phi, X)$

Note: the most general scalar tensor theory
with <u>second order</u> equations of motion

Scalars: Lesson

- Even just one degree of freedom is rich.
- We can be systematic.
- Need to control for higher derivatives...
- ... yet higher derivatives are useful for screening.

Vectors: Einstein-Aether

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}(R + M^{\alpha\beta}{}_{\mu\nu}\nabla_\alpha U^\mu \nabla_\beta U^\nu)$$

Not the measured G

$$M^{\alpha\beta}{}_{\mu\nu} \equiv c_1 g^{\alpha\beta}g_{\mu\nu} + c_2\delta^\alpha_\mu\delta^\beta_\nu + c_3\delta^\alpha_\nu\delta^\beta_\mu + c_4 U^\alpha U^\beta$$

Timelike vector: $U^\alpha U_\alpha = -1$

Nordtvedt, 1972
Jacobson & Mattingley, 2001

Simplest case: gauge field

$$M^{\alpha\beta}{}_{\mu\nu}\nabla_\alpha A^\mu \nabla_\beta A^\nu \sim F\star F$$

Vectors: Einstein-Aether

Prototypical model for *dynamical* Lorentz violation.

$U^\alpha U_\alpha = -1$ constraint that can be solved as $U^0(g_{\alpha\beta}, U^i)$

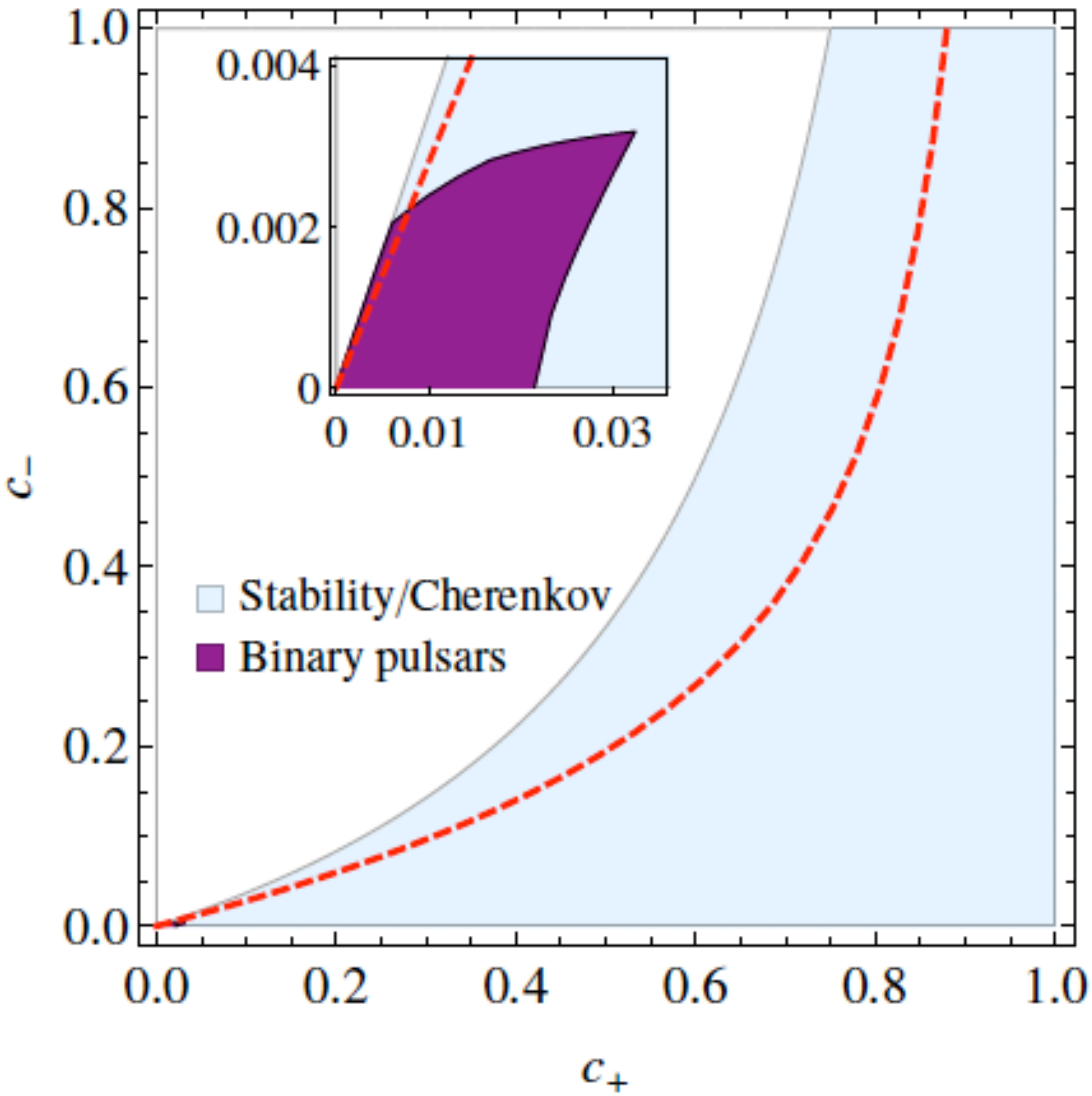
$$G_N = \frac{2G_*}{2-(c_1+c_4)}$$ Newtonian

$$G_C = \frac{2+c_1+c_2+c_3}{2(1-c_1-c_3)} G_N$$ Cosmological

Carroll & Lim 2004

c_i constraints from stability, positivity and gravitational Cherenkov.

Vectors: Einstein-Aether



$$c_\pm = c_1 \pm c_3$$

Yagi et al (2013)

Vectors

- Horava-Lifschitz can (in certain regimes) be shown to be equivalent to A-E.
- Despite a "canonical" action and minimal coupling, there are non-trivial effects.
- General procedure of introducing Lorentz violation dynamically.

Vectors: massive gravity

Massive gravity: weak field

$$S = \int d^D x - \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

Fierz-Pauli Action

Static, spherically symmetric solution:

$$h_{\mu\nu}dx^\mu dx^\nu = -B(r)dt^2 + C(r)dr^2 + A(r)r^2 d\Omega^2$$

$$\begin{aligned} B(r) &= -\frac{2M}{3M_P}\frac{1}{4\pi}\frac{e^{-mr}}{r}, \\ C(r) &= -\frac{2M}{3M_P}\frac{1}{4\pi}\frac{e^{-mr}}{r}\frac{1+mr}{m^2r^2}, \\ A(r) &= \frac{M}{3M_P}\frac{1}{4\pi}\frac{e^{-mr}}{r}\frac{1+mr+m^2r^2}{m^2r^2} \end{aligned}$$

Modified gravitation

Vectors: massive gravity

$m \to 0$ does not give the correct massless limit

GR

$$h_{00} = \frac{M}{2M_{Pl}}\frac{1}{4\pi r}$$
$$h_{ij} = \frac{M}{2M_{Pl}}\frac{1}{4\pi r}\delta_{ij}$$

massive gravity

$$h_{00} = \frac{2M}{3M_{Pl}}\frac{e^{-mr}}{4\pi r}$$
$$h_{ij} = \frac{M}{3M_{Pl}}\frac{e^{-mr}}{4\pi r}\delta_{ij}$$

$$\Phi_{GR} = -\frac{GM}{r}$$
$$\Phi_m = -\frac{4}{3}\frac{GM}{r}$$

yet lensing angle $\alpha_m = \alpha_{GR} = \frac{4GM}{b}$

Redefining G leads to 3/4 mismatch

Vectors: massive gravity

Why bother?

- Technically natural (a la t'Hooft)- a small parameter such that the $m \to 0$ restores a symmetry (GC invariance in this case) remains small.
- Massive gravity may be used to degravitate (i.e. supress effect of long wavelength sources).

Vectors: massive gravity

Massive Gravity: mass term as a high pass filter

Analogy: $$S = \int d^D x \; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu - mA_\mu \partial^\mu \phi - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + A_\mu J^\mu$$

Solve $\Box\phi + m\,\partial \cdot A = 0$ and integrate out ϕ

Non-local but gauge invariant action

Hinterbichler (2012)

$$S = \int d^D x \; -\frac{1}{4}F_{\mu\nu}\left(1 - \frac{m^2}{\Box}\right)F^{\mu\nu} + A_\mu J^\mu$$

Important on large scales (small $\Box$)

Vectors: massive gravity

Massive Gravity: non-linear theory

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}R_g + \frac{M_f^2}{2}\int d^4x\sqrt{-f}R_f$$
$$- m^2 M_g^2 \int d^4x\sqrt{-g}\sum_{n=0}^{4}\beta_n e_n(\sqrt{g^{-1}f}) + S_m$$

Hassan-Rosen Bigravity

$$M_f \neq 0$$

DRGT

$$M_f = 0$$

where we define $\sqrt{g^{-1}f}\sqrt{g^{-1}f} = g^{\mu\lambda}f_{\lambda\nu}$ with eigenvalues λ_n

and
$$e_0 = 1$$
$$e_1 = \sum_{i=1}^{4}\lambda_i$$
$$e_2 = \sum_{i,j;i<j}\lambda_i\lambda_j = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$$
$$e_3 = \sum_{i,j,k;i<j<k}\lambda_i\lambda_j\lambda_k = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4$$
$$e_4 = \sum_{i,j,k,l;i<j<k<l}\lambda_i\lambda_j\lambda_k\lambda_l = \lambda_1\lambda_2\lambda_3\lambda_4 = \det(\sqrt{g^{-1}f}).$$

Vectors: massive gravity

- Only an effective theory valid up to:

$$\Lambda_3 = (M_{Pl} m^2)^{\frac{1}{3}} \sim 1000 \text{ km}$$

- Not clear if bigravity is well-posed (too many degrees of freedom).

- Simplest DRGT does not give flat FRW universe.

- Can be mapped onto galileons.

Higher Derivative Theories

EFTs seem to lead to higher derivative theories:

$$
\begin{aligned}
-\frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g}} \quad = \quad & \lambda + \frac{M_p^2}{2} R + a_1 R_{\mu\nu} R^{\mu\nu} \\
& + a_2 R^2 + a_3 R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} + a_4 \Box R \\
& + \frac{b_1}{m^2} R^3 + \frac{b_2}{m^2} R R_{\mu\nu} R^{\mu\nu} + \frac{b_3}{m^2} R_{\mu\nu} R^{\nu\lambda} R_{\lambda}{}^{\mu}
\end{aligned}
$$

Why are they “higher derivative”: $R \sim \partial^2 g + \partial g \partial g$

$$R^2 \sim (\partial^2 g)^2 + \cdots$$

Problems: arbitrary choice of coefficients leads to Ostragradski instability *and* only effective at Planck scale.

Higher Derivative Theories

Higher derivative theories: $\mathcal{L} = \sqrt{-g} f(R)$

Field equations:

$$f_R R_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} - f_{R;\mu\nu} + g_{\mu\nu} \Box f_R = \frac{\chi}{2} T_{\mu\nu}$$

Conformal transformation $\bar{g}_{\mu\nu} = f_R g_{\mu\nu}$ and $\phi \equiv \sqrt{\frac{3}{\chi}} \ln f_R$

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{\chi}{2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{g}^{\rho\sigma}\phi_{,\rho}\phi_{,\sigma} - \bar{g}_{\mu\nu}V\right) + \frac{\chi}{2}\bar{T}_{\mu\nu}$$

I.e. conformally equivalent to Einstein but non-minimal coupling or scalar tensor with no Ostragradski instability.

Higher Derivative Theories

Examples of $\mathcal{L} = \sqrt{-g} f(R)$

Hu-Sawicki: $R - \dfrac{\mu R_c}{1 + \left(\frac{R}{R_c}\right)^n}$

Starobinsky: $R - \mu R_c \left[1 - \left(1 + \dfrac{R^2}{R_c^2} \right)^{-n} \right]$

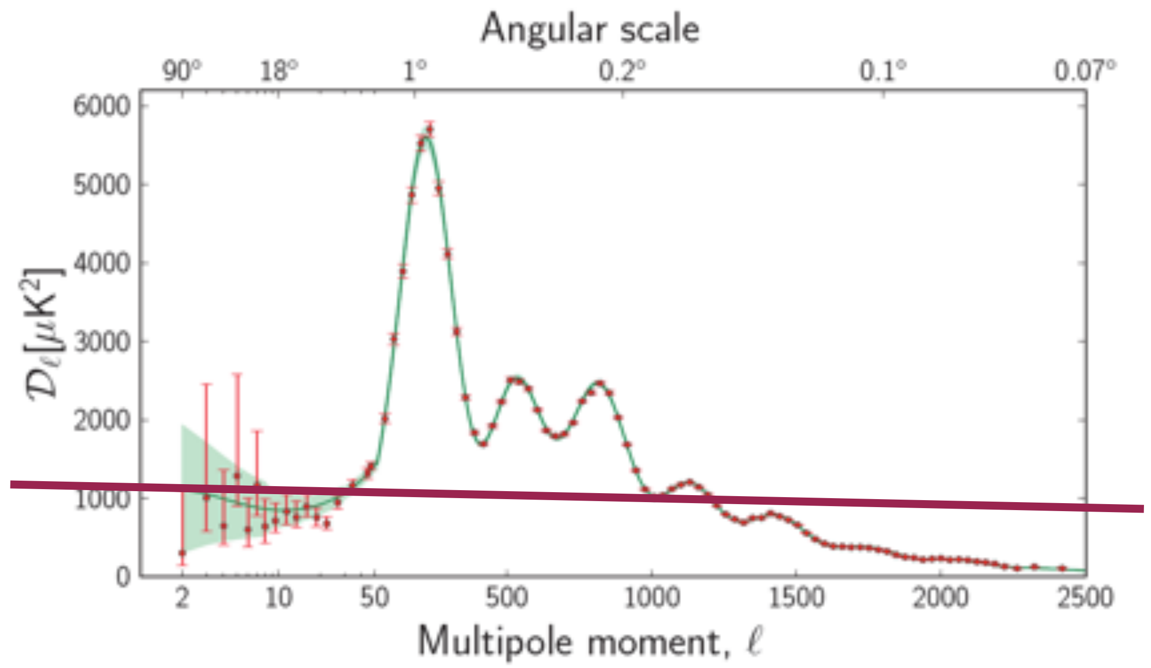
Secretely have a cosmological constant:

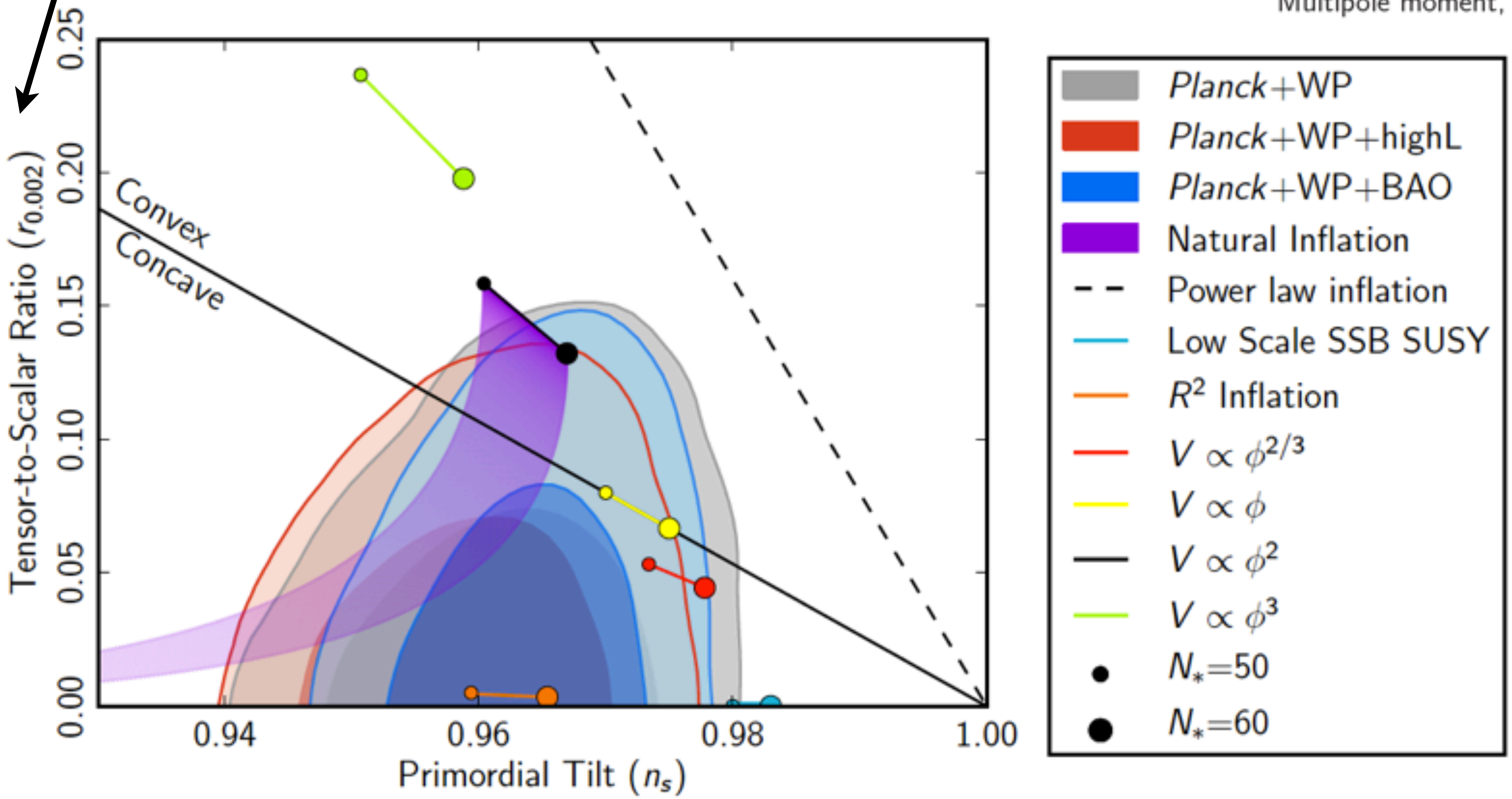
$$R - 2\Lambda + \frac{C}{R^n} \cdots$$

Initial Conditions

Primordial Gravitational Waves

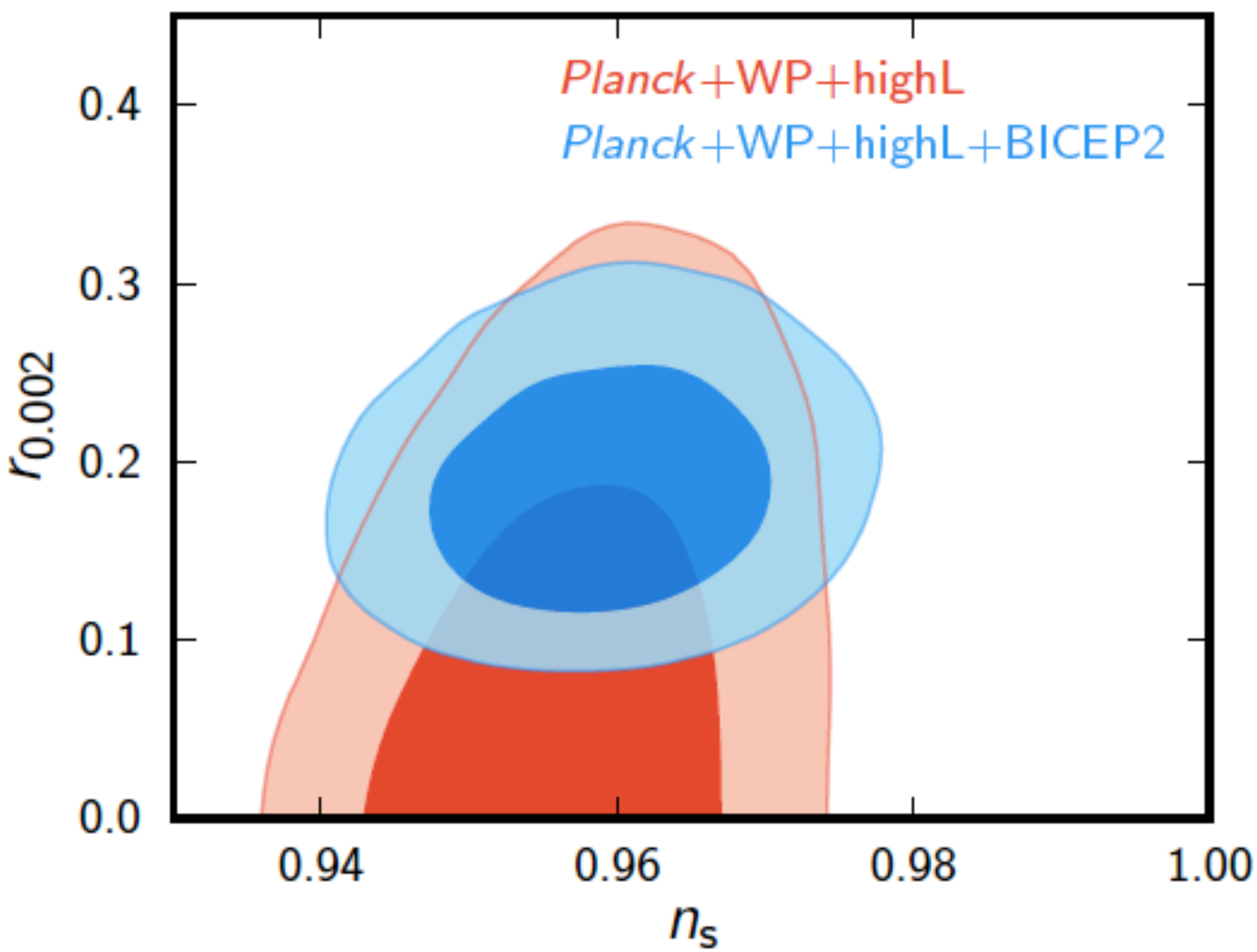
Primordial Tilt





Planck XXII

But ...


Planck+WP+highL
Planck+WP+highL+BICEP2
0.4
0.3
0.2
0.1
0.0
$r_{0.002}$
0.94
0.96
0.98
1.00
n_s

BICEP2

Non-locality

Non-local gravity

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R+\alpha\frac{R}{\Box}R+\beta R_{\mu\nu}R^{\mu\nu}+\zeta\frac{R_{\mu\nu}}{\Box}R^{\mu\nu}\cdots\right)$$

Integrating out ultra-light (or massless) d.o.f.

e.g.- one loop effective action for gravity

Barvinsky & Vilkovisky 1995

Late time/large scale effect with $\alpha \sim 10$ and $\frac{R}{\Box} \propto \ln\frac{t_0}{t_{\rm eq}}$

Generalize $$\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[1+f\left(\frac{R}{\Box}\right)\right]R$$

Deser & Woodard

Non-Locality

Non-local gravity- the action is not enough

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(1+\alpha\frac{R}{\Box^2}\right)R$$

Lagrange multiplier

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(1+\phi R)\,R+\lambda\left(\Box^2\phi-R\right)$$

Maggiore, 2013

Solve $\Box^2\phi = R$ but <u>also</u> $\phi = 0$ if $R = 0$.

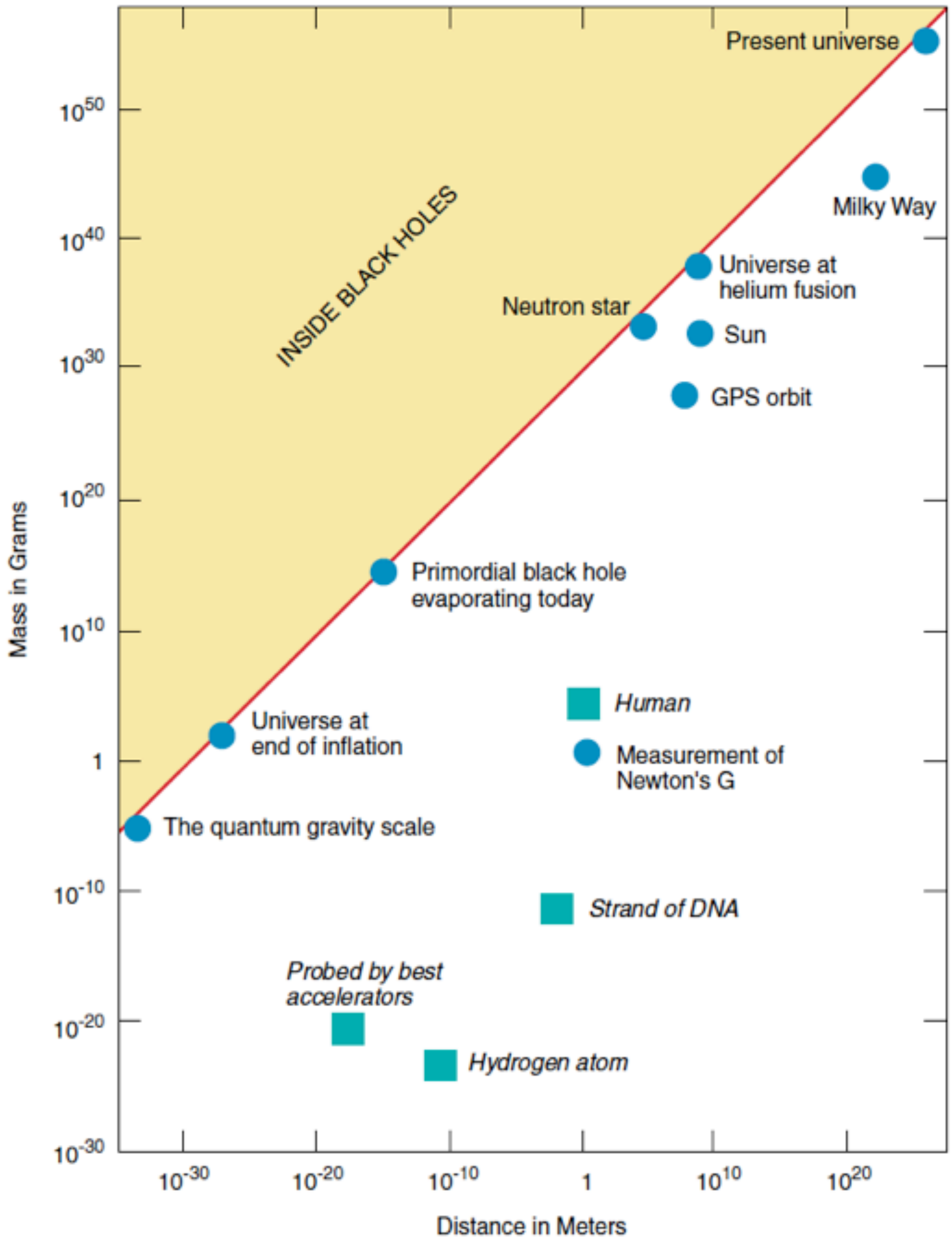
Additional boundary conditions needed but no extra degrees of freedom

Potential problems: causality, instability, ...

Non-Locality

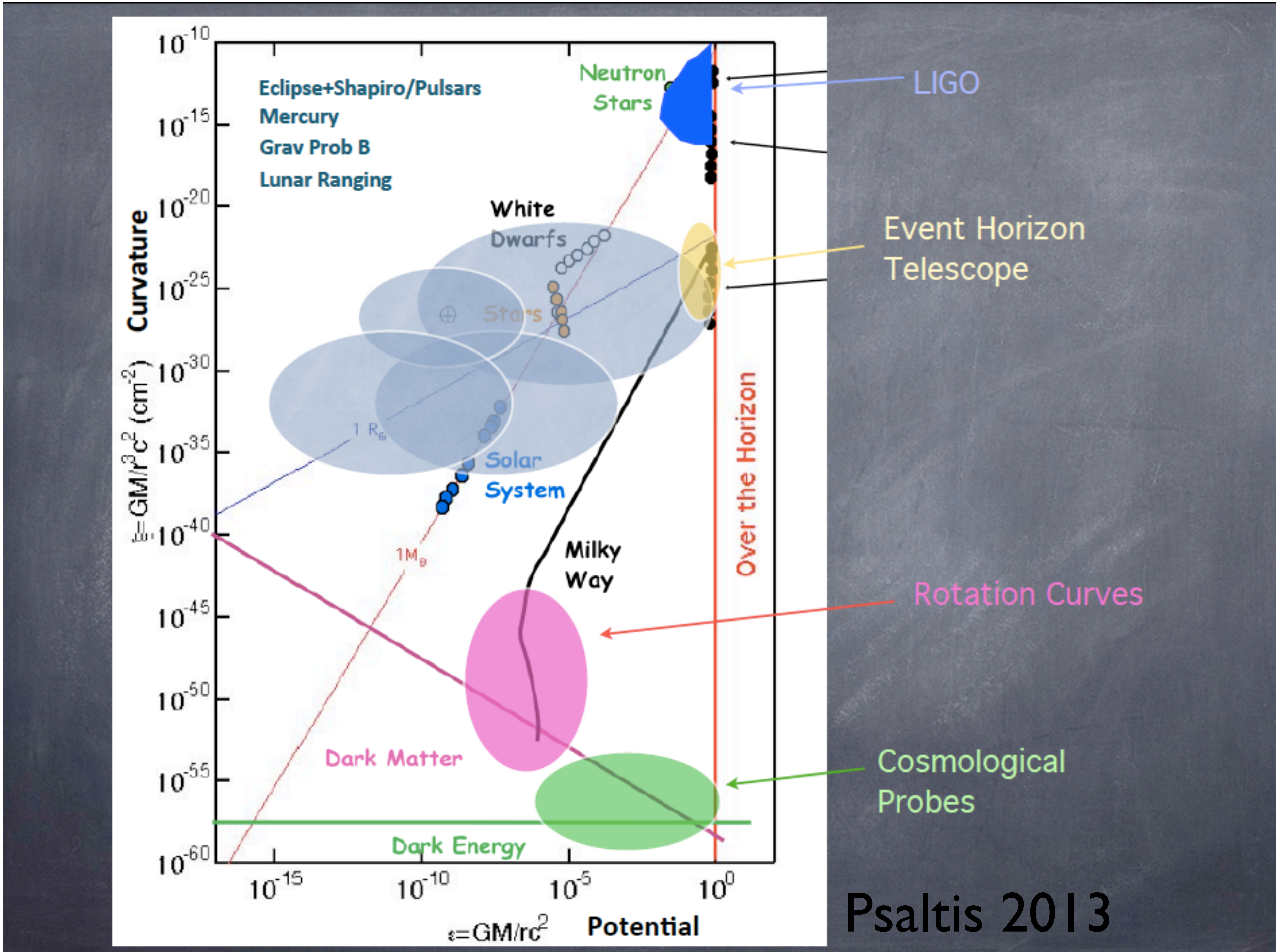
- Will not appear in an EFT- integrating out *massless* modes.
- No extra degrees of freedom.
- Action is not enough- causality!
- Generic theory will have instabilities.

Testing Gravity.



"... by pushing a theory to its extremes, we also find out where the cracks in its structure might be hiding."
John Wheeler

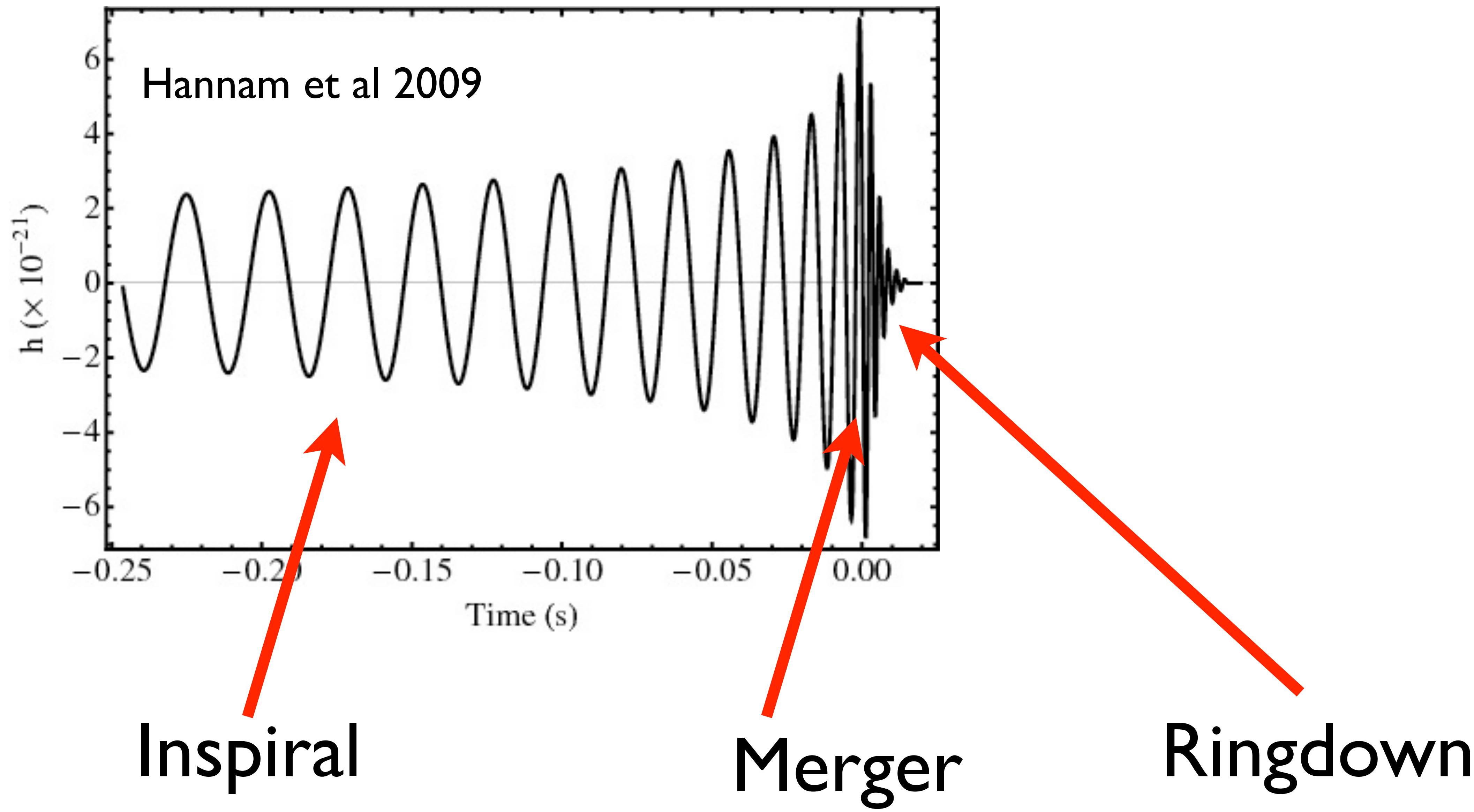
Decadal Survey 2000



Psaltis 2013

Future Tests: Gravity Waves

Binary Merger



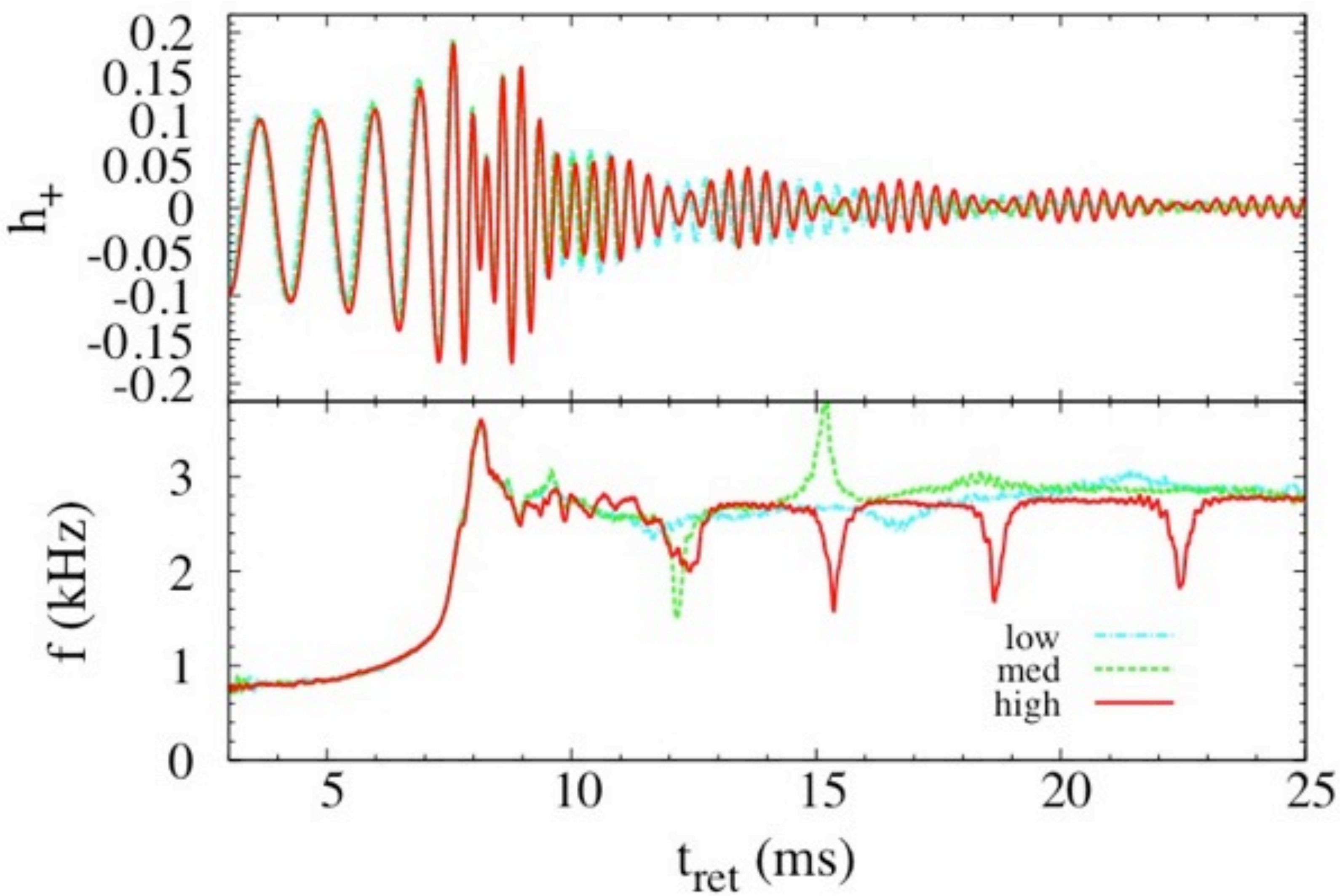
All will be sensitive to gravity ...

Future Tests: Gravity Waves

Binary Merger: sponataneous scalarization

Damour, Esposito-Farese 1993

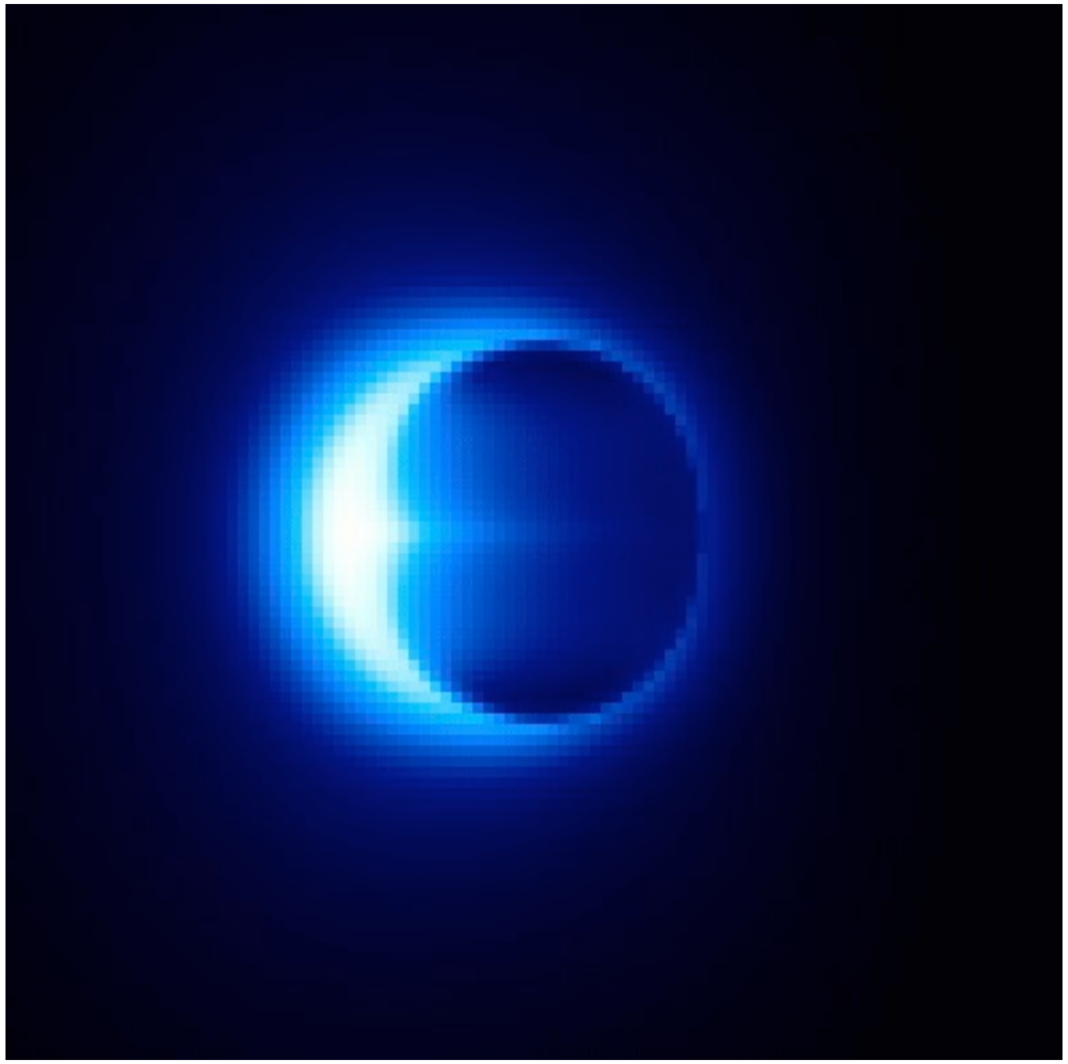
Scalar-Tensor: the scalar field (i.e. "G"!) is excited



Shibata 2013

Future Tests: Event Horizon Telescope

Long Baseline imaging of Sgr A*- $R_S \sim 10$ microarcseconds



Broderick & Psaltis

Shadow depends on spin, mass <u>and</u> theory of gravity.

Test the no-hair theorem!

The Universe: background cosmology

$$ds^2 = a^2 \gamma_{\mu\nu} dx^\mu dx^\nu$$

FRW equations

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta} \longrightarrow \mathcal{H}^2 = \frac{8\pi G}{3} a^2 \rho$$

<u>Any</u> theory (modified gravity or otherwise)

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta} + U_{\alpha\beta}$$

$$\searrow$$

$$\rho_X(\tau), P_X(\tau)$$

The Universe: background cosmology

We measured distances:

Hubble: $D_H = \frac{c}{H_0} = 3000\ h^{-1}\ \mathrm{Mpc}$

Comoving: $D_C = \int_t^{t_0} \frac{c dt'}{a(t')} = c \int_a^1 \frac{da}{a^2 H(a)}$

Transverse: $D_M = \begin{cases} \frac{D_H}{\sqrt{\Omega_k}} \sinh[\sqrt{\Omega_k} D_C/D_H] & \text{for } \Omega_k > 0 \\ D_C & \text{for } \Omega_k = 0 \\ \frac{D_H}{\sqrt{|\Omega_k|}} \sin[\sqrt{|\Omega_k|} D_C/D_H] & \text{for } \Omega_k < 0 \end{cases}$

Luminosity: $D_L = (1+z) D_M$

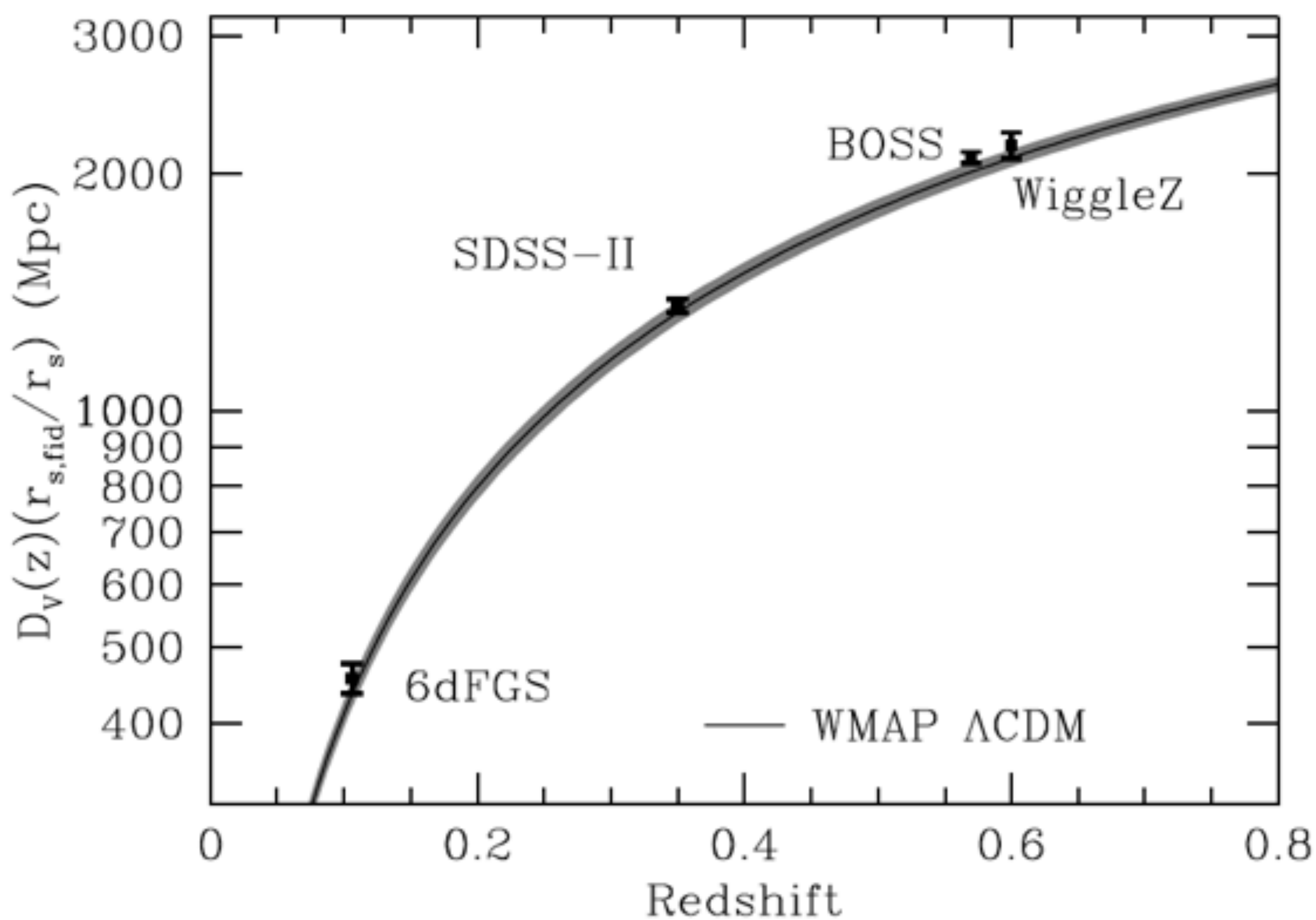
Angular Diameter: $D_A = \frac{D_M}{1+z}$

The Universe: background cosmology

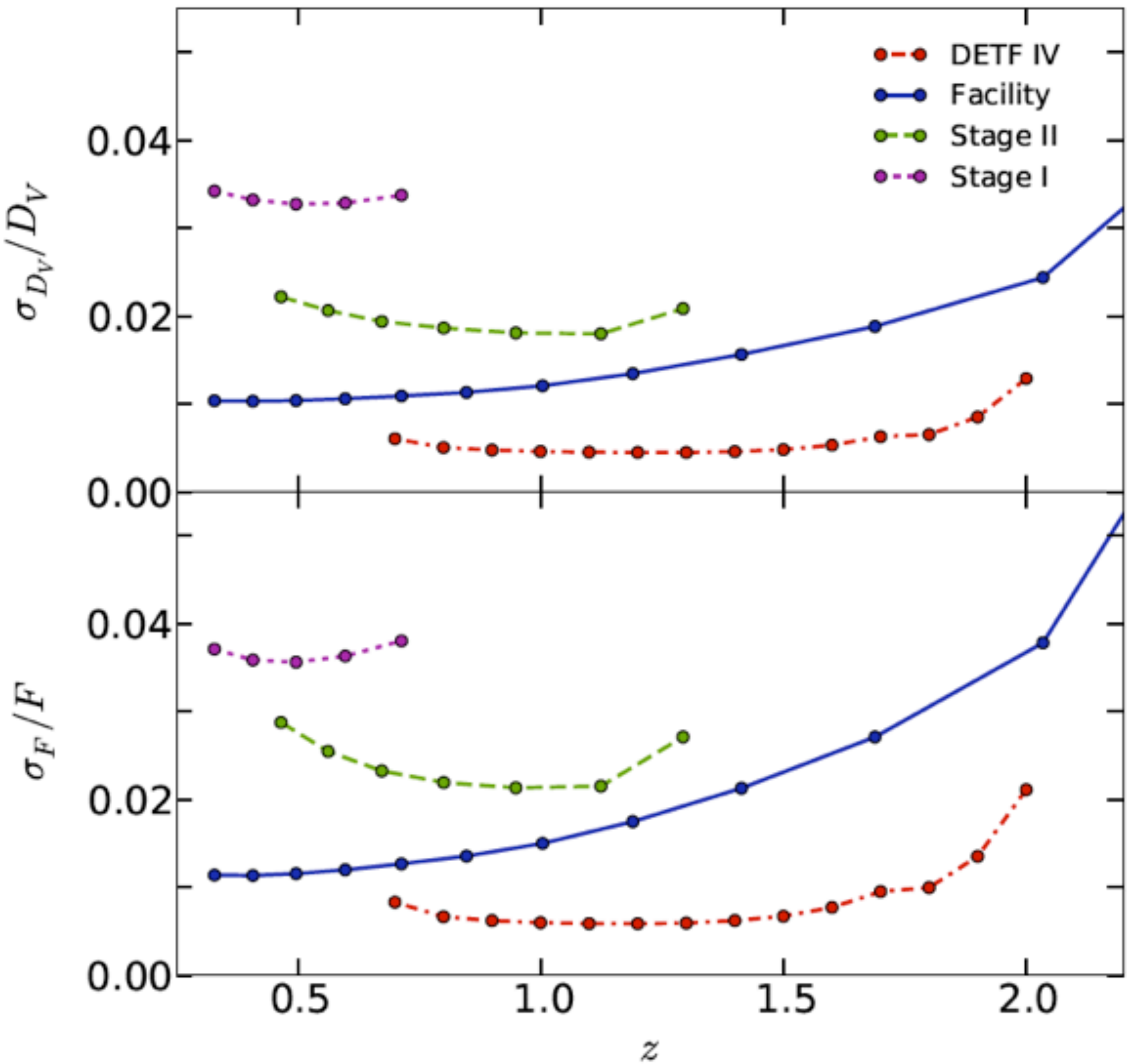
BAO:

Volume distance: $D_V(z) = \left[(1+z)^2 D_A^2 \frac{cz}{H(z)}\right]^{\frac{1}{3}}$

Alcock-Paczinski: $F(z) = (1+z) D_A(z) H(z)/c.$

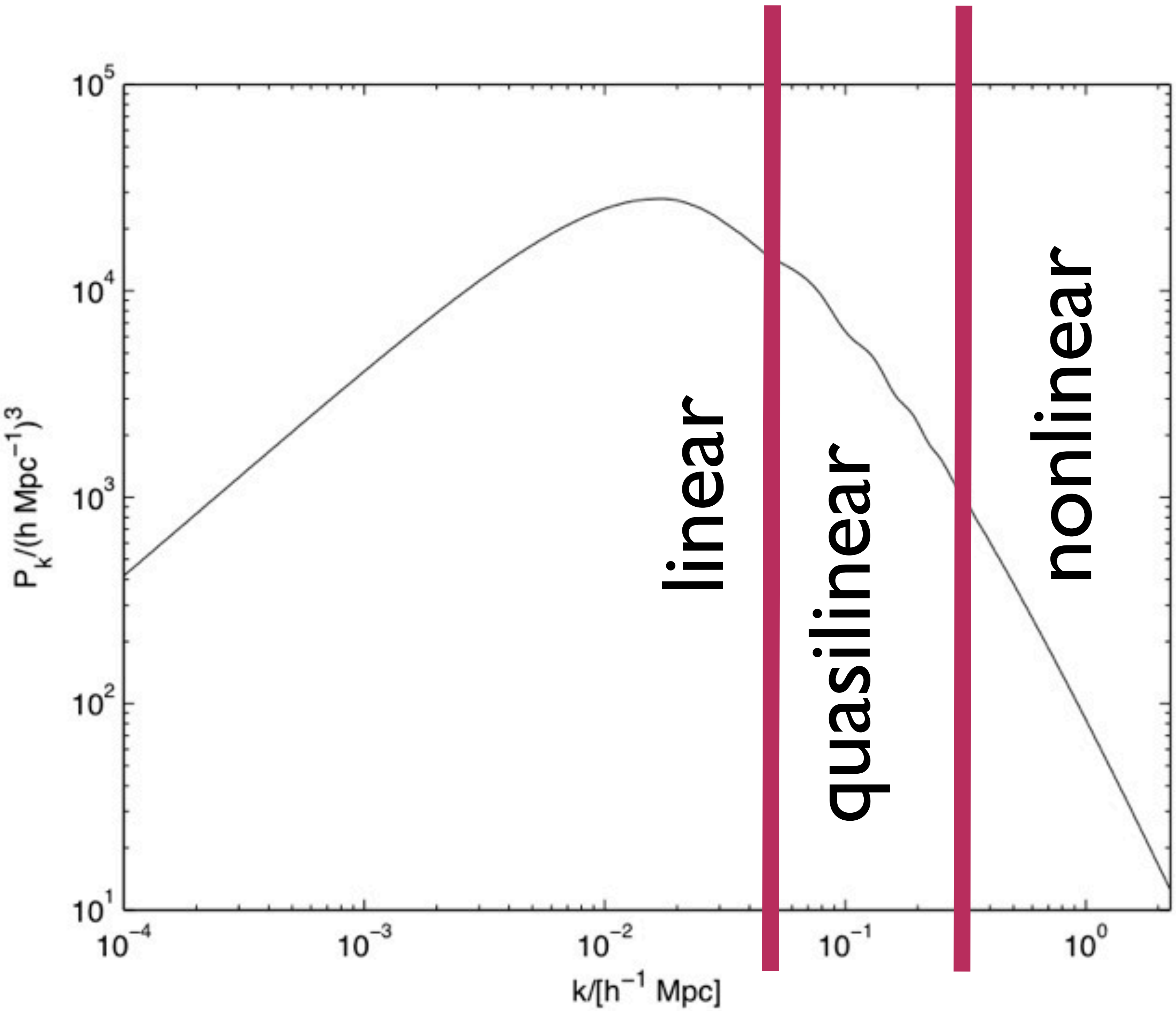


The Universe: background cosmology



Bull *et al* (2014)

The Universe: large scale structure



Linear Perturbation Theory $(10 - 10,000 h^{-1} Mpc)$

$$ds^2 = a^2(\gamma_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu$$

Diffeomorphism invariance $\longrightarrow$ Gauge invariant Newtonian potentials

$$(\hat{\Phi}, \hat{\Psi})$$

$$\rho \to \rho(\tau)[1 + \delta(\tau, \mathbf{r})]$$

$$\hat{\Gamma} = \frac{1}{k}\left(\dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi}\right)$$

$$\delta G_{\alpha\beta} = 8\pi G \delta T_{\alpha\beta}$$

$$\delta G^{(gi)}_{00} : \; 2\vec{\nabla}^2\hat{\Phi} - 6\mathcal{H}k\hat{\Gamma} = 8\pi G a^2 \rho \delta^{(gi)}$$

$$\delta G^{(gi)}_{0i} : \; 2k\hat{\Gamma} = 8\pi G(\rho + P)\theta^{(gi)}$$

$$\delta G^{(gi)}_{ij} : \; \hat{\Phi} - \hat{\Psi} = 8\pi G a^2(\rho + P)\Sigma^{(gi)}$$

(+ $\delta G^{(gi)}_{ii}$ equation)

Extending Einstein's equations

$$\delta G_{\mu\nu} = 8\pi G \delta T^M_{\mu\nu} + \delta U_{\mu\nu}$$

$$\downarrow$$

Linear in $\hat{\Phi}, \hat{\Gamma}, \hat{\chi}, \dot{\hat{\chi}}$

Baker, Ferreira, Skordis 2012
Bloomfield, Flanagan, Park, Watson 2012
Gleyzes, Gubitosi, Piazza, Vernizzi 2013
Pearson, Battye 2011

ArXiv:1209.2117

Extending Einstein's equations

Key: Matter + Metric + New degree of freedom

$$-a^2\delta G_0^{0\,(gi)} = \kappa a^2 G\,\rho_M \delta_M^{(gi)} + \alpha_0 k^2 \hat{\chi} + \alpha_1 k \dot{\hat{\chi}} + A_0 k^2 \hat{\Phi} + F_0 k^2 \hat{\Gamma}$$

ArXiv:1209.2117

Extending Einstein's equations

Key: Matter + Metric + New degree of freedom

$$-a^2\delta G^{0\,(gi)}_{0} = \kappa a^2 G\,\rho_M \delta^{(gi)}_M + \alpha_0 k^2\hat{\chi} + \alpha_1 k\dot{\hat{\chi}} + A_0 k^2\hat{\Phi} + F_0 k^2\hat{\Gamma}$$

Functions of time (and scale).

$$-a^2\delta G^{0\,(gi)}_{i} = \nabla_i\left[\kappa a^2 G\,\rho_M(1+\omega_M)\theta^{(gi)}_M + \beta_0 k\hat{\chi} + \beta_1\dot{\hat{\chi}} + B_0 k\hat{\Phi} + I_0 k\hat{\Gamma}\right]$$

$$a^2\delta G^{i\,(gi)}_{i} = 3\,\kappa a^2 G\,\rho_M \Pi^{(gi)}_M + \gamma_0 k^2\hat{\chi} + \gamma_1 k\dot{\hat{\chi}} + \gamma_2\ddot{\hat{\chi}} + C_0 k^2\hat{\Phi} + C_1 k\dot{\hat{\Phi}} + J_0 k^2\hat{\Gamma} + J_1 k\dot{\hat{\Gamma}}$$

$$a^2\delta G^{i}_{j} = D^i_j\left[\kappa a^2 G\,\rho_M(1+\omega_M)\Sigma_M + \epsilon_0\hat{\chi} + \frac{\epsilon_1}{k}\dot{\hat{\chi}} + \frac{\epsilon_2}{k^2}\ddot{\hat{\chi}} + D_0\hat{\Phi} + \frac{D_1}{k}\dot{\hat{\Phi}} + K_0\hat{\Gamma} + \frac{K_1}{k}\dot{\hat{\Gamma}}\right]$$

ArXiv:1209.2117

Extending Einstein's ... uations

Key: Matter + Metric ... lom

$$-a^2\delta G^{0\,(gi)}_0 = \kappa a^2 G \ldots$$

$$+ \ldots$$

...ime

$$-a^2\delta G^{0\,(gi)}_i \ldots \beta_1\dot{\hat{\chi}}\big]$$

$$\ldots\hat{\chi} + \gamma_1 k\dot{\hat{\chi}} + \gamma_2\ddot{\hat{\chi}}$$

$$\ldots J_0 k^2\hat{\Gamma} + J_1 k\dot{\hat{\Gamma}}$$

$$a^2\delta G^i_j = \ldots + \omega_M)\Sigma_M \quad +\epsilon_0\hat{\chi} + \tfrac{\epsilon_1}{k}\dot{\hat{\chi}} + \tfrac{\epsilon_2}{k^2}\ddot{\hat{\chi}}\big]$$

$$\ldots + \tfrac{D_1}{k}\dot{\hat{\Phi}} \quad +K_0\hat{\Gamma} + \tfrac{K_1}{k}\dot{\hat{\Gamma}}$$

Integrability

↓

7 to 9 free functions of time

ArXiv:1209.2117

... but "Integrability condition" can help

$$U_{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta S_U}{\delta g^{\alpha\beta}}$$

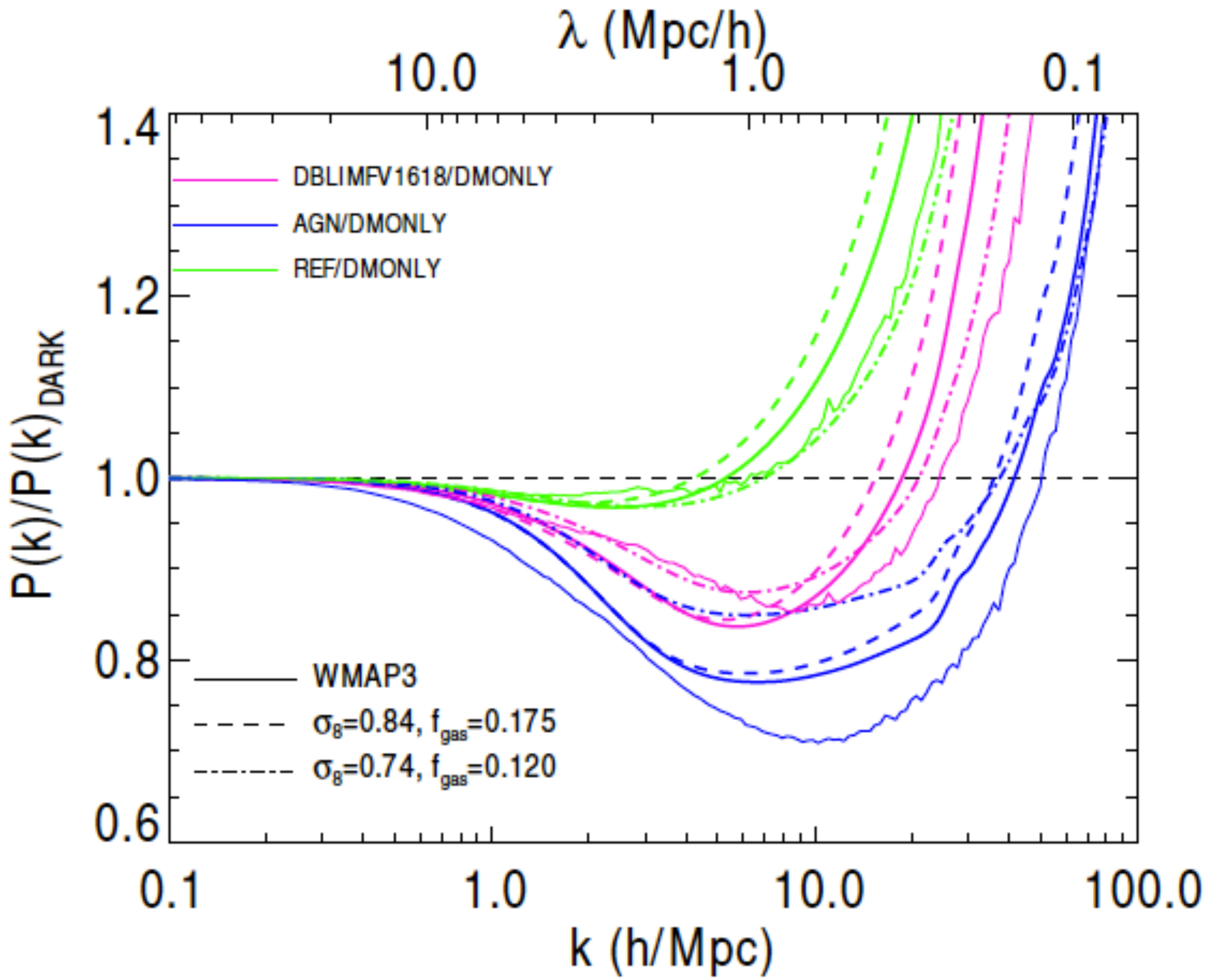
Use general principles to restrict S_U

Example- assume locality, scalar field, etc leads to only <u>7 free functions of time.</u>

Bloomfield, Flanagan, Park, Watson 2012
Gleyzes, Gubitosi, Piazza, Vernizzi 2013
Pearson, Battye 2011

What about the non-linear regime?

Baryon, feedback and bias



Semboloni et al 2012

And now to what we observe: Light vs Matter

- For a perturbed line element of the form:

$$ds^2 = a^2(\tau)[-(1+2\Phi)d\tau^2 + (1-2\Psi)\gamma_{ij}dx^i dx^j]$$

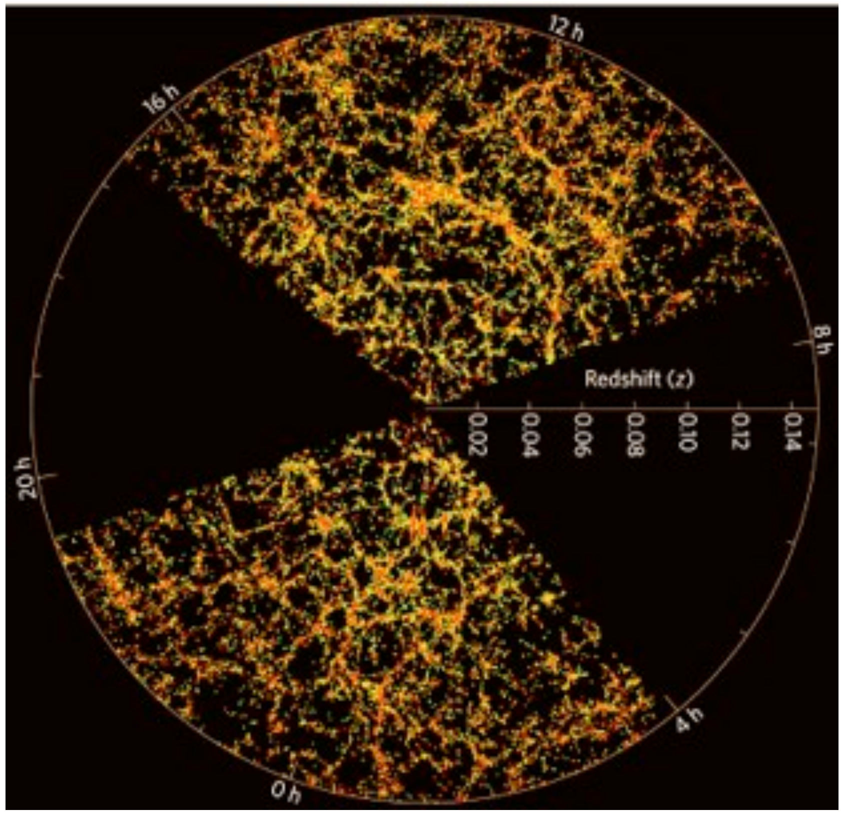
the equations of motion are:

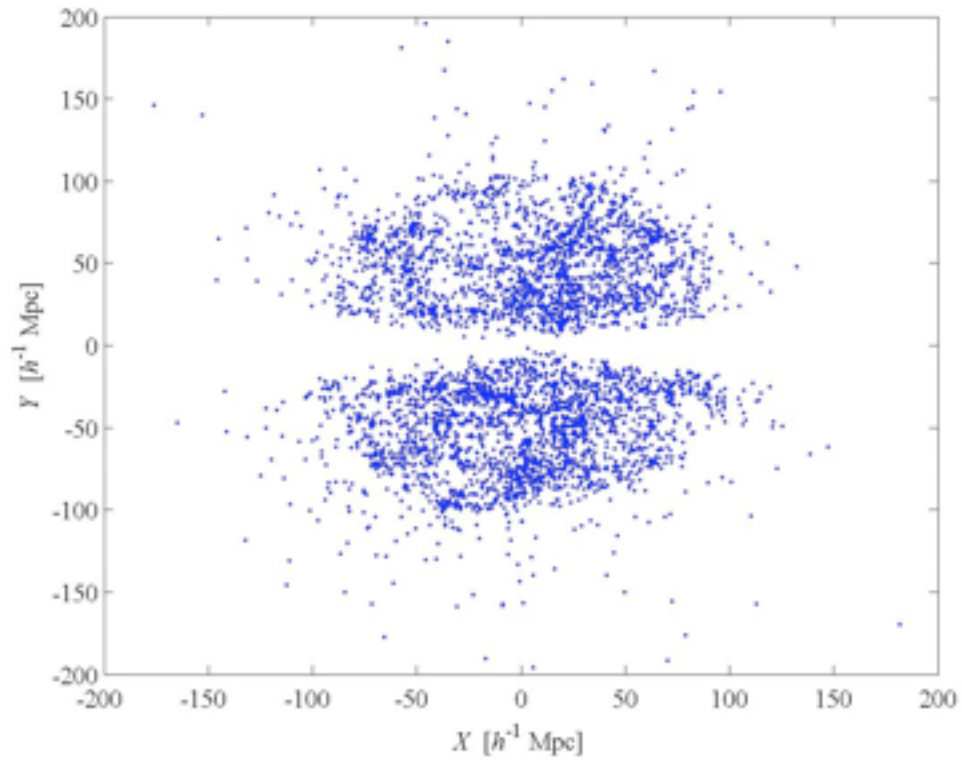
$$\frac{1}{a}\frac{d(a\mathbf{v})}{d\tau} = -\nabla\Phi$$ (non-relativistic particles)

$$\frac{d\mathbf{v}}{d\tau} = -\nabla_\perp(\Phi + \Psi)$$ (relativistic particles)

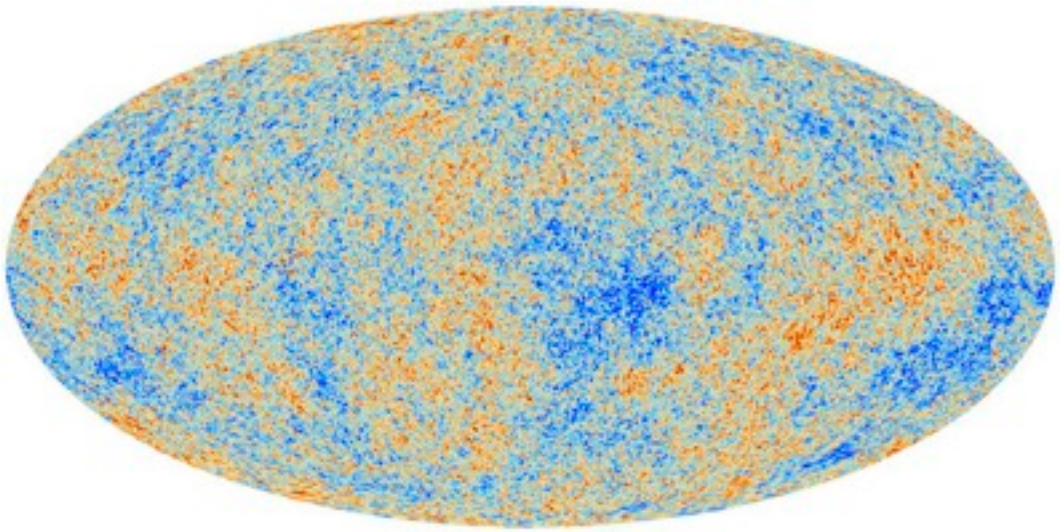
What we observe.

$\delta, \vec{v}$

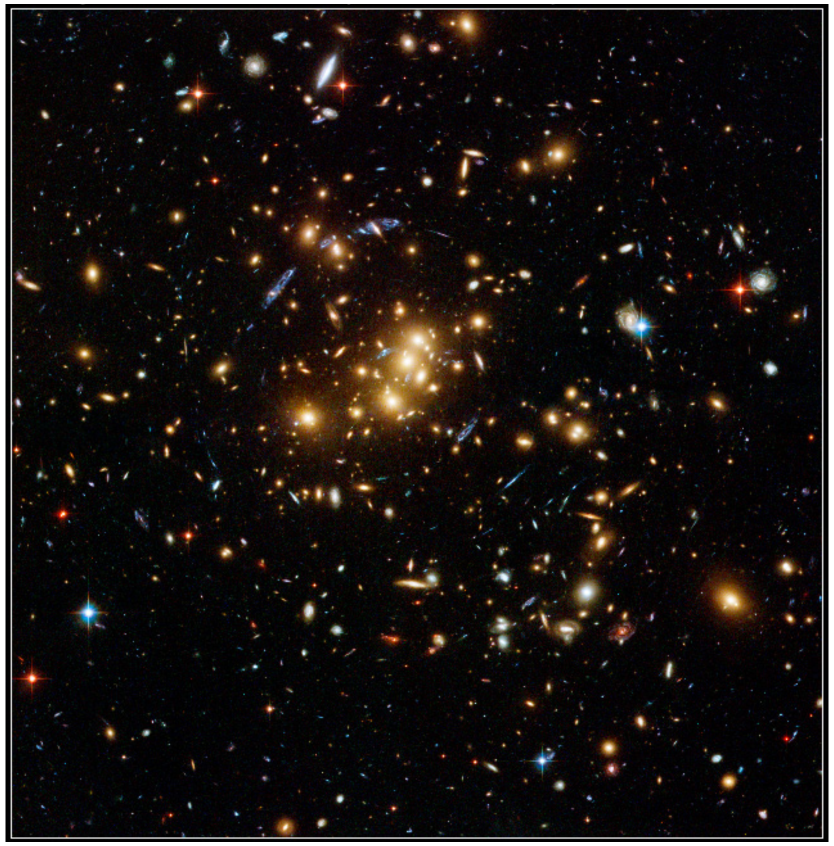




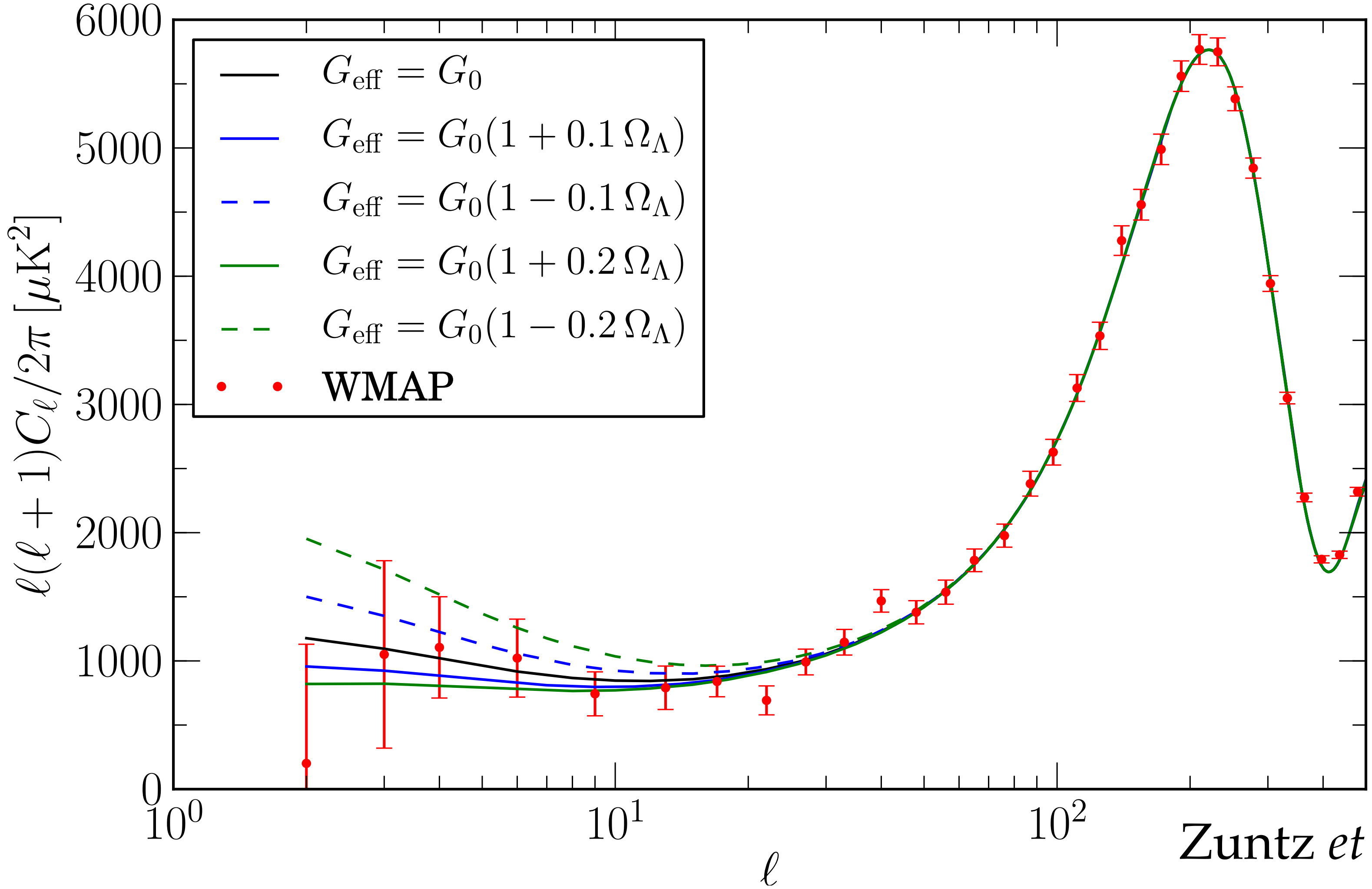
$\vec{v}$



Φ, Ψ



Large Scales: the problem with cosmic variance

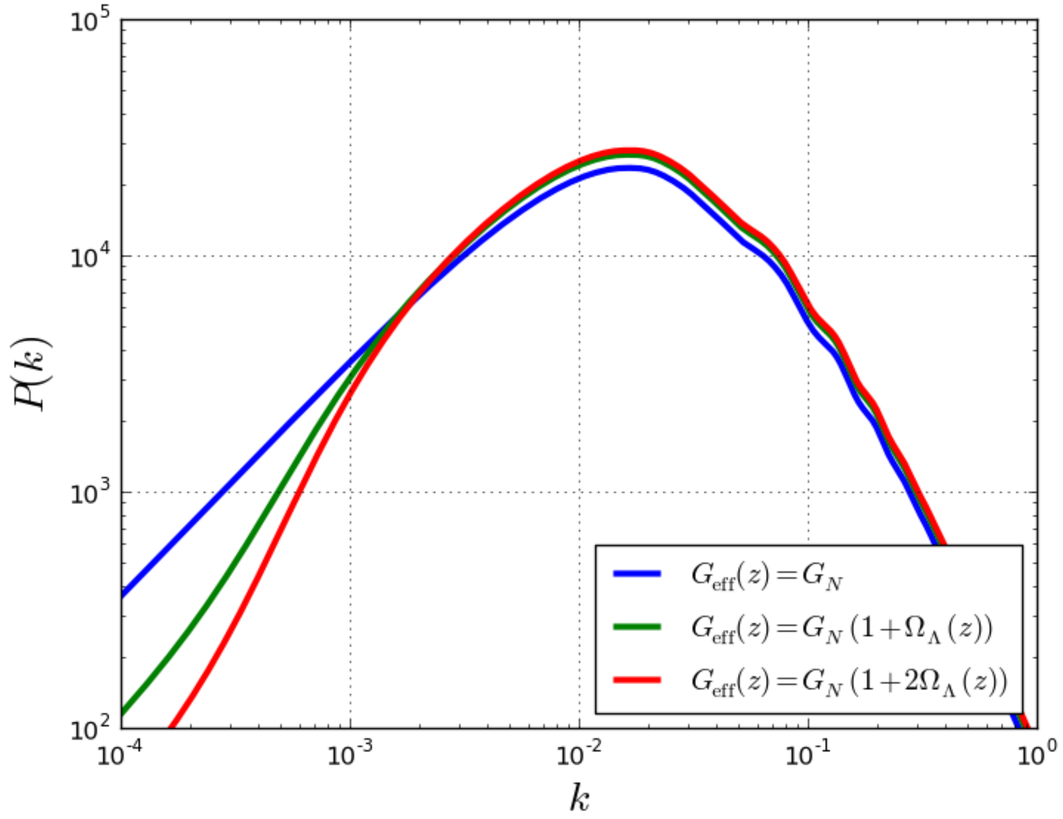


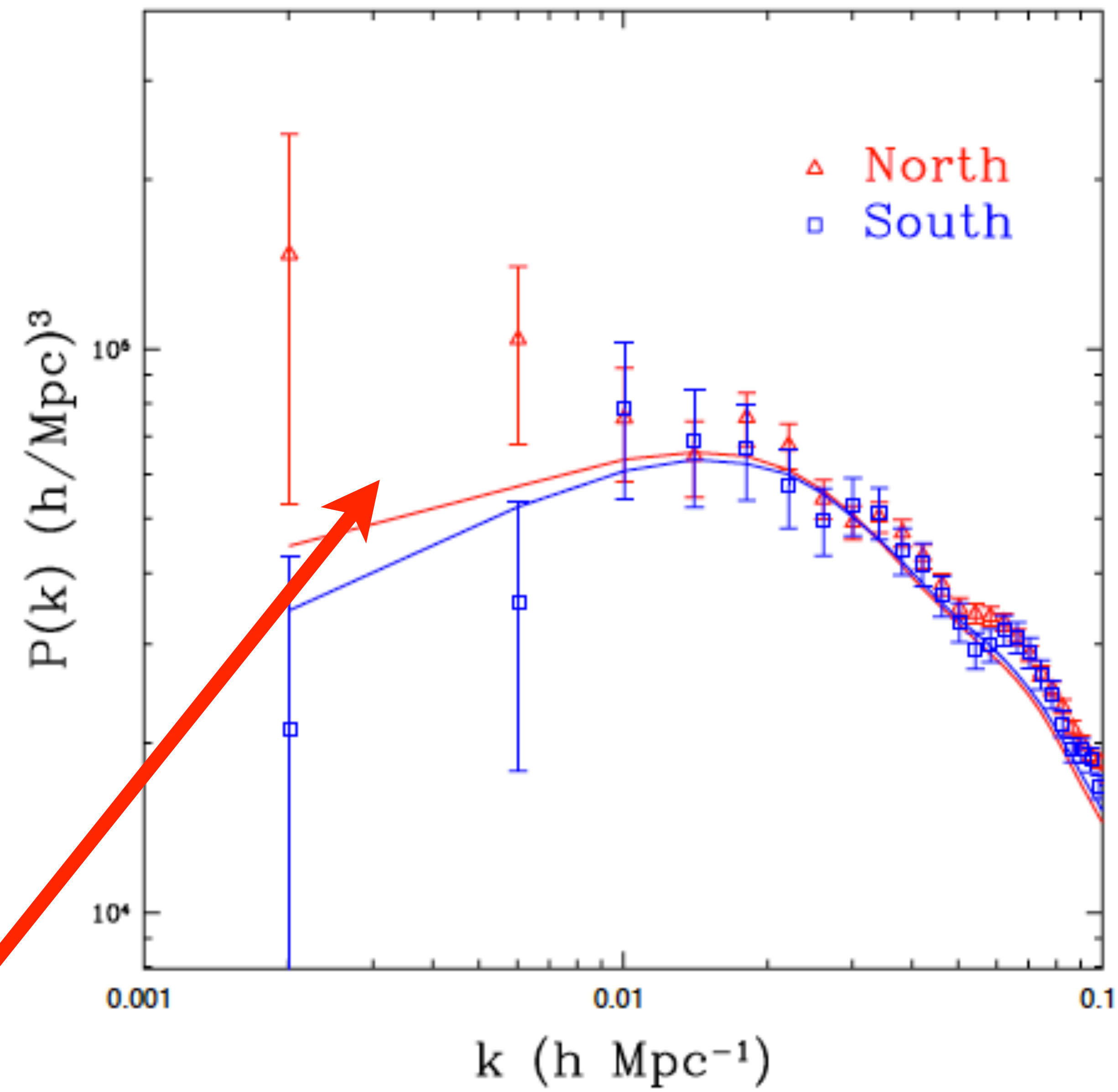
Zuntz *et al.* 2011

ISW- late time effects on large scales

$$\propto \int (\dot{\Phi} + \dot{\Psi})\,d\eta$$

Large scales: the problem with the Galaxy





Systematic effect due to stellar densities

Ross et al (BOSS) 2012

Not so large scale: "quasi-static" regime

A preferred length scale- the horizon

$$\mathcal{H}^{-1} \equiv \left(\frac{\dot{a}}{a}\right)^{-1} \propto \tau \simeq 3000h^{-1}\mathrm{Mpc}$$

Focus on scales such that $k\tau \gg 1$

Most surveys $\leq 300h^{-1}\mathrm{Mpc}$

$$-k^2\Phi = 4\pi G\mu a^2\rho\Delta$$

$$\gamma\Psi = \Phi$$

Zhang, Liguori, Bean and Dodelson
Caldwell, Cooray and Melchiorri
Amendola, Kunz and Sapone
Bertschinger and Zukin
Amin, Blandford and Wagoner
Pogosian, Silvestri, Koyama and Zhao
Bean and Tangmatitham

Not so large scale: "quasi-static" regime

Revisit the linearized equations of motion:

Approximate time derivatives... $\dot{X} \sim \mathcal{H}X$

... and then discard. $\mathcal{H}X \ll kX$

What if ... $\dot{X} \sim MX$ and $M \sim k$?

Remarkably general but must be checked case by case!

Not so large scale: "quasi-static" regime

The "quasi-static" functions reduce to a simple form

$$\mu = \mu_0(a)\left[1+\left(\frac{M_1(a)}{k}\right)^2\right]$$

$$\gamma = \gamma_0(a)\left[1+\left(\frac{M_2(a)}{k}\right)^2\right]$$

DeFelice et al 2011
Baker et al 2012
Silvestri et al 2013

If non-zero (M_1, M_2) are detected, a signature of something truly new: tensors, screening, breakdown in the quasi-static approximation, etc.

Goal: to use k and z dependent measurements of (γ, μ) to constrain PPF functions

Example: Growth of Structure

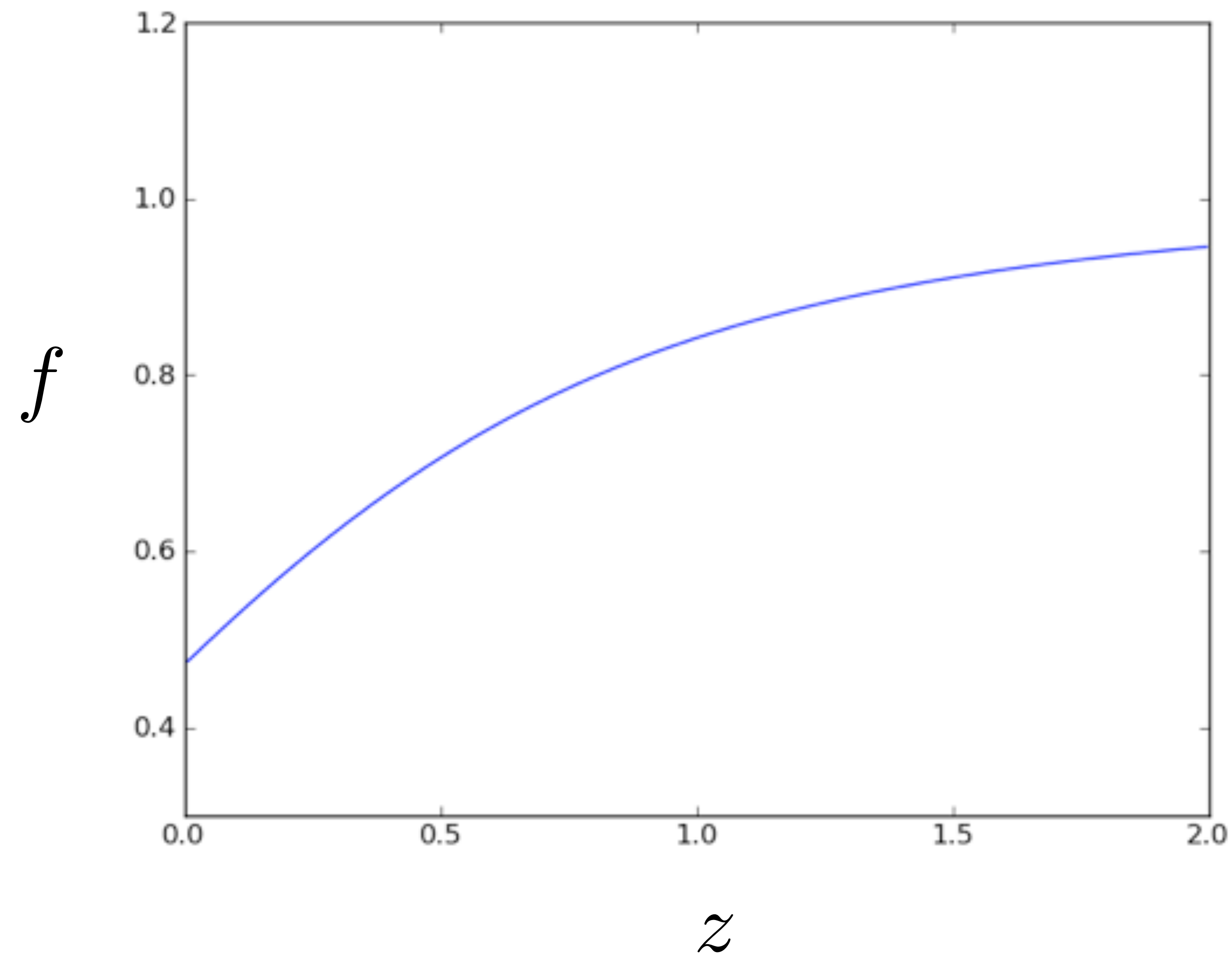
- Evolution of CDM energy density perturbations:

$$\ddot{\delta}_M + \mathcal{H}\dot{\delta}_M - 3\ddot{\Phi} - 3\mathcal{H}\dot{\Phi} + k^2\Psi = 0$$

- The growth rate of structure is quantified via f:

$$f(k,a) = \frac{d\ln\delta_M(k,a)}{d\ln a}$$

- In GR $\delta_M \propto a$ during matter domination, so $f = 1$ (independent of k for linear scales).



Example: Growth of Structure

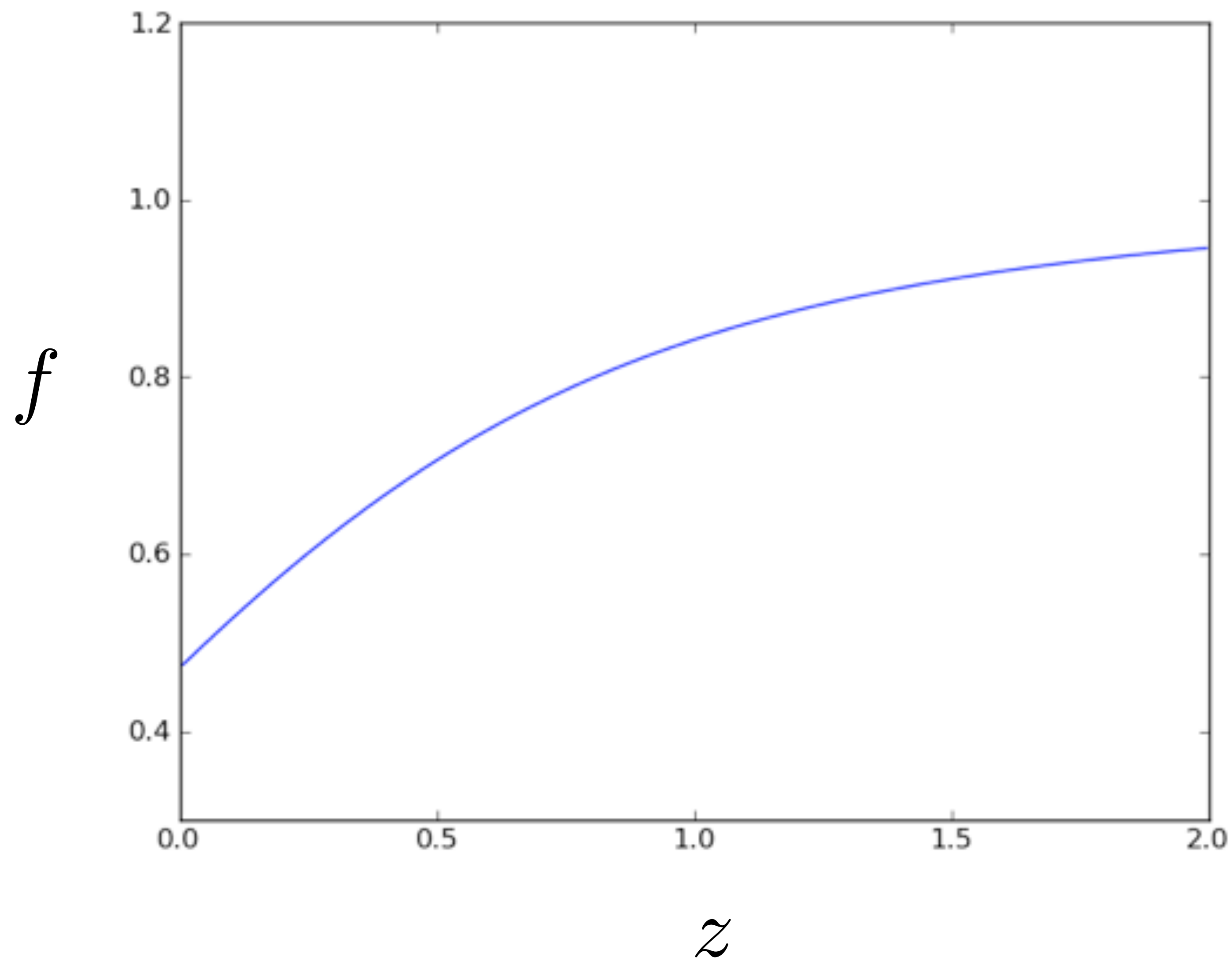
- Evolution of CDM energy density perturbations:

$$\ddot{\delta}_M + \mathcal{H}\dot{\delta}_M - 3\ddot{\Phi} - 3\mathcal{H}\dot{\Phi} + k^2\Psi = 0$$

Relation to δ_M has changed. Relation to Φ has changed.

- The growth rate of structure is quantified via *f* :

$$f(k,a) = \frac{d\ln\delta_M(k,a)}{d\ln a}$$



- In GR $\delta_M \propto a$ during matter domination, so $f = 1$ (independent of *k* for linear scales).

Growth of Structure

f satisfies a simple ODE

$$\frac{df}{d\ln a} + qf + f^2 = \frac{3}{2}\Omega_M \xi$$

with $q = \frac{1}{2}[1 - 3w(1 - \Omega_M)]$ and $\xi = \frac{\mu}{\gamma}$

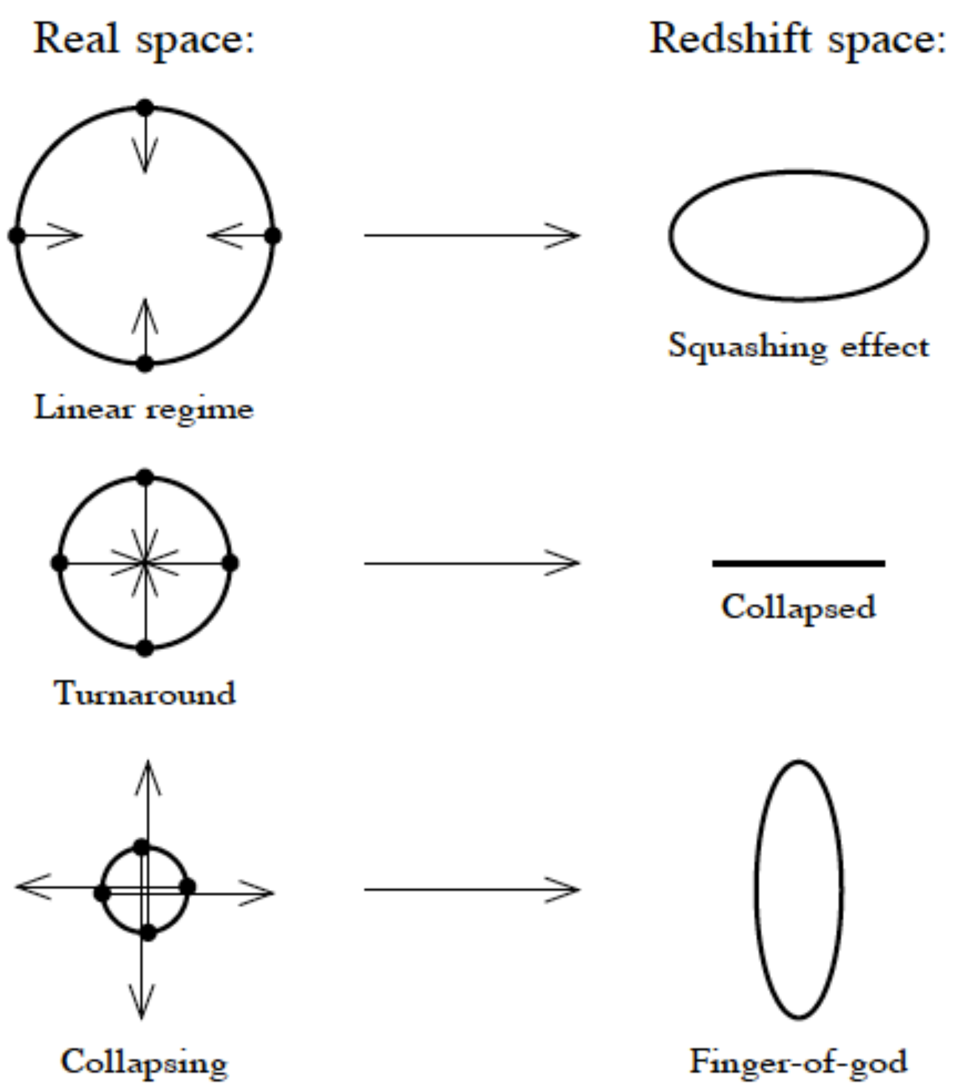
$\xi = 1$ for GR

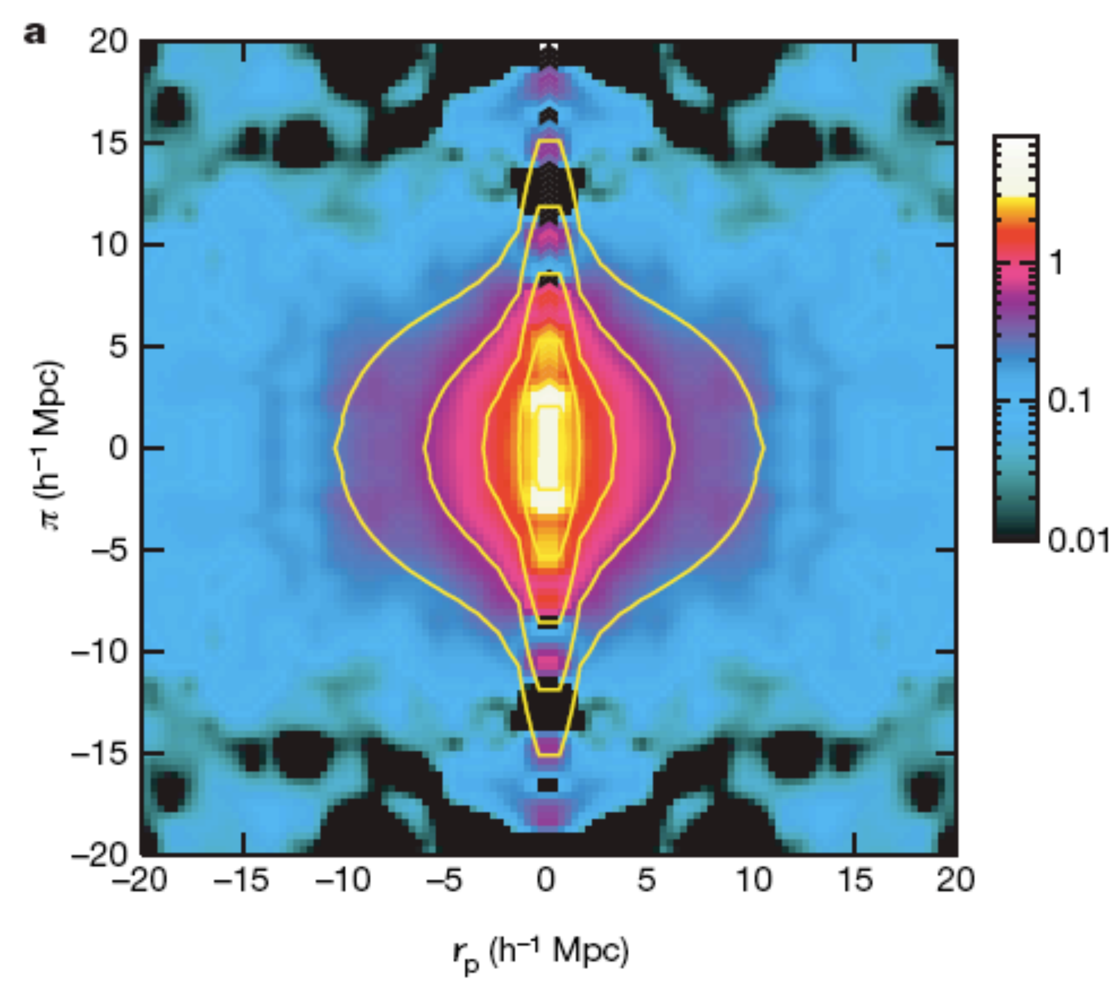
$\xi \neq 1$ not GR

Scalar Tensor $\xi \sim 1 + \frac{1}{\omega\phi}$

See Tessa Baker
(talking in Session B at 2:30 PM)

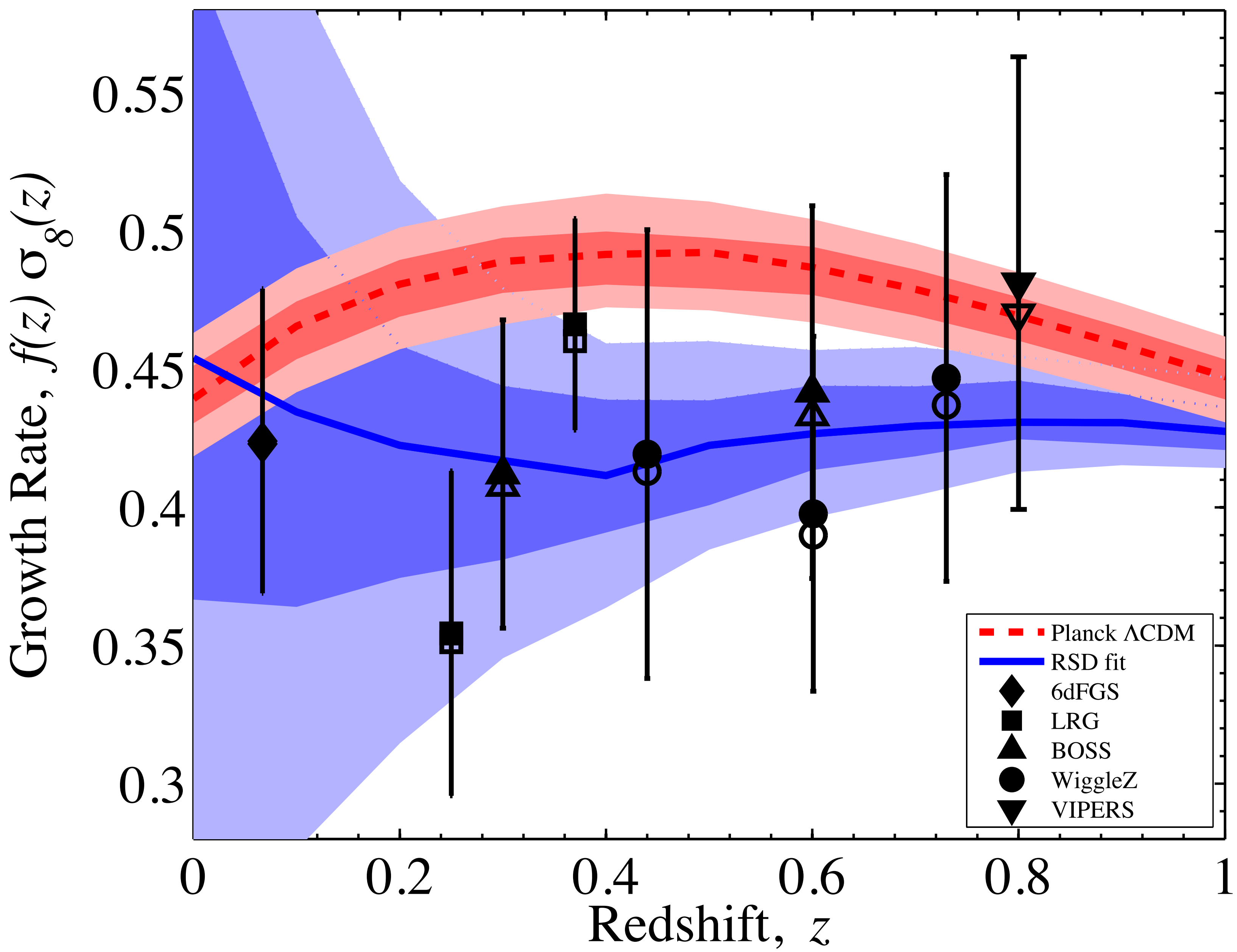
Growth of structure: Redshift Space Distortions





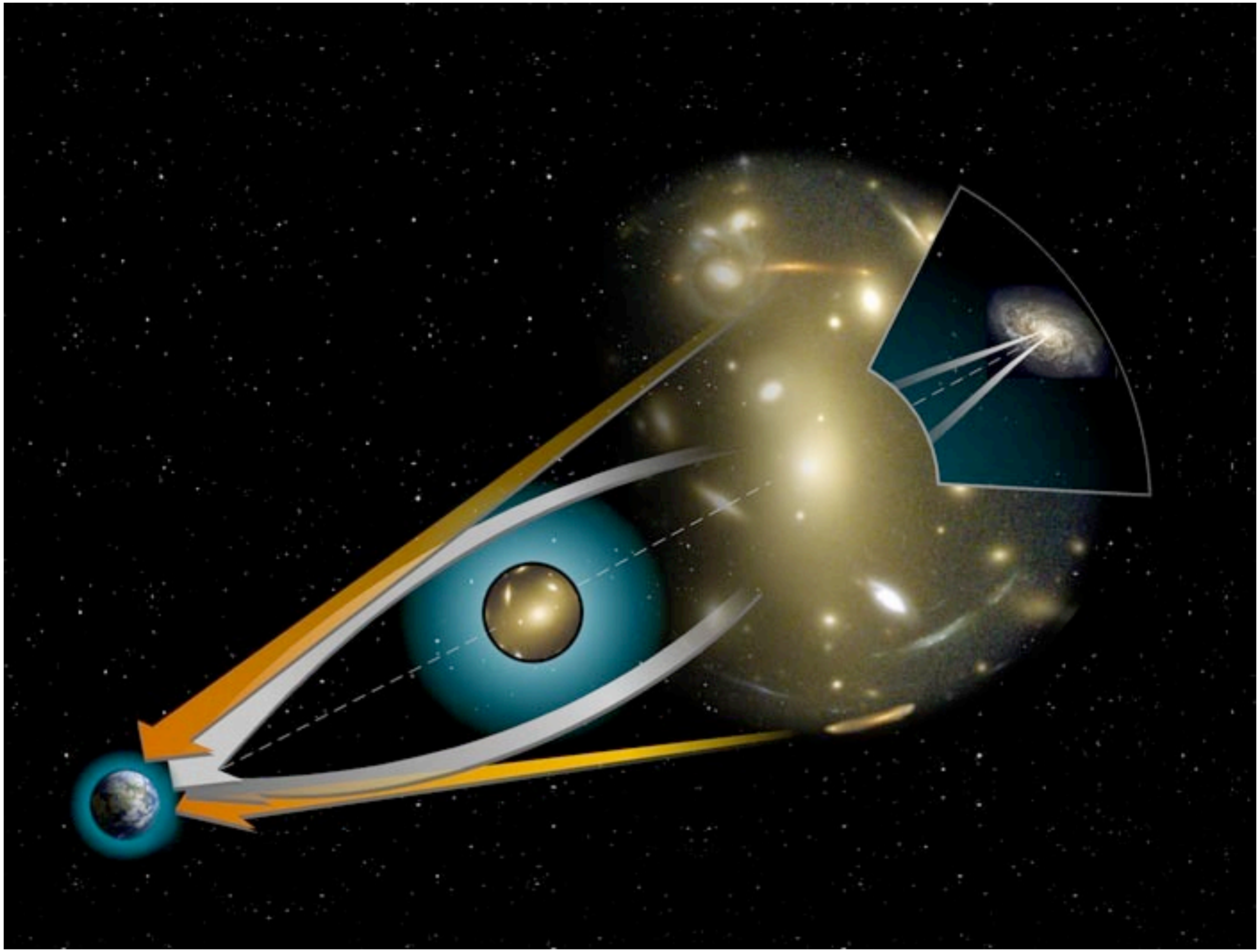
Guzzo et al 2008

Growth of structure: Redshift Space Distortions



Macaulay et al
ArXiv:1303:6583

Weak Lensing



Weak Lensing

Convergence
Power Spectrum

$$P_\kappa^{GR}(\ell) = \frac{\ell^4}{4}\int_0^{\chi_\infty} d\chi\, \frac{g(\chi)^2}{\chi^6}\, P_{\Phi_{GR}}\left(\frac{\ell}{\chi}\right)$$

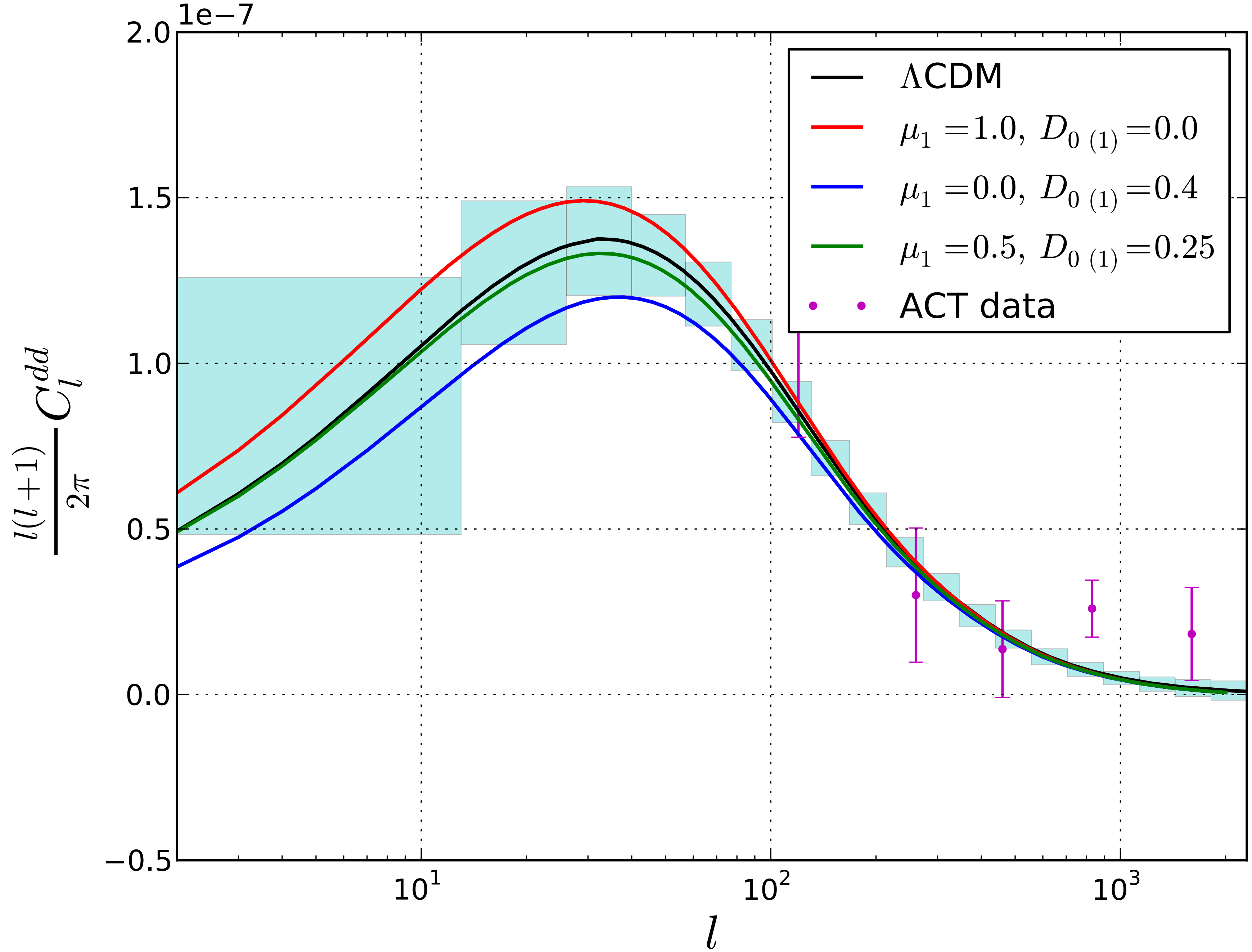
$$g(\chi) = 2\chi\int_\chi^{\chi_\infty} d\chi' \left(1-\frac{\chi}{\chi'}\right) W\left(\chi'\right)$$

Source distribution

$$\begin{aligned} P_{\Phi_{GR}}(k) &= \langle|\Phi_{GR}(k)|^2\rangle \\ P_{\Phi_{MG}}(k) &= \frac{1}{2}\left(2+\frac{1}{\gamma}\right)\langle|\Phi_{MG}(k)|^2\rangle \\ \nabla^2\Phi_{MG} &= 4\pi G\mu a^2\rho\Delta_{MG} \end{aligned}$$

Multiple dependencies on (μ, γ)

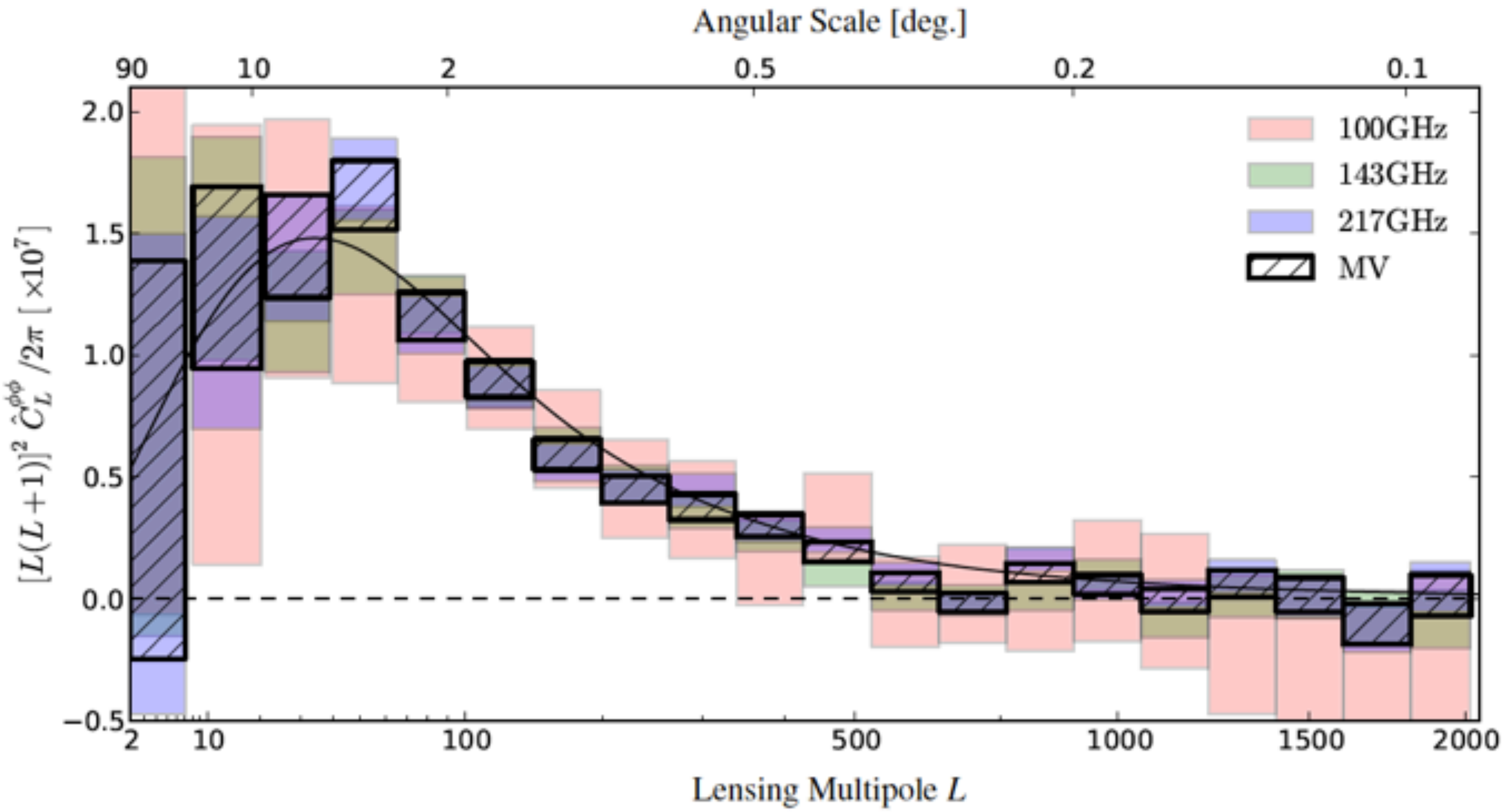
CMB Lensing



Power spectrum of the CMB lensing potential

Planck forecast errors

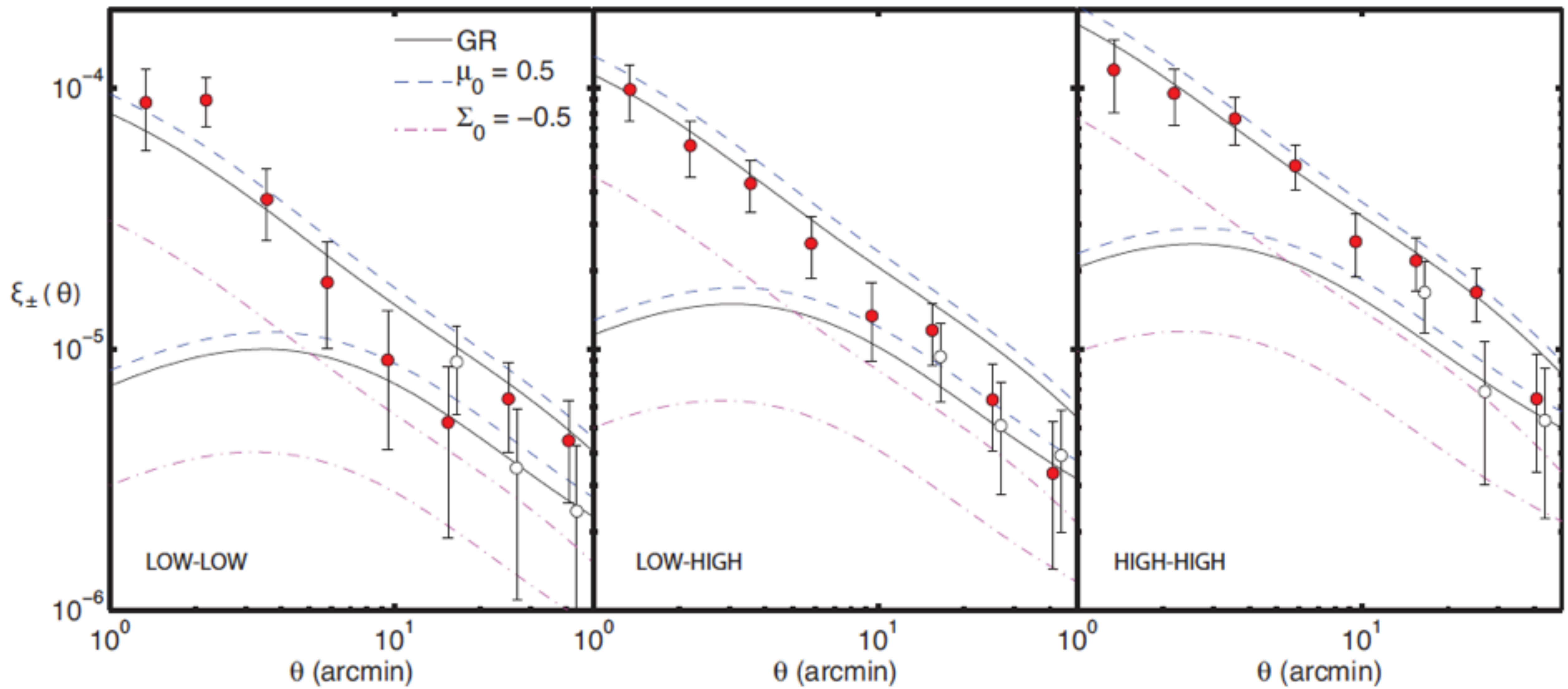
Weak Lensing of the CMB



$$\propto \int (\Phi + \Psi) d\eta$$

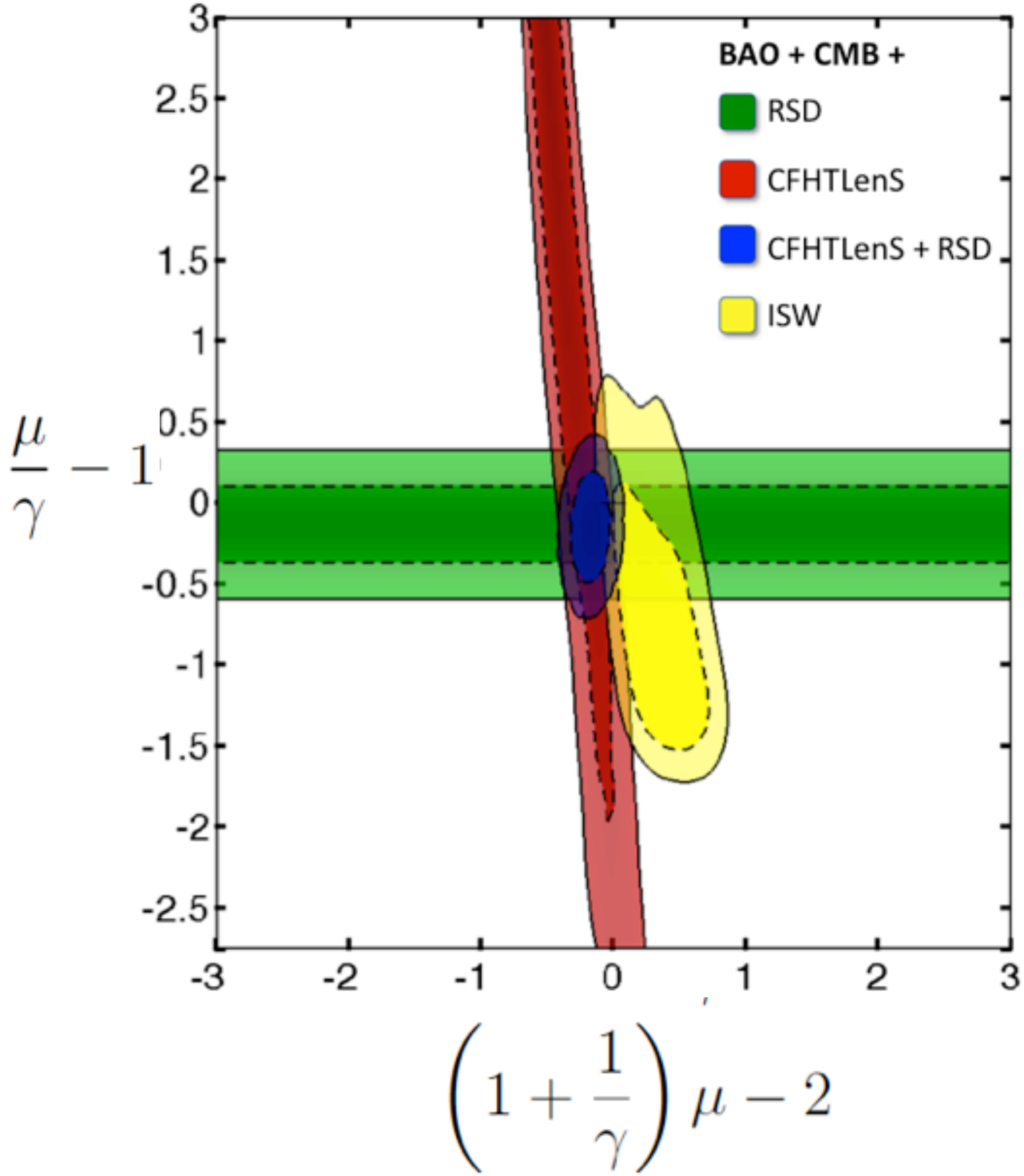
Planck 2013

Galaxy Weak Lensing



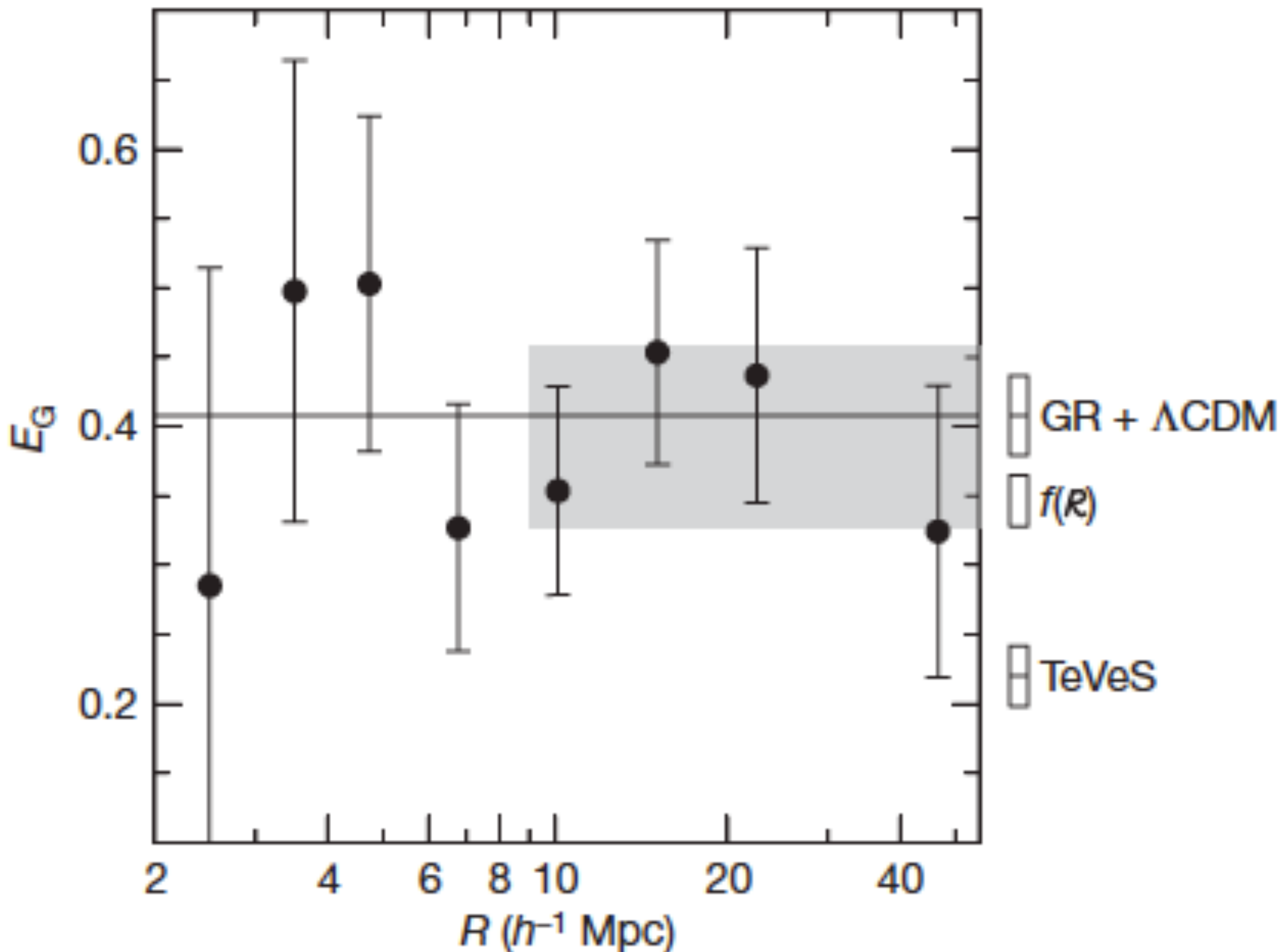
Simpson et al 2012
(CFHTLens)

Galaxy Weak Lensing



Simpson et al 2012
(CFHTLens)

Cross correlating data sets



Reyes et al 2010

The Future

EUCLID

MAPPING THE GEOMETRY OF THE DARK UNIVERSE

The Square Kilometre Array

The International Radio Telescope for the 21st Century

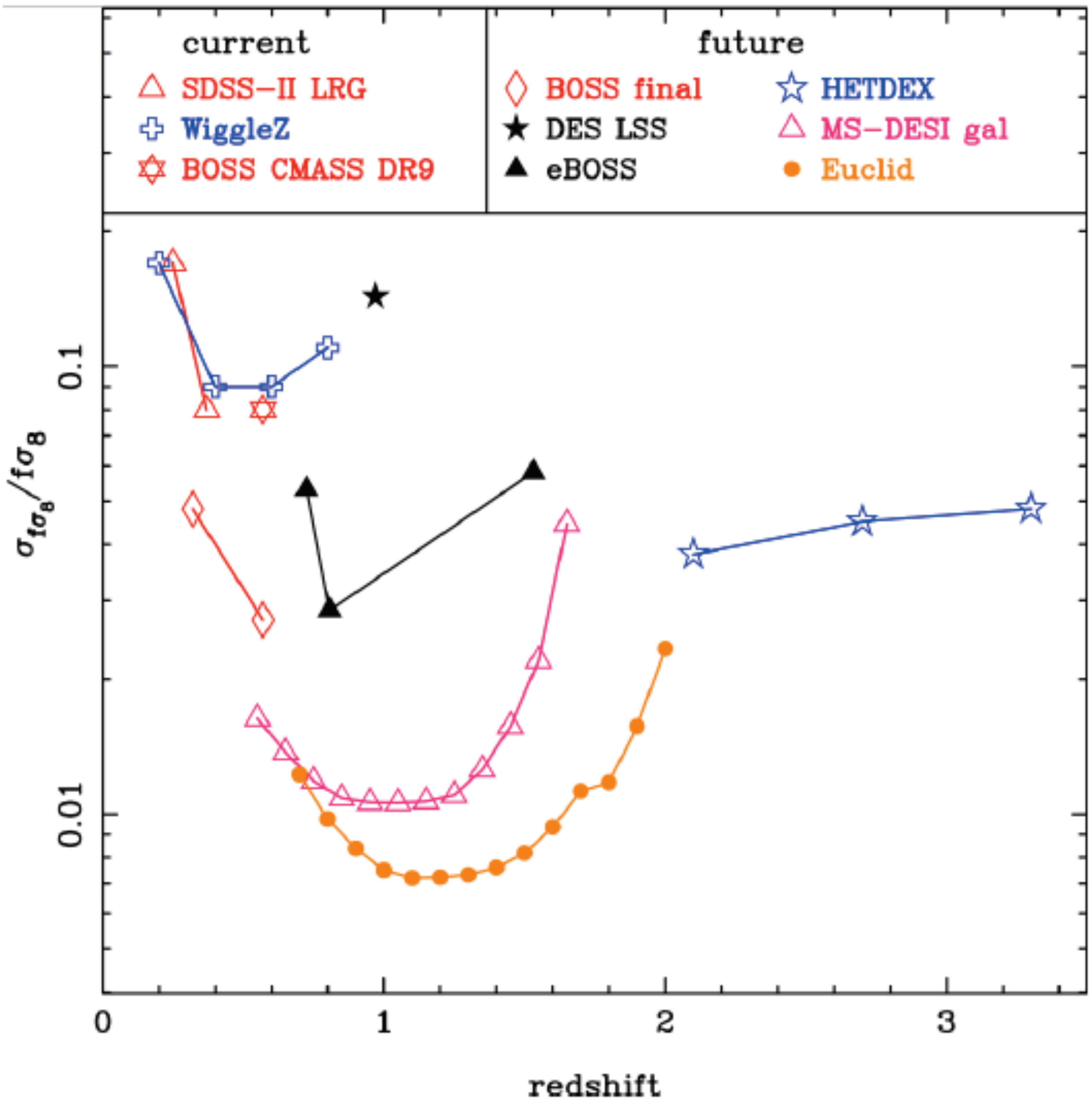
LSST

Large Synoptic Survey Telescope

The Future is now

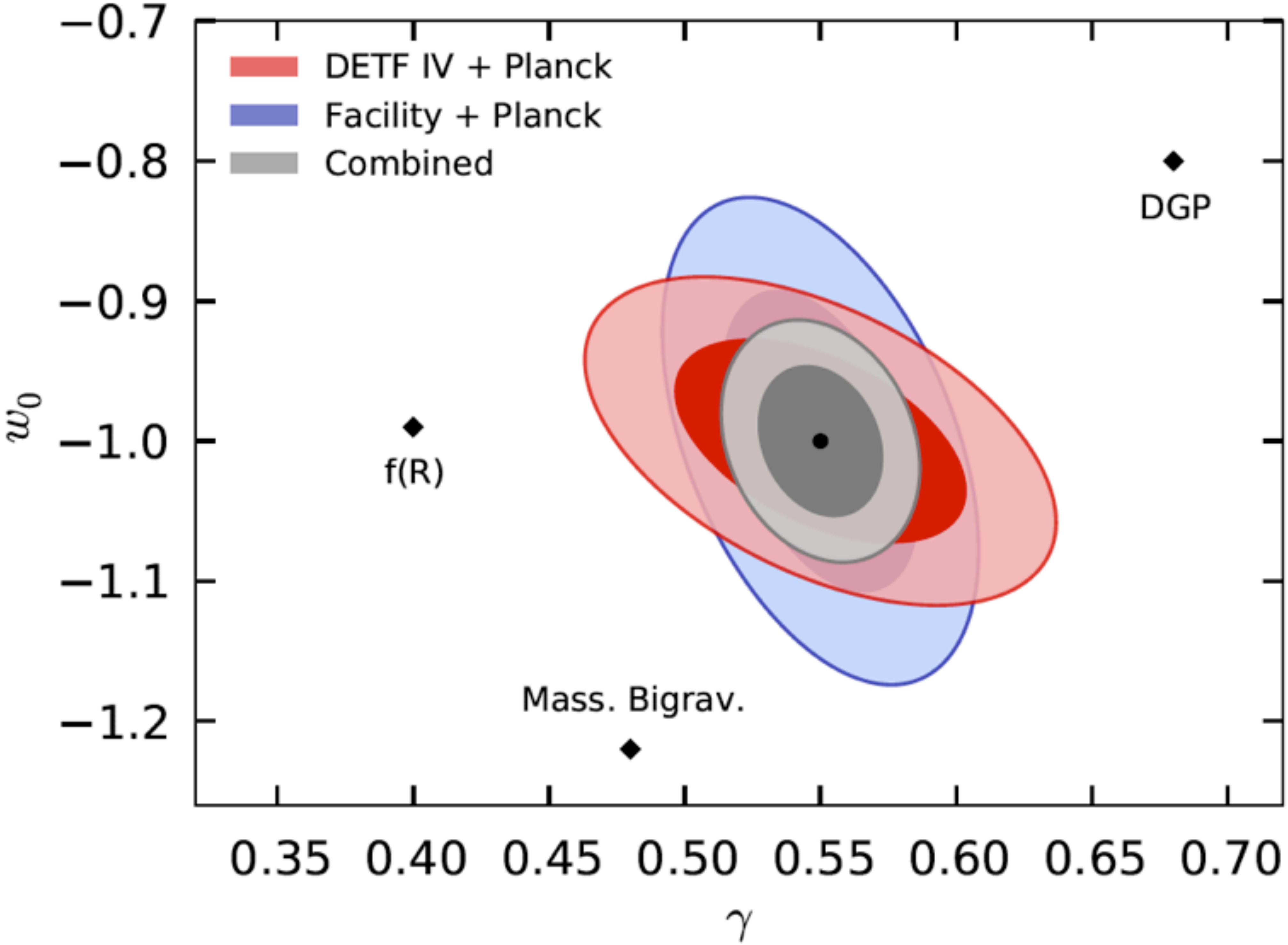
Data Type	Now	Soon	Future
Photo-z:LSS (weak lensing)	DES, RCS, KIDS	HSC	LSST, Euclid, SKA
Spectro-z (BAO, RSD, ...)	BOSS	MS-DESI,PFS,HETDEX, Weave	Euclid, SKA
SN Ia	HST, Pan-STARRS, SCP, SDSS, SNLS	DES, J-PAS	JWST,LSST
CMB/ISW	WMAP	Planck	
sub-mm, small scale lensing, SZ	ACT, SPT	ACTPol,SPTPol, Planck, Spider, Vista	CCAT, SKA
X-Ray clusters	ROSAT, XMM, Chandra	XMM, XCS, eRosita	
HI Tomography	GBT	Meerkat, Baobab, Chime, Kat 7	SKA

The Future



Percival 2013

BAOs + RSDs



$$f = \Omega_M^\gamma$$

What does this mean?

Example: Jordan-Brans-Dicke Theory

$$S = \int \sqrt{-g} d^4x \left[\phi R - \frac{\omega}{\phi} (\nabla \phi)^2 \right]$$

Cosmology

Now: $\frac{1}{\omega} < 6 \times 10^{-3}$ Avillez & Skordis 2013

Euclid: $\frac{1}{\omega} < 3 \times 10^{-4}$ (RSDs only) Baker, Ferreira & Skordis, 2013

Solar System

Now: $\frac{1}{\omega} < 1 \times 10^{-4}$ Cassini

Summary

- The large scale structure of the Universe can be used to test gravity (different eras probe different scales).
- There is an immense landscape of gravitational theories (how credible or natural is open for debate).
- We need a unified framework to test gravity
- Focus on linear scales at late times (for now).
- Non-linear scales can be incredibly powerful but much more complicated
- Need new methods and observations to access the really large scales (is HI tomography the future?).
- There are a plethora of new experiments to look forward to.