Galaxies and Clusters

The Large scale Structure of the Universe

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Large Scale Structure

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Structure of the universe on large scale:

Large Scale Structure

Structure of the universe on large scale:

Newton...

Large Scale Structure

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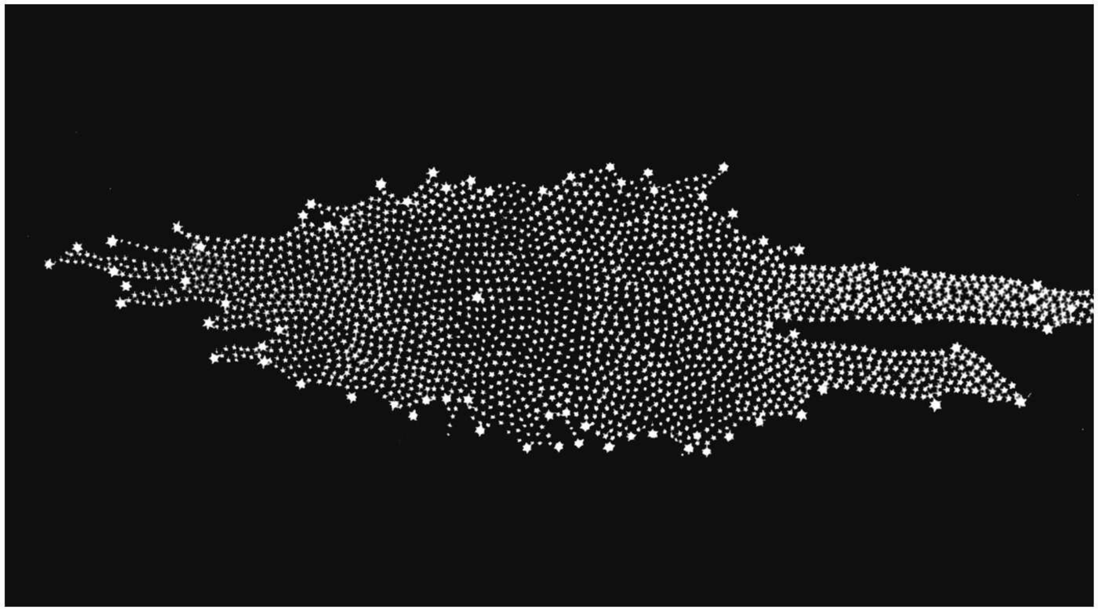
XIXth century



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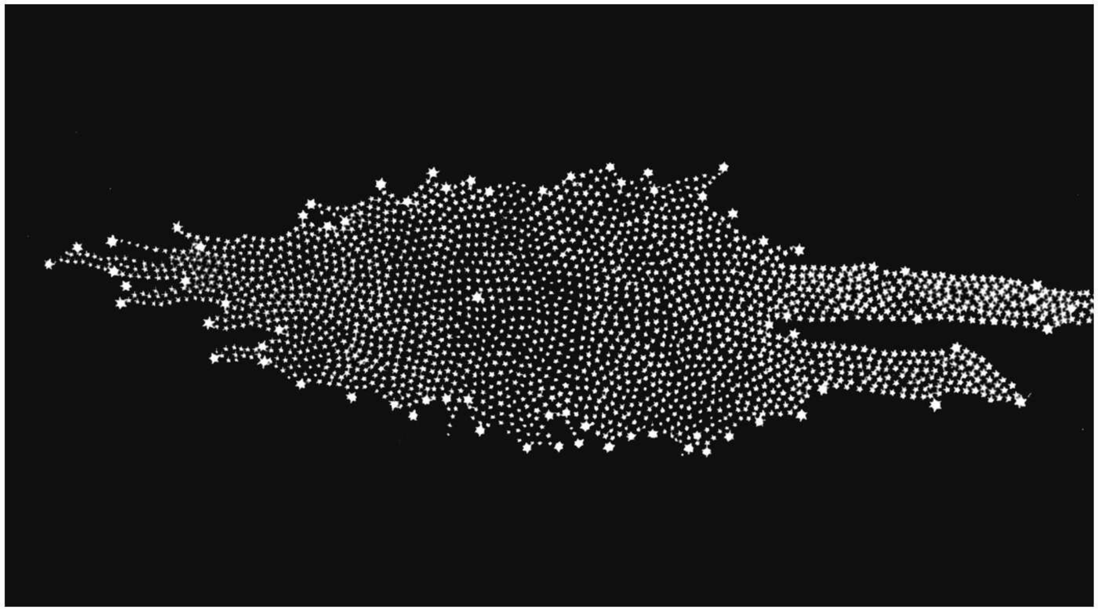

Einstein

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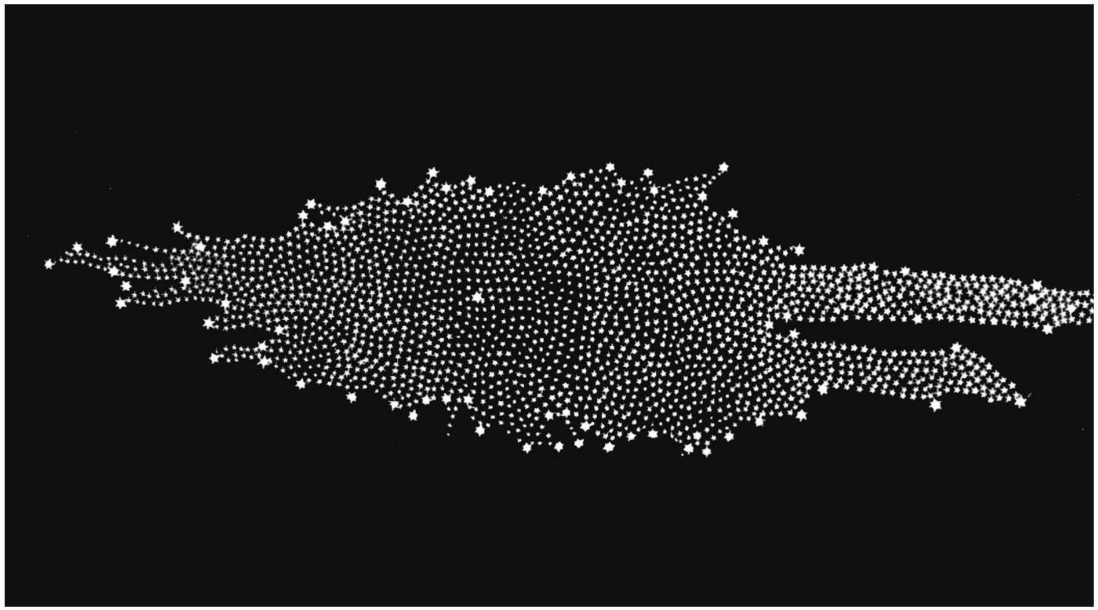

Einstein Cosmological Principle

Great debate

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Nature of Nebulae

Shapley/Curtis

Great debate

Nature of Nebulae

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Solved by Hubble

Problematic

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Large scale structure of the Universe

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- Description of large scale structure of galaxies

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- Description of large scale structure of galaxies
- Infer largescale structure of the contents (baryons, DM, DE,...)
- and their evolution.

Problematic

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Physics of galaxy formation processes $\leftrightarrow$ "Clustering"

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- Physics of the early Universe

Test for the homogeneity

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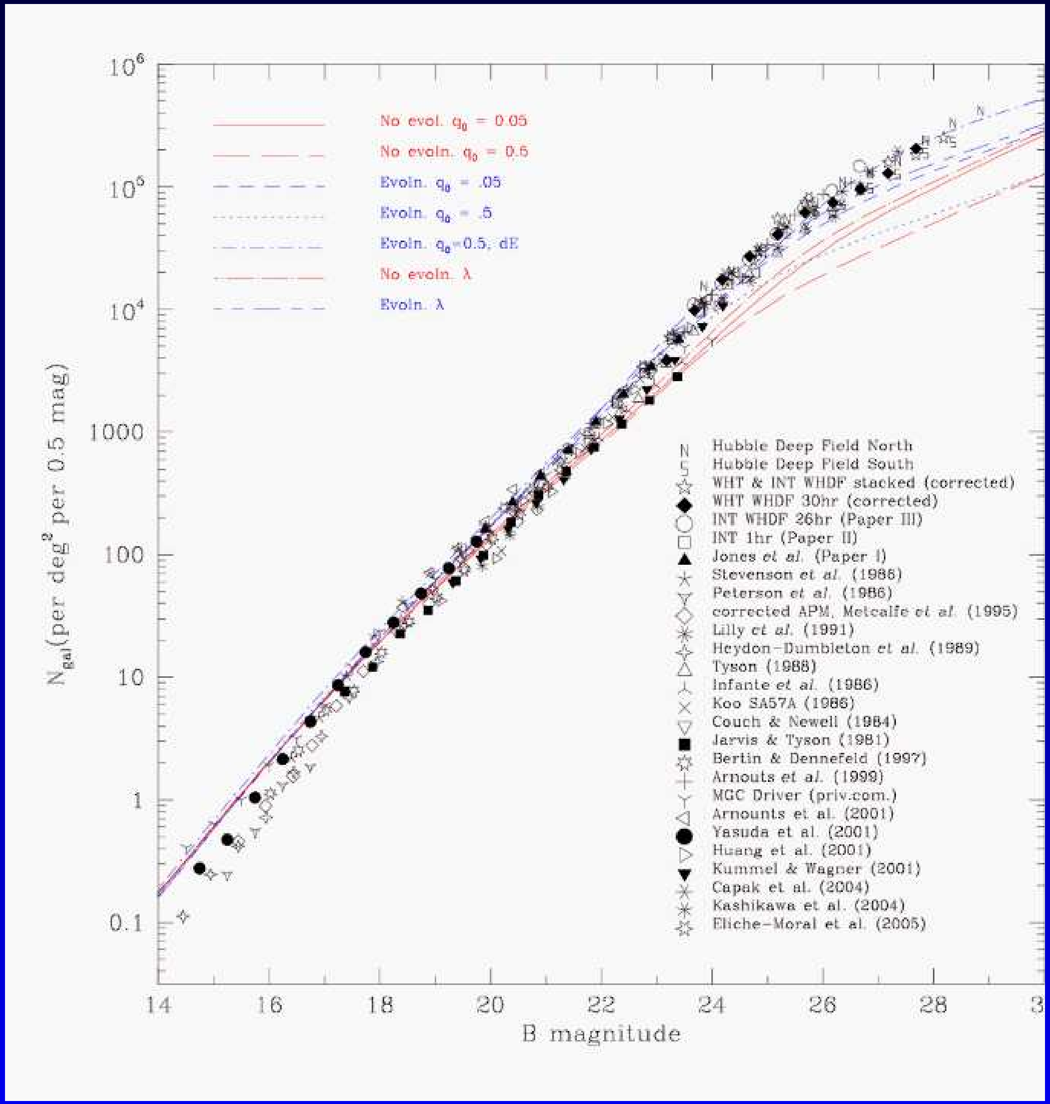
- does not garanty Roberston Walker...

Homogeneity

From galaxies number counts:



Galaxy Surveys

Distribution of galaxies = matter distribution.

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2D surveys : solid angle Ω, m_0

$$m = M + 5.\log(D_L(z)) + 25 + k(z)$$

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D in Mpc.

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- Sloan telescope: CCD, all digital

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Telescope time consuming.

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access to distance:

$$v = cz = H_0 D$$

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Spectra are very demanding on telescope time...

Typically 1 hour/spectrum.

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1980: CfA survey ~ 2000 galaxies

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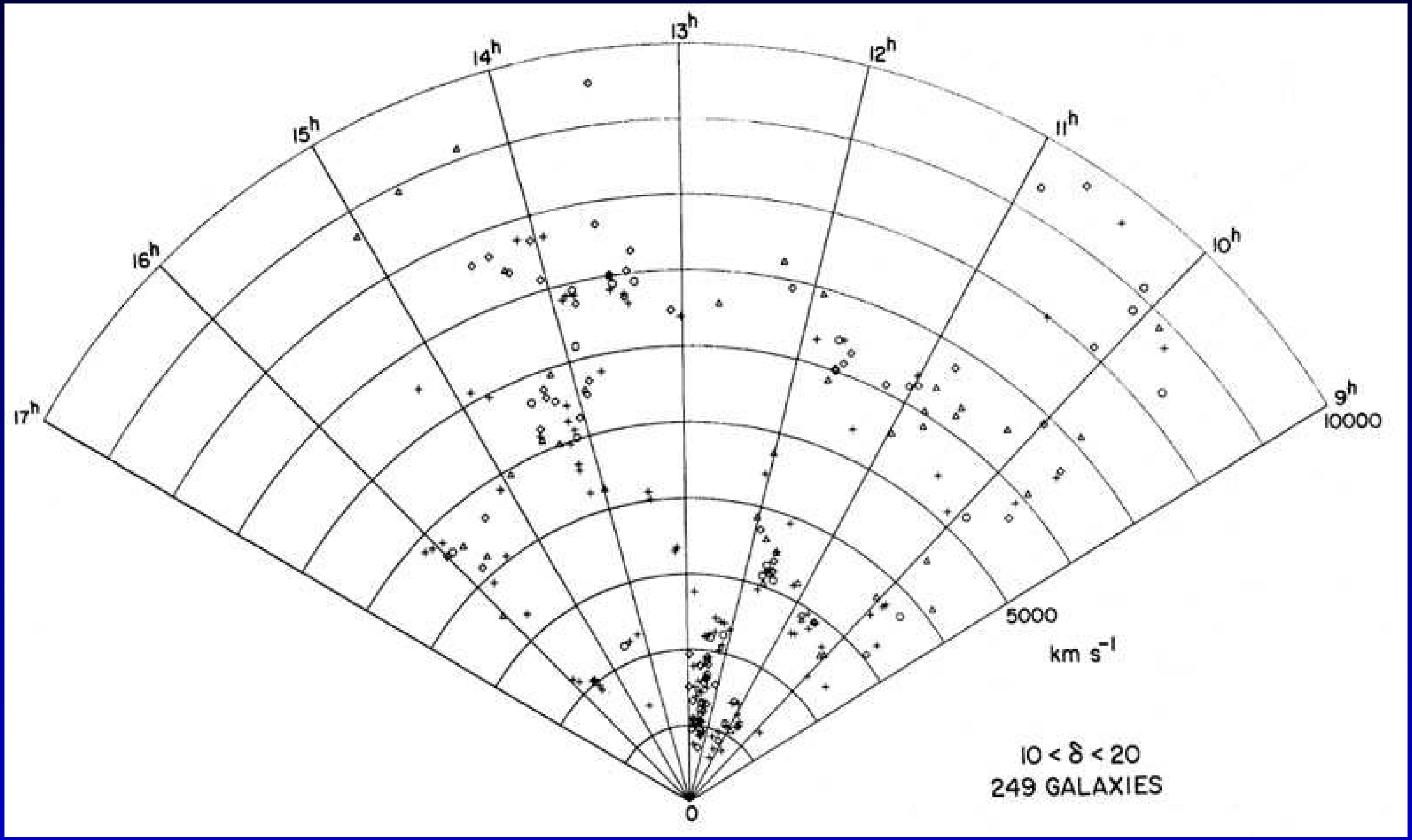

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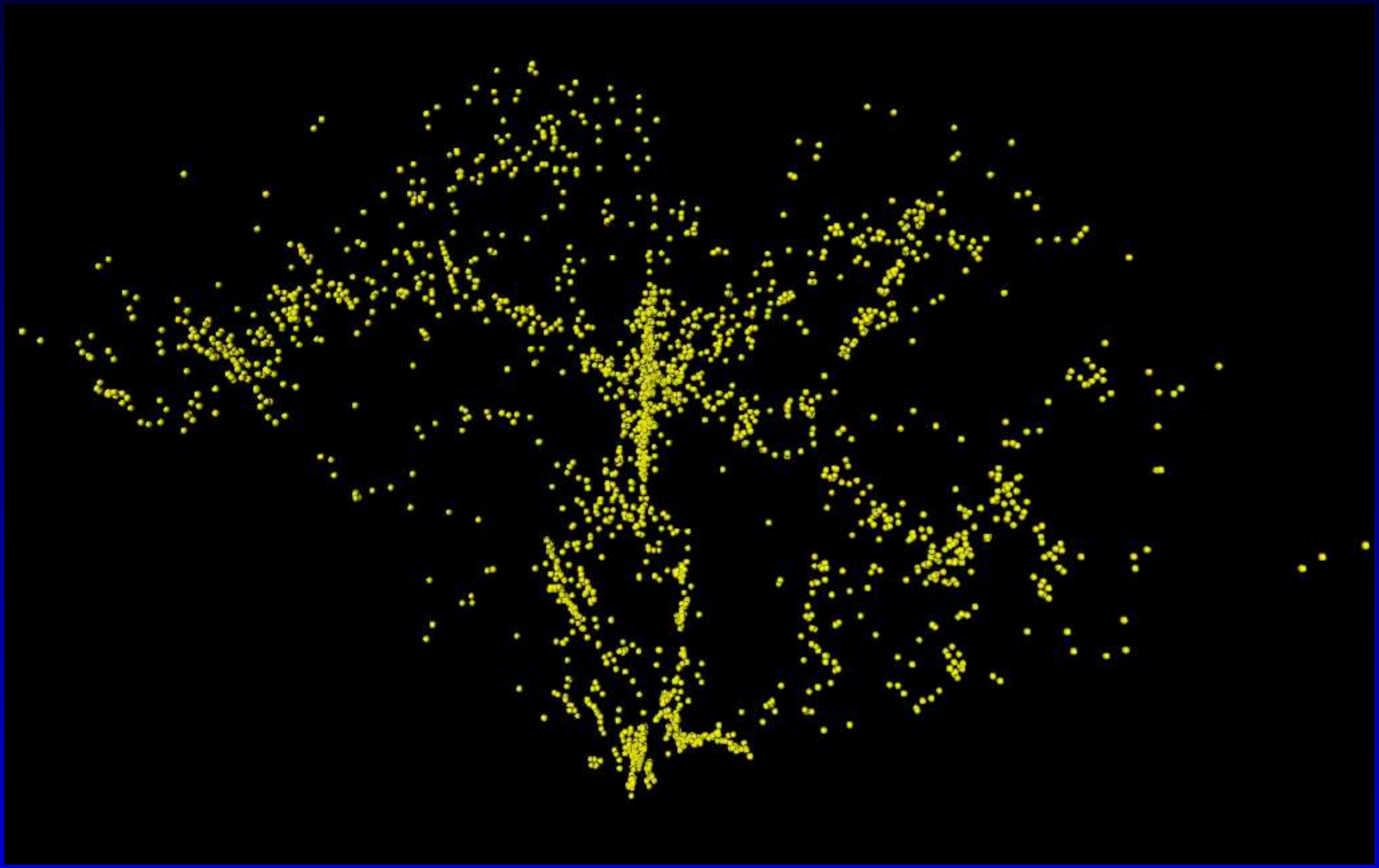

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Increase stat, increase scales: 2 in size, 2^3 in Volume, 2^3 in number...

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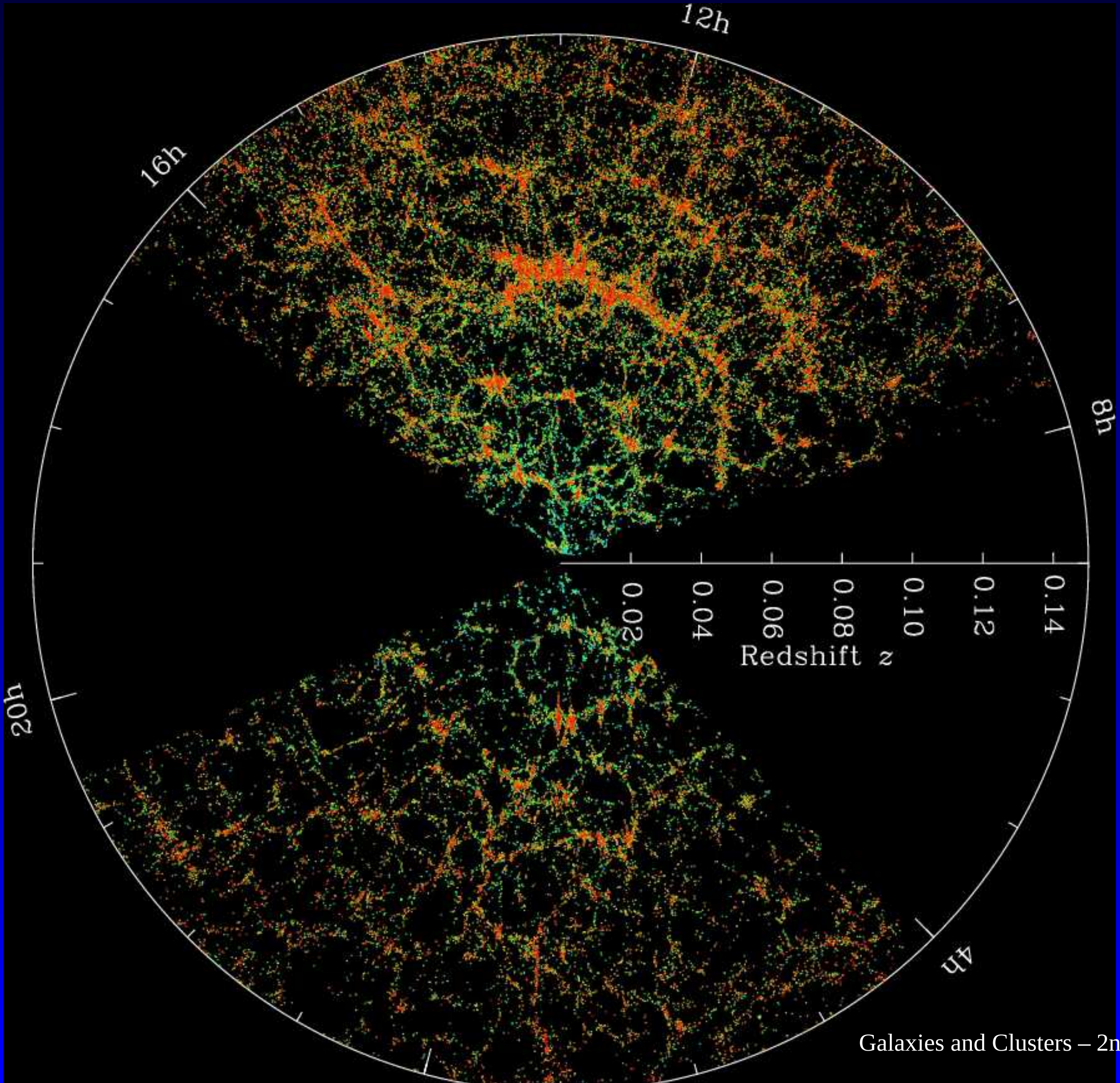

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A given galaxy (L or M) can be detected up to some maximum distance $d_{max}(L)$:

$$5.\log(d_{max}(L) = m_0 - M_\odot - 25. + 2.5\log(L/L_\odot) - k(z)$$

where $d_{max}(L)$ is in Mpc.

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Defines a volume:

$$d < d_{max}(L) \rightarrow V_{max} = 1/3\Omega d^3_{max}$$

Luminosity function

$$\phi(L)$$

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In a given survey:

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estimation:

$$\phi(L) dL = \Delta N / V_{max}(L)$$

Inhomogeneities are to be corrected for.

Schechter expression

Classical universal form (Schechter, 1974)

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$$\phi(L)dL = \phi^* \left(\frac{L}{L^*}\right)^{\alpha} \exp(-L/L^*)\frac{dL}{L^*}$$

$\phi(L)dL$: measurements

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2dF

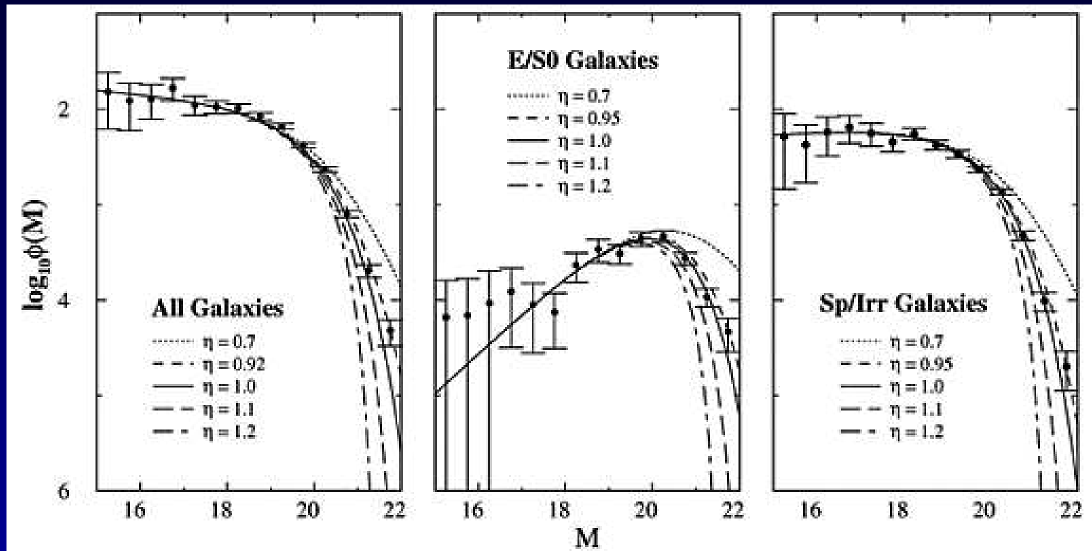

Figure 1. Luminosity function from the Stromlo-APM redshift survey. Fits in the three panels are to (a) the full galaxy sample, (b) the early-type subsample, and (c) the late-type galaxy subsample. The solid curves show Schechter function ($\eta = 1$). The dotted lines in each panel show how the fits change for the selected values of η with the other parameter assuming the same values estimated in the quoted reference. In all these cases, the exponential Schechter function is a critical curve within a large class of possible power law distributions. Curves with $\eta > 1$ fall off before the exponential while those with $\eta < 1$ fall more slowly.

$\phi(L)dL$: measurements

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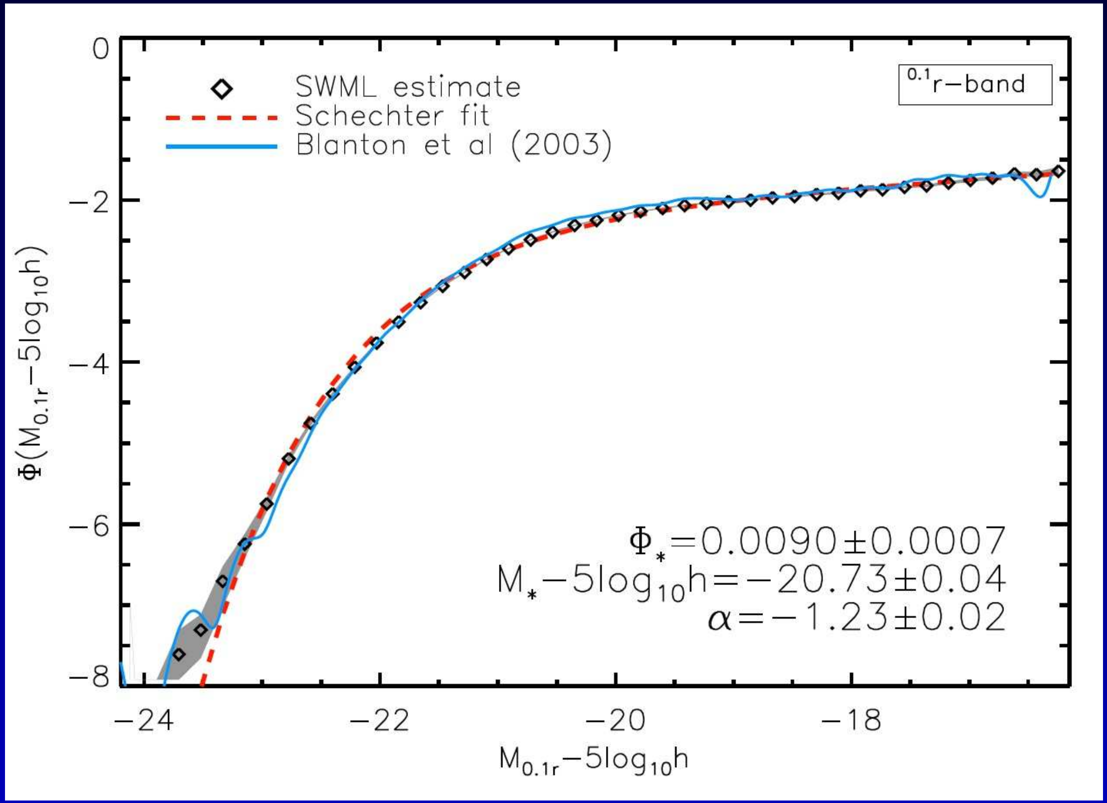
SDSS



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$$M^* = -19.66(\pm 0.07) + 5.\log(h)$$

$$\alpha = -1.21 \pm 0.03$$

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SDSS:

$$\phi^* = 0.009(h^{-1}\mathrm{Mpc})^{-3}$$

$$M^* = -20.73(\pm 0.07) + 5.\log(h)$$

$$\alpha = -1.23 \pm 0.02$$

(beware of the band used)

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Number density of Galaxies:

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diverges for $\alpha < -1$!!!

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Mean luminosity density:

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in short: $\overline{n} = \phi^*$ and $L = L^*$

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M/L ratio:

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n.a.

$$(M/L)_c = \frac{\overline{\rho}_c}{\overline{\rho}_L} \sim 1400h$$

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Mean stellar mass density:

$$M/L)_{Sp} \sim 2 - 5h$$

and

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i.e.

$$\Omega_{*} \sim 0.005$$

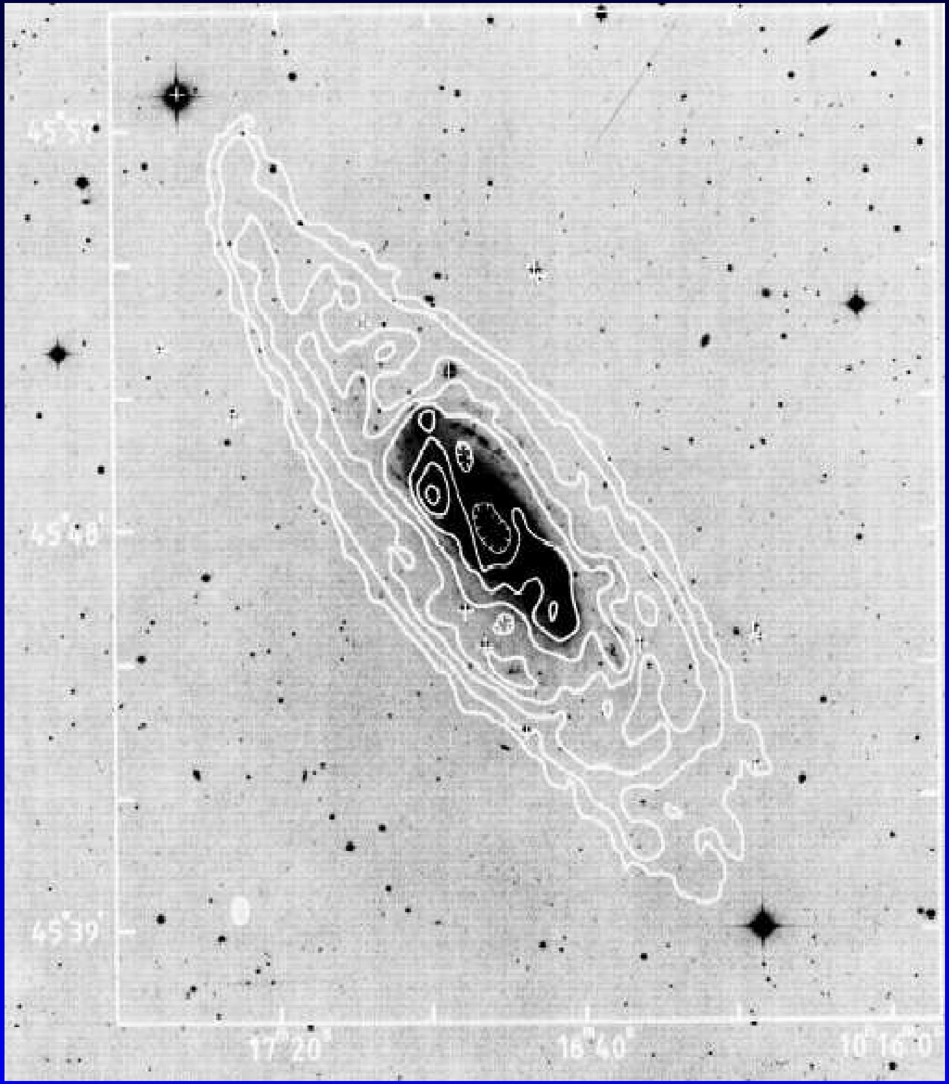
Dark Matter

Robust evidence I: galaxy rotation curves

Typical galaxy NGC 3198 (may mean best case...)

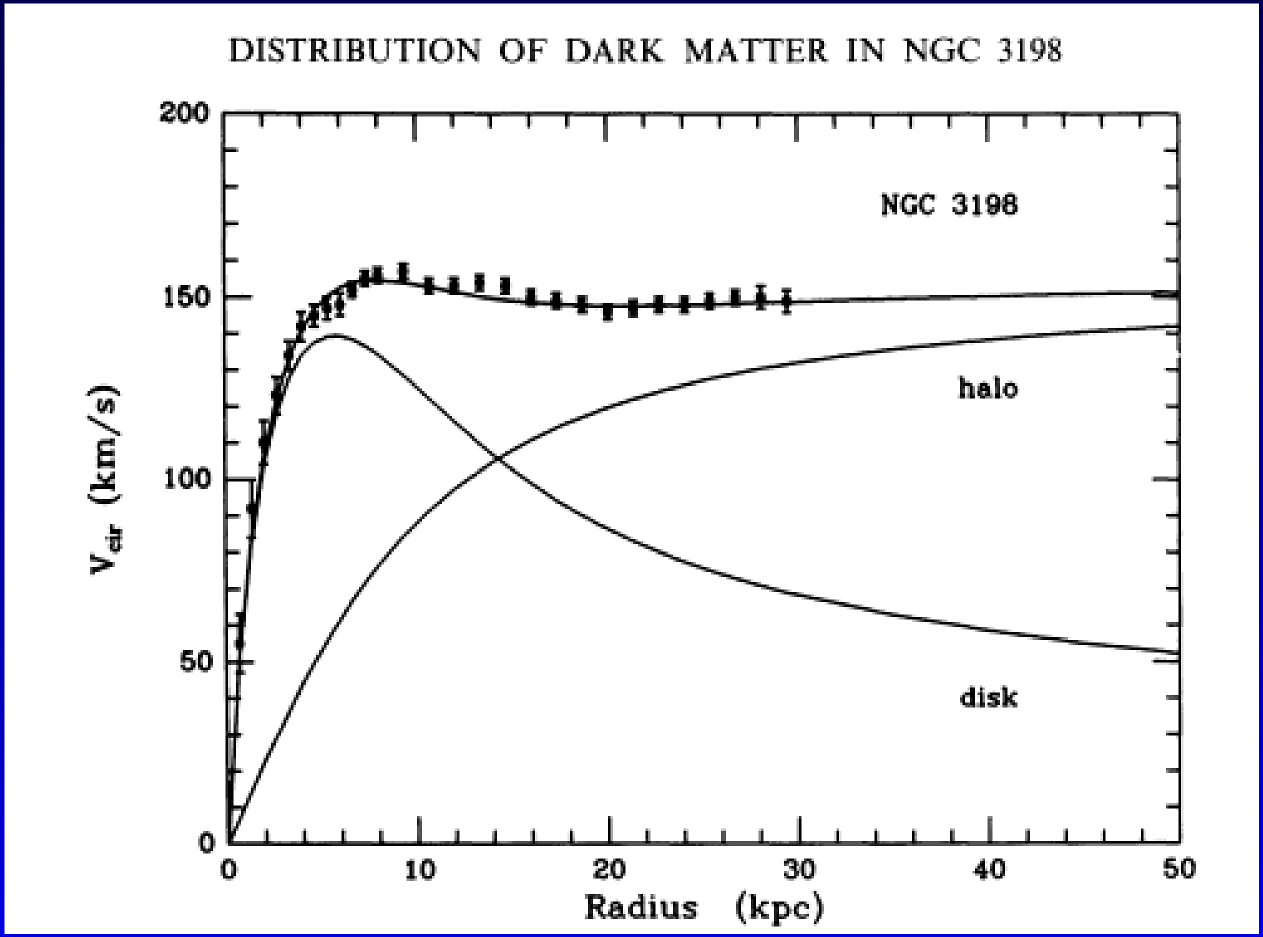
Dark Matter

NGC 3198: optical + HI view



Dark Matter

NGC 3198 : rotation curve



Dark Matter: Galaxies

"Observed" amount of dark matter in galaxies:

$$\frac{M_{tot}}{M_{vis}} \approx 5 - 10$$

so :

$$\Omega_{gal} \approx 0.025 - 0.05$$

Note : we do not know how far galaxies extend.

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- This includes uncertainties in the photometry, redshift...

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$$s(z) = \frac{n(> L(z))}{n(> L_0)}$$

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- Purity:

$$p(z) = 1 - \frac{\tilde{n}(> L(z))_{\rm false}}{n(> L_0)}$$

Large Scale Structure

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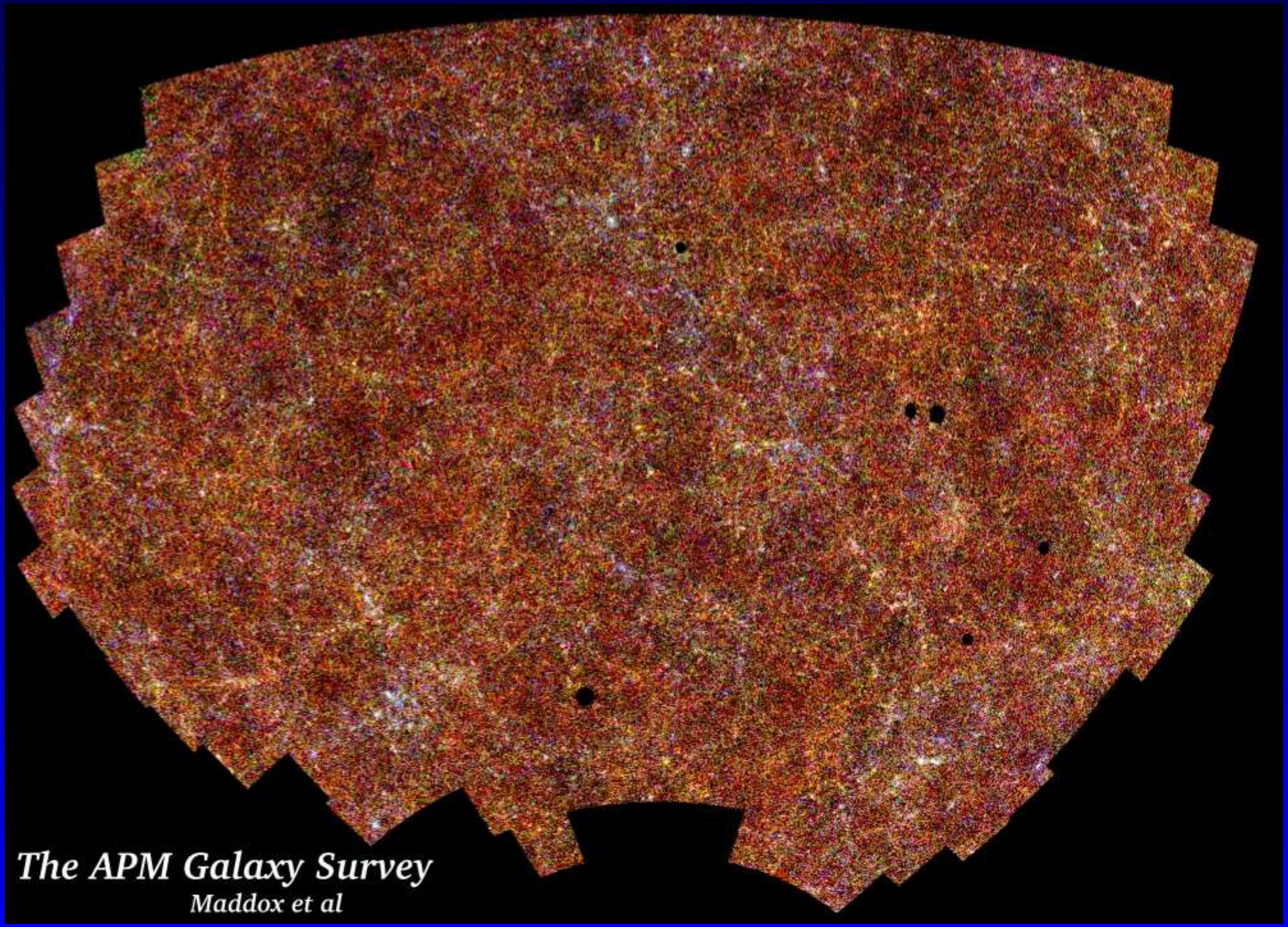
$> 1925 \Longrightarrow\sim$ Galaxy distribution on large scale.



Two point correlation function

Starting hypothesis: point process with average galaxy number density $\overline{n}$

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Isotropy:

$$\xi(\vec{r}) = \xi(r)$$

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Symetric way:

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$$
\begin{aligned}
dP_{12} &= \overline{n}^2 dV_1 dV_2 (1 + \xi(r_{12})) \\
&= < dN_1 dN_2 >
\end{aligned}
$$

For a continuous field

$$\xi(r) = \frac{< (\rho(x+r) - \overline{\rho})(\rho(x) - \overline{\rho}) >}{\overline{\rho}^2}$$

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Some point process may not be the Poisson model of any continuous field $\rho(x)$

High order correlation functions

3 points

$$dP_{123} = \overline{n}^3 dV_1 dV_2 dV_3 (1 + \xi(r_{12}) + \xi(r_{23}) + \xi(r_{31}) + \zeta(r_{12}, r_{23}, r_{31})) = < dN_1 dN_2 dN_3 >$$

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4 points

$$dP_{1234} = \overline{n}^4 dV_1 dV_2 dV_3 dV_4 \,(1 + \xi + ... + \zeta(r_{12}, r_{23}, r_{31}) + ... + \eta(r_{12}, r_{23}, r_{34}, r_{41}))$$

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N points...

$$dP_{12...N} = \overline{n}^p dV_1 dV_2 ... dV_N (1 + ... + \xi^N (r_{12}, ... r_{N1}))$$

High order correlation functions

3 points

$$dP_{123} = \overline{n}^3 dV_1 dV_2 dV_3 (1 + \xi(r_{12}) + \xi(r_{23}) + \xi(r_{31}) + \zeta(r_{12}, r_{23}, r_{31})) =< dN_1 dN_2 dN_3 >$$

4 points

$$dP_{1234} = \overline{n}^4 dV_1 dV_2 dV_3 dV_4 \, (1 + \xi + ... + \zeta(r_{12}, r_{23}, r_{31}) + ... + \eta(r_{12}, r_{23}, r_{34}, r_{41}))$$

N points...

$$dP_{12...N} = \overline{n}^p dV_1 dV_2 ... dV_N (1 + ... + \xi^N (r_{12}, ... r_{N1}))$$

provide all statistical information on the distribution

Power Spectrum I

$$\delta(\vec{x}) = \int \delta(\vec{k}) \exp(i\vec{k}\vec{x}) d^3\vec{k}$$

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So the power spectrum:

$$P(\vec{k}) = \delta(\vec{k})\delta^*(\vec{k})$$

is the FT of $\xi(r)$.

Power Spectrum II

$$\xi(r) = \int P(k) k^2 \frac{\sin(kr)}{kr} dk$$