Galaxies and Clusters, 2

The Large scale Structure of the Universe

Alain Blanchard

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$$V = \sum_i dV_i$$

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$$\begin{aligned} < N^2 > &= \sum_{i,j} < n_i n_j > \\ &= \sum_i < n_i^2 > + \sum_{i \neq j} < n_i n_j > \\ &= \overline{n} V + \int \overline{n}^2 dV_1 dV_2 (1 + \xi(r_{12})) \end{aligned}$$

Count in Cell III

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In short: $\sigma_V^2 \approx \overline{\xi}$

Normalization and Bias

Amplitude of fluctuations are usually referred for a sphere of $8h^{-1}$Mpc:

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one might have more complicated relation between galaxies and DM, and the bias can be a function of scale:

$$b(r)$$

Angular correlation function

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from $\xi(r)$ one can get $w(\theta)$

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$$w(\theta) = \frac{1}{D_*} \frac{\int_0^{+\infty} dy y^4 \psi(y)^2 \int_{-\infty}^{+\infty} du \xi((u^2 + D_*^2 y^2 \theta^2)^{1/2})}{\left(\int_0^{+\infty} y^2 \psi(y) dy\right)^2}$$

(Peebles, 1980, LSS of the Universe)

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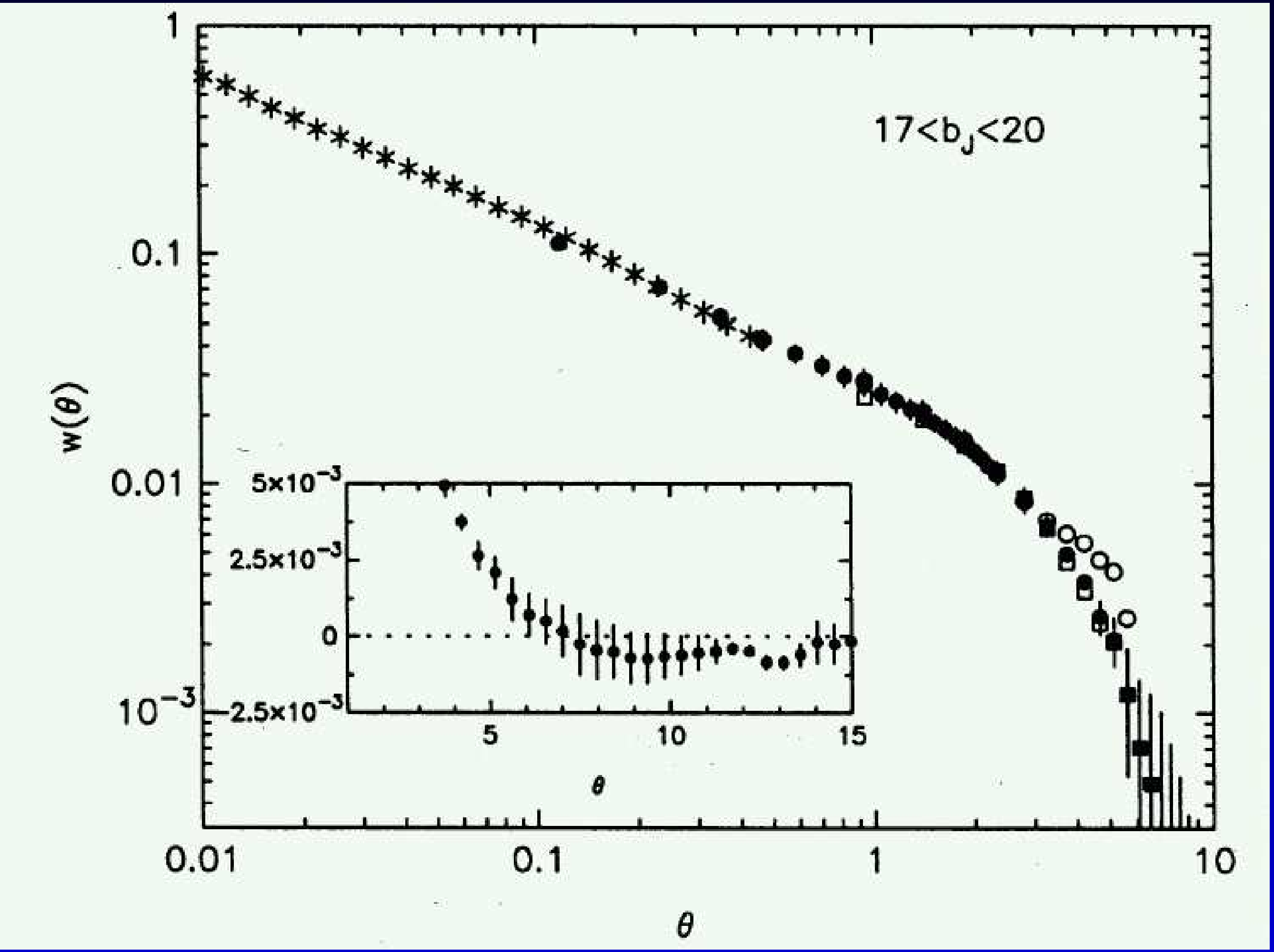
(Peebles, 1980, LSS of the Universe) So finally:

$$w(\theta) = \frac{1}{D_*} W(D_*\theta)$$

Observations



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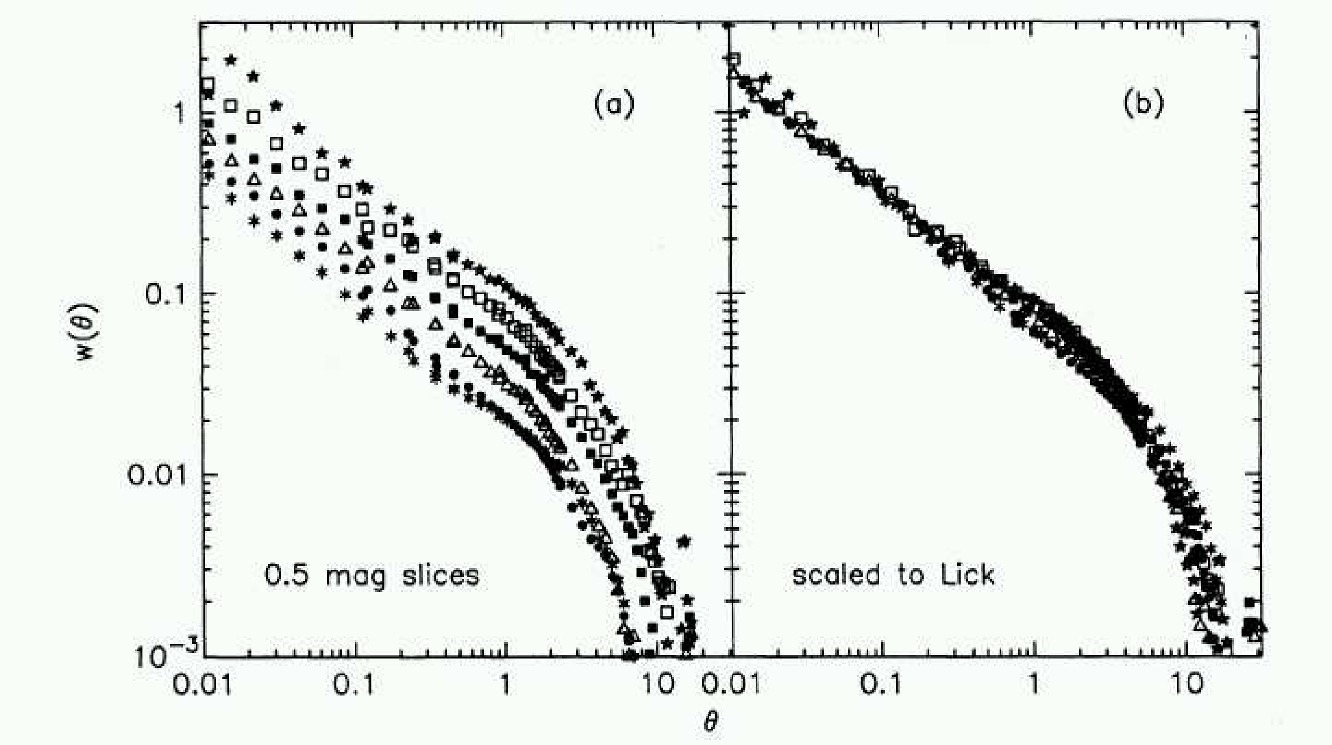

Observed properties

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Three point correlation function:

$$\xi^{(3)}(r_a, r_b, r_c) = Q(\xi(r_a)\xi(r_b)+\xi(r_b)\xi(r_c)+\xi(r_c)\xi(r_a))$$

avec :

$$Q \sim 1.27$$

Dependence

Ex: luminosity:

$$dP = \phi(M) dM dV$$

and:

$$dP_{12}(r) = (\phi(M_1)\phi(M_2) + \Gamma(M_1, M_2, r)) dM_1 dV_1 dM_2 dV_2$$

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By definition:

$$dP(r) = dN(r) = \overline{n} dV (1 + \xi(r))$$

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$$\xi(r) = \frac{\sum_i dN_i(r)}{\overline{n} \sum_i dV_i} - 1. = \frac{N_{dd}(r)}{\overline{n} \sum_i dV_i} - 1.$$

$N_{dd}(r)$ is the number of pairs of galaxies with separation between r and $r + dr$.

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(n_p being the density of random particules within the survey limits)

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(n_p being the density of random particules within the survey limits)so:

$$\sum dV_i = \frac{1}{n_p} N_{dr}(r)$$

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pair conservation implies:

$$\int_{\sim V} \xi(r) dV = 0$$

so ξ is forced to become negative on some scale.

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Hamilton (1993)

$$\xi(r) = \frac{N_{dd}(r) N_{rr}(r)}{N_{dr}^2(r)} - 1.$$

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Landy and Szalay (1993):

$$\xi(r) = 1. + \left(\frac{n_p}{\overline{n}}\right)^2 \frac{N_{dd}(r)}{N_{rr}(r)} - 2. \left(\frac{N_p}{N_g}\right) \frac{N_{dr}(r)}{N_{rr}(r)}$$

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Increases the noise, improves the volume surveyed.

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Troubles with $\xi(r)$:

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Introducing r_p, π

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Introducing r_p, π

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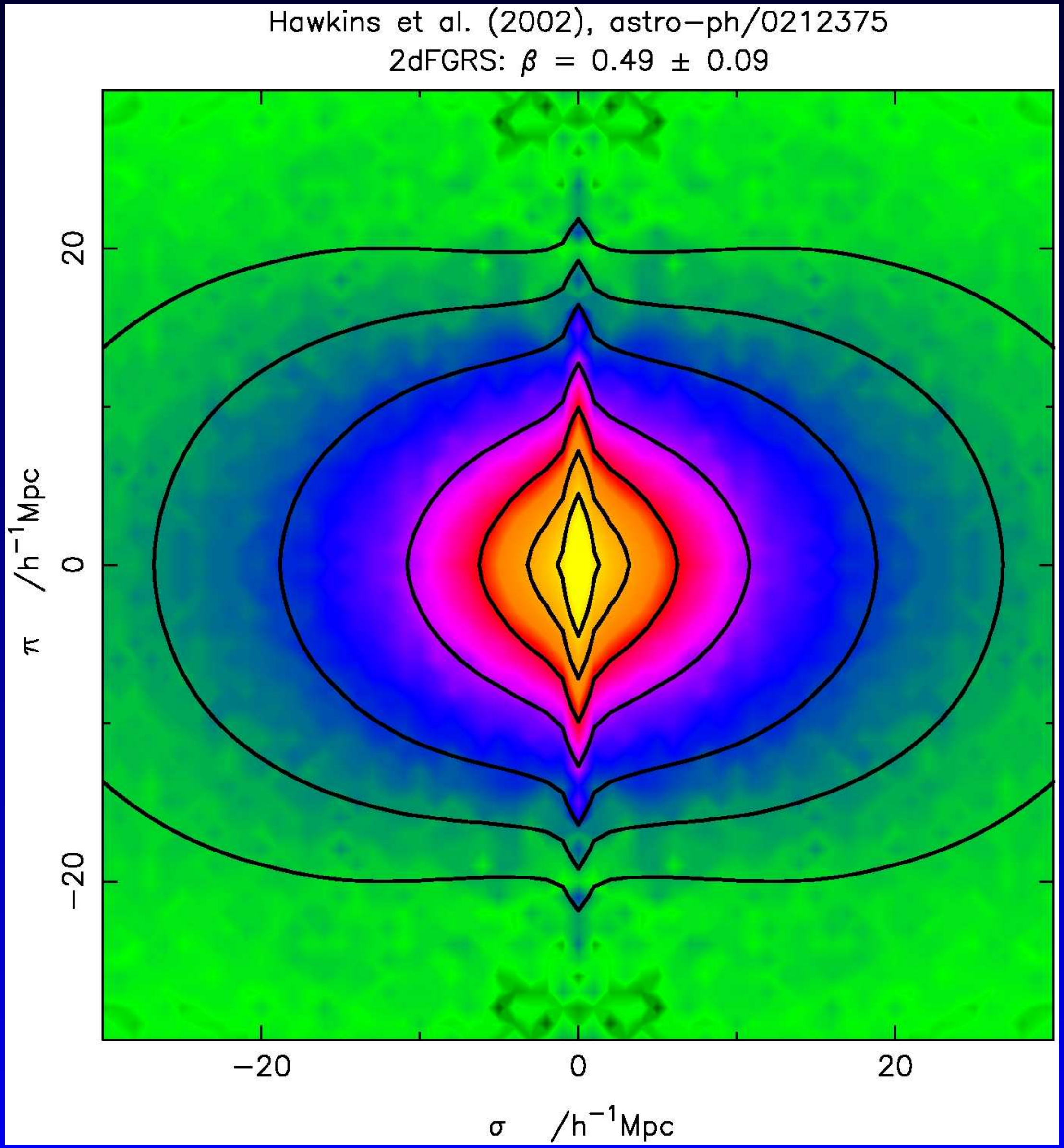
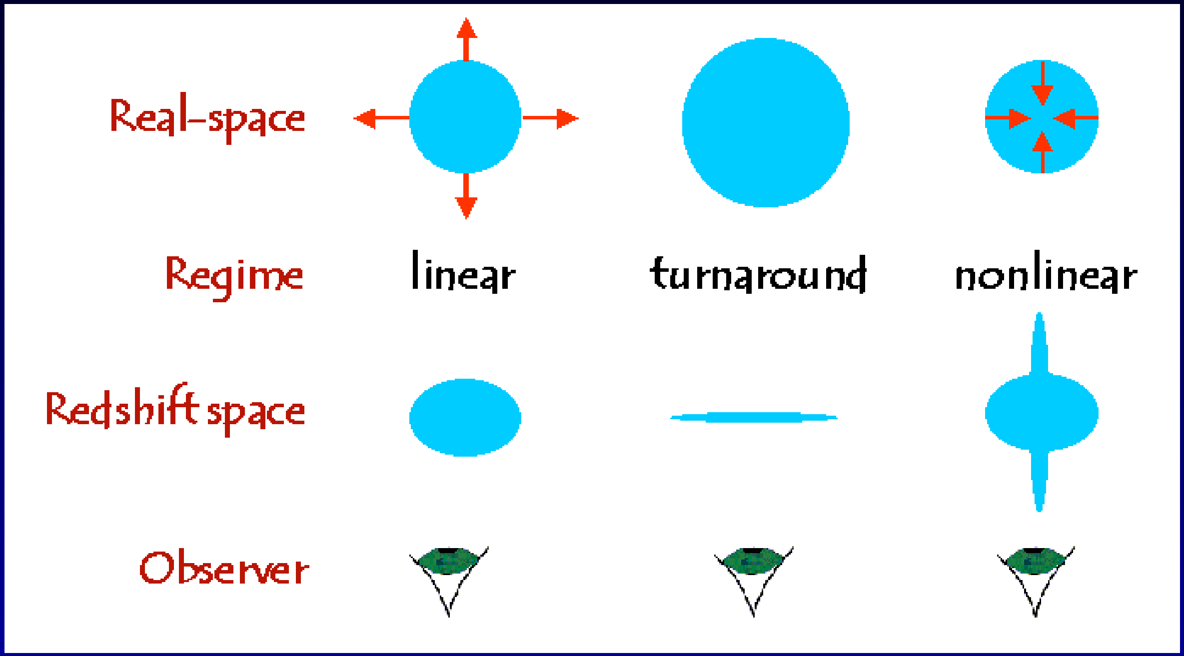
projected:

$$w_p(r_p) = 2\int_0^{+\infty} \xi(r_p, \pi) d\pi$$

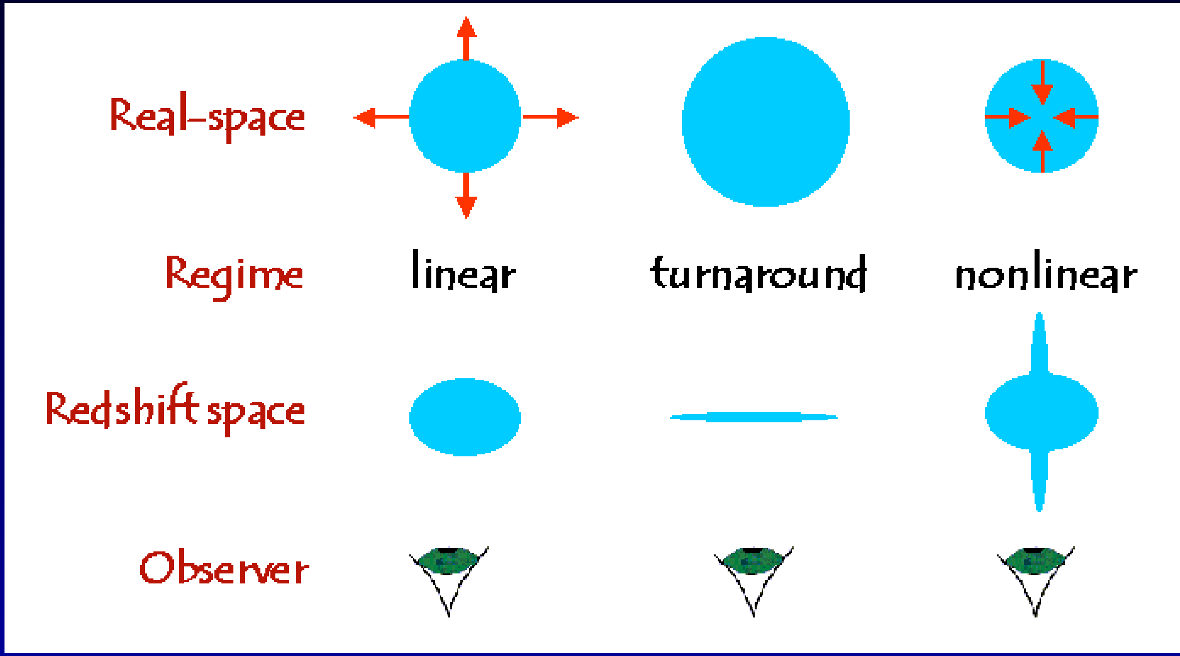
Redshift Space Distorsion: 2dF



Origin

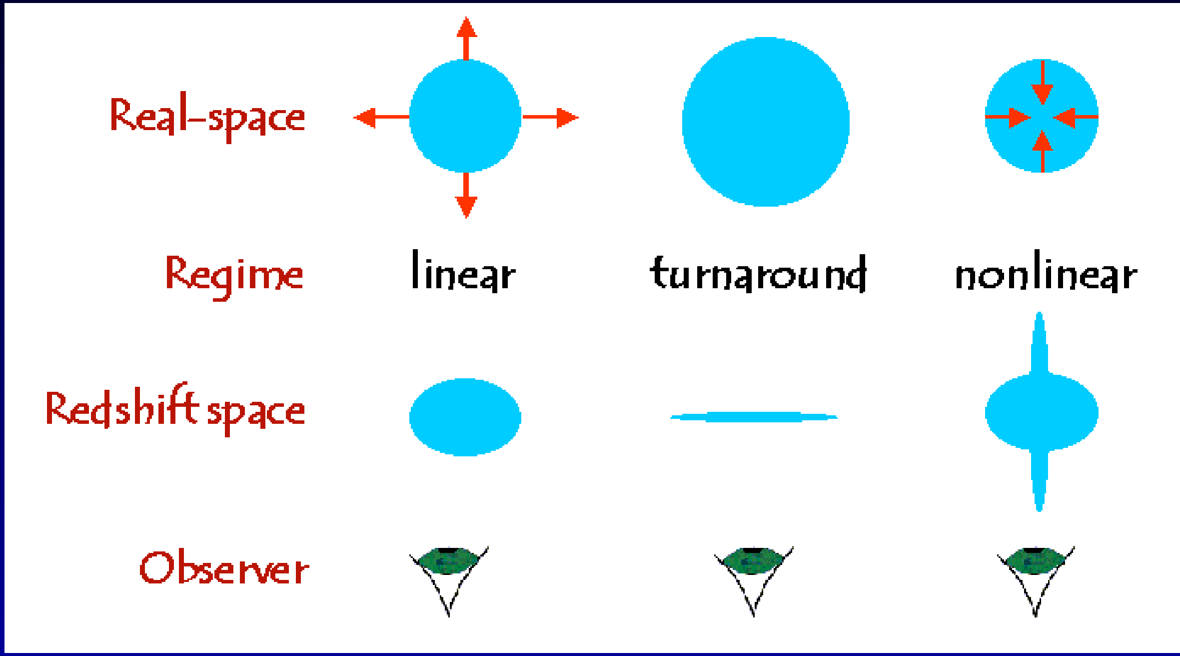

Real-space
Regime
linear
turnaround
nonlinear
Redshift space
Observer

Origin



Pairwise velocity on small scales

Origin



Pairwise velocity on small scales
Redshift distorsion proportional to

$$\beta = f\delta\rho/\rho$$

with

$$f = \frac{d\ln D}{d\ln a}$$

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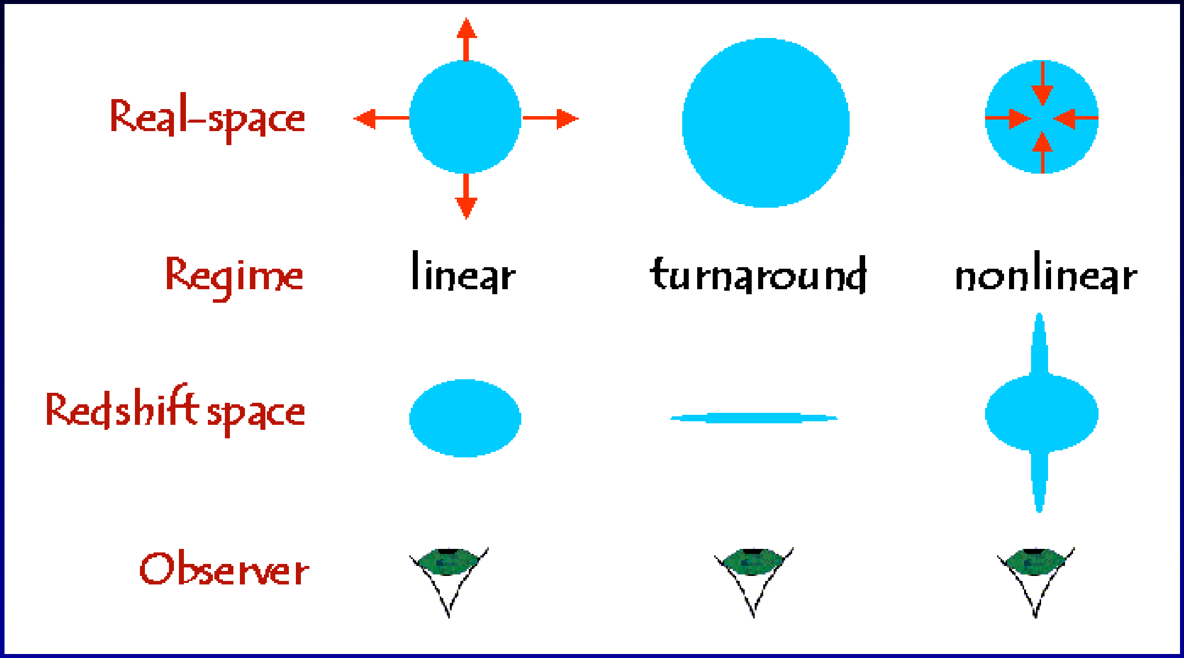

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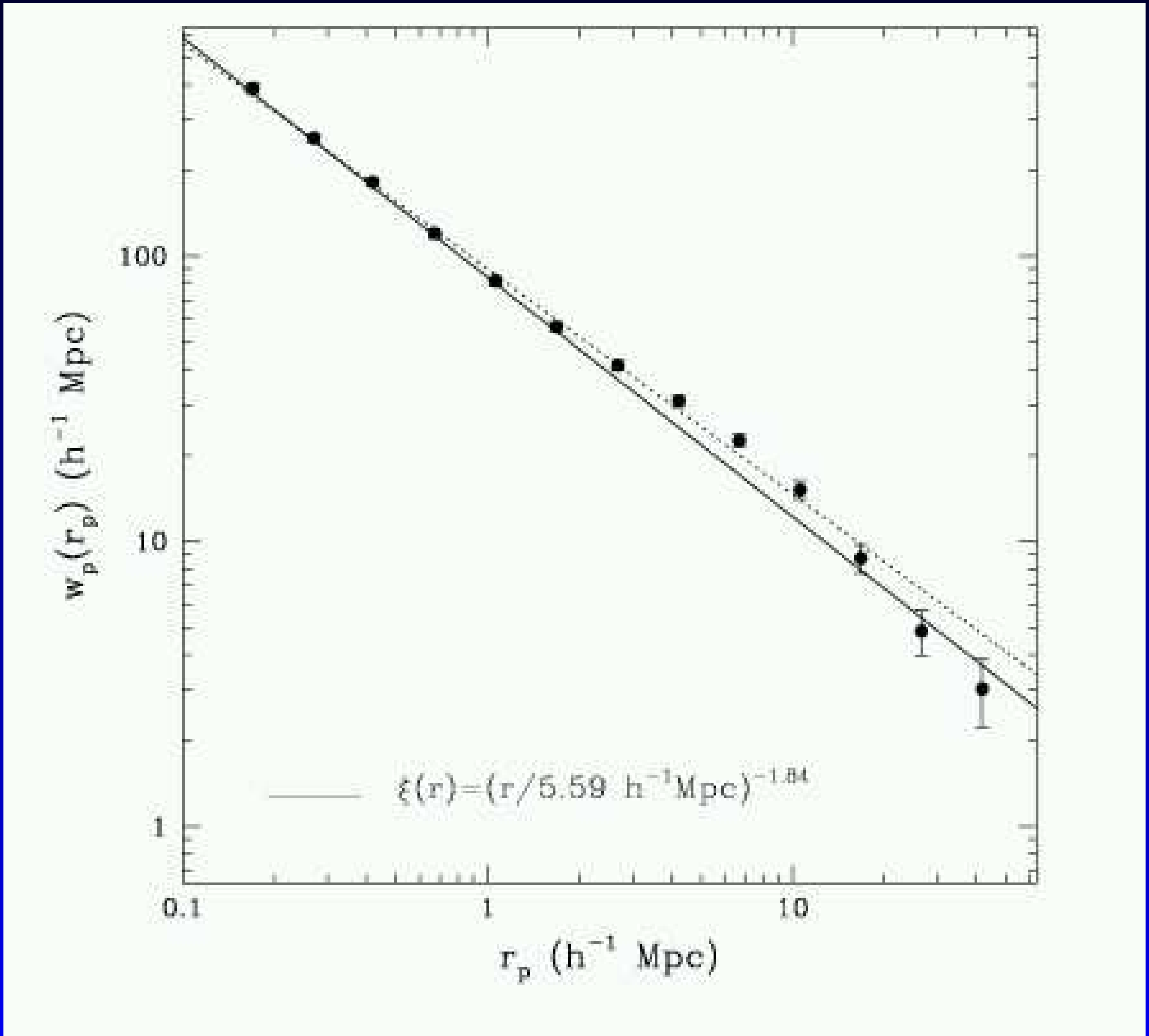
Observations: SDSS



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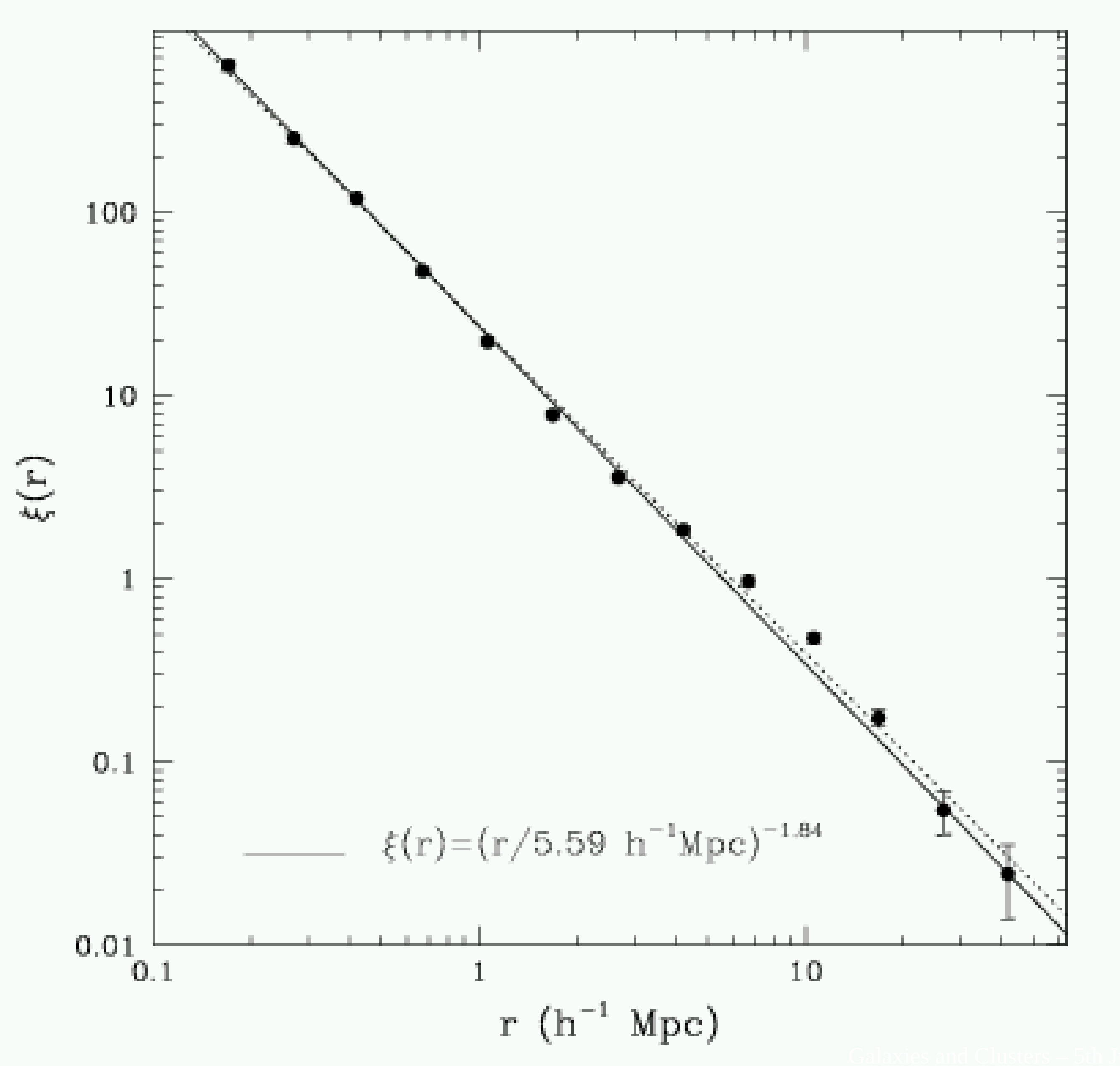


Observed properties

SDSS, all

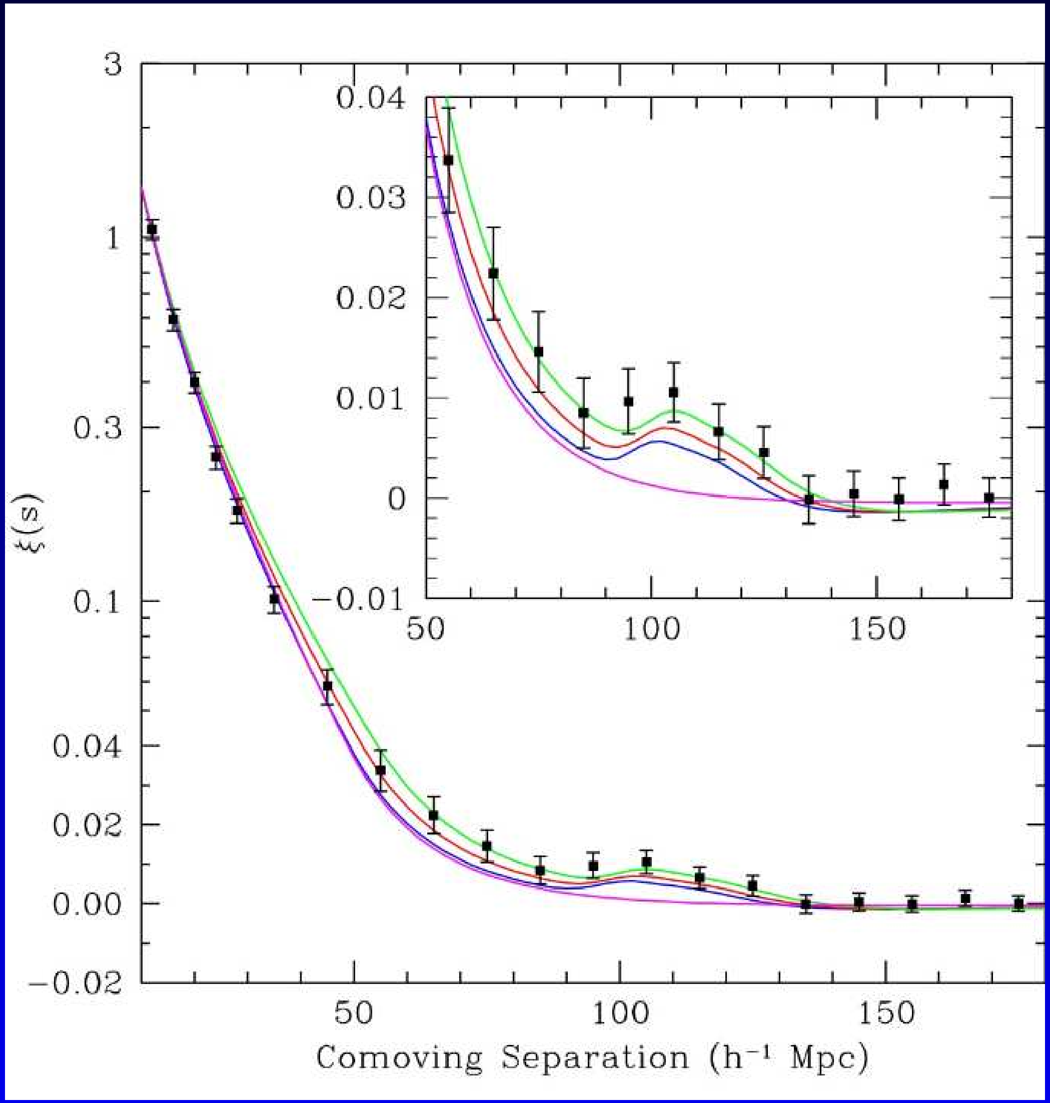
For powerlaw $\xi(r) \propto (r/r_0)^{-\gamma}$:

$$\gamma \sim 1.84 \text{ and } r_0 \sim 5.59h^{-1}\text{Mpc}$$

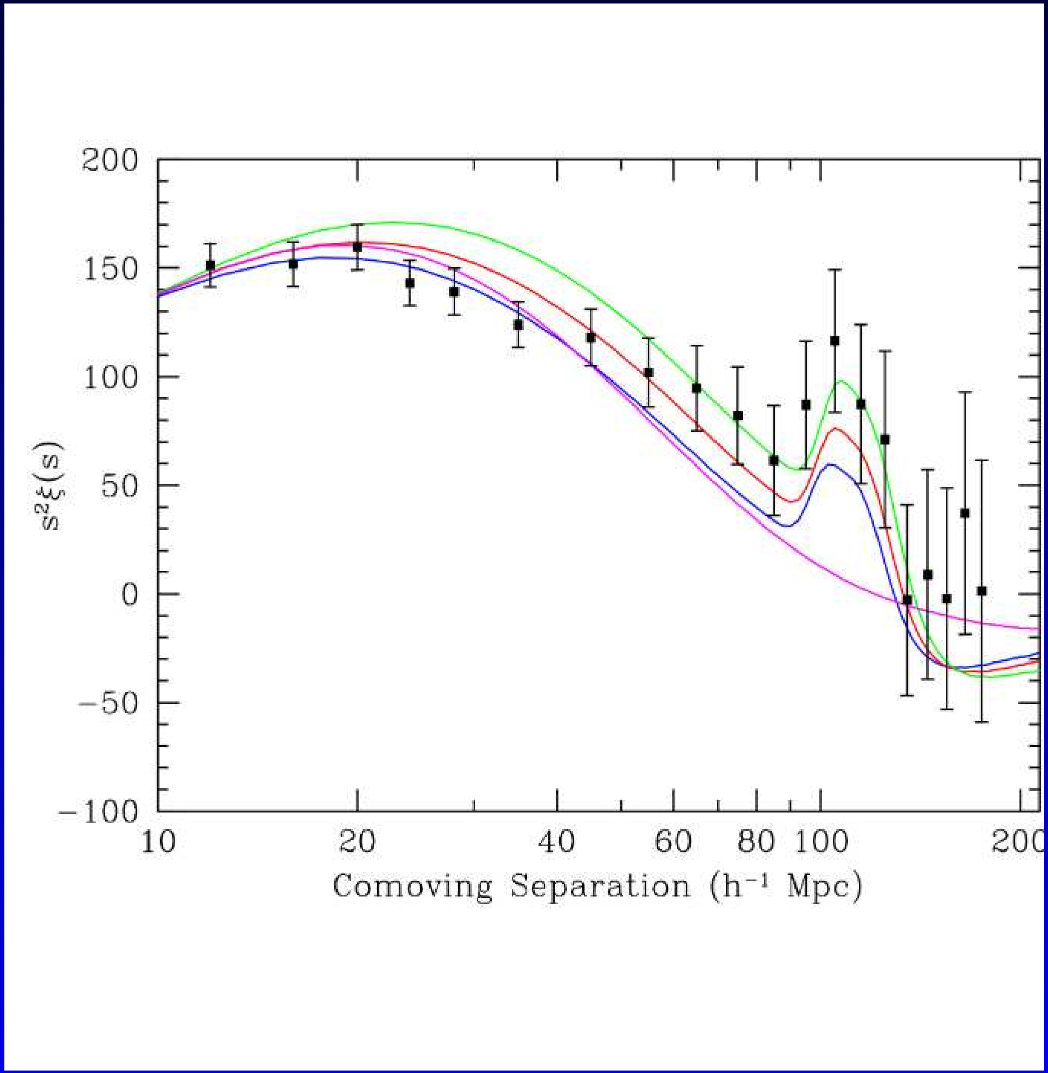
SDSS, LRG

$$r_0 \sim 10.h^{-1}\text{Mpc}$$

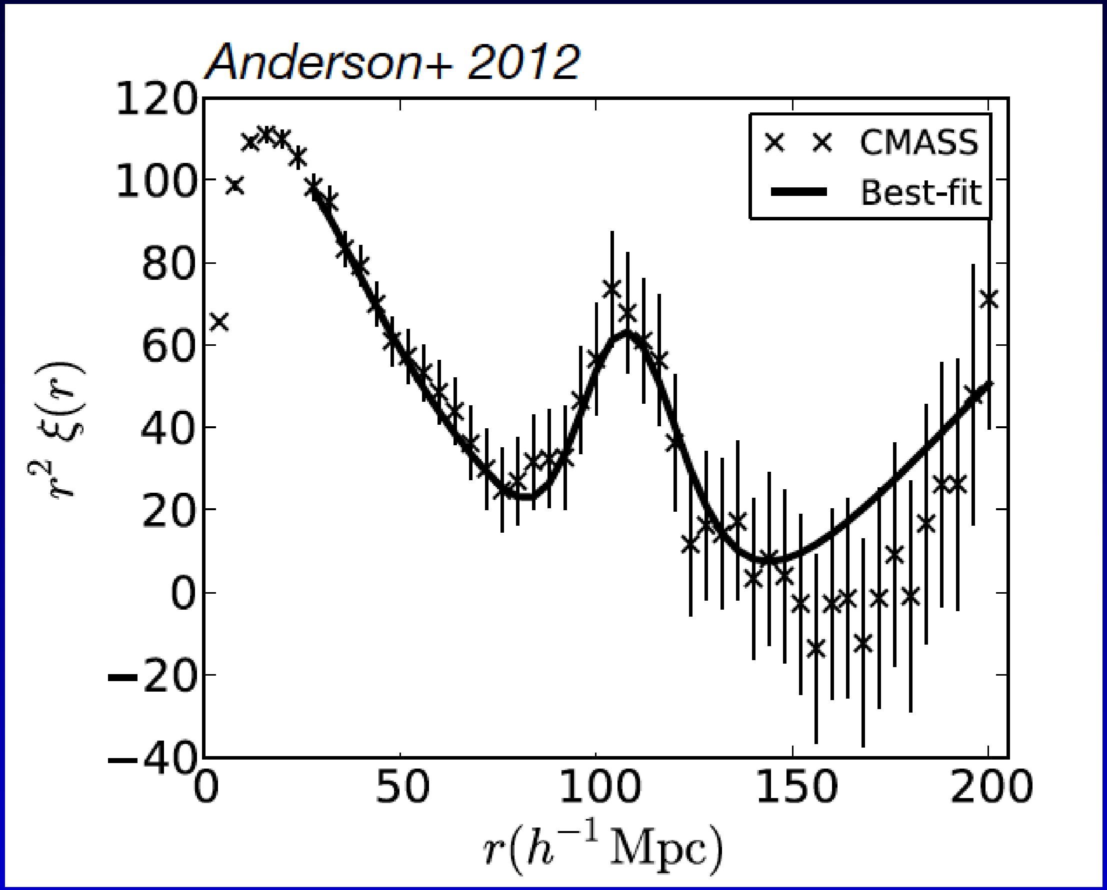
Observations SDSS, LRG



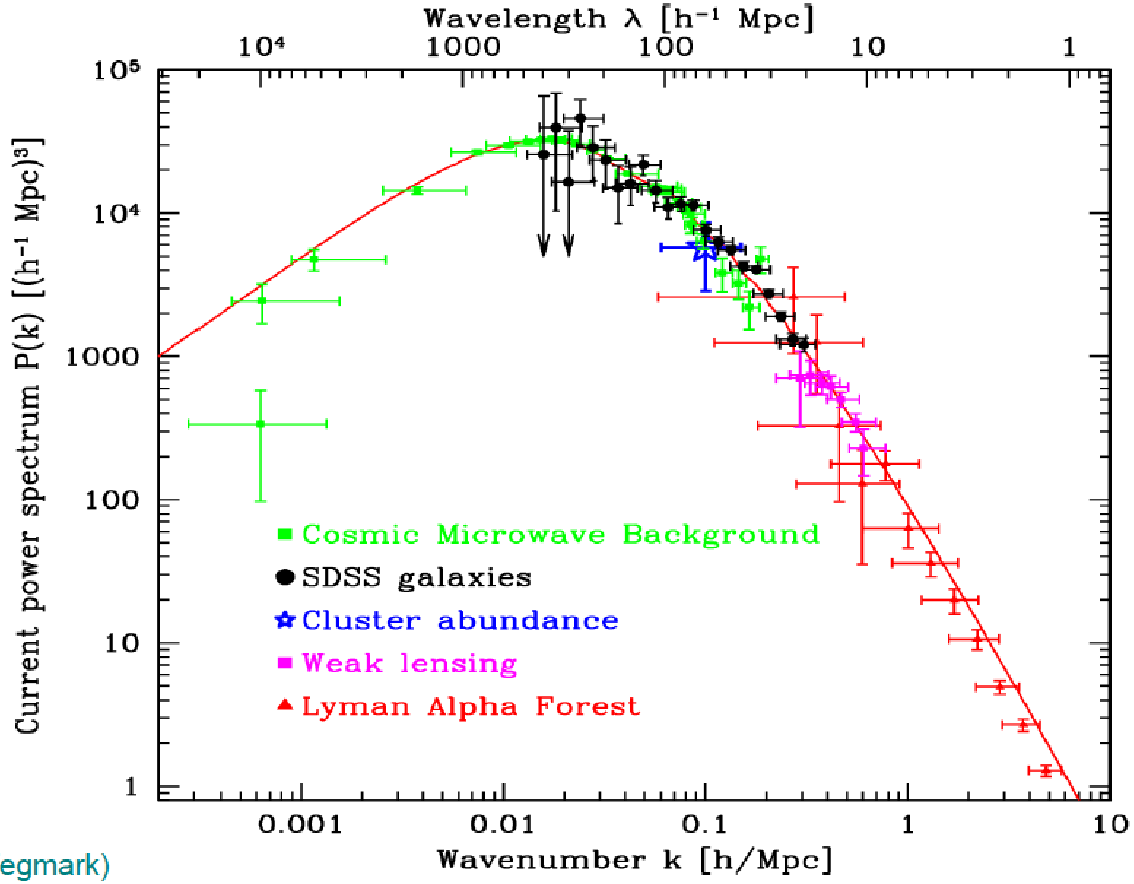
Observations SDSS, LRG



Observations SDSS: Boss II



Observations: the Power Spectrum



(figure from Max Tegmark)

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That is:

$$\frac{G\delta M}{R} \propto \frac{R^3\sqrt{\xi(R)}}{R} \propto \frac{R^{3-\gamma/2}}{R} = R^{2-\gamma/2}$$

not really probe by galaxy surveys...