Boosting the Universe:
Observational consequences of our galaxy's motion through the CMB

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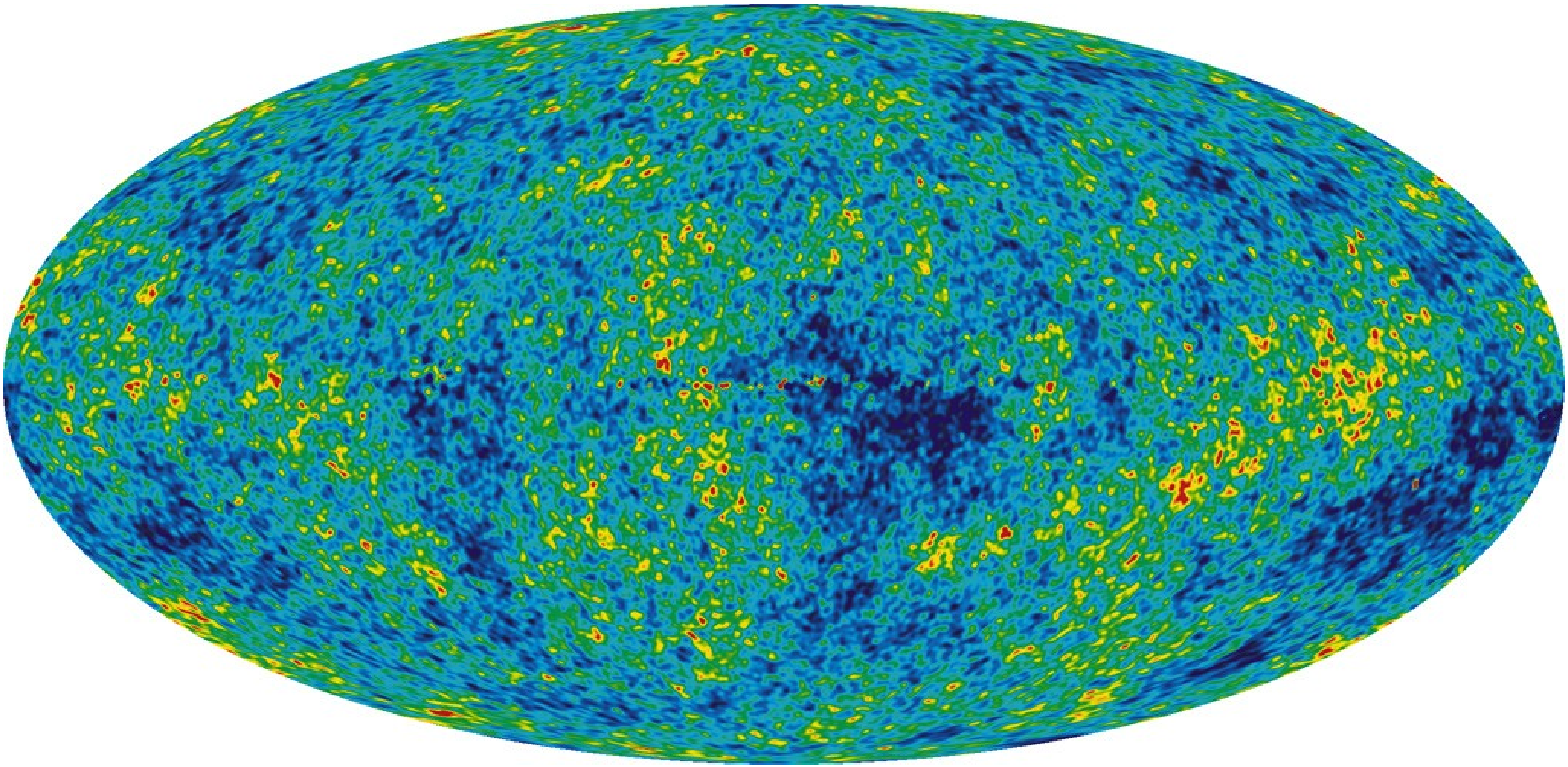
The CMB Sky



Credit: WMAP/NASA Science Team

Background

- Temperature maps are decomposed into spherical harmonics and their coefficients

$$\Delta T(\mathbf{n}) = \sum_{l>0} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\mathbf{n})$$

- The coefficients can be written in terms of the temperature fluctuations

$$a_{lm} = \int d\Omega_{\mathbf{n}}\, \Delta T(\mathbf{n}) Y^*_{lm}(\mathbf{n})$$

- If the coefficients are gaussian, we can exploit their properties to find the power spectrum

$$\langle a_{lm} \rangle = 0$$

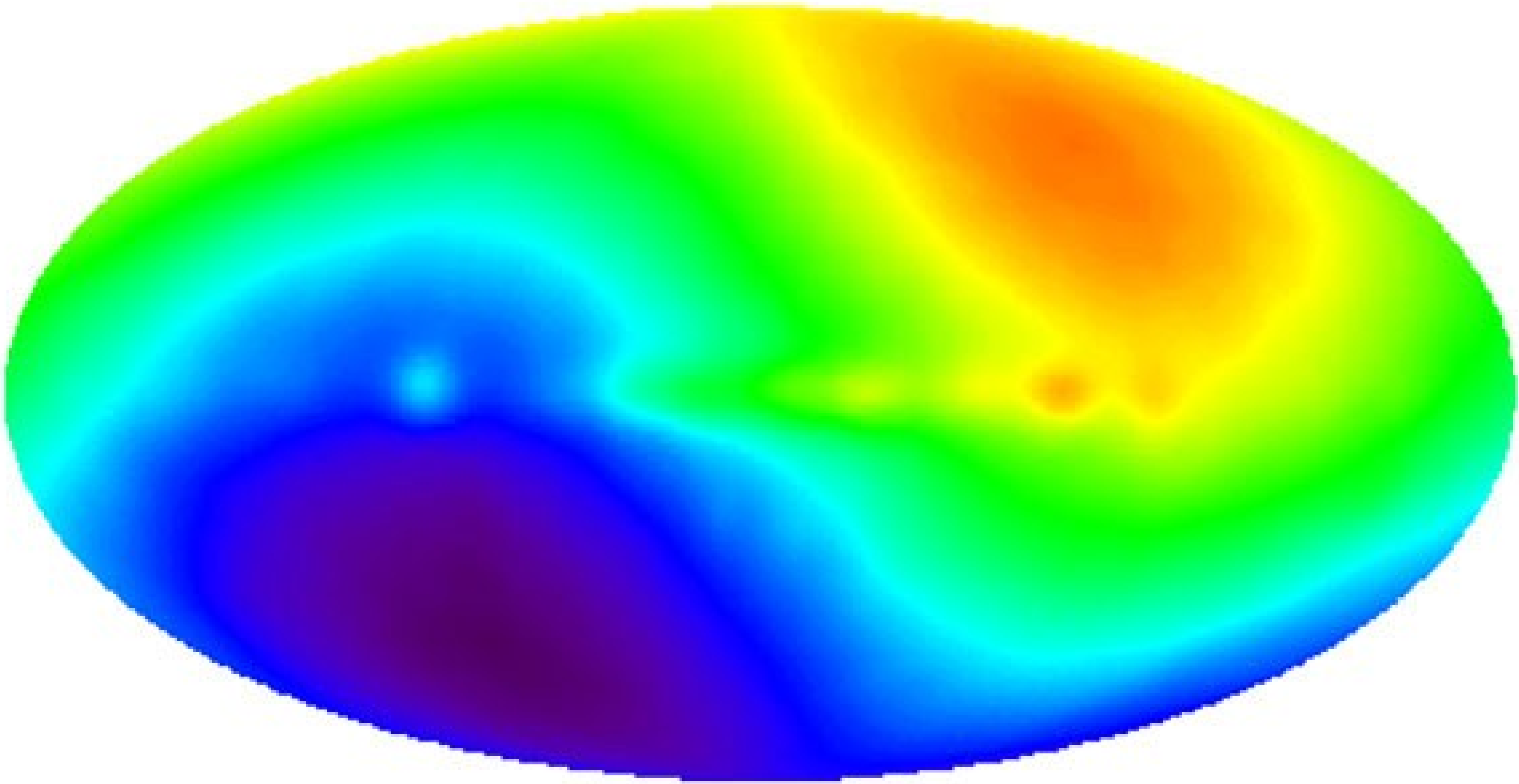
$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell,\ell'}\ \delta_{m,m'}\ C_\ell$$

- We can define an unbiased estimator of the C_l's in terms of the coefficients and use them to plot the power spectrum

$$\mathcal{C}_\ell \equiv \frac{1}{2\ell+1} \sum_m |a_{\ell m}|^2$$

However, many systematic and other effects must be accounted for before we can trust the data we are analyzing, such as...

The CMB dipole



From this we infer the Earth's motion to be β=.00123 through the CMB rest frame

The Lorentz boost effect has two pieces:

the Doppler effect

$$\nu' = \gamma\nu(1 + \beta\cos\theta)$$

and the aberration effect

$$\cos\theta' = \frac{\cos\theta + \beta}{1 + \beta\cos\theta}$$

where $\cos\theta \equiv \hat{\mathbf{n}} \cdot \hat{\mathbf{v}}$

Invariant quantity:

$$\frac{I'}{\nu'^3} = \frac{I}{\nu^3}$$

$$I'(\nu, \hat{\mathbf{n}}') = \gamma^3 \left(1 + \beta \cos\theta\right)^3 I(\nu, \hat{\mathbf{n}})$$

•Decompose this into spherical harmonics and their coefficients (as with the temperature fluctuations)

•Do an expansion to linear order in β

•Integrate over frequency

Which gives...

$$a'_{\ell m} = a_{\ell m} + \beta \xi^{+}_{\ell m} a_{(\ell+1)m} + \beta \xi^{-}_{\ell m} a_{(\ell-1)m}$$

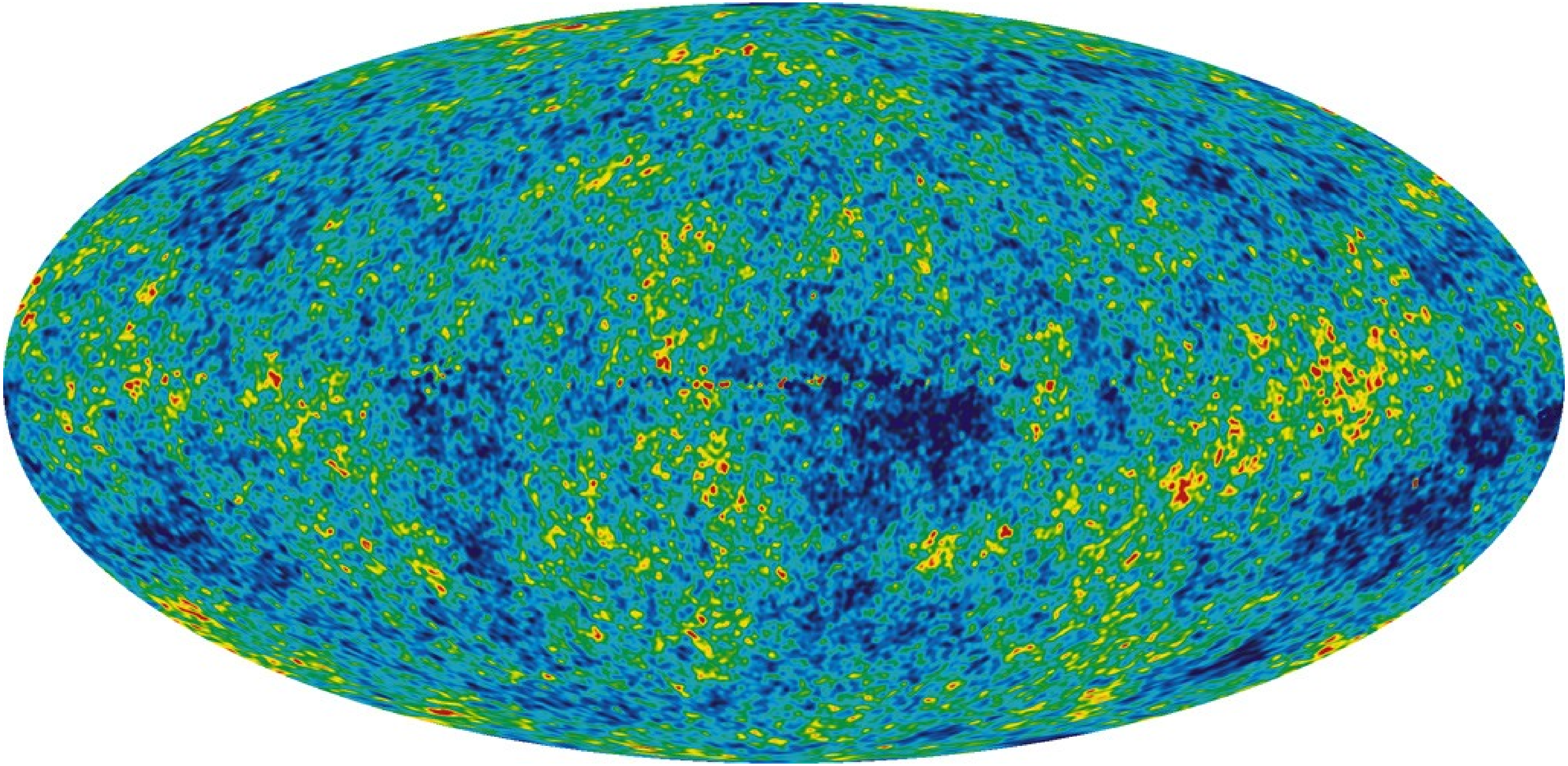
with

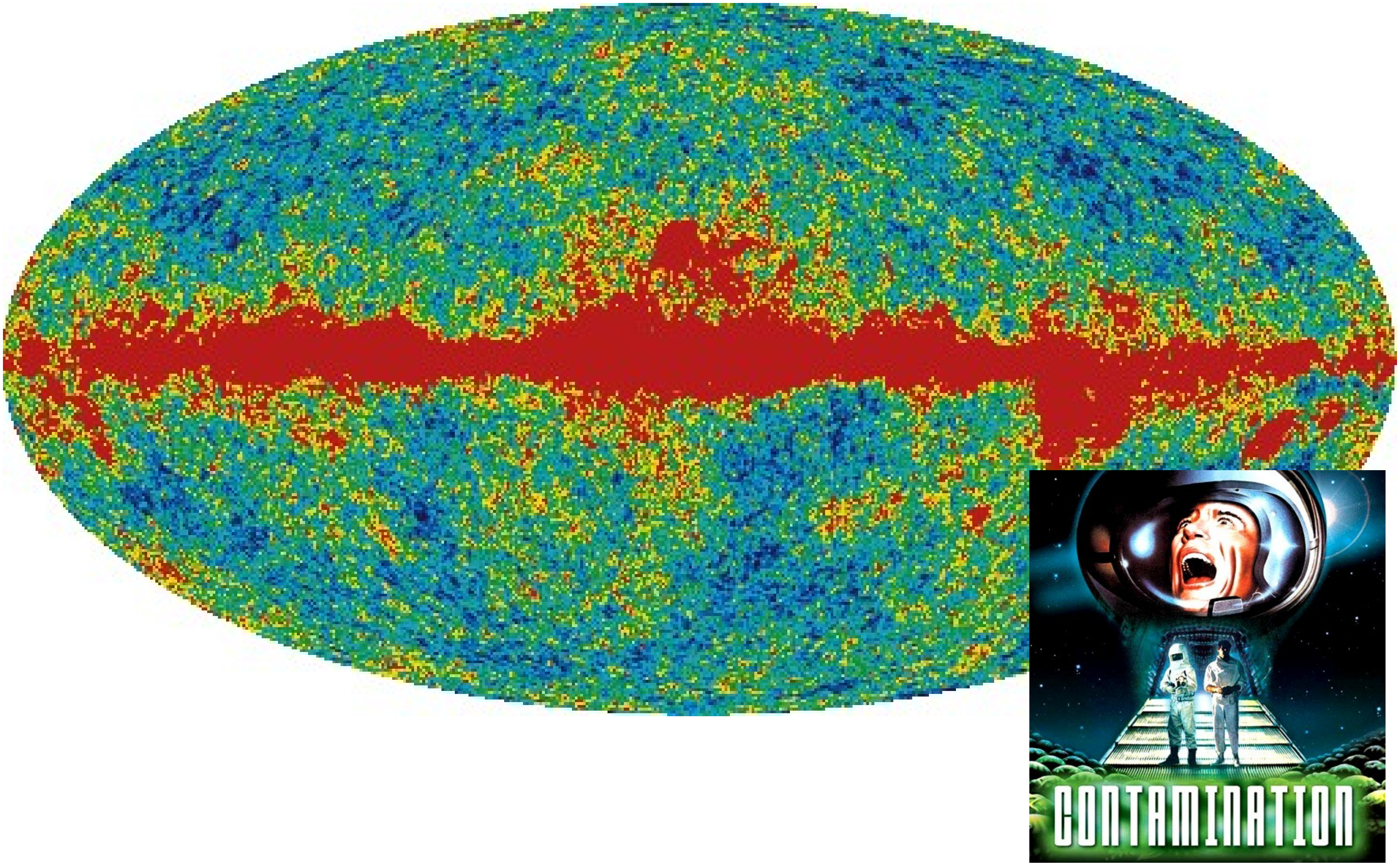
$$\xi^{+}_{\ell m} = -(\ell - 2)\sqrt{\frac{\ell^2 - m^2}{(2\ell+1)(2\ell-1)}} \qquad \xi^{-}_{\ell m} = (\ell + 3)\sqrt{\frac{\ell^2 - m^2}{(2\ell+1)(2\ell-1)}}$$

from these we find $\langle \mathcal{C}'_\ell \rangle \approx C_\ell (1 + 4\beta^2 + \mathcal{O}(\beta^3))$

Challinor & von Leeuwen (2001)
Pereira, AY, Stuke, Starkman (2010)

Not the whole story




CONTAMINATION

Finding the pseudo-power spectrum

$$\Delta\tilde{T}(\mathbf{n}) = \sum_{\ell>0}\sum_{m=-\ell}^{\ell} a_{\ell m} W(\mathbf{n}) Y_{\ell m}(\mathbf{n})$$

$$\tilde{a}_{\ell_1 m_1} = \sum_{\ell_2, m_2} a_{\ell_2 m_2} K_{\ell_1 m_1 \ell_2 m_2}$$

$$K_{\ell_1 m_1 \ell_2 m_2} \equiv \sum_{\ell_3, m_3} w_{\ell_3 m_3} (-1)^{m_2} \left[\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}\right]^{\frac{1}{2}} \times \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & -m_2 & m_3 \end{pmatrix}$$

So for boosted, masked Cls...

$$\langle \tilde{\mathcal{C}}'_{\ell_1} \rangle = \frac{1}{2\ell_1+1} \sum_{m_1=-\ell_1}^{\ell_1} |\tilde{a}'_{\ell_1 m_1}|^2$$

$$= \frac{1}{2\ell_1+1} \sum_{m_1=-\ell_1}^{\ell_1} \sum_{\ell_2 m_2} \sum_{\ell_3 m_3} \langle a'_{\ell_2 m_2} a^{*\prime}_{\ell_3 m_3} \rangle K_{\ell_1 m_1 \ell_2 m_2} K^{*}_{\ell_1 m_1 \ell_3 m_3}$$

Remember: $a'_{\ell m} = a_{\ell m} + \beta \xi^{+}_{\ell m} a_{(\ell+1)m} + \beta \xi^{-}_{\ell m} a_{(\ell-1)m}$

We will no longer have a cancellation of the βl terms when calculating the power spectrum and the series doesn't converge.

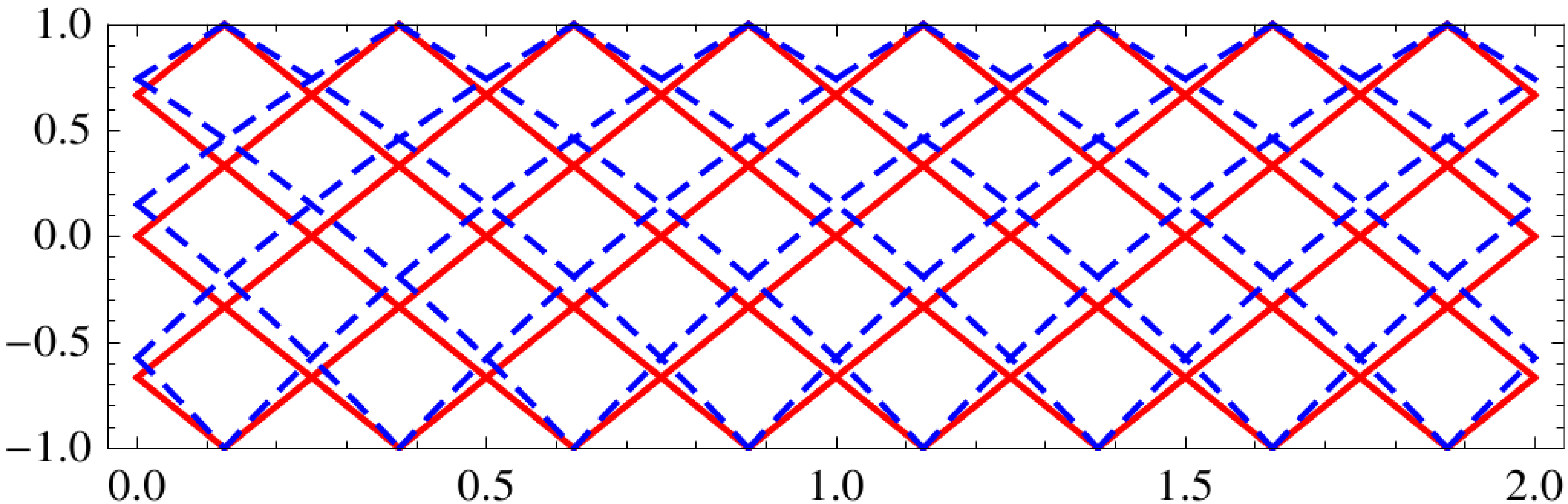
Therefore, accounting for the Lorentz boost effect <u>must</u> be done in real space.

How will we do this?

We compute a boost matrix, which transforms the an unboosted temperature map into a boosted one:

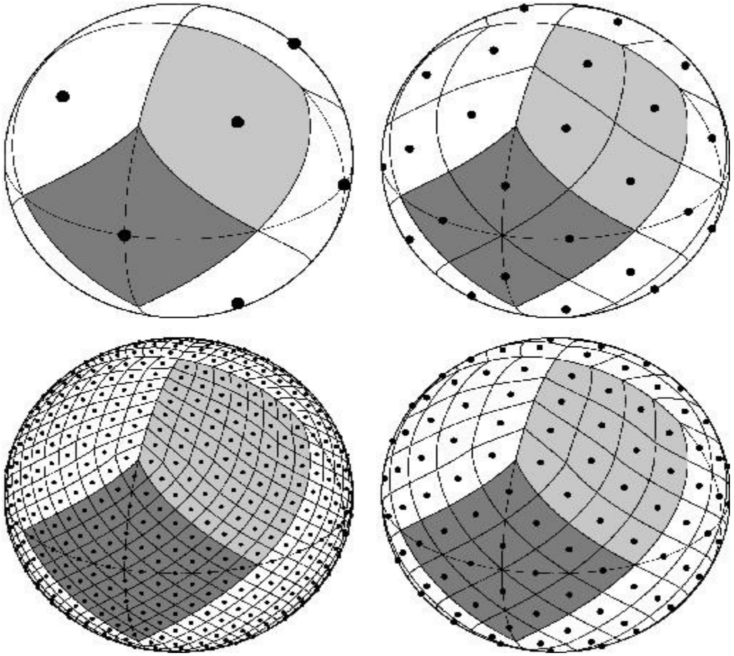
$$\Delta T(p') = \sum_{p} \Lambda_{pp'} \Delta T(p)$$

First, we must characterize the way boosted and unboosted pixels overlap in HEALPix pixelization



Blue dashed = boosted, Red = rest−frame

Actual pixelizations are more complicated with multiple pixel shapes (note the difference in the equatorial and polar base pixels)



Fractional area overlaps are calculated by randomly throwing points into pixels, keeping track of the new pixel they have been boosted to, and dividing by the total number of points.

This introduces some new complications:

- The size of the boost matrix is exactly dependent on the number of pixels in your map (usually very large) and the accuracy is dependent on β (very small)

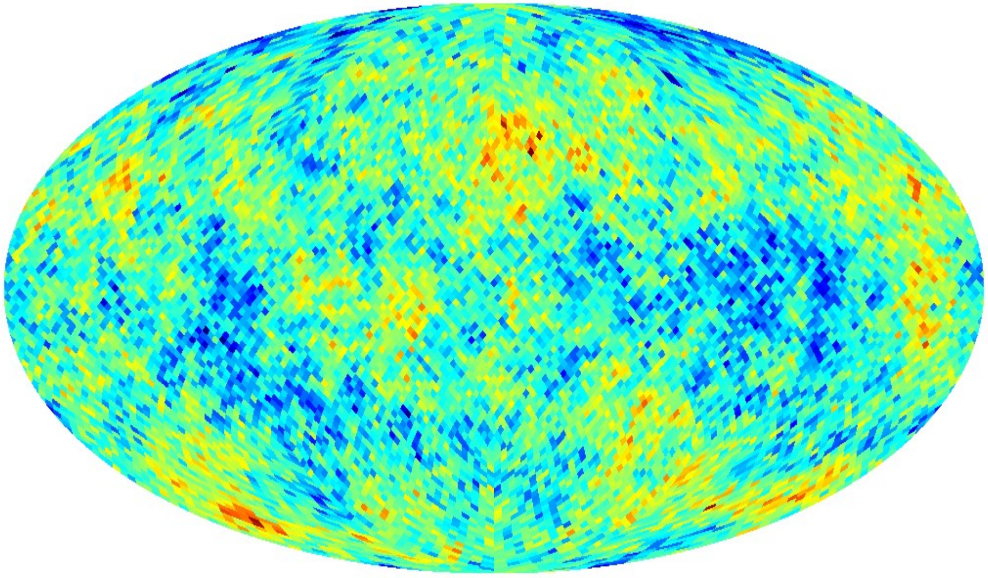
For a map which has resolution Nside=512 has ~10^6 pixels, we use

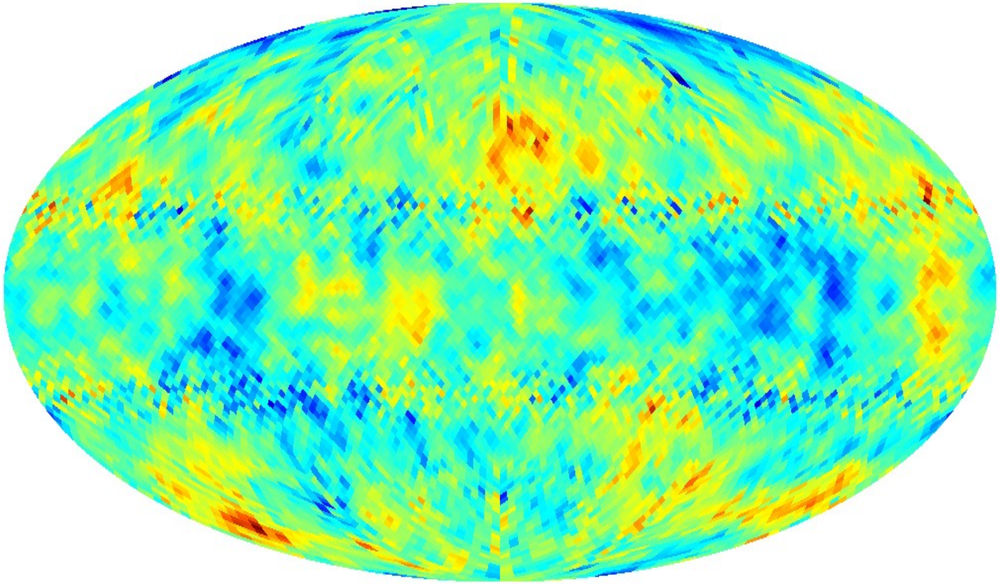
10^{13} operations to achieve an acceptable error level

- Accounting for the boost in real space causes a smoothing effect, which lowers the scale one trusts for a given resolution

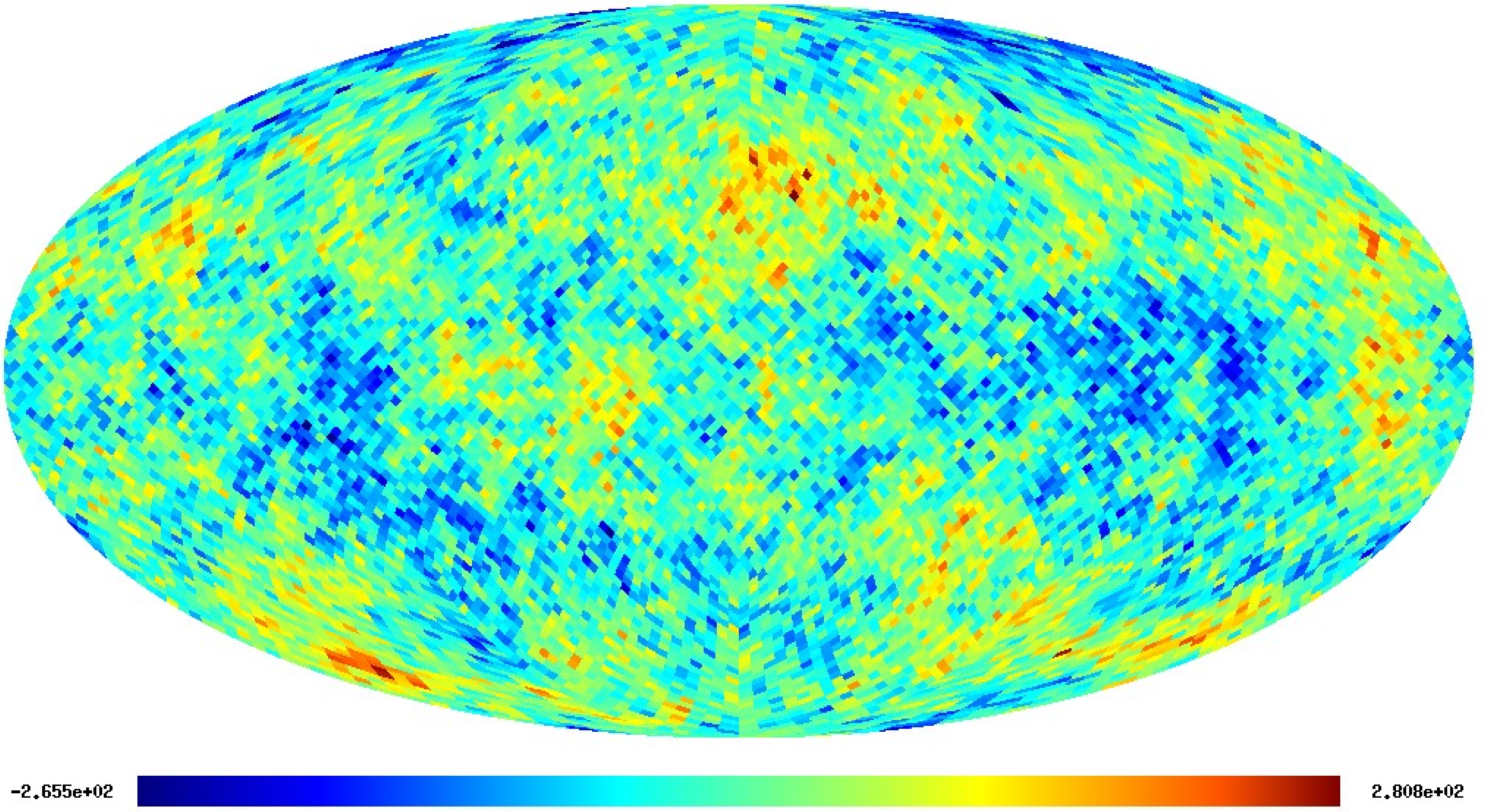
We should get $\Lambda^{-1}\Lambda = \mathbf{1}$

...but we don't.

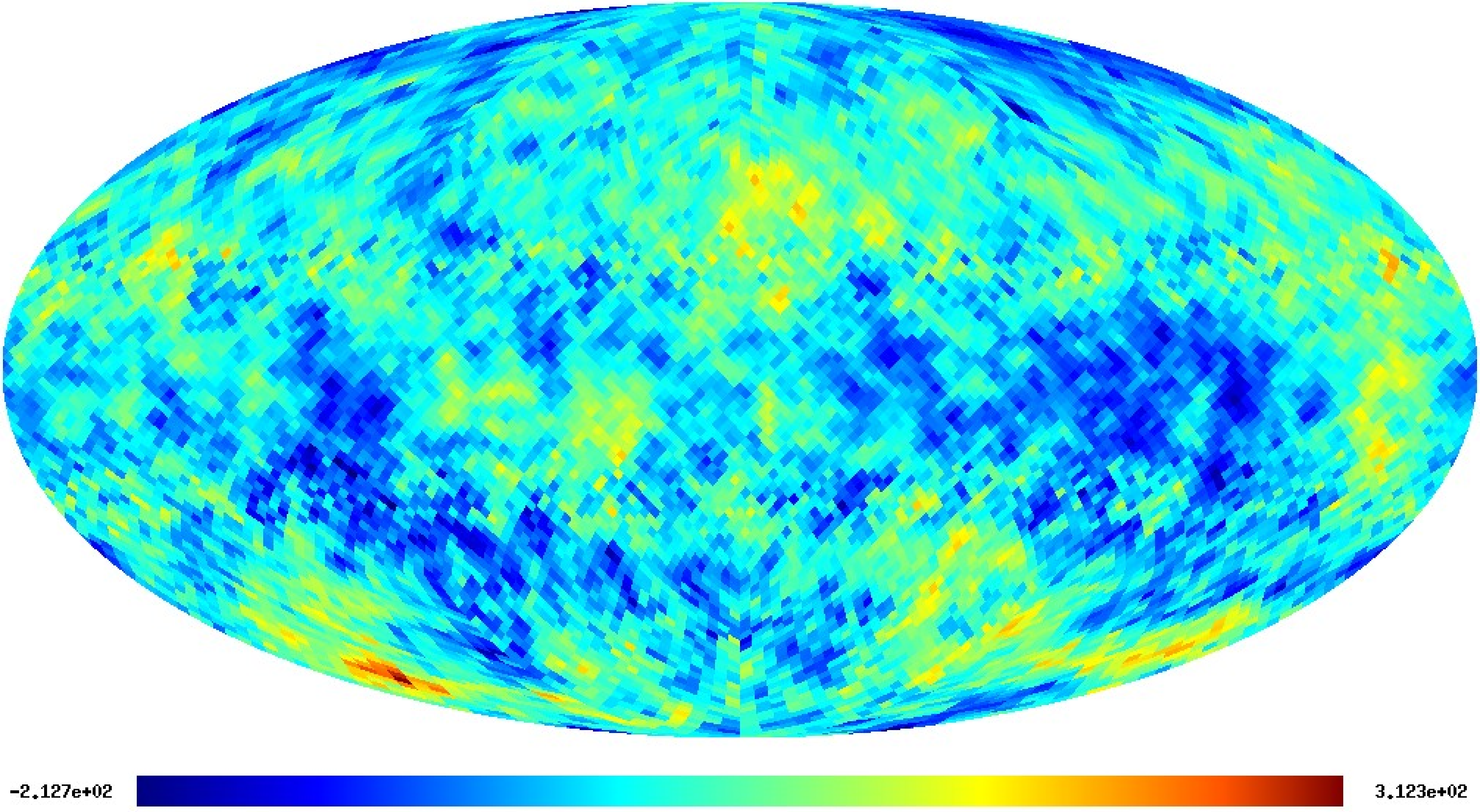


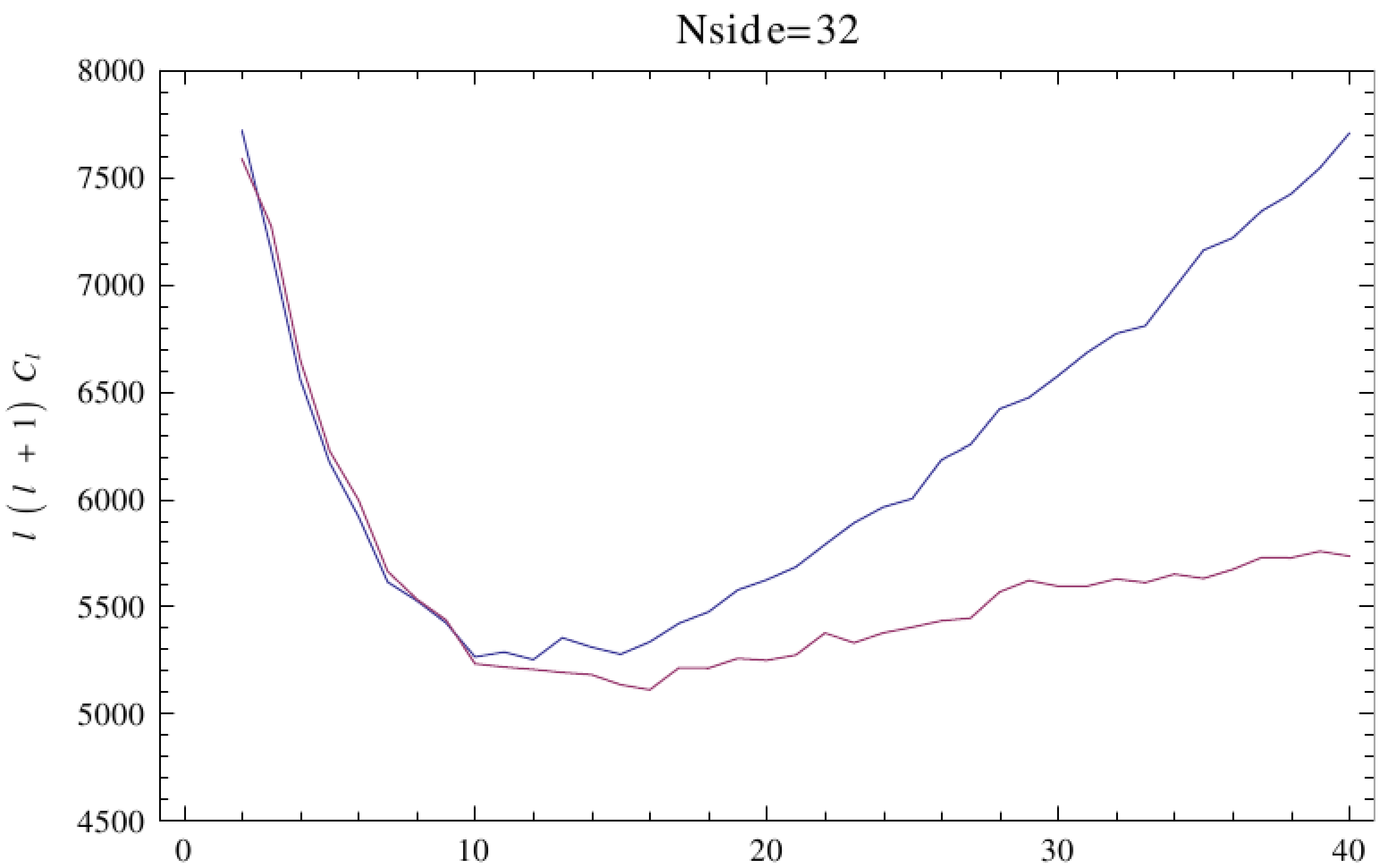


Nside = 32, β = .1



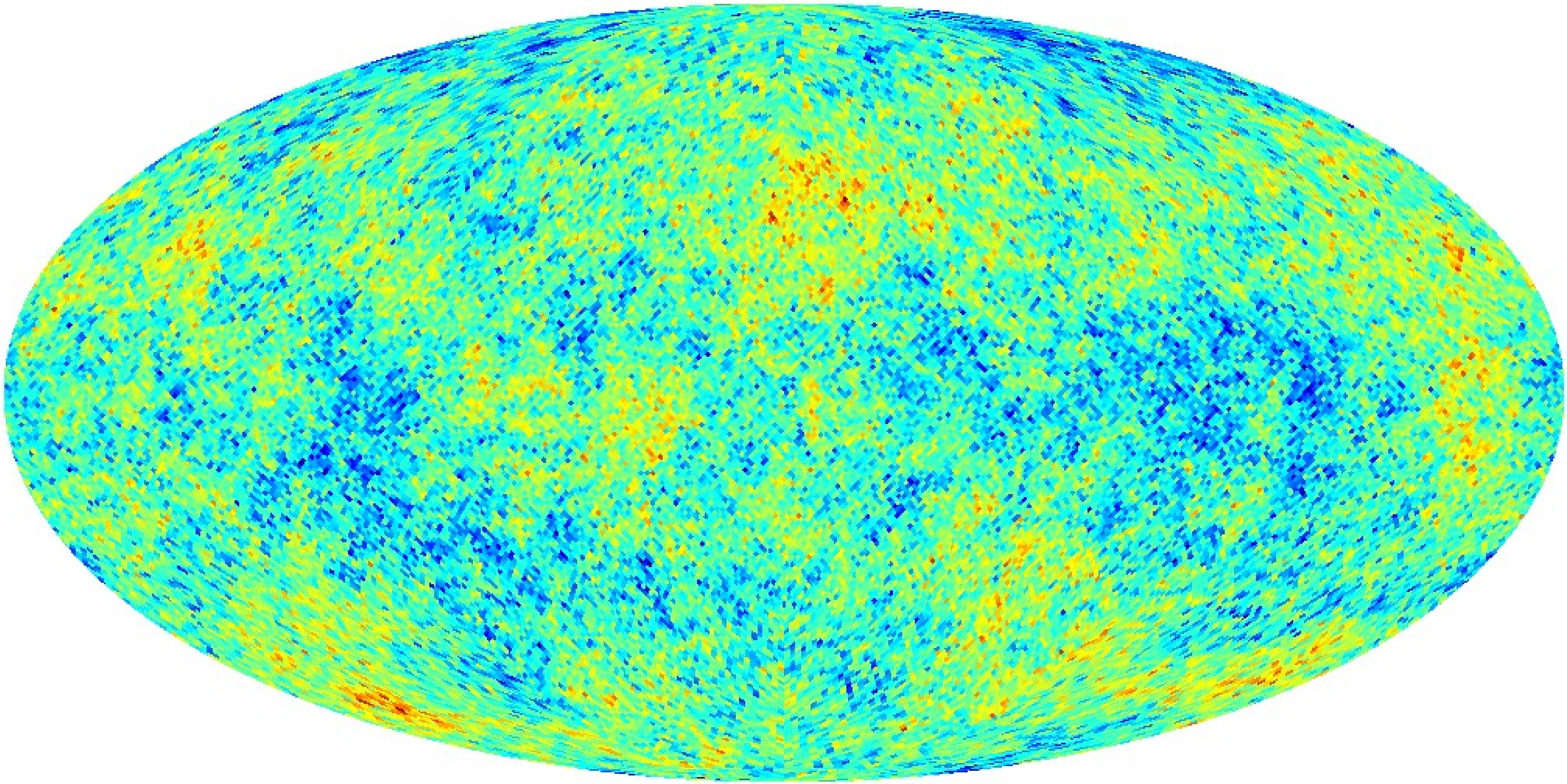
Nside = 32, β = .1



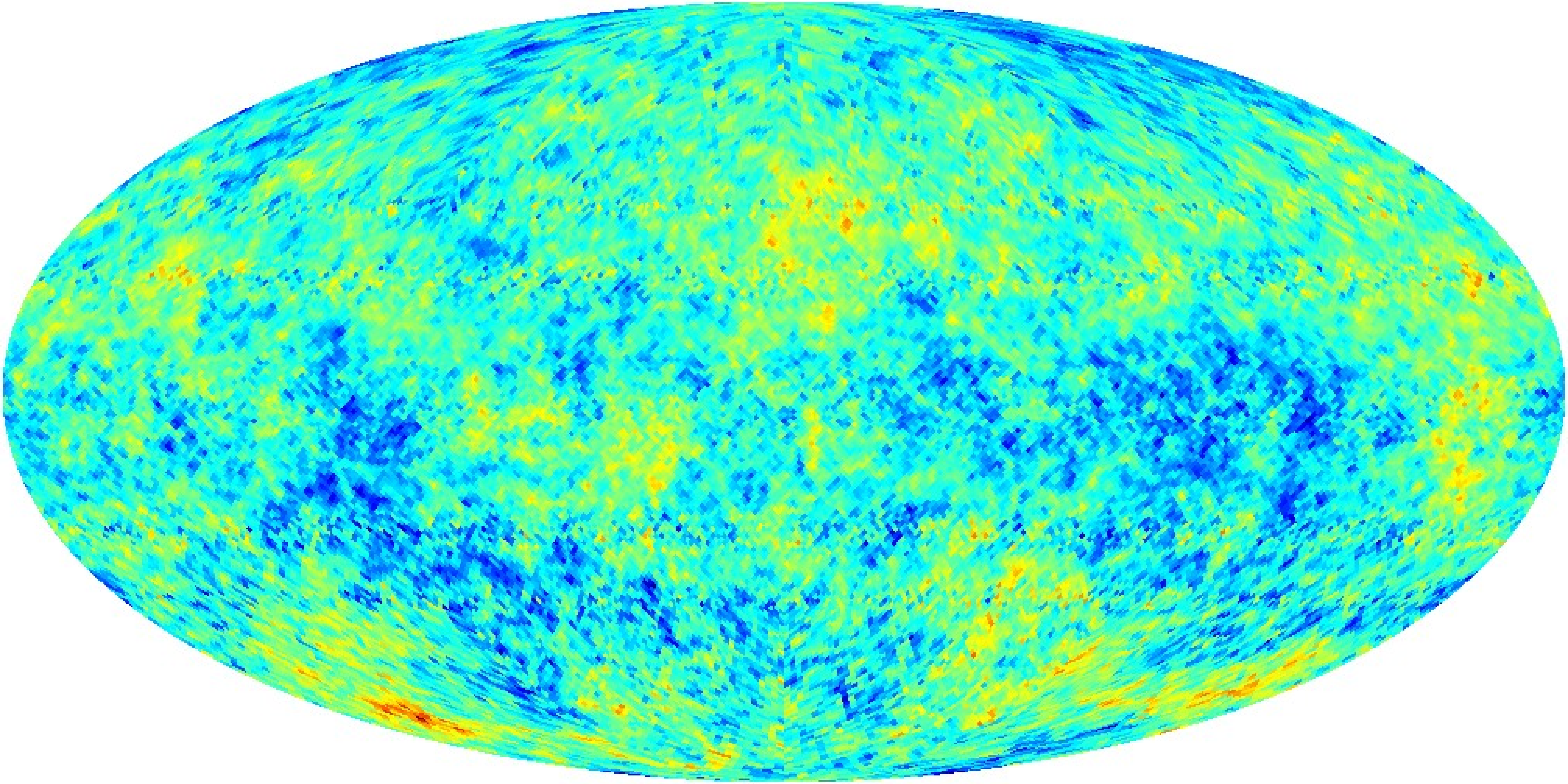


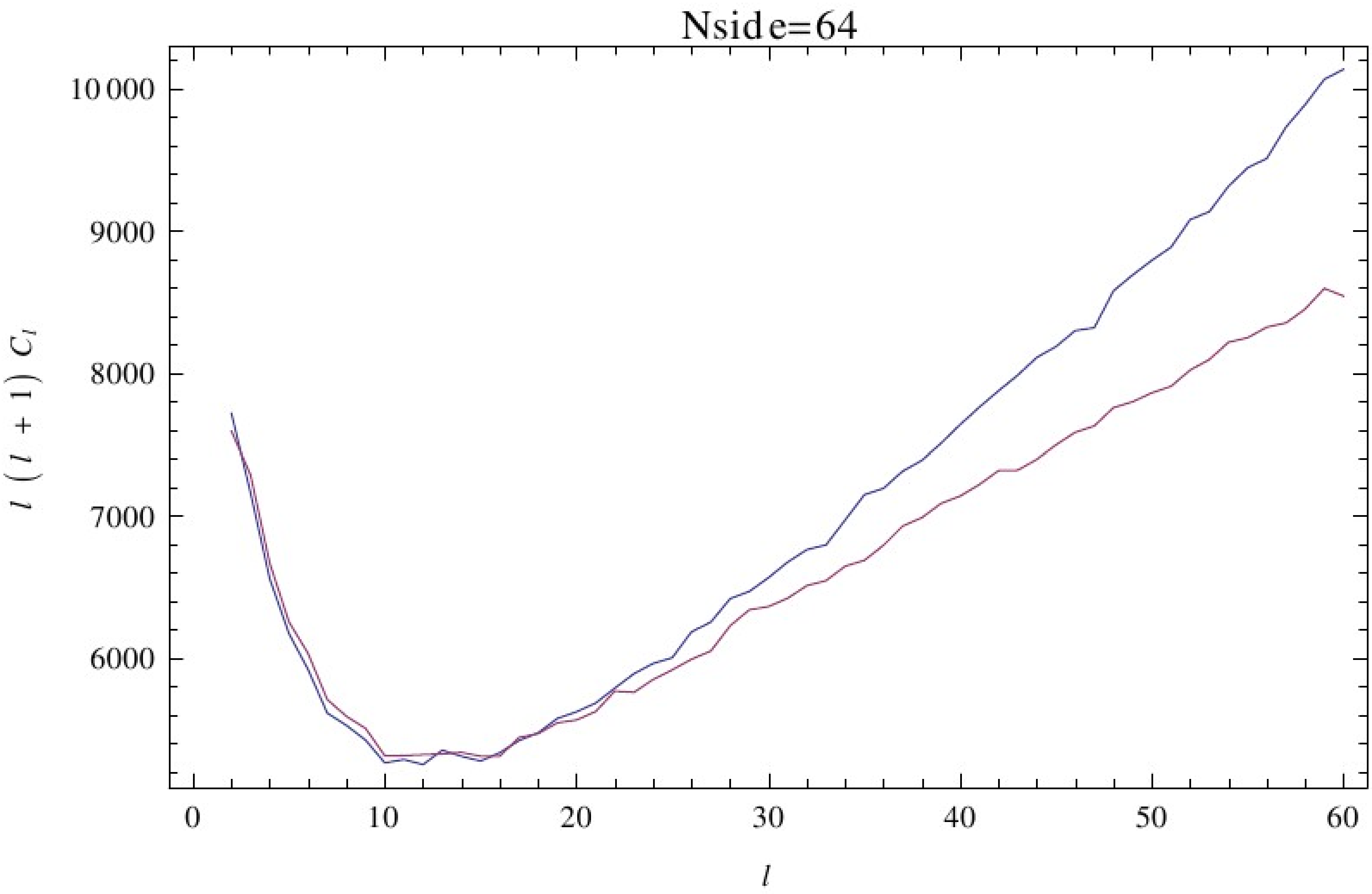
Nside=32
8000
7500
7000
6500
6000
5500
5000
4500
l (l + 1) C_l
0
10
20
30
40

Nside = 64, β = .1



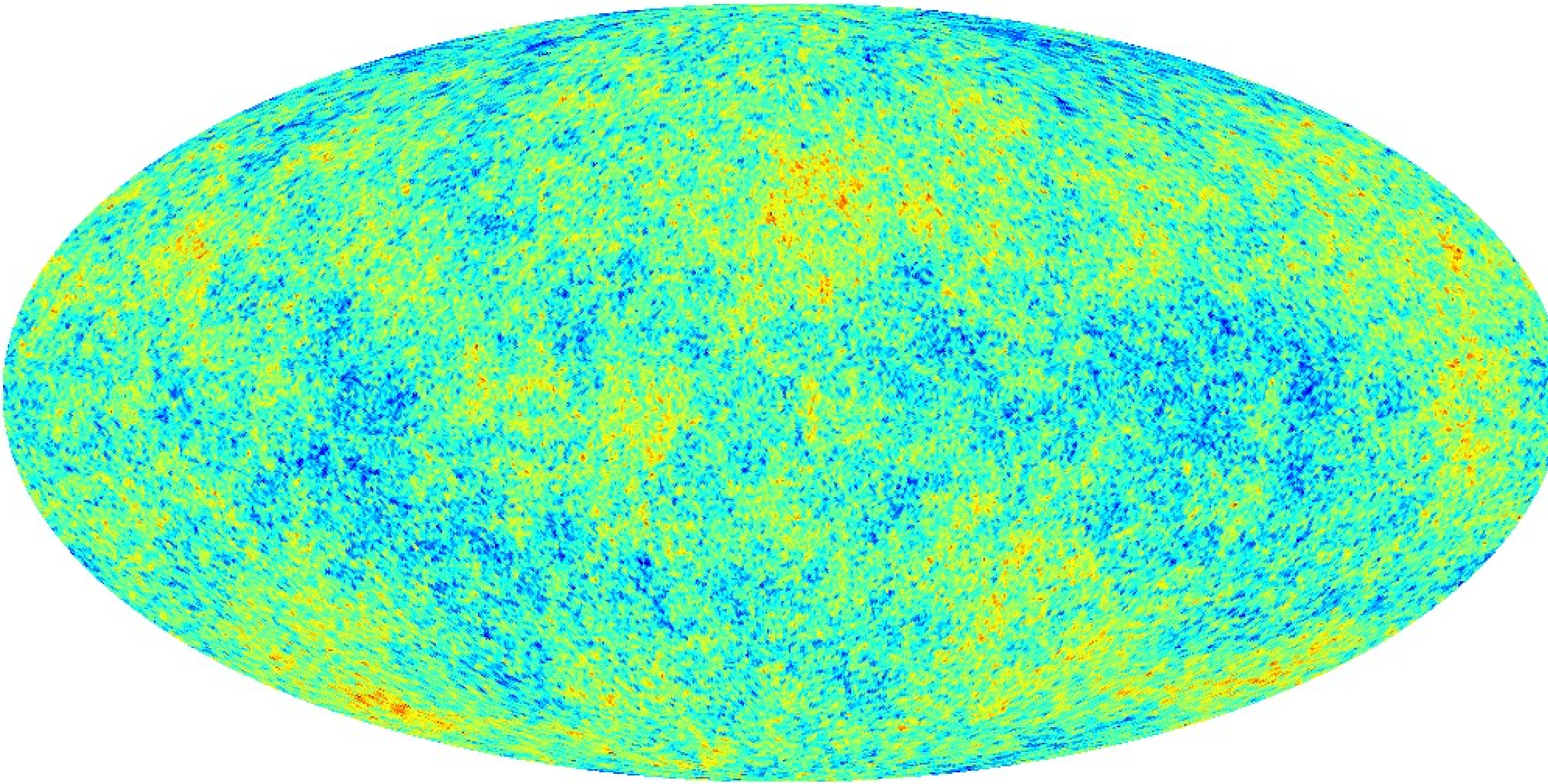
Nside = 64, β = .1



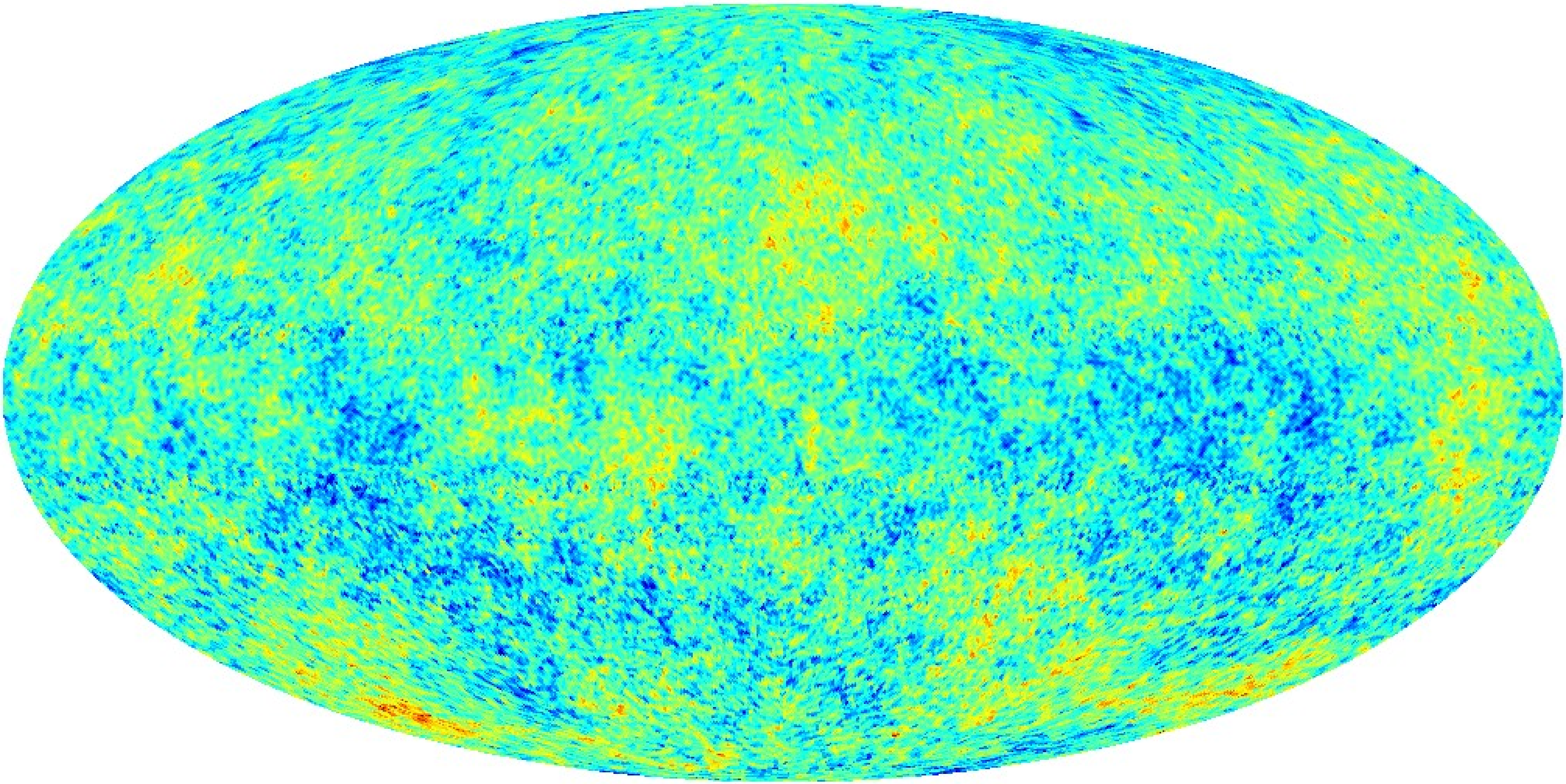


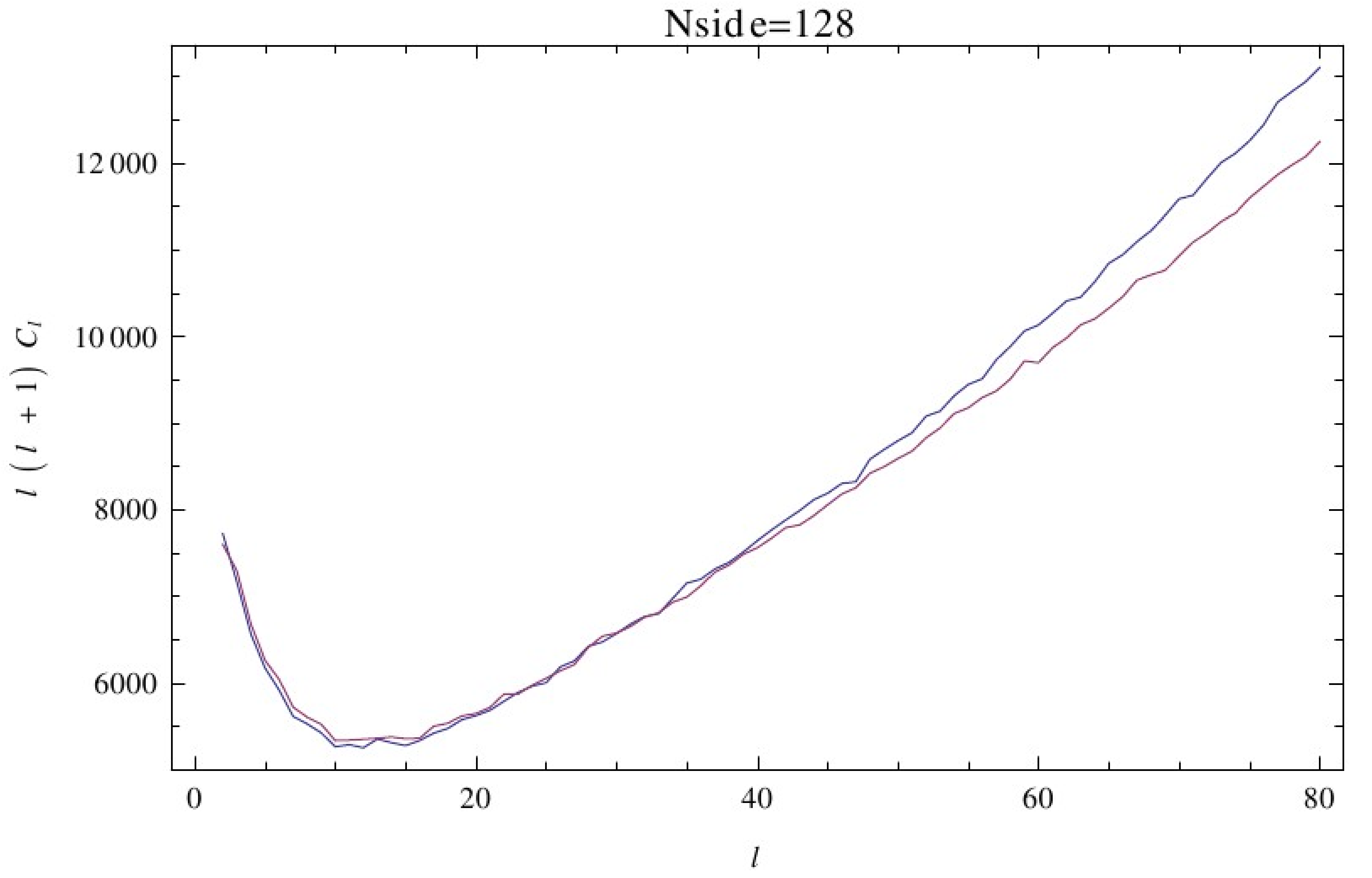
Nside=64
10 000
9000
8000
7000
6000
l (l + 1) C_l
0
10
20
30
40
50
60
l

Nside = 128, β = .1



Nside = 128, β = .1





Higher resolution maps than expected are necessary for analyzing de-boosted data

Conclusions

- Unlike the full sky effect, masked and boosted maps have complicated mode mixing
  - This shows a need to account for the boost effect in real space rather than l-space
- Straight forward, but adds complications such as smoothing effects and long computation time for area matrices
- Effects will be present for $\beta l \sim 1$ (important for Planck) on incomplete skies. Look for results in our forthcoming paper.
- We have extended this work to polarization, where pinning down mode mixing is extremely important. Stay tuned...