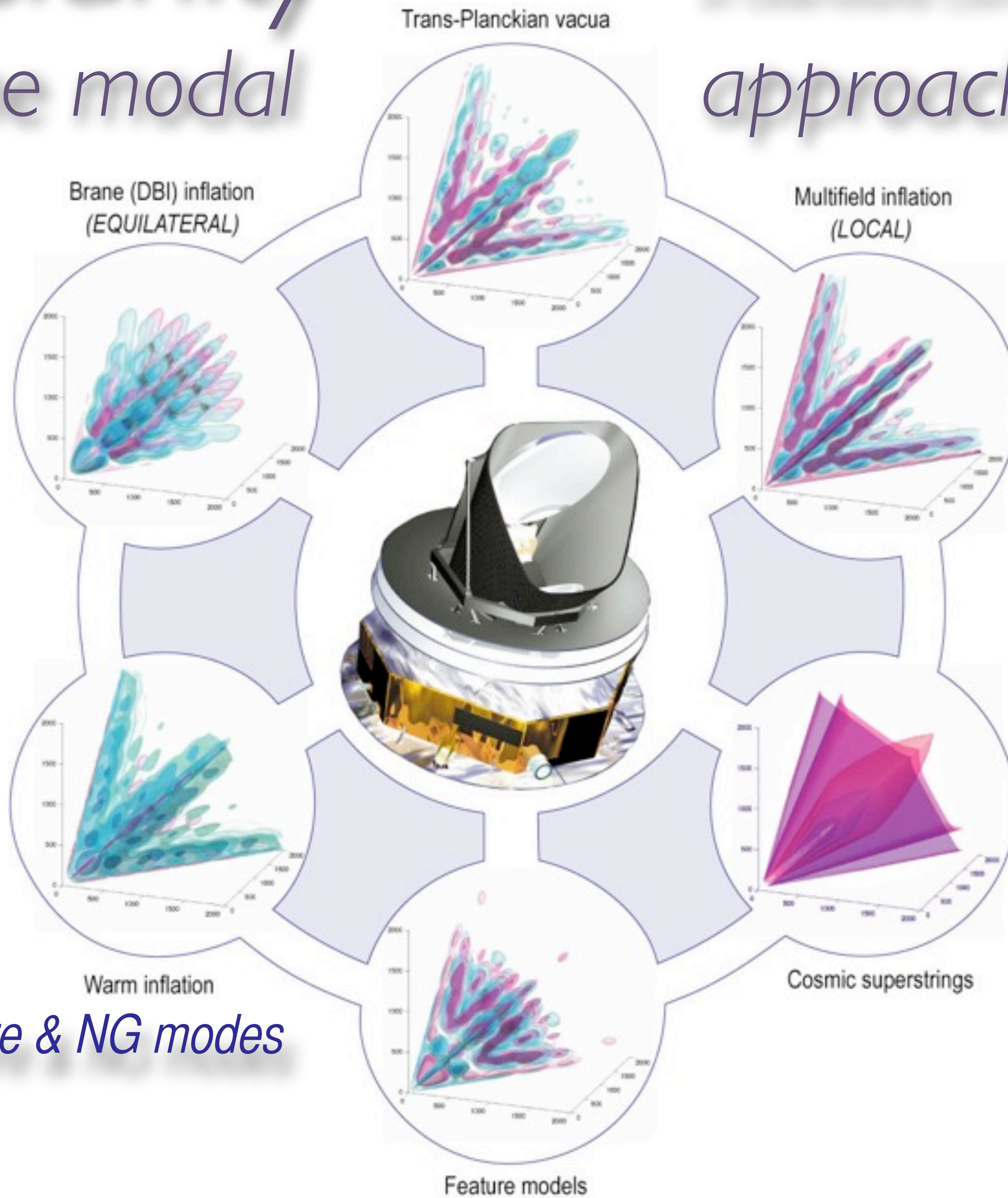
Non-Gaussianity

Applications of the modal approach

The Azores School on Observational Cosmology

James Fergusson & Paul Shellard

Centre for Theoretical Cosmology, DAMTP University of Cambridge



LECTURE 3

Non-Gaussian simulations

Optimality: inverse covariance

Polyspectra reconstruction

Contaminants – PCA analysis

Afterword: Large scale structure & NG modes

Interim conclusions ...

Non-Gaussianity

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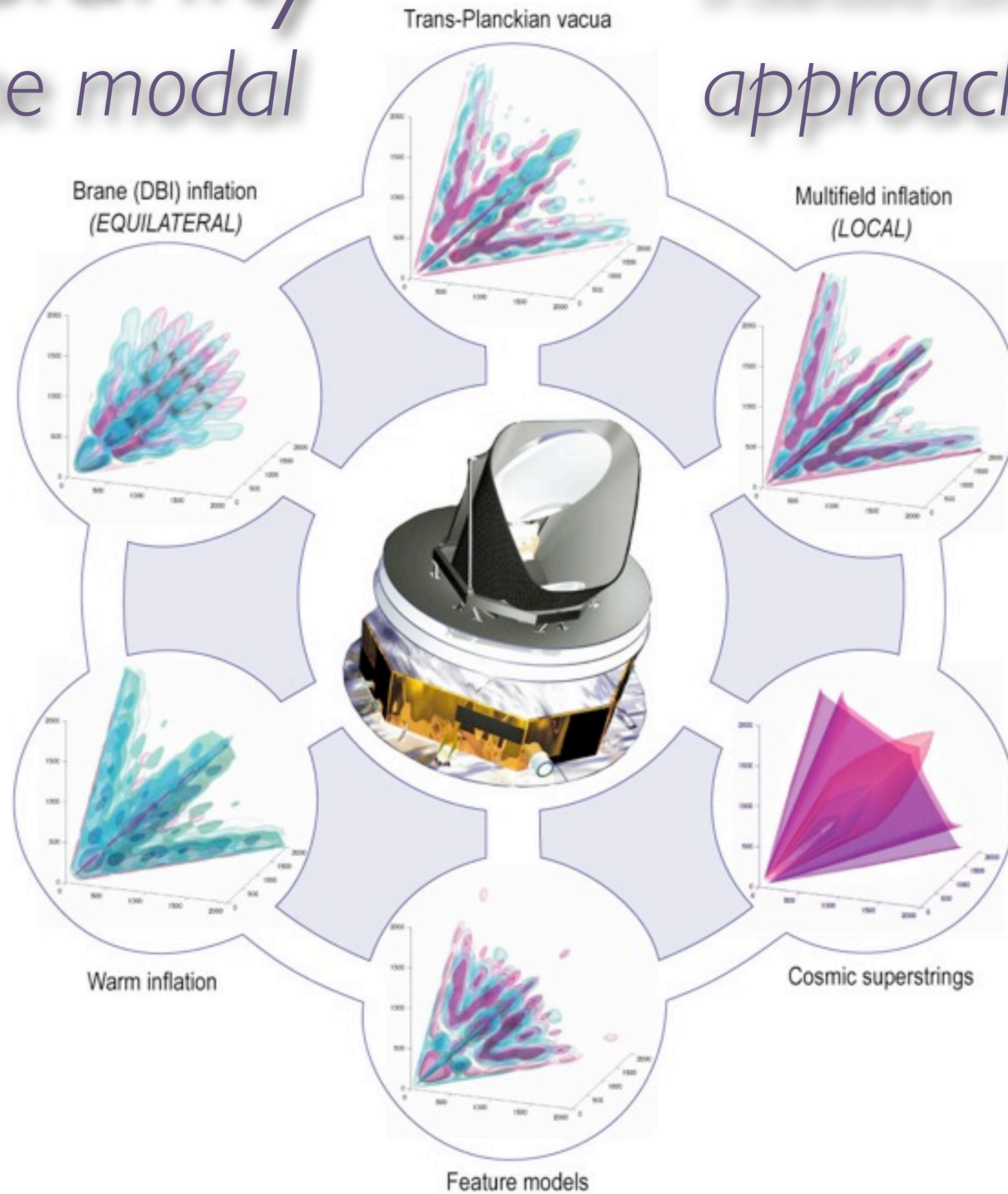
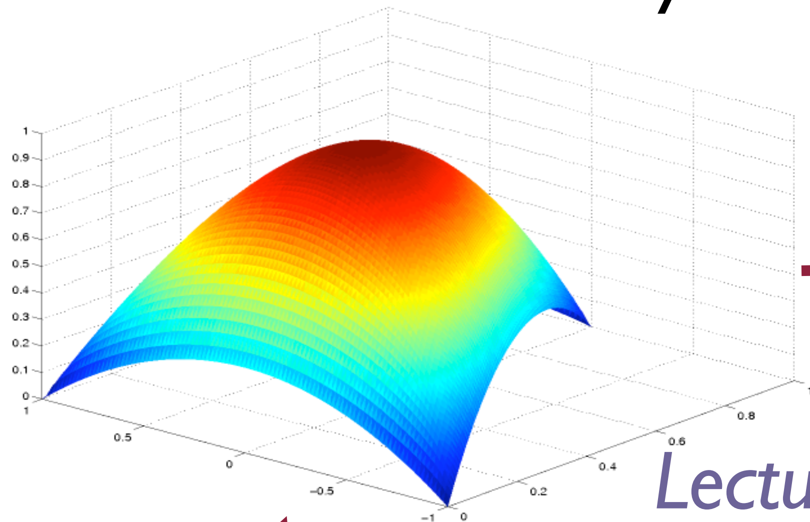
Centro de Astrofísica da Universidade do Porto



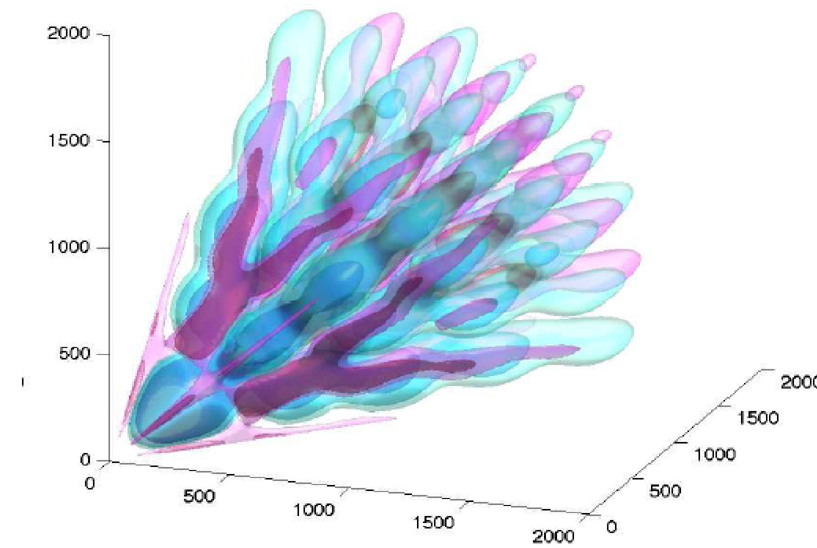
Overview

Primordial non-Gaussianity



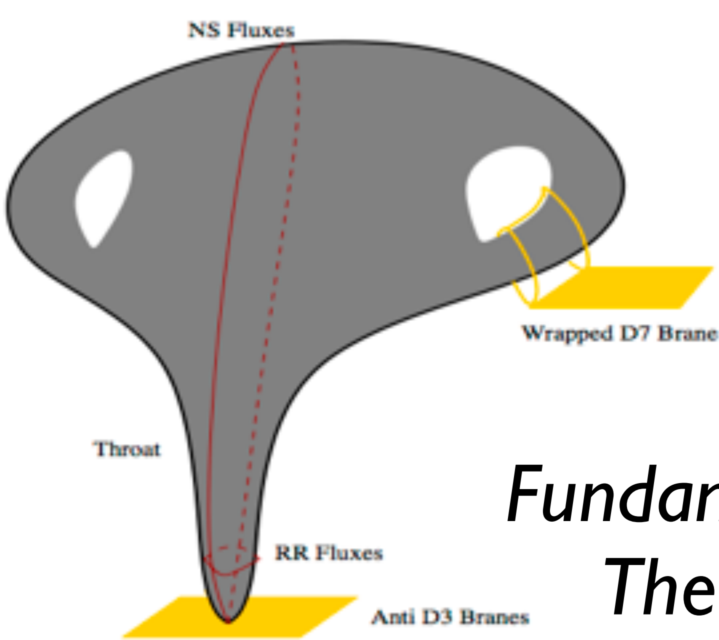
Lecture 1

CMB (or LSS) fingerprint

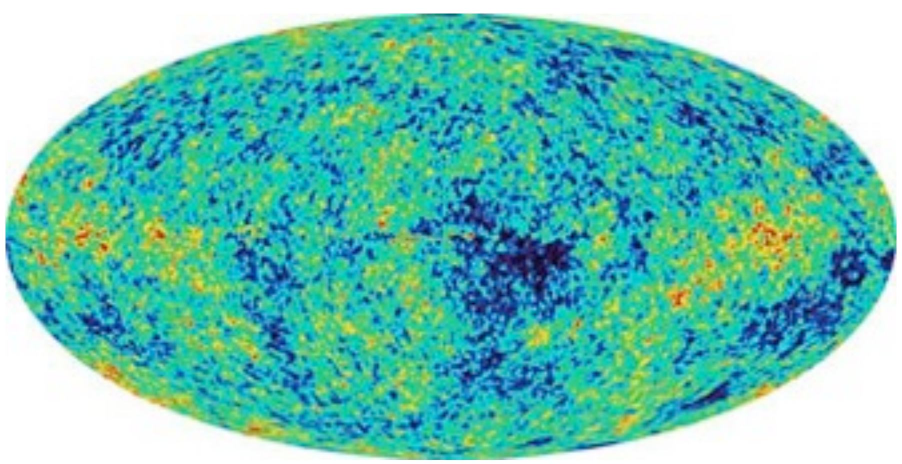


Lecture 2

Lecture 3



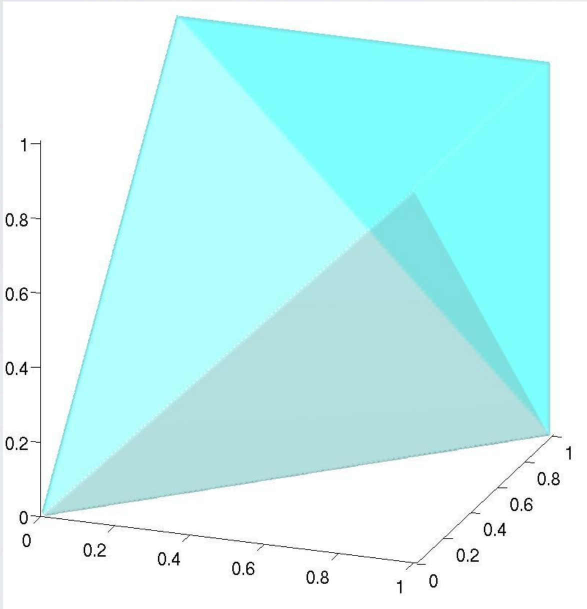
Fundamental Theory

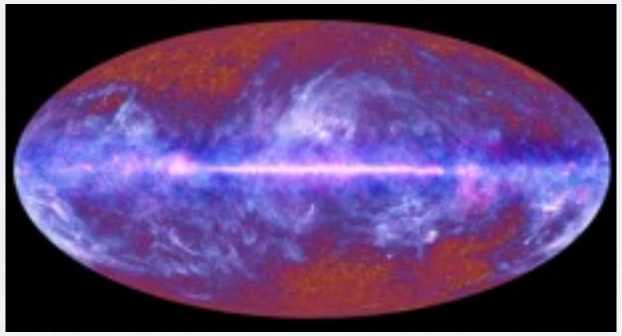


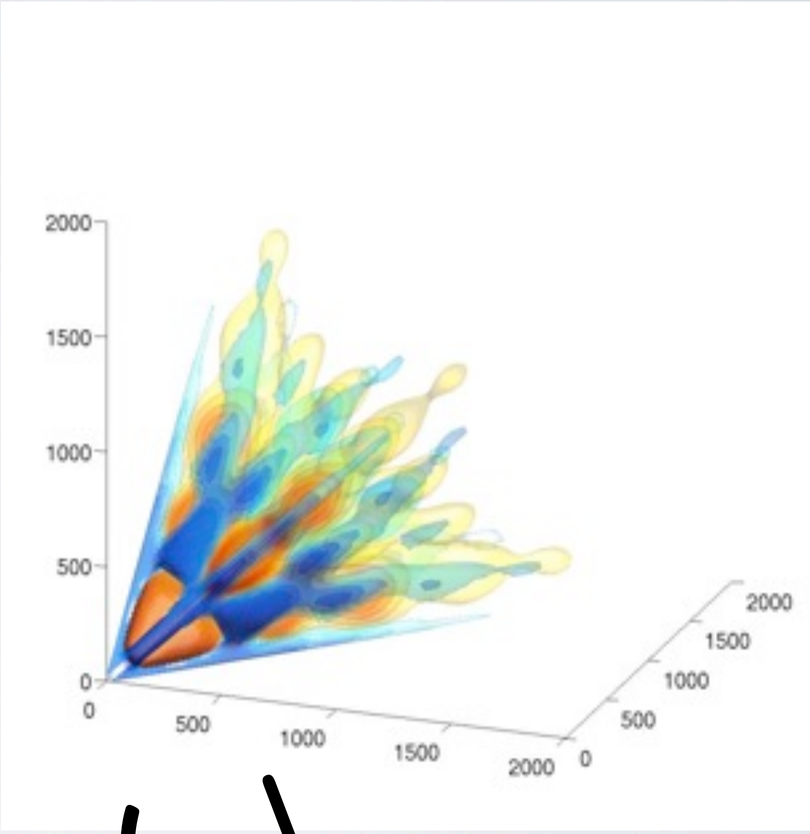
Observational Data

RECAP

Separability = Tractability
Basis = Good







$$\mathcal{E} = \frac{1}{N}\sum_n \alpha_n \beta_n \longleftarrow \Gamma = \bar{\mathcal{R}}\tilde{\mathcal{R}}^T \longrightarrow \mathcal{E} = \frac{\sum \bar{\alpha}\bar{\beta}}{\sum \bar{\alpha}^2}$$

APPLICATIONS

The advantage of the modal approach is in the power of an orthonormal basis.

This allows us to do much more than estimation

- Simulations
- Inverse covariance
- Reconstruction
- Contaminants

SIMULATION

This method can also be used to simulate maps with a given bispectrum and trispectrum

$$a_{lm} = a_{lm}^G + \frac{1}{6} F_{NL} a_{lm}^B + \frac{1}{24} G_{NL} a_{lm}^T$$

$$a_{lm}^B = \sum_{l_i m_i} \int Y_{l_1 m_1} Y_{l_2 m_2} Y_{l_3 m_3} b_{l_1 l_2 l_3} \frac{a_{l_2 m_2}^G}{C_{l_2}} \frac{a_{l_3 m_3}^G}{C_{l_3}}$$

$$a_{lm}^T = \sum_{l_i m_i} \int Y_{l_1 m_1} Y_{l_2 m_2} Y_{l_3 m_3} Y_{l_4 m_4} t_{l_1 l_2 l_3 l_4} \frac{a_{l_2 m_2}^G}{C_{l_2}} \frac{a_{l_3 m_3}^G}{C_{l_3}} \frac{a_{l_4 m_4}^G}{C_{l_4}}$$

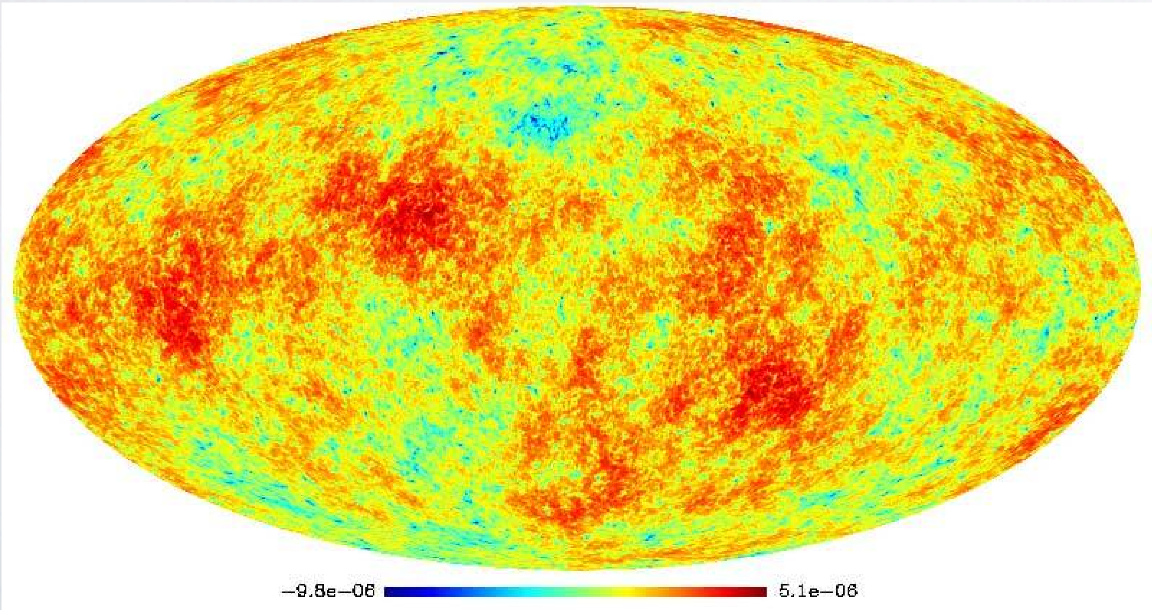
Note: If you want to use both simultaneously you need to calculate the trispectrum minus the spurious trispectrum generated from the bispectrum squared

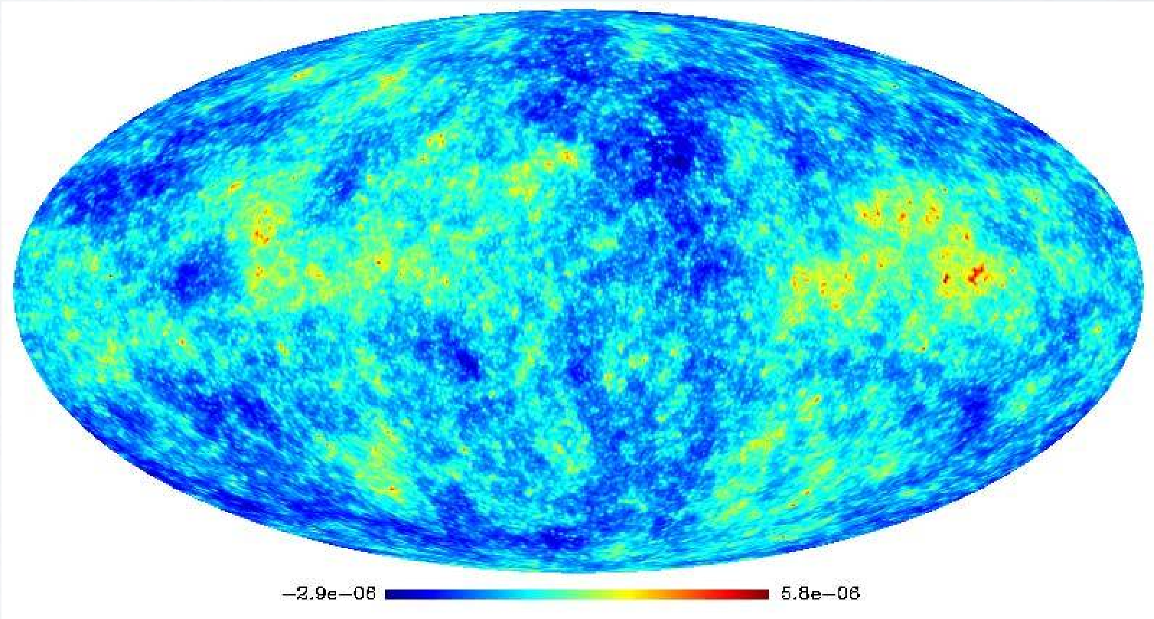
SIMULATION

Neither part interferes with the other. Using the expansion the nonGaussian contributions can be easily calculated

$$a^B_{lm} = \sum_n \bar{\alpha}^Q_n \frac{q_l^{\{i}}{v_l\sqrt{C_l}} \int d^2\hat{\mathbf{n}} Y_{lm}(\hat{\mathbf{n}}) M^j(\hat{\mathbf{n}}) M^{k\}}(\hat{\mathbf{n}})$$

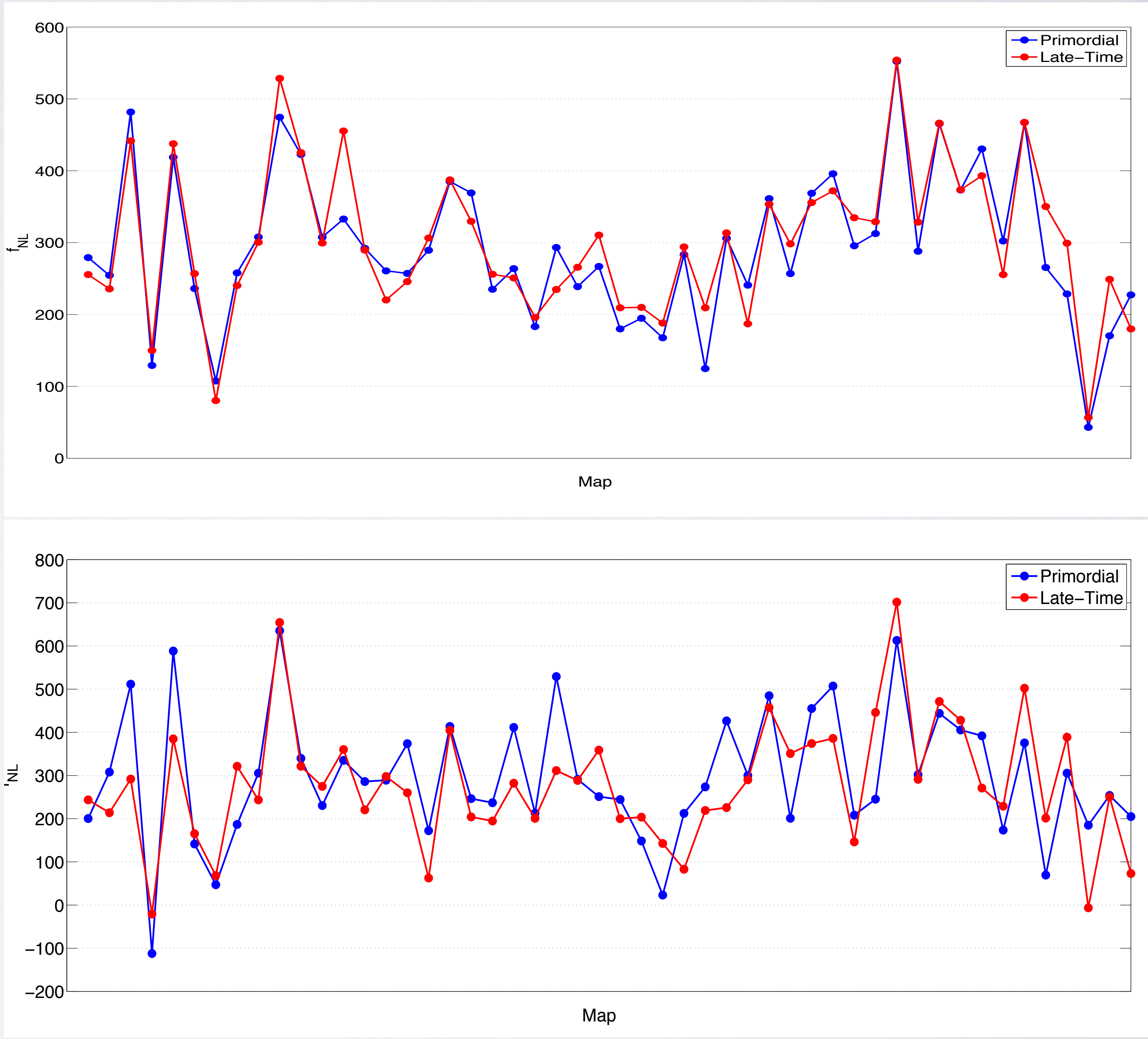
$$M^i(\hat{\mathbf{n}}) = \sum_{lm} \frac{q_l^i Y_{lm}(\hat{\mathbf{n}}) a^G_{lm}}{v_l\sqrt{C_l}}$$





SIMULATION

To test the accuracy of the method we simulated maps using both the primordial and CMB decompositions and then applied both the primordial and CMB estimators to both sets to produce consistent results



	Ideal simulations		WMAP5 simulations	
	Average	St. Dev.	Average	St. Dev.
Primordial estimator	292.9	104.8	297.7	152.1
Late-time estimator	300.6	104.9	278.7	160
Internal st. dev.	38.5		102.6	

INVERSE COVARIANCE

In general it is very hard to calculate the inverse covariance matrix. If we perform the same modal decomposition on the covariance

$$\alpha = \mathcal{R}\mathcal{A}$$

$$\beta = \mathcal{R}\mathcal{B} \longrightarrow \mathcal{P}\mathcal{B} = \mathcal{R}^T\beta$$

$$\zeta \ = \ \mathcal{R}\mathcal{C}\mathcal{R}^T$$

$$\begin{aligned}
\mathcal{E} \ &\equiv \ \frac{\alpha^T\zeta^{-1}\beta}{\alpha^T\zeta^{-1}\alpha} \\
&= \ \frac{(\mathcal{R}\mathcal{A})^T\mathcal{R}\mathcal{C}^{-1}\mathcal{R}^T\mathcal{R}\mathcal{B}}{\mathcal{R}\mathcal{A}^T\mathcal{R}\mathcal{C}^{-1}\mathcal{R}^T\mathcal{R}\mathcal{A}} = \frac{\mathcal{A}^T\mathcal{P}\mathcal{C}^{-1}\mathcal{P}\mathcal{B}}{\mathcal{A}^T\mathcal{P}\mathcal{C}^{-1}\mathcal{P}\mathcal{A}}
\end{aligned}$$

INVERSE COVARIANCE

Hang on, we defined $\zeta = \mathcal{R}\mathcal{C}\mathcal{R}^T$ but used

$$\zeta^{-1} = \mathcal{R}\mathcal{C}^{-1}\mathcal{R}^T$$

While R is rectangular, it does have a right inverse, $\mathcal{R}\mathcal{R}^T = I$, and as it's orthonormal the inverse is just its transpose

$$\zeta^{-1} = \left(\mathcal{R}\mathcal{C}\mathcal{R}^T\right)^{-1} = \mathcal{R}\left(\mathcal{C}^{-1} + \mathcal{Z}_\perp\right)\mathcal{R}^T$$

and $\mathcal{Z}_\perp$ is an arbitrary matrix which is perpendicular to the subspace and can be ignored

INVERSE COVARIANCE

We can understand the effect of the projection by considering

$$\mathcal{A} = \left[\begin{array}{c} \mathcal{A}_{\|} \\ 0 \end{array} \right] \qquad \mathcal{B} = \left[\begin{array}{c} \mathcal{B}_{\|} \\ \mathcal{B}_{\perp} \end{array} \right] \qquad \mathcal{C}^{-1} = \left[\begin{array}{cc} \mathcal{C}^{-1}_{\|} & \mathcal{C}^{-1}_{\times} \\ \mathcal{C}^{-1}{}^{T}_{\times} & \mathcal{C}^{-1}_{\perp} \end{array} \right]$$

$$\mathcal{X}_{\|} \equiv \mathcal{P}\mathcal{X}$$

$$\mathcal{X}_{\perp} \equiv (I - \mathcal{P})\mathcal{X}$$

$$\mathcal{M}_{\|} \equiv \mathcal{P}\mathcal{M}\mathcal{P}$$

$$\mathcal{M}_{\perp} \equiv (I - \mathcal{P})\mathcal{M}(I - \mathcal{P})$$

$$\mathcal{M}_{\times} \equiv \mathcal{P}\mathcal{M}(I - \mathcal{P})$$

INVERSE COVARIANCE

We can understand the effect of the projection by considering

$$\bar{\mathcal{E}} = \frac{\mathcal{A}_{\parallel}\left(\mathcal{C}_{\parallel}^{-1}\mathcal{B}_{\parallel} + \mathcal{C}_{\times}^{-1}\mathcal{B}_{\perp}\right)}{\mathcal{A}_{\parallel}^{T}\mathcal{C}_{\parallel}^{-1}\mathcal{A}_{\parallel}}$$

$$\mathcal{E} = \frac{\mathcal{A}_{\parallel}\mathcal{C}_{\parallel}^{-1}\mathcal{B}_{\parallel}}{\mathcal{A}_{\parallel}^{T}\mathcal{C}_{\parallel}^{-1}\mathcal{A}_{\parallel}}$$

The difference is the projection of contamination from the orthogonal space into the subspace

INVERSE COVARIANCE

If we remember our discussion of the linear term we proved that

$$\zeta = \frac{1}{6}\left\langle \beta\beta^T \right\rangle$$

$$\begin{aligned}
\langle \beta_n\, \beta_{n'} \rangle = & \sum_{l_i m_i l'_i m'_i} \left\langle \left(\mathcal{R}_{n l_1 l_2 l_3} \frac{a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} - 3\, C_{l_1 m_1, l_2 m_2} a_{l_3 m_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}} \right) \right. \\
& \left. \times \left(\frac{a_{l'_1 m'_1} a_{l'_2 m'_2} a_{l'_3 m'_3} - 3\, C_{l'_1 m'_1, l'_2 m'_2} a_{l'_3 m'_3}}{\sqrt{C_{l'_1} C_{l'_2} C_{l'_3}}} \mathcal{R}_{n l'_1 l'_2 l'_3} \right) \right\rangle \\
= & \sum_{l_i m_i l'_i m'_i} \frac{\mathcal{R}_{n l_1 l_2 l_3} \mathcal{R}_{n' l'_1 l'_2 l'_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3} C_{l'_1} C_{l'_2} C_{l'_3}}} \left[6 \langle a_{l_1 m_1} a_{l'_1 m'_1} \rangle \langle a_{l_2 m_2} a_{l'_2 m'_2} \rangle \langle a_{l_3 m_3} a_{l'_3 m'_3} \rangle \right. \\
+ & \; 9 \langle a_{l_1 m_1} a_{l_2 m_2} \rangle \langle a_{l'_1 m'_1} a_{l'_2 m'_2} \rangle \langle a_{l_3 m_3} a_{l'_3 m'_3} \rangle - 9\, C_{l_1 m_1, l_2 m_2} \langle a_{l'_1 m'_1} a_{l'_2 m'_2} \rangle \langle a_{l_3 m_3} a_{l'_3 m'_3} \rangle \\
- & \; 9 \langle a_{l_1 m_1} a_{l_2 m_2} \rangle C_{l'_1 m'_1, l'_2 m'_2} \langle a_{l_3 m_3} a_{l'_3 m'_3} \rangle + 9\, C_{l_1 m_1, l_2 m_2} C_{l'_1 m'_1, l'_2 m'_2} \langle a_{l_3 m_3} a_{l'_3 m'_3} \rangle + ... \left. \right] \\
= & \; 6 \sum_{l_i m_i l'_i m'_i} \mathcal{R}_{n l_1 l_2 l_3} \frac{C_{l_1 m_1, l'_1 m'_1} C_{l_2 m_2, l'_2 m'_2} C_{l_3 m_3, l'_3 m'_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3} C_{l'_1} C_{l'_2} C_{l'_3}}} \mathcal{R}_{n' l'_1 l'_2 l'_3} \\
= & \; 6 \mathcal{R} \mathcal{C} \mathcal{R}^T
\end{aligned}$$

INVERSE COVARIANCE

Also as all covariance matrices are symmetric positive definite they have a Cholesky decomposition

$$\zeta = \tilde{\lambda}\,\tilde{\lambda}^T$$

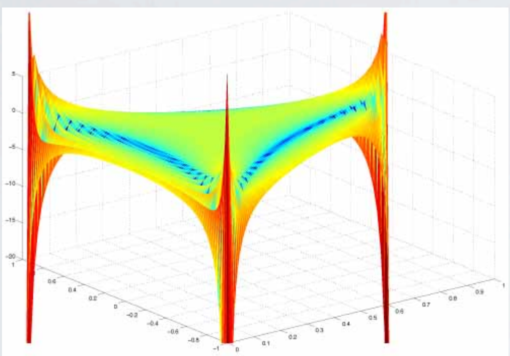
And we can absorb the covariance into our modes. This amounts to a re-orthogonalisation to an uncorrelated orthonormal basis

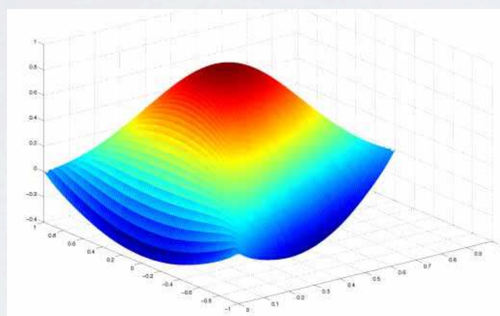
$$\alpha' = \tilde{\lambda}^{-1}\alpha \quad \beta' = \tilde{\lambda}^{-1}\beta$$

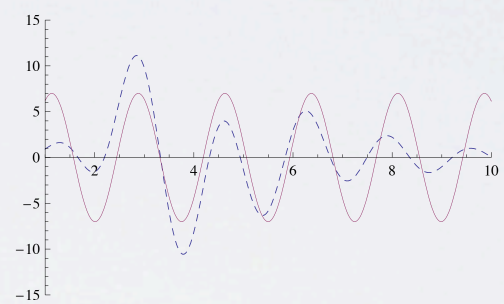
$$\mathcal{E} = \frac{\alpha'^T\beta'}{\alpha'^T\alpha'}\,, \quad \zeta' = I$$

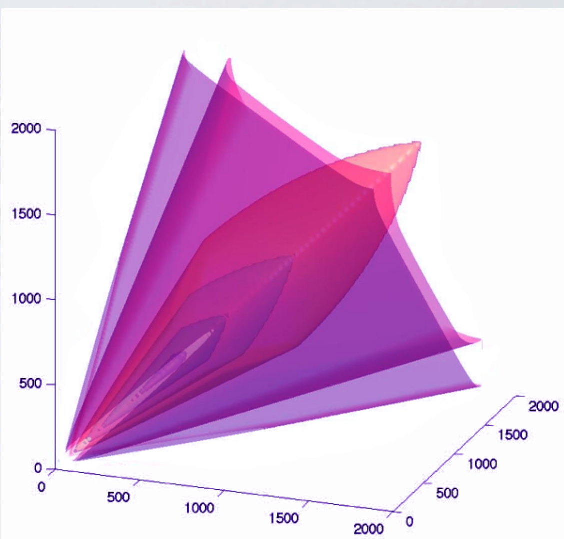
RECONSTRUCTION

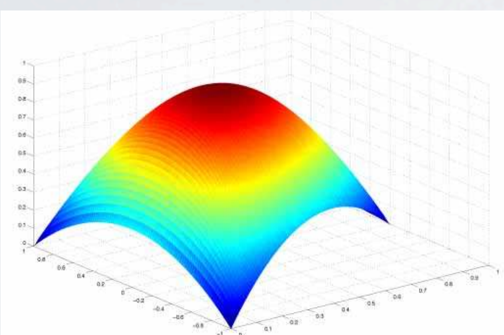
There is no "standard model" of non-Gaussianity so our goal should not be to obtain the tightest error bars on f_{NL} but to decide which f_{NL} we should be trying to constrain.

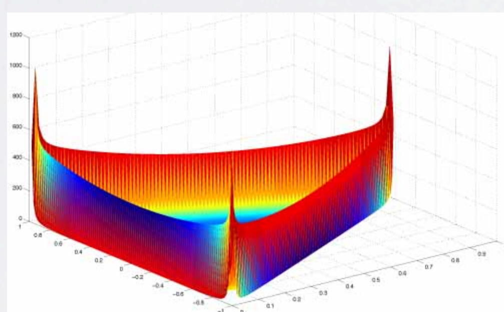


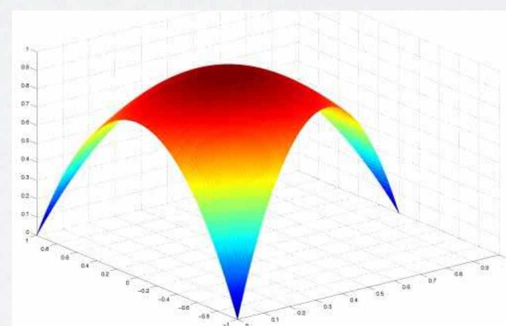


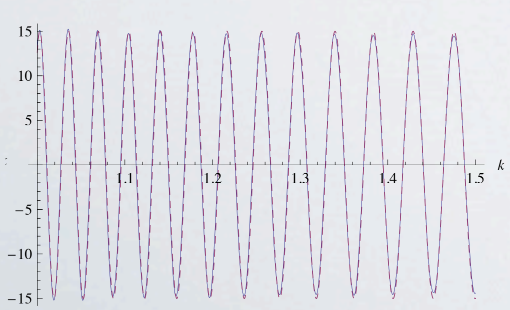


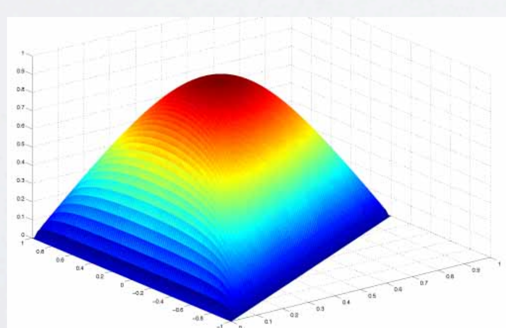


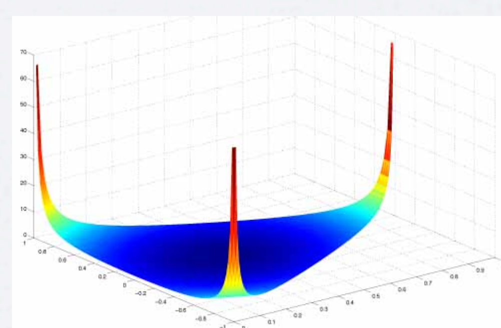


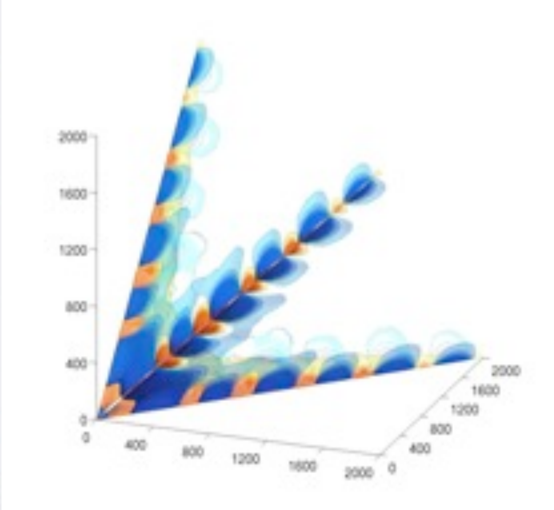






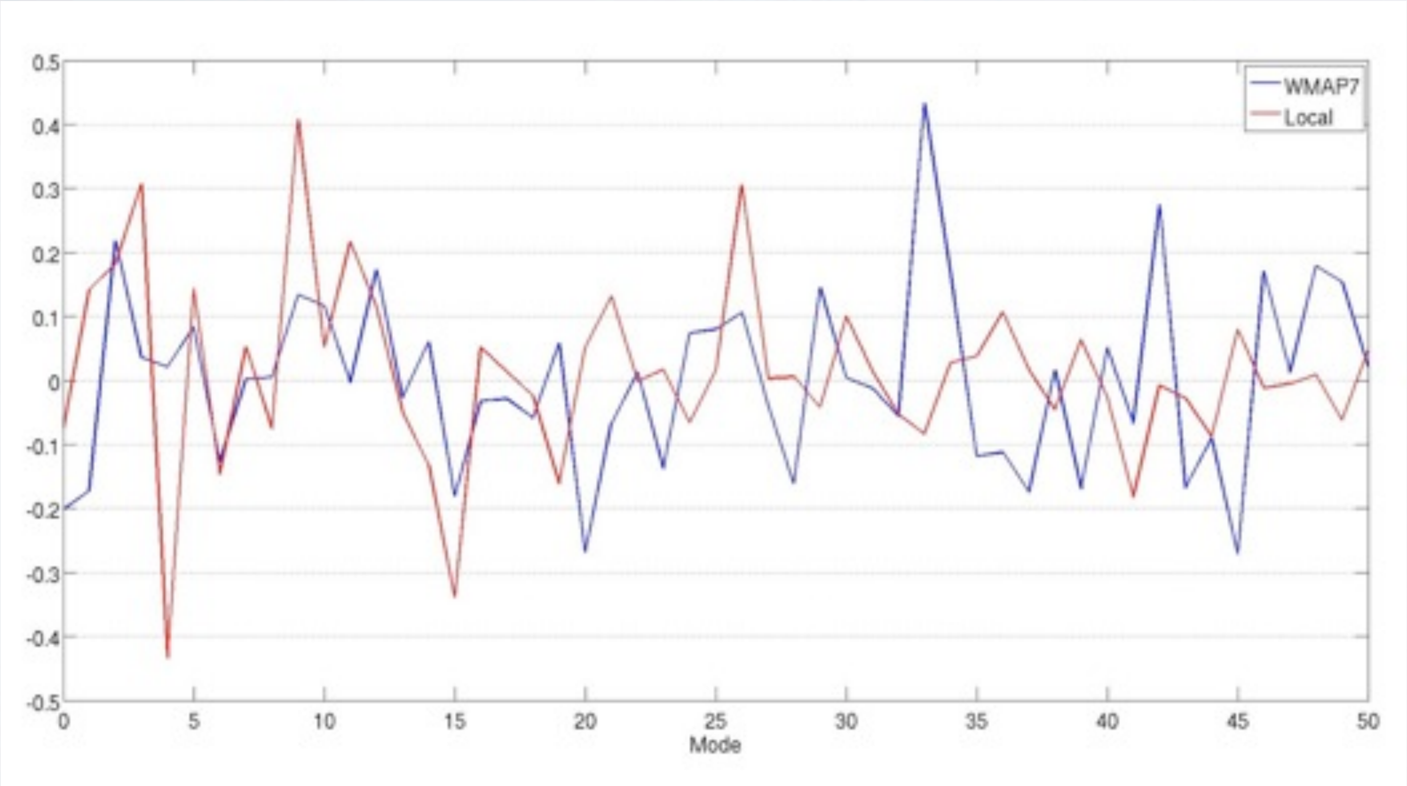


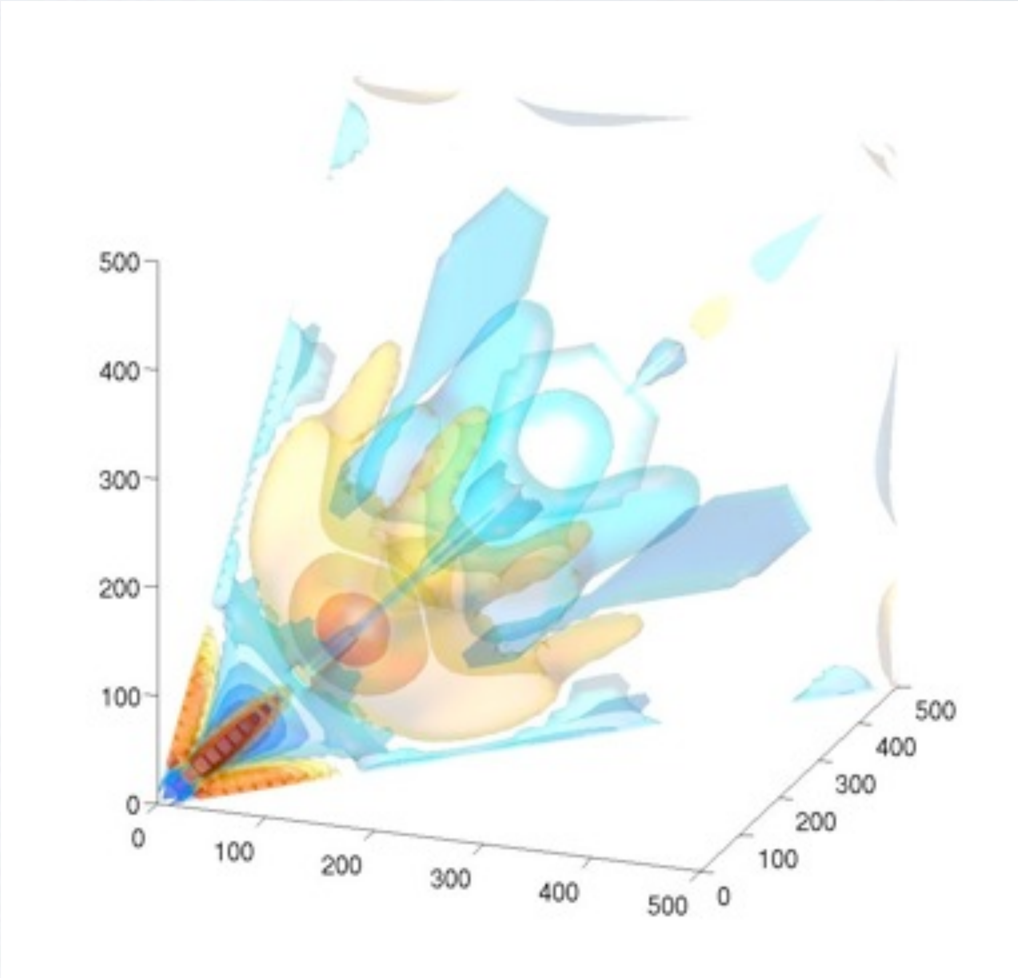




RECONSTRUCTION

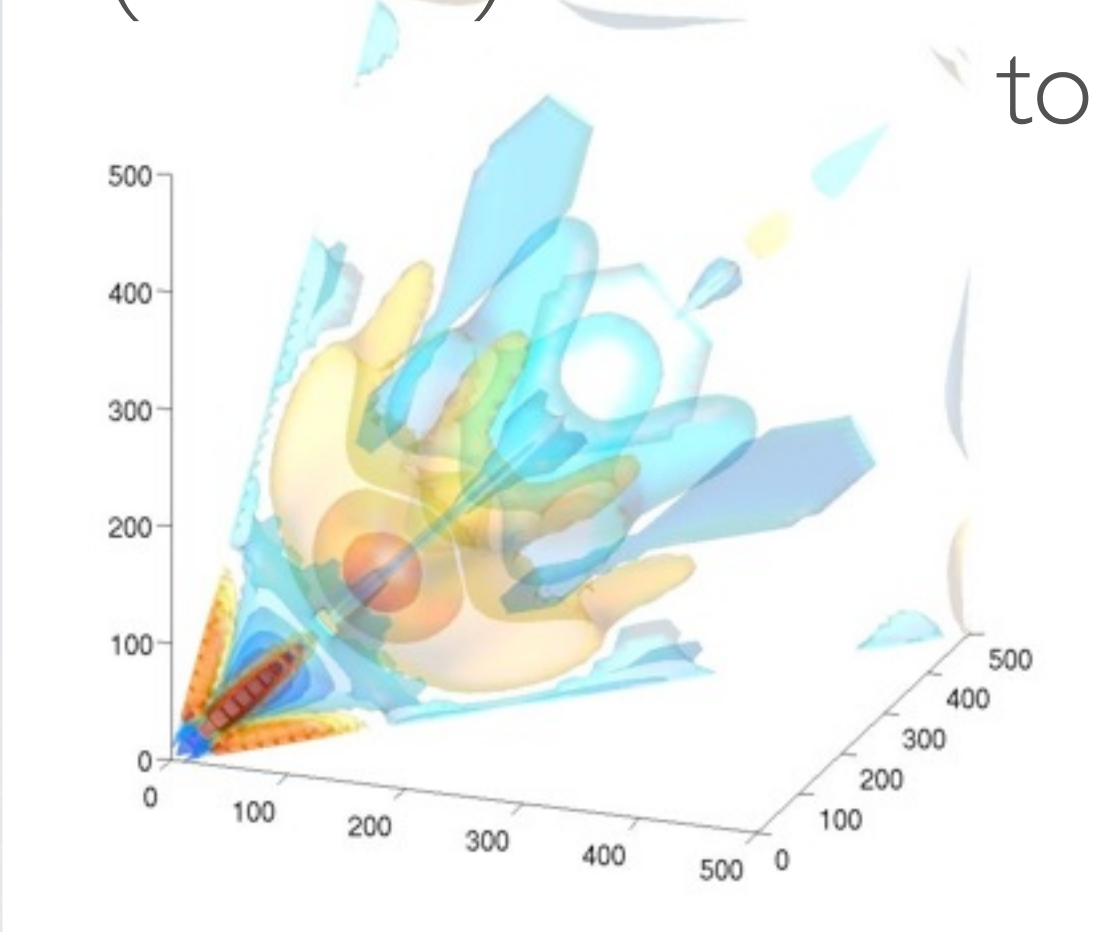
We have $\langle\beta\rangle = \alpha$ so can reconstruct the best fit bispectrum to the data by using the β as our α. If we have constructed a primordial basis as well then we can use Γ to find the best fit primordial bispectrum

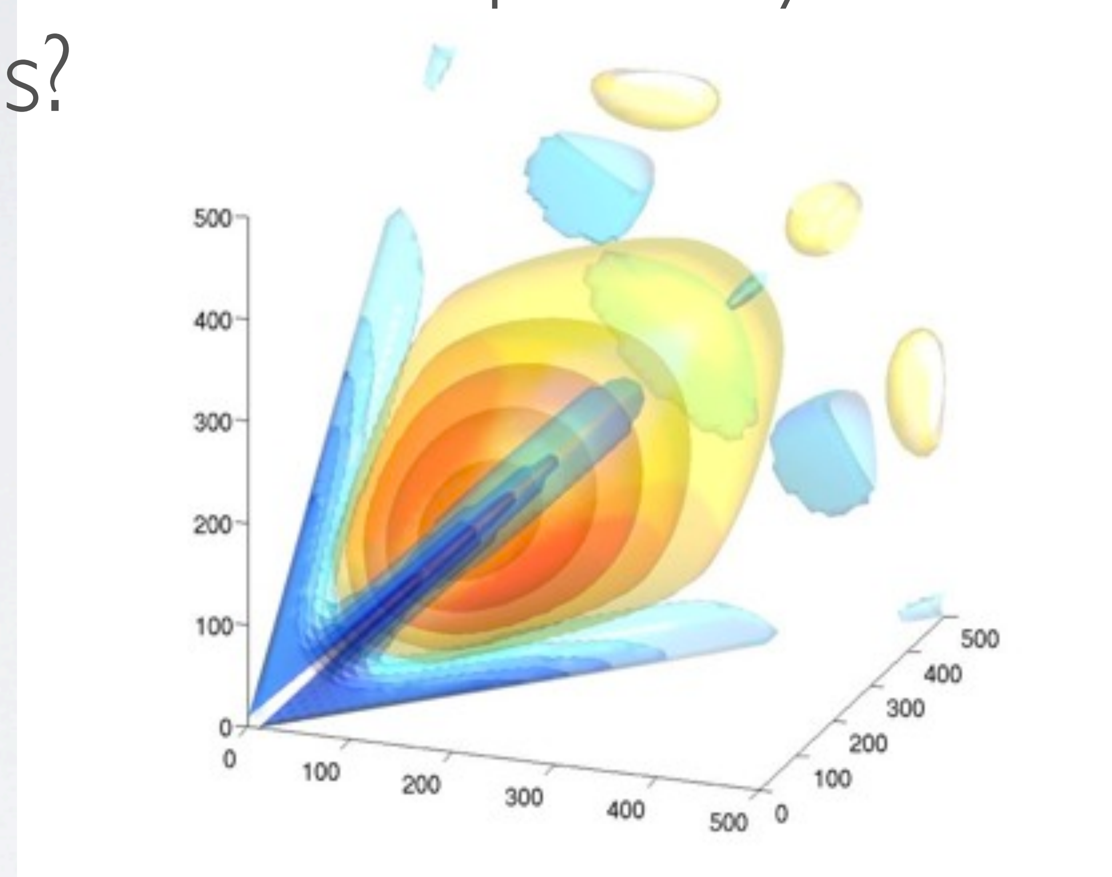


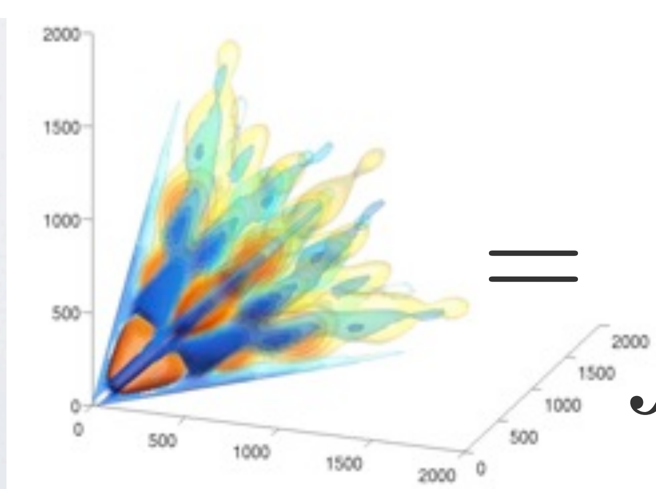
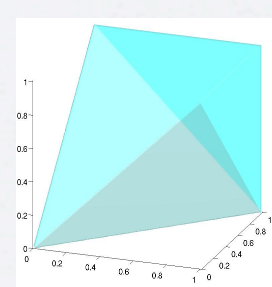


RECONSTRUCTION

First (and last) exercise: What primordial shape do you need to fit this?

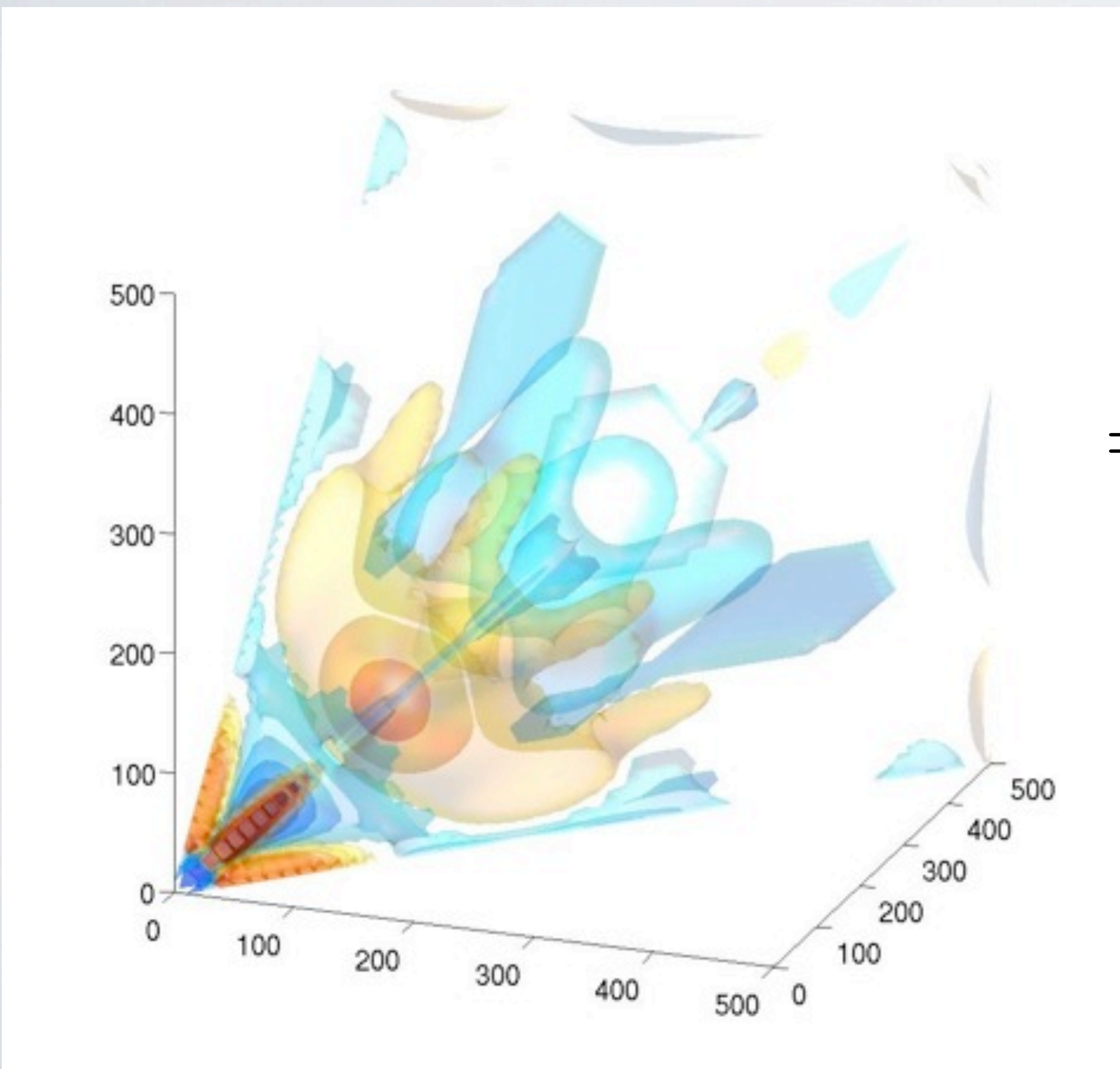




$$\Bigg($$  $$= \int_{\mathcal{V}_k}$$  $$\times \Delta \Bigg)$$

RECONSTRUCTION

Solution?



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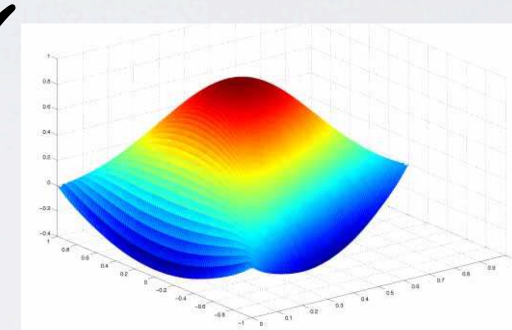

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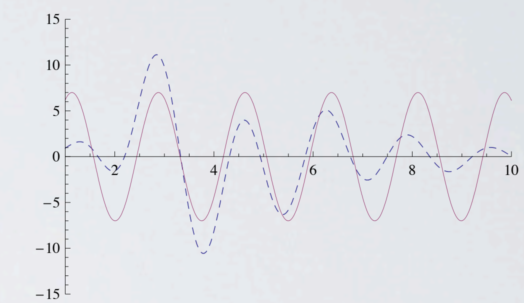

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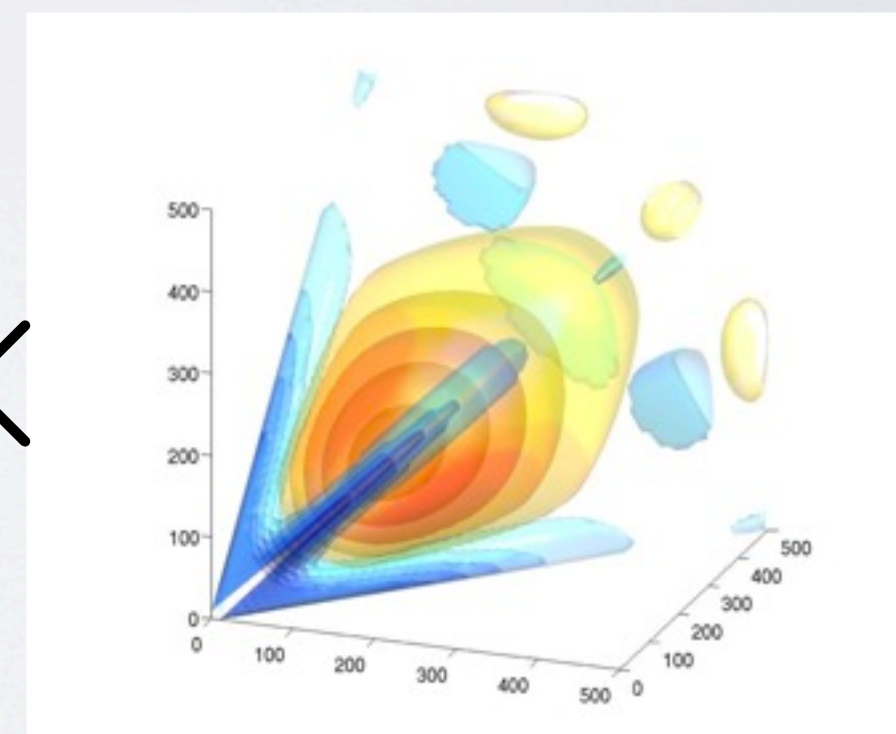
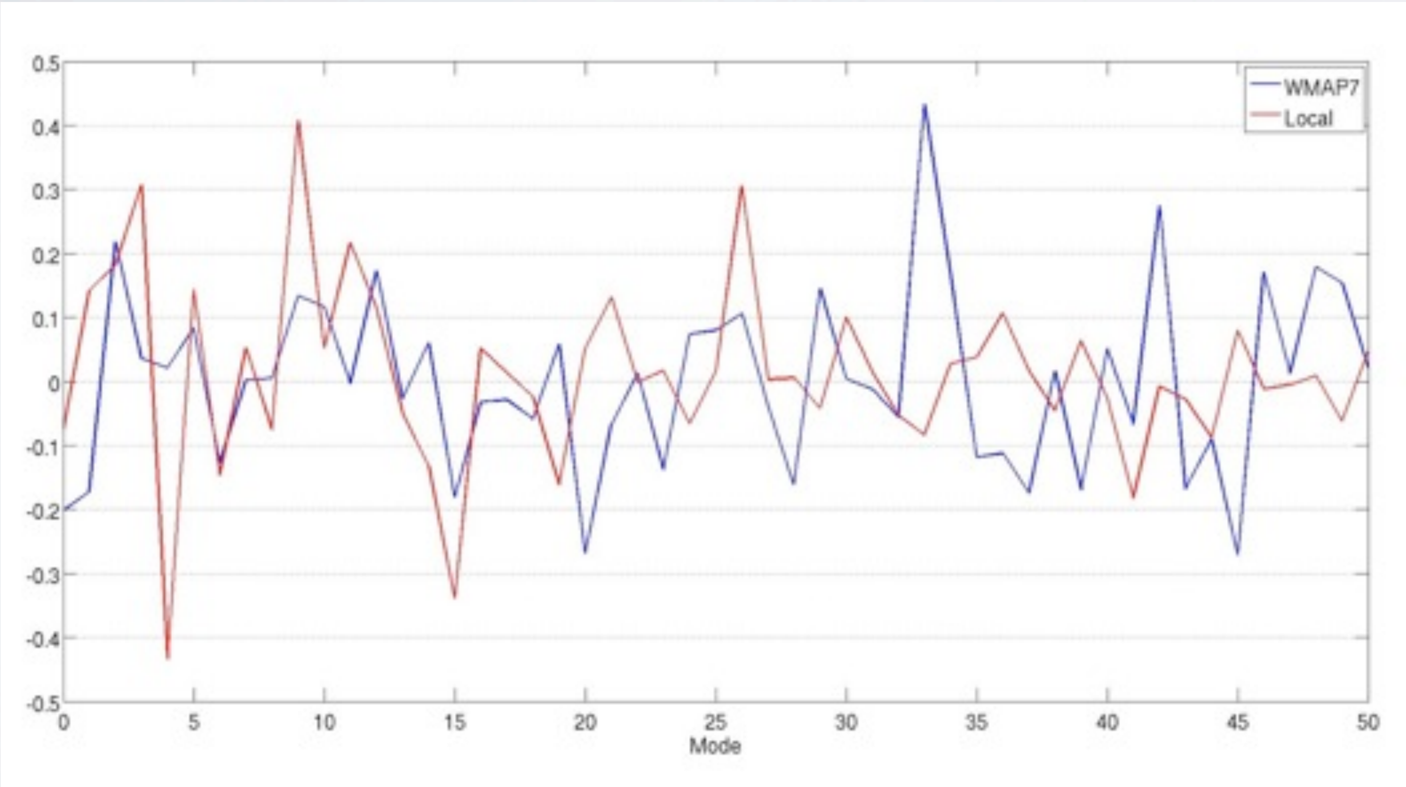
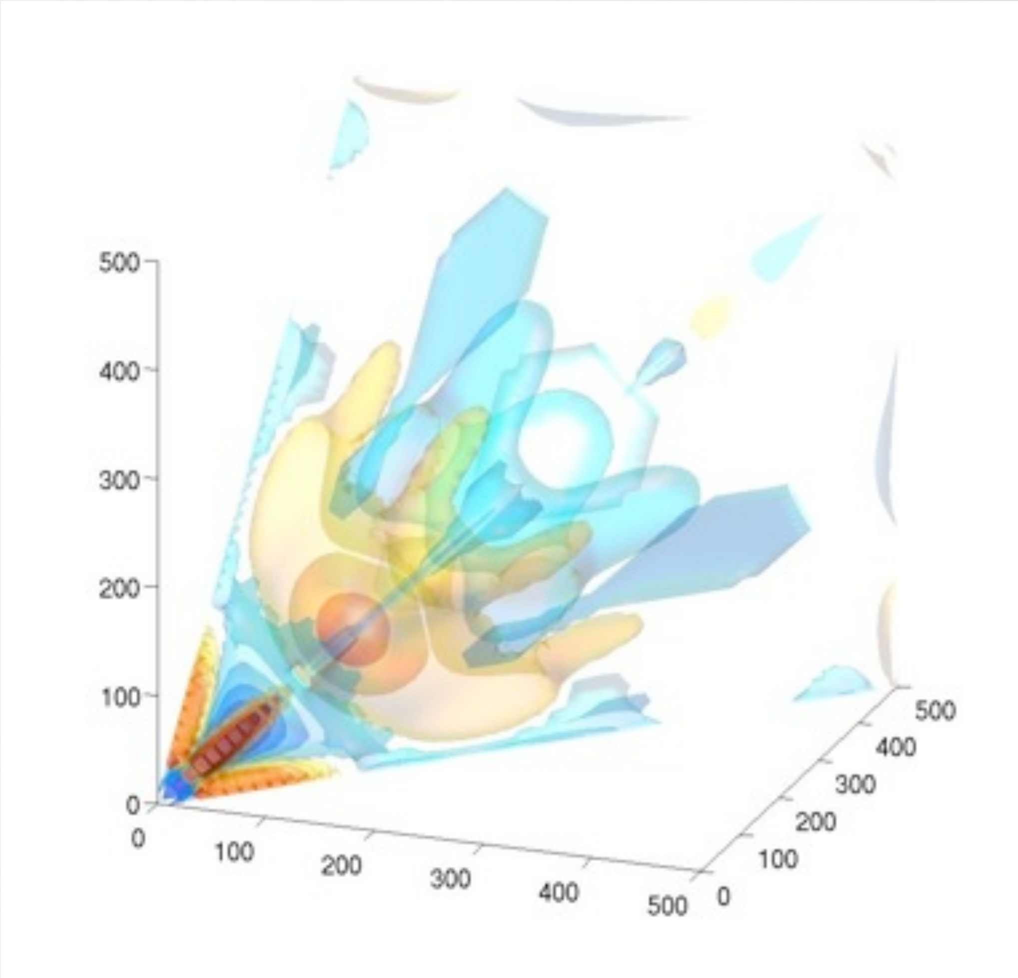


RECONSTRUCTION

This is a *a posteriori* method. So we must be very careful with our interpretation. How do we know if this is a real bispectrum or if it is just a reconstruction of noise?





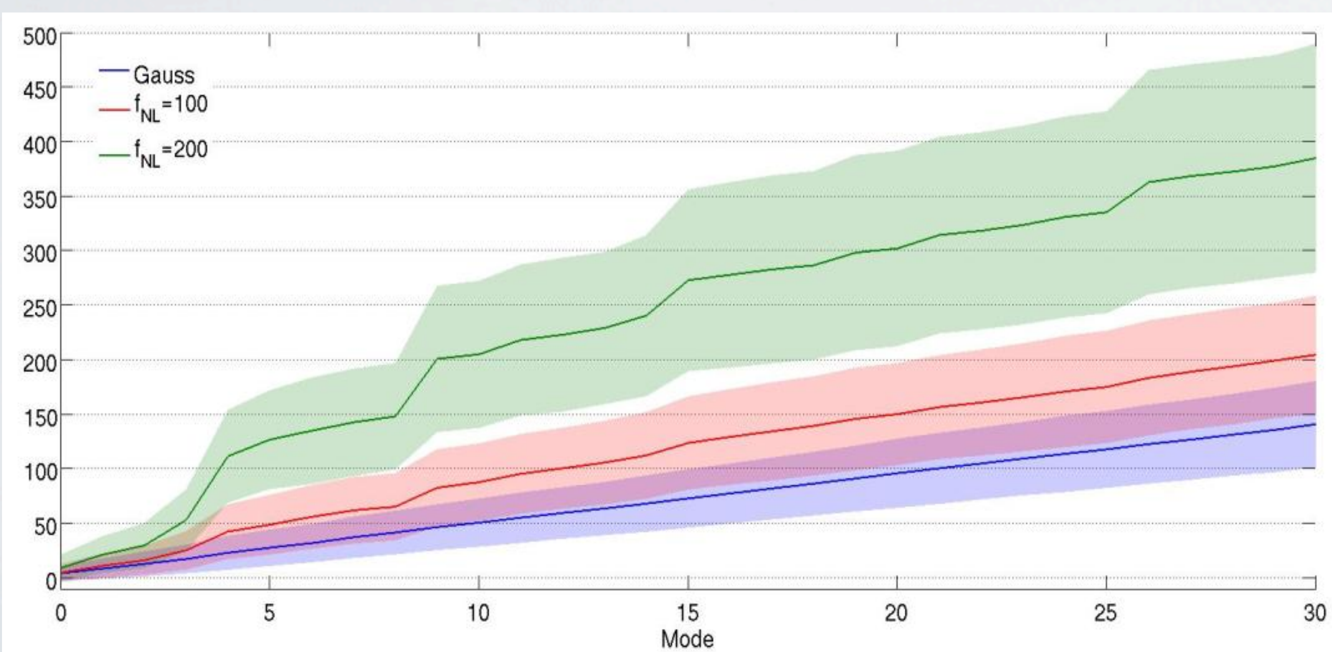
RECONSTRUCTION

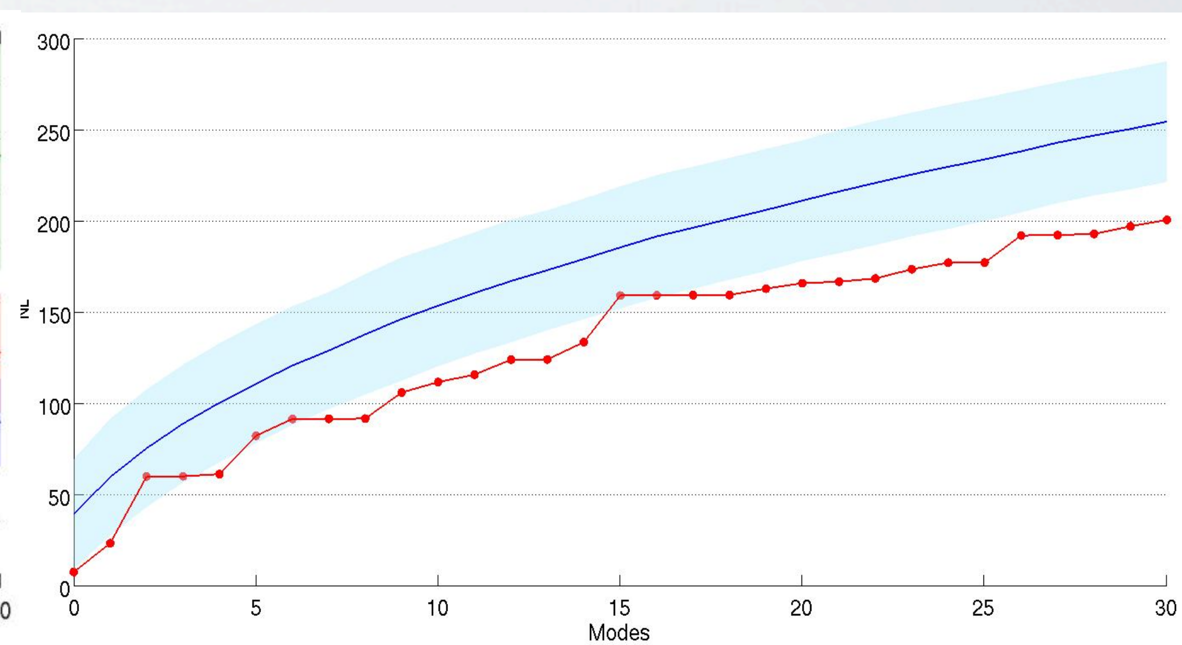
We can perform a blind search on the amplitude

$$F_{NL}^2 = \sum_{n=0}^{N} \beta_n^2$$

$$F_{NL}^2 = 6N \qquad \text{(Gaussian)}$$

$$\delta F_{NL}^2 = 6\sqrt{2N}\ \text{(Gaussian)}$$





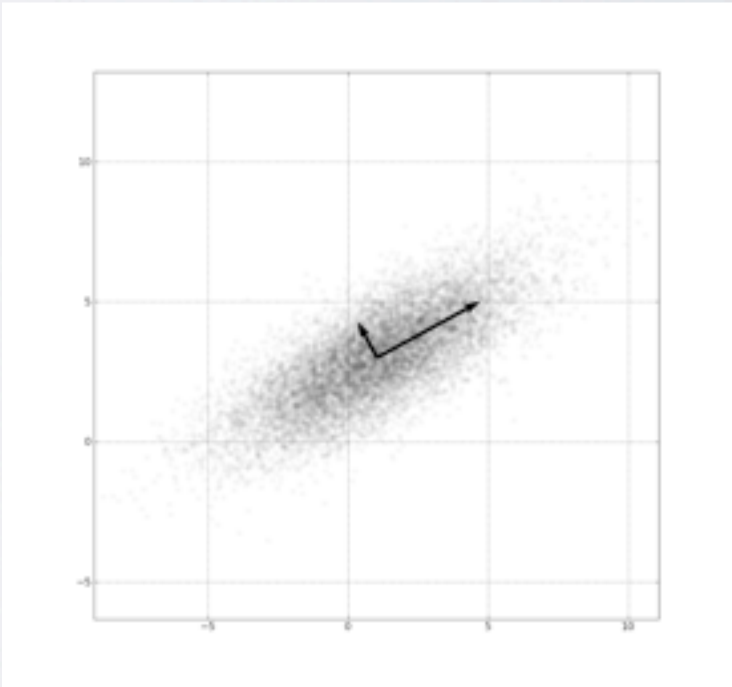
CONTAMINANTS

As we expect the covariance matrix to be the identity we can use principle component analysis to identify the shape of contaminants.

We first calculate the covariance matrix for beta from simulations

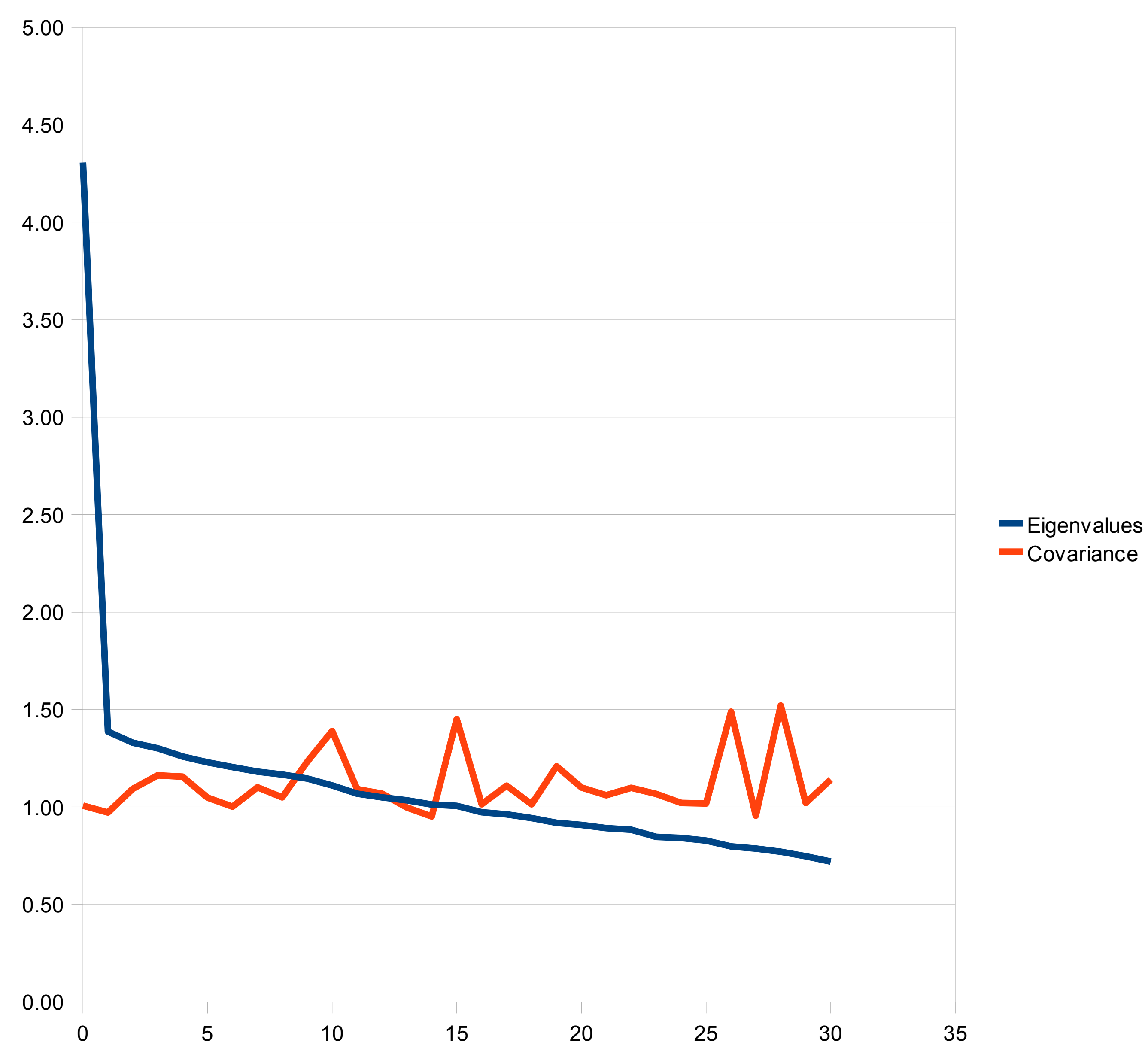
$$V \zeta V^T = D$$

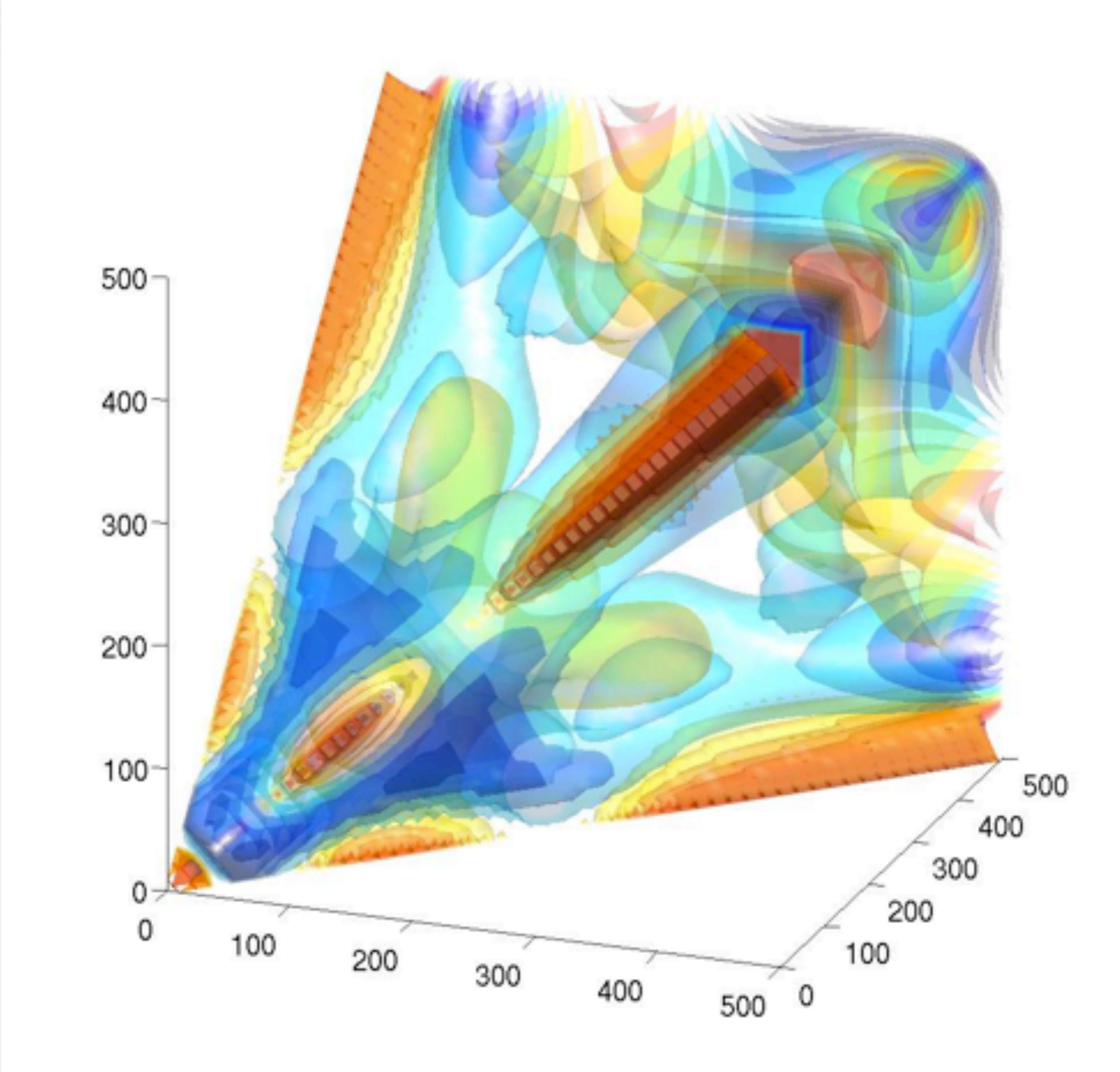
And then find the rotation which diagonalises it. This is equivalent to performing an eigen decomposition. The result is that you obtain a new orthonormal basis but now your modes are uncorrelated and ordered from greatest to least variance.



CONTAMINANTS

WMAP inhomogeneous noise

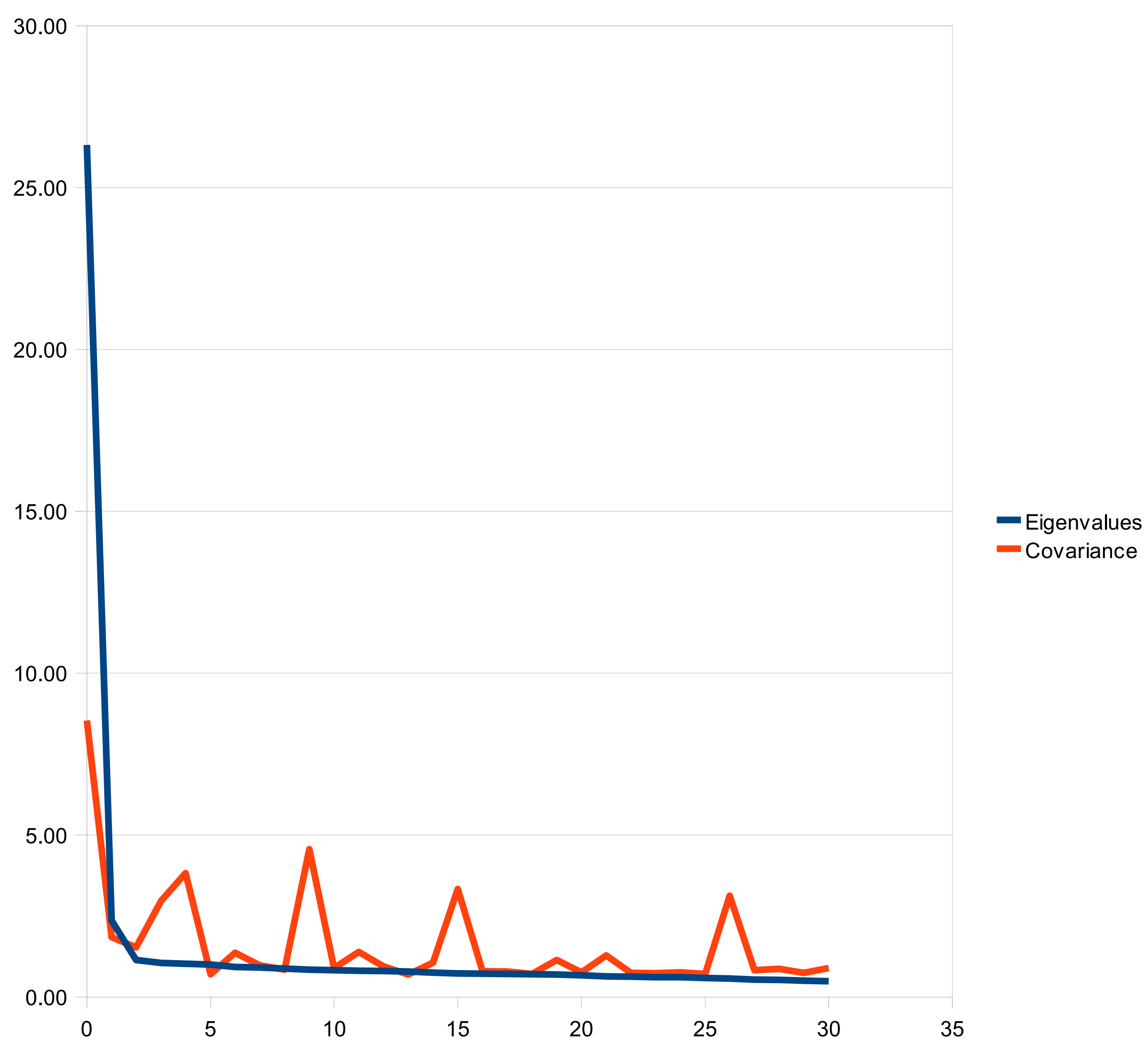


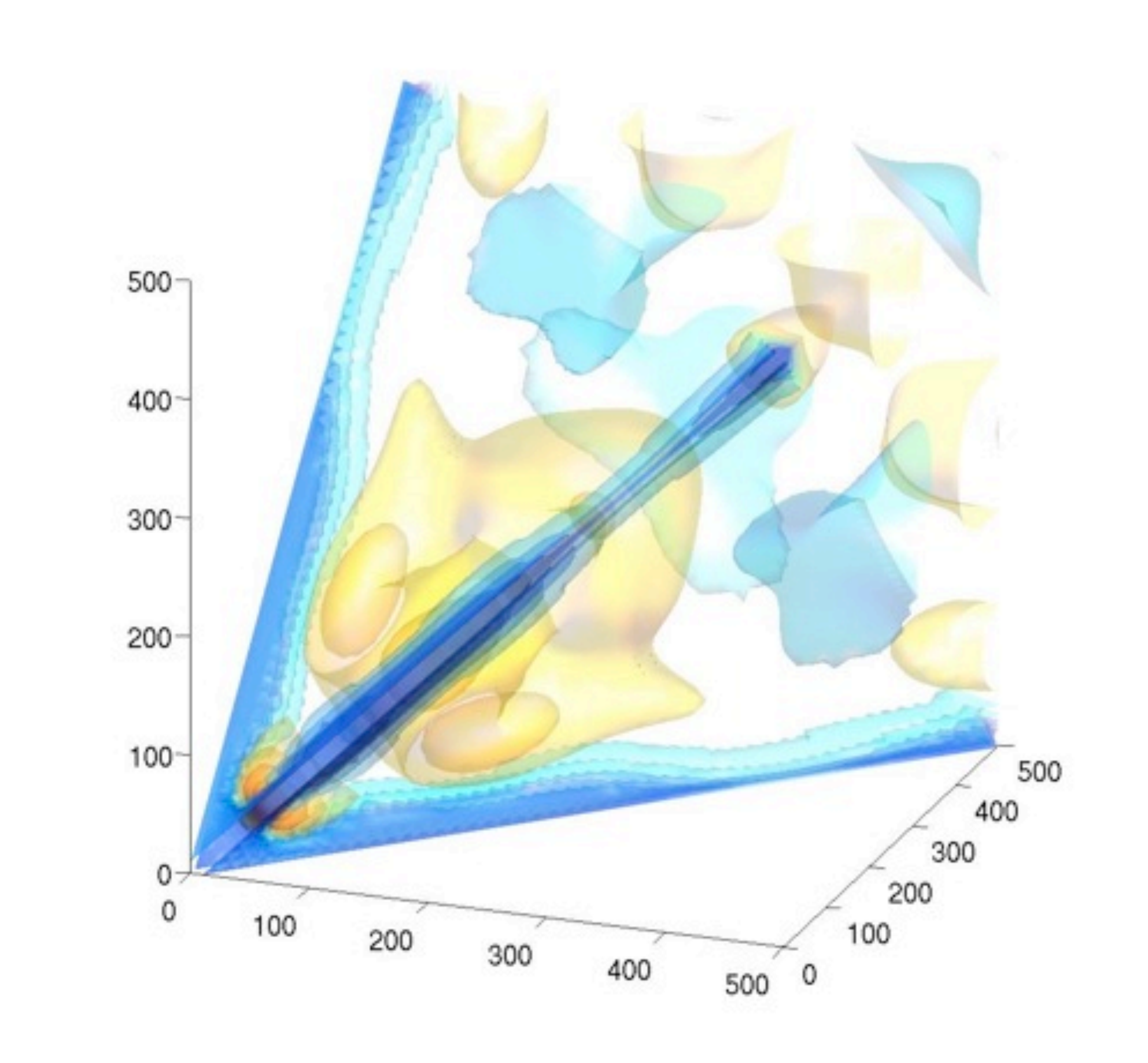


Monday, 5 September 2011

CONTAMINANTS

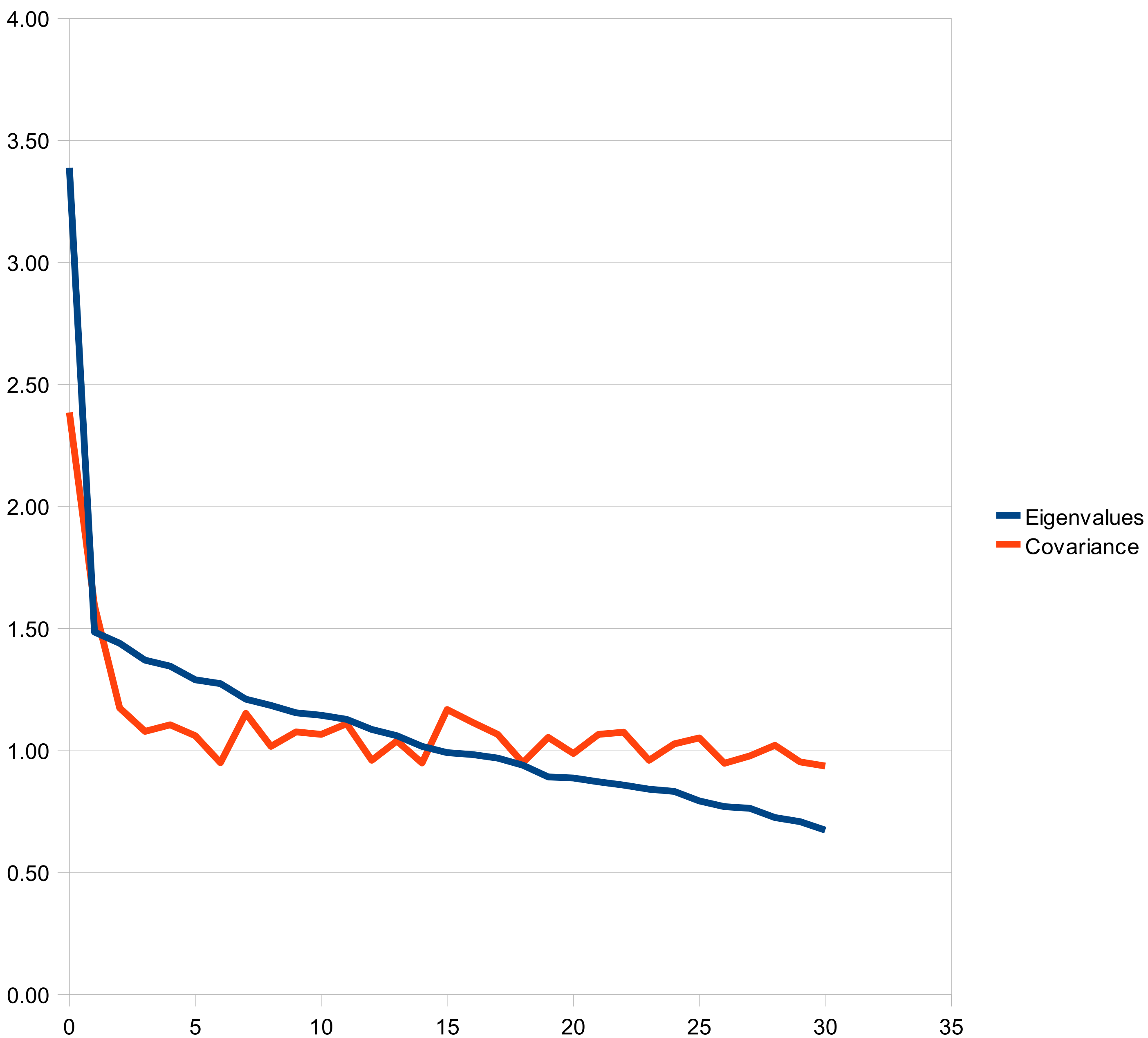
WMAP Mask

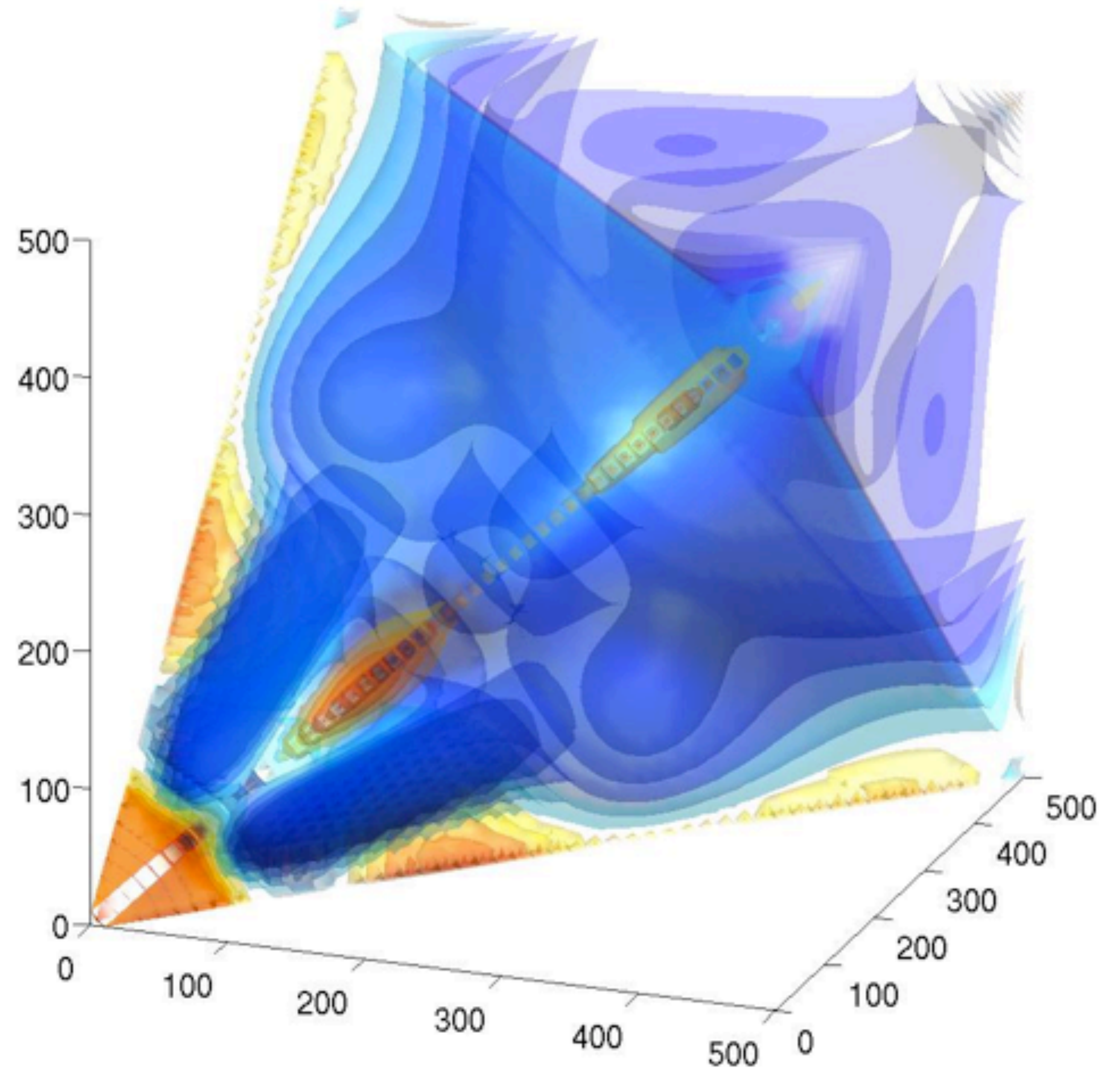




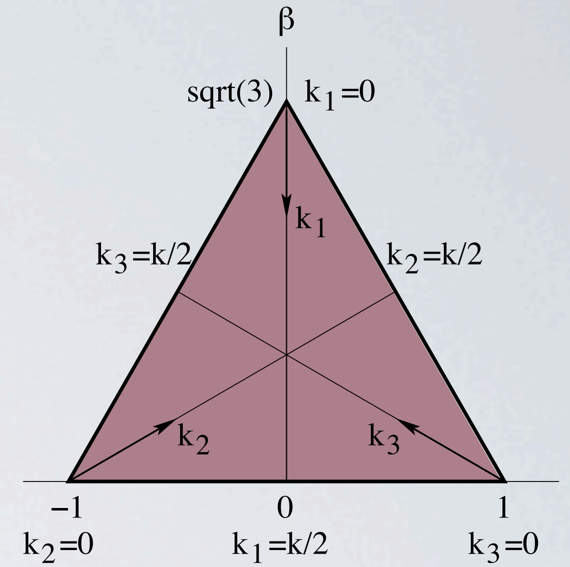
CONTAMINANTS

Point sources





AFTERWORD: NG FRAGMENTATION



What to do with a new NG shape function?

1. Plot it in appropriate 2D or 3D coordinates
2. Check cross-correlation with other standard shapes

$$F(S,S') = \int_{\mathcal{V}_k} S(k_1,k_2,k_3)S'(k_1,k_2,k_3)\omega(k_1,k_2,k_3)d\mathcal{V}_k\,,$$

arXiv:0812.3413

3. Normalise consistently relative to local shape using F_{NL}

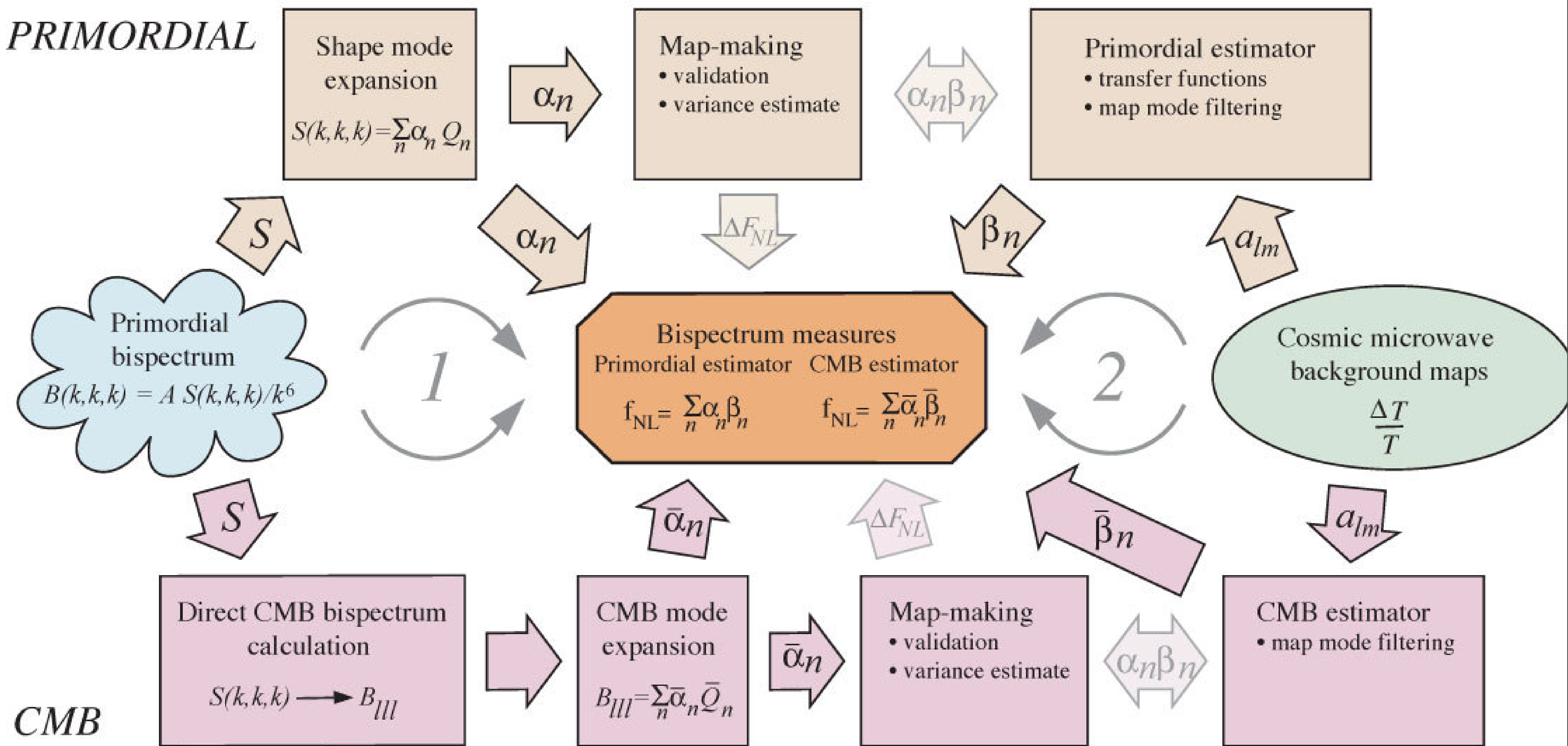
$$F_{\rm NL} = \frac{1}{N\bar{N}_{\rm loc}} \sum_{l_i m_i} \mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3} b_{l_1 l_2 l_3} \frac{a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}\,,$$

arXiv:0912.5516

4. Predict/calculate standardised eigenmode coefficients

- *await late-time and primordial mode coefficient CMB constraints*

CMB PIPELINES



Monday, 5 September 2011

CMB conclusions

- NG calculational techniques well-developed
- Growing number of primordial NG shapes
- No significant evidence for CMB NG ... yet
- General modal WMAP bispectrum constraints
- Useful for characterising contaminants, secondaries etc
- First near-optimal WMAP trispectrum constraints
- Planck analysis underway ....

First Planck cosmology papers due end 2012

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First Planck cosmology papers due end 2012

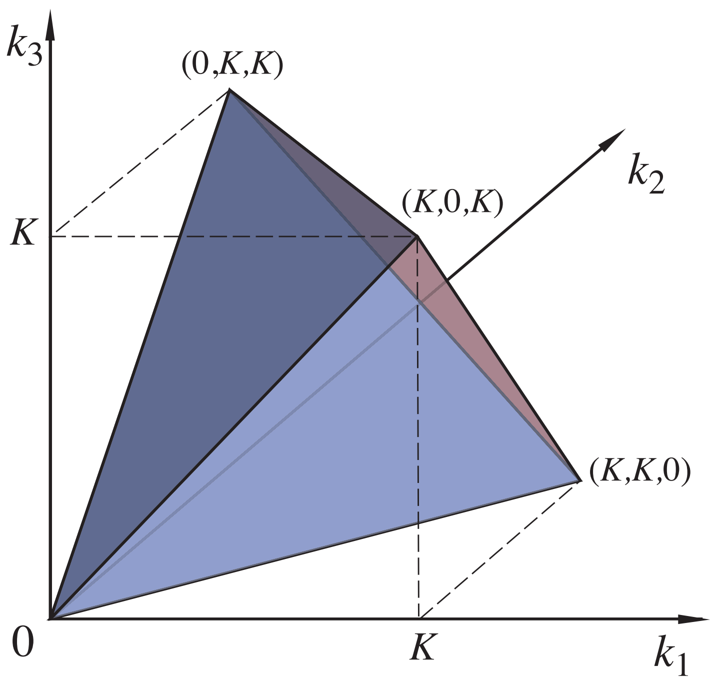
And now for something completely different:

- Postscript on large-scale structure ....

Modal Polyspectra Estimation

THEORY

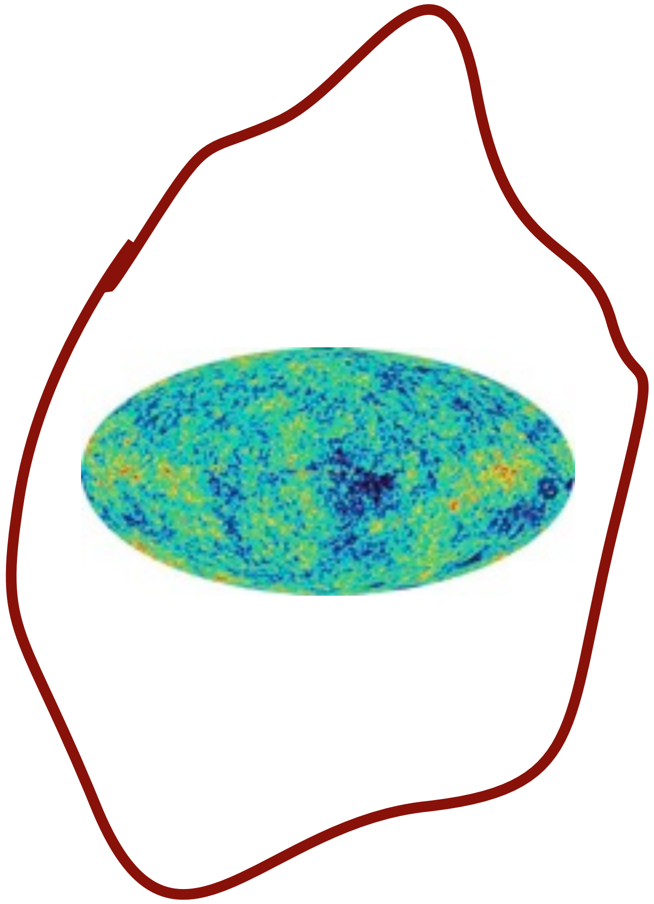
Space of (primordial) isotropic polyspectra (k-space)



Expand any model with primordial modes

OBSERVATION

Space *V of all* possible polyspectra



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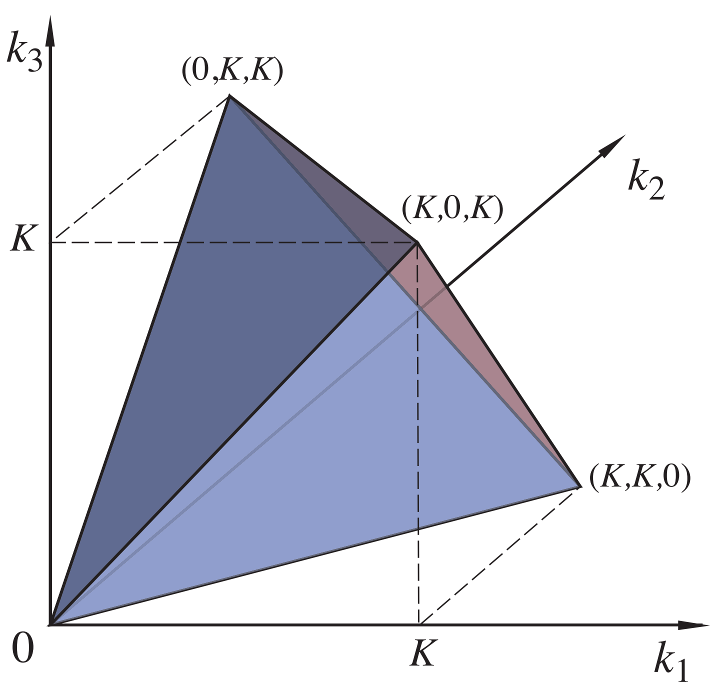

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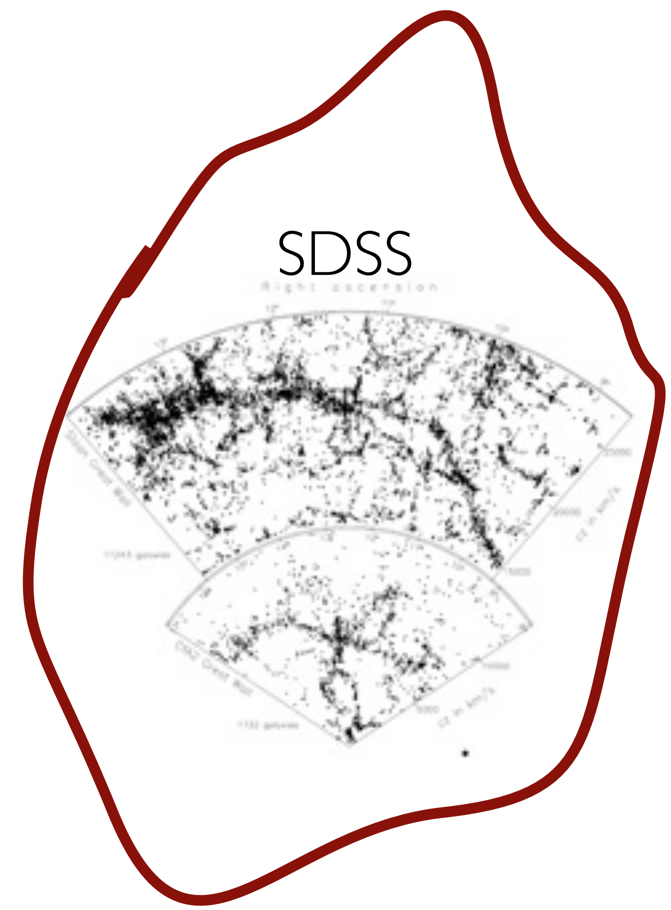

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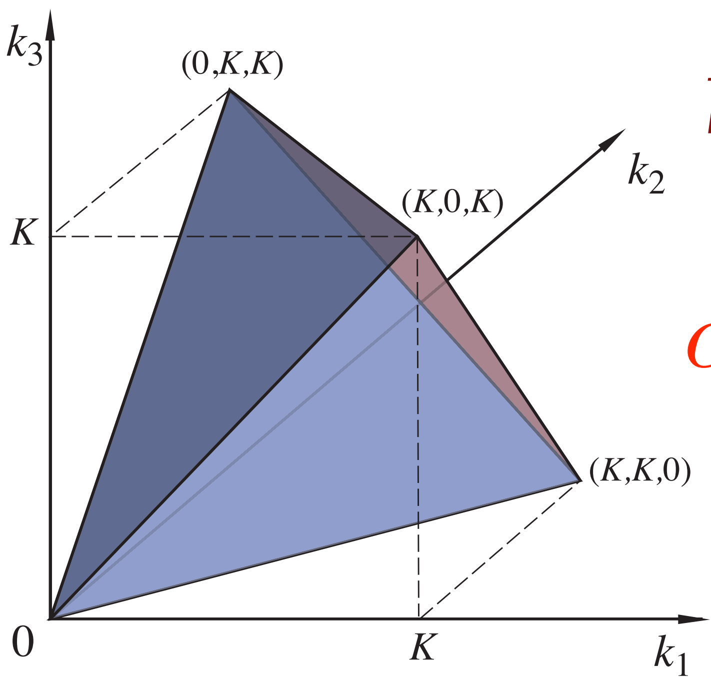

Mode transfer functions

$\alpha_n \rightarrow \bar{\alpha}_n$

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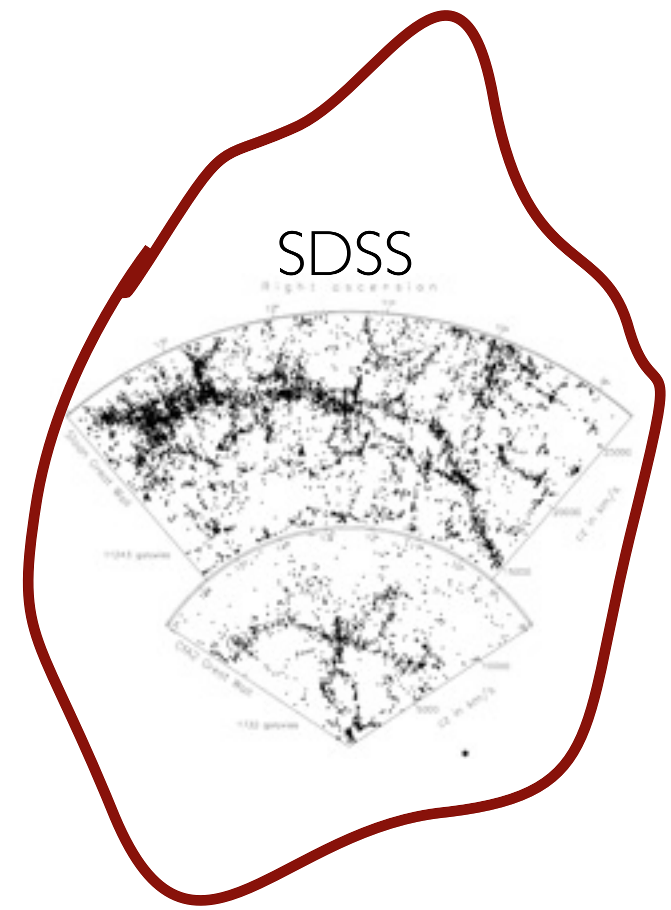

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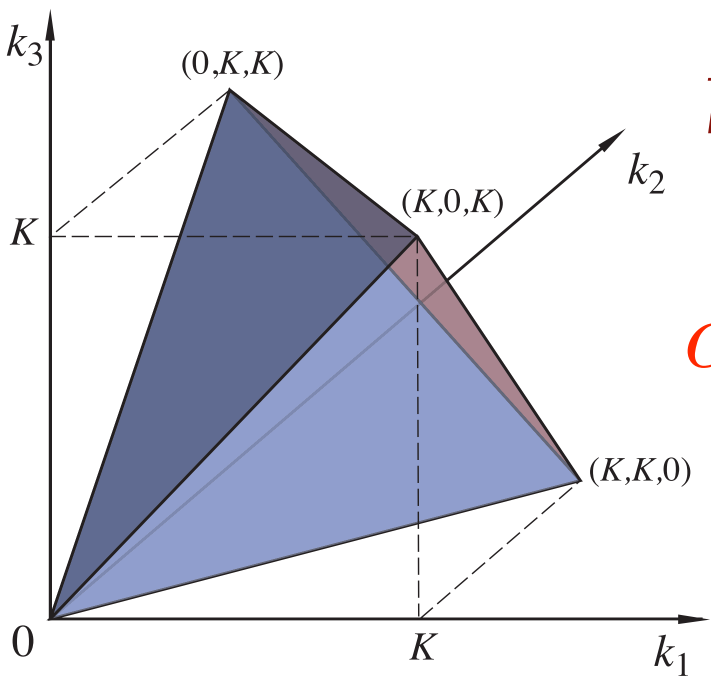

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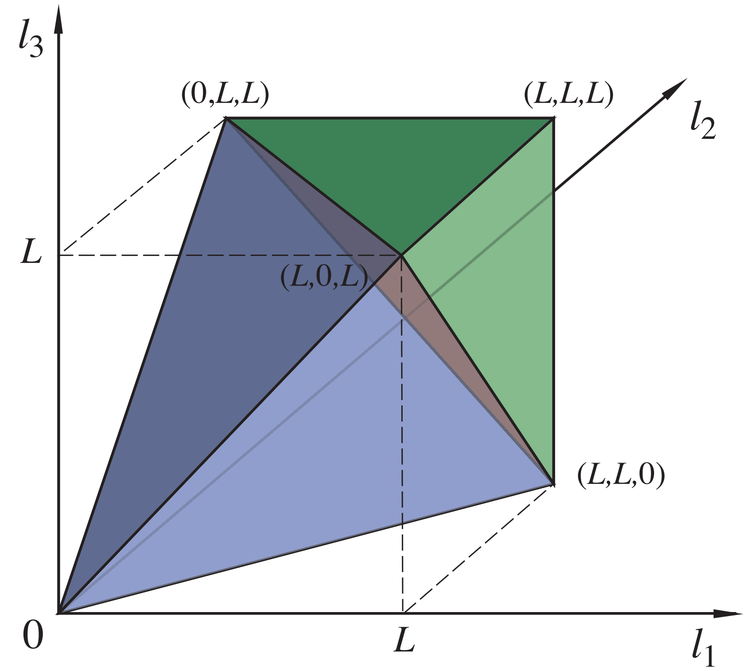
Projected space V_P of late-time polyspectra



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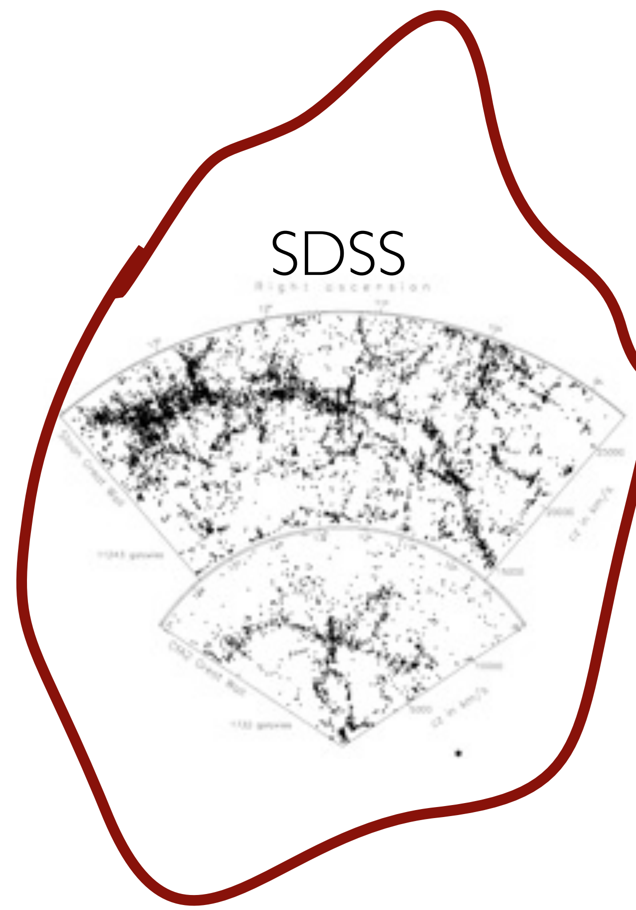

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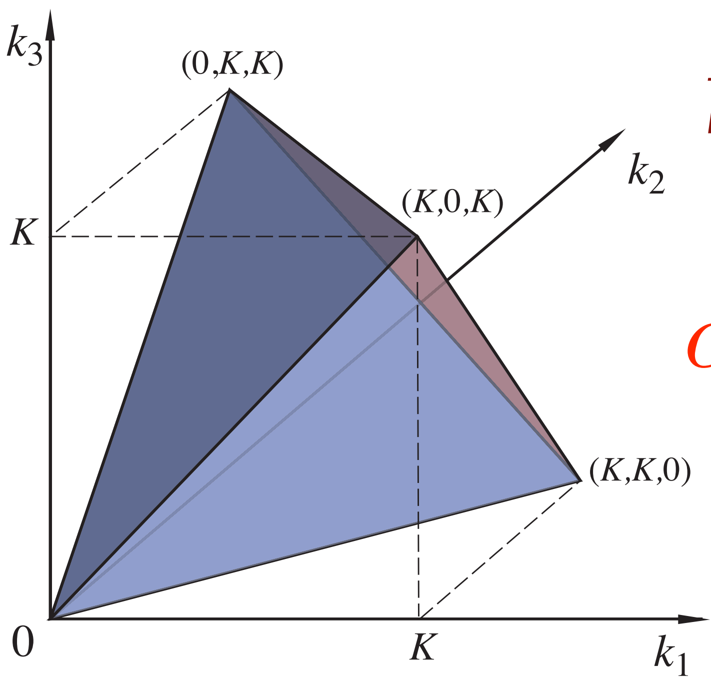

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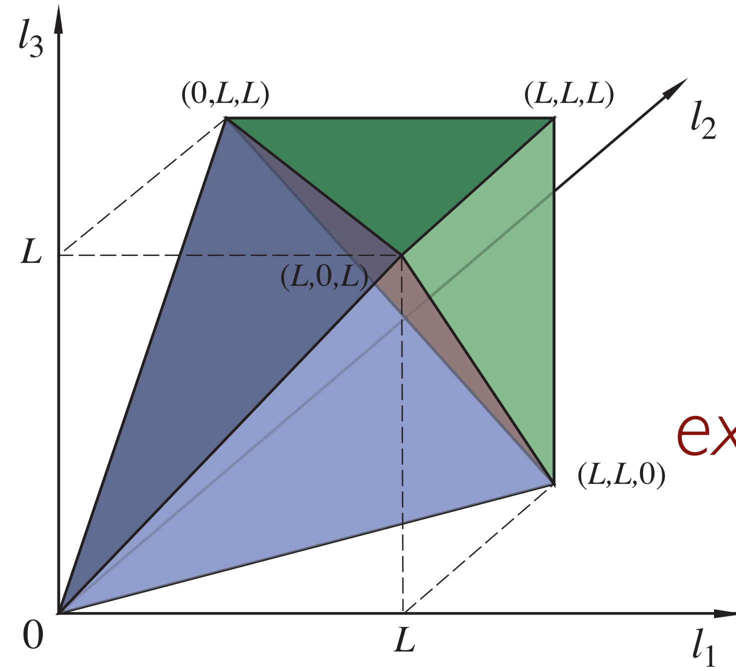

Data mode extraction

$\bar{\beta}_n$

<u>OBSERVATION</u>

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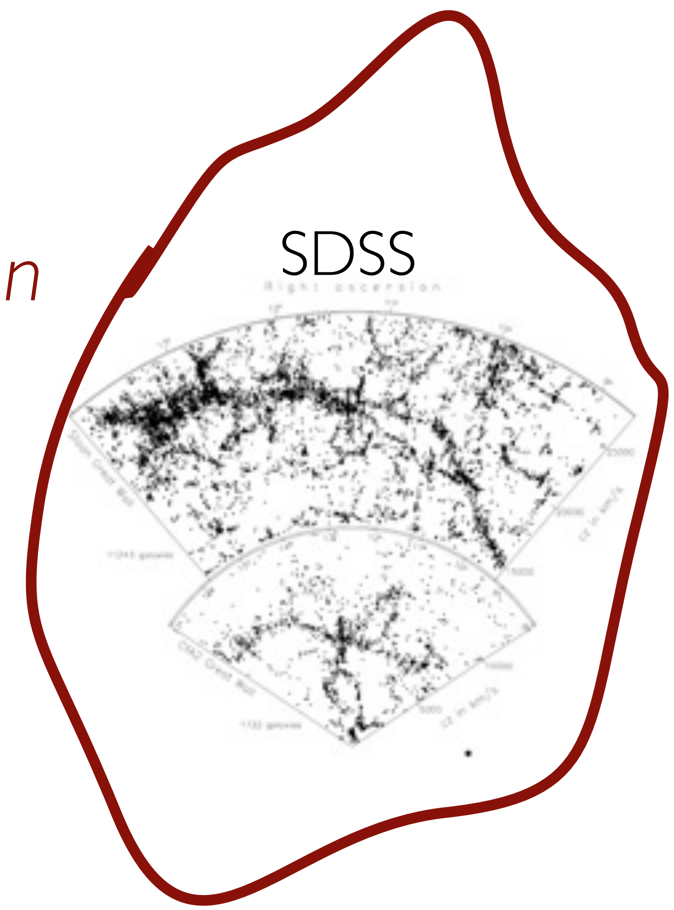

Filter with sufficient separable eigenmodes

Modal Polyspectra Estimation

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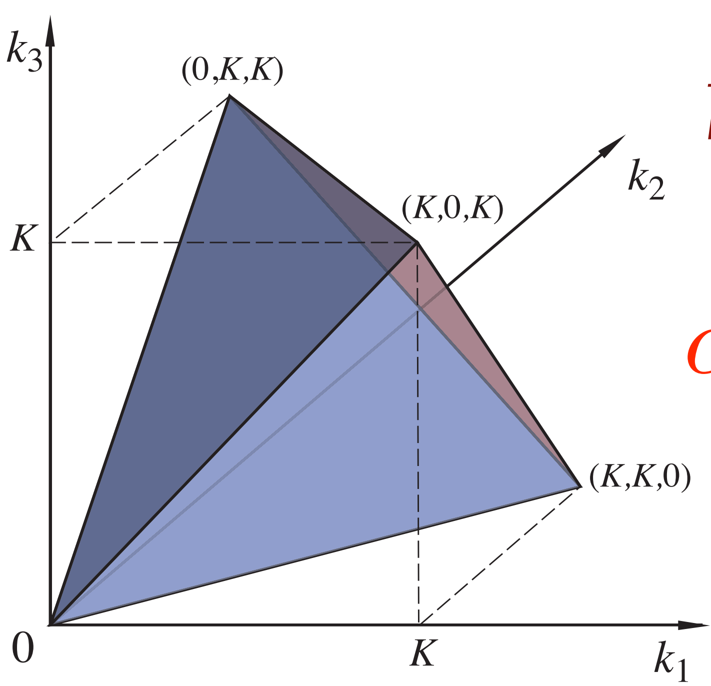

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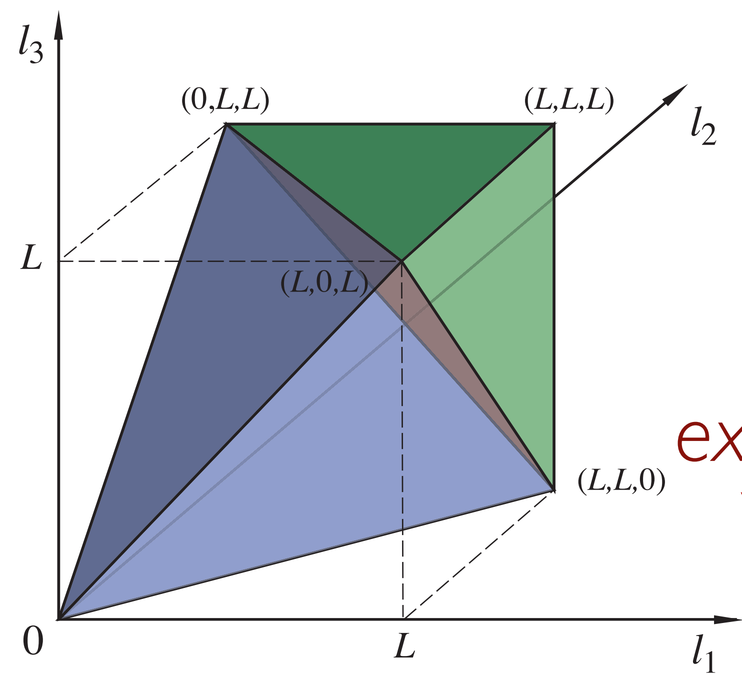

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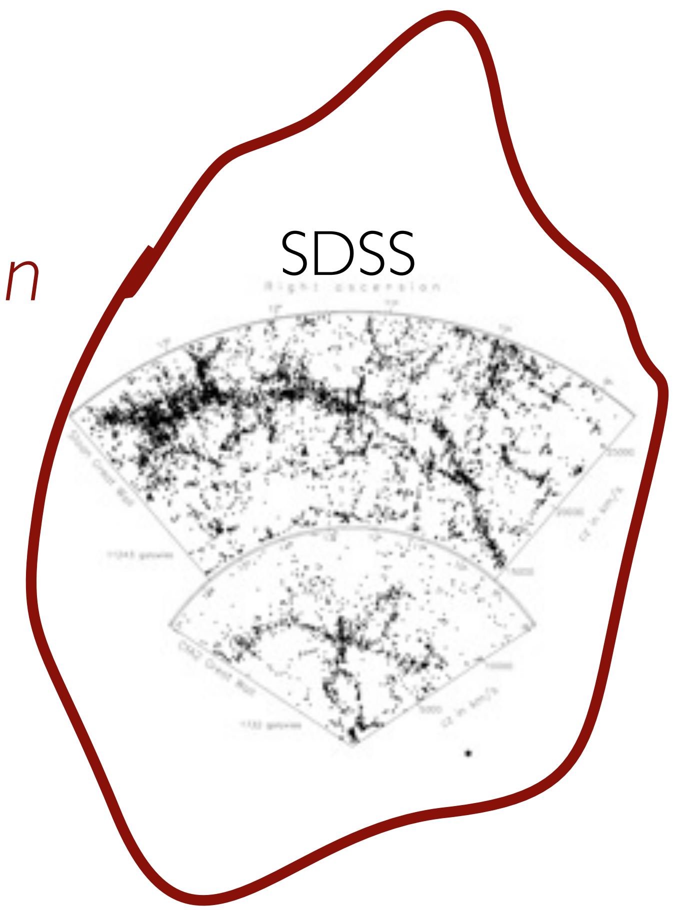

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$\bar{\beta}_n$

Filter with sufficient separable eigenmodes

Modal estimator

$$\mathcal{E} = \frac{\sum_n \bar{\alpha}_n^R \bar{\beta}_n^R}{\sum_n (\bar{\alpha}_n^R)^2}$$

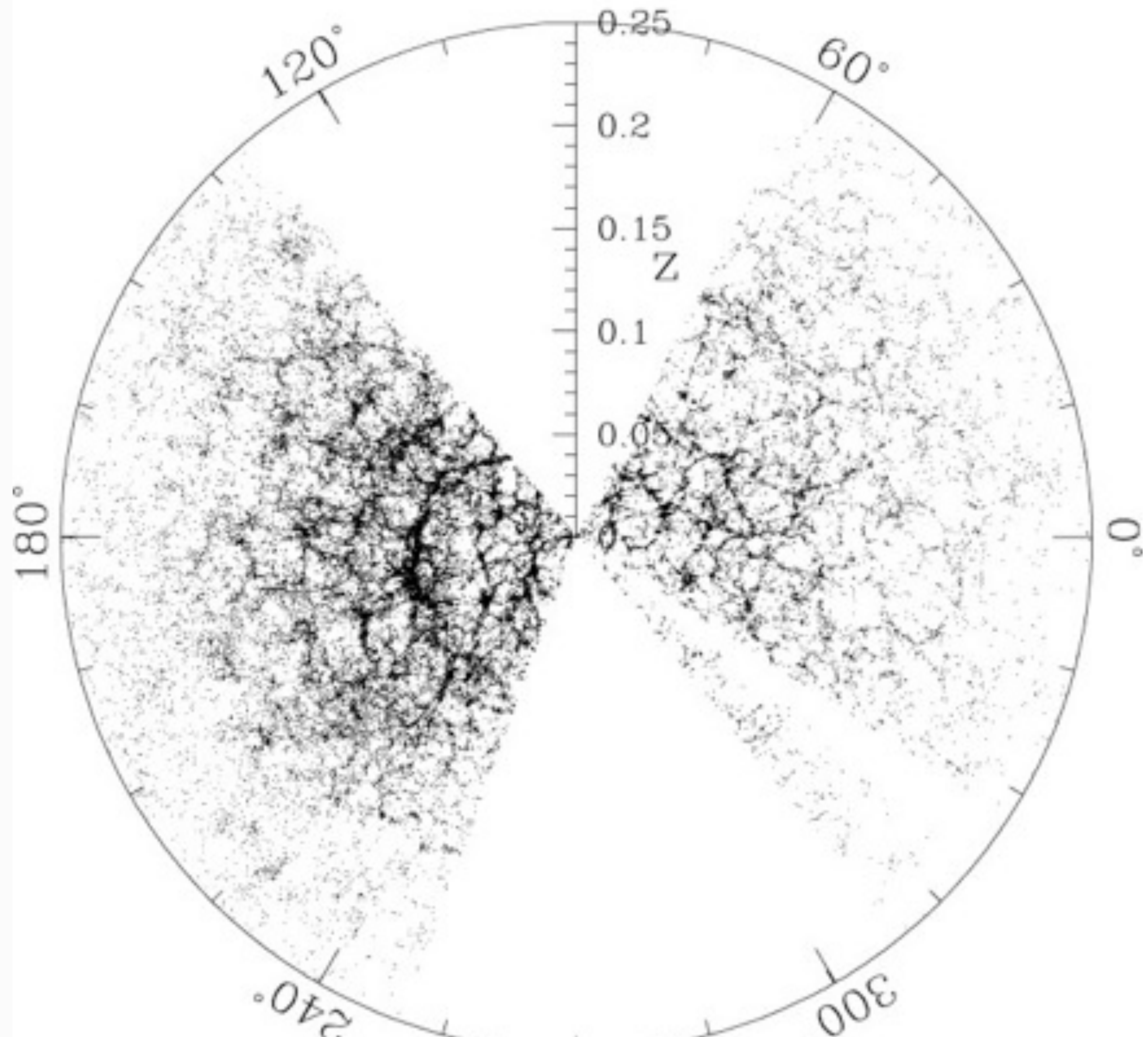
Large-scale structure

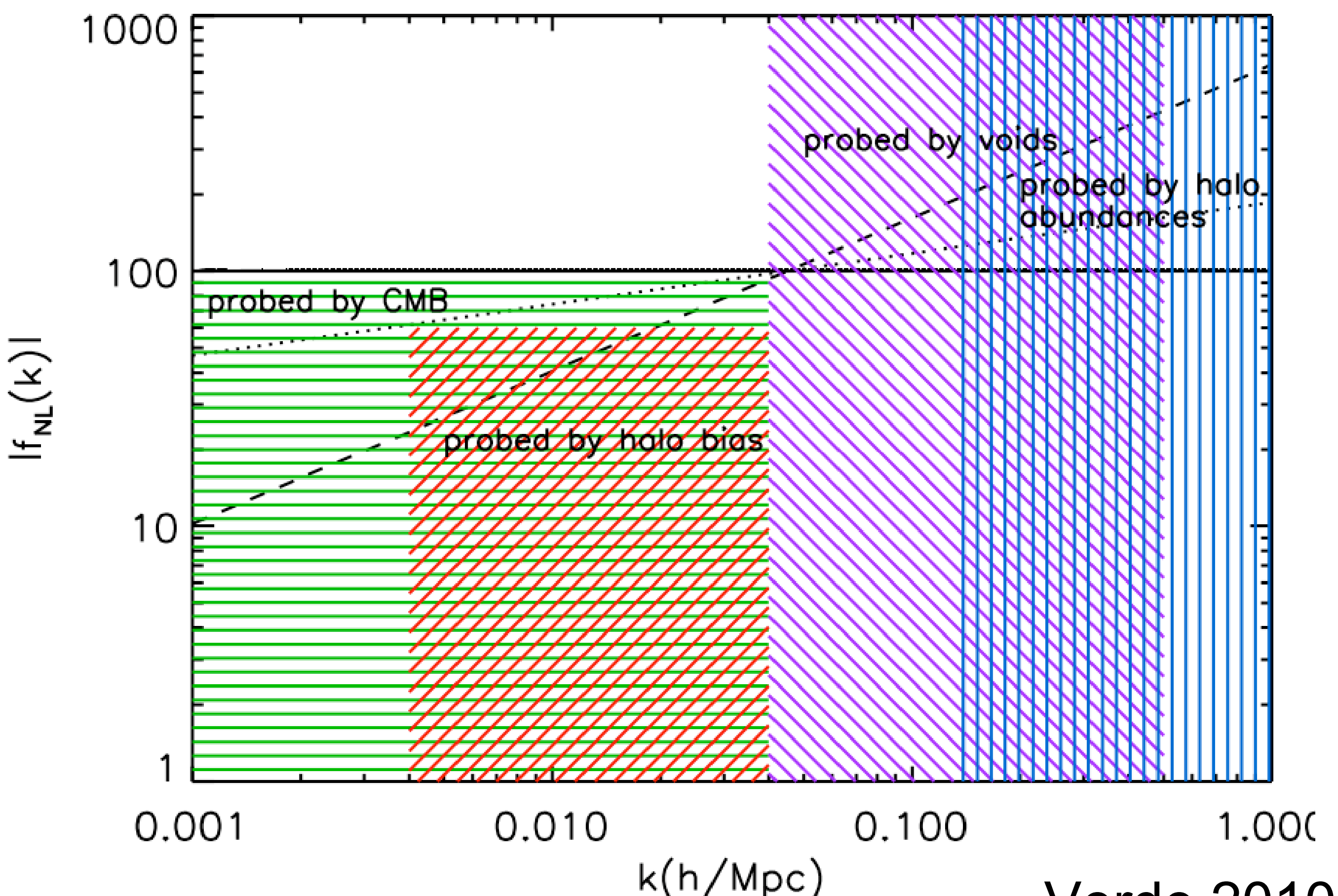
- Complementary length scales
- In principle, vast 3D data sets

Present: SDSS, Wigglez, 6dF, etc

Underway: Pan-STARRS, DES, etc

Future: SKA, Euclid ... $\Delta f_{NL} = 1$





Verde 2010

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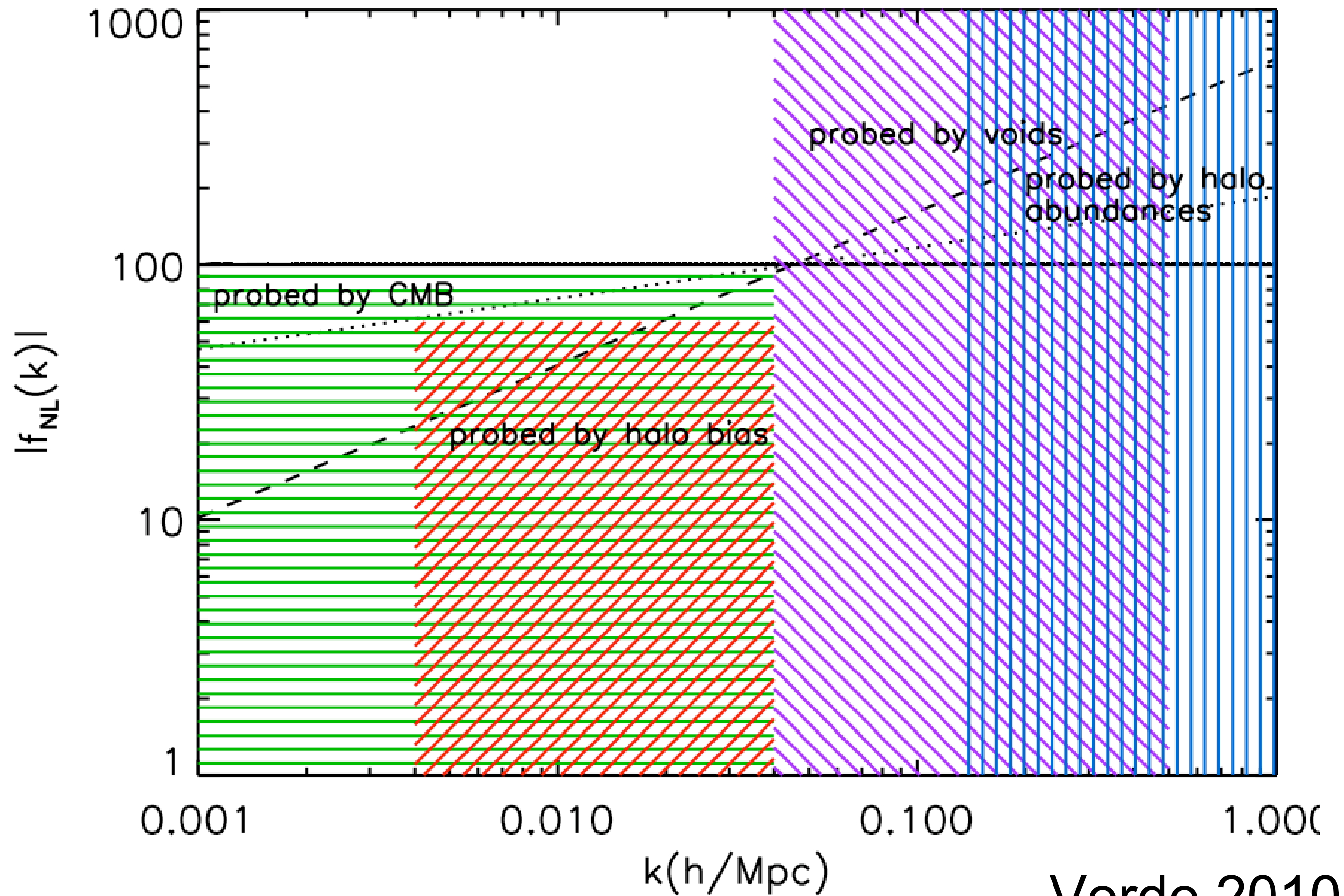

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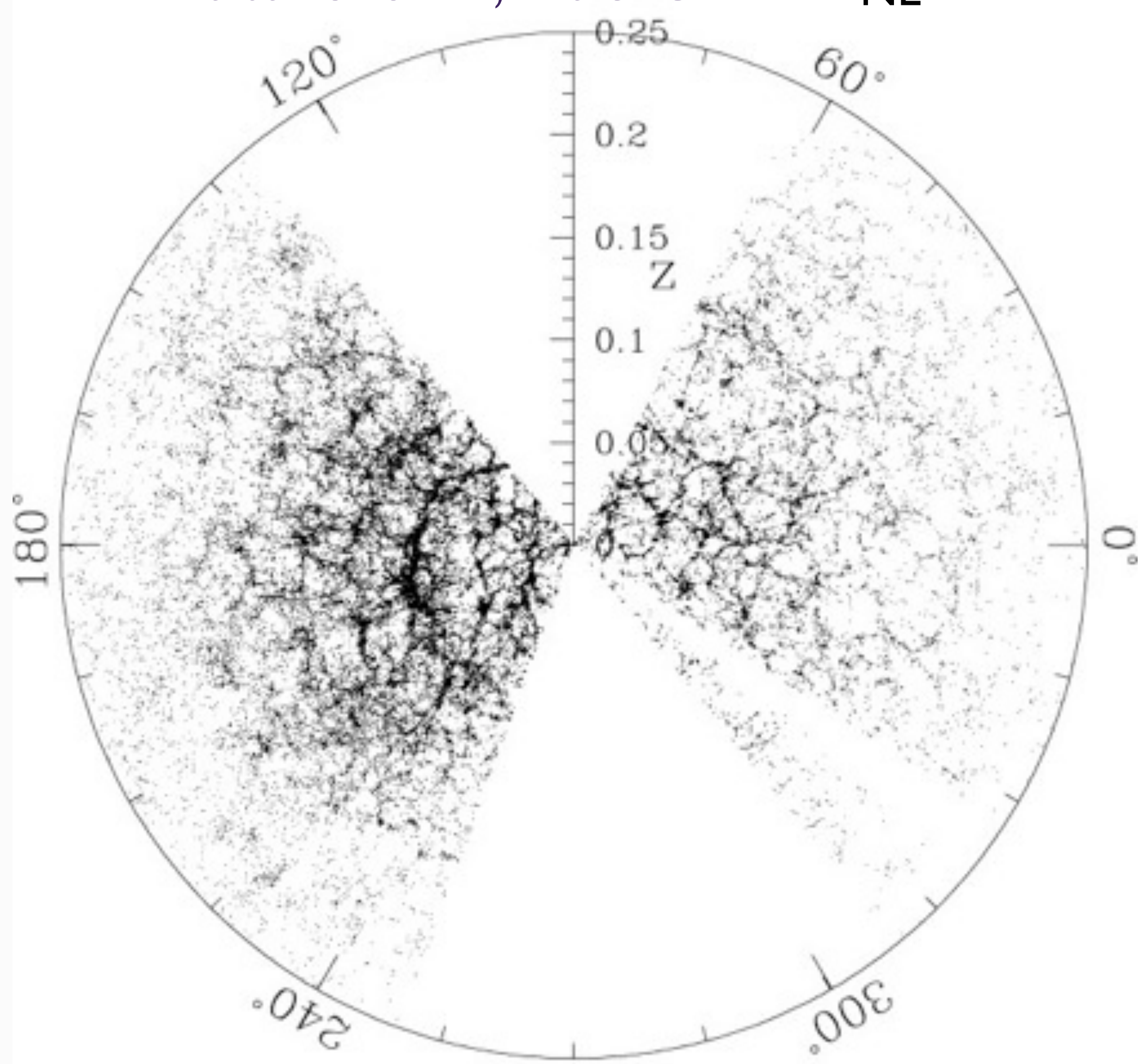


- In practice, challenging systematics

  E.g. evolution, stars, sky brightness, color offset

- Fully nonlinear analysis required

  Gravitational nonlinearity and bias effects appear in higher order polyspectra

  Computationally intensive: N-body sims-based

Approaches to LSS Non-Gaussianity

Direct calculation of higher-order correlators or polyspectra

Venerable history - *Groth & Peebles 1977; see Liguori, Sefusatti et al, 2010 review.*

Real-space correlators tackled on SDSS, Wigglez etc datasets see e.g. Wigglez poster.

Computationally challenging - operations naively scale as N^p (p polyspectra order)

Abundance of rare peaks

Interesting, controversial, not optimal ***e.g. Hoyle et al 10 Chongchitan & Silk, 10,11; Enqvist et al, 10***

Scale-dependent halo bias

Dalal, Dore, Huterer, Shirokov 2007

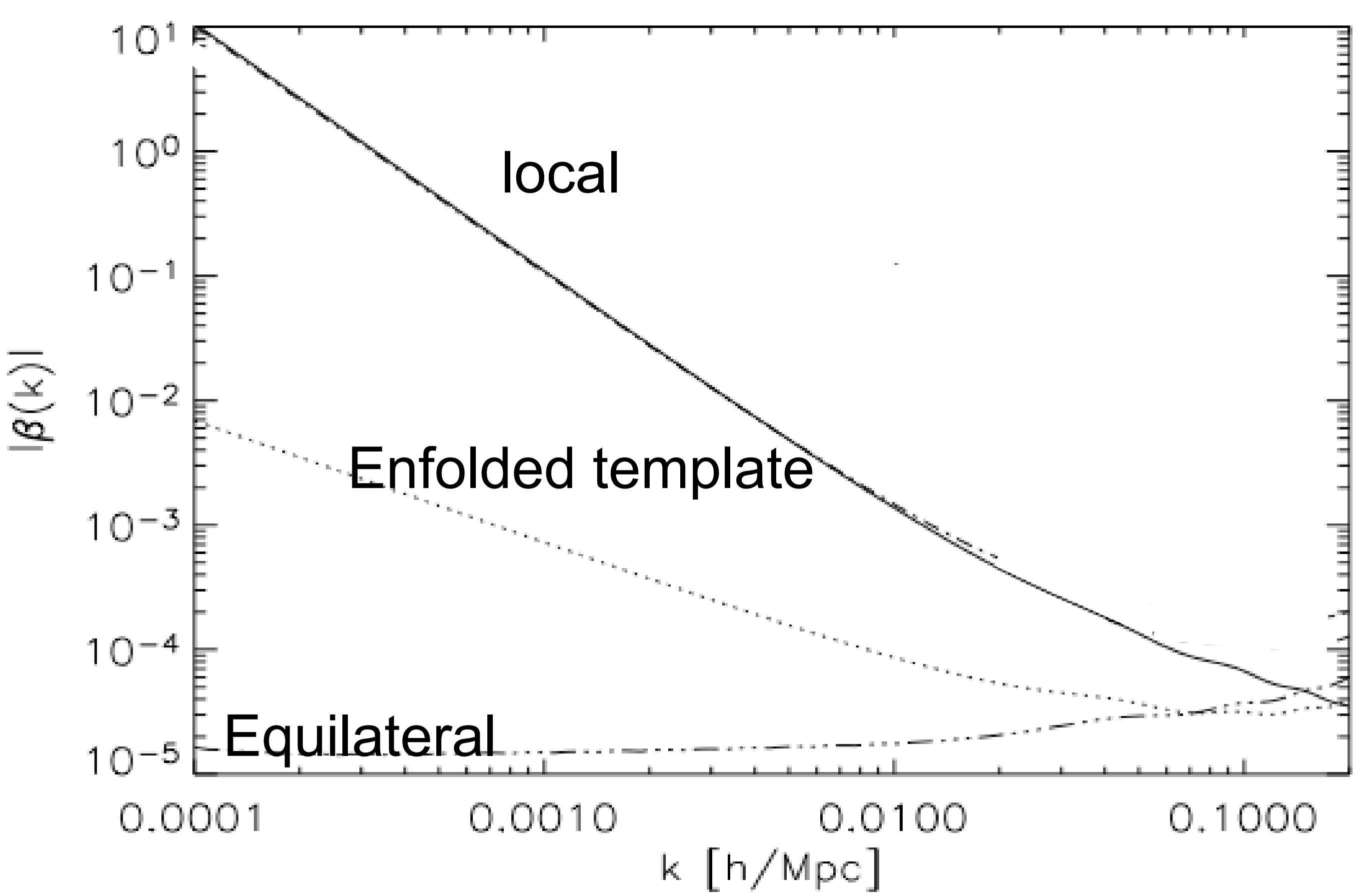
Amplification of galaxy bias in $P_g(k)$:

$$\frac{\Delta b}{b^{L,G}} = \beta(k)\frac{\Delta_c(z)}{D(z)}$$

Peak-background split formulation
Desjacques et al 2009; see also Grossi et al

But also pertains to ***galaxy bispectrum***

Bispectrum S/N wins for large surveys
see ***e.g. Sefusatti et al, 2010 etc etc***



Verde & Matarresse,

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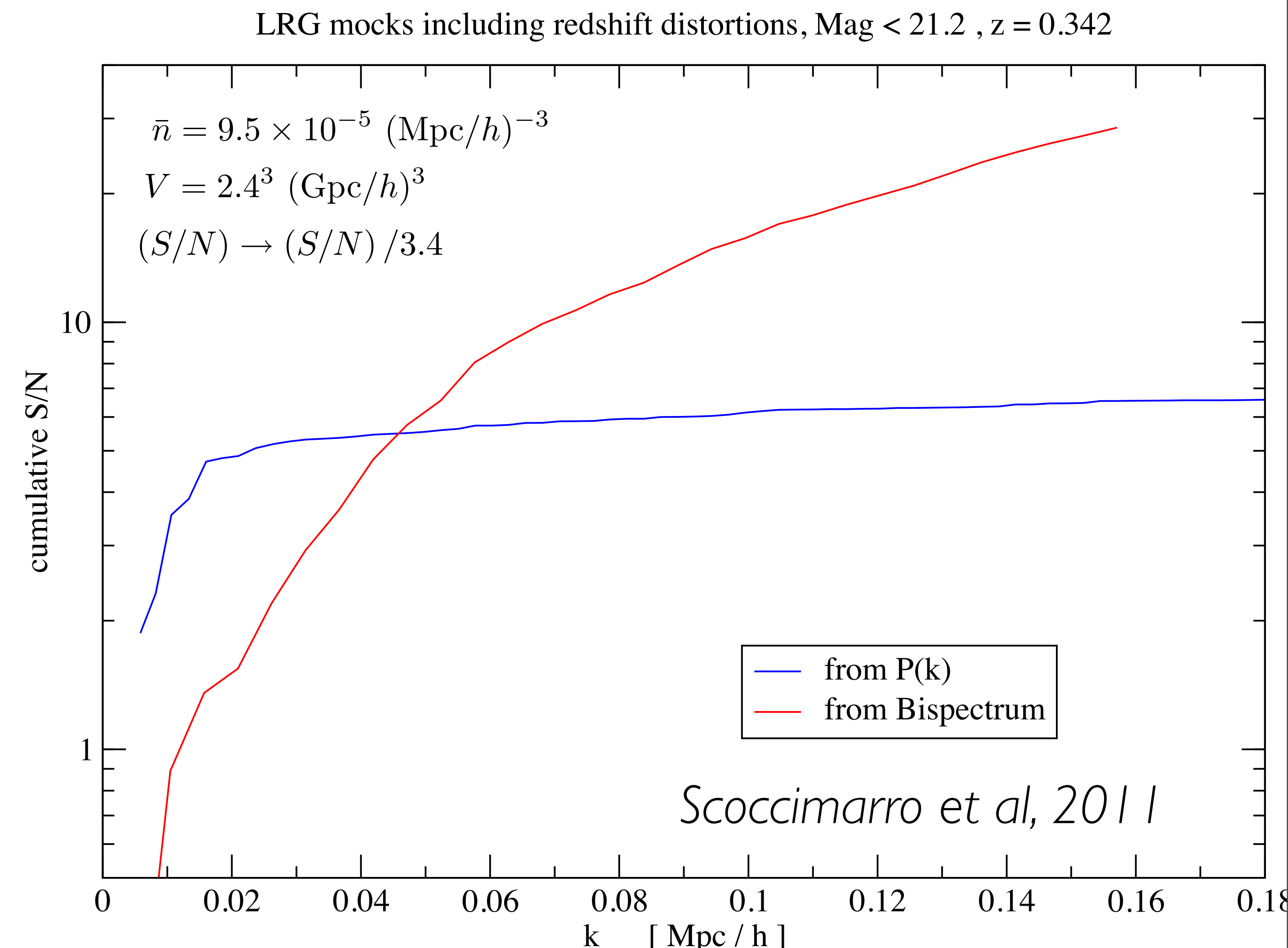
Signal to Noise f_{NL}=100 (local)



Scoccimarro et al, 2011

Large-scale structure polyspectra

Fergusson, Regan & EPS, arXiv:1008.1730

Bispectrum for 3D galaxy/matter density distribution

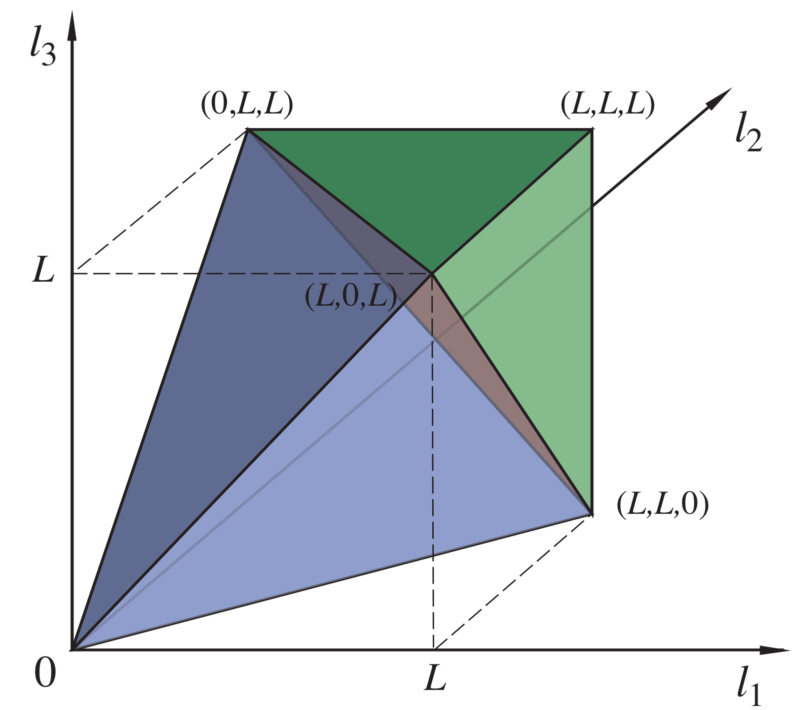
$$\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3)$$

General large-scale structure estimator

$$\mathcal{E} = \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3k_3}{(2\pi)^3} \frac{(2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3)}{P(k_1)P(k_2)P(k_3)} \left[\delta^{obs}_{\mathbf{k}_1} \delta^{obs}_{\mathbf{k}_2} \delta^{obs}_{\mathbf{k}_3} - 3\langle \delta^{sim}_{\mathbf{k}_1} \delta^{sim}_{\mathbf{k}_2} \rangle \delta^{obs}_{\mathbf{k}_3}\right]$$

Defines inner product on tetrapyd

$$\langle B_i, B_j \rangle \equiv \frac{V}{\pi} \int_{\mathcal{V}_B} dk_1 dk_2 dk_3 \frac{k_1 k_2 k_3 \, B_i(k_1, k_2, k_3) \, B_j(k_1, k_2, k_3)}{P(k_1)P(k_2)P(k_3)}$$



Computationally very demanding

Operations $\sim 10^3 \times L^6 \sim 10^{21}$

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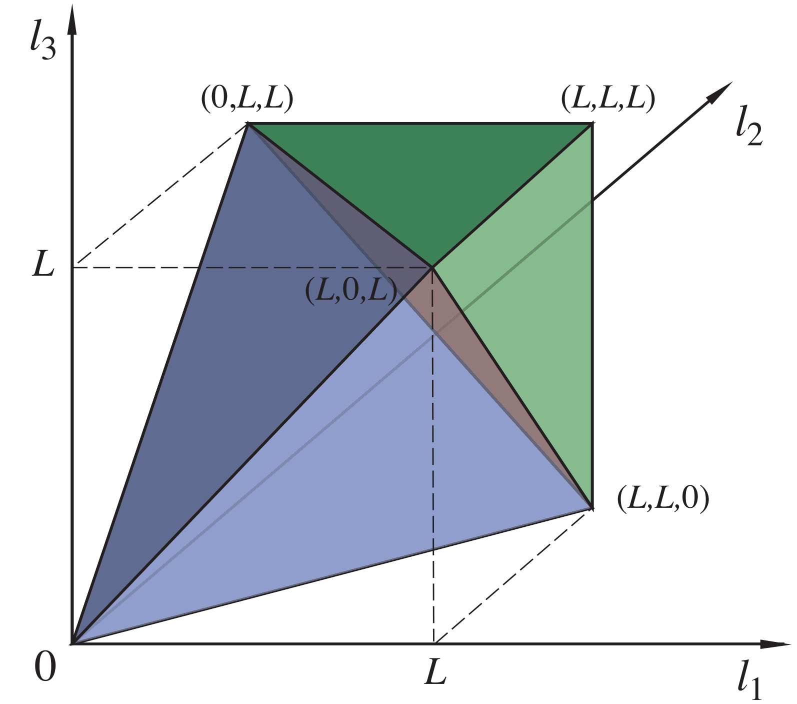

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Aside: 3D LSS trispectrum estimator

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LSS modal estimator

Separable mode expansion for bispectrum (or trispectrum)

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arXiv:1008.1730
arXiv:1108.3813

The bi-/trispectrum estimator becomes simply

$$\mathcal{E} = \sum_n \alpha_n^{\mathcal{Q}}\, \beta_n^{\mathcal{Q}}$$

with coefficients from data filtered by individual modes - Operations ~ L^3

$$\beta_n^{\mathcal{Q}} = \int d^3x\, M_r(\mathbf{x})\, M_s(\mathbf{x})\, M_t(\mathbf{x}) \qquad M_r(\mathbf{x}) = \int d^3k\, \frac{\delta_{\mathbf{k}}^{obs} q_r(k)\, e^{i\mathbf{k}\cdot\mathbf{x}}}{\sqrt{kP(k)}}$$

Contributions from nonlinear gravity - characterise with N-body simulations

$$\omega B(k_1,k_2,k_3) = \sum_n (\alpha_n^G + F_{\rm NL}\alpha_n^B + \tau_{NL}\alpha_n^T)\,\mathcal{R}_n(k_1,k_2,k_3)$$

Deploy a phenomenological estimator $\quad \mathcal{E}(F_{\rm NL}) = \sum (\alpha_n(F_{\rm NL}) - \beta_n)^2$

<u>*Computational breakthrough*</u> - general analysis bi-/trispectrum ~ $10^3 \times L^3 \sim 10^{12}$

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Universal LSS Initial Conditions

Arbitrary N-body i.c.s $\Phi = \Phi^G + \frac{1}{6}F_{\rm NL}\Phi^B + \frac{1}{24}\tau_{NL}\Phi^T$

Regan, Schmittfull, EPS & Fergusson, 1108.3813

$$\Phi^B(\mathbf{k}) = \int \frac{d^3\mathbf{k}'}{(2\pi)^3}\frac{d^3\mathbf{k}''}{(2\pi)^3}\frac{(2\pi)^3\delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'')B(k,k',k'')\Phi^G(\mathbf{k}')\Phi^G(\mathbf{k}'')}{P(k)\,P(k')+P(k')\,P(k'')+P(k)\,P(k'')},$$

$$= \sum_n \alpha_n \sqrt{\frac{P(k)}{k}} q_{\{r}(k) \int d^3\mathbf{x} e^{i\mathbf{k}.\mathbf{x}} M_s(\mathbf{x}) M_{t\}}(\mathbf{x}),$$

see also Verde et al. '10,'11 and Scoccimarro et al, '11

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Highly efficient working bispectra & trispectra pipeline (w. estimators)

(1024^3 i.c. sims 1 hour on 6 cores)

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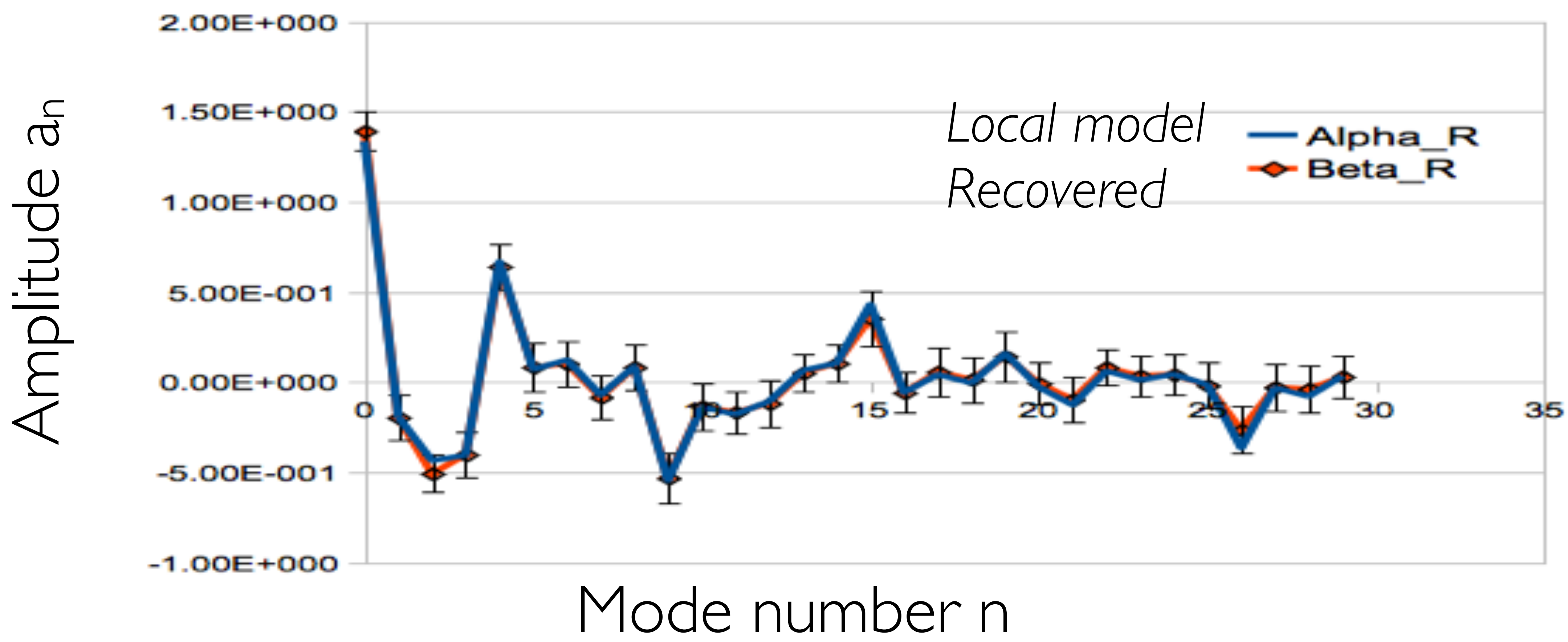
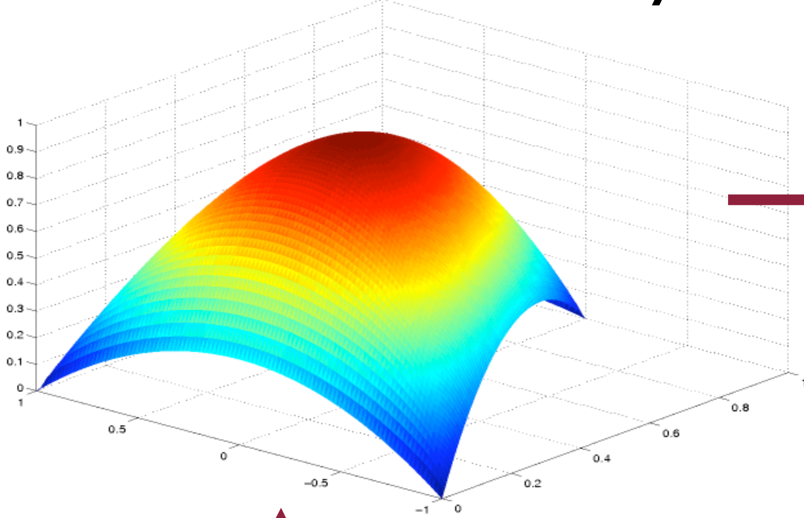
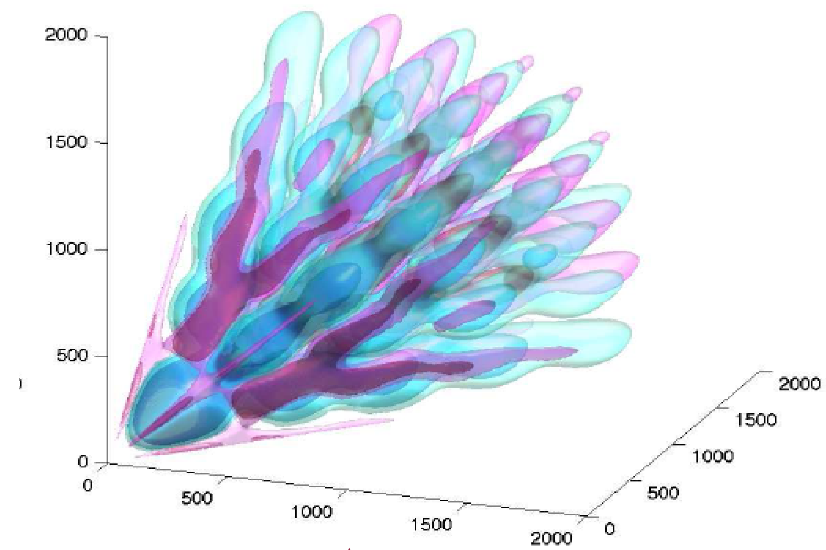


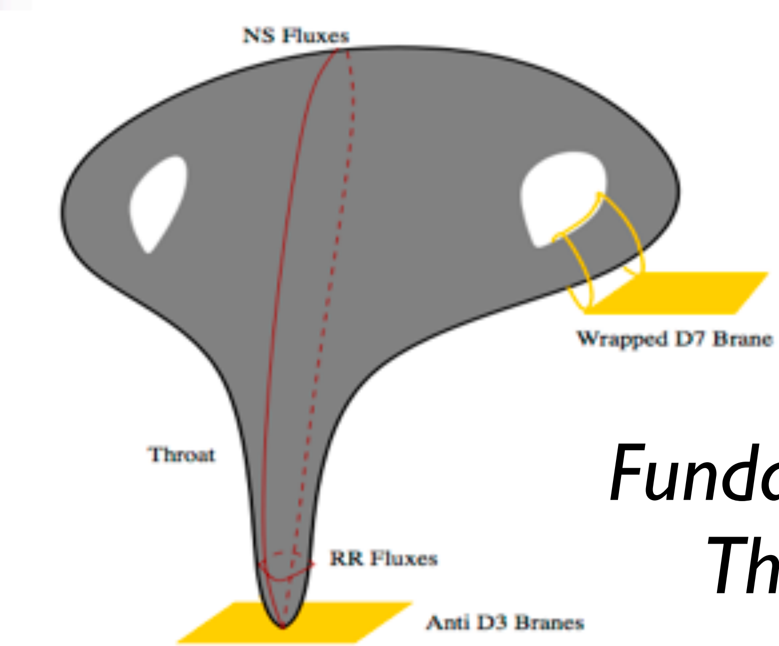
CONCLUSIONS

Primordial non-Gaussianity

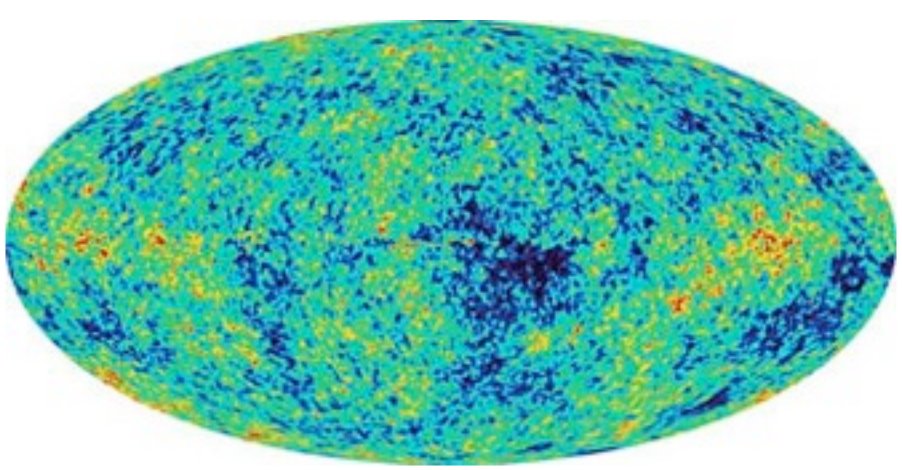


CMB (or LSS) fingerprint





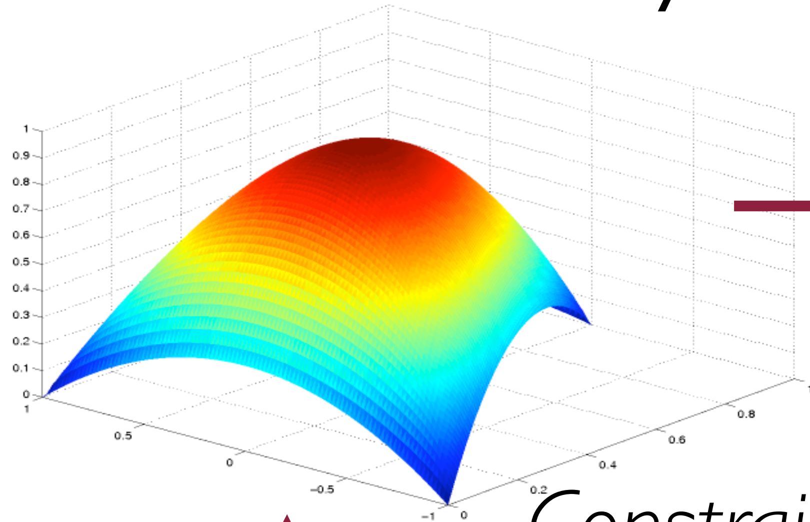
Fundamental Theory



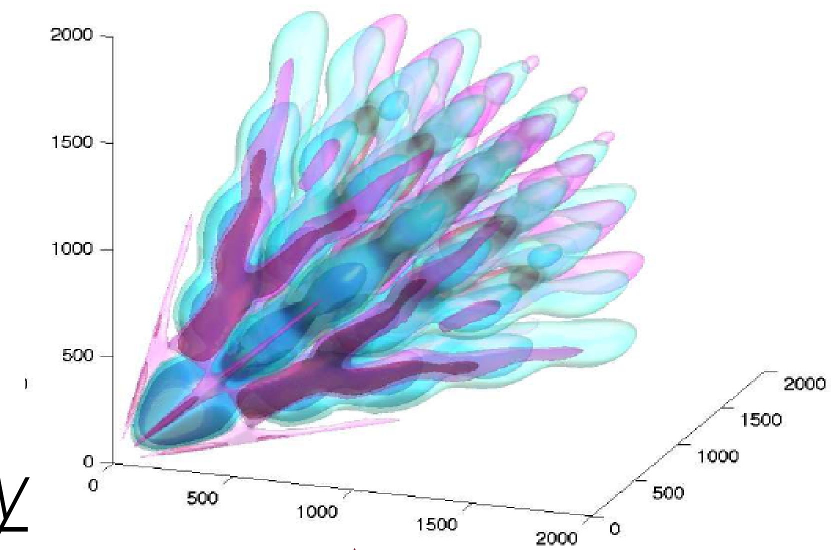
Observational Data

CONCLUSIONS

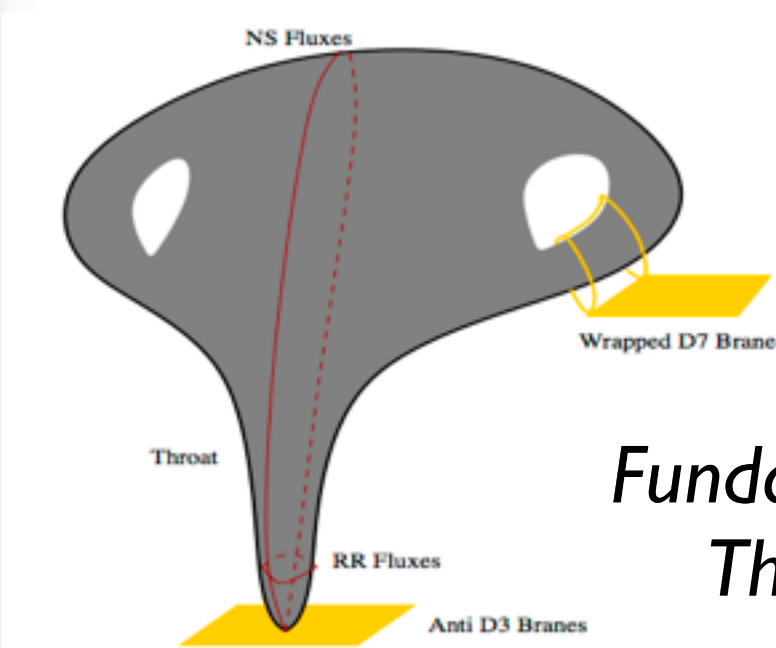
Primordial non-Gaussianity



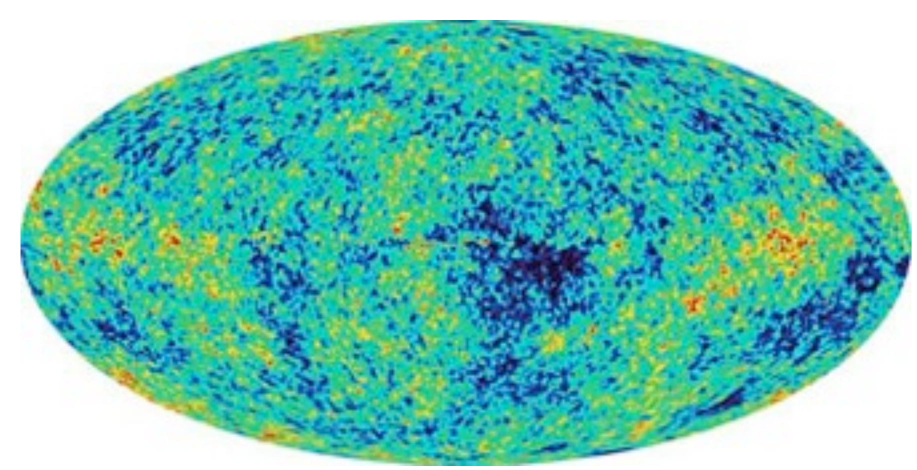
CMB (or LSS) fingerprint



Constraints on Non-Gaussianity



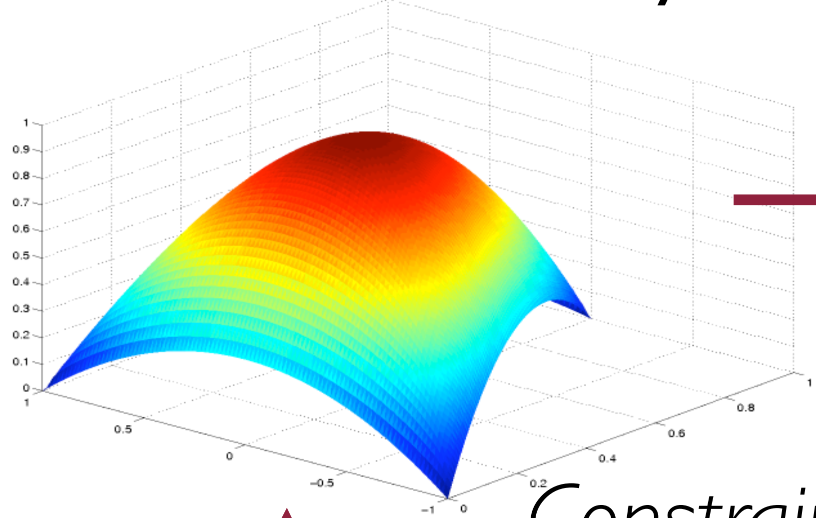
Fundamental Theory



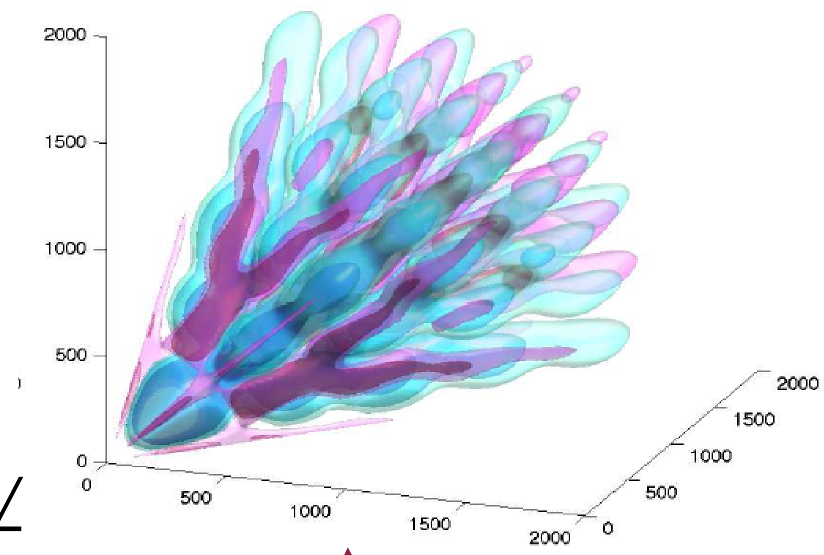
Observational Data

CONCLUSIONS

Primordial non-Gaussianity

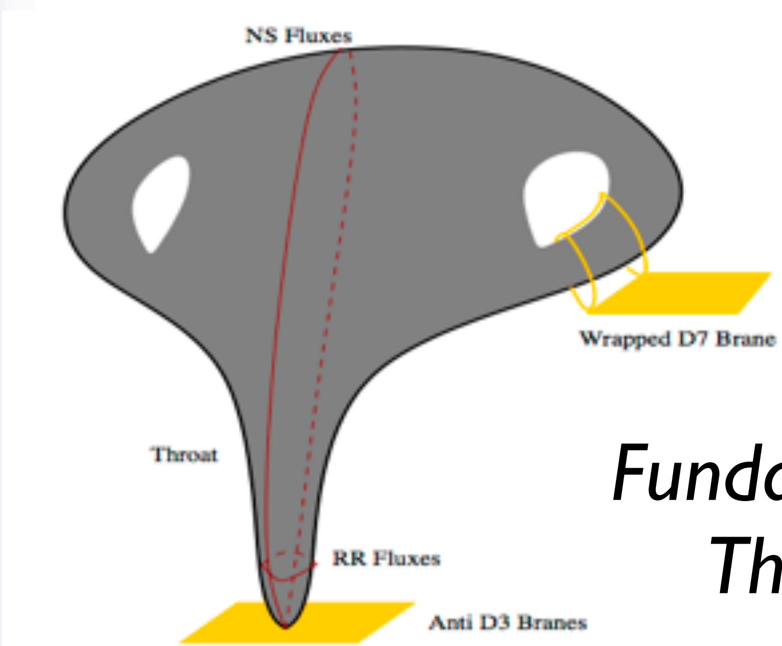


CMB (or LSS) fingerprint

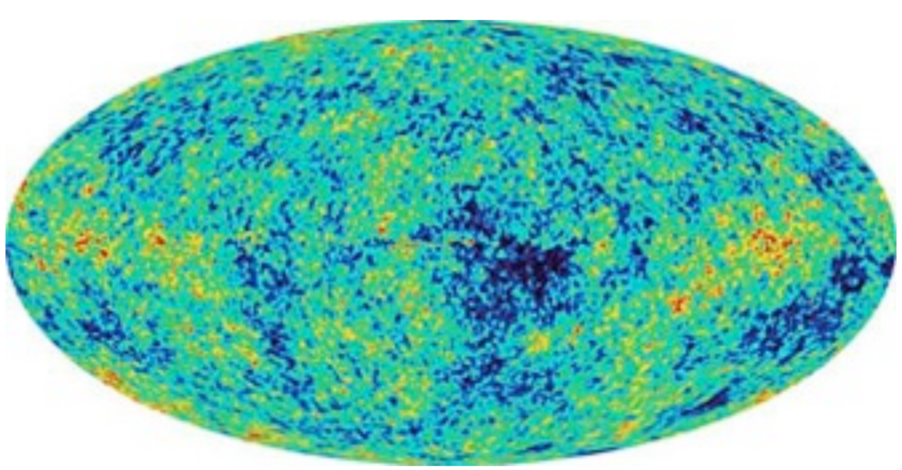


Constraints on Non-Gaussianity

COBE few parts in 100



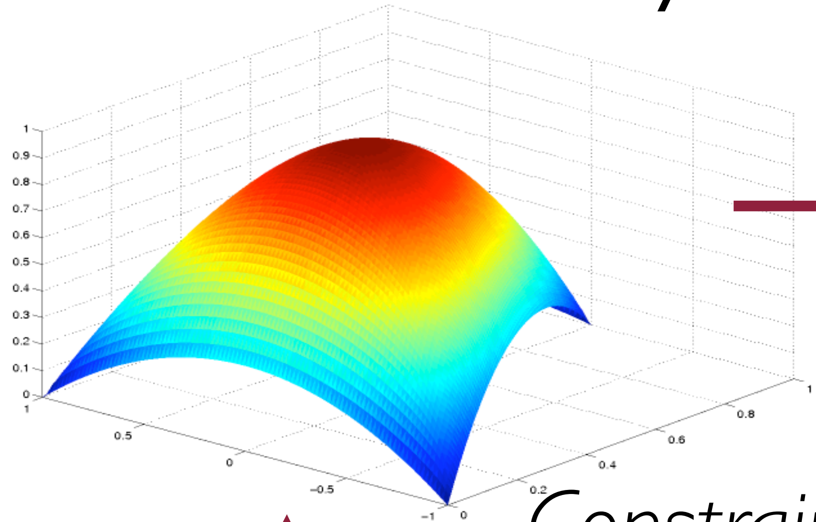
Fundamental Theory



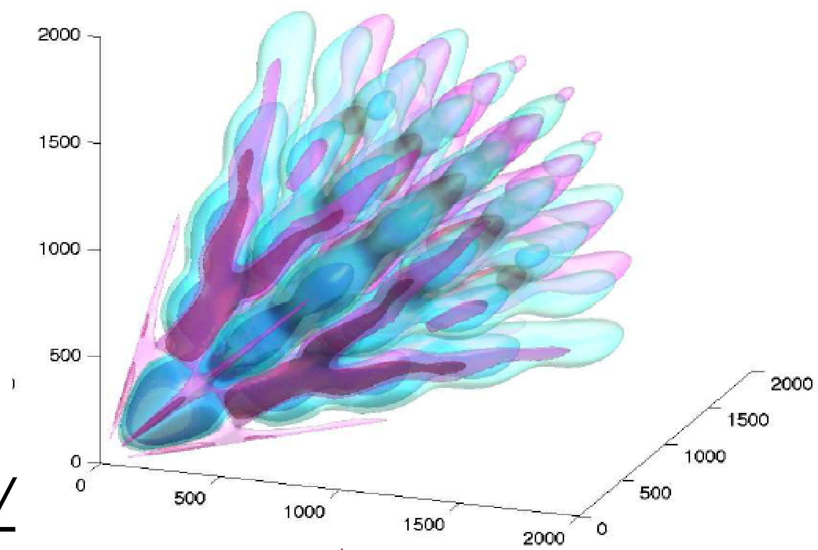
Observational Data

CONCLUSIONS

Primordial non-Gaussianity



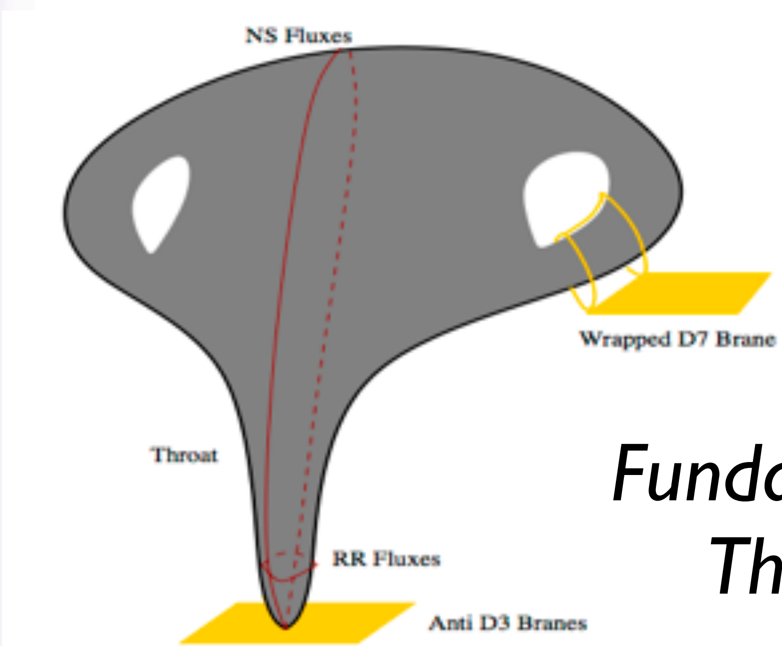
CMB (or LSS) fingerprint



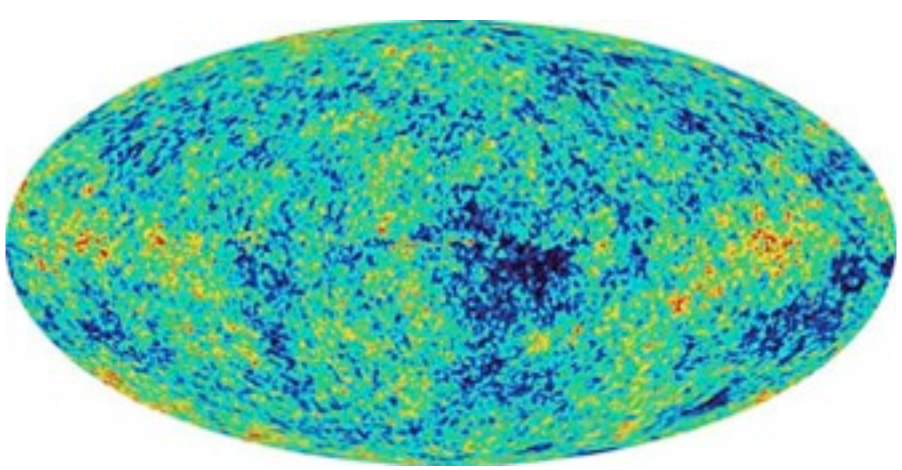
Constraints on Non-Gaussianity

COBE few parts in 100

WMAP 2-3 parts in 10,000



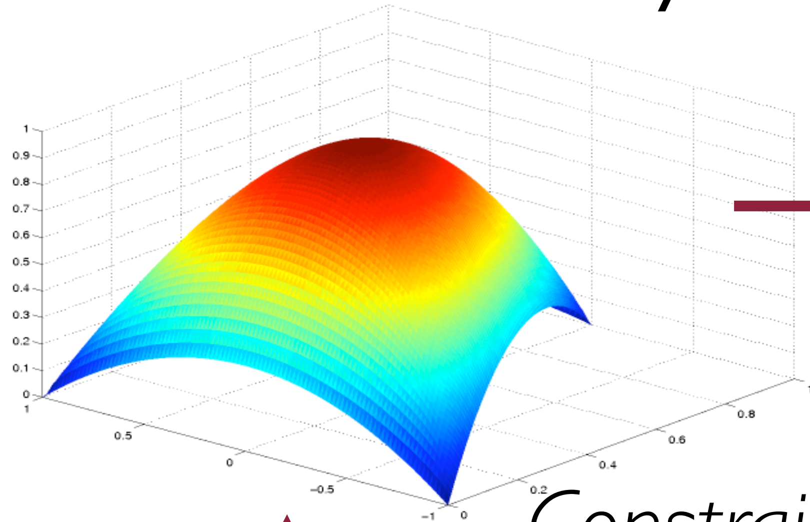
Fundamental Theory



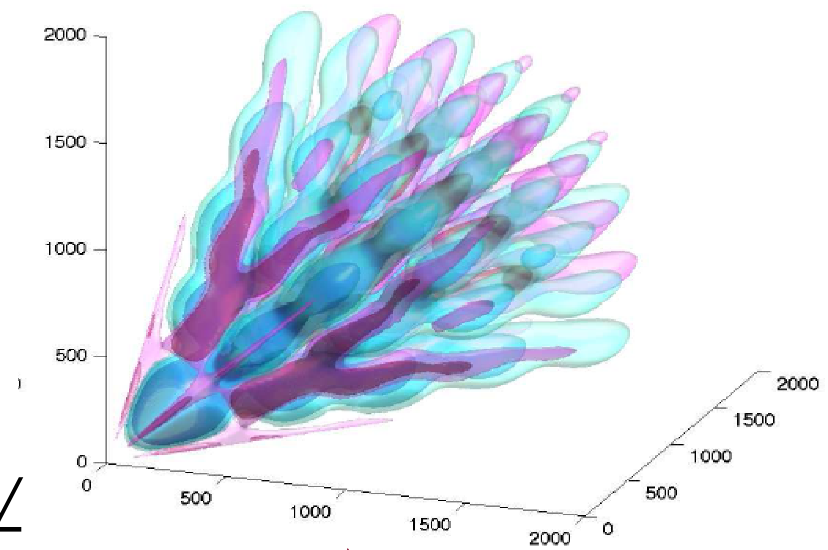
Observational Data

CONCLUSIONS

Primordial non-Gaussianity

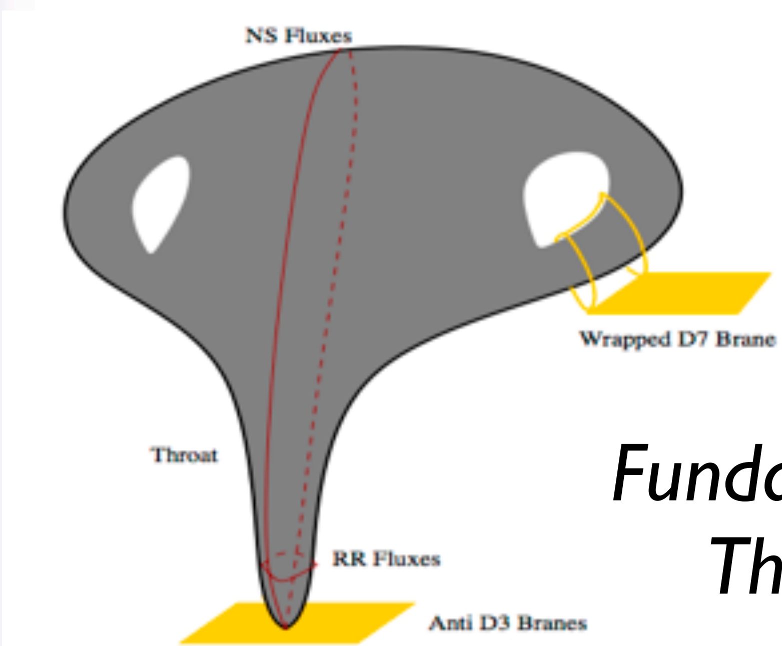


CMB (or LSS) fingerprint

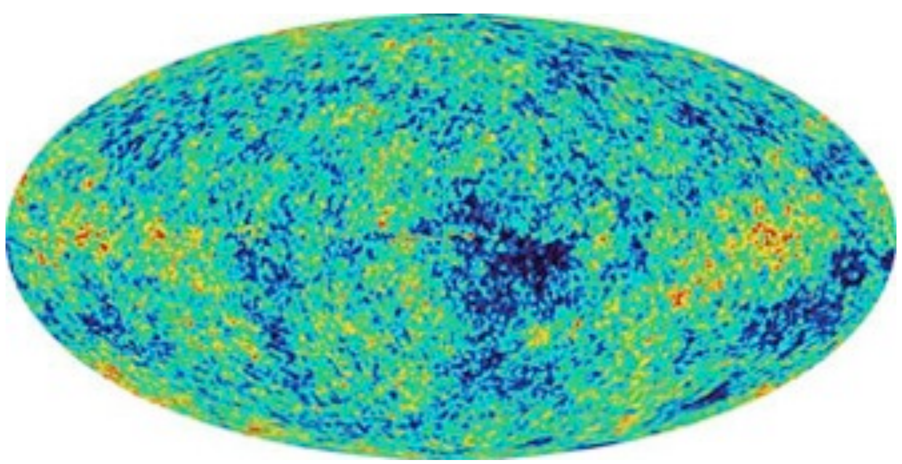


<u>*Constraints on Non-Gaussianity*</u>

COBE	few parts in 100
WMAP	2-3 parts in 10,000
Planck	few parts in 100,000



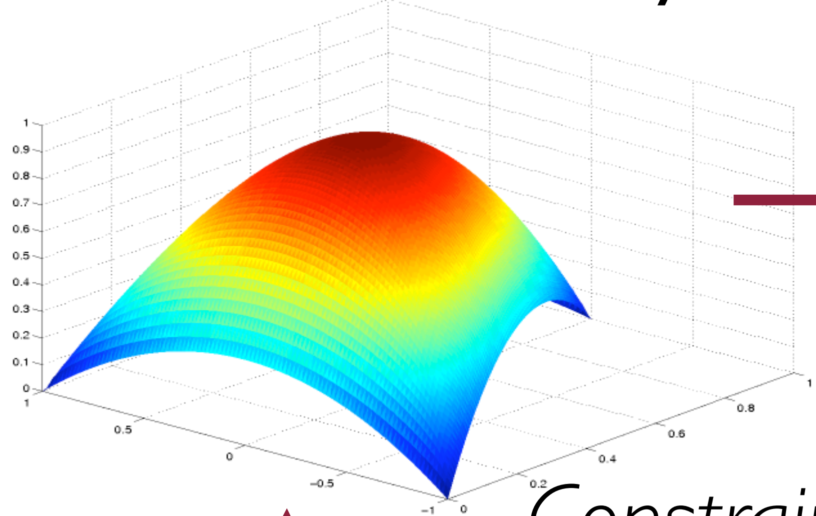
Fundamental Theory



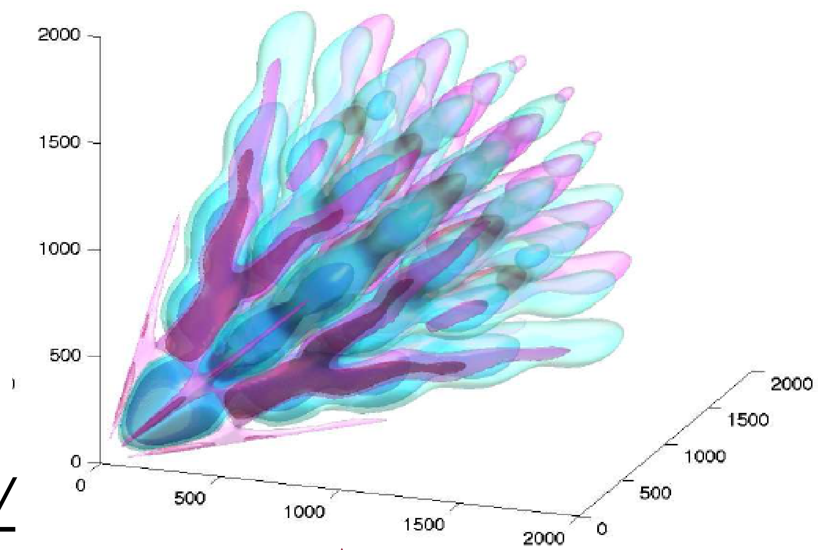
Observational Data

CONCLUSIONS

Primordial non-Gaussianity



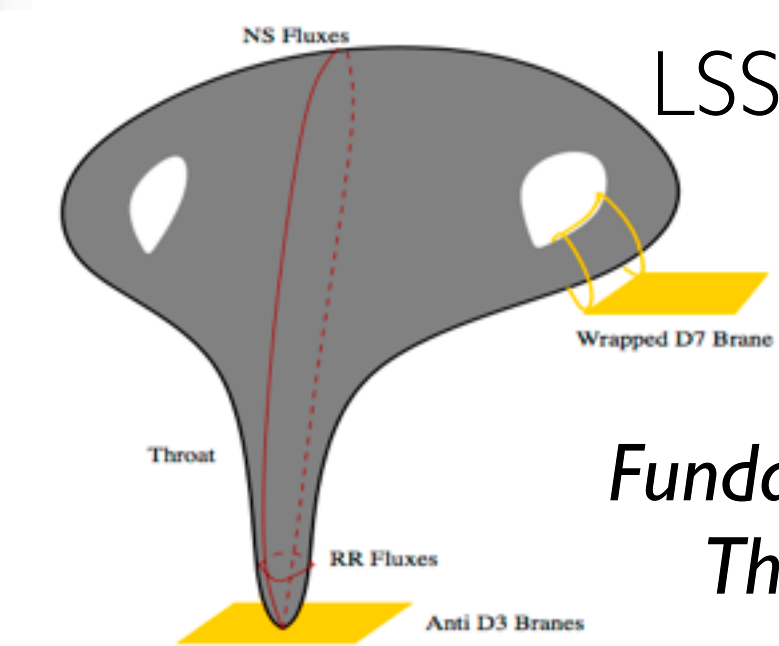
CMB (or LSS) fingerprint



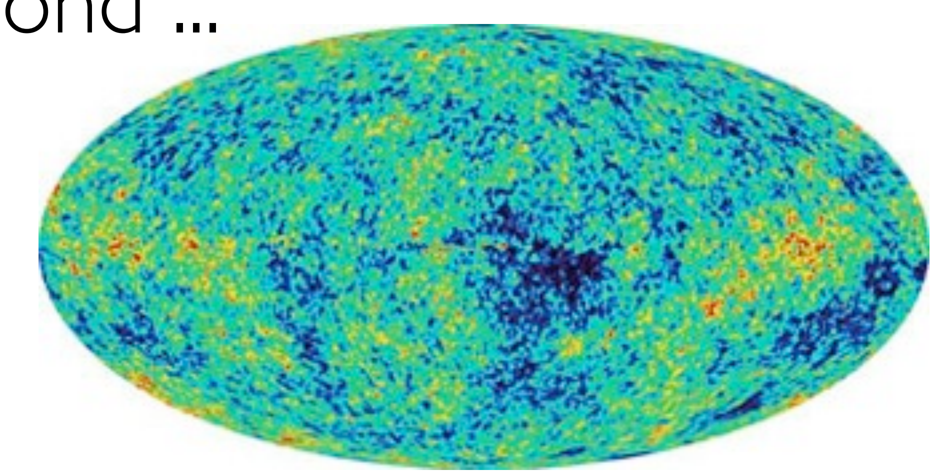
<u>*Constraints on Non-Gaussianity*</u>

COBE	few parts in 100
WMAP	2-3 parts in 10,000
Planck	few parts in 100,000

LSST/Euclid/SKA and beyond ...



Fundamental Theory



Observational Data