Dark Energy

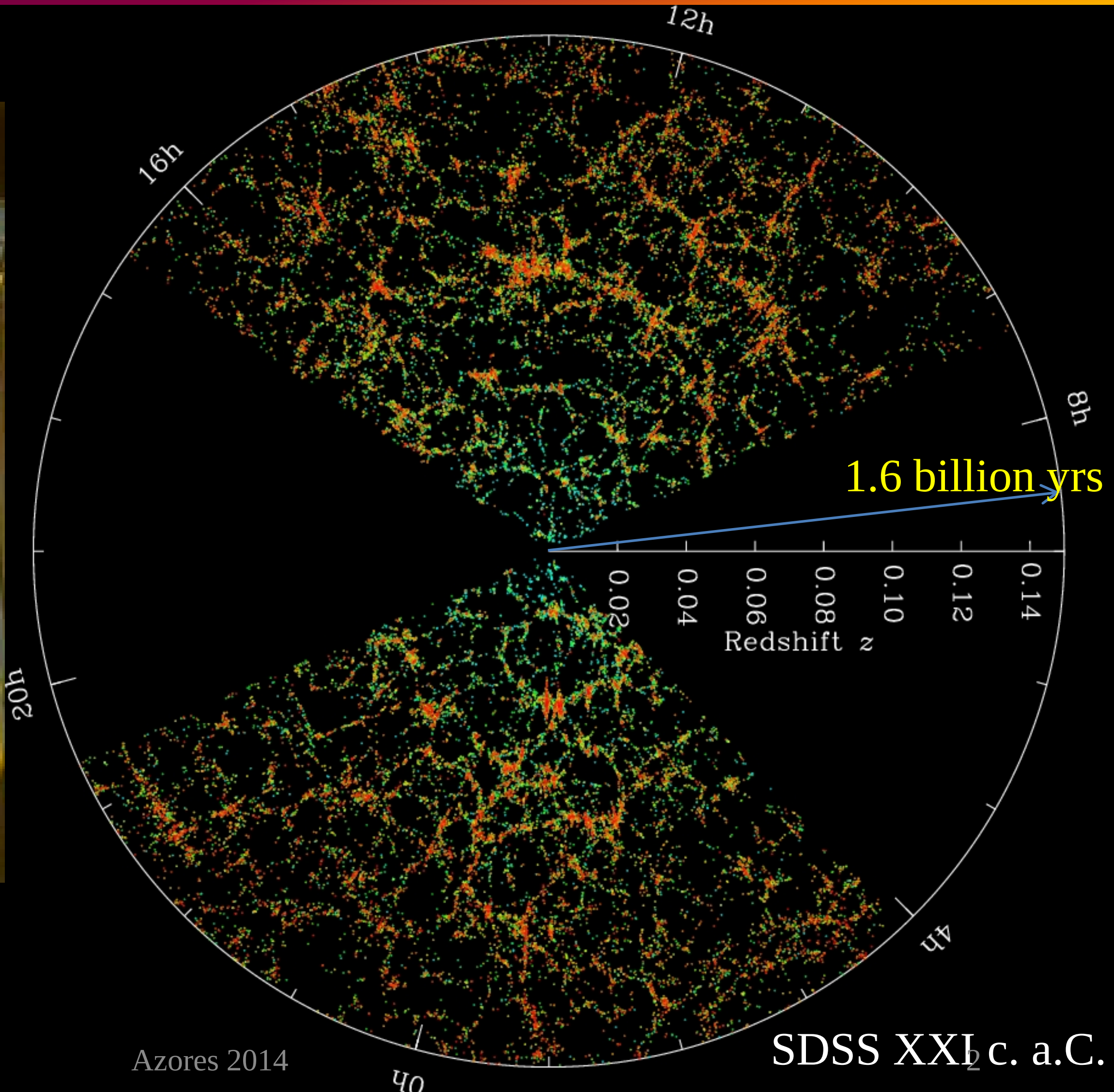
A Journey From Light to Darkness

Luca Amendola

University of Heidelberg

Maps of the World

Kosmas IV c. a.C



SDSS XXI c. a.C.

Outline

- Lecture 1 Discovery of Acceleration
- Lecture 2 Dark Energy
- Lecture 3 Testing Dark Energy

Texts

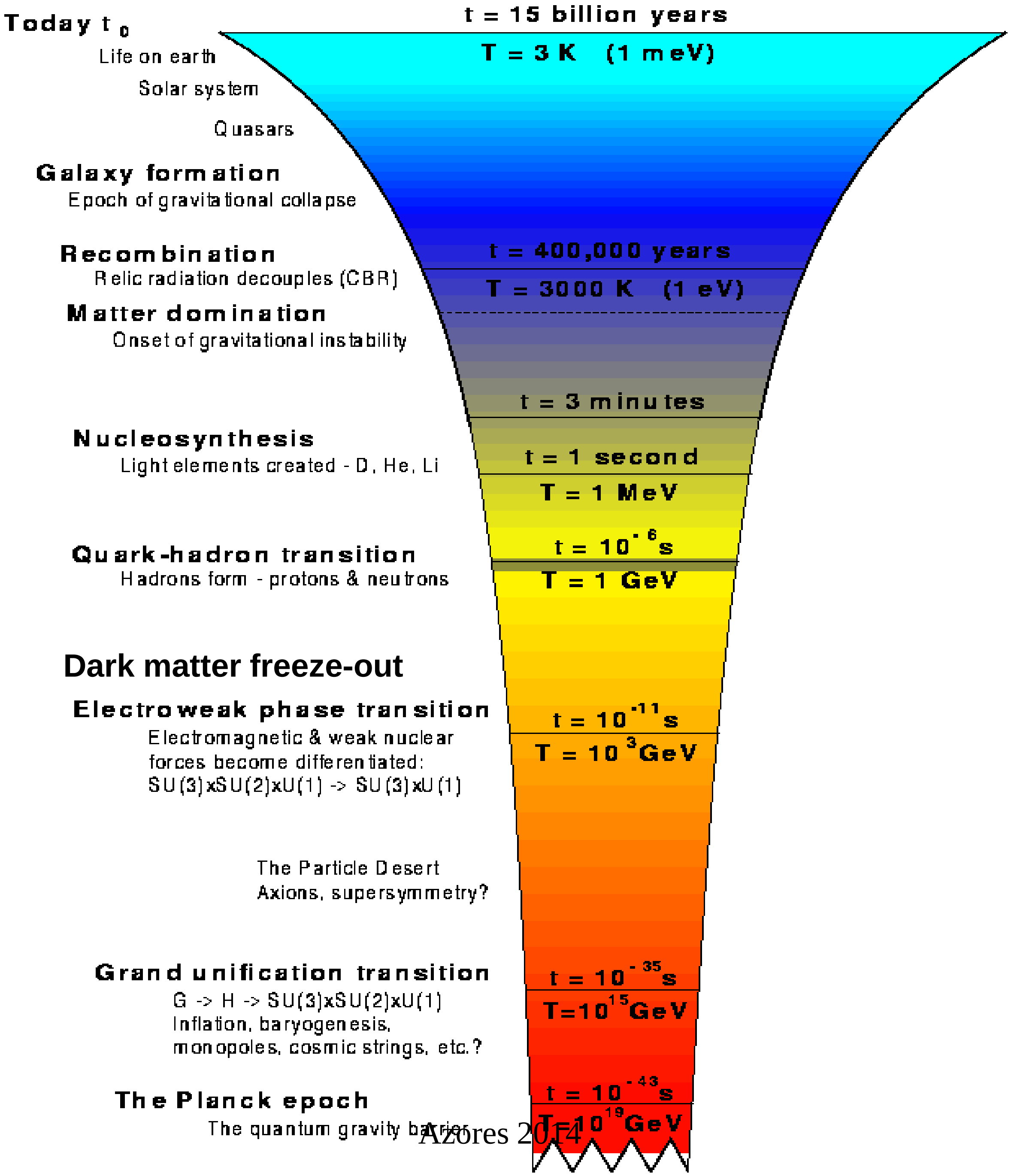
- **Dodelson**, Modern Cosmology
- **Amendola & Tsujikawa**, Dark Energy. Theory and Observations, CUP
- **Euclid Theory WG**, Cosmology and Fundamental Physics with the Euclid Satellite, arXiv 1206.1225

Not covered here but you can talk to me about…

- **Modification of gravity (Horndeski, bimetric)**
- **CMB, B-mode polarization, Large scale structure**
- **Statistical methods (Bayesian methods, robustness, non gaussian Fisher matrices)**
- **Euclid mission**

Where’s the matter?

- **Baryons**
- **Dark Matter**
- **Dark Energy**



Today t_0
Life on earth
Solar system
Quasars
Galaxy formation
Epoch of gravitational collapse
Recombination
Relic radiation decouples (CBR)
Matter domination
Onset of gravitational instability
Nucleosynthesis
Light elements created - D, He, Li
Quark-hadron transition
Hadrons form - protons & neutrons
Dark matter freeze-out
Electroweak phase transition
Electromagnetic & weak nuclear
forces become differentiated:
SU(3)xSU(2)xU(1) -> SU(3)xU(1)
The Particle Desert
Axions, supersymmetry?
Grand unification transition
G -> H -> SU(3)xSU(2)xU(1)
Inflation, baryogenesis,
monopoles, cosmic strings, etc.?
The Planck epoch
The quantum gravity barrier
t = 15 billion years
T = 3 K (1 meV)
t = 400,000 years
T = 3000 K (1 eV)
t = 3 minutes
t = 1 second
T = 1 MeV
$t = 10^{-6}$ s
T = 1 GeV
$t = 10^{-11}$ s
$T = 10^{3}$ GeV
$t = 10^{-35}$ s
$T=10^{15}$ GeV
$t = 10^{-43}$ s
$T=10^{19}$ GeV

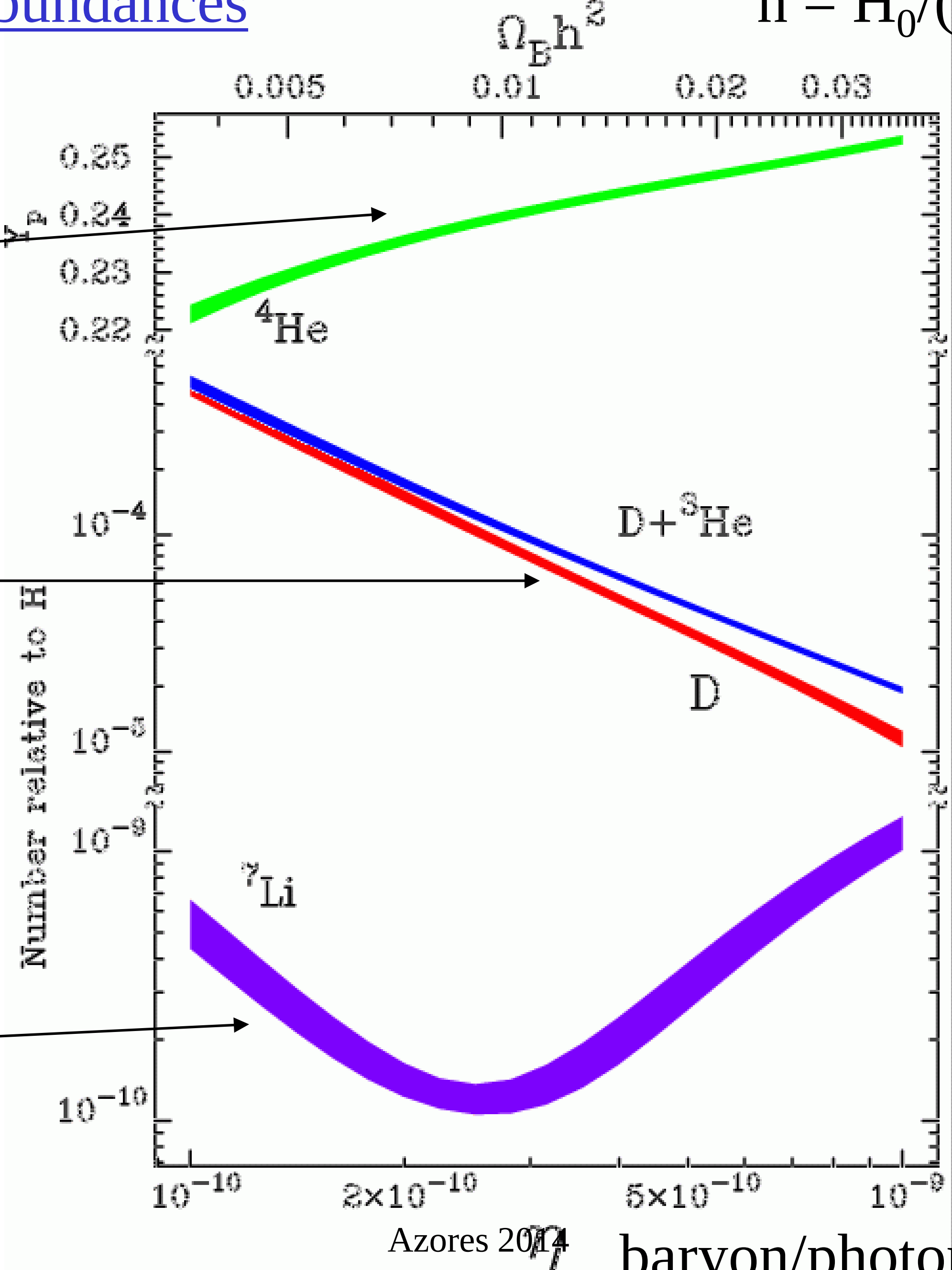
BBN predicted abundances

$h = H_0/(100\ \mathrm{km/sec/Mpc})$

Fraction of baryonic mass in He^4

Deuterium to Hydrogen ratio

Lithium to Hydrogen ratio



baryon/photon ratio

Light Element abundances depend mainly on the density of baryons in the Universe

BBN & the Baryon Density

Light element abundances are concordant if the baryon (neutron+proton) to photon ratio is about

$$\eta = n_b/n_{photon} = 6 \text{ x } 10^{-10} \quad \text{or} \quad \Omega_b h^2 = 0.02$$

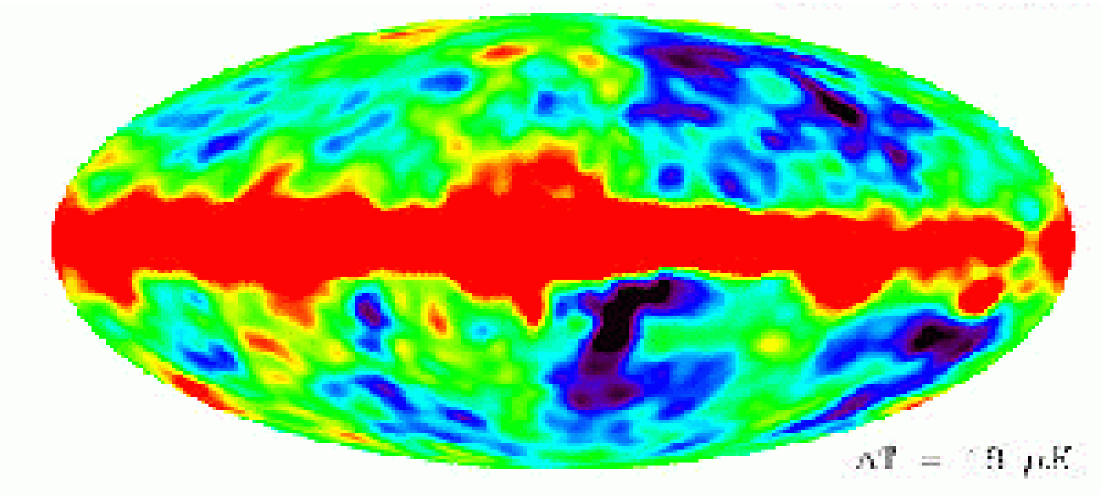
(We can make the conversion from η to $\Omega_b\, h^2$ since we know the present density of CMB photons,

$$n_{photon} = 420 \text{ per cm}^3,$$

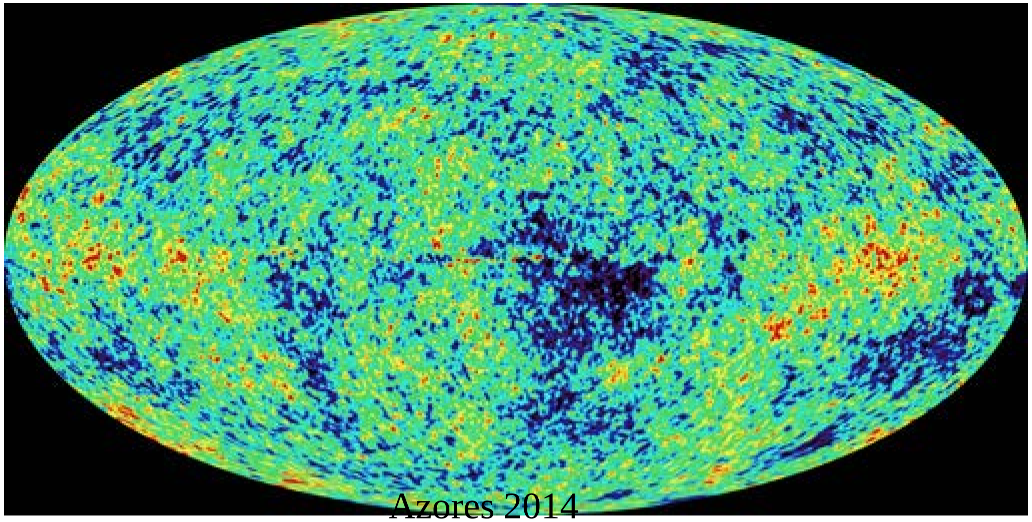
very precisely from the CMB Temperature.)

20 years of CMB

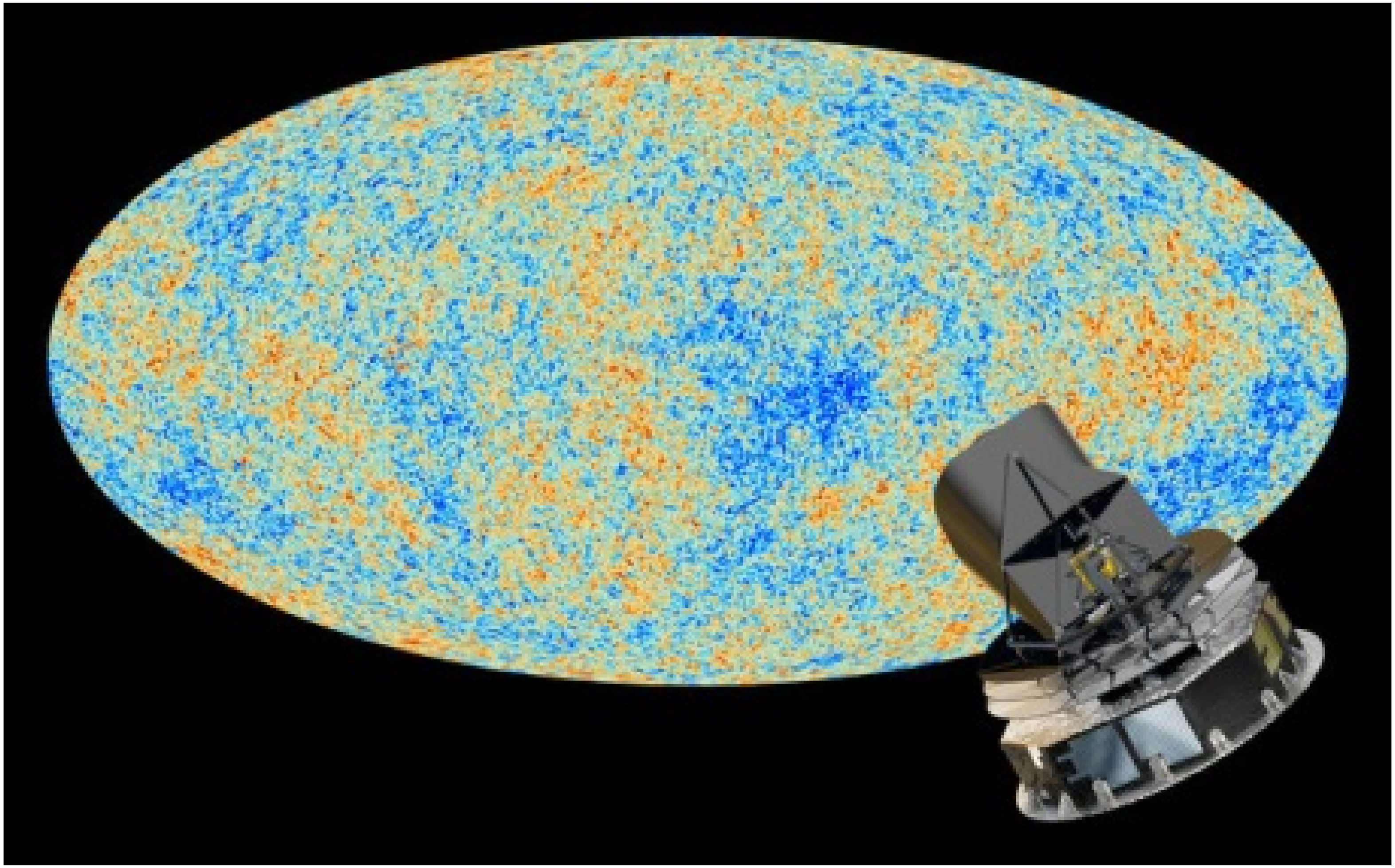
COBE
1990



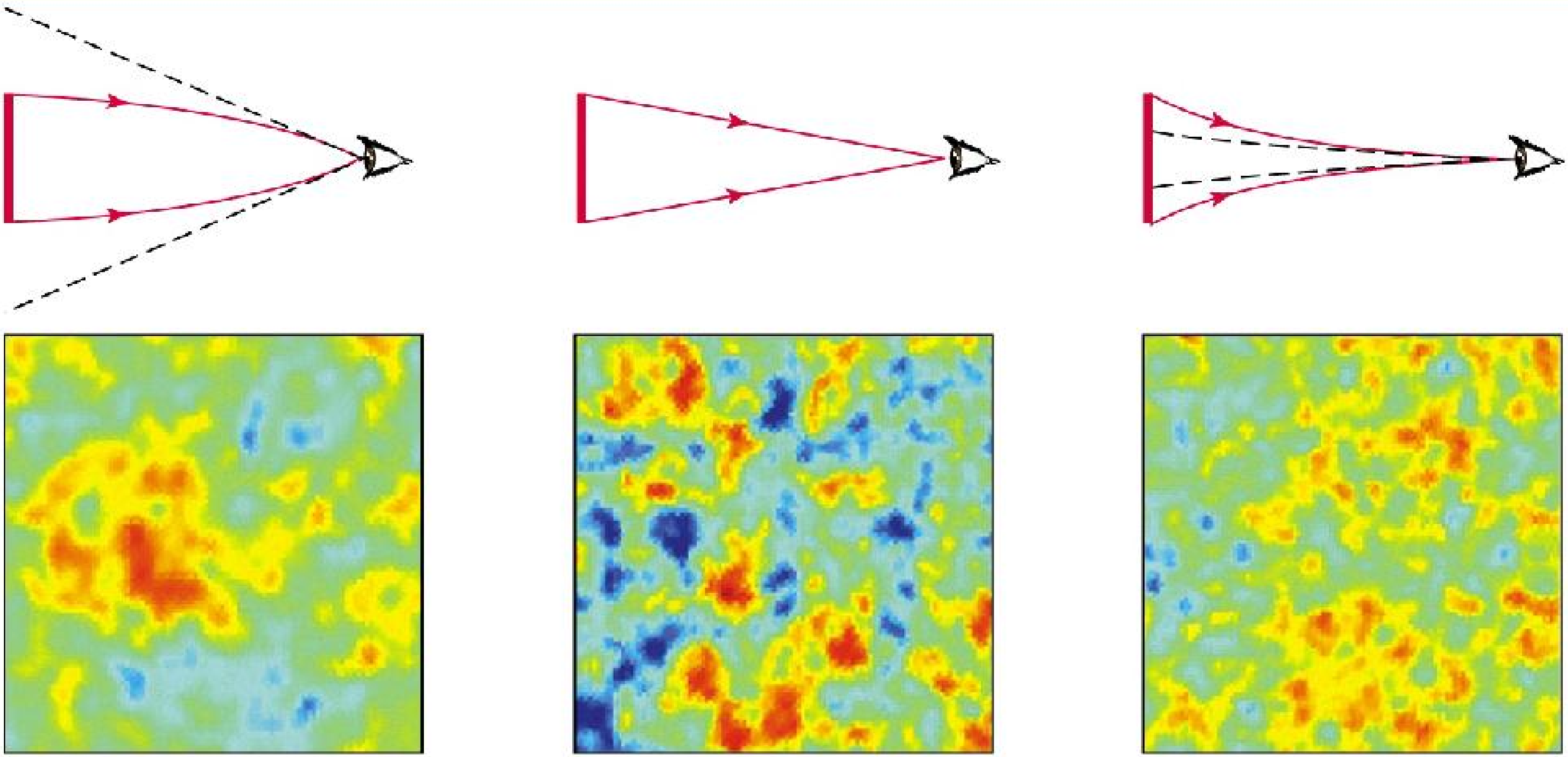
WMAP
2007



Maps of Planck



Seeing the Sound Horizon



a If universe is closed, "hot spots" appear larger than actual size

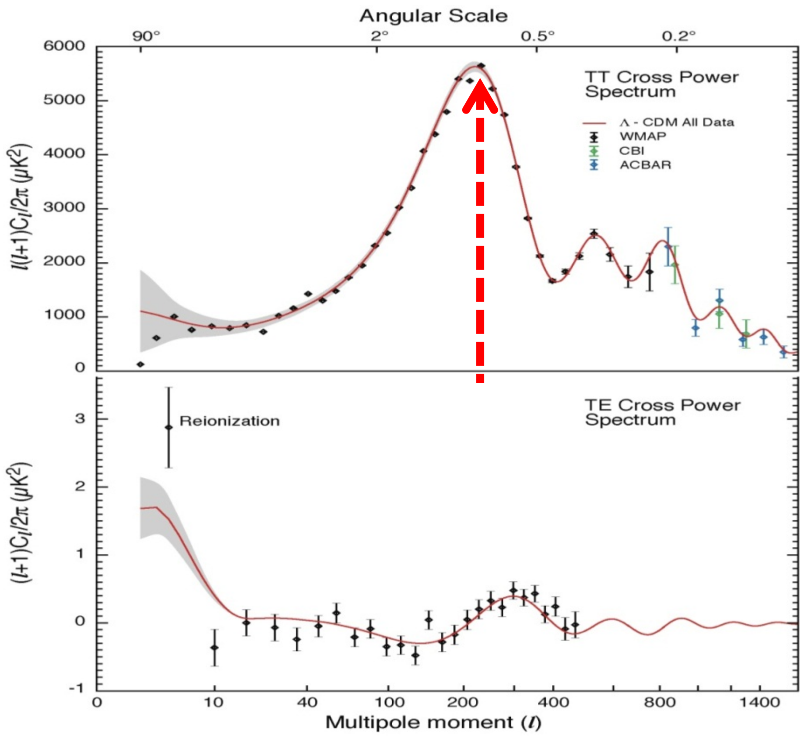
b If universe is flat, "hot spots" appear actual size

c If universe is open, "hot spots" appear smaller than actual size

$$\Omega_k < 0$$

$$\Omega_k > 0$$

Acoustic peaks probe space curvature

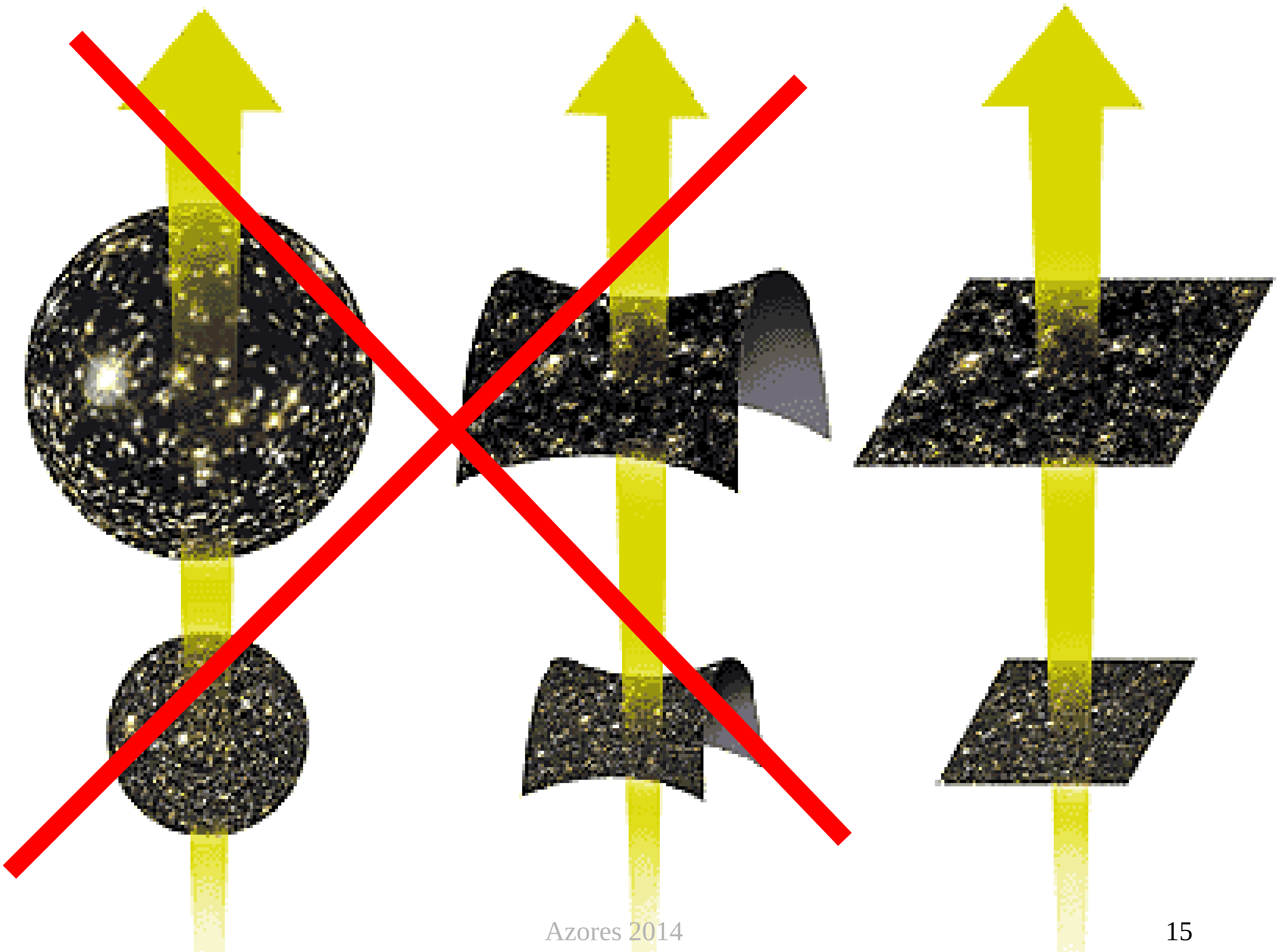


$$C_l = \frac{1}{2l+1}\sum_m |a_{lm}|^2$$

$$\Omega_{tot} = 1.02 \pm 0.02$$

WMAP+Planck

A Flat World…



So…where's the matter?

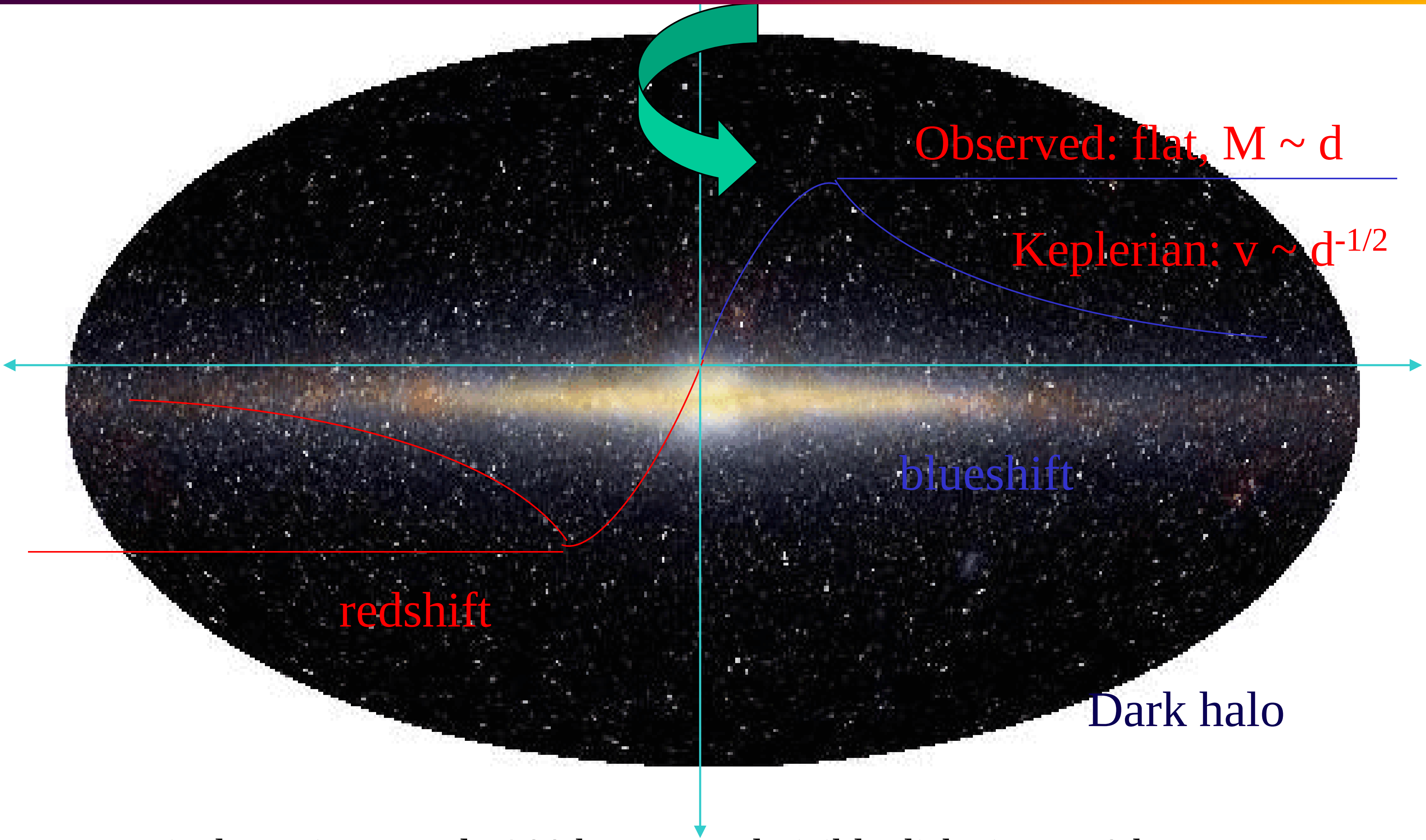
So far we have measured two precious numbers

$\Omega_b = 0.04$ Baryon density

$\Omega_{tot} = 1$ Total matter density

What makes up 96% of the stuff?

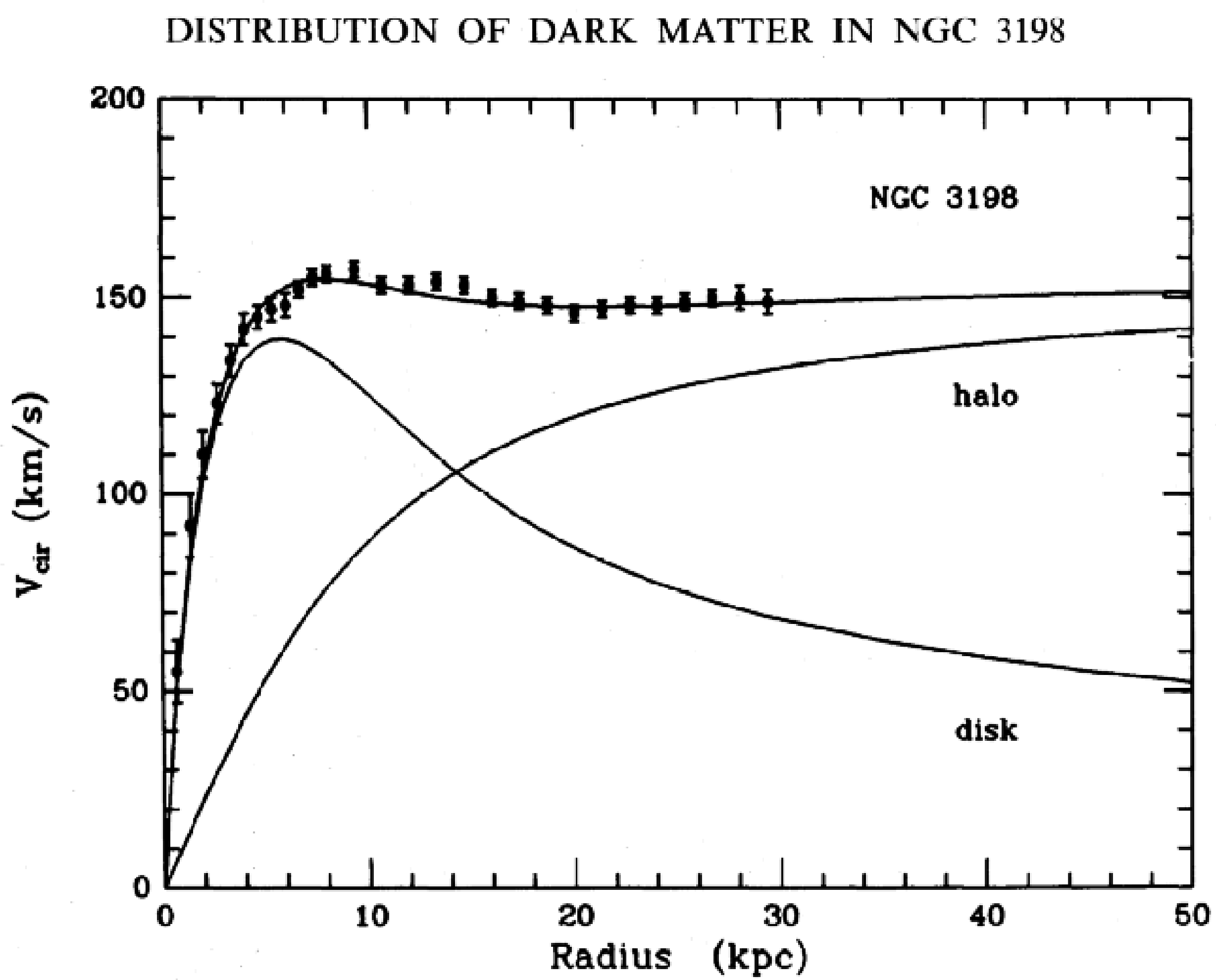
Dark matter



Typical rotation speed ~200 km/sec and visible disk size ~ 10 kpc

Mass ~ 10^{11} M_{sun}

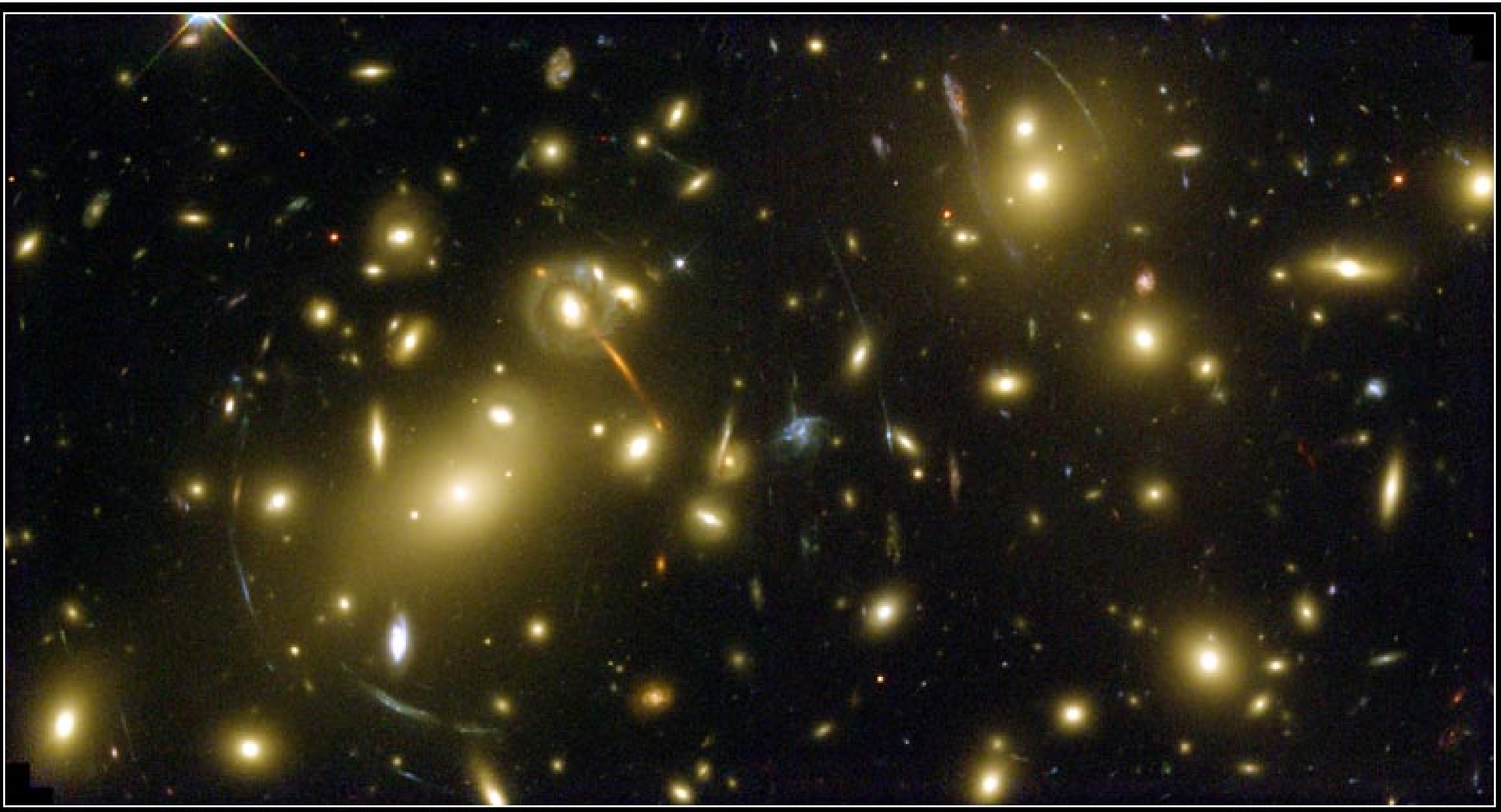
Dark matter


DISTRIBUTION OF DARK MATTER IN NGC 3198
NGC 3198
halo
disk
200
150
100
50
0
0
10
20
30
40
50
Radius (kpc)
V_{cir} (km/s)

Clusters of Galaxies: Size ~ Megaparsec (Mpc)

Mass ~ 10^{15} M_{sun}

Largest gravitationally bound objects: galaxies, gas, dark matter



Galaxy Cluster Abell 2218 HST • WFPC2

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

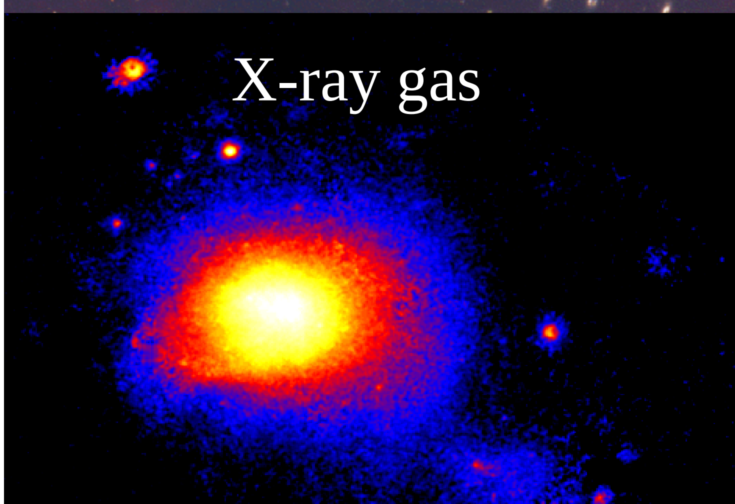
Where's the matter?

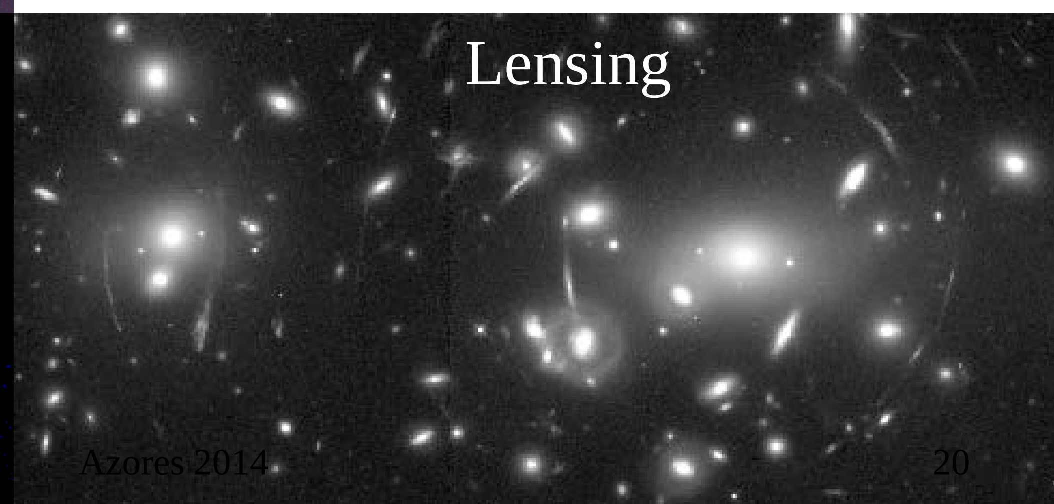
$$\Omega_m \sim 0.3$$

Cluster baryons

$$f_b = \frac{M_{gas}}{M_{TOT}} = \frac{M_b}{M_{TOT}} = \frac{\Omega_b}{\Omega_m}$$

- $f_b \sim$ 10-20%
- $\Omega_b h^2 = 0.02$ (BBN/CMB)
- $\Omega_m \sim 0.3$





Cosmology Executive Summary

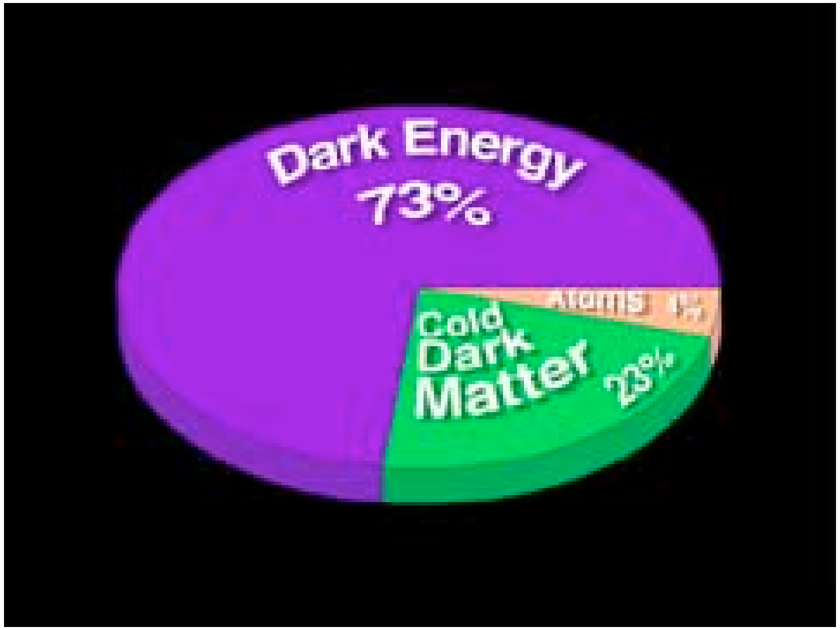
Baryons 4%

Dark matter 26%

Massive neutrinos: 0.1%

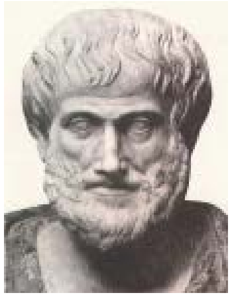
Spatial curvature: very close to 0

Something else: 70%



Back to the classics

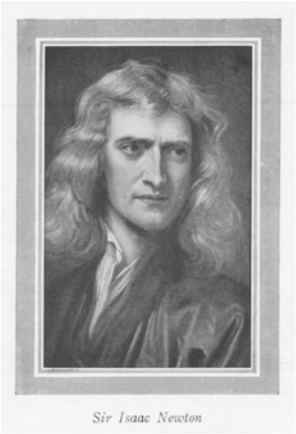
Historical perspective, circa 350 b.c.e.



d that the **sky** falls on our head?

- **Aristotle**'s answer: quintessence

Historical perspective, circa 1700 c.e.



- Gravity is always attractive: how to avoid that the **stars** fall on our head?
- **Newton**'s answer: God's initial conditions

Historical perspective, circa 1900 c.e.

of repulsive gravity, by modifying the equations of General Relativity.

Original GR equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\textit{geometry} \qquad \textit{matter}$$

$$T_{\mu}^{\nu} = \begin{Bmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{Bmatrix}$$

- These are the most general equations that are

1. covariant
2. covariantly conserved
3. second order in the metric
4. reducing to Newton at low energy

Box on $T_{\mu n}$

Basic hydrodynamic equations for a non-relativistic fluid at rest:

$$\begin{aligned} \dot{r} &= 0 \\ —p &= 0 \end{aligned} \qquad (1)$$

where the energy density $r = nmc^2$ and the pressure is $p_i = nmv_i^2$. If we define the matrix

$$T_{\mu n} = diag(r, p, p, p)$$

then, more simply

$$\frac{\partial T^{\mu n}}{\partial x^{\mu}} \text{⌘} T^{\mu n}_{,\mu} = 0$$

The relativistic version is the only tensor that depends on $r, p, u^{\mu} = dx^{\mu}/ds, g_{\mu n}$ and reduces to this limit in the Minkowski space

$$T^{\mu n} = (r + p)u^{\mu} u^{n} - pg^{\mu n} \qquad (2)$$

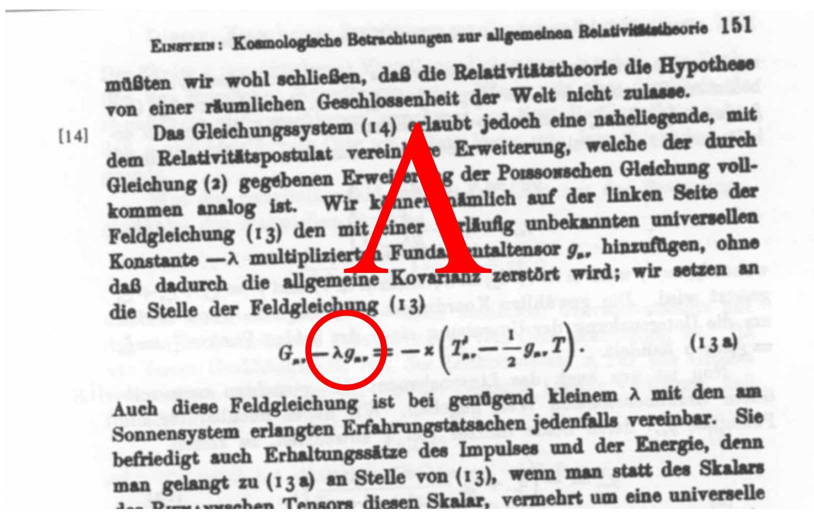
Einstein's equations are complete only when a relation between p and r is given: the equation of state:

$$p = wr$$

- Neglecting instead the fourth condition, we can add a (*small*) term $\square g_{\mu n}$ and rewrite the equations as

$$R_{\mu n} - \frac{1}{2} R g_{\mu n} - \square g_{\mu n} = 8 p G T_{\mu n}$$

- The new term is the cosmological constant.



Λ

EINSTEIN: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie 151

müßten wir wohl schließen, daß die Relativitätstheorie die Hypothese von einer räumlichen Geschlossenheit der Welt nicht zulasse.

[14] Das Gleichungssystem (14) erlaubt jedoch eine naheliegende, mit dem Relativitätspostulat vereinbare Erweiterung, welche der durch Gleichung (2) gegebenen Erweiterung der Poissonschen Gleichung vollkommen analog ist. Wir können nämlich auf der linken Seite der Feldgleichung (13) den mit einer vorläufig unbekannten universellen Konstante $-\lambda$ multiplizierten Fundamentaltensor $g_{\mu\nu}$ hinzufügen, ohne daß dadurch die allgemeine Kovarianz zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \qquad (13a)$$

Auch diese Feldgleichung ist bei genügend kleinem λ mit den am Sonnensystem erlangten Erfahrungstatsachen jedenfalls vereinbar. Sie befriedigt auch Erhaltungssätze des Impulses und der Energie, denn man gelangt zu (13a) an Stelle von (13), wenn man statt des Skalars des Riemannschen Tensors diesen Skalar, vermehrt um eine universelle

Einstein 1917

- The big idea of recent years has been to move the new term from right to left

$$R_{\mu n} - \frac{1}{2} R g_{\mu n} = 8p G T_{\mu n} + \Box g_{\mu n}$$

thereby introducing a new form of matter

$$T_{\mu n(\Box)} = \left(\frac{\Box}{8p} \right) g_{\mu n}$$

- This matter has a fundamental property. Writing

$$T^{n}_{\mu(\Box)} = \left(\frac{\Box}{8p} \right) d^{n}_{\mu}$$

or

$$\begin{pmatrix} r & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} = \begin{pmatrix} \frac{\Box}{8p} & 0 & 0 & 0 \\ 0 & \frac{\Box}{8p} & 0 & 0 \\ 0 & 0 & \frac{\Box}{8p} & 0 \\ 0 & 0 & 0 & \frac{\Box}{8p} \end{pmatrix}$$

one gets immediately

$$p_{\Box} = -\frac{\Box}{8p}, \quad r_{\Box} = \frac{\Box}{8p}$$

that is, the cosmological constant has negative pressure (if $\Box > 0$).

- Introducing the equation of state

$$p = w r$$

one has that the cosmological constant has a negative eq. of state

$$w = -1$$

As a comparison, the eq. of state of matter (dust or cold dark matter) is

$$p = mv^2 \square 0 \; ! \quad w = 0$$

while for radiation

$$p = r/3 \; ! \quad w = 1/3$$

A repulsive gravity

- What a negative pressure has to do with a repulsive gravity?
- Homogeneous and isotropic Friedmann metric

$$ds^2 = dt^2 - a^2 \left(\frac{dr^2}{1 - kr^2} + r^2 \sin q \, df^{\,2} + r^2 dq^2 \right)$$

For a single perfect fluid, the ten Einstein equations reduce to two equations for the scale factor and the energy density (here we put for simplicity $k = 0$ and always assume $a_0 = 1$)

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8p}{3} r \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4p}{3}(r + 3p) = -\frac{4p}{3} r \, (1 + 3w) \tag{4}$$

From the second one it appears that if

$$w < -1/3$$

then we get accelerated expansion. Therefore the cosmological constant (or any fluid with $w < -1/3$) accelerates the expansion ! "repulsive gravity". We call this hypothetical fluid *Dark Energy.*

- Consider now only the cosm. constant

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho_\Lambda = \frac{\Lambda}{3}$$

from which

$$a = a_0 e^{\sqrt{\frac{\Lambda}{3}}\,t}$$

This accelerated expansion is a prototype of primordial inflation (de Sitter metric).

- Generally speaking, there are at least three components (plus curvature) so that dynamics is more complicate:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\left(\rho_\gamma + \rho_M + \rho_\Lambda\right) - \frac{k}{a^2}$$

$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0$$

Ordinary matter (baryons plus dark matter) conserves energy during expansion, so that we have four different behaviors

$$\rho_\gamma \propto a^{-4}$$

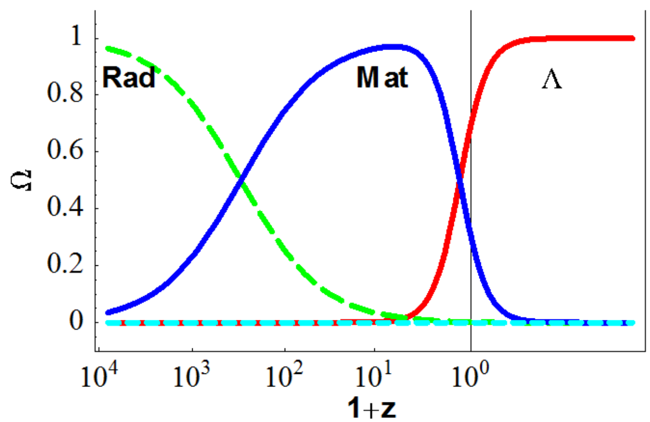
$$\rho_M \propto a^{-3}$$

$$\rho_k \equiv \frac{k}{a^2} \propto a^{-2}$$

$$\rho_\Lambda \propto a^0$$

- In general, therefore, we have

rad. ! matter ! curvature ! cosm.const.



$$\boxtimes_i \;⌘\; \frac{r_i}{r_t}$$

Quantistic interpretation

- Think of a field, eg a scalar field, as a series of classical oscillators. Then, every oscillator contributes an energy due to the sum of its potential and kinetic energy.
- When at rest, every oscillator has only its potential energy of the lowest level, that we can always put to zero.
- Quantistically, however, the state of minimum is not at zero energy but rather

$$E_0 = \frac{1}{2}\hbar\omega$$

- Therefore, for a field, the total zero-point energy is

$$E_0 = \sum_i \frac{1}{2}\hbar\omega_i$$

summing over all possible modes. Summing over $k_i = 2\pi/\lambda_i$ where $\lambda_i = L/n_i$ are all the wavelengths of the modes contained in a box of size L, we obtain $dn_i = dk_i L/2\pi$ modes in the range dk_i, so that

$$E_0 = \frac{1}{2}\hbar L^3 \int \frac{d^3k}{(2\pi)^3}\omega_k$$

- where the oscillation frequency is in relation to the particle's mass:

$$w^2 = k^2 + m^2/\hbar^2$$

The total energy density integrating up to a cut-off frequency k_{max} is then

$$r_{vacuum} = lim \frac{E}{L^3} = \hbar \frac{k_{max}^4}{16p^2}$$

- The energy diverges at the high frequencies (ultraviolet divergence). We must suppose then that there is k_{max} beyond which a new interaction modifies the system.
- The problem is, which k_{max} ?. If we assume as limit the Planck energy

$$E_{Planck} = 10^{19}\, GeV$$

we get

$$r_{vacuum} = 10^{92} g/cm^3$$

Now, the experimental limit is

$$r = 3H^2/8pG \,'\, 10^{-29} g/cm^3$$

then, the theoretical estimate is off by 120 orders of magnitude!
- This fundamental theoretical problem is still open.

From observations to theory

- What we really observe in cosmology is light from sources and from backgrounds.
- How do we connect these observables to cosmological quantities like $\rho_m, \rho_\gamma, k, a(t), H_0$ etc?
- First, define

$$\Omega_M = \frac{8\pi\rho_0}{3H_0^2}, \qquad \Omega_\Lambda = \frac{8\pi\rho_\Lambda}{3H_0^2}, \qquad \Omega_k = \frac{8\pi k}{3H_0^2}$$

and note that

$$1 = \Omega_M + \Omega_\Lambda + \Omega_k$$

so rewrite Friedman equation as ($a_0 = 1$)

$$H^2 = H_0^2(\Omega_m a^{-3} + \Omega_\Lambda a^0 + \Omega_k a^{-2})$$

- Then, generalize it to several components:

$$\begin{aligned} H^2 &= H_0^2(\Omega_m a^{-3(1+w_m)} + \Omega_\Lambda a^{-3(1+w_\Lambda)} + \dots) \\ &= H_0^2 \sum_i \Omega_i a^{-3(1+w_i)} = H_0^2 E(a)^2 \end{aligned}$$

Cosmic Inventory

name	density	EOS w
baryons	0.04	≈ 0
CDM	0.26	≈ 0
radiation	0.0001	1/3
Massive neutrinos	<0.05	≈ 0
Cosm. const.	0.70	-1
curvature	<0.03	-1/3
Other ?	?	?

First basic observable: Age of the Universe

- The age of the universe can be deduced from the Friedmann equation:

$$\left(\frac{da}{dt}\right)^2 = H_0^2 a^2 E(a)^2$$

we get

$$\left(\frac{dz}{H_0\,dt}\right) = (1+z)E(z)$$

and finally

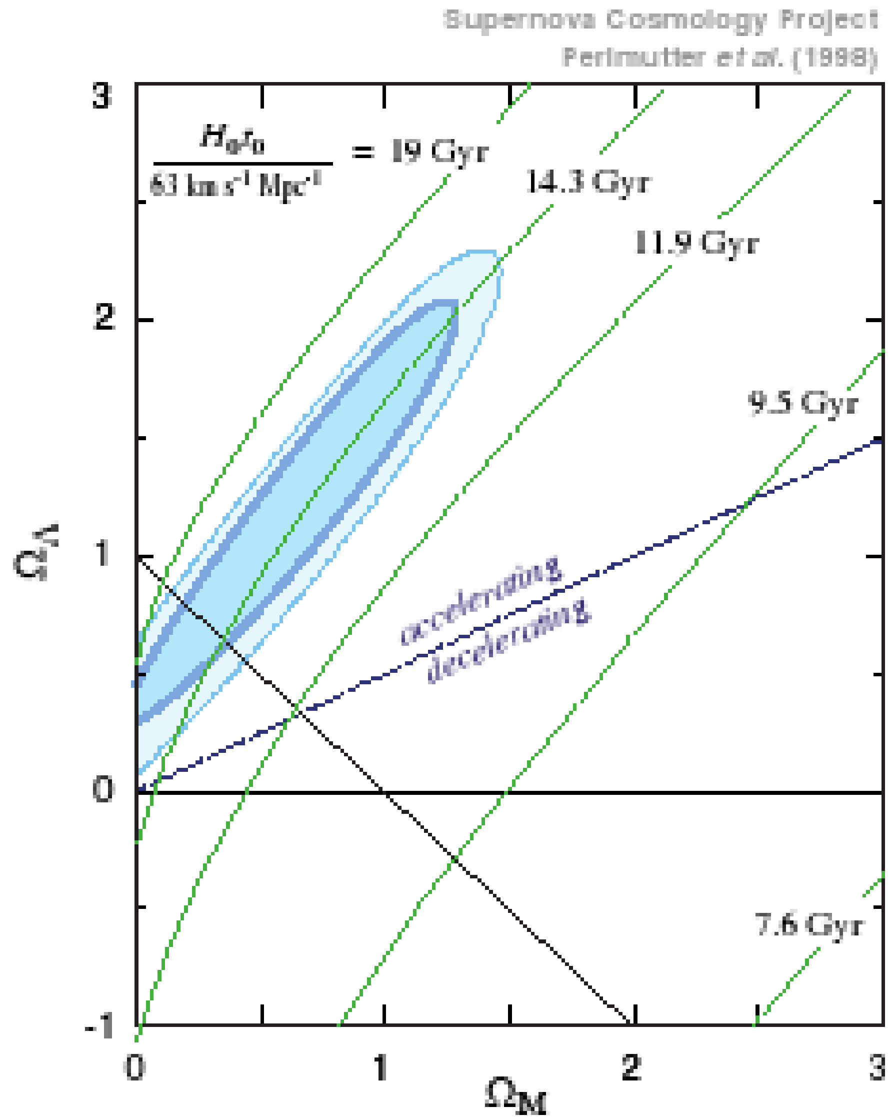
$$t_0 - t_1 = H_0^{-1}\int_0^{z_1}\frac{dz}{(1+z)E(z)}$$

Notice that the Hubble constant is

$$H_0^{-1} = \frac{1}{100\,h\,km/sec/Mpc} = 9.76\,h^{-1}\,Gyr$$

For $z_1 \to \infty$ we get then the age of the universe.

- The effect of the cosmological constant , when $\Omega_{tot} = \Omega_M + \Omega_\Lambda$ is fixed, is to increase the cosmological age.



Best fit age of universe: $t_o = 14.5 \pm 1\ (0.63/h)$ Gyr

Second basic observable: Luminosity distance

- From flat Friedmann's metric

$$ds^2 = c^2 dt^2 - a^2 dr^2$$

and integrating along the null geodesics, we get the proper distance which is what you would measure with fixed rods

$$r = \int \frac{c\,dt}{a(t)} = c \int \frac{da}{\dot{a} a} = c \int \frac{dz}{H(z)}$$

→ generalized Hubble law: measuring distances means measuring cosmology.

- If we compare the energy L emitted by a source at proper distance r with flux f arriving at the observer, we define the luminosity distance $d(z)$ such that

$$f = \frac{L}{4\pi r^2 (1+z)^2} = \frac{L}{4\pi d^2}$$

- The two extra factors of $1+z$ take into account the loss of energy due to redshift and the spread of energy due to the relative dilatation of the emission time versus observer's time. We get

$$d(z) = r(1+z) = cH_0^{-1}(1+z) \int_0^{z_1} \frac{dz}{E(z)}$$

where $cH_0^{-1} = \frac{300.000 km/sec}{100 h km/sec/Mpc} = 3000 h^{-1} Mpc$.

Exact expression

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\varphi^2\right)$$

$$r_d = H_0^{-1} S\left[c \int_0^{z_d} \frac{dz'}{E(z')}\right] = H_0^{-1} S\left[c(\tau_0 - \tau_d)\right]$$

where

$$S(R) = \begin{matrix} |\Omega_k|^{-1/2}\sin(|\Omega_k|^{1/2}R) & (k=1) \\ R & (k=0) \\ |\Omega_k|^{-1/2}\sinh(|\Omega_k|^{1/2}R) & (k=-1) \end{matrix} \qquad (10)$$

- Remember our "reference" cosmology

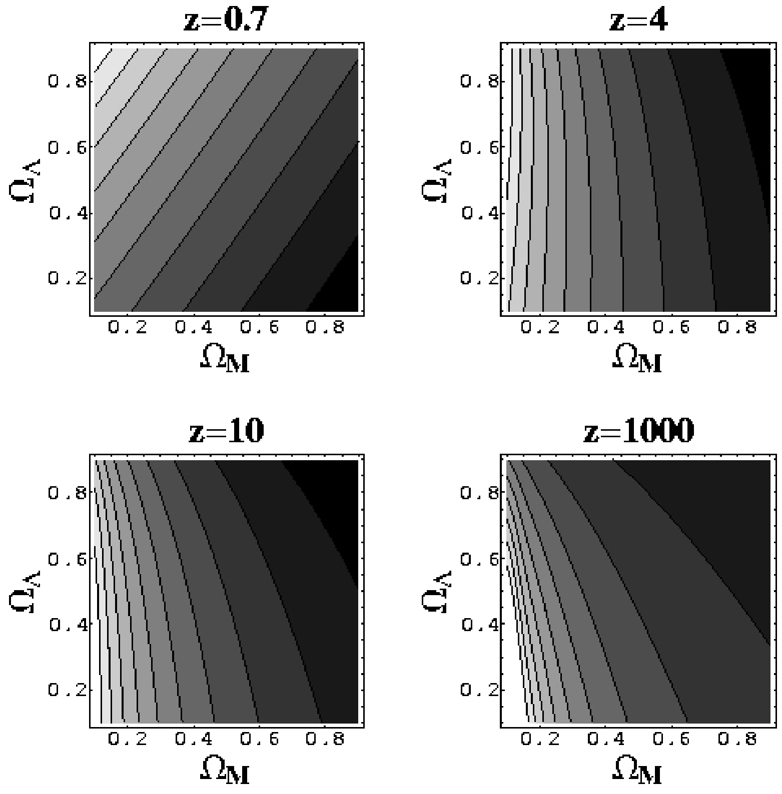
$$E^2(z) = \Omega_M(1+z)^3 + \Omega_\Lambda + \Omega_K(1+z)^2$$

- The luminosity distance therefore depends upon the cosmological constant and, like for the age, increases for Ω_Λ increasing. Therefore, a larger cosm. const. induces a smaller luminosity of the standard candles.

- Suppose we have a source of known absolute luminosity $M = -2.5 \log L + const$. Then one defines instead of the flux f an apparent magnitude $m = -2.5 \log f + const$ as

$$m - M = 25 + 5 \log d(z; \Omega_M, \Omega_\Lambda)$$

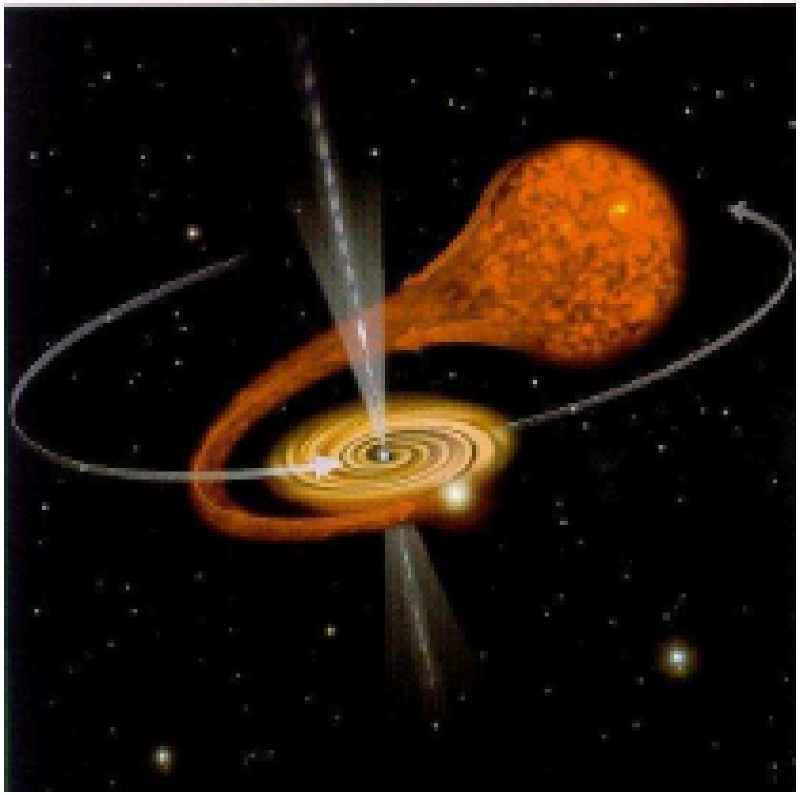
If M is the same for every object, then the apparent magnitude gives directly $d(z)$ and is then possible to test for the presence of a cosmological constant.



Curves of constant luminosity distance

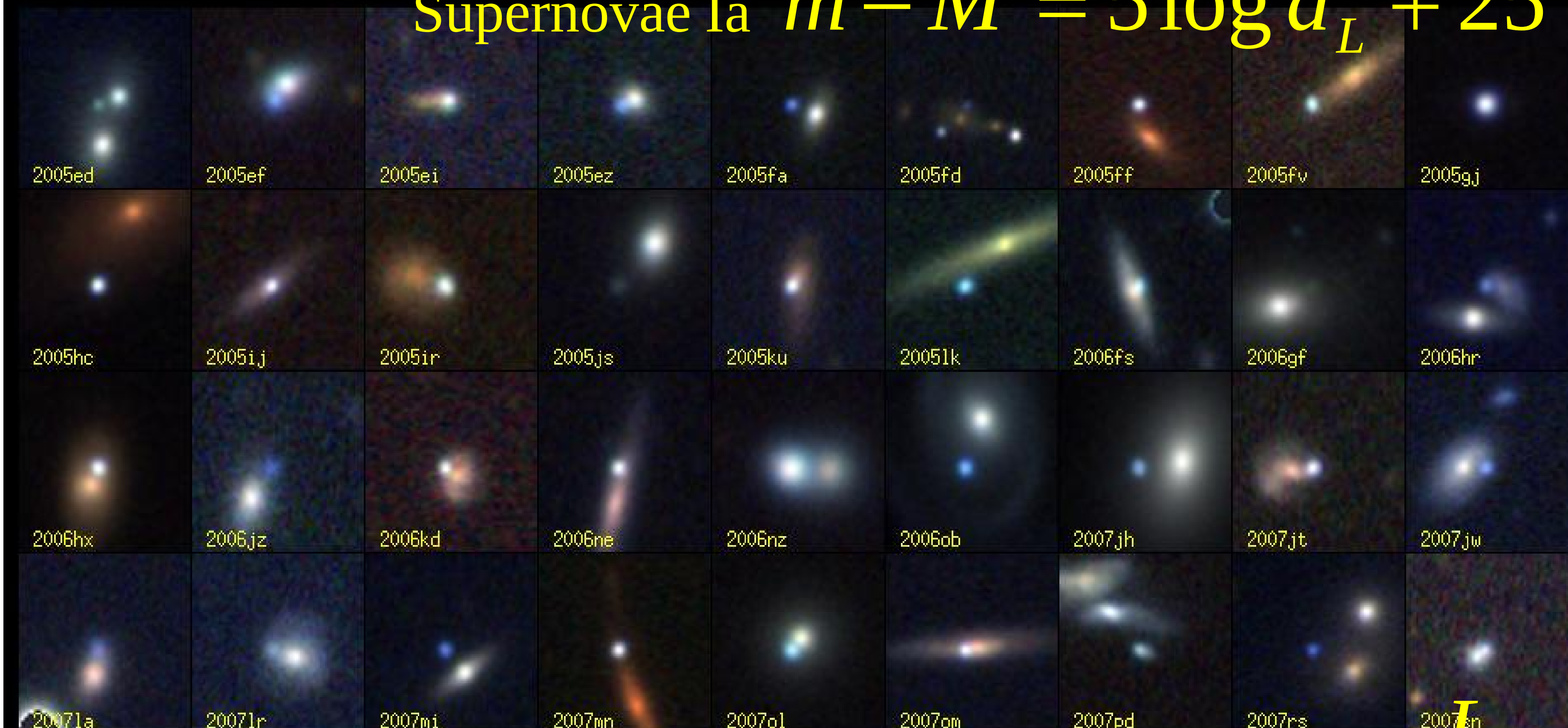
Standard candles

- Are there **standard candles** in nature ?
- The best such thing so far are **supernovae Ia**.

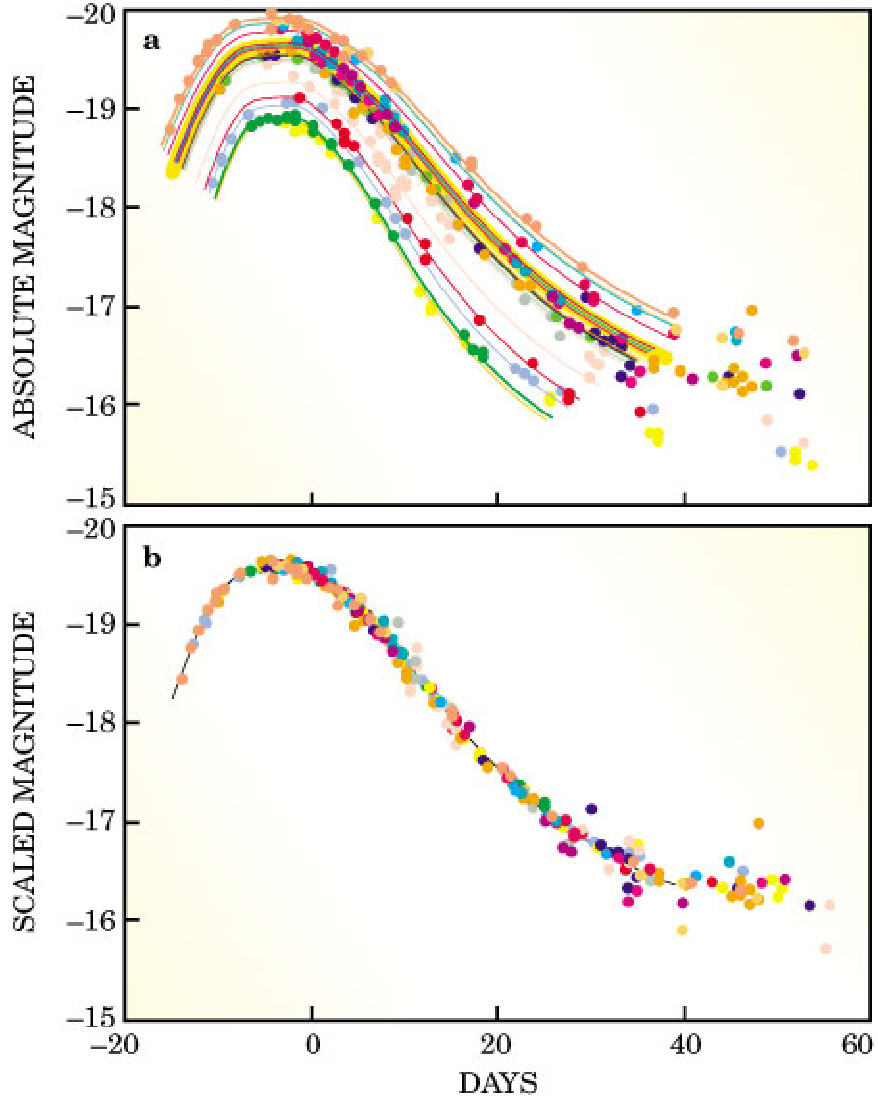


Lighthouses in the dark

Supernovae Ia $m - M = 5\log d_L + 25$



$$d^2 = \frac{L}{4\pi f}$$

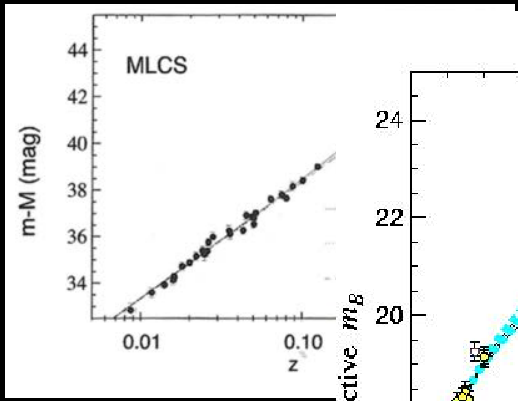


Phillips, Hamuy, et al.
1993, 1995

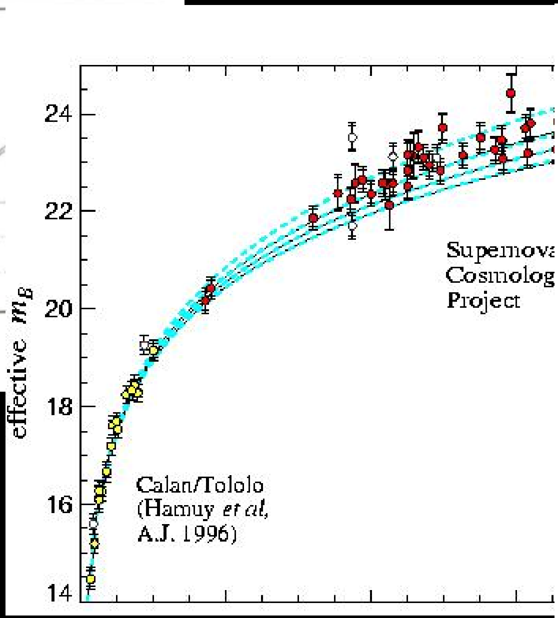
- Then, we compare $m_{obs}(z)$ with

$$m_{theor}(z) = M + 25 + \log d(z; \Omega_M, \Omega_\Lambda, ..)$$

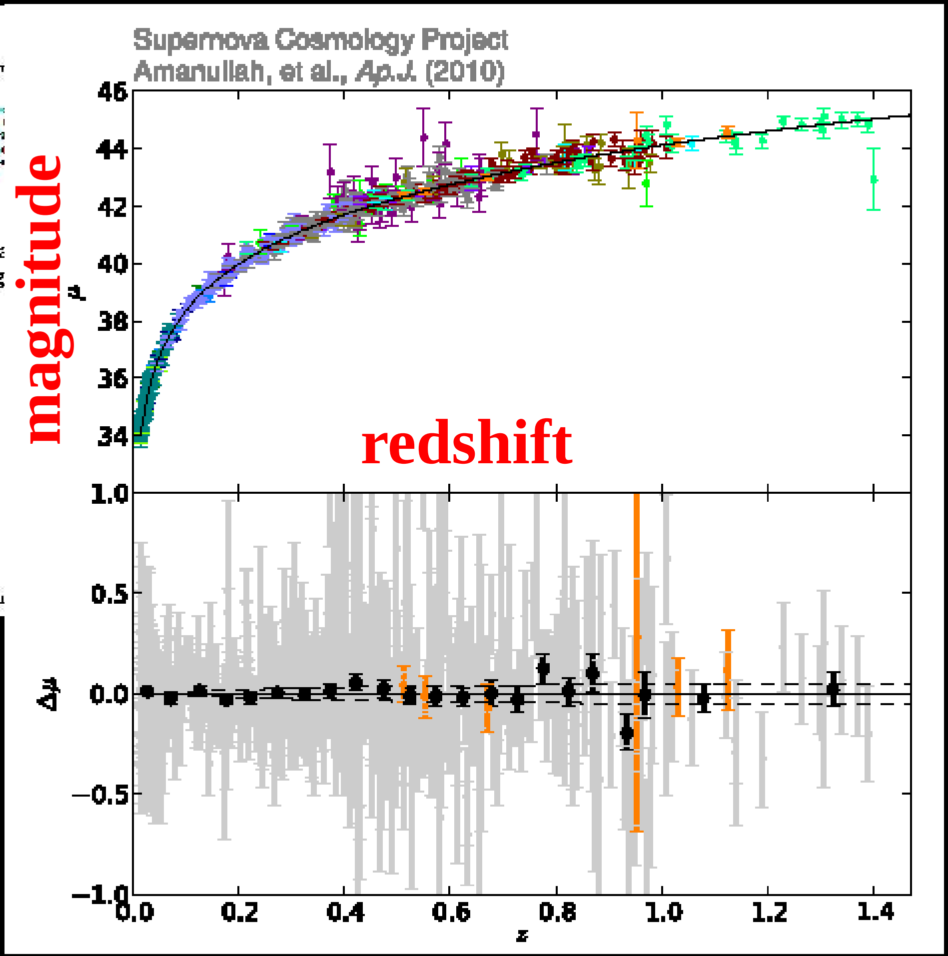
Hubble diagram



1997



1998



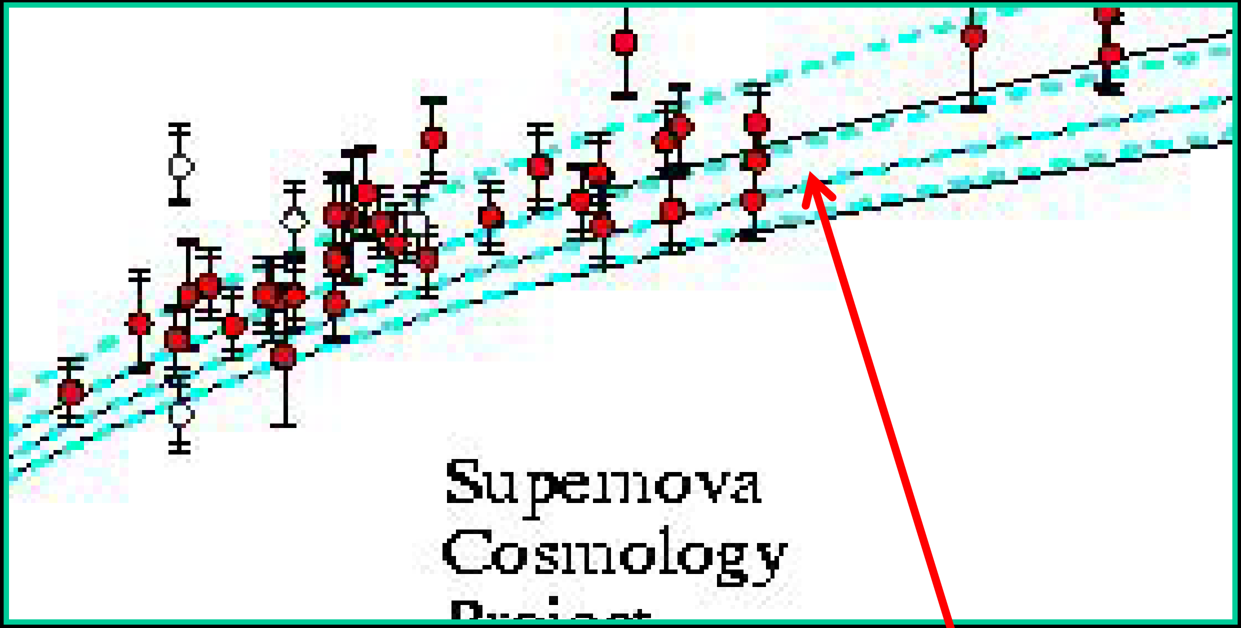
2010

Bug or feature?

The SNIa are dimmer than expected in a ordinary matter universe!

BUT:

- Dependence on progenitors?
- Contamination?
- Environment?
- Host galaxy?
- Dust?
- Lensing?
- Unknowns?



Ordinary matter

Cosmological explanation

There is however a simple cosmological solution

Local Hubble law

$$r(z) = \frac{z}{H_0}$$

$$\longrightarrow$$

$$r(z) = \int \frac{dz}{H(z)}$$

Global Hubble law

If H(z) in the past is smaller (i.e. acceleration), then r(z) is larger: larger distances (for a given redshift) make dimmer supernovae

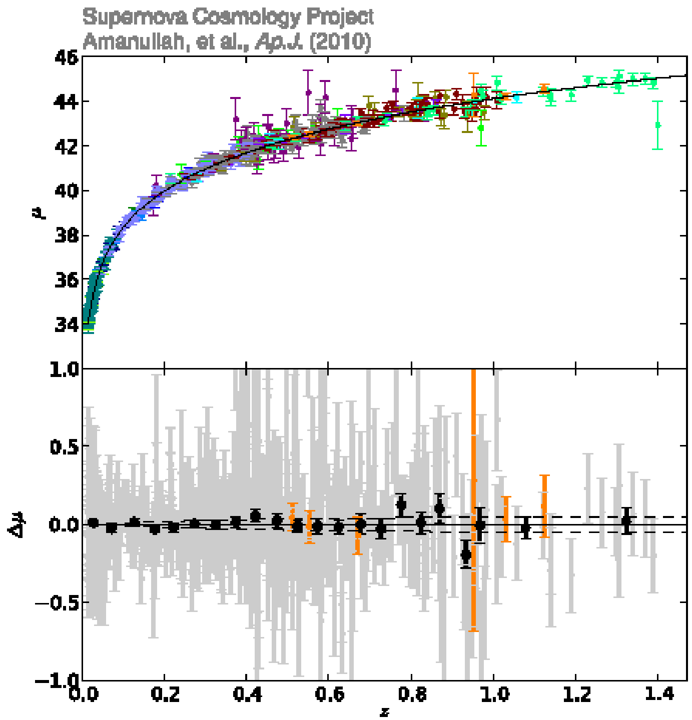
- Remember our “reference” cosmology

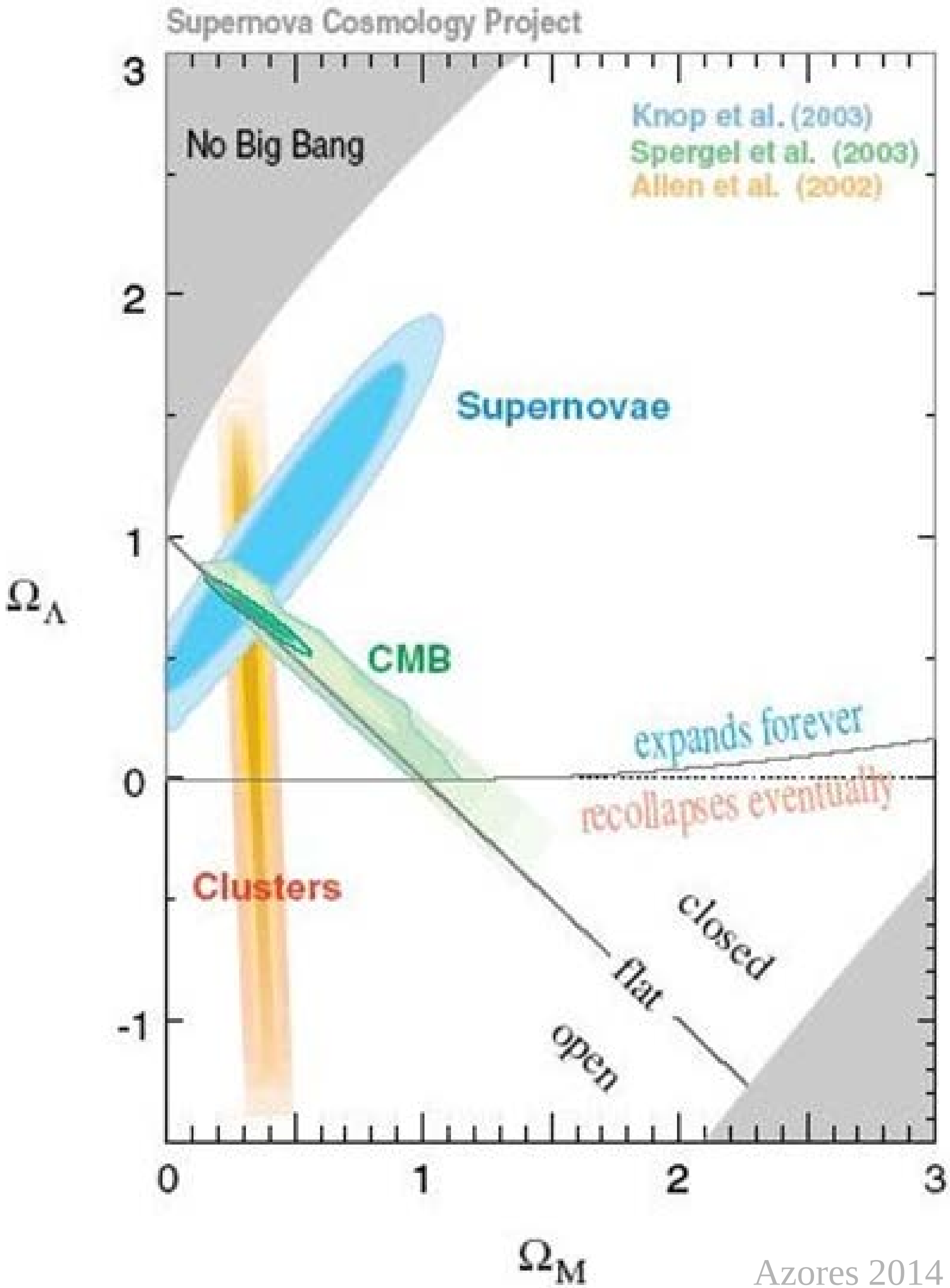
$$E^2(z) = \Omega_M(1+z)^3 + \Omega_\Lambda + \Omega_K(1+z)^2$$

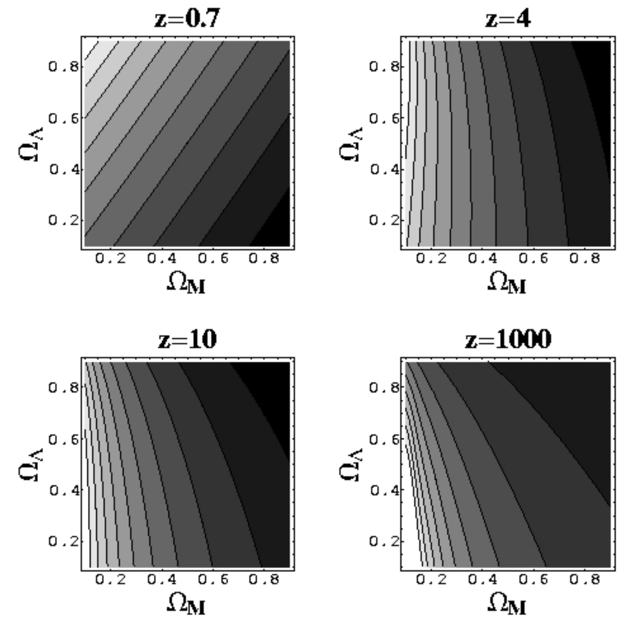
Fit with

$$m(z) = const + 5\log d_L(z)$$

$$d_L(z) \approx (1+z)\int \frac{dz}{H(z)}$$




Supernova Cosmology Project
No Big Bang
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)
Supernovae
CMB
Clusters
expands forever
recollapses eventually
closed
flat
open
Ω_Λ
Ω_M
3
2
1
0
-1
0
1
2
3


z=0.7
z=4
z=10
z=1000
Ω_Λ
Ω_M
0.2
0.4
0.6
0.8

From Dark Energy to Dark Force

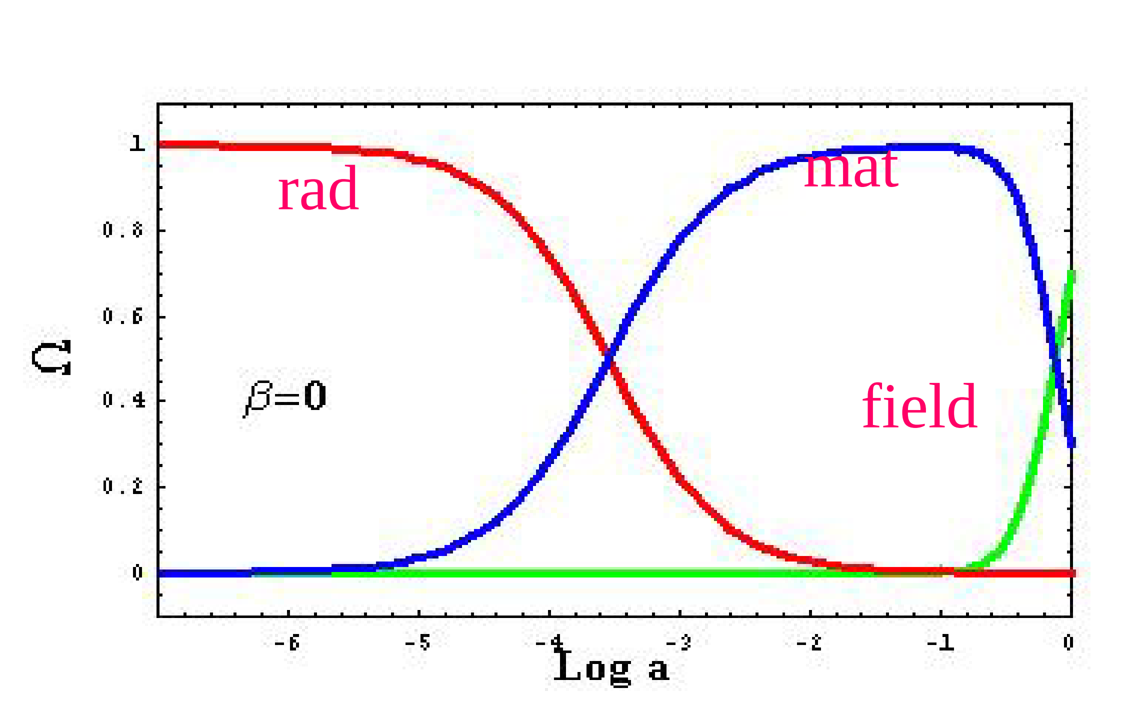
The two problems of the cosmological constant:

1. The fine tuning problem

2. The coincidence problem

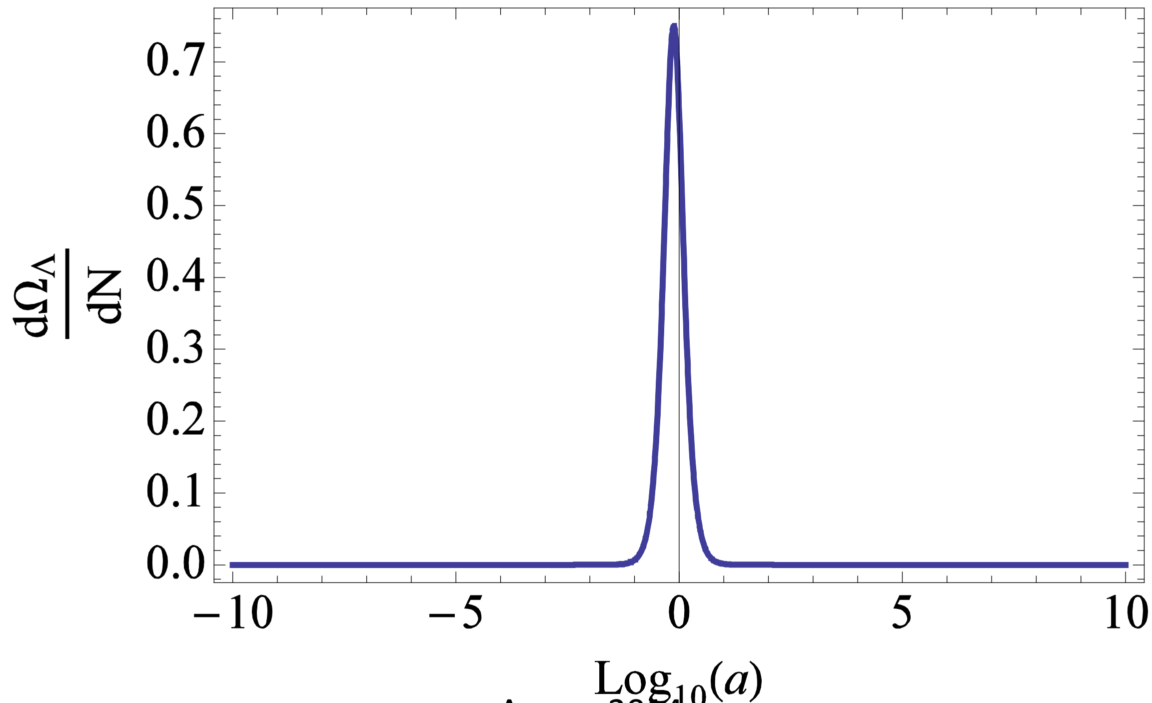
Why now?

The coincidence problem



Why now?

The coincidence problem



Beyond the cosmological constant

- A cosm. constant, as we have seen, has a negative pressure and does not fluctuate.
- But any fluid with pressure $p = wr$ such that $w < -1/3$ has in fact similar properties.
- The simplest case is a scalar field f. Pressure and energy for a potential $V(f)$ are

$$p = \frac{1}{2}\dot{f}^2 - V(f), \quad r = \frac{1}{2}\dot{f}^2 + V(f)$$

So that the conservation equation

$$\dot{r} + 3H(r + p) = 0$$

is in fact the Klein Gordon equation

$$\ddot{f} + 3H\dot{f} + V^0 = 0$$

The eq. of state is

$$w = \frac{\frac{1}{2}\dot{f}^{2} - V(f)}{\frac{1}{2}\dot{f}^{2} + V(f)}$$

kin. energy dominates	$w\,!\;\;1$	stiff fluid
equipartition	$w\,!\;\;0$	dust
pot. energy dominates	$w\,!\;\;-1$	cosm. const.
negative kin. energy	$w < -1$	phantom

- In general, the equation of state will vary with time.
- As a first approximation

$$\begin{aligned} w &= w_0 + w_1 z \\ w &= w_0 + w_1(a - 1) \end{aligned}$$

Scalar field Action

$$S = \int \sqrt{-g}\, d^4 x \left[\frac{1}{16\pi G} R + L_\phi \right]$$

$$L_\phi = -\frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi)$$

Energy-momentum tensor

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_\phi)}{\delta g^{\mu\nu}} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi) \right] .$$

Klein-Gordon equation

$$\phi_{;\mu}^{;\nu} - V'(\phi) = 0$$

- This matter component, denoted dark energy or quintessence, has properties similar to the cosm. constant but

1. it fluctuates at large scale
2. is composed of particles of microscopical mass

$$m = V_{,\phi\phi}^{1/2} = H\hbar = 10^{-32} eV$$

(the energy of a particle with Compton wavelength is equal to the horizon scale 3000 Mpc !)

3. gives an expansion different from the cosm. constant, and therefore in principle observable with the same methods as above

- The crucial point is that a scalar field does not cluster because its "sound speed"

$$c_s^2 = \frac{dp}{d\rho}$$

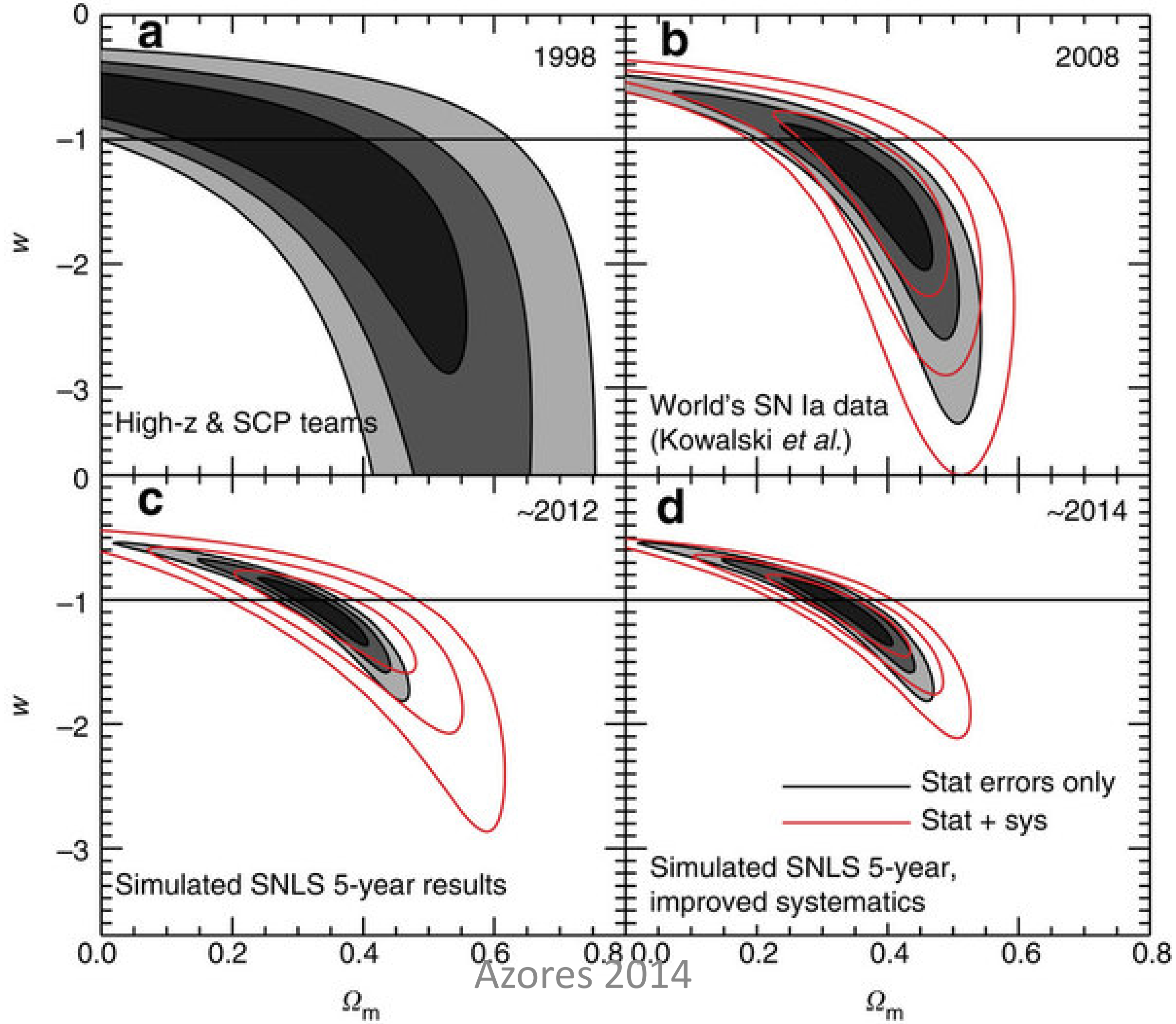
is equal to the speed of light. That is, its own pressure resists gravitational collapse. Perturbing the Klein-Gordon equation in Fourier space:

$$\ddot{\varphi} + 2H\dot{\varphi} + c_s^2 k^2 \varphi + a^2 U''\varphi = 0$$

At small scales, k dominates and the solution for φ oscillates acoustically around zero instead of growing. Therefore, a scalar field is a good candidate for dark energy.

- Repeat the SNIa fit with

$$E^2(z) = \Omega_M(1+z)^3 + \Omega_{DE}(1+z)^{3+3w_{DE}}$$



Linear perturbations

- General problem

$$d(R_{\mu n} - \frac{1}{2} g_{\mu n} R) = 8 p d(T_{\mu n})$$

- This introduces, at linear level, the following quantities:

$$dr\,, dp, dv, dg_{\mu n}$$

- The general perturbed metric can be written as

$$g_{\mu n} = g_{\mu n}^{(0)} + g_{\mu n}^{(1)}$$

where in all generality

$$g_{\mu n}^{(1)} = a^2 \begin{pmatrix} 2y & w_i \\ w_i & 2f\, d_{ij} + h_{ij} \end{pmatrix}$$

- However, this is far too general for what we need. First, as any tensor field, the metric tensor can be written as a sum of terms that depend on purely scalar, vector and tensor quantities. For instance

$$w \;\text{⌘}\; w^{k} + w^{?} = \text{—}w_{s} + w^{?} \qquad (6)$$

and

$$h_{ij} \;\text{⌘}\; h d_{ij}/3 + h^{k}_{ij} + h^{?}_{ij} + h^{T}_{ij} \qquad (7)$$

Only the scalar quantities couple to dr so we consider only these now.

- Then, we can choose any reference frame, i.e. we can change the coordinates to $y^{\mu} = f(x^{n})$! 4 conditions on the metric coefficients. However, we would like to keep the unperturbed part $g^{(0)}_{\mu n}$ as it is. Then we can subject the perturbed part to 4 extra conditions: this is called *gauge choice*.
- One of the simplest choice is called *longitudinal* or *Newtonian*: we put $w_i = h = 0$ and obtain finally

$$g^{(1)}_{\mu n} = a^{2} \;\checkmark\; \begin{matrix} 2 & 0 \\ 0 & 2\Phi d_{ij} \end{matrix} \;\blacklozenge \qquad (8)$$

- Then we get the general perturbation equation. In particular, we also obtain $= \Phi$ if the fluid is a perfect fluid (no anisotropic stress).

- Newtonian regime: small scales with respect to horizon H^{-1}, small velocities, for a single component

$$
\begin{aligned}
\dot{d} &= -\text{—}v \\
\dot{v}_i &= Hv_i - \text{—}\Phi \\
4\,\Phi &= -4pa^2 r\, d
\end{aligned}
$$

- Deriving the first we get for the density perturbations $d = (r - r_M)/r_M$

$$\ddot{d} + 2H\dot{d} = 4pr_M d \text{ ⌘ } \frac{3}{2}H^2 d$$

$!$ evolution of a density contrast under a gravitational potential proportional to r_M.

- To solve the equation is sufficient to know the behavior of $H(t)$ and $r(t)$. If there is only matter, $H^2 = H_0^2 a^{-3}$ and the solutions are

$$d \leftarrow a, \quad d \leftarrow a^{-3/2}$$

- Dark energy changes the growth of perturbations.

Growth of perturbations

- If there is a smooth component as the cosmological constant (for which $\delta_\Lambda = 0$) then all it changes is that $H^2 = H_0^2 \Omega_m a^{-3}$ where $\Omega_m < 1$: weaker gravity forcing. If $\Omega_m \simeq const.$ then

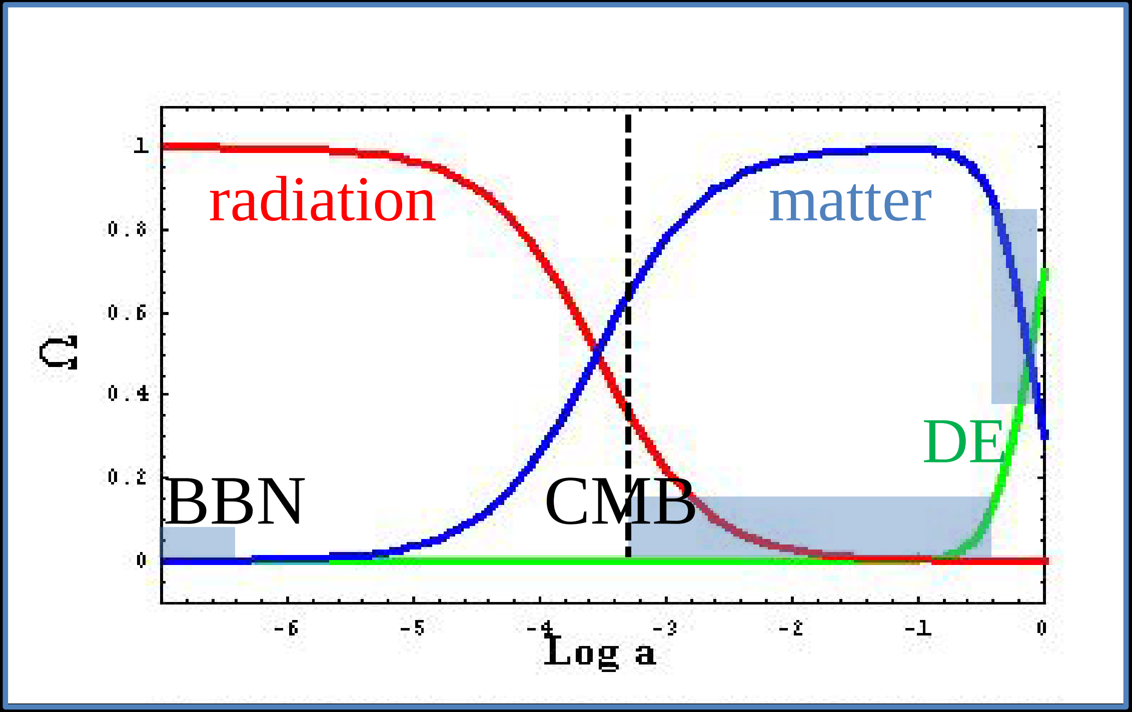
$$\begin{aligned} \delta &\propto a^p \\ p &= \frac{1}{4}\left(-1 \pm \sqrt{1 + 24\Omega_m}\right) \end{aligned}$$

- If the cosmological constant is the dominating component, the second member vanishes (there is no potential gradient) : $\delta \simeq$ const.
- A better way to study perturbation growth is to parametrize the growth in this way

$$\frac{d \log \delta}{d \log a} \simeq \Omega_m(a)^\gamma, \quad \gamma \simeq 0.55$$

Time view

We know so little about the evolution of the universe!

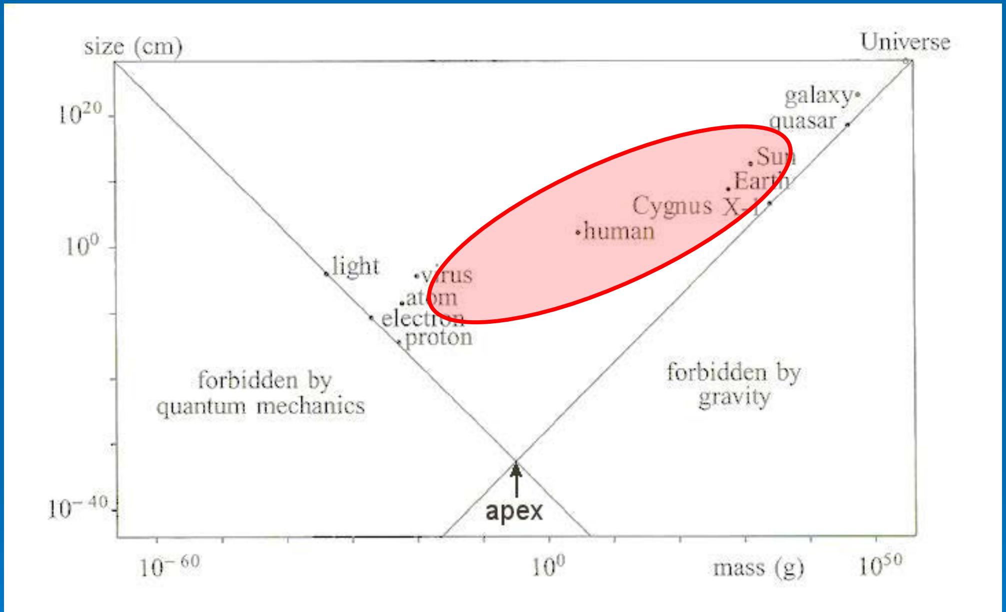


Gravity: what and what not

- Gravity is universal, long range and unscreened
- Is the force responsible for the structure and evolution of the Universe
- is governed by the well-tested Einstein theory
- is a force mediated by a spin-2 massless particle universally coupled to all fields

Is that all?

- However, we only directly test gravity within the solar system, at the present time, and with “baryons”

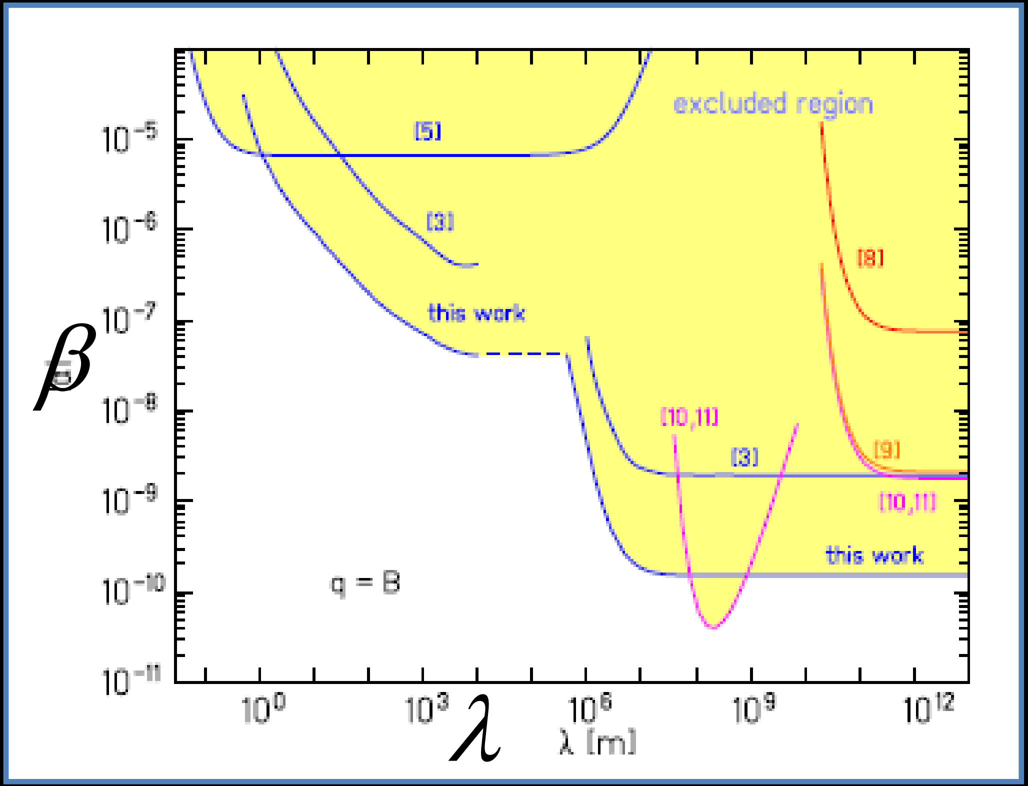


On Space and Time, Edited by Shahn Majid

...and we have yet to catch a graviton!

Testing Gravity

$$\frac{GM}{r} \rightarrow \frac{GM}{r}(1+\beta e^{-r/\lambda})$$



Schlamminger et al 2008

A systematic approach to testing MG

Observations:

- Isotropy
- Large abundance
- Slow evolution
- Weak clustering

Theory:

- Scalar field?
- $\Omega_{DE} \approx \Omega_m$
- $w_{eff} \approx -1$
- $c_s \approx 1$

The past ten years of DE research

$$\int dx^4 \sqrt{-g}\left[R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter} \right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter} \right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter} \right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi, \frac{1}{2}\phi_{,\mu}\phi^{,\mu})R + G_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter} \right]$$

Cosmological constant, Dark energy w=const, Dark energy w=w(z),quintessence, scalar-tensor model, coupled quintessence, k-essence, f(R), Gauss-Bonnet, Galileons, KGB,

The Horndeski Lagrangian

The most general 4D scalar field theory with second order equation of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

$$\begin{aligned}
\mathcal{L}_2 &= \underline{K(\phi, X)}, \\
\mathcal{L}_3 &= \underline{-G_3(\phi, X)}\Box\phi, \\
\mathcal{L}_4 &= \underline{G_4(\phi, X)}\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \\
\mathcal{L}_5 &= \underline{G_5(\phi, X)}\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)\right].
\end{aligned}$$

- First found by Horndeski in 1975
- rediscovered by Deffayet et al. in 2011
- no ghosts, no classical instabilities
- it modifies gravity!
- it includes f(R), Brans-Dicke, k-essence, Galileons, clustering DE etc etc
- Invariant under conformal and disformal transformations

The next ten years of DE research

Combine observations of background, linear and non-linear perturbations to reconstruct as much as possible the Horndeski & Massive Gravity model

… or to rule them out!

The Great Horndeski Hunt

Let us assume we have only

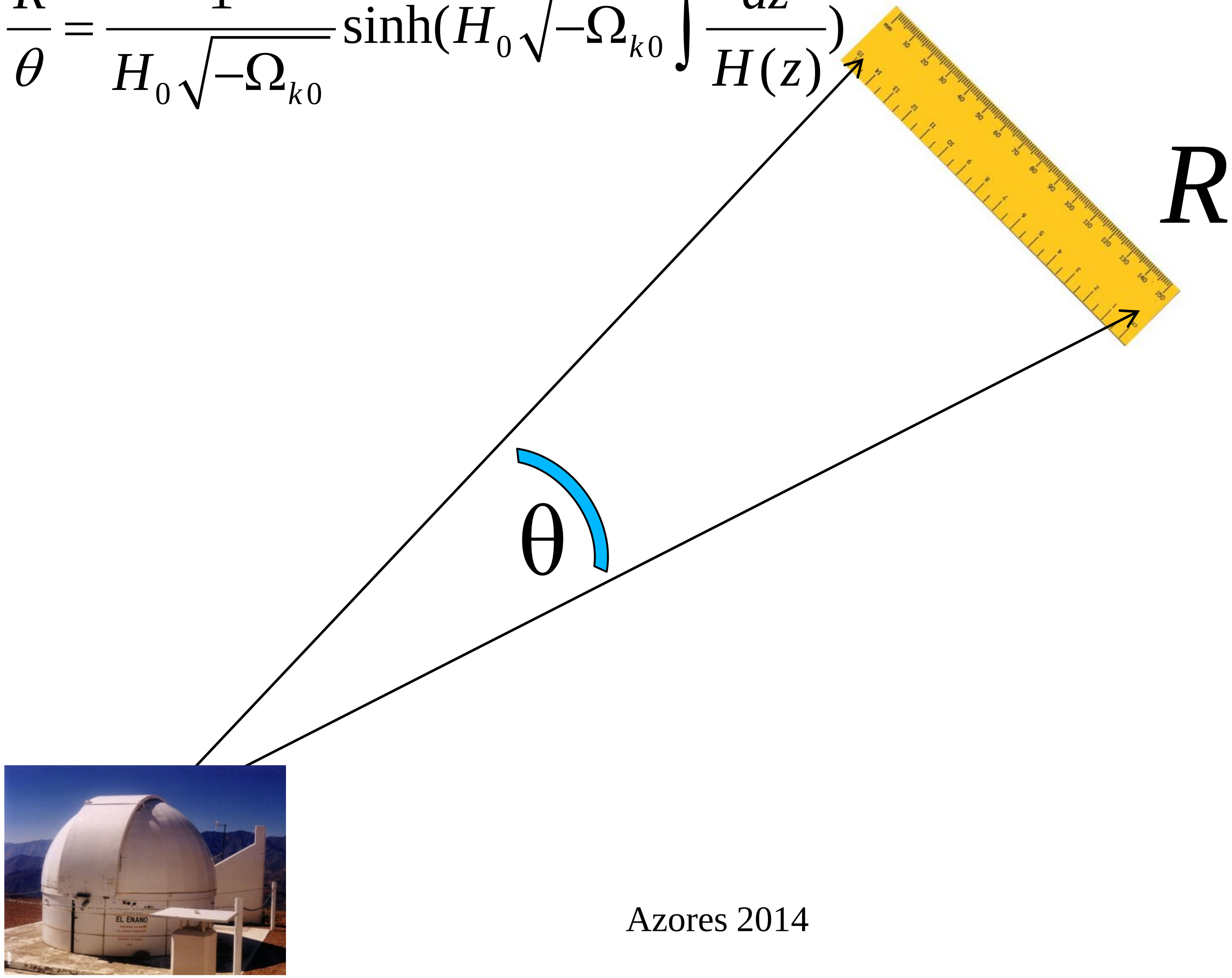
1) a perturbed FRW metric

2) pressureless matter

3) the Horndeski field

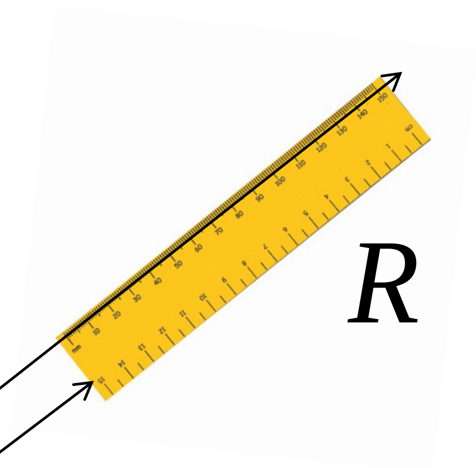
Standard rulers

$$D(z) = \frac{R}{\theta} = \frac{1}{H_0\sqrt{-\Omega_{k0}}} \sinh(H_0\sqrt{-\Omega_{k0}} \int \frac{dz}{H(z)})$$

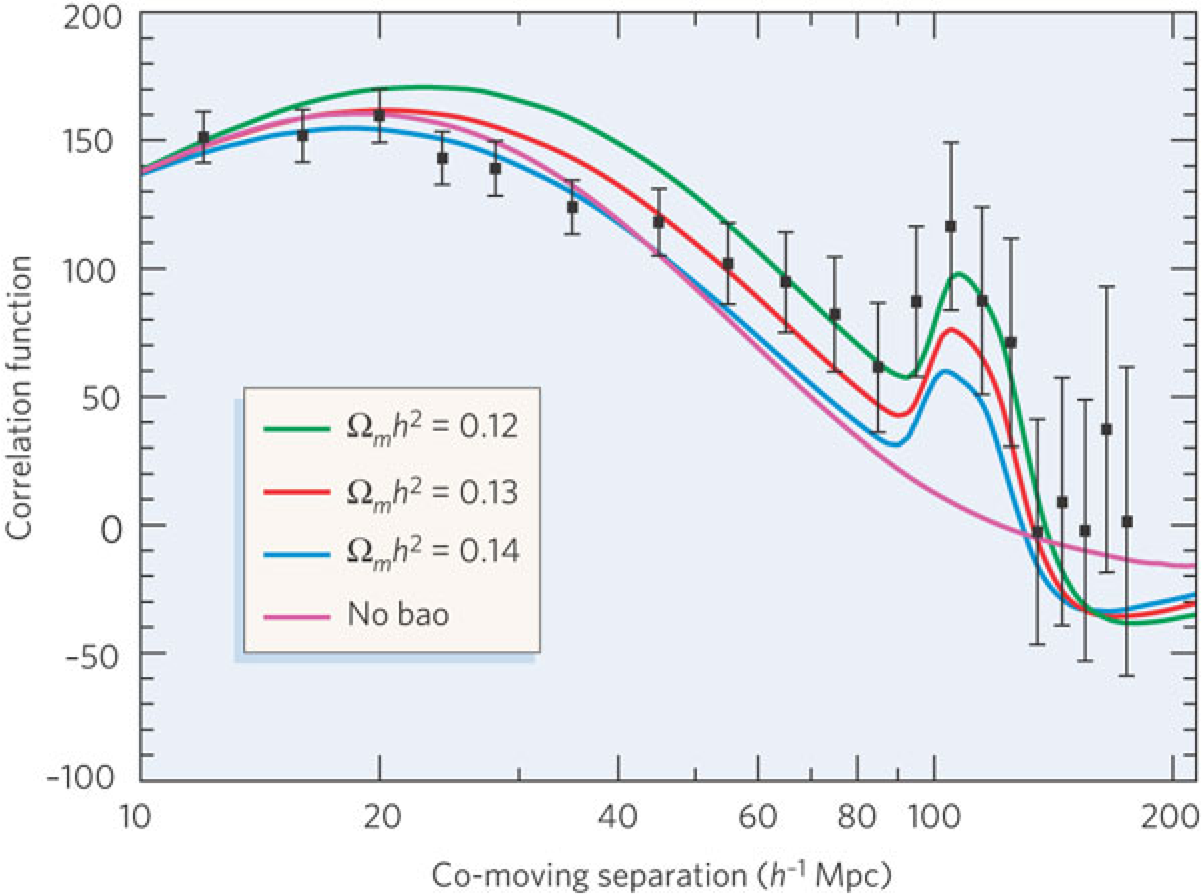


Standard rulers

$$H(z) = \frac{dz}{R}$$



BAO ruler



Charles L. Bennett

Nature 440, 1126-1131(27 April 2006)

Background: SNIa, BAO, ...

Then we can measure H(z) and

$$D(z) = \frac{1}{H_0\sqrt{-\Omega_{k0}}}\sinh(H_0\sqrt{-\Omega_{k0}}\int\frac{dz}{H(z)})$$

and therefore we can reconstruct the full FRW metric

$$ds^2 = dt^2 - \frac{a(t)^2}{\left(1-\frac{\Omega_{k0}}{4}r^2\right)^2}(dx^2+dy^2+dz^2)]$$

Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation

$$\nabla^2\Psi = 4\pi G\rho_m\delta_m$$

- anisotropic stress

$$1 = -\frac{\Psi}{\Phi}$$

Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation $$\nabla^2\Psi = 4\pi G\, Y(k,a)\rho_m \delta_m$$
- anisotropic stress $$\eta(k,a) = -\frac{\Phi}{\Psi}$$

Modified Gravity at the linear level

- standard gravity

$$Y(k,a)=1$$
$$\eta(k,a)=1$$

- scalar-tensor models

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{2(F+F'^2)}{2F+3F'^2}$$
$$\eta(a)=1+\frac{F'^2}{F+F'^2}$$

Boisseau et al. 2000
Acquaviva et al. 2004
Schimd et al. 2004
L.A., Kunz &Sapone 2007

- f(R)

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{1+4m\frac{k^2}{a^2R}}{1+3m\frac{k^2}{a^2R}},\quad \eta(a)=1+\frac{m\frac{k^2}{a^2R}}{1+2m\frac{k^2}{a^2R}}$$

Bean et al. 2006
Hu et al. 2006
Tsujikawa 2007

- DGP

$$Y(a)=1-\frac{1}{3\beta};\quad \beta=1+2Hr_cw_{DE}$$
$$\eta(a)=1+\frac{2}{3\beta-1}$$

Lue et al. 2004;
Koyama et al. 2006

- massive bi-gravity

$$Y(a)=....$$
$$\eta(a)=....$$

see F. Koennig and L. A. 2014

Modified Gravity at the linear level

In the quasi-static limit, every Horndeski model is characterized at linear scales by the two functions

$$\eta(k,a) = h_2 \left(\frac{1+k^2 h_4}{1+k^2 h_5} \right)$$

$$Y(k,a) = h_1 \left(\frac{1+k^2 h_5}{1+k^2 h_3} \right)$$

k = wavenumber

$\boldsymbol{h_i}$ **= time-dependent functions**

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

Modified Gravity at the linear level

$$h_1 \equiv \frac{w_4}{w_1^2} = \frac{c_T^2}{w_1}, \qquad h_2 \equiv \frac{w_1}{w_4} = c_T^{-2},$$

$$h_3 \equiv \frac{H^2}{2XM^2} \frac{2w_1^2 w_2 H - w_2^2 w_4 + 4w_1 w_2 \dot{w}_1 - 2w_1^2(\dot{w}_2 + \rho_m)}{2w_1^2},$$

$$h_4 \equiv \frac{H^2}{2XM^2} \frac{2w_1^2 H^2 - w_2 w_4 H + 2w_1 \dot{w}_1 H + w_2 \dot{w}_1 - w_1(\dot{w}_2 + \rho_m)}{w_1},$$

$$h_5 \equiv \frac{H^2}{2XM^2} \frac{2w_1^2 H^2 - w_2 w_4 H + 4w_1 \dot{w}_1 H + 2\dot{w}_1{}^2 - w_4(\dot{w}_2 + \rho_m)}{w_4},$$

$$\begin{aligned}
w_1 \equiv & 1 + 2\left(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}XHG_{5,X}\right), \\
w_2 \equiv & -2\dot{\phi}\left(XG_{3,X} - G_{4,\phi} - 2XG_{4,\phi X}\right) + \\
& + 2H\left(w_1 - 4X\left(G_{4,X} + 2XG_{4,XX} - G_{5,\phi} - XG_{5,\phi X}\right)\right) - \\
& - 2\dot{\phi}XH^2\left(3G_{5,X} + 2XG_{5,XX}\right), \\
w_3 \equiv & 3X\left(K_{,X} + 2XK_{,XX} - 2G_{3,\phi} - 2XG_{3,\phi X}\right) + 18\dot{\phi}XH\left(2G_{3,X} + XG_{3,XX}\right) - \\
& - 18\dot{\phi}H\left(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2 G_{4,\phi XX}\right) - \\
& - 18H^2\left(1 + G_4 - 7XG_{4,X} - 16X^2 G_{4,XX} - 4X^3 G_{4,XXX}\right) - \\
& - 18XH^2\left(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2 G_{5,\phi XX}\right) + \\
& + 6\dot{\phi}XH^3\left(15G_{5,X} + 13XG_{5,XX} + 2X^2 G_{5,XXX}\right), \\
w_4 \equiv & 1 + 2\left(G_4 - XG_{5,\phi} - XG_{5,X}\ddot{\phi}\right).
\end{aligned}$$

De Felice et al. 2011; L.A. et al.,PRD, arXiv:1210.0439, 2012

Yukawa Potential

$$\eta(k,a) = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

Momentum space

$$\nabla^2\Psi = 4\pi G\, Y(k,a)\rho_m\delta_m$$

$$\Psi = -\frac{GM}{r}h_2\left(1+\frac{h_4-h_5}{h_5}e^{-r/\sqrt{h_5}}\right) = -\frac{\bar{G}M}{r}\left(1+Qe^{-mr}\right)$$

Real space

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

Quasi-static approximation

$$c_s^2 k^2 \square \; a^2 H^2$$

$$\phi_{;\mu}\phi^{;\mu} - V' = Q\rho$$

$$\phi \rightarrow (\delta\phi)e^{ikx}$$

$$-\delta\ddot{\phi} - k^2\delta\phi - V''\delta\phi = Q(\delta\rho)$$

$$k^2\delta\phi + m^2\delta\phi = -Q(\delta\rho)$$

From a wave equation:

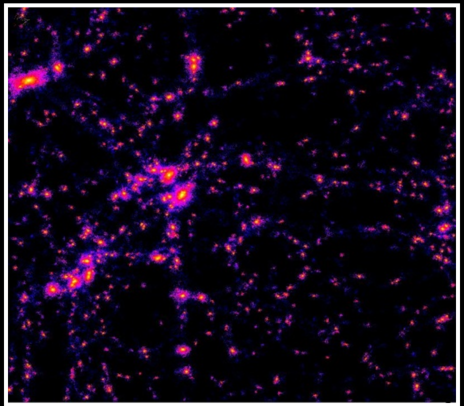
$$\begin{aligned} E_{\delta\phi} \;\equiv\; & D_1\ddot{\Phi} + D_2\ddot{\delta\phi} + D_3\dot{\Phi} + D_4\dot{\delta\phi} + D_5\dot{\Psi} + D_6\frac{k^2}{a^2}\dot{\chi} \\ & + \left(D_7\frac{k^2}{a^2} + D_8\right)\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + \left(D_{10}\frac{k^2}{a^2} + D_{11}\right)\Psi + D_{12}\frac{k^2}{a^2}\chi = 0\,, \end{aligned}$$

To a "Poisson" equation:

$$B_7\frac{k^2}{a^2}\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + A_6\frac{k^2}{a^2}\Psi \;\simeq\; 0\,,$$

Reconstruction of the metric

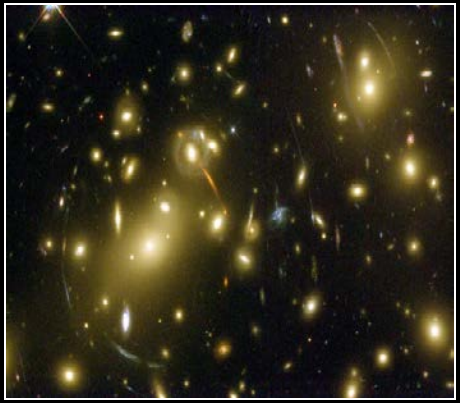
$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$



massive particles respond to Ψ

$$\delta'' + (1+\frac{H'}{H})\delta' = \nabla^2\Psi$$

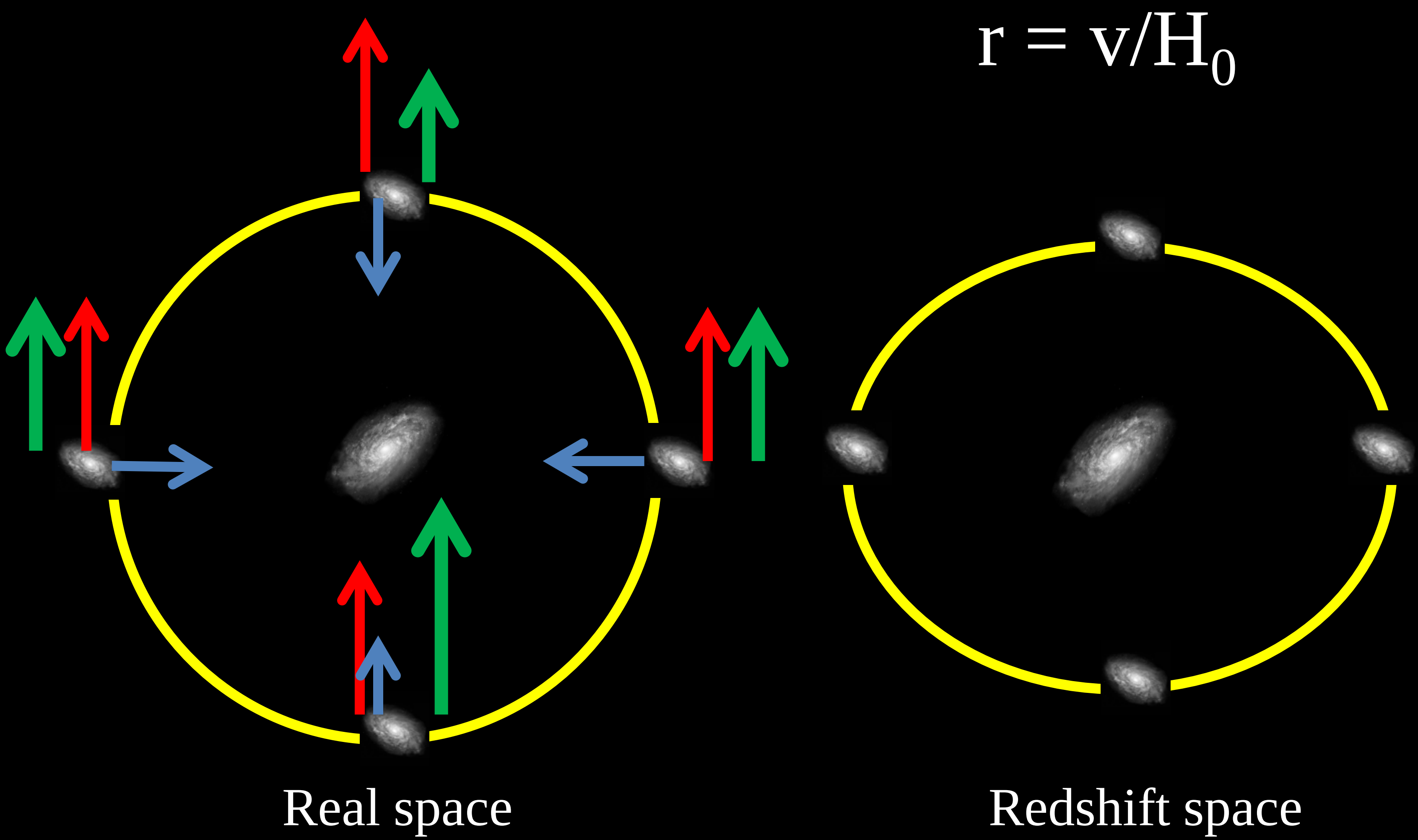
$$\delta = \frac{\delta\rho}{\rho}$$

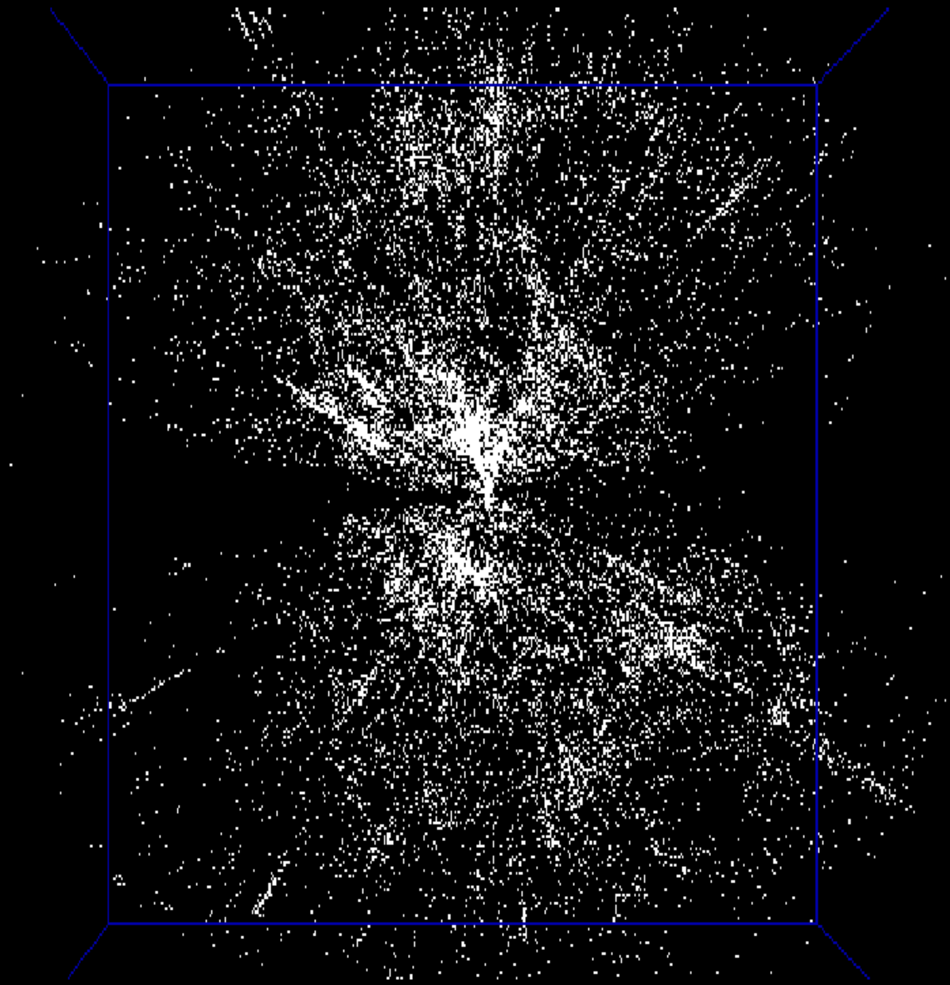


massless particles respond to Φ-Ψ

$$\alpha = \int \nabla_{perp}(\Psi - \Phi)dz$$

Peculiar velocities





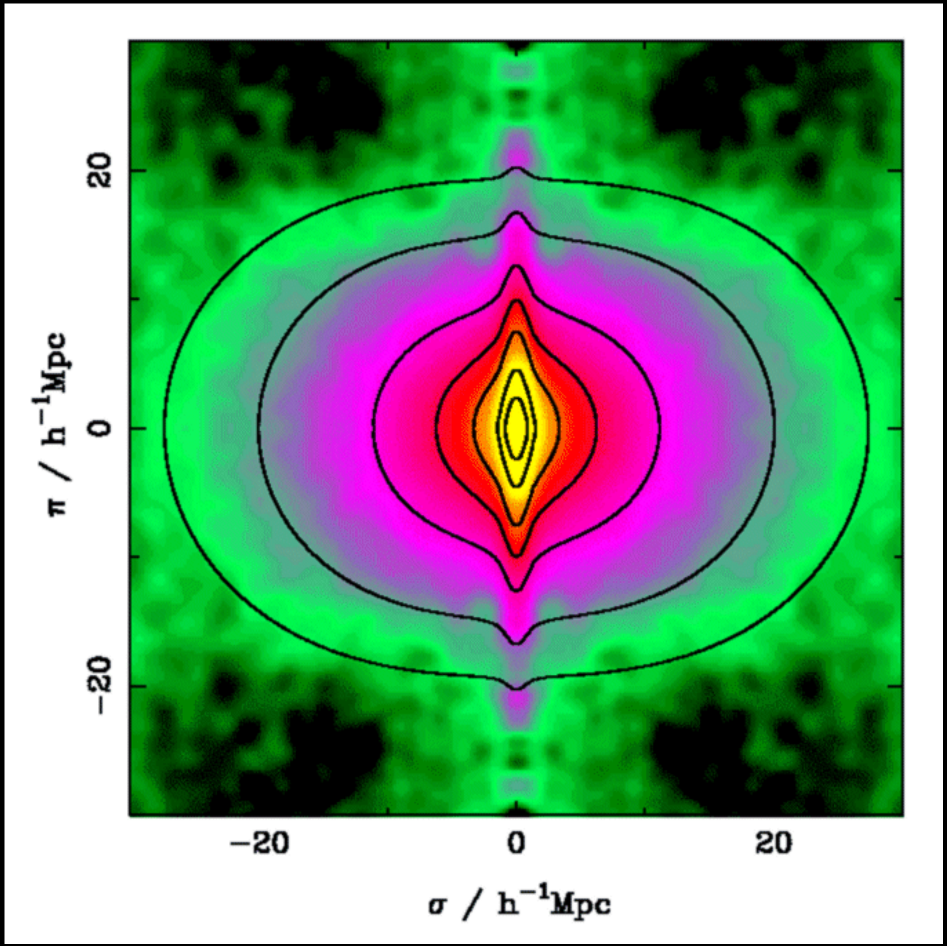
Peculiar velocities

$$P_z = (1 + \beta\mu^2)^2 P_r$$

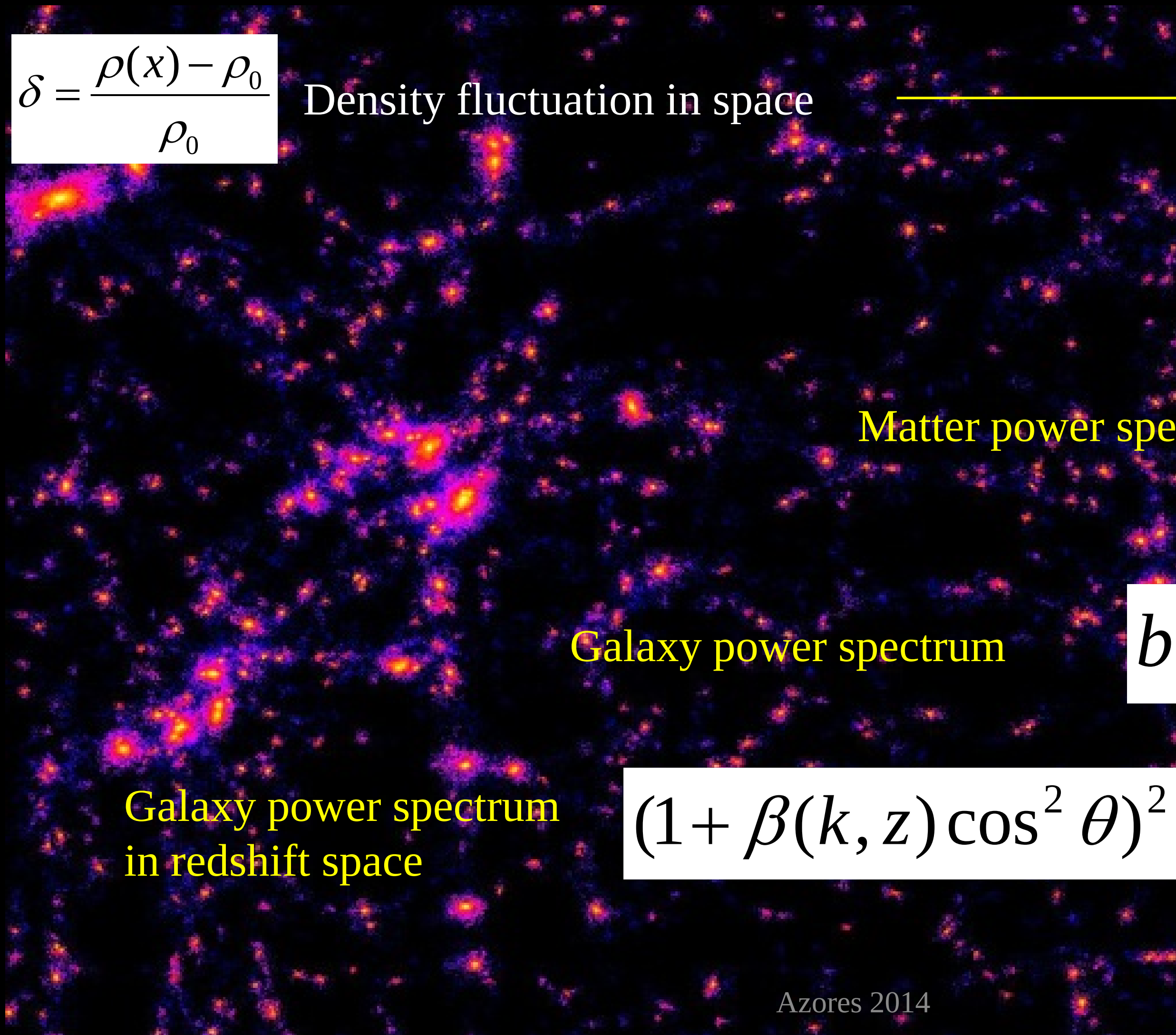
redshift distortion parameter

$$\beta = \frac{\delta'}{\delta b}$$

Kaiser 1987



Reality check



$$\delta = \frac{\rho(x) - \rho_0}{\rho_0}$$ Density fluctuation in space → $$\left\langle {\delta_k}^2 \right\rangle = P(k, z)$$

Matter power spectrum $$P_{matter}(k, z)$$

Galaxy power spectrum $$b^2(k, z) P_{matter}(k, z)$$

Galaxy power spectrum in redshift space $$(1 + \beta(k, z)\cos^2\theta)^2 b^2(k, z) P_{matter}(k, z)$$

Deconstructing the galaxy power spectrum

Redshift distortion

Galaxy clustering

Line of sight angle

$$\delta_{gal}(k, z, \mu) = G b \sigma_8 \left(1 + \frac{f}{b}\mu^2\right)\delta_{t,0}(k)$$

$$\left(f = \frac{\delta'}{\delta}\right)$$

Growth function

Galaxy bias

normalization

Present mass power spectrum

Three linear observables: A, R, L

clustering

μ=0
Amplitude
A

$$\delta_{gal}(k,z,0) = Gb\sigma_8\delta_{m,0}(k) \equiv A$$

μ=1
Redshift distortion
R

$$\delta_{gal}(k,z,1) = G\sigma_8 f\,\delta_{m,0}(k) \equiv R$$

lensing

Lensing
L

$$k^2\Phi_{lens} = k^2(\Psi - \Phi) = -\frac{3}{2}\Sigma G\Omega_m\sigma_8\delta_{m,0}(k) \equiv L$$

$$\Sigma = Y(1+\eta)$$

The only model-independent ratios

Redshift distortion/Amplitude

$$P_1 = \frac{R}{A} = \frac{f}{b}$$

Lensing/Redshift distortion

$$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f}$$

Redshift distortion rate

$$P_3 = \frac{R'}{R} = \frac{f'}{f} + f$$

Expansion rate

$$E = \frac{H}{H_0}$$

How to combine them to test the theory?

Summarizing....

Matter conservation equation independent of gravity theory

$$\delta_m'' + (1 + \frac{\mathcal{H}'}{\mathcal{H}})\delta_m' = -k^2\Psi$$

Observables

$$P_2 = \frac{L}{R} = \frac{\Omega_{m0}Y(1+\eta)}{f} \qquad P_3 = \frac{R'}{R} = \frac{f'}{f} + f \qquad E = \frac{H}{H_0}$$

The anisotropic stress is directly observable

A unique combination of model independent observables

$$\frac{3P_2(1+z)^3}{2E^2\left(P_3+2+\frac{E'}{E}\right)}-1=\eta$$

Observables

Theory

Testing the entire Horndeski Lagrangian

A unique combination of model independent observables

$$\frac{3P_2(1+z)^3}{2E^2\left(P_3+2+\frac{E'}{E}\right)}-1=\eta=h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

Observables

Theory

L.A., M. Motta, I. Sawicki,
M. Kunz, I. Saltas, 1210.0439
1305.0008

Horndeski Lagrangian: not too big to fail

$$g(z,k) \equiv \frac{(REa^2)'}{LEa^2}$$

$$2g_{,k}g_{,kkk} - 3(g_{,kk})^2 = 0$$

If this relation is falsified, the Horndeski theory is rejected*

L.A., M. Motta, I. Sawicki,
M. Kunz, I. Saltas, 1210.0439

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy

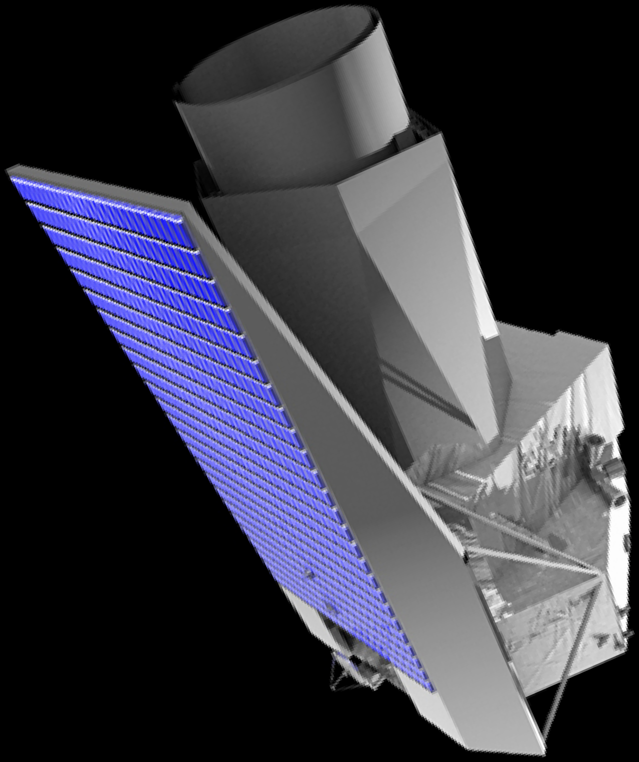
15,000 square degrees

70 million redshifts, 2 billion images

Median redshift z = 1

PSF FWHM ~0.18’’

>1000 peoples, >10 countries



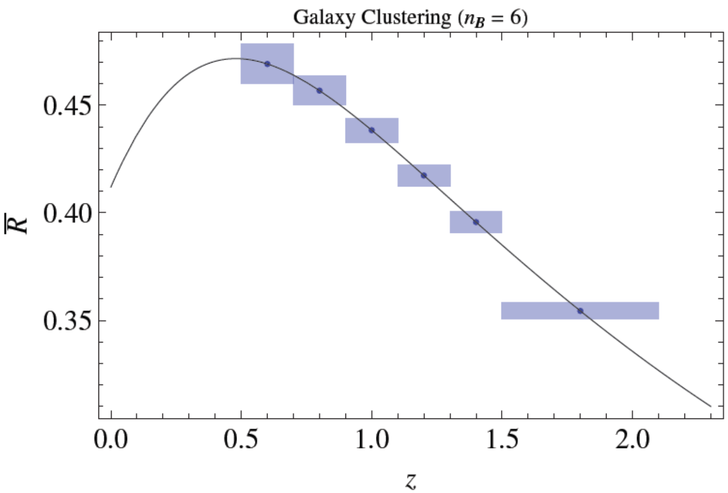
Euclid satellite

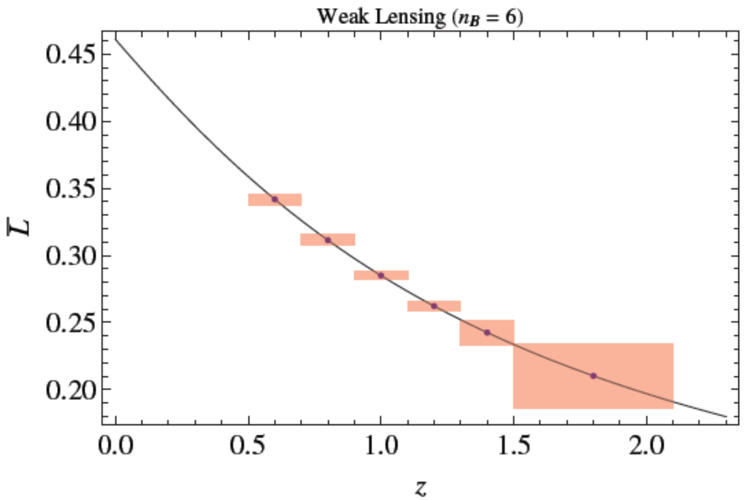
arXiv Red Book 1110.3193

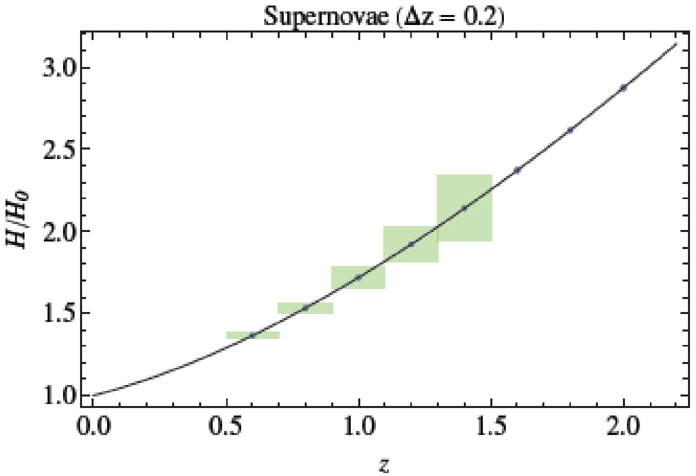
arXiv Theory Review 1206.1225

Euclid forecasts...

Combining galaxy clustering, weak lensing and SN....







100k SN (LSST)

$$G\sigma_8 f \delta_{m,0}(k) \equiv R$$

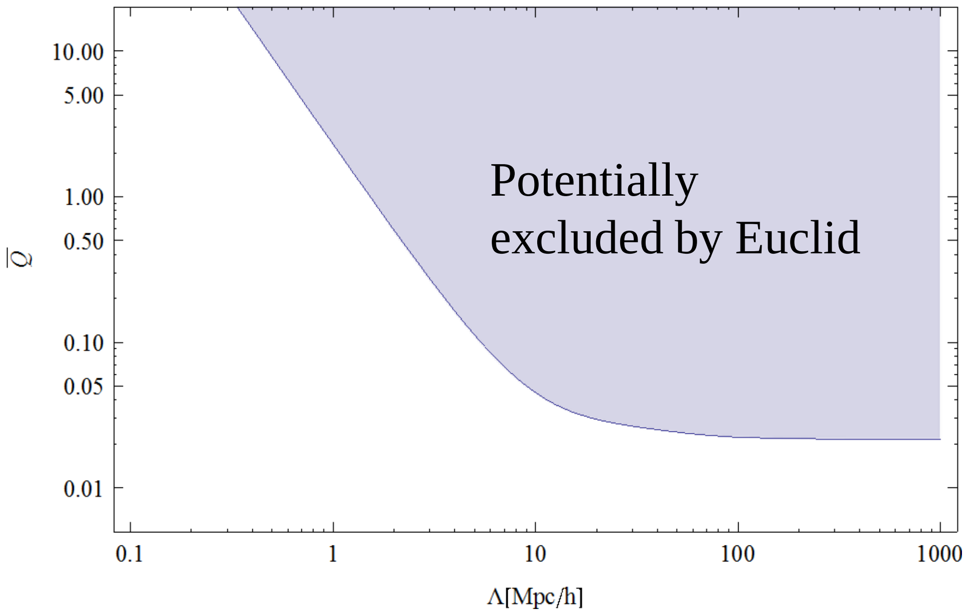
$$-\frac{3}{2}\Sigma G\Omega_m \sigma_8 \delta_{m,0}(k) \equiv L$$

A cosmological exclusion plot

Model 4: η has the Horndeski form
Error on h2, (h4-h5) in h/Mpc

$$\eta = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

$$\eta = -\frac{\bar{G}\mu}{r}\left(1+Qe^{-r/\lambda}\right)$$



Azores 2014

The fourfold way to gravity screening

$$G^* = G(1 + \alpha e^{-m_\varphi r})$$

$$m_\varphi, \alpha \left\{ \begin{array}{l} \text{depend on time} \\ \text{depend on space} \\ \text{depend on local density} \\ \text{depend on species} \end{array} \right.$$

Screening mechanisms

$$G^* = G(1+\alpha e^{-m_\varphi r})$$

$$m = m(\phi)$$

The field ϕ obeys a Poisson equation

$$\nabla^2\phi + m^2\phi = \alpha^{1/2}\rho\delta$$

So the solution is something like

$$\phi = \phi(\rho\delta) = \phi(\text{local density})$$

$$G^* = G(1+\alpha e^{-m_\varphi(\rho)r})$$

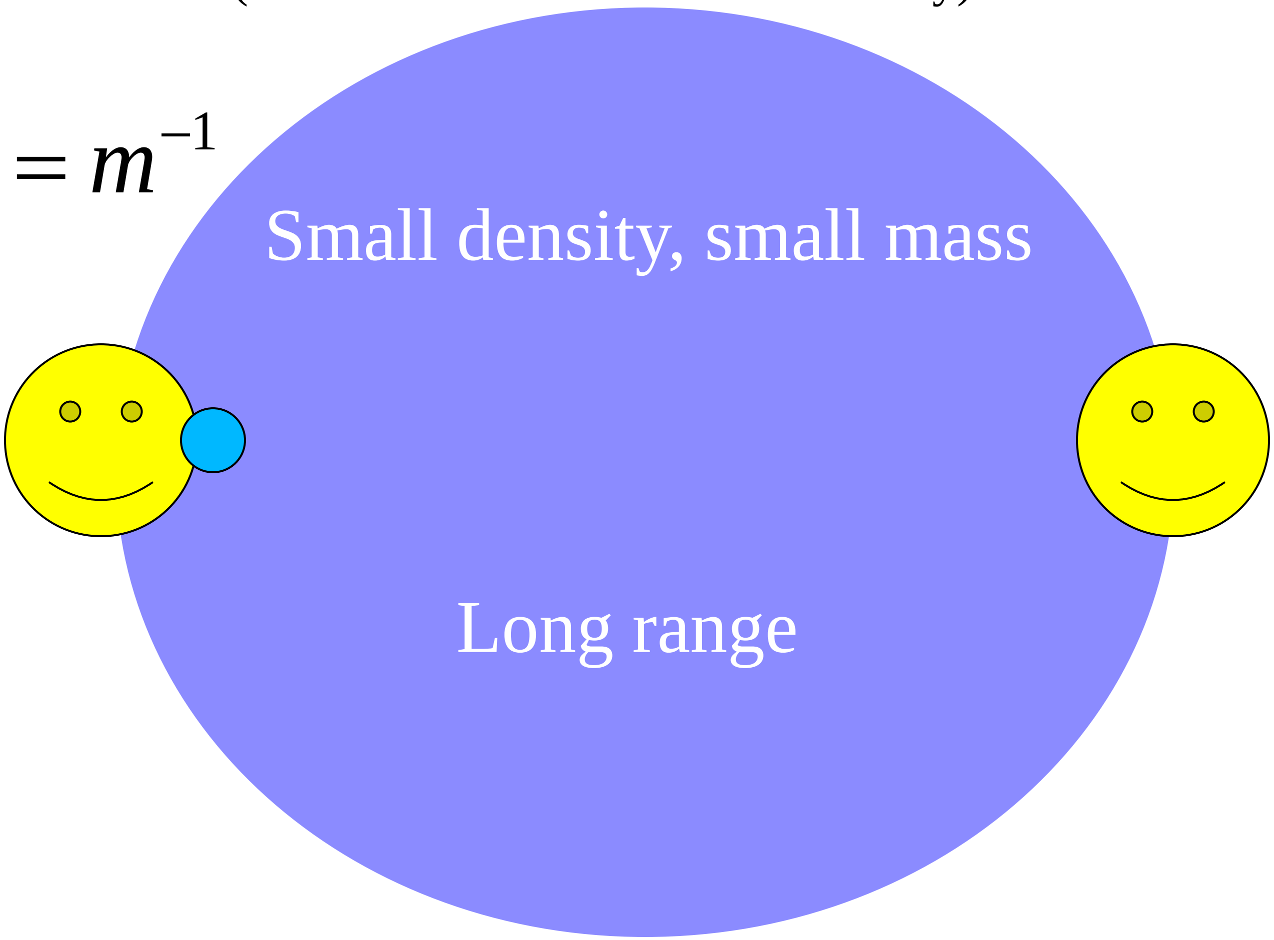
A density-dependent range!

Khoury Weltman 2003

Screening mechanisms

(mass increases with the local density)

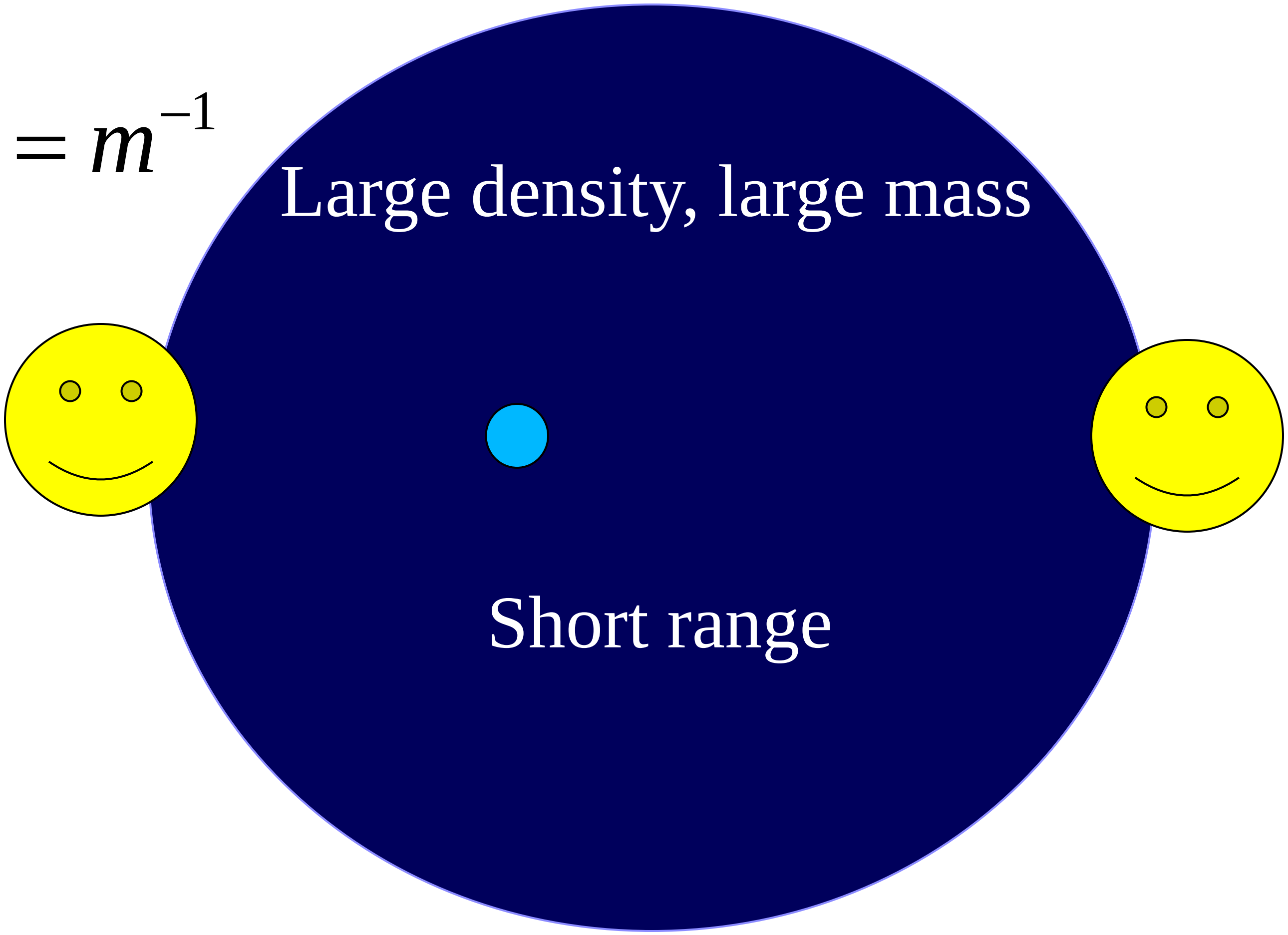
$$\text{range} = m^{-1}$$



Screening mechanisms

(mass increases with the local density)

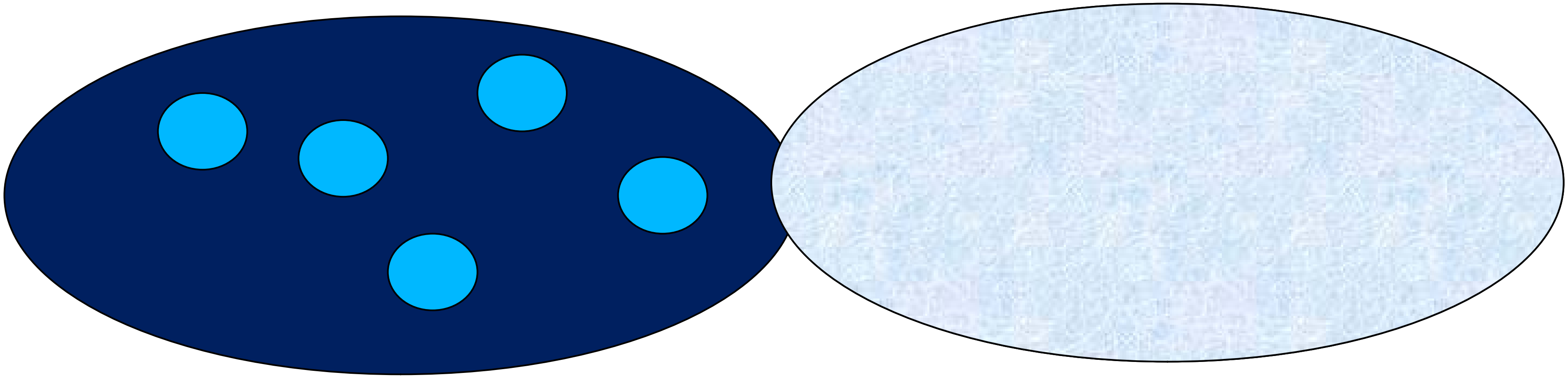
$$\text{range} = m^{-1}$$



Under the carpet..

Problem of non-linearity:
screening effects mix linear and non-linear scales

same density contrast



different physics