# Non-Gaussianity

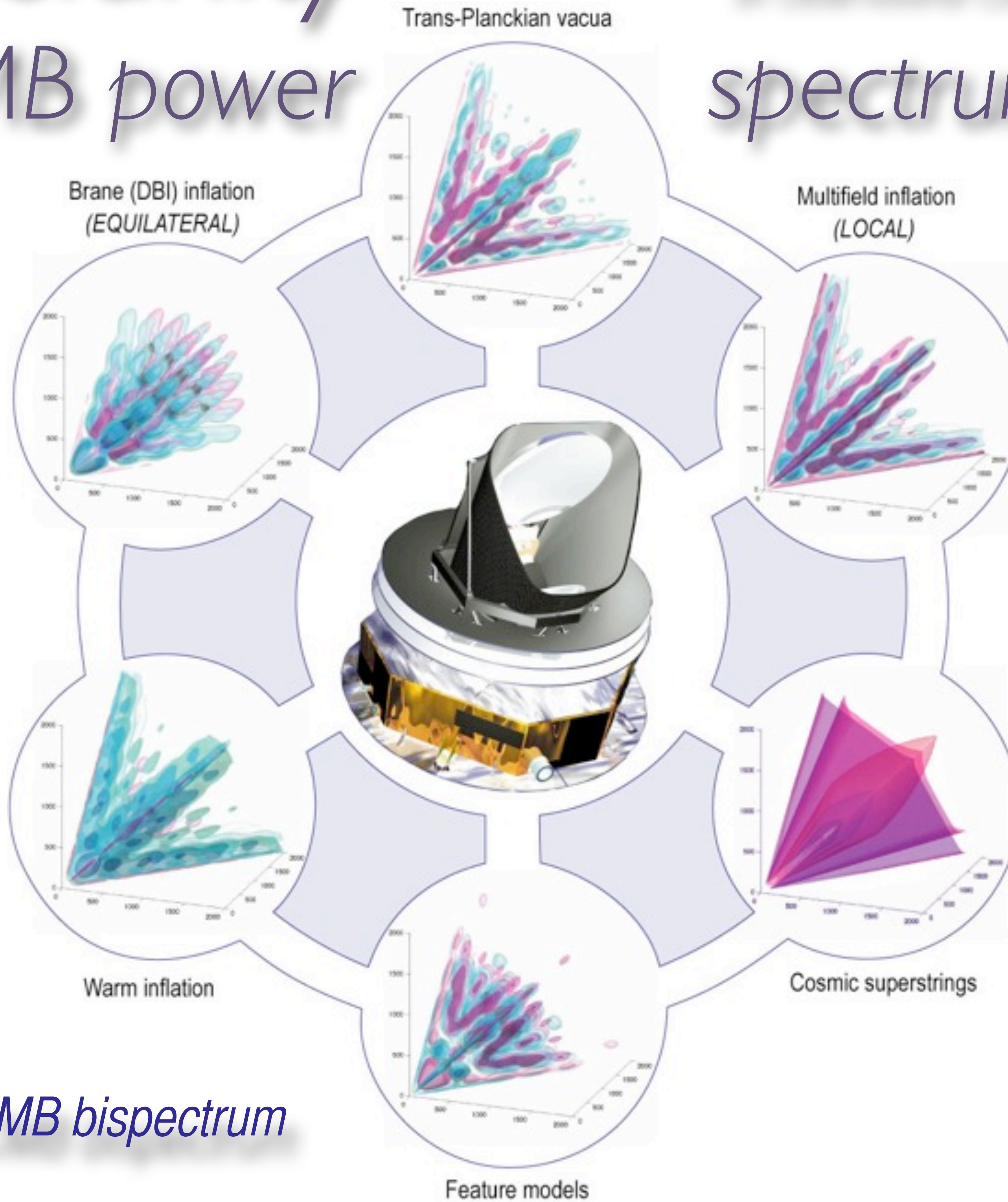
## Beyond the CMB power spectrum

The Azores School on Observational Cosmology

Centro de Astrofísica da Universidade do Porto

*James Fergusson & Paul Shellard*

*Centre for Theoretical Cosmology, DAMTP University of Cambridge*




*LECTURE 1*

- *Review: CMB and Inflation*
- *Inflationary fluctuations & Randomness*
- *Primordial non-Gaussianity*
- *The CMB bispectrum*
- *CMB bispectrum estimation*
- *General modal approach to CMB bispectrum*

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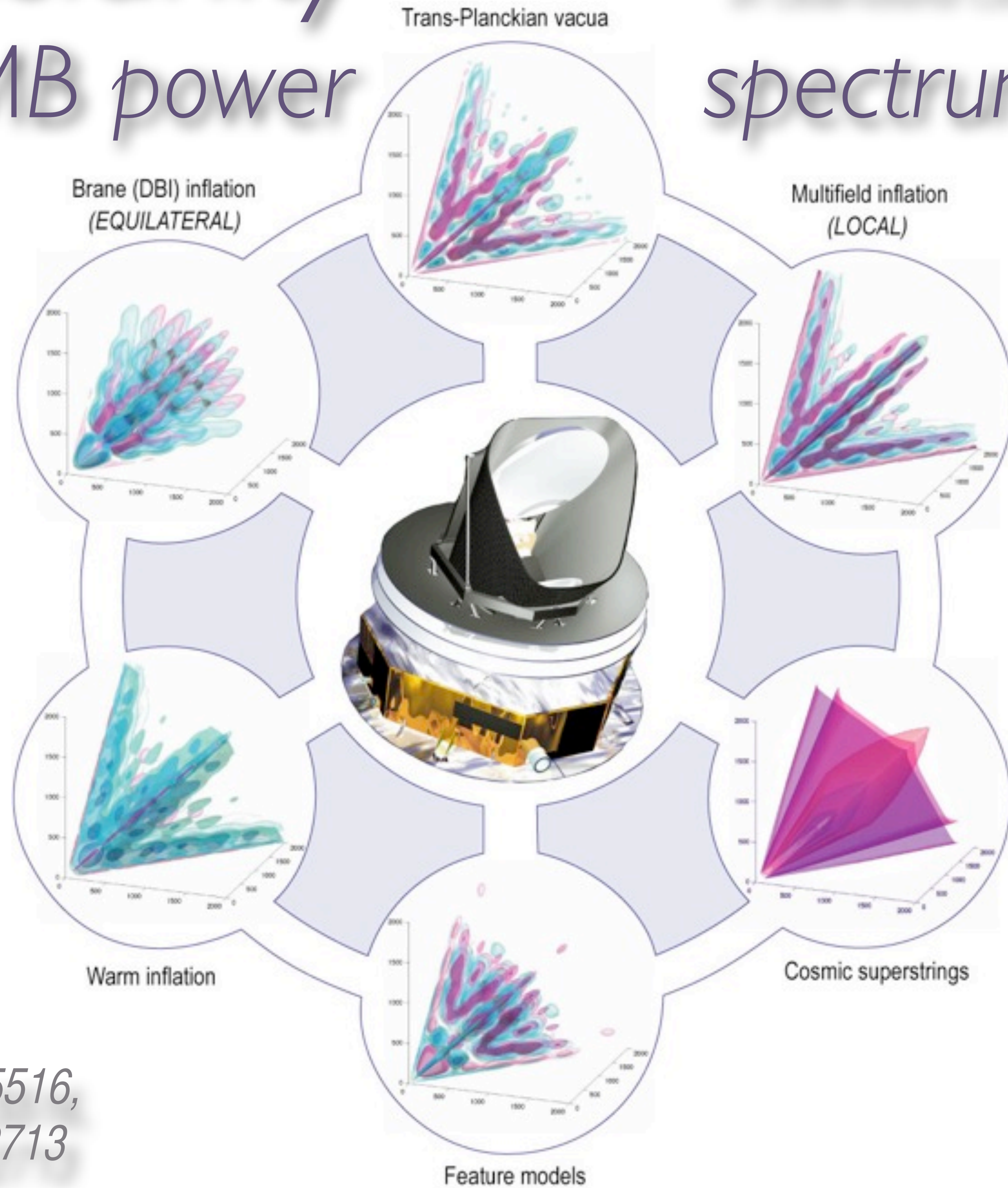
*James Fergusson*
*& Paul Shellard*

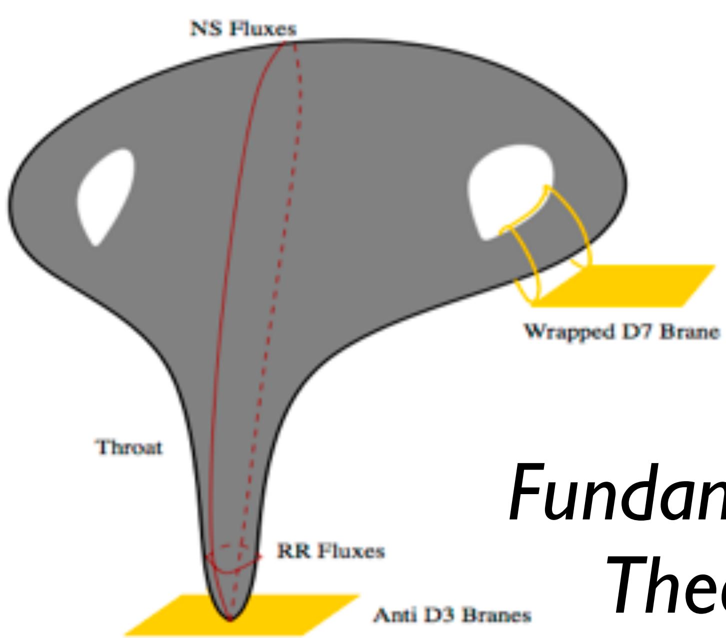
*Centre for Theoretical Cosmology, DAMTP University of Cambridge*

*work with Michele Liguori*
*Donough Regan*
*& Marcel Schmittfull*
*also Hiro Funakoshi*

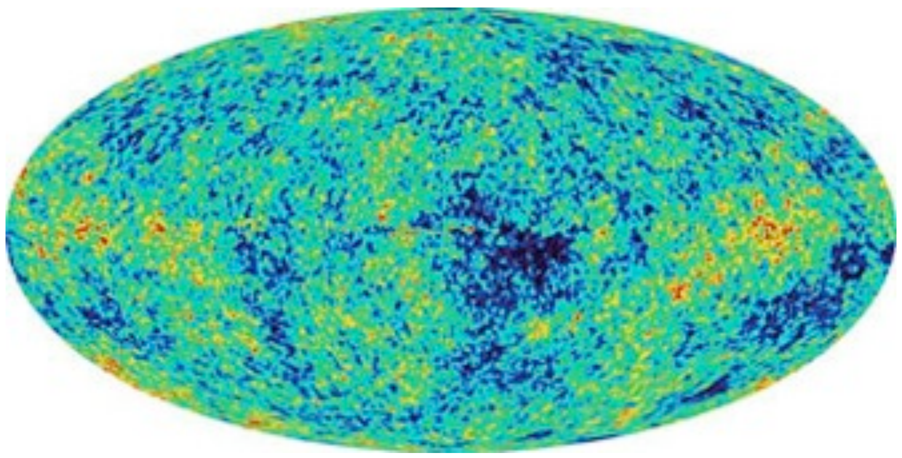

*arXiv:1006.1642*
*arXiv:1012.6039*
*arXiv:1108.3813*
*arXiv: 1105.2791*
*arXiv:1008.1730*
*(arXiv:1004.2915), arXiv:0912.5516, arXiv:0812.3413, astro-ph/0612713*



# Motivation




*Fundamental Theory*



*Observational Data*

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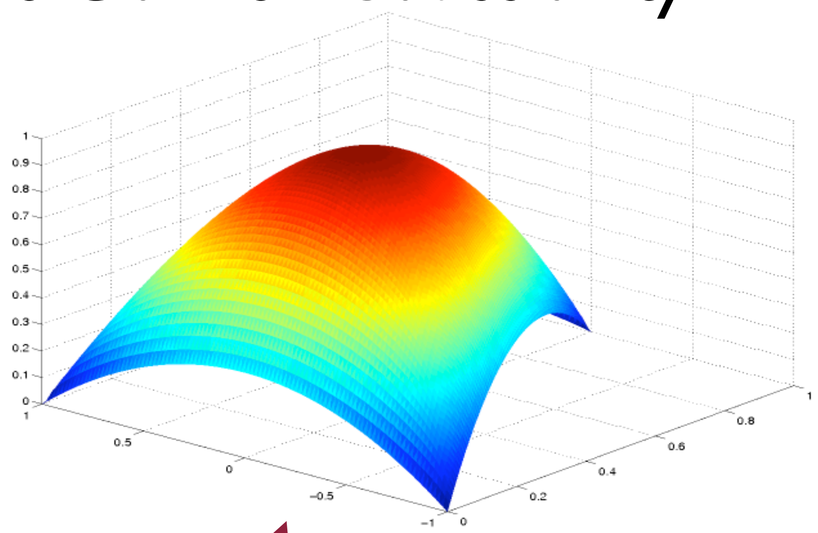
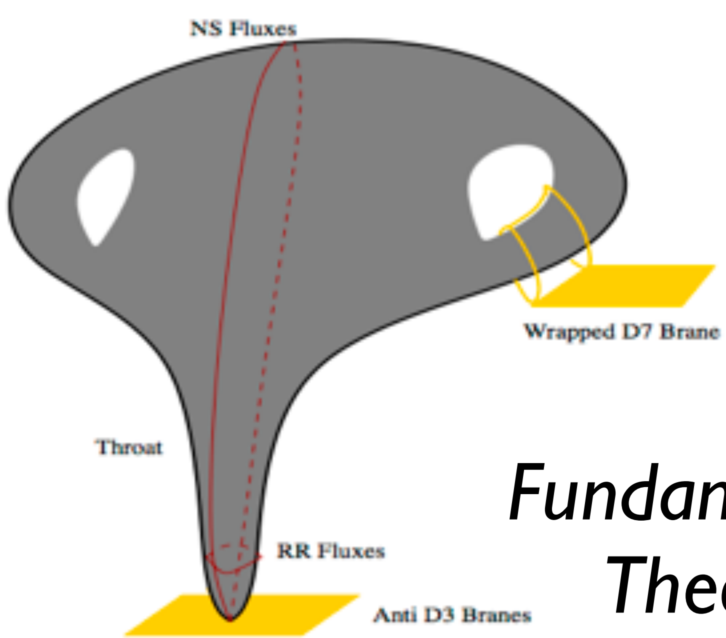
*Primordial non-Gaussianity*



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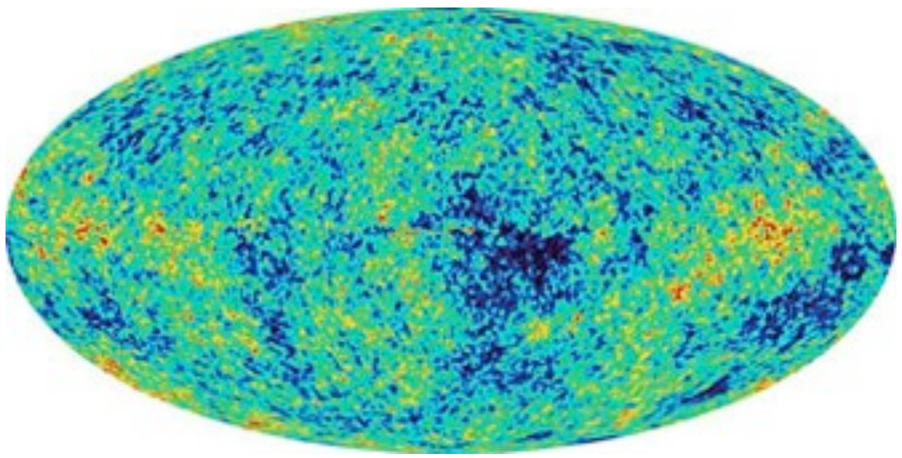

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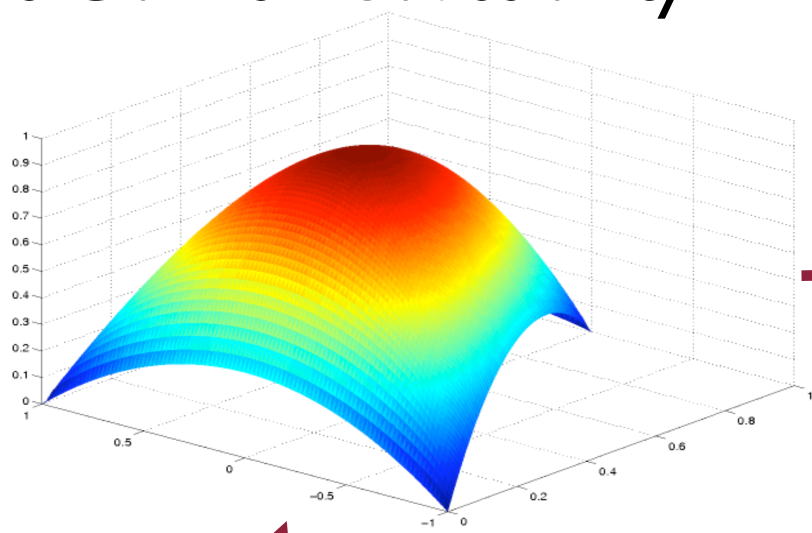
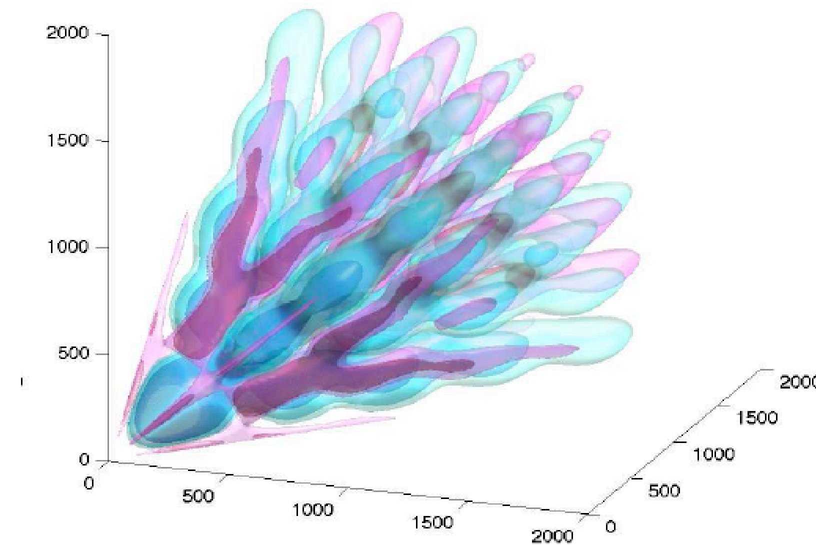
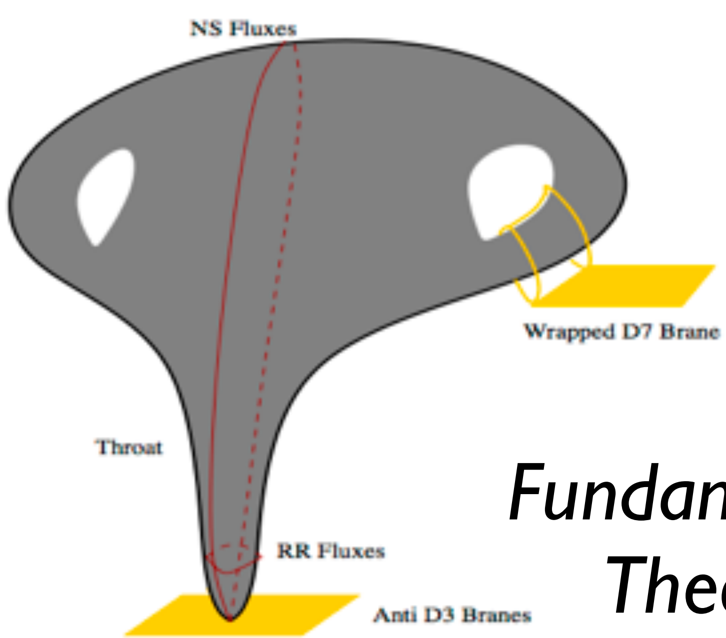


*CMB (or LSS) fingerprint*

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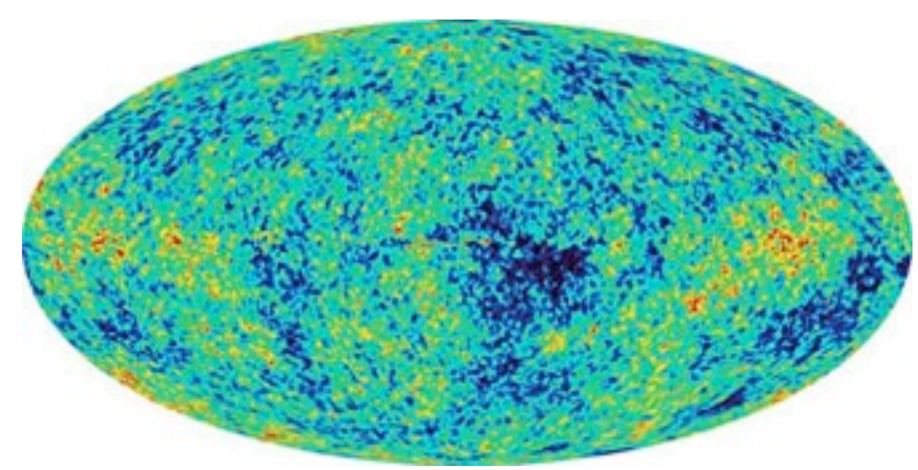

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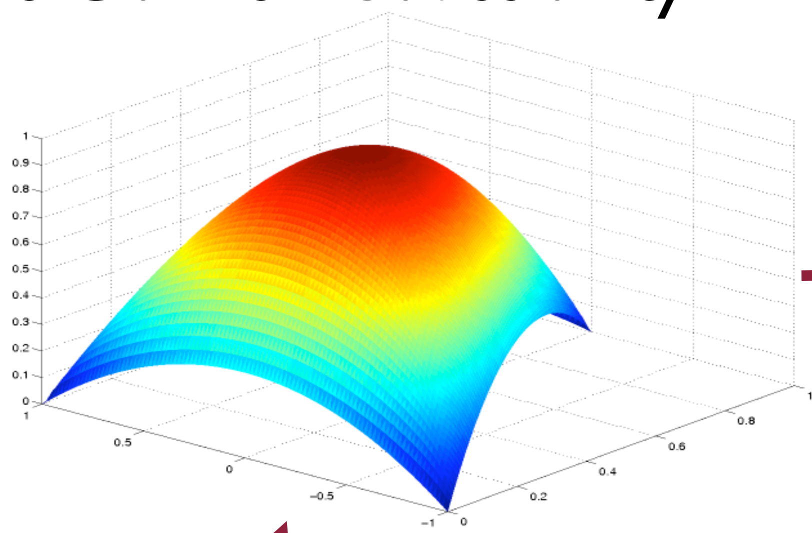

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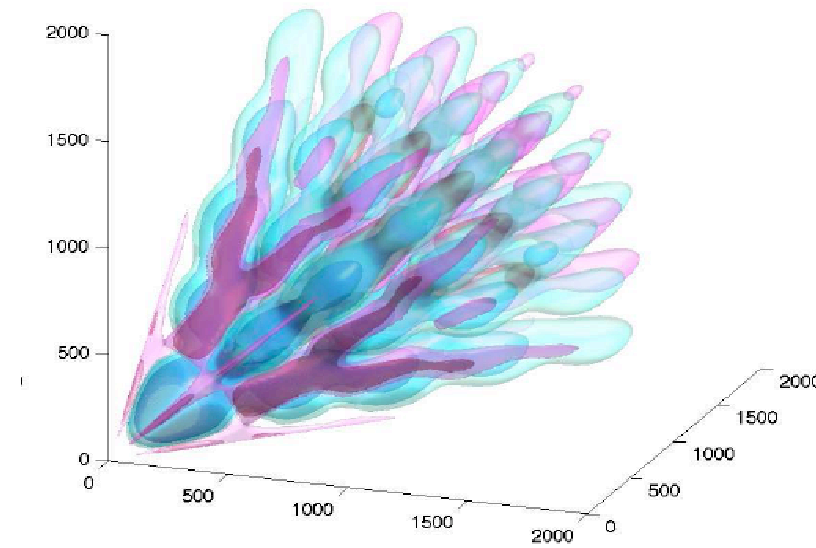

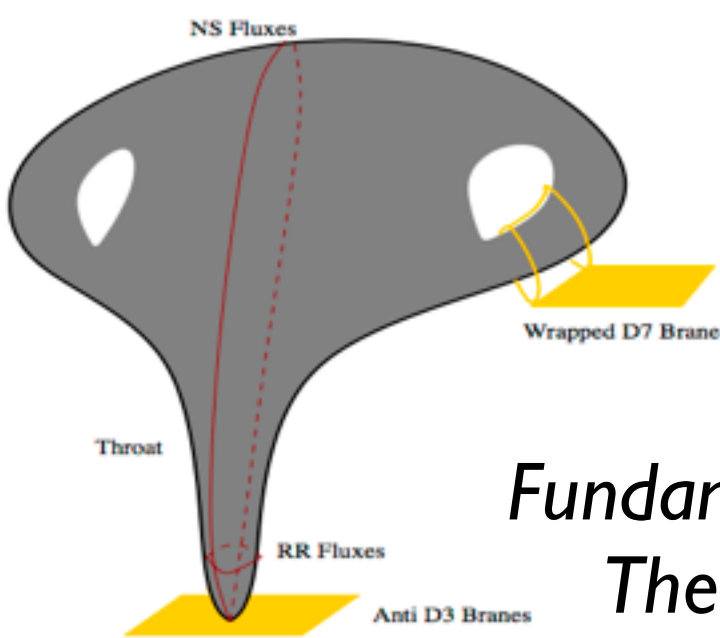


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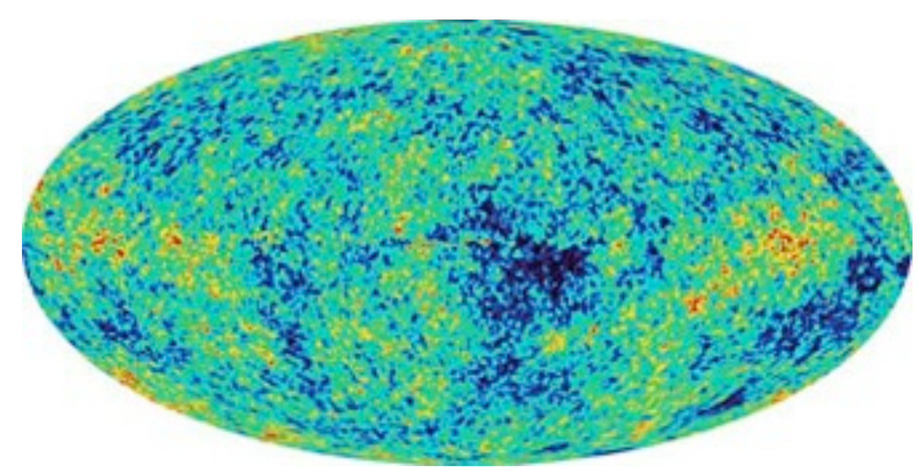

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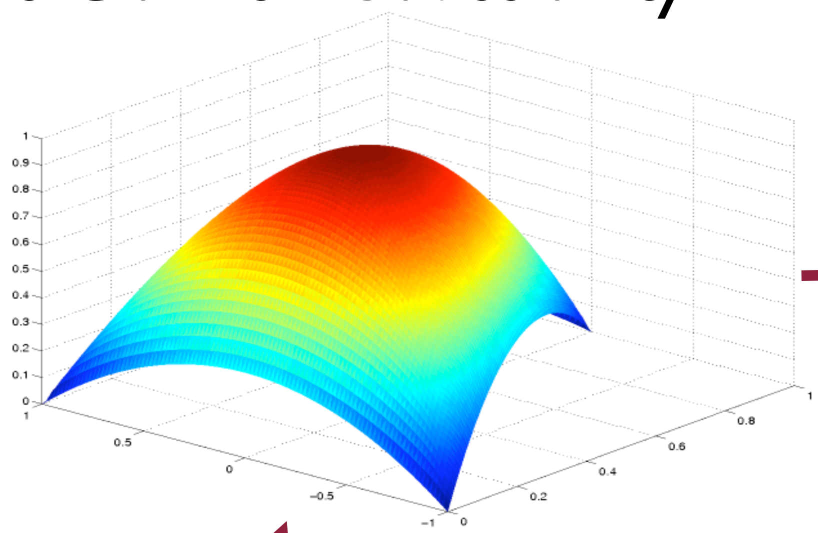

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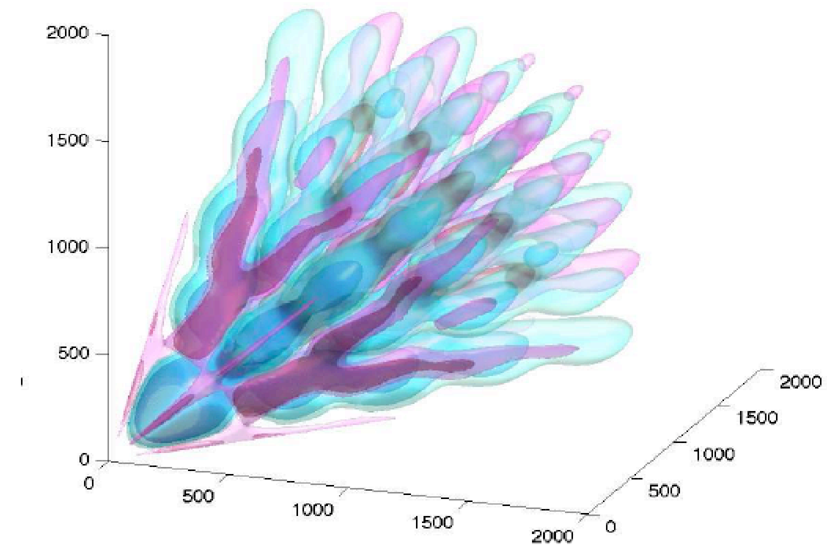

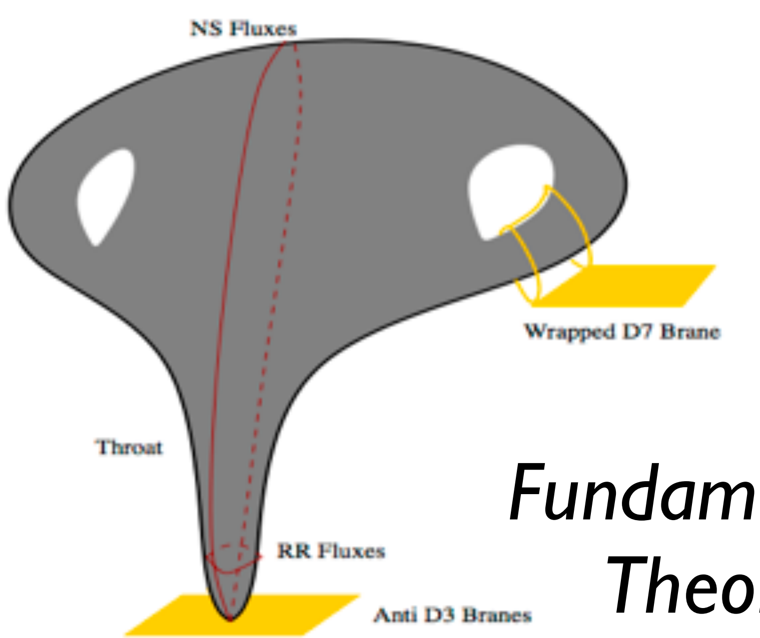


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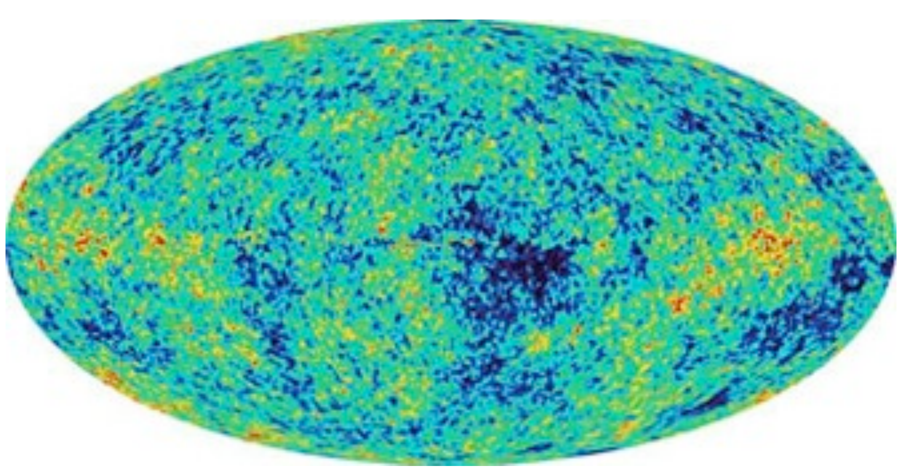

*Observational Data*

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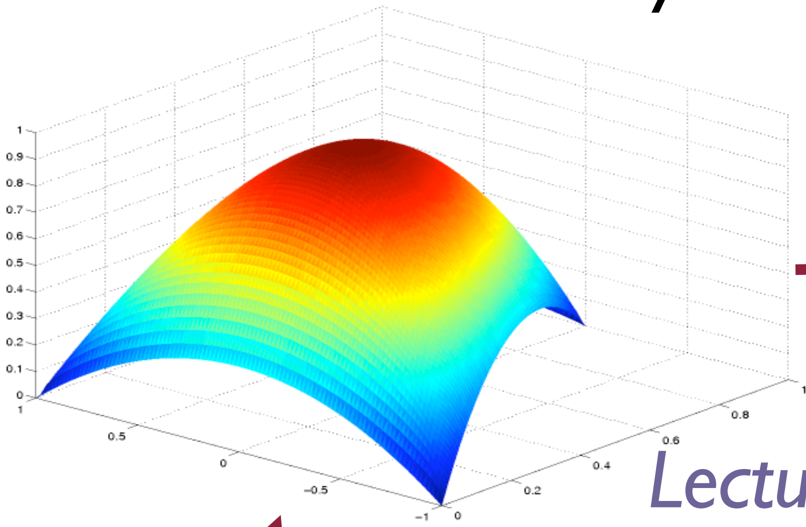

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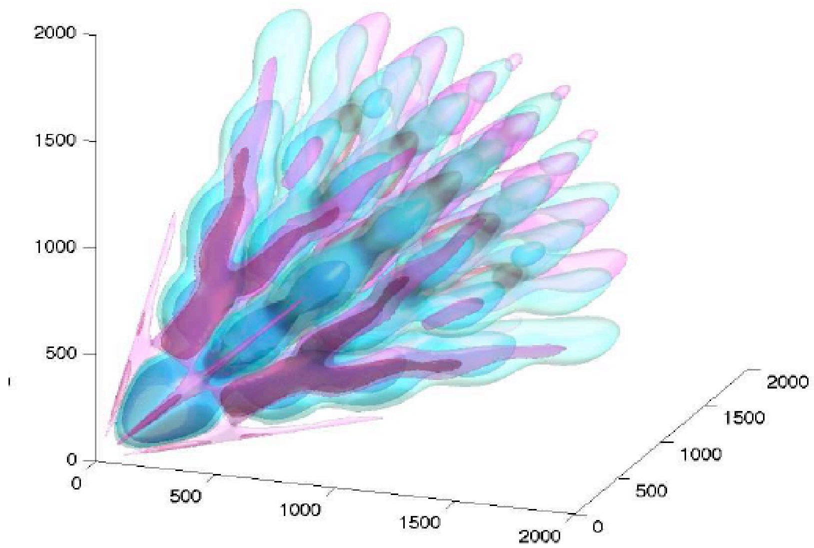
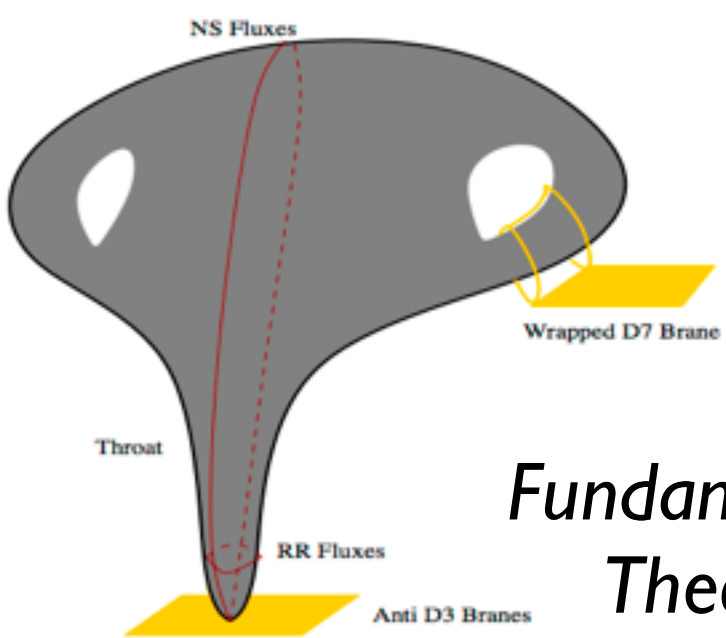
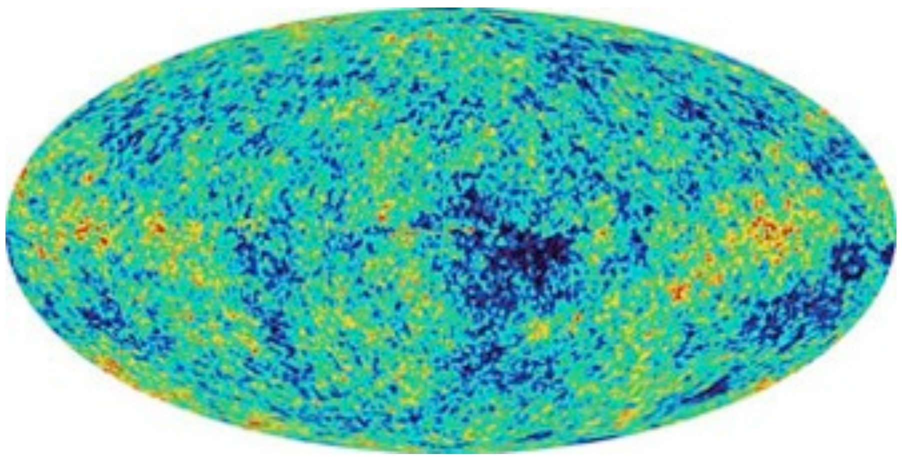


*Lecture 1*

*Lecture 2*




*Lecture 3*



*Fundamental Theory*

*Observational Data*

# Motivation

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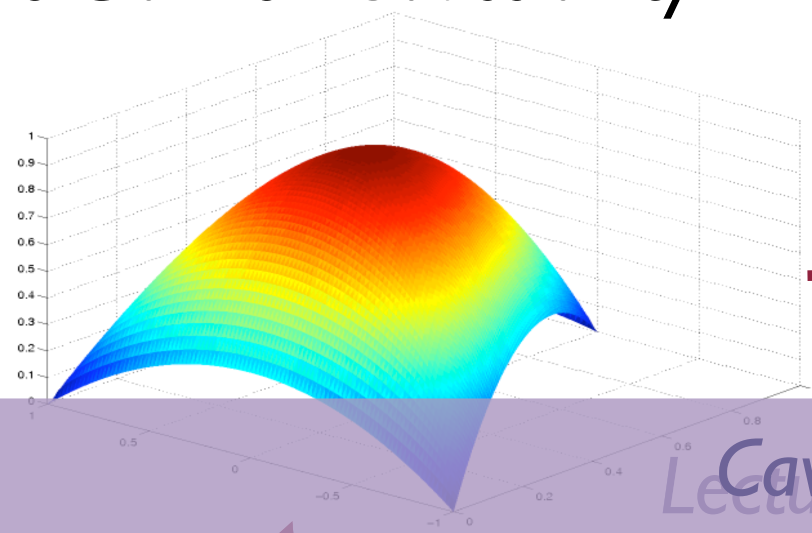

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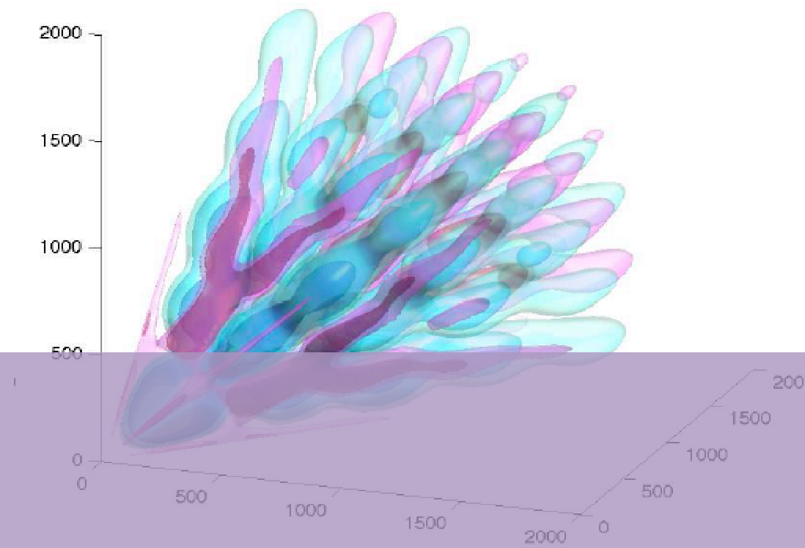



Lecture 1

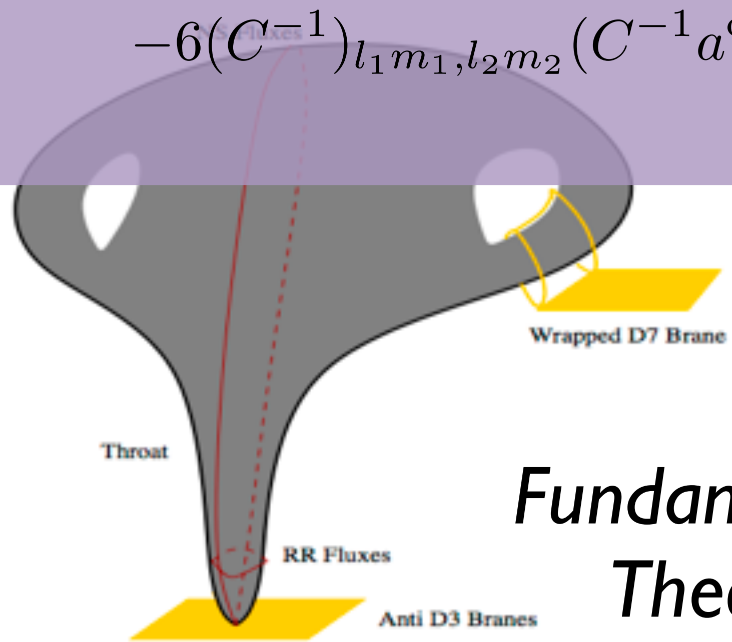
## Caveat

Lecture 2

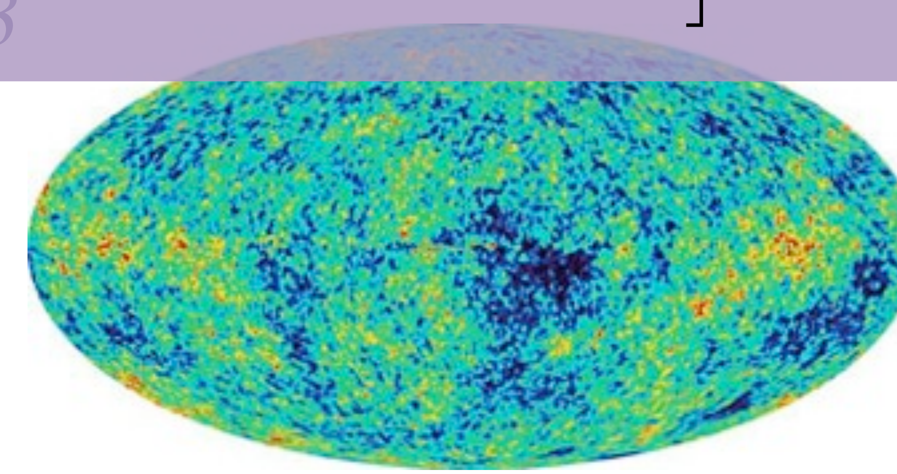
$$\mathcal{E}^{\rm general} = \frac{f_{\rm sky}}{\tilde{N}} \sum_{l_i m_i} \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} a_{l_4 m_4} \rangle_c \Big[ (C^{-1} a^{\rm obs})_{l_1 m_1} (C^{-1} a^{\rm obs})_{l_2 m_2} (C^{-1} a^{\rm obs})_{l_3 m_3} (C^{-1} a^{\rm obs})_{l_4 m_4}$$

$$- 6 (C^{-1})_{l_1 m_1, l_2 m_2} (C^{-1} a^{\rm obs})_{l_3 m_3} (C^{-1} a^{\rm obs})_{l_4 m_4} + 3 (C^{-1})_{l_1 m_1, l_2 m_2} (C^{-1})_{l_3 m_3, l_4 m_4} \Big],$$

Lecture 3

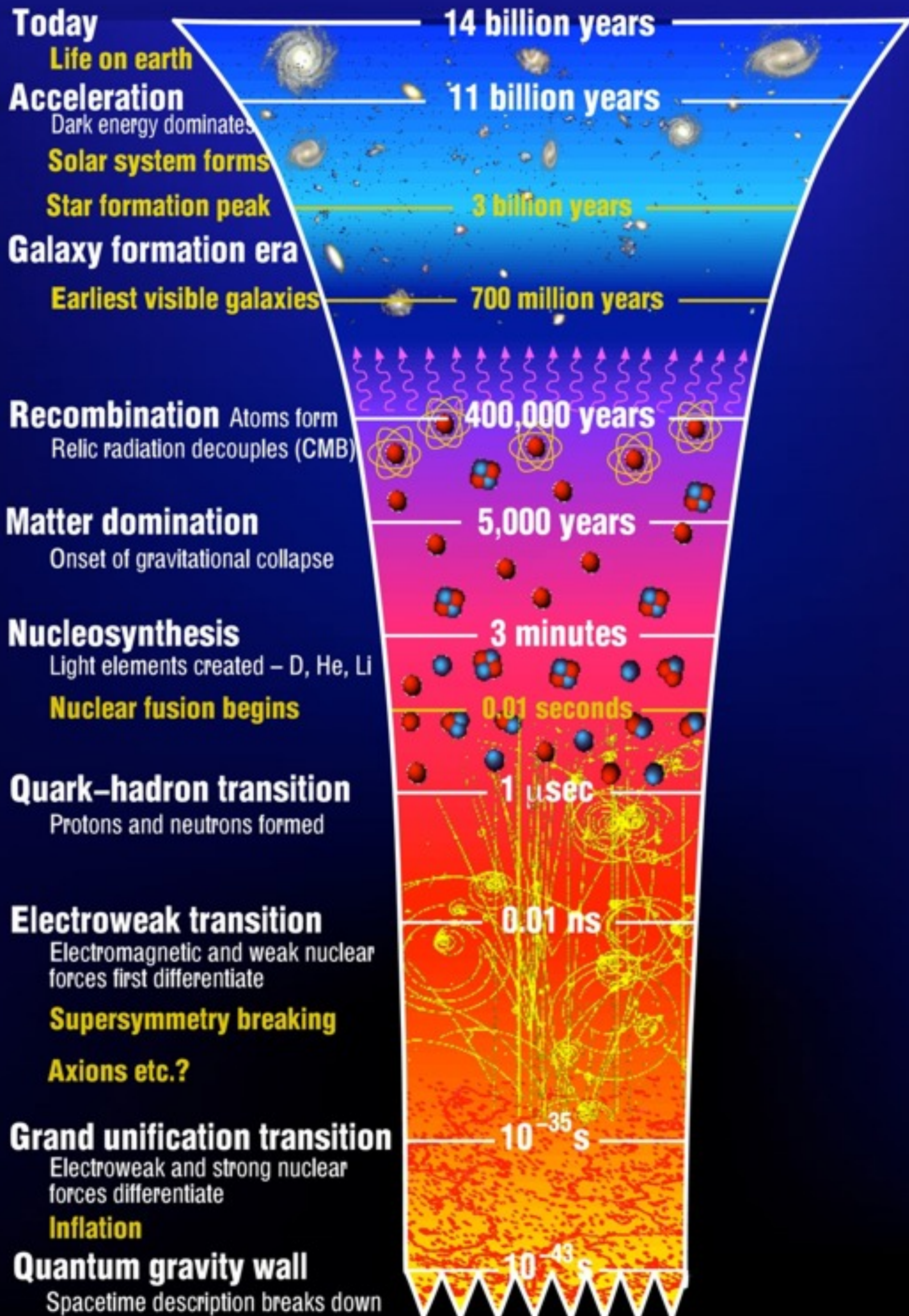



*Fundamental Theory*



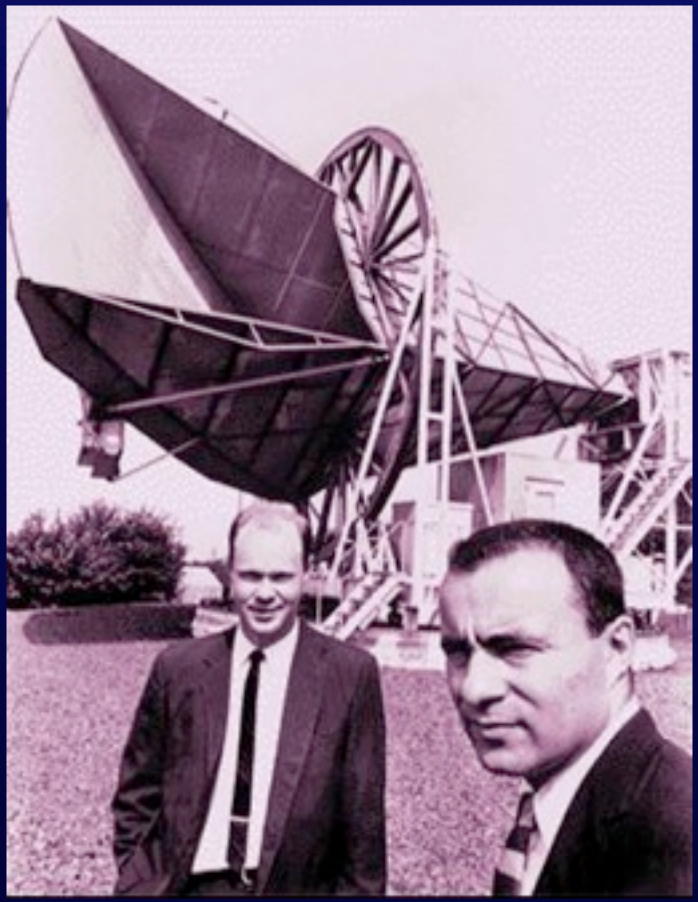
*Observational Data*

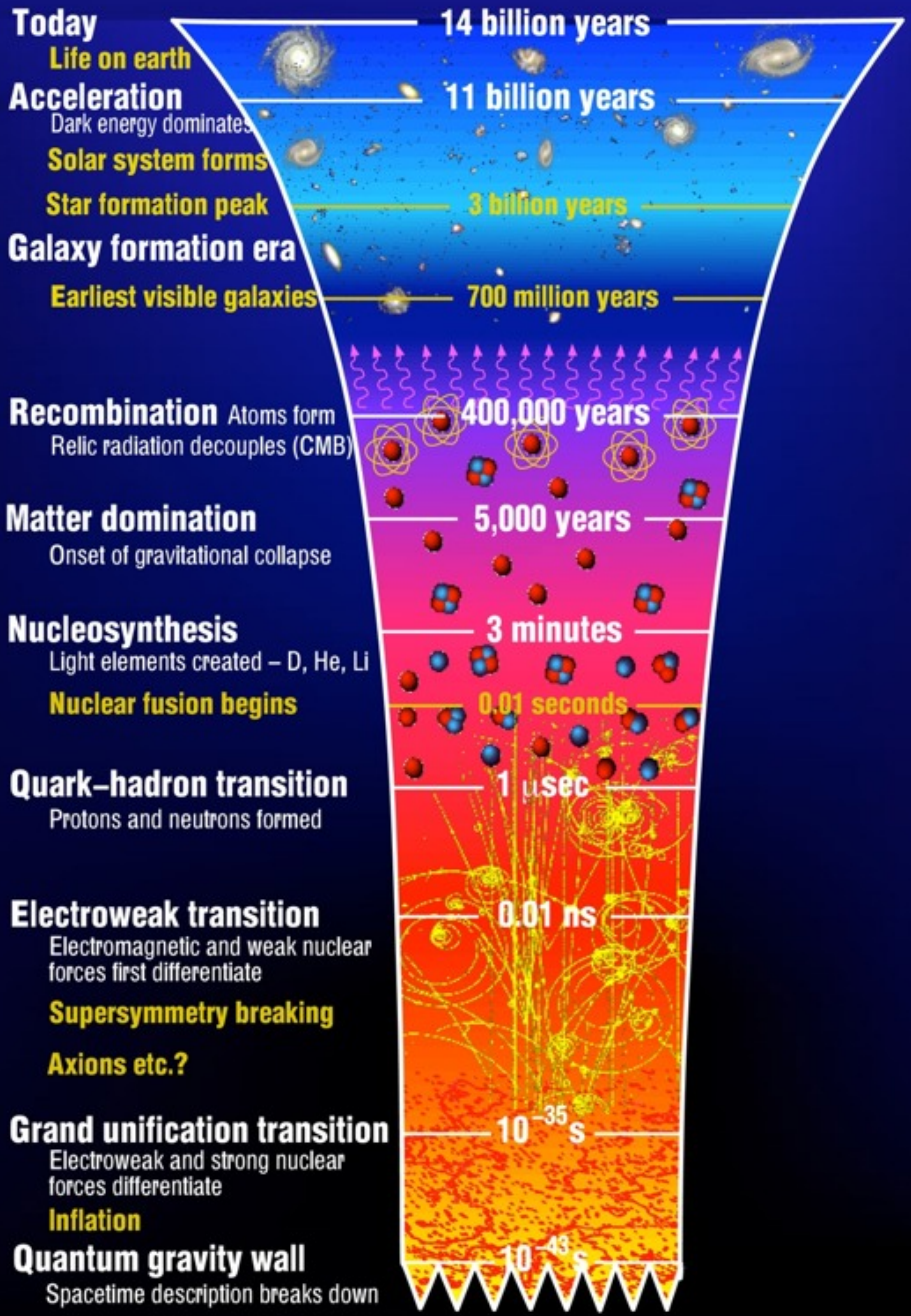
# The standard cosmology …



# *The standard cosmology ...*

## *CMB discovery - 1965*

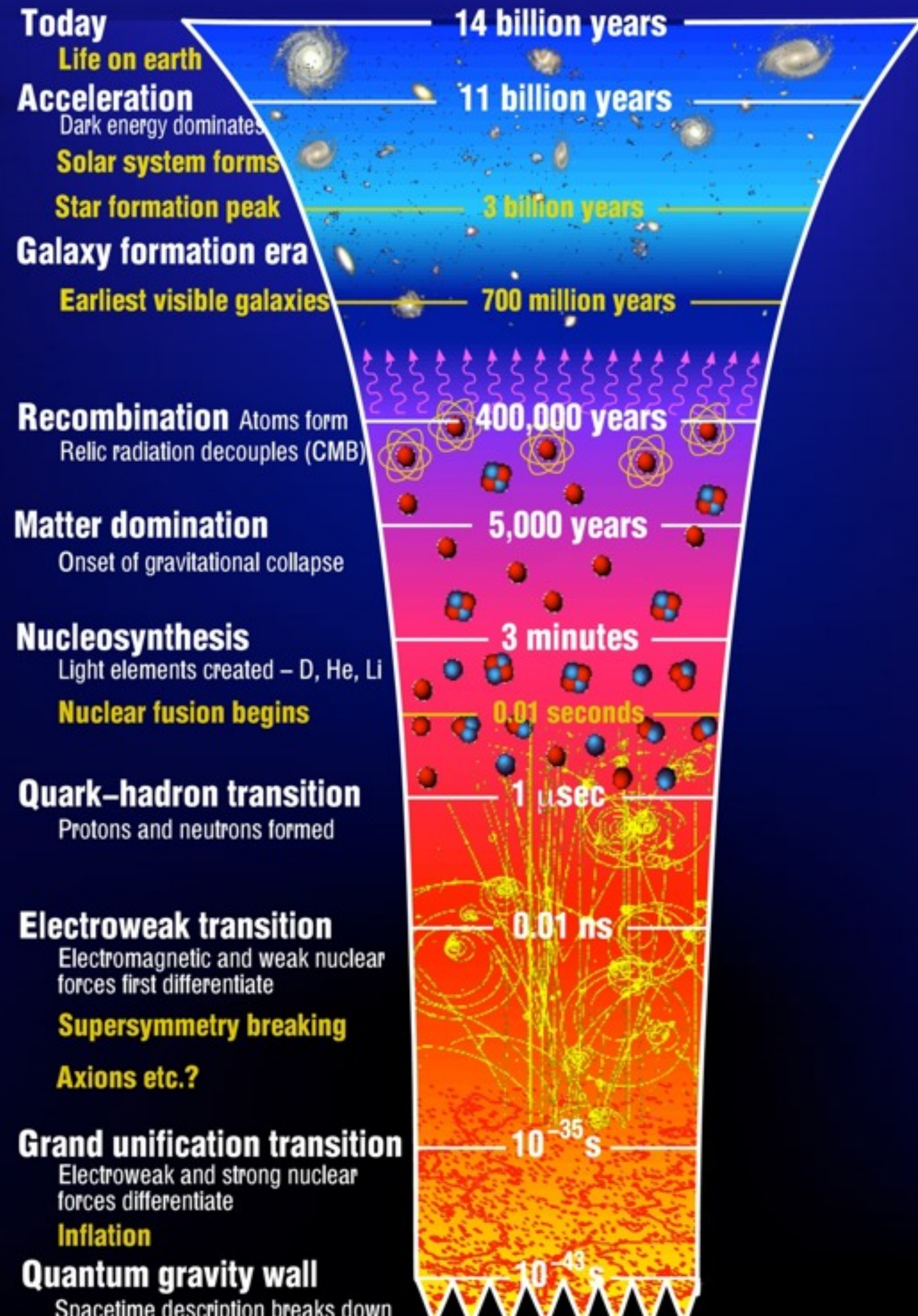




# The standard cosmology ...

*CMB discovery - 1965*

*Precision cosmology - WMAP 2003*

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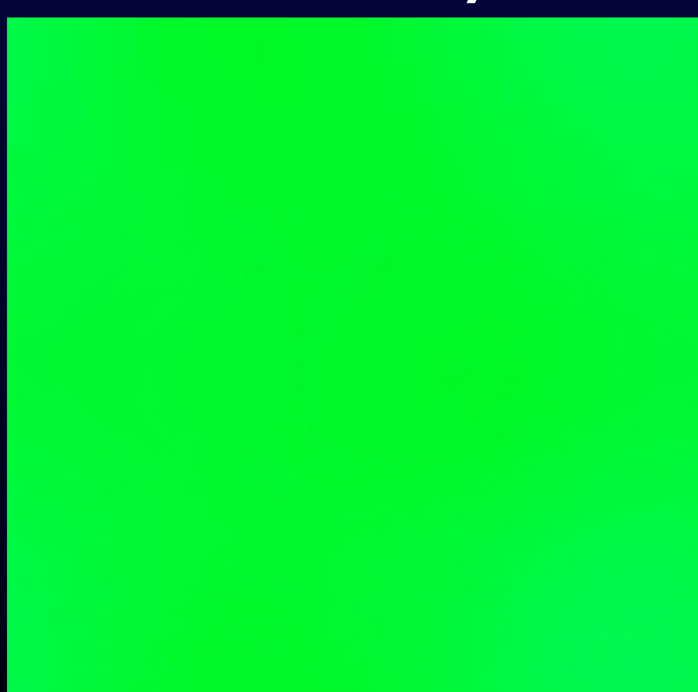
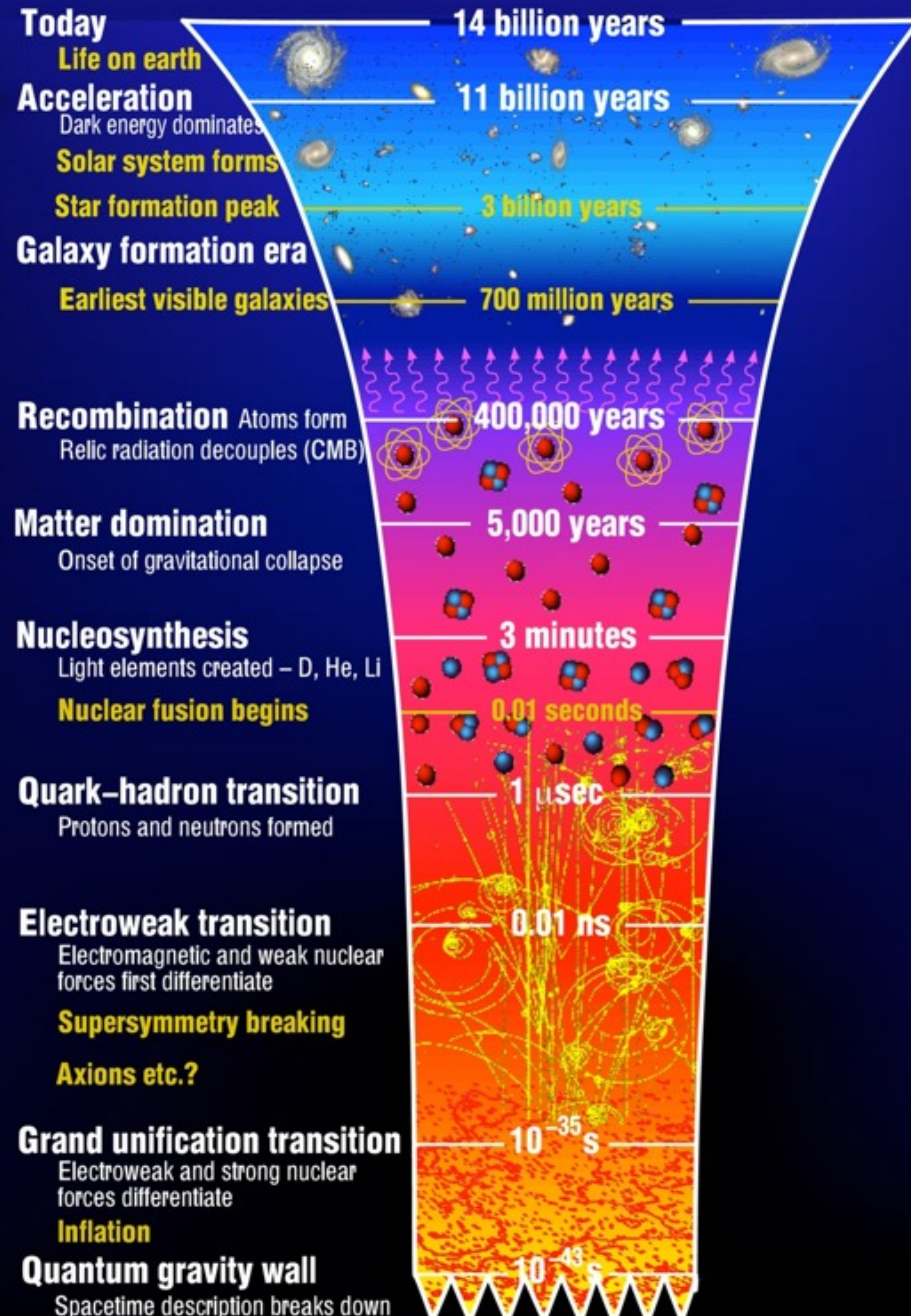
Launched 30 June 2001

*With inflationary fluctuations ....*





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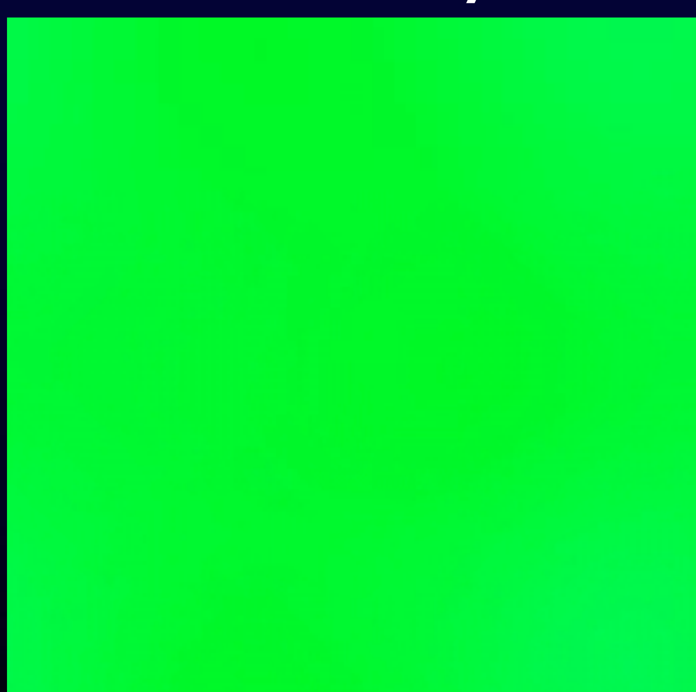
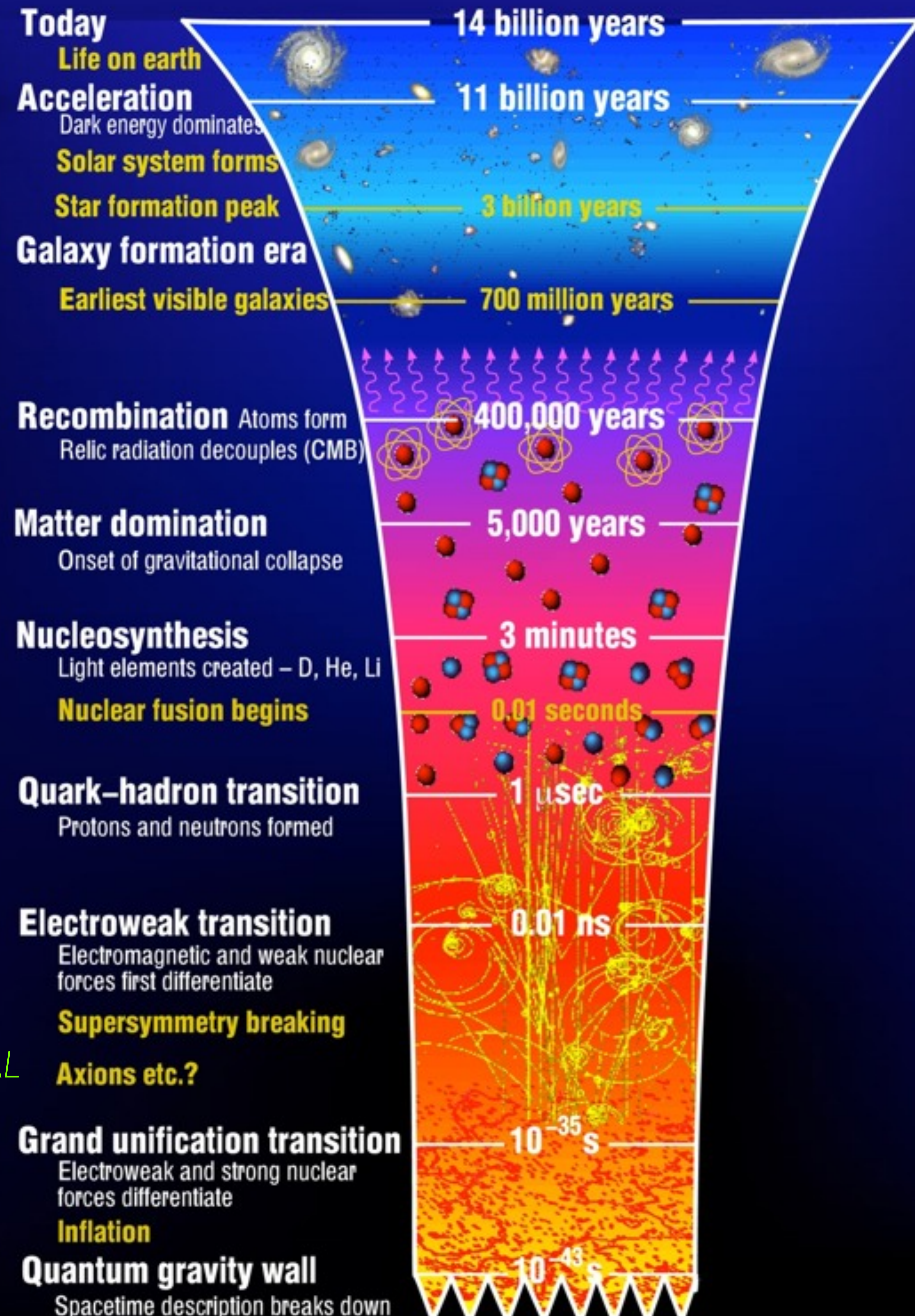


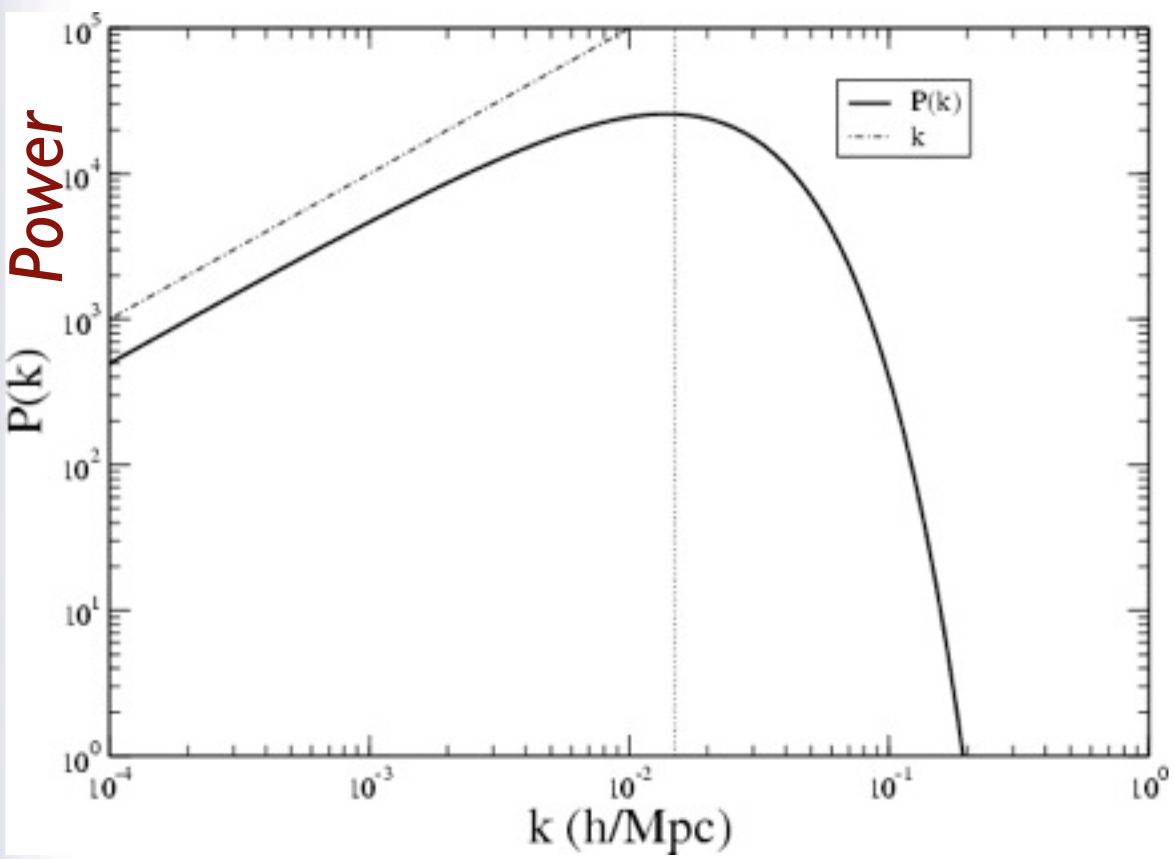
FUNDAMENTAL THEORY



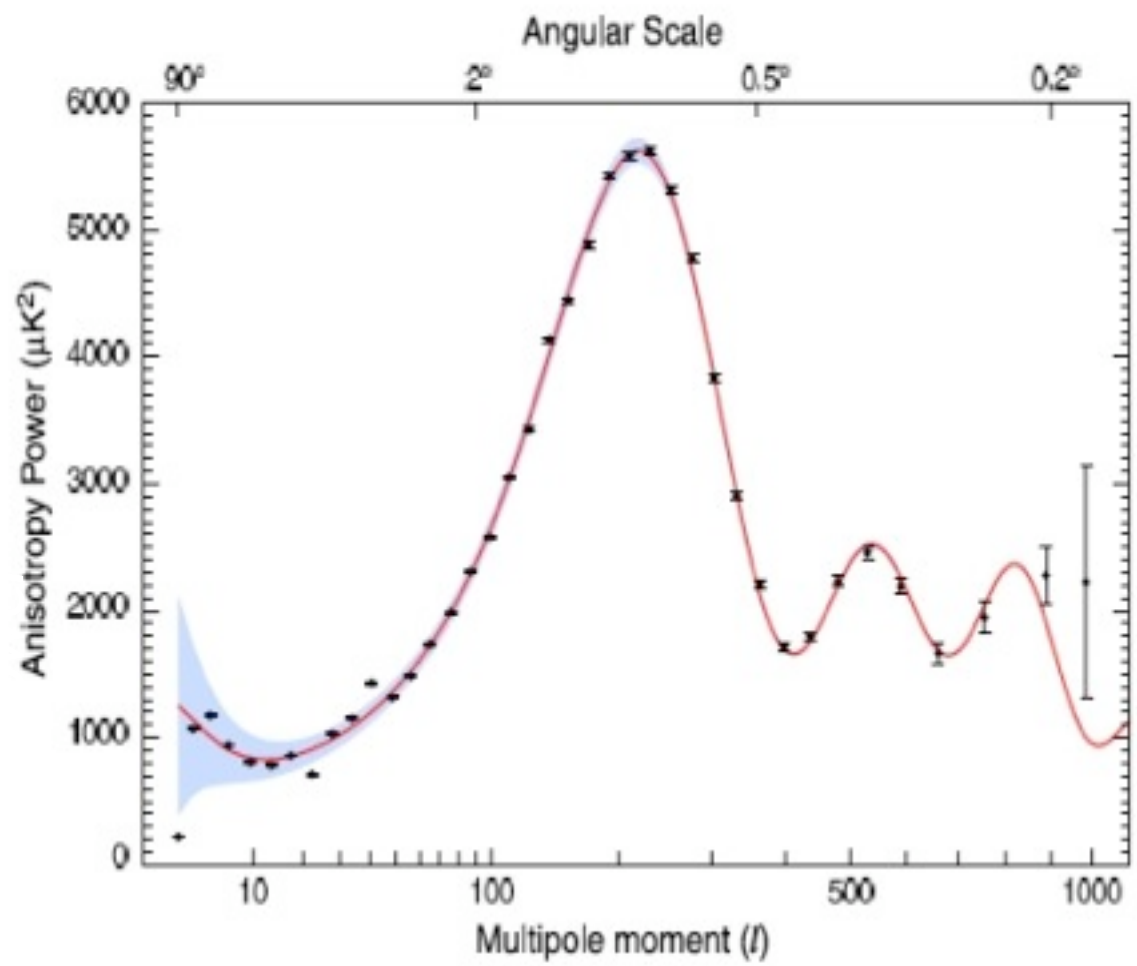
# WMAP data analysis

***Why is the CMB so good for cosmology?*** *Answer:* ***Linearity!***

*Assume simple primordial model - inflation (small scale-free seeds)*




Simple initial power spectrum




CMB angular power spectrum

*Transfer function $\Delta_l(k)$ depends on $H_0$, $\Omega$, $\Omega_\Delta$, $\Omega_M$, $\Omega_B$, etc.*

# WMAP data analysis

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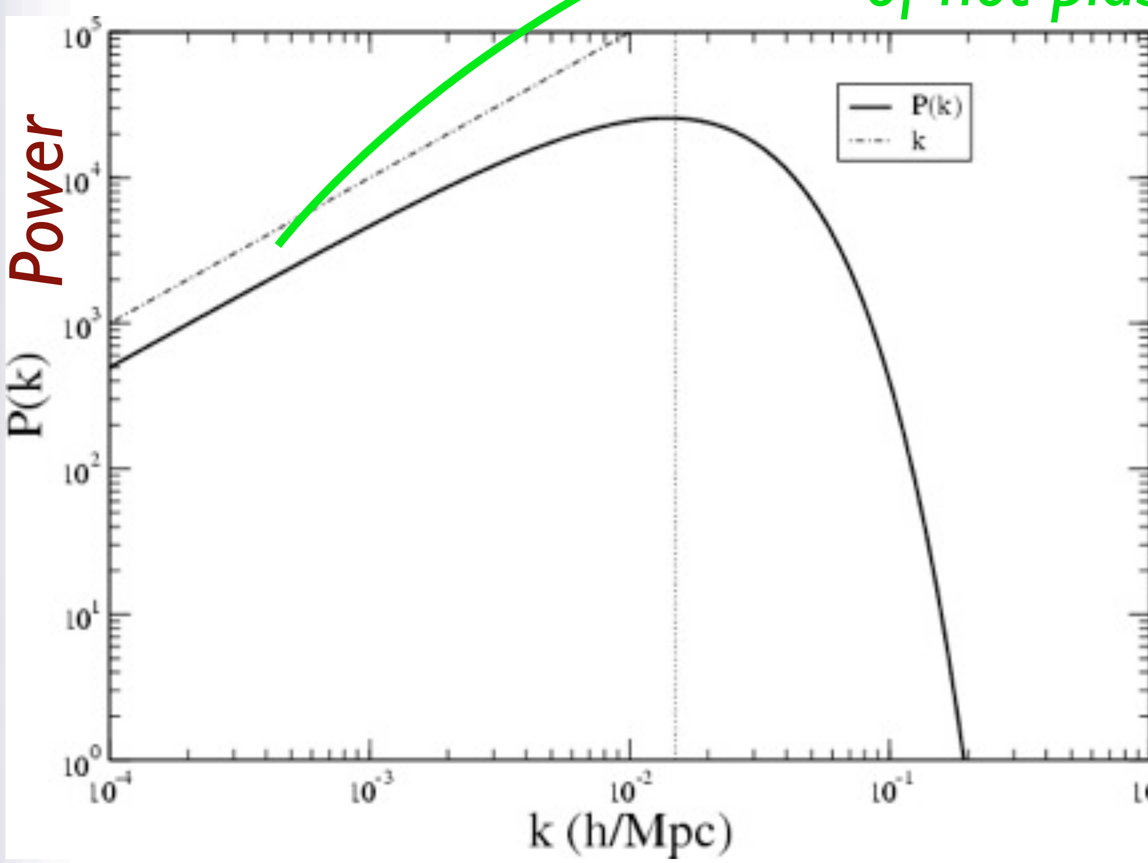
Transfer function $\Delta_l(k)$

*acoustic oscillations of hot plasma*

*Power*

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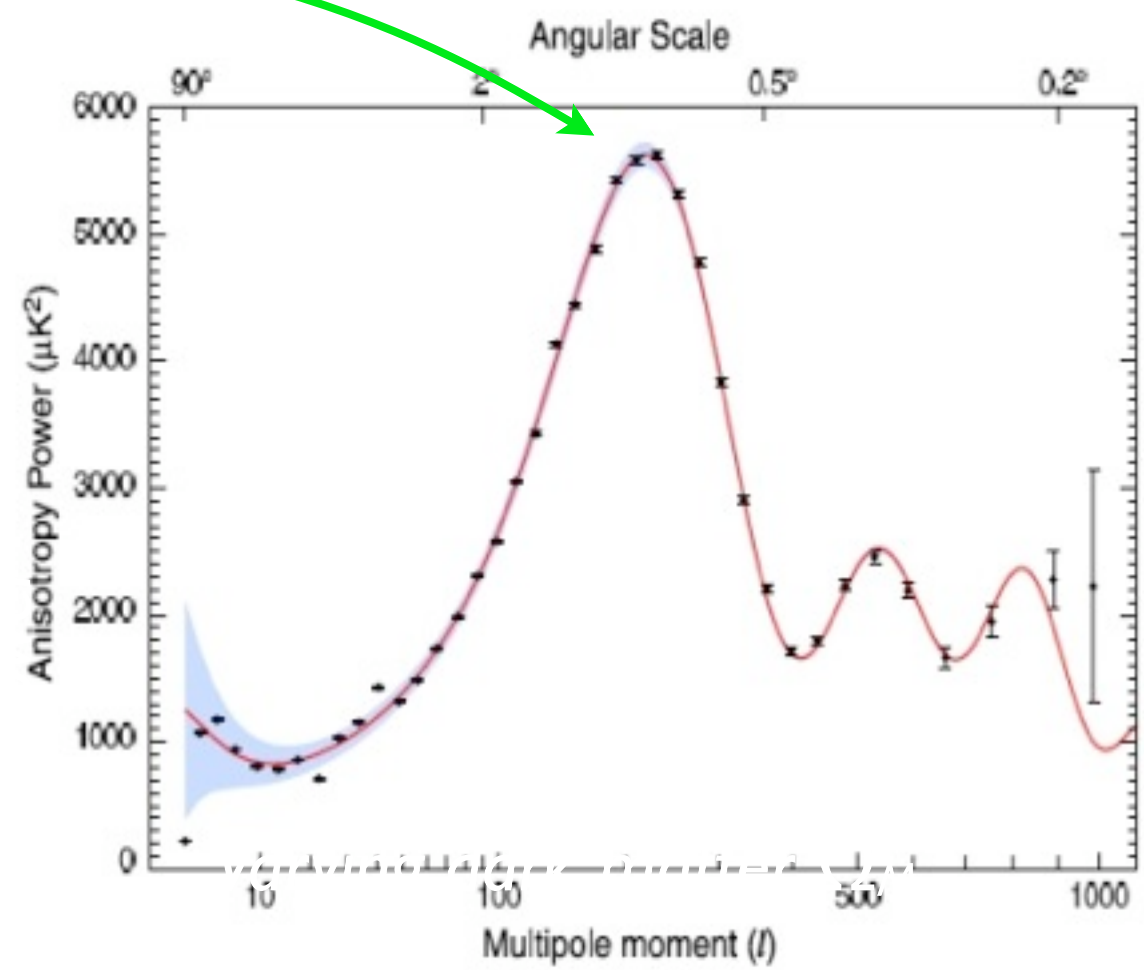


CMB angular power spectrum

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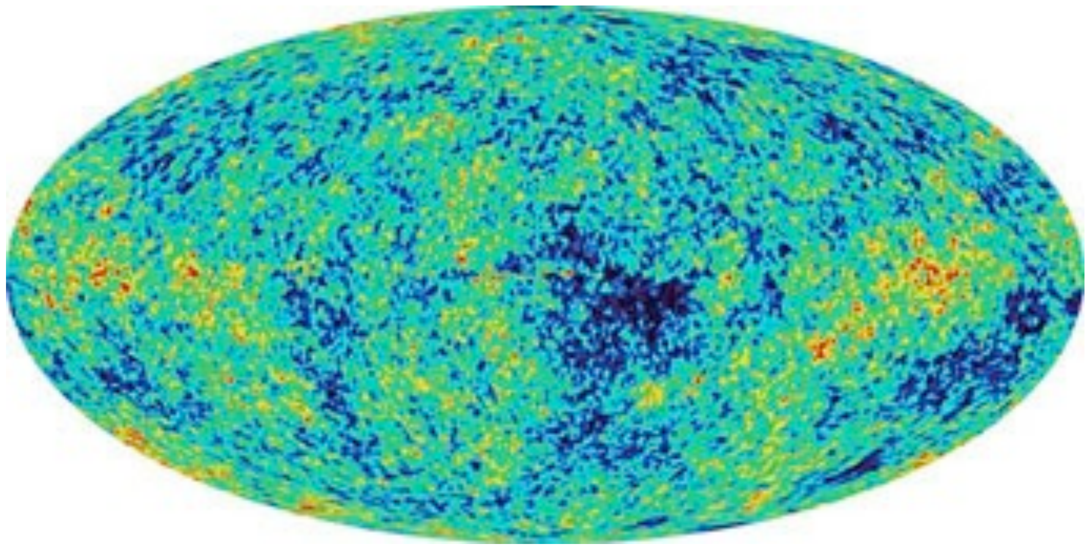
# A Gaussian Universe?

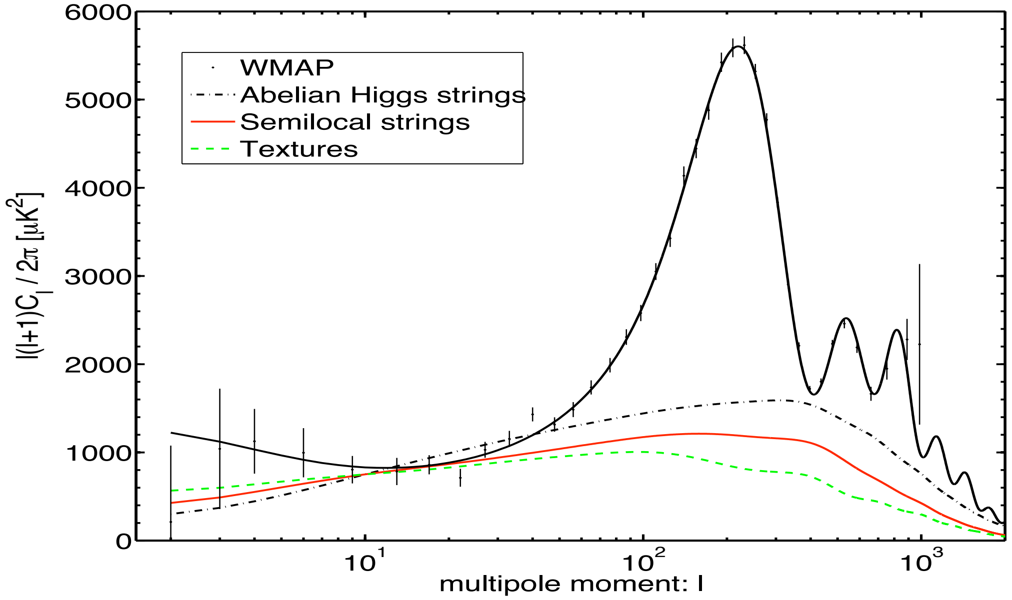
*The triumph of the inflationary concordance model ...*

*A self-consistent concordance model with (WMAP 7yr results):*

4.56(±0.16)% baryons, 22.7(±1.4)% dark matter, 72.8(±1.5)% dark energy with spatial flatness (to 1%) and an age of 13.75(±0.11) billion years.

*All based on the two-point correlator (power spectrum)*






I.e. massive compression of data (e.g. WMAP $10^6$ pixels to 1000 l's)

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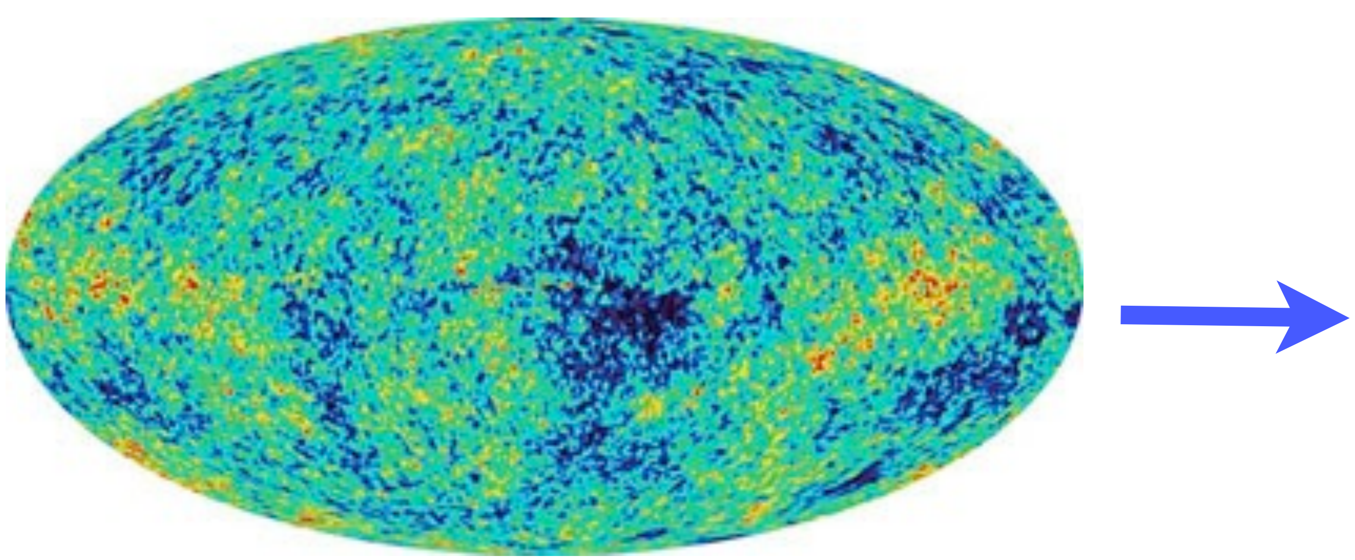

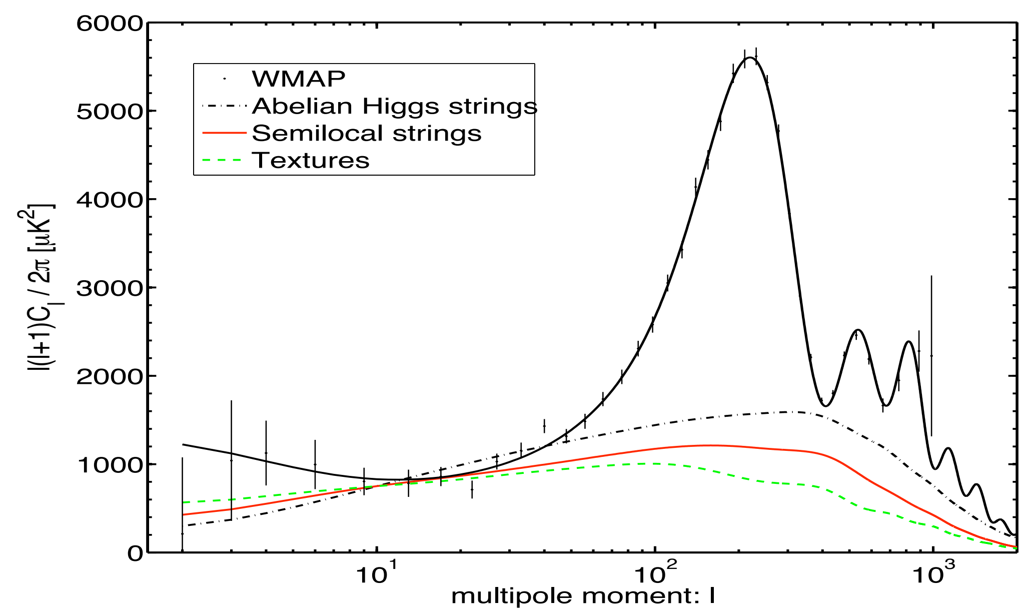


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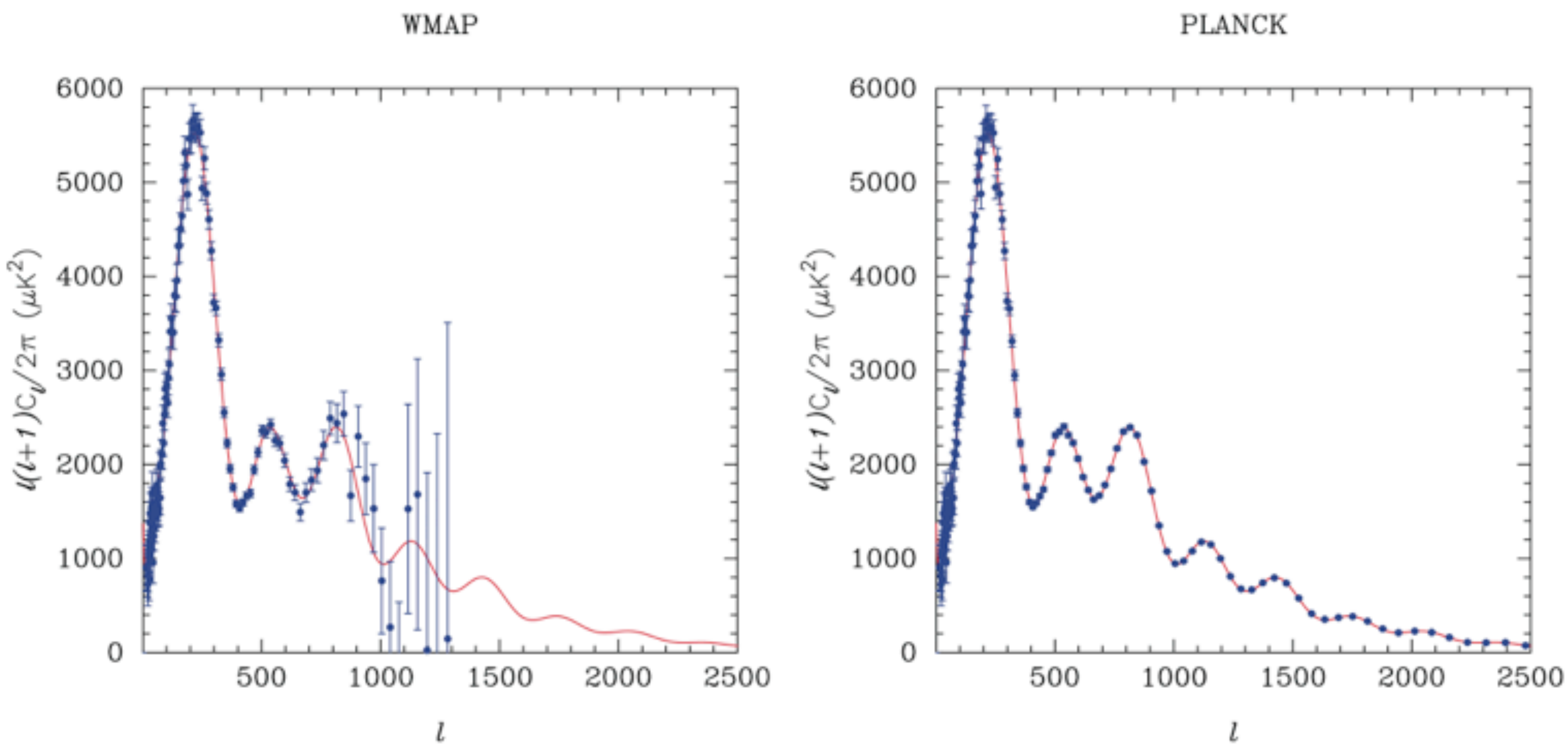
*But is this all there is to be learnt about the early universe?*

# CMB Future - Planck

Launched in May 2009, the ESA Planck satellite, is currently mapping the Universe at high resolution - first cosmology results Jan 2013



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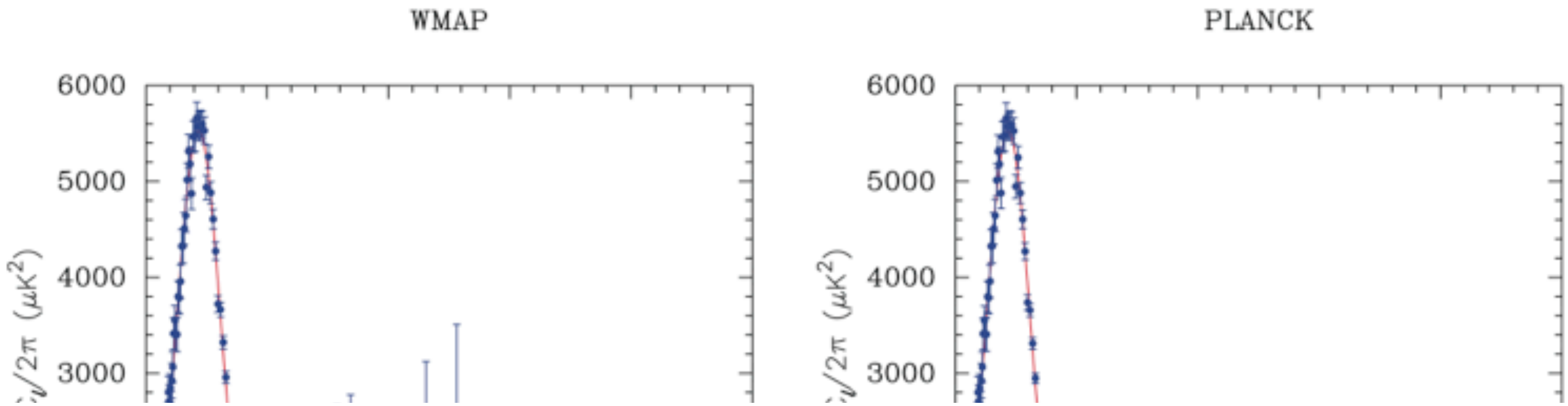
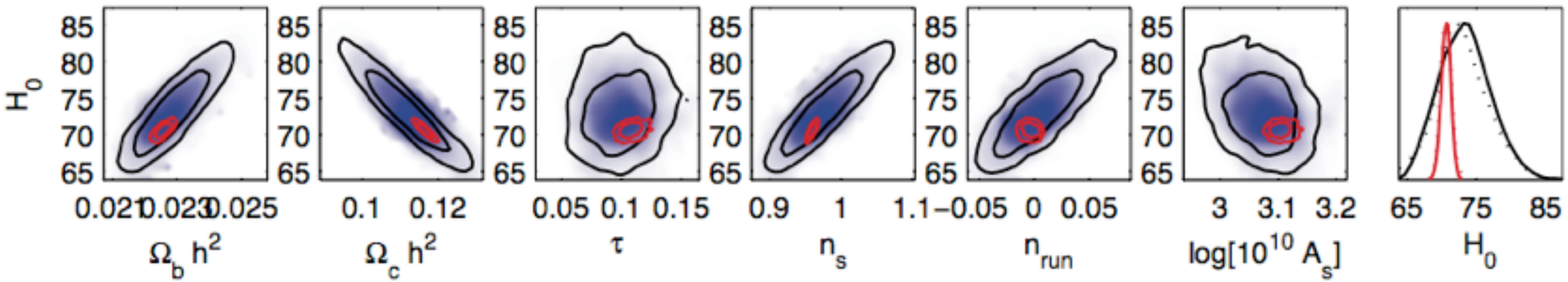



*Typical example with Hubble parameter likelihoods (4 x smaller errors)*



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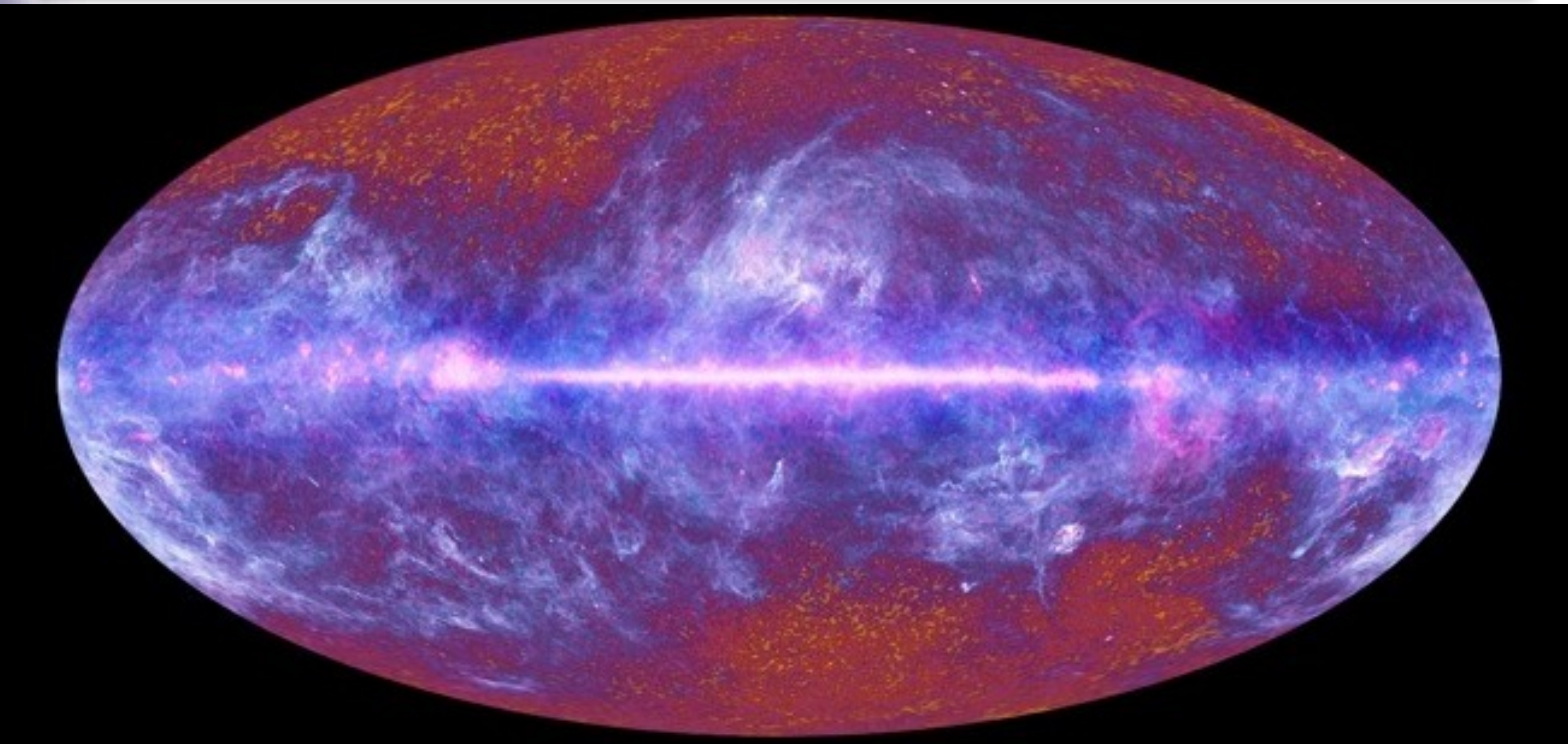

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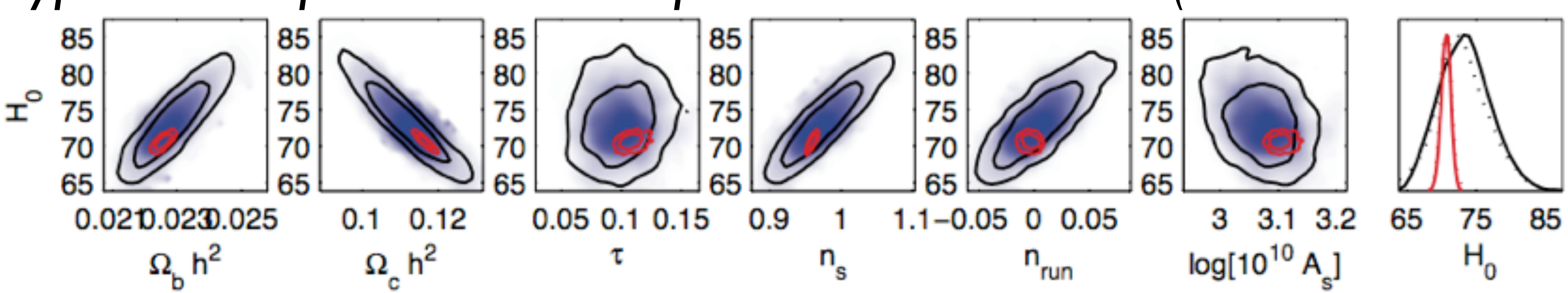
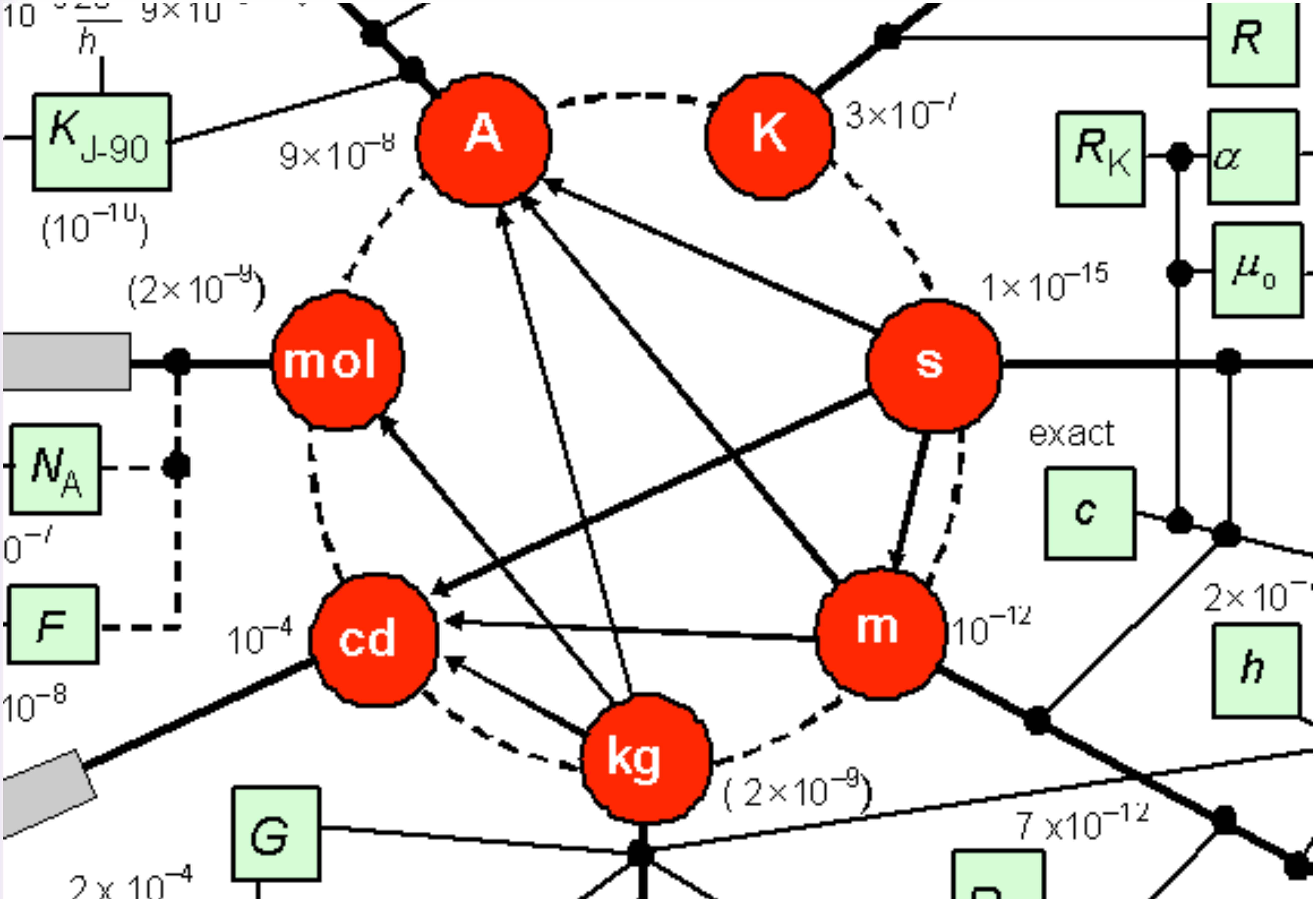


# Only a cosmic standards lab?



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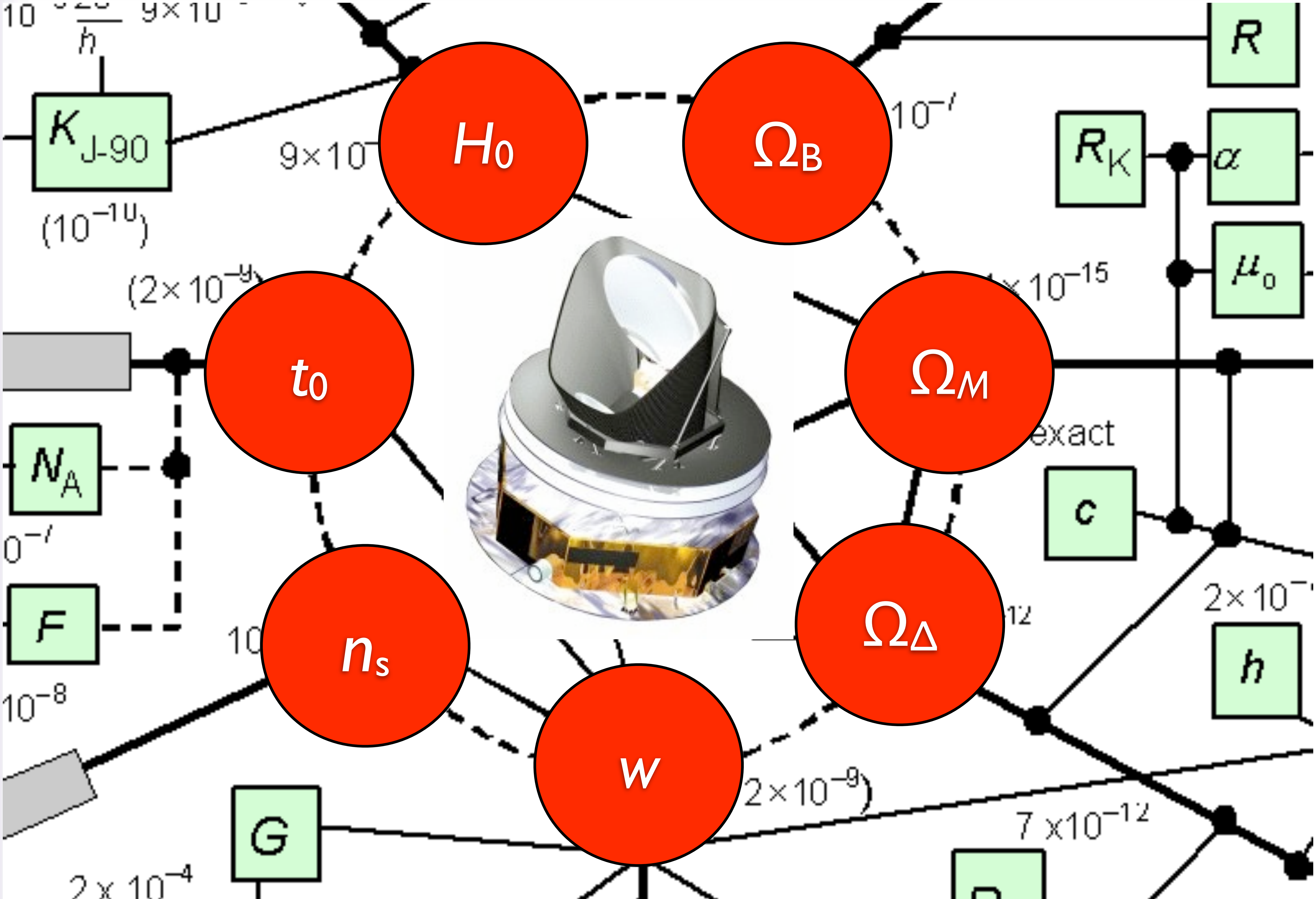
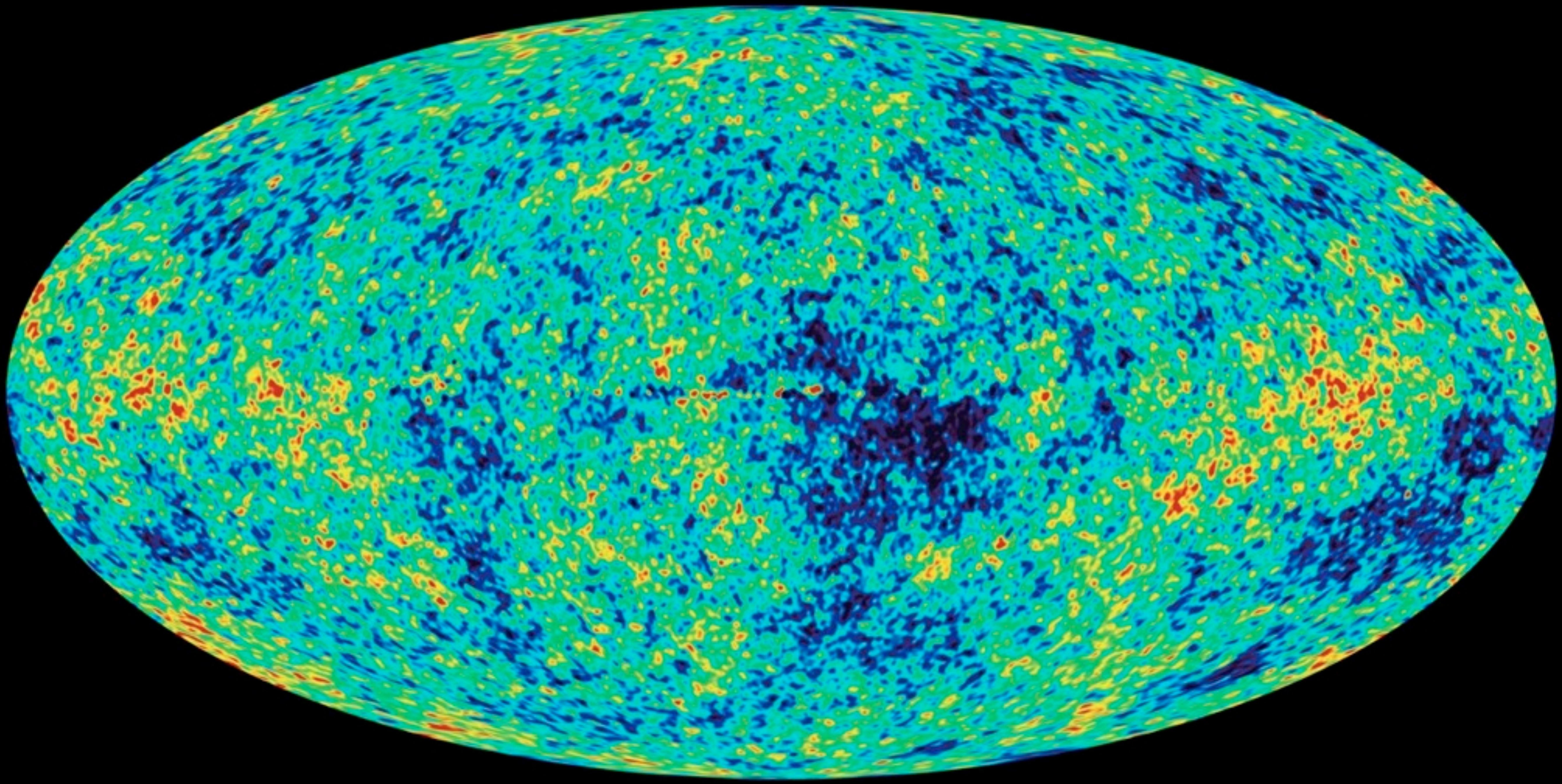
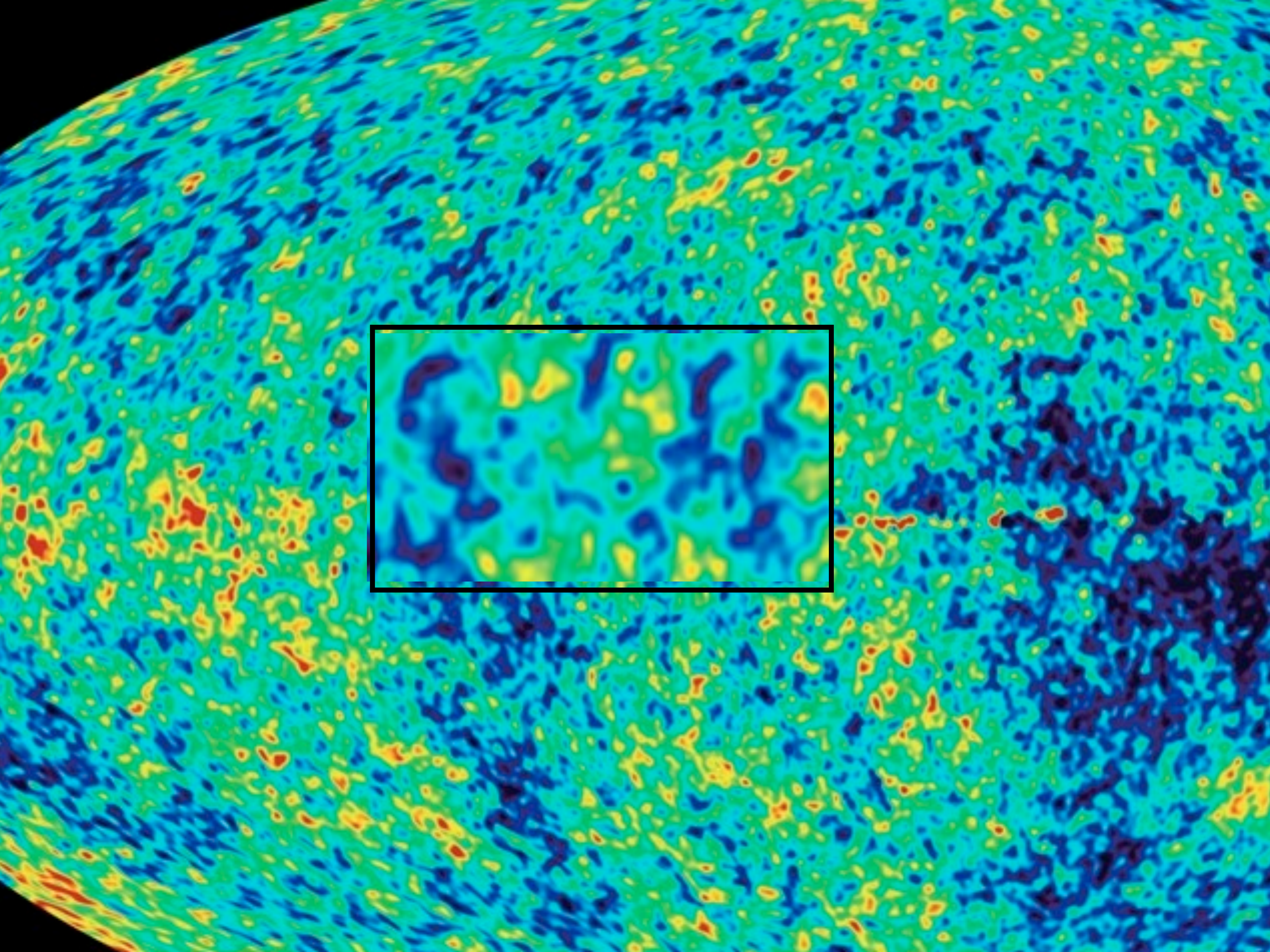
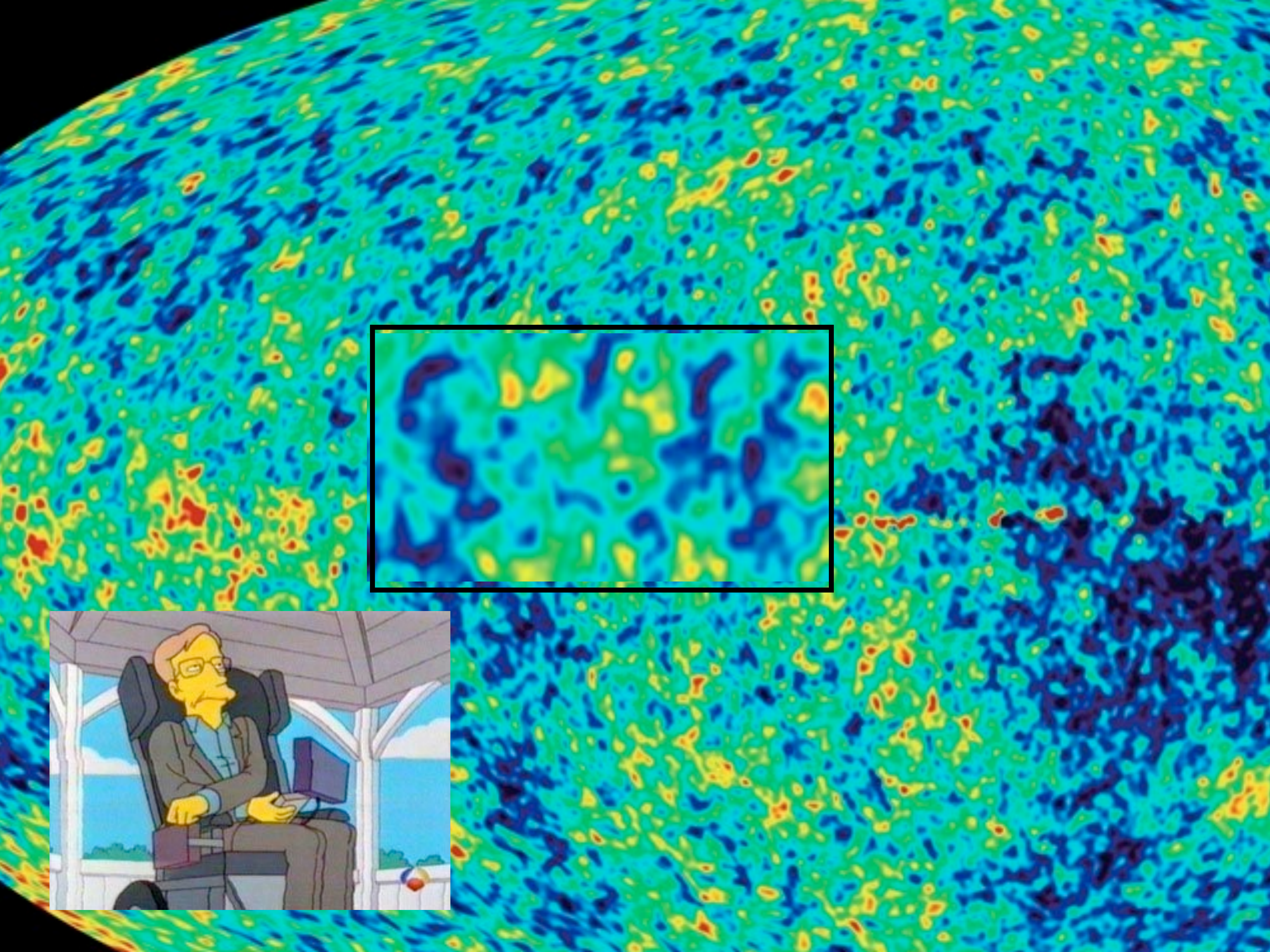
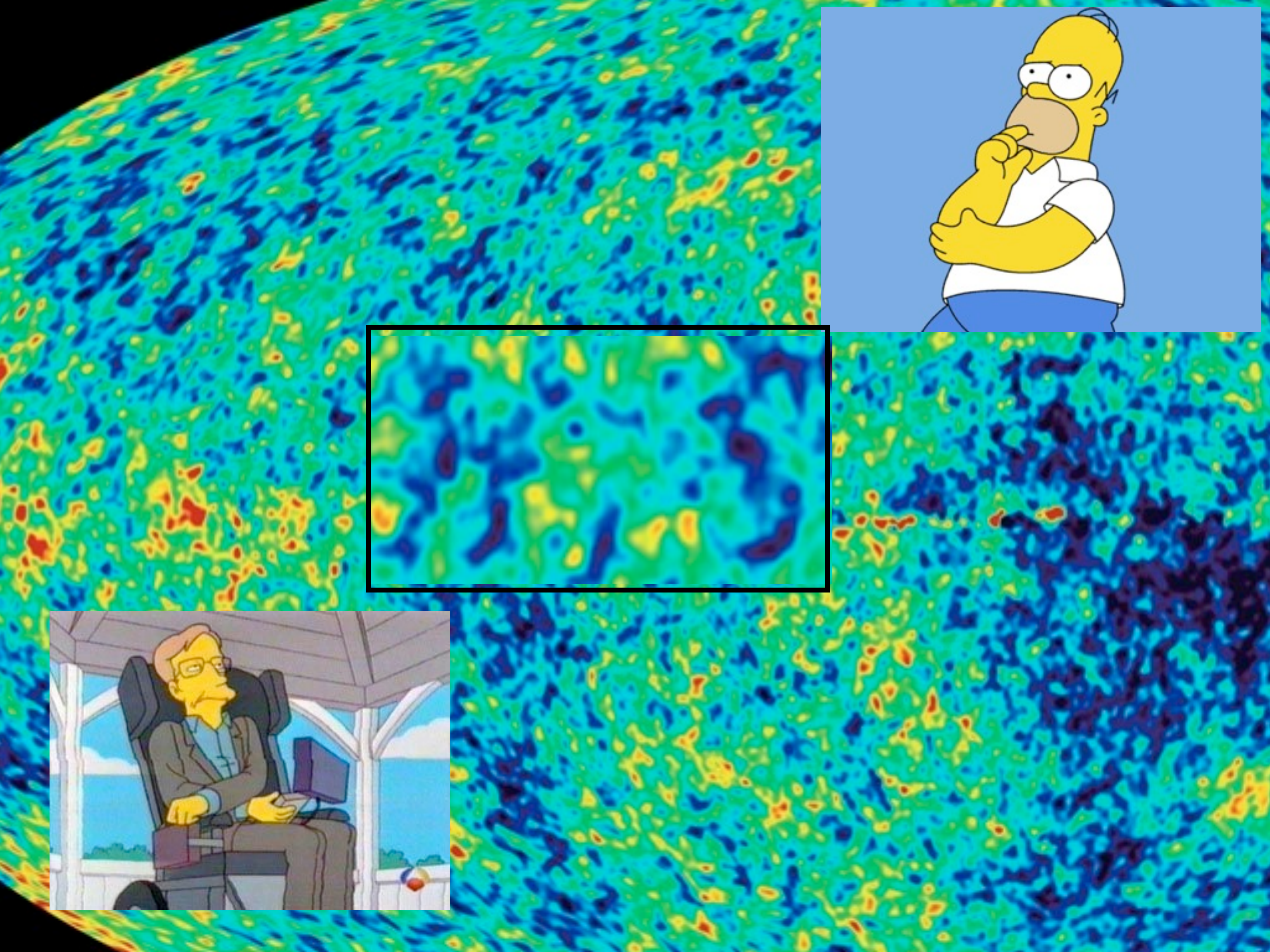


# *Messages in the WMAP sky?*









# INFLATION

Inflation is defined as a period of accelerated expansion $\ddot{a} > 0$

The simplest example has equation of state $P = -\rho c^2$

so the energy conservation equation yields $\dot{\rho} = 0 \Rightarrow \rho = \text{const}$

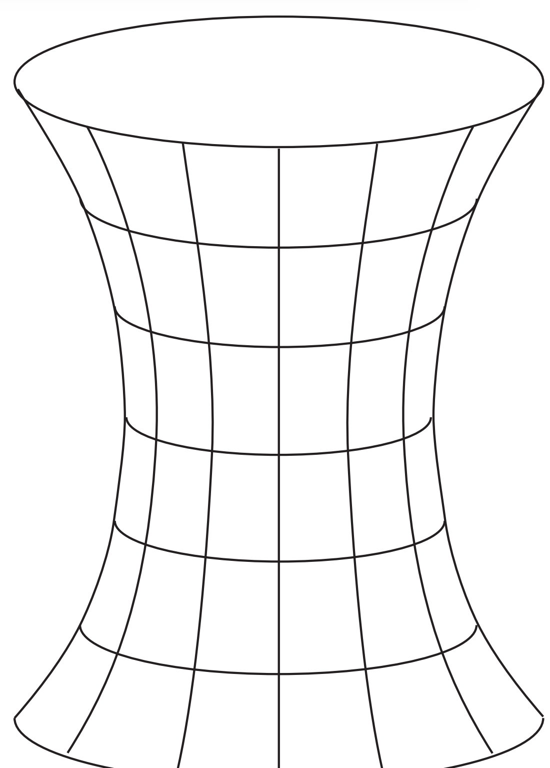
The Friedmann equation gives $\equiv \left(\frac{c^2}{8\pi G}\right)\Lambda$

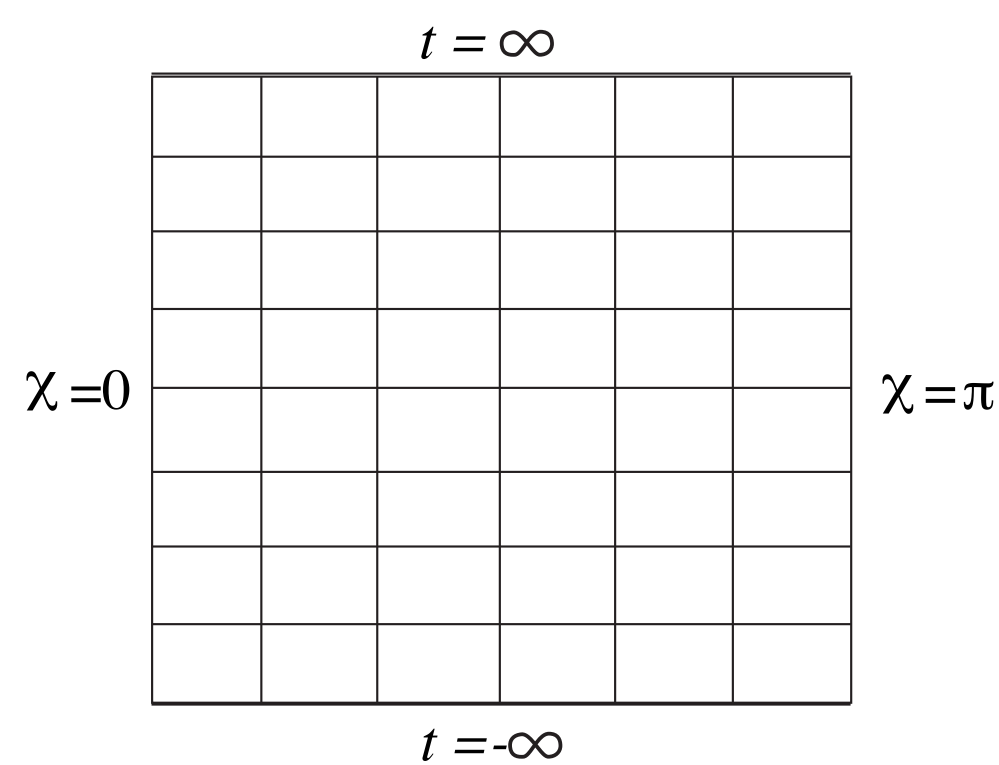
$$\left(\frac{\dot{a}}{a}\right)^2 = c^2\frac{\Lambda}{3} \qquad \Rightarrow \boxed{a(t) = e^{H(t-t_0)}}$$

with Hubble parameter

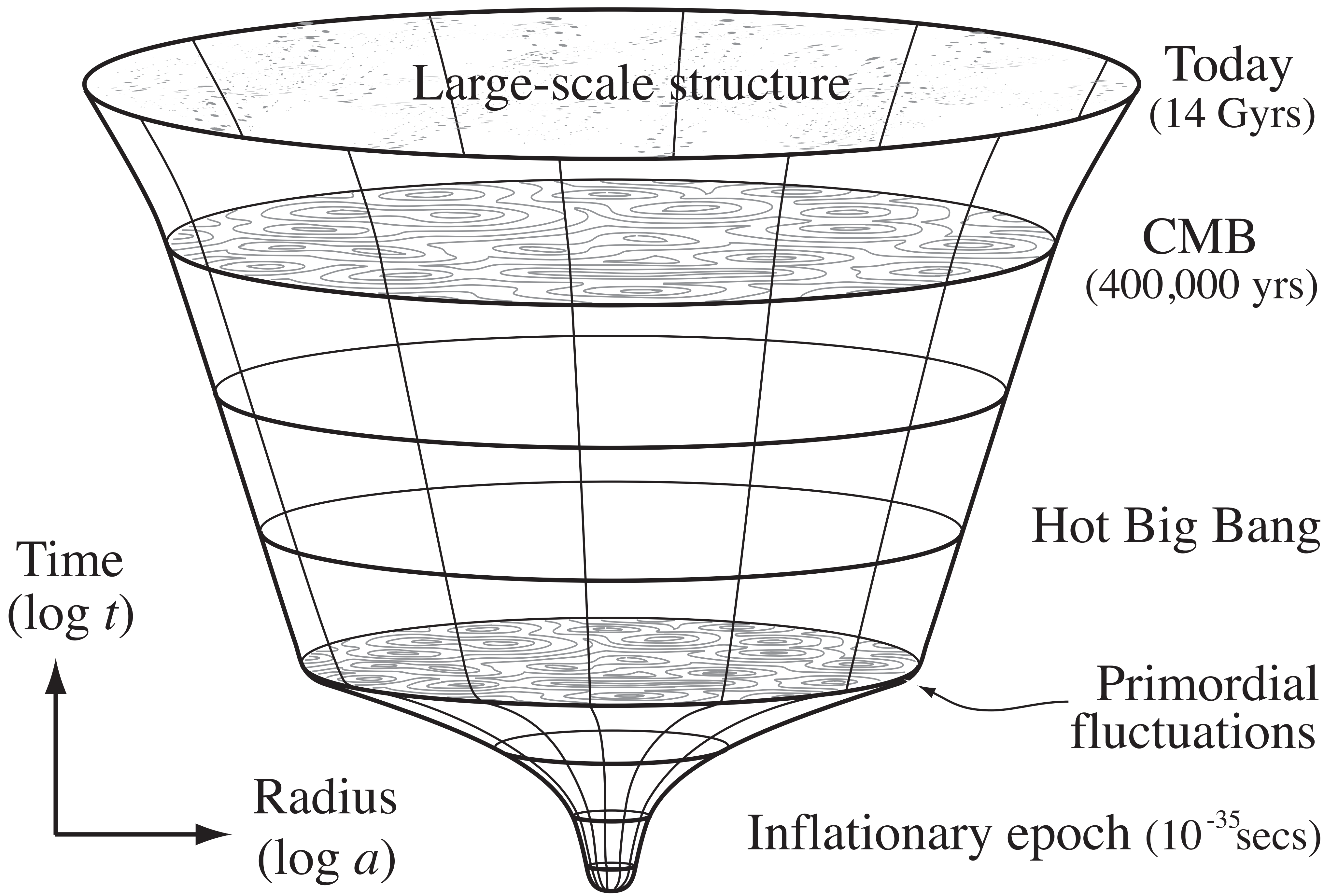
$$H = c\sqrt{\frac{\Lambda}{3}}$$

*De Sitter space*
*conformal diagram*



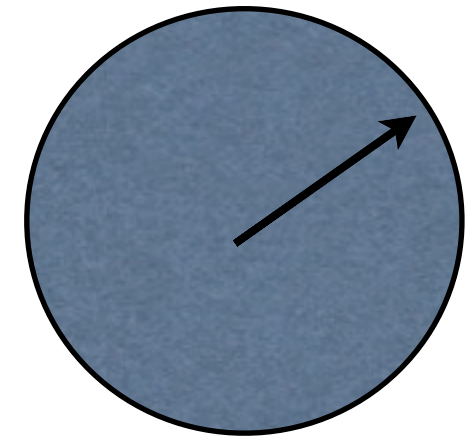


# Inflationary Hot Big Bang model



# Quantum fluctuations

An accelerating universe has an event horizon



$$d_{\rm E}(t) = c\int_t^\infty e^{-H_{\rm I}t'}dt' = cH_{\rm I}^{-1}$$

Heisenberg's Uncertainty principle with $\Delta x \approx cH_{\rm I}^{-1}$ implies $\Delta p$

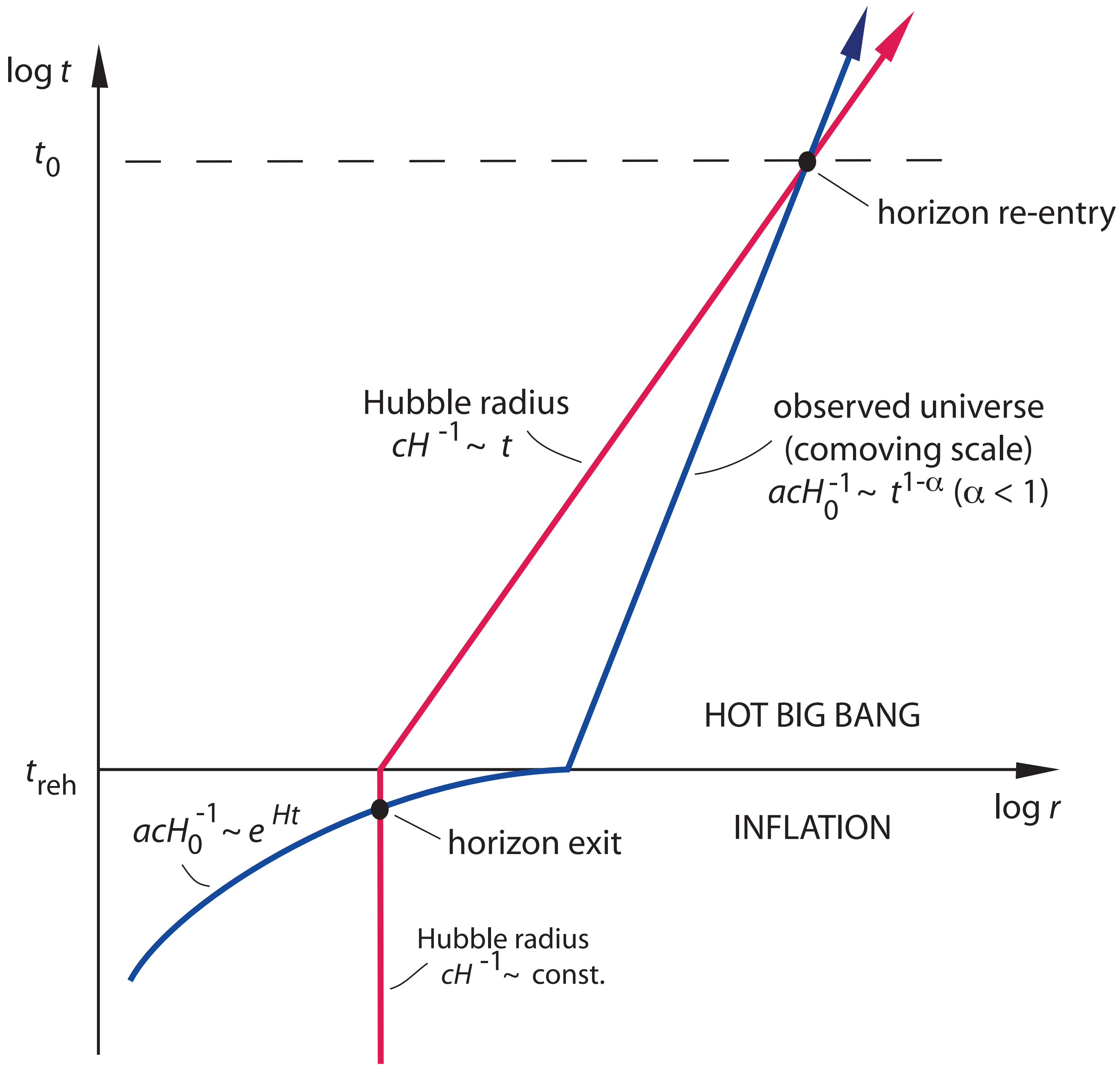
$$\Delta E \;\approx\; \frac{c}{\hbar}\dot\phi^2\mathcal{V} \;\approx\; \frac{c}{\hbar}\left(\frac{\Delta\phi}{\Delta t}\right)^2(cH_{\rm I}^{-1})^3 \;\approx\; \frac{c^4}{\hbar}(\Delta\phi)^2H_{\rm I}^{-1}$$

Hence $\Delta E\Delta t \approx (c^4/\hbar)(\Delta\phi)^2H_{\rm I}^{-2} \approx \hbar,$ implies inflaton

fluctuations $$\boxed{(\Delta\phi)^2\big|_{\rm I} \approx \left(\frac{\hbar H_{\rm I}}{c^2}\right)^2}$$

These exit the horizon and "freeze" as classical spacetime fluctuations

# Perturbation evolution




*The observed universe*
*(comoving scale evolution):*

*At reheating well outside the Hubble radius,*

$$\frac{a(t_{\mathrm{reh}})}{a(t_0)} cH_0^{-1} \gg cH^{-1}(t_{\mathrm{reh}})$$

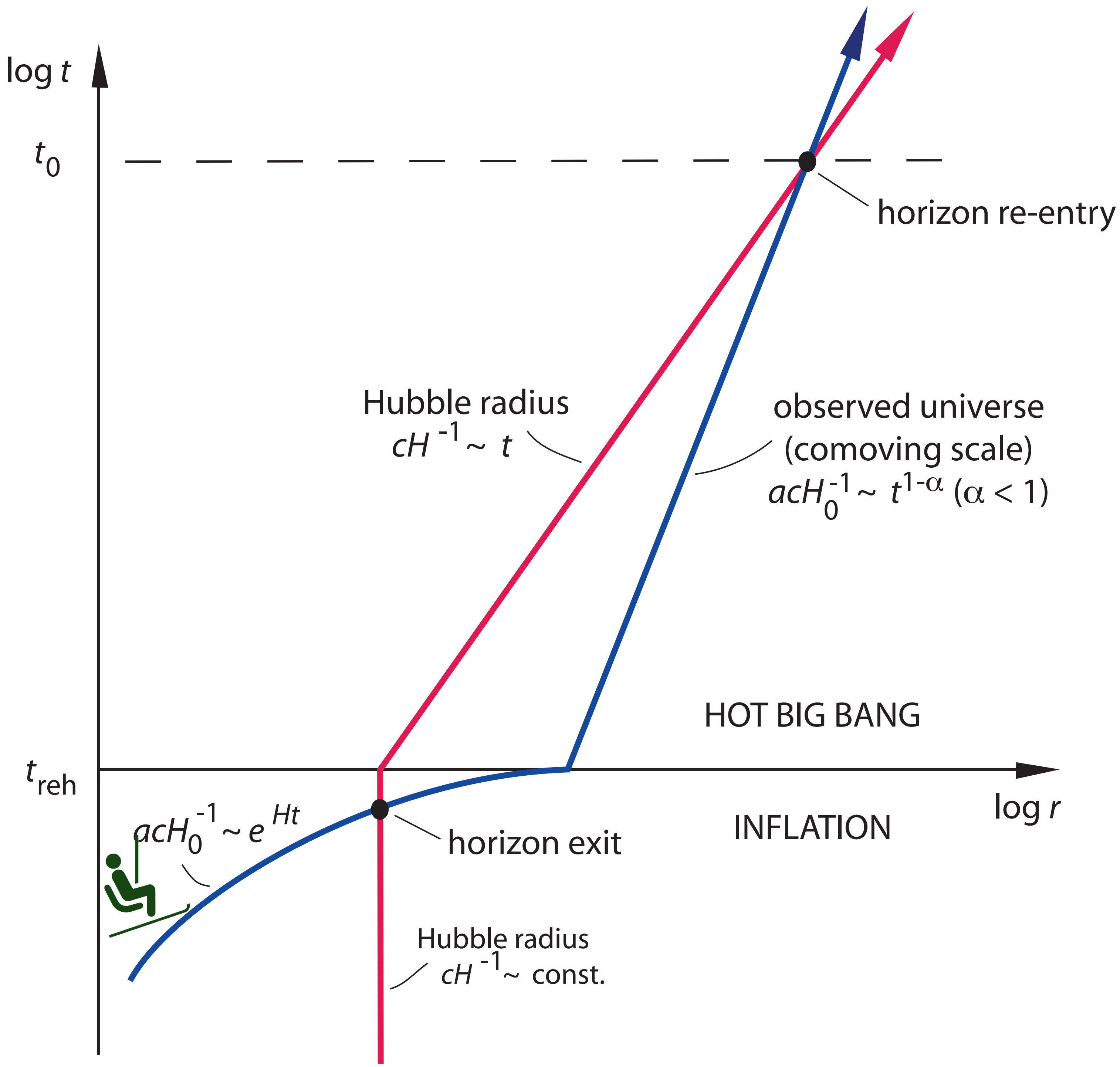
*but matches the Hubble radius earlier during inflation,*

$$\frac{a(t_{\mathrm{I}})}{a(t_{\mathrm{reh}})} \frac{a(t_{\mathrm{reh}})}{a(t_0)} cH_0^{-1} = cH_{\mathrm{I}}^{-1}$$

*that is, at a time $t_I$ defined by*

$$a(t_{\mathrm{I}})H_{\mathrm{I}} = a(t_0)H_0$$

# Perturbation evolution

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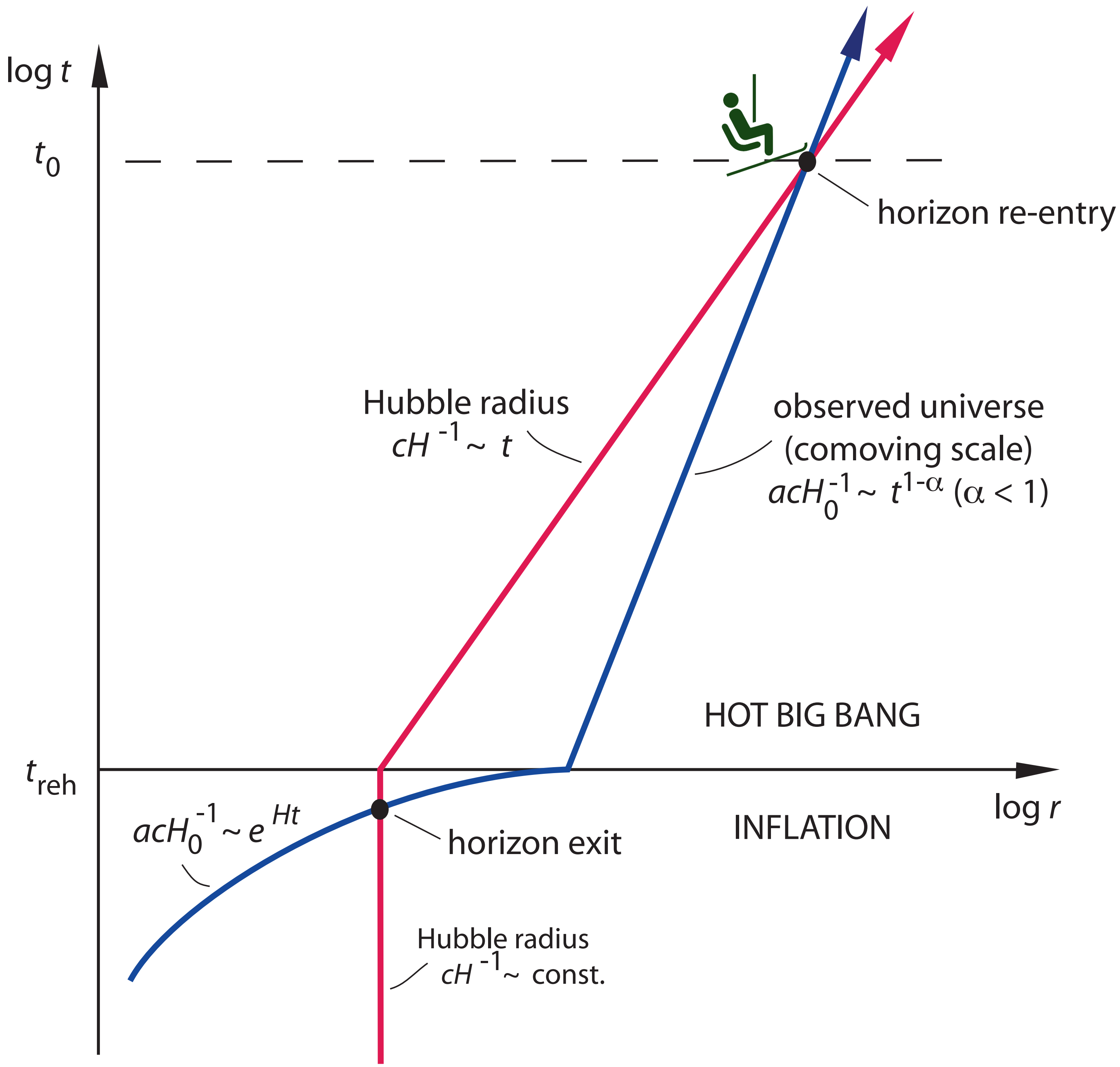


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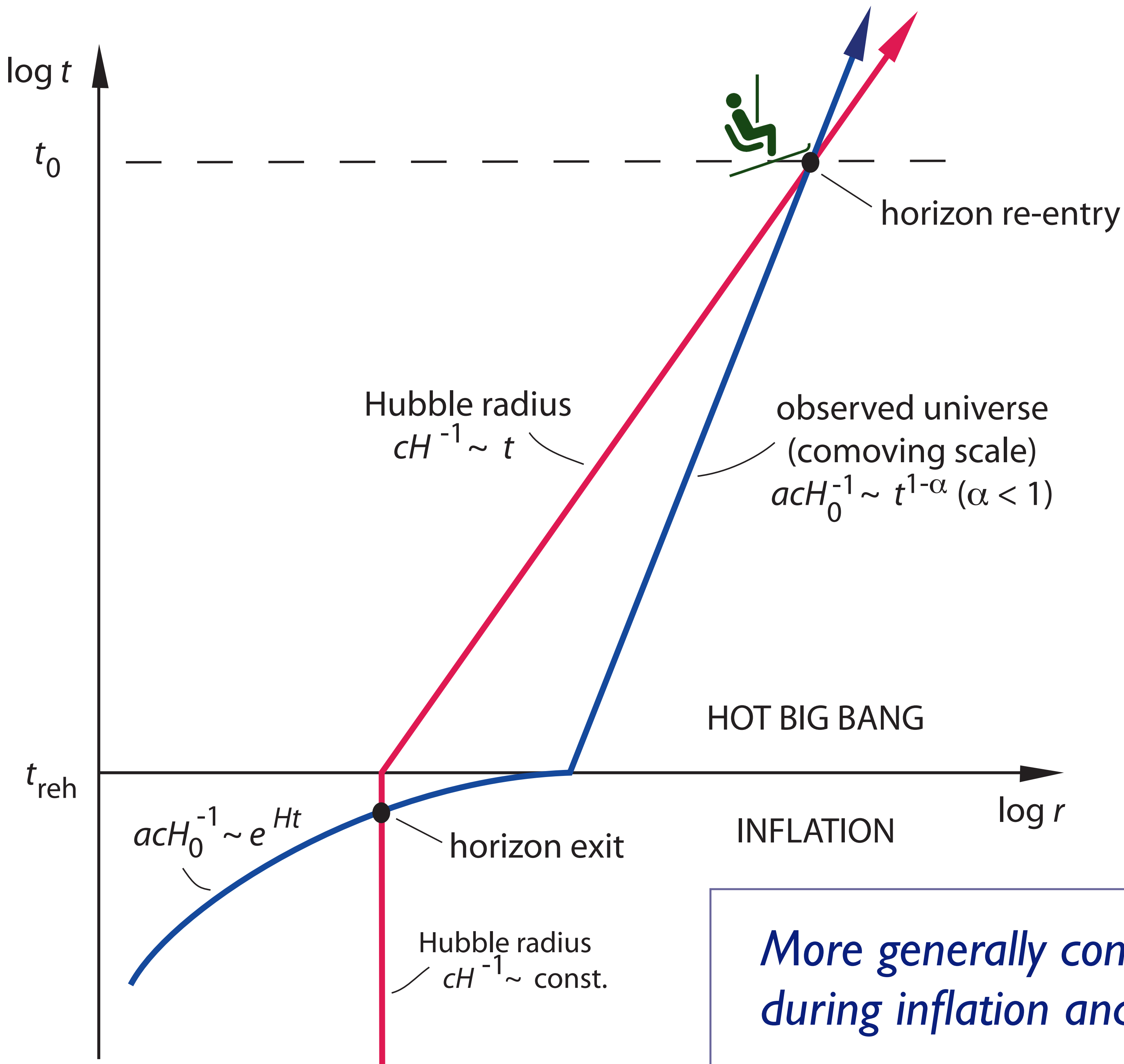


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*More generally comoving lengthscale x exits at time $t_I$ during inflation and then re-enters at time $t_H$:*

$$a(t_{\rm I})H_{\rm I} = a(t_{\rm H})H(t_{\rm H}) = cx^{-1}$$

# Slow-roll inflation

An extended period of inflation ensues if slow roll conditions apply

$$\dot{\phi}^2 \ll V(\phi) \qquad |\ddot{\phi}| \ll | 3H\dot{\phi}|, \; |V'| \qquad \leftrightarrow \qquad \epsilon, \; \eta \ll 1$$

or equivalently if the inflaton potential is flat and featureless (approx slow roll params)

$$\epsilon_V = \tfrac{1}{2} M_{pl}^2 \left( \frac{V'}{V} \right)^2 , \qquad \eta_V = M_{pl}^2 \frac{V''}{V} \qquad \rightarrow \qquad \epsilon_V, \; \eta_V \ll 1$$

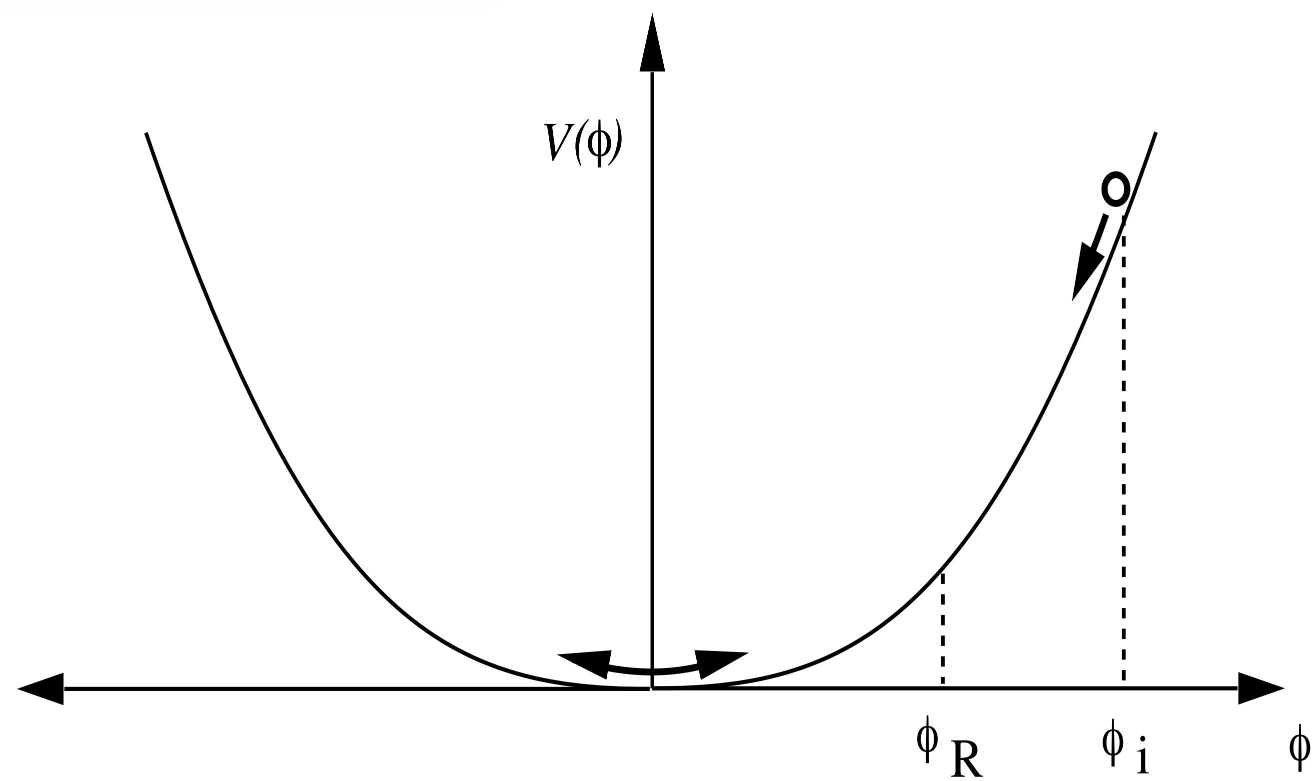
The slow roll equations then admit quasi-exponential expansion

$$H^2 = \frac{1}{3M_{pl}^2} V(\phi) \, , \qquad 3H\dot{\phi} = -\frac{dV}{d\phi}$$

Inflation ends and reheating occurs:

$$\tfrac{1}{2}\dot{\phi}_R^2 \approx V(\phi_R) \qquad \text{or} \qquad \epsilon(\phi_R) = 1$$

The total number of e-foldings is



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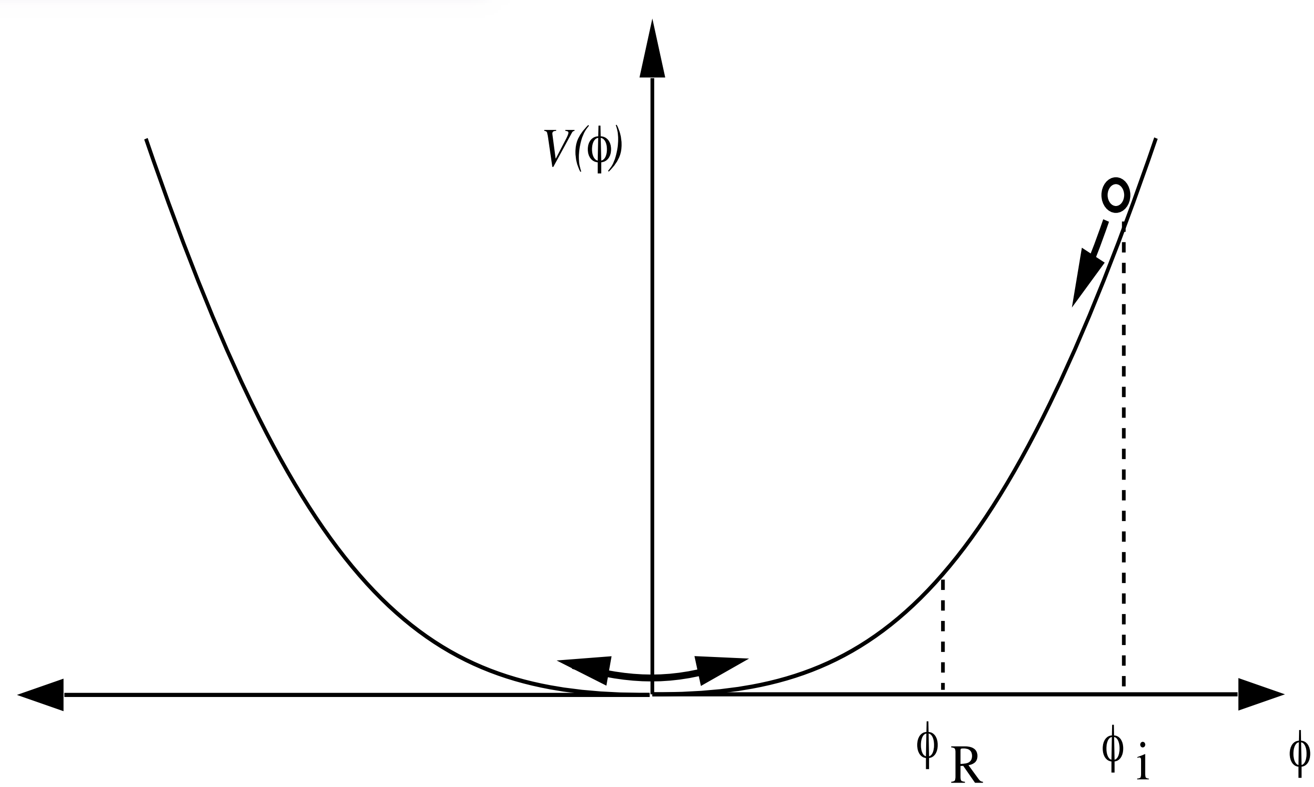
$$\begin{aligned}\mathcal{N} &= \ln[a(t_i)/a(t_R)] = \int_{t_i}^{t_R} H\,dt = \int_{\phi_i}^{\phi_R}\frac{H}{\dot{\phi}}d\phi \\ &\approx \frac{1}{M_{pl}^2}\int_{\phi_i}^{\phi_R}\frac{V}{V'}d\phi\end{aligned}$$



# Slow roll spectral index

Amplitude of scalar perturbations for slow roll

$$\Delta_\zeta^2(k) = \frac{1}{24\pi^2}\frac{H^2}{M_{pl}^2}\frac{1}{\epsilon}\bigg|_{k=aH} \quad\longrightarrow\quad \Delta_\zeta^2(k) \approx \frac{1}{24\pi^2}\frac{V}{M_{pl}^2}\frac{1}{\epsilon_V}\bigg|_{k=aH}$$

Spectral index in the slow roll approximation

$$n_s - 1 \equiv \frac{d\ln\Delta_\zeta^2}{d\ln k} \quad\longrightarrow\quad n_s - 1 \approx 2\eta - 4\epsilon \approx 2\eta_V - 6\epsilon_V$$

so for simple free field potential

$$\epsilon_V \approx \eta_V \approx \frac{1}{\mathcal{N}} \text{ for } V(\phi) = \tfrac{1}{2}m^2\phi^2 \quad\longrightarrow\quad n_s \approx 1 - \frac{2}{\mathcal{N}} \approx 0.96$$

and more generally $\quad V(\phi) = \Lambda^4\left(\frac{\phi}{\mu}\right)^p \quad\longrightarrow\quad n_s \approx 0.98 - \frac{p}{100}$

*Aside:* Hamilton-Jacobi slow-roll params $\quad \epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{2M_{pl}^2}\frac{\dot{\phi}^2}{H^2} \qquad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}$

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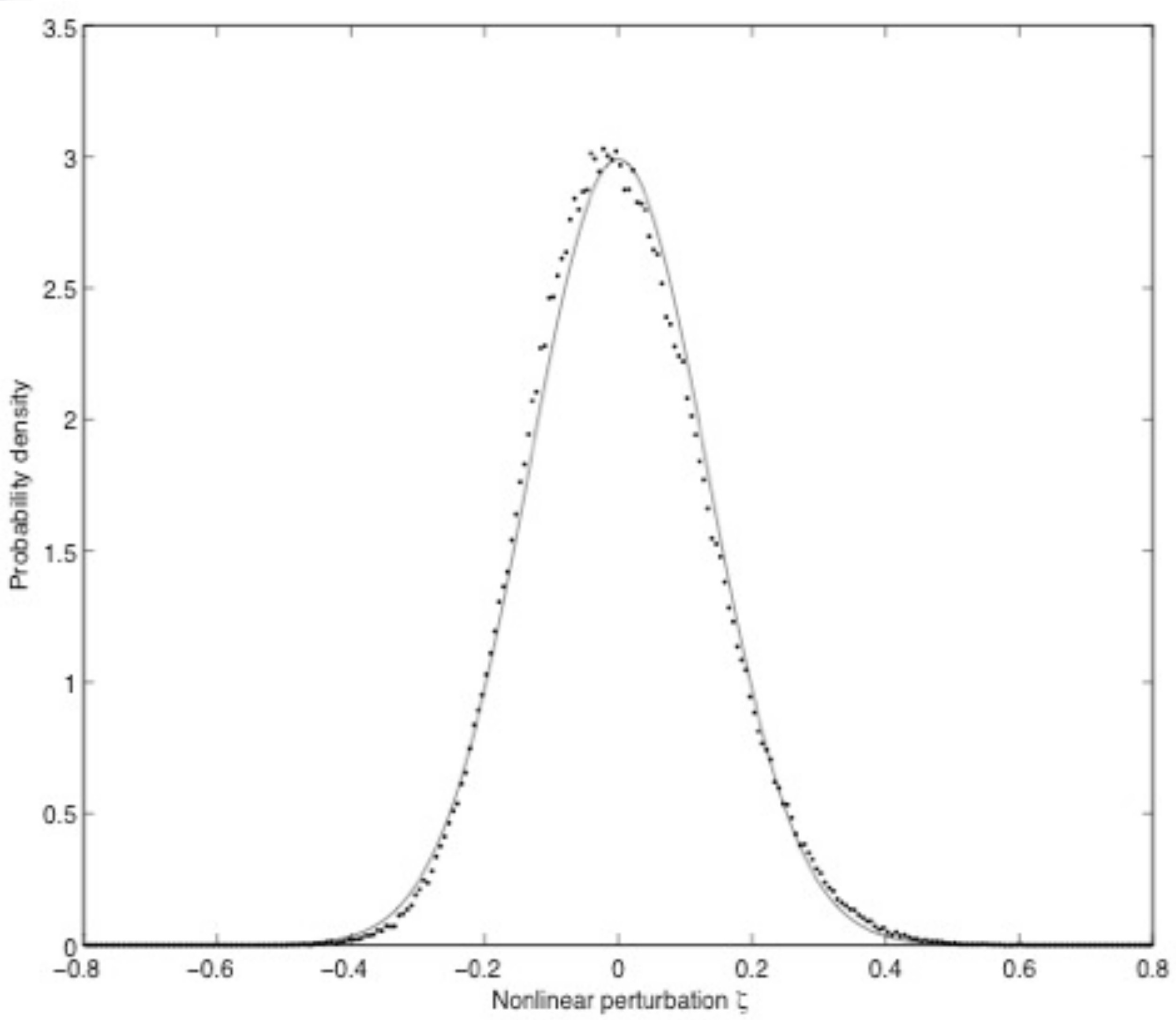
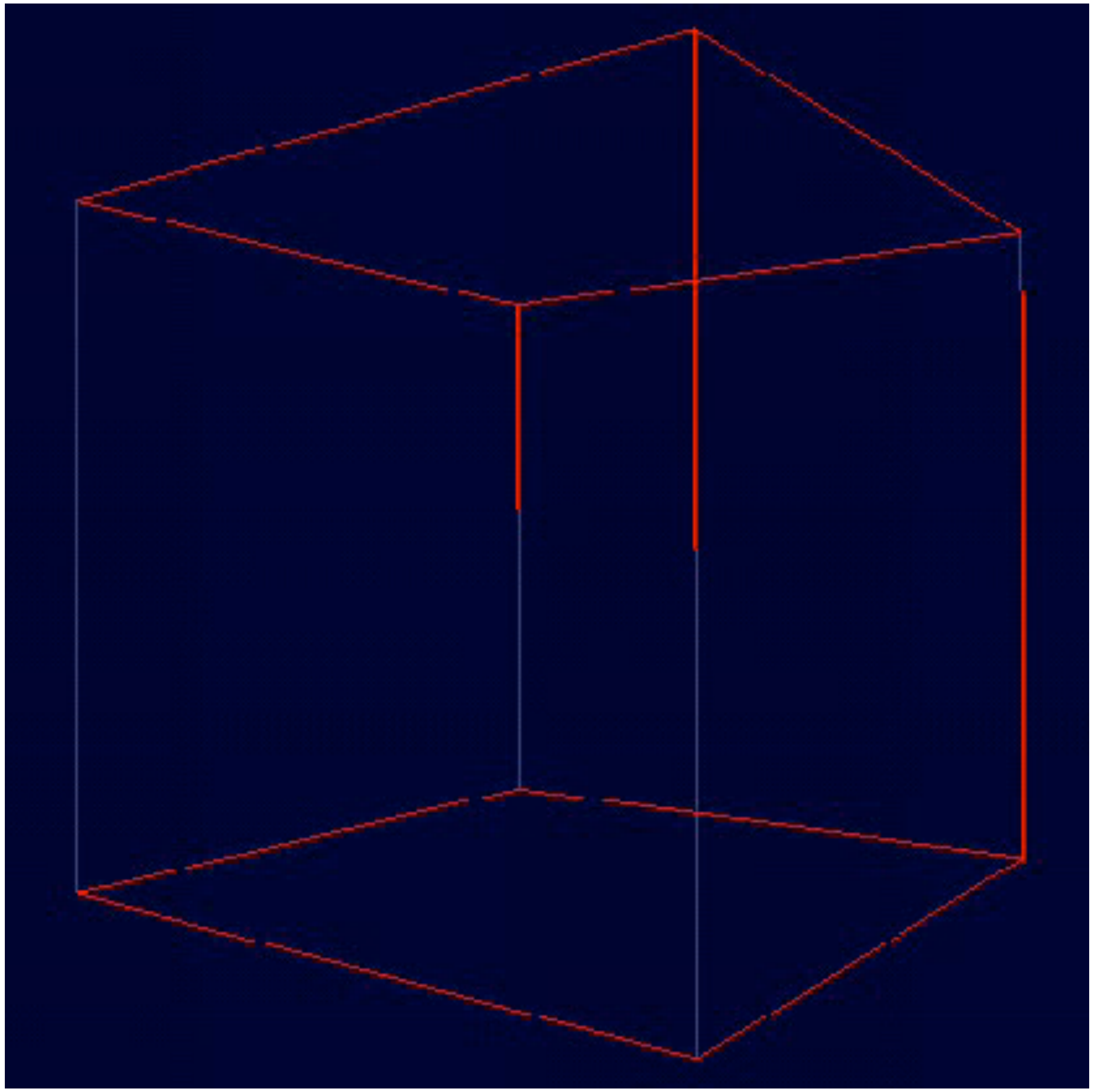
# Probability density function

Numerical results ...



Small NG signal for single field inflation

Significant $f_{NL}$ for multifield inflation (see later)

Ongoing stochastic simulations/theory *(Funakoshi & EPS, 2011)*

# Randomness primer

*Tossing a coin:*

Heads

Tails

# Randomness primer

## Pascal's triangle

*Tossing a coin:*

Heads

Tails

| | |
|---|---|
| N = 0 | 1 |
| N = 1 | 1 1 |
| N = 2 | 1 2 1 |
| N = 3 | 1 3 3 1 |
| N = 4 | 1 4 6 4 1 |
| N = 5 | 1 5 10 10 5 1 |
| N = 6 | 1 6 15 20 15 6 1 |

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***Pascal's triangle***

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| N = 0 | | | | | | | 1 | | | | | | | |
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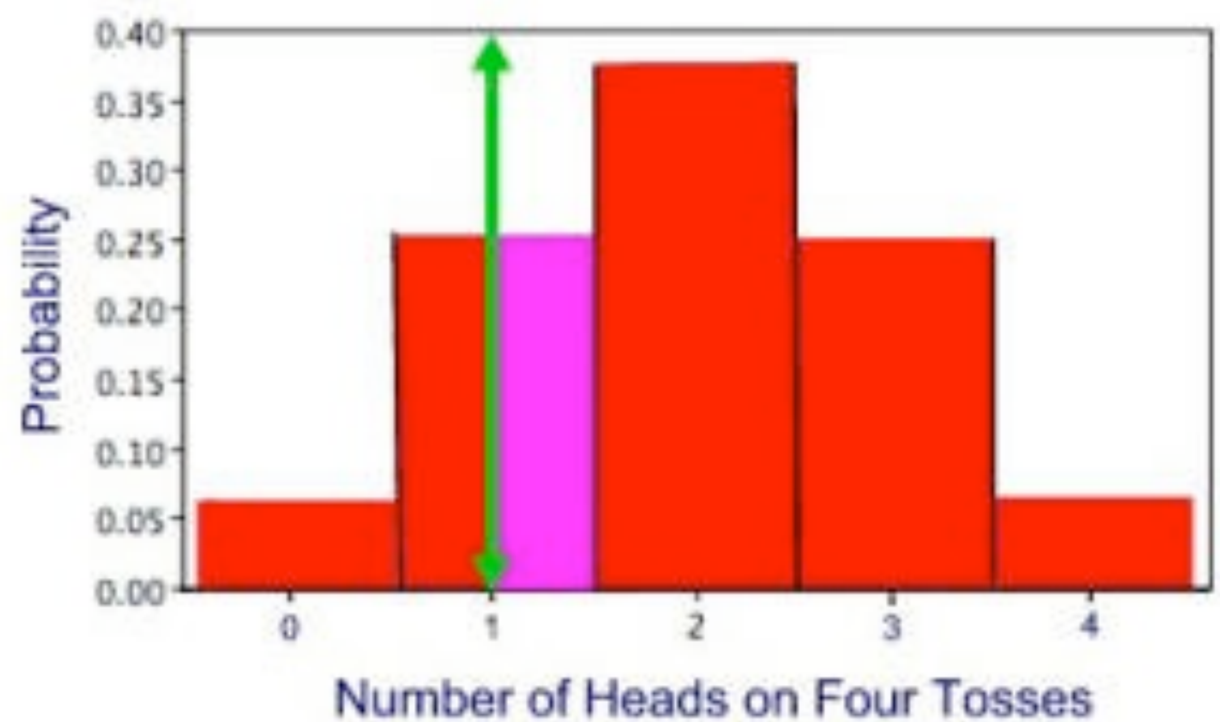
| N = 2 | | | | | 1 | | 2 | | 1 | | | | | |
| N = 3 | | | | 1 | | 3 | | 3 | | 1 | | | | |
| N = 4 | | | 1 | | 4 | | 6 | | 4 | | 1 | | | |
| N = 5 | | 1 | | 5 | | 10 | | 10 | | 5 | | 1 | | |
| N = 6 | 1 | | 6 | | 15 | | 20 | | 15 | | 6 | | 1 | |

*Plot of binomial distn for different no. of tosses:*



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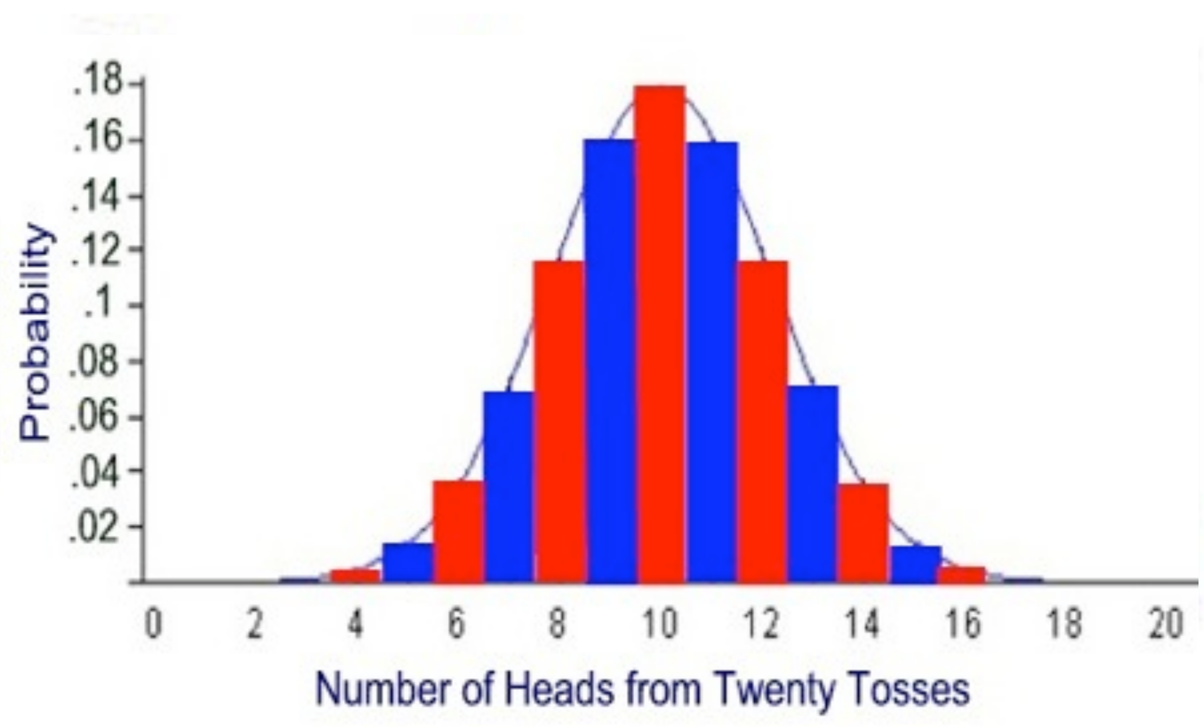

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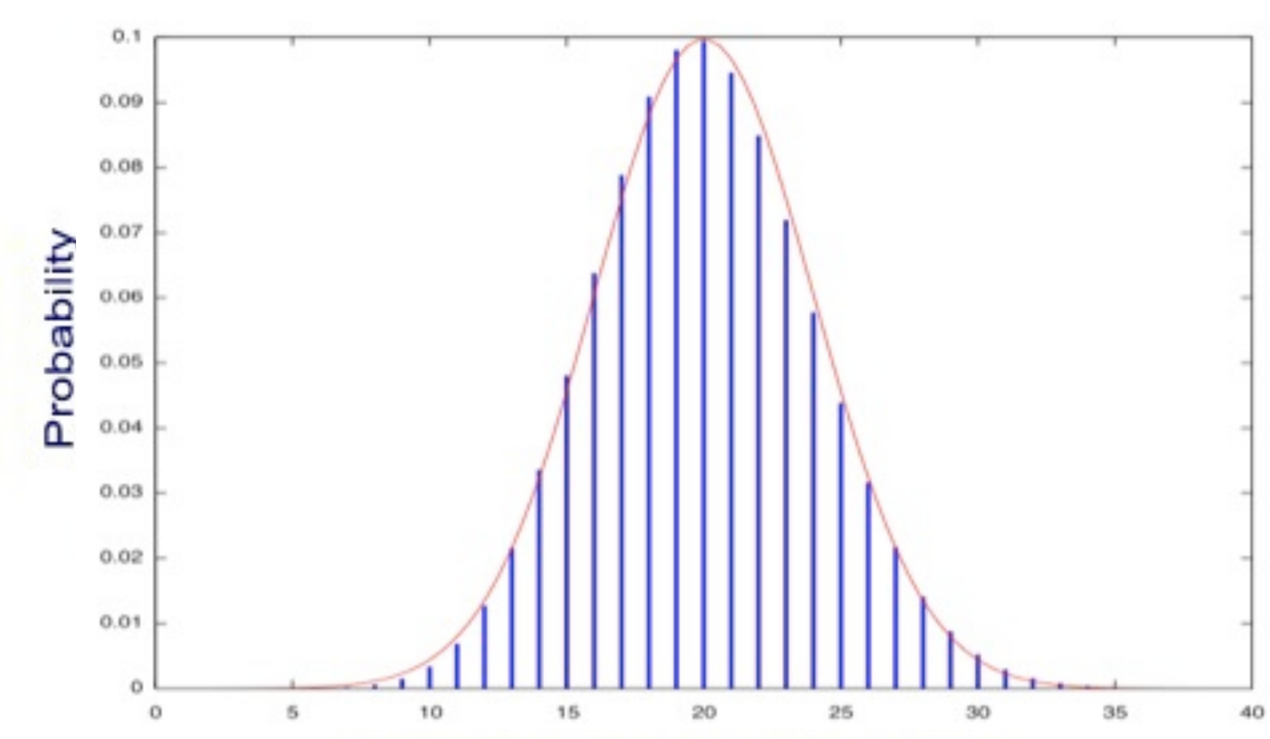
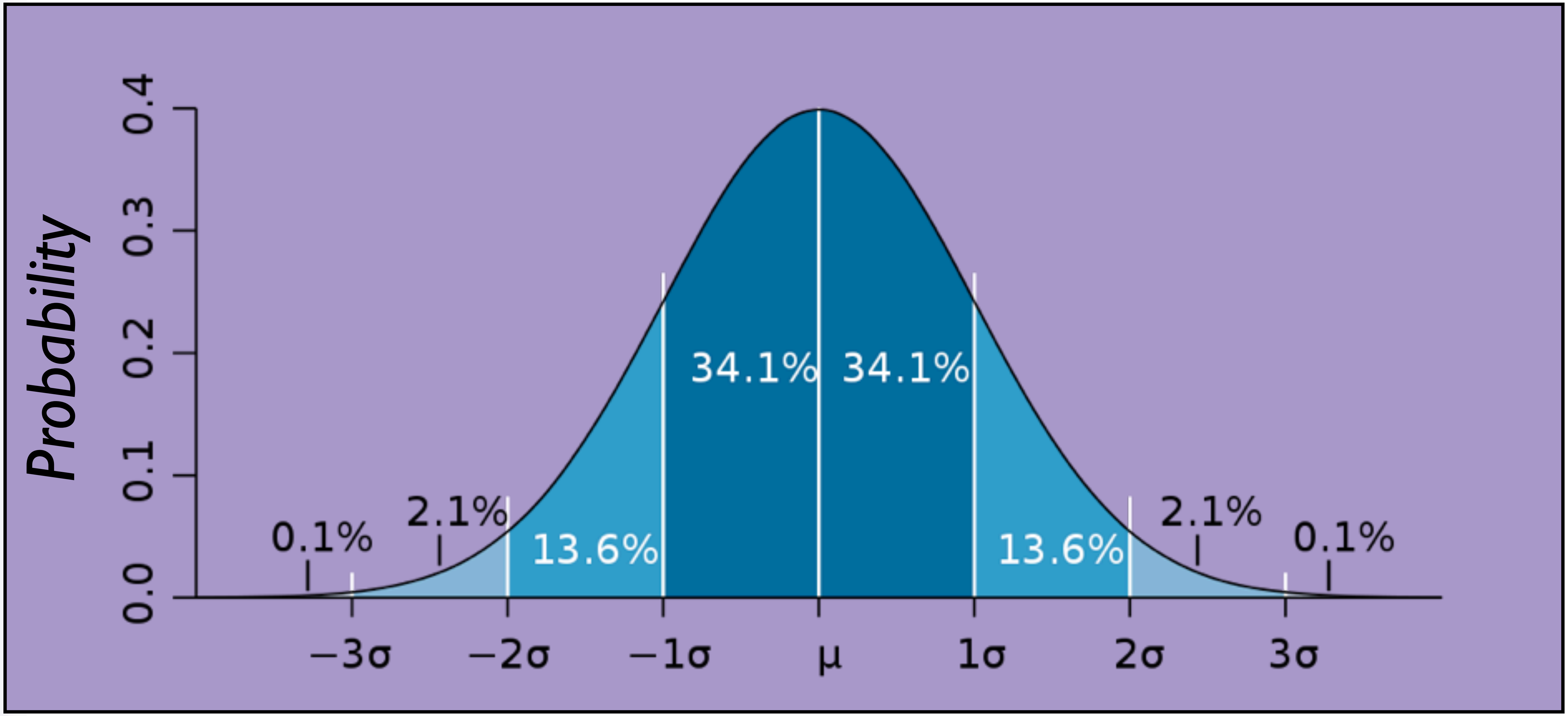



*Approaches "bell curve" or normal distribution (when rescaled)*

# Normal or Gaussian distribution




Determined only by mean μ and standard deviation

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

Central limit theorem: any independent random process

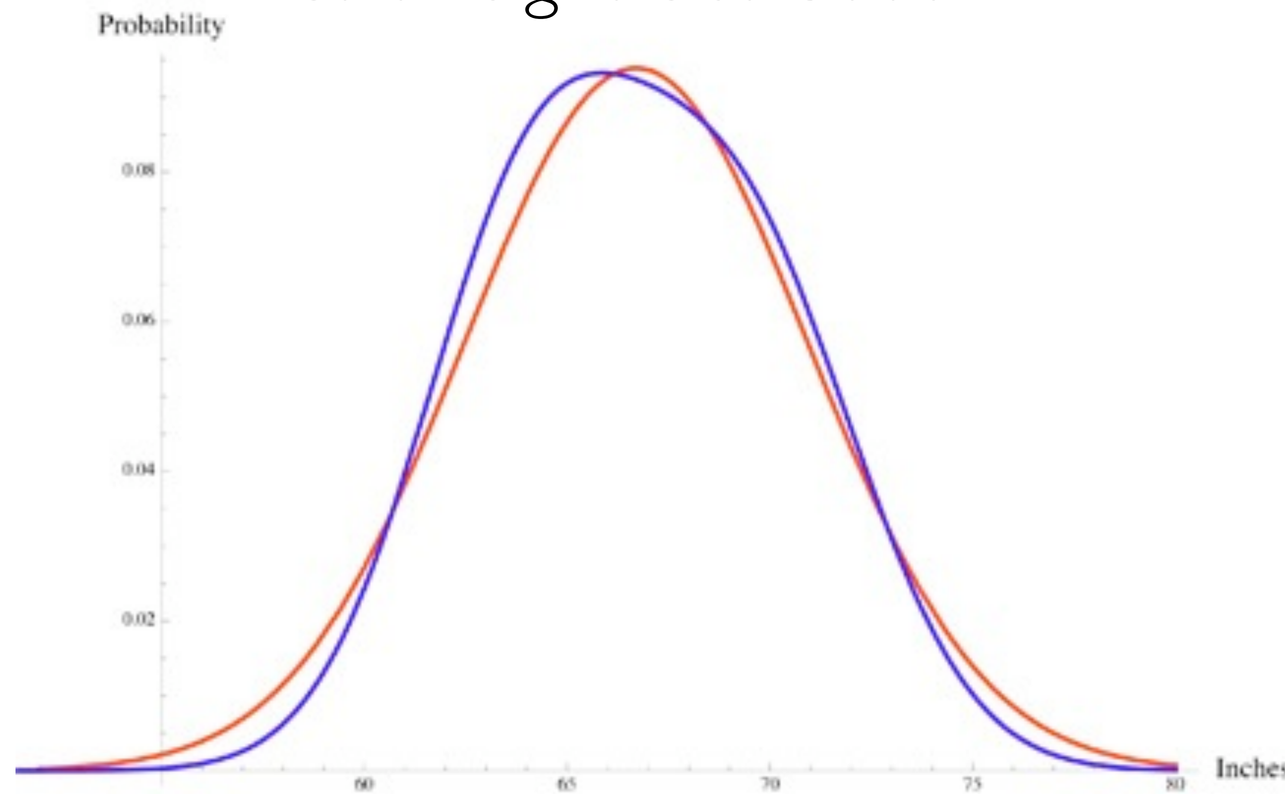
*Gauss 1809*

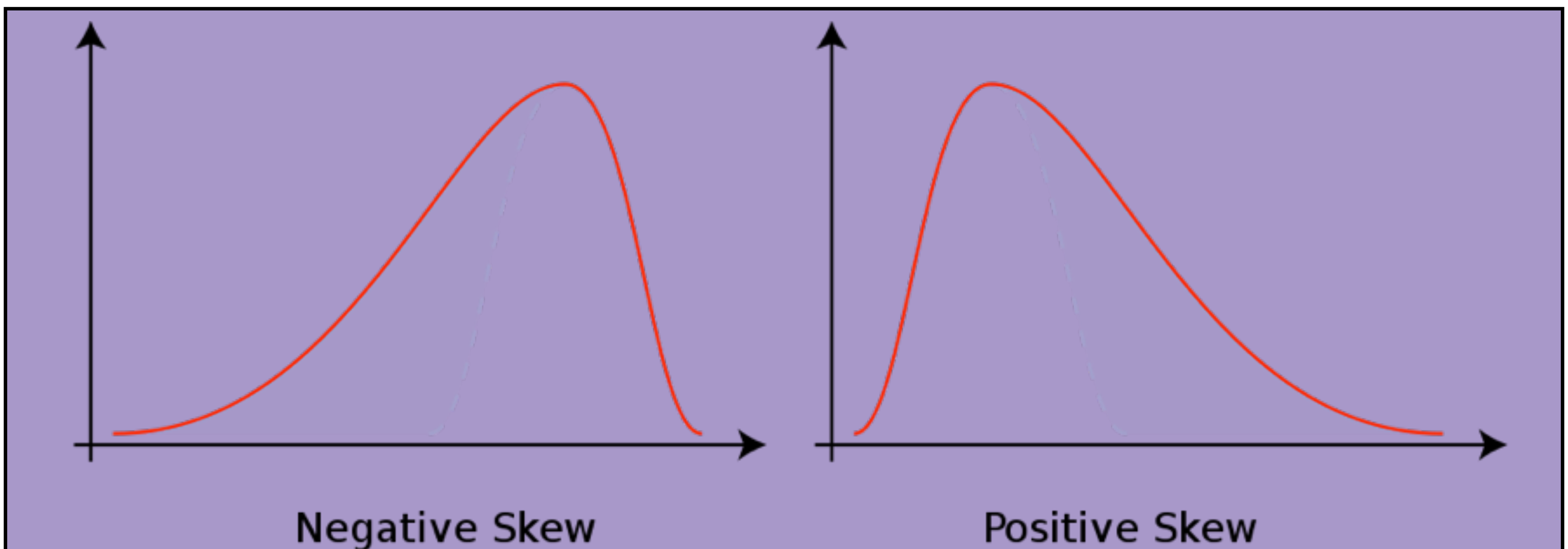
# Non-randomness: Skewness

The skewness measures cubic deviation from normal distribution:

$$\gamma_1 \equiv \left\langle \left( \frac{\Delta T}{T} \right)^3 \right\rangle$$

*Adult height distribution*



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For the CMB this is the sum over a 3D object - the bispectrum $b_{l_1 l_2 l_3}$

$$\gamma_1 = \frac{1}{4\pi} \sum_{l_i} h^2_{l_1 l_2 l_3} b_{l_1 l_2 l_3}$$

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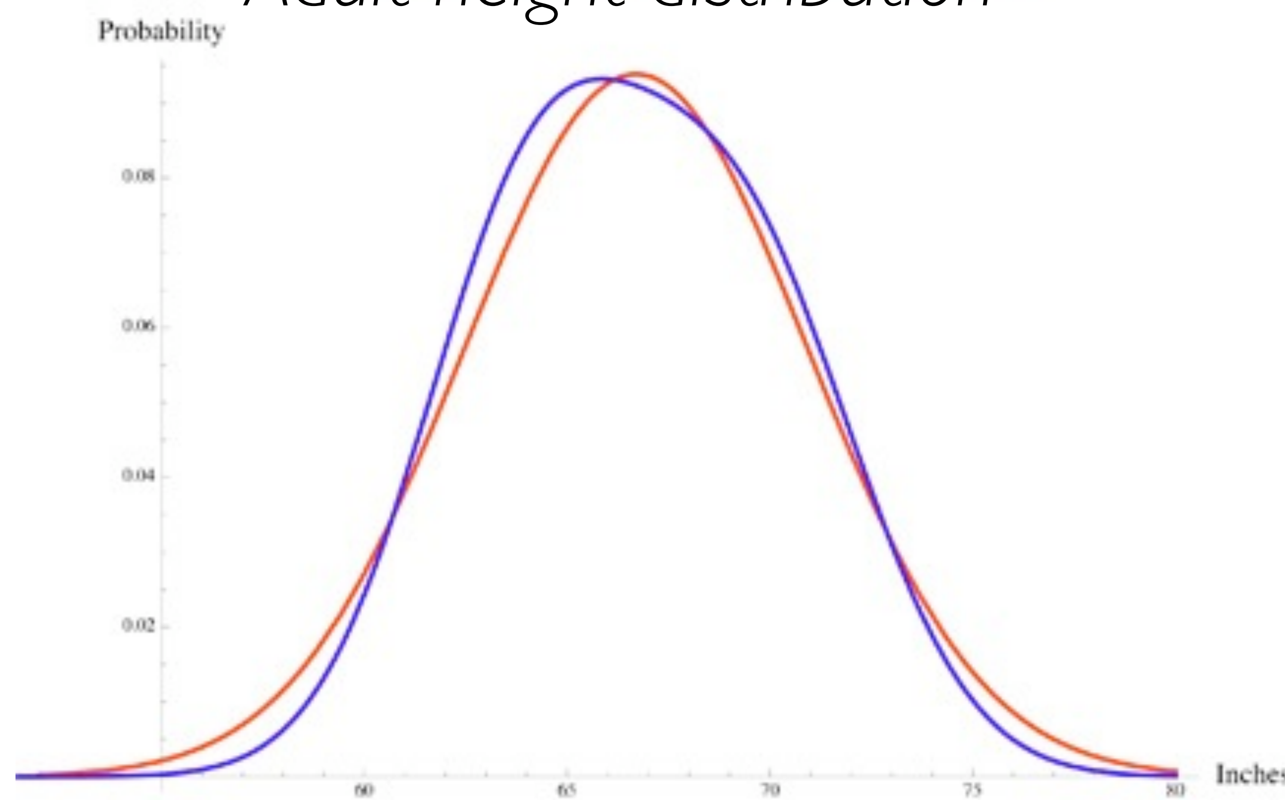
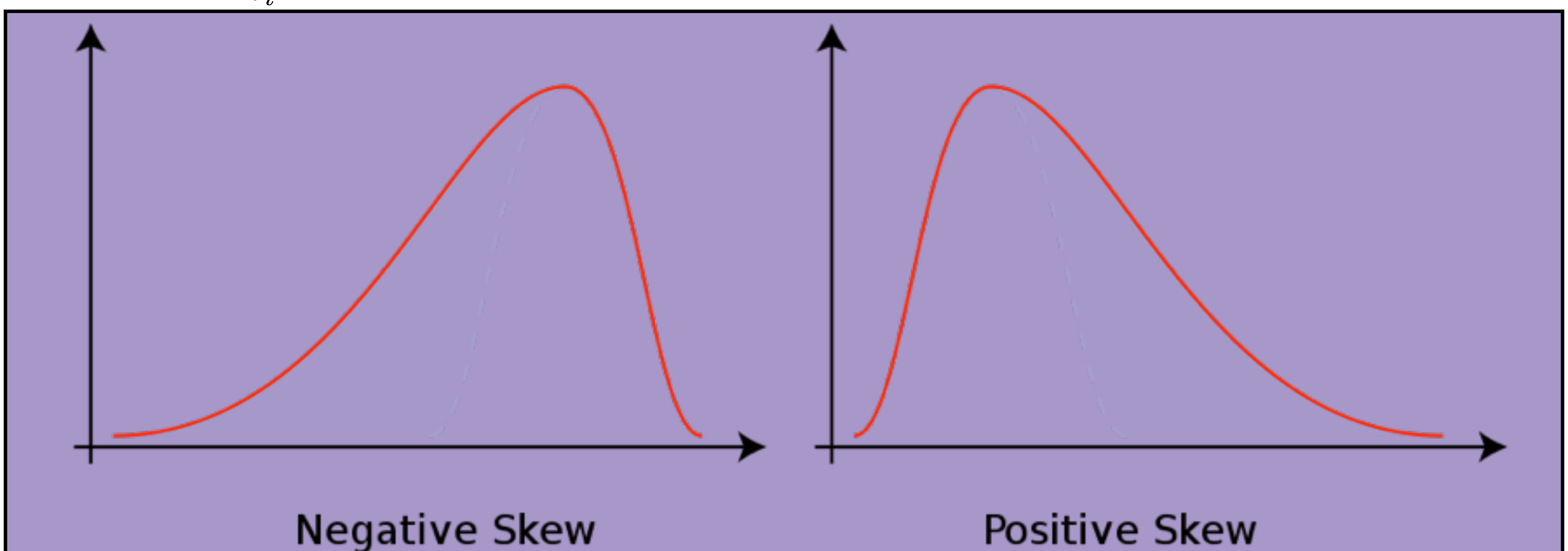
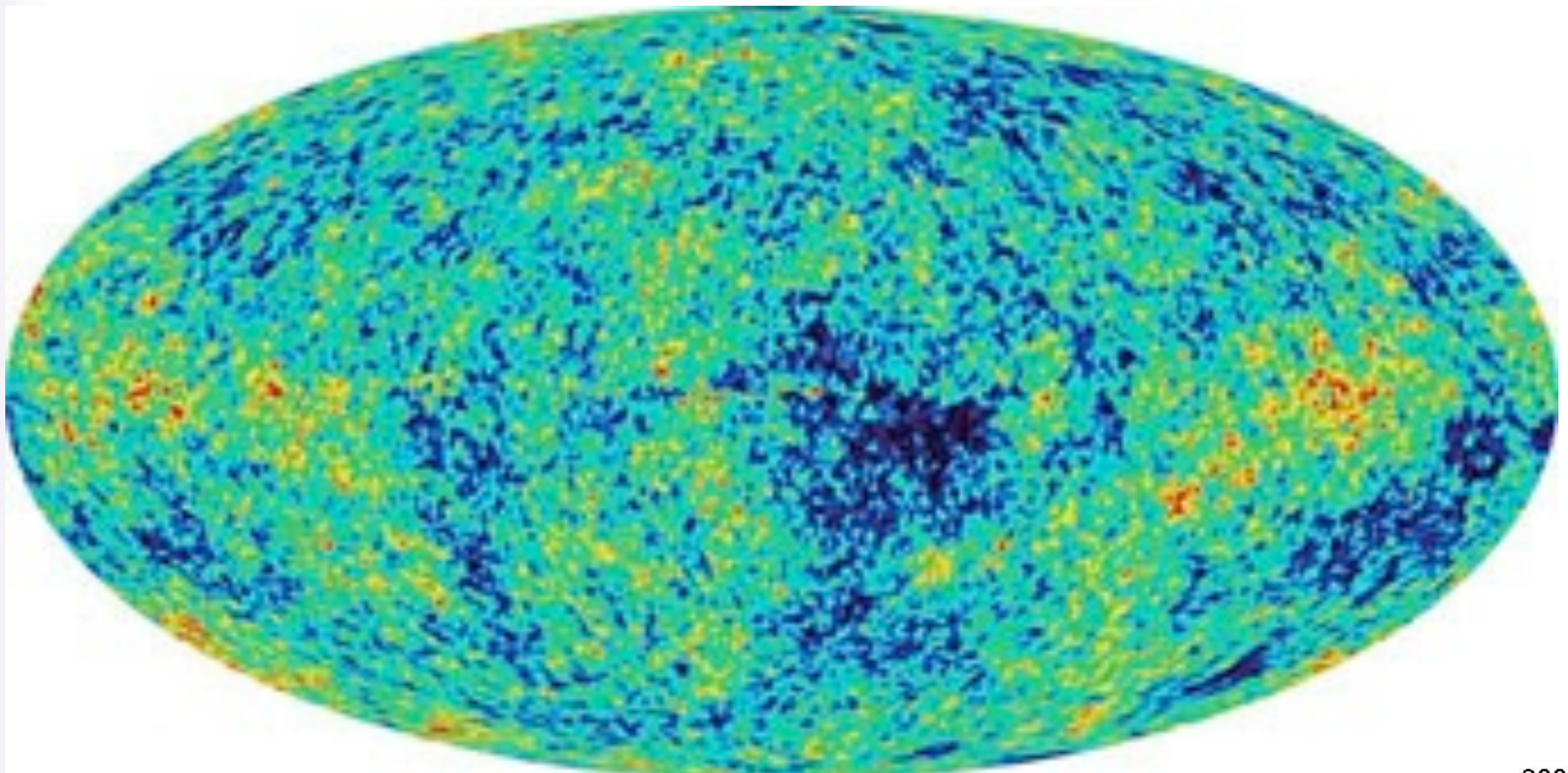


# Non-randomness: the Bispectrum

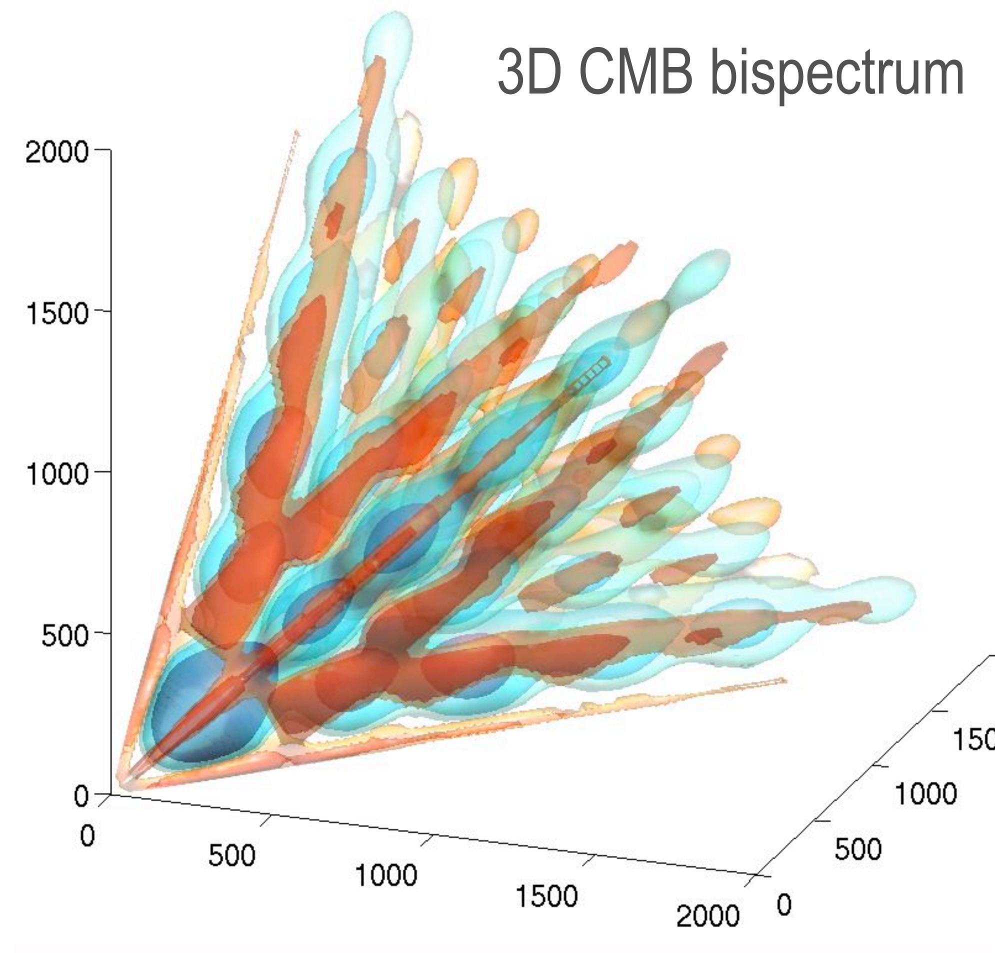


We search for (isotropic) non-random correlations at three points,
i.e. summing over similar triangles with side lengths representing 3 coords.

Any non-random bispectrum

$$B_{l_1 l_2 l_3} \sim \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle$$

is then a 3D object constructed from the $a_{lm}$'s with multipoles $l_i$ making triangles (the new coords)
$B_{l_1 l_2 l_3}$ exists on a tetrahedron



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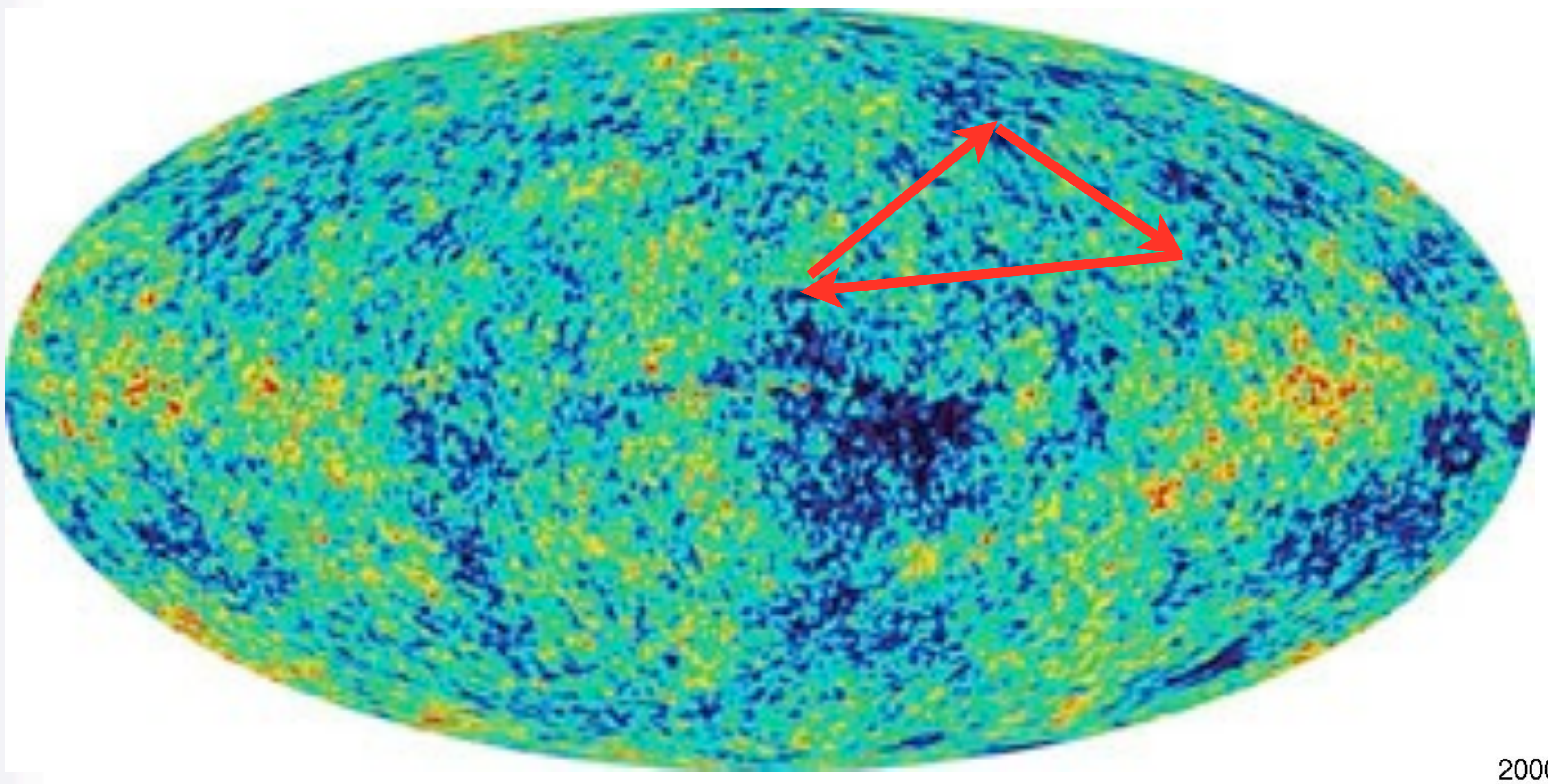

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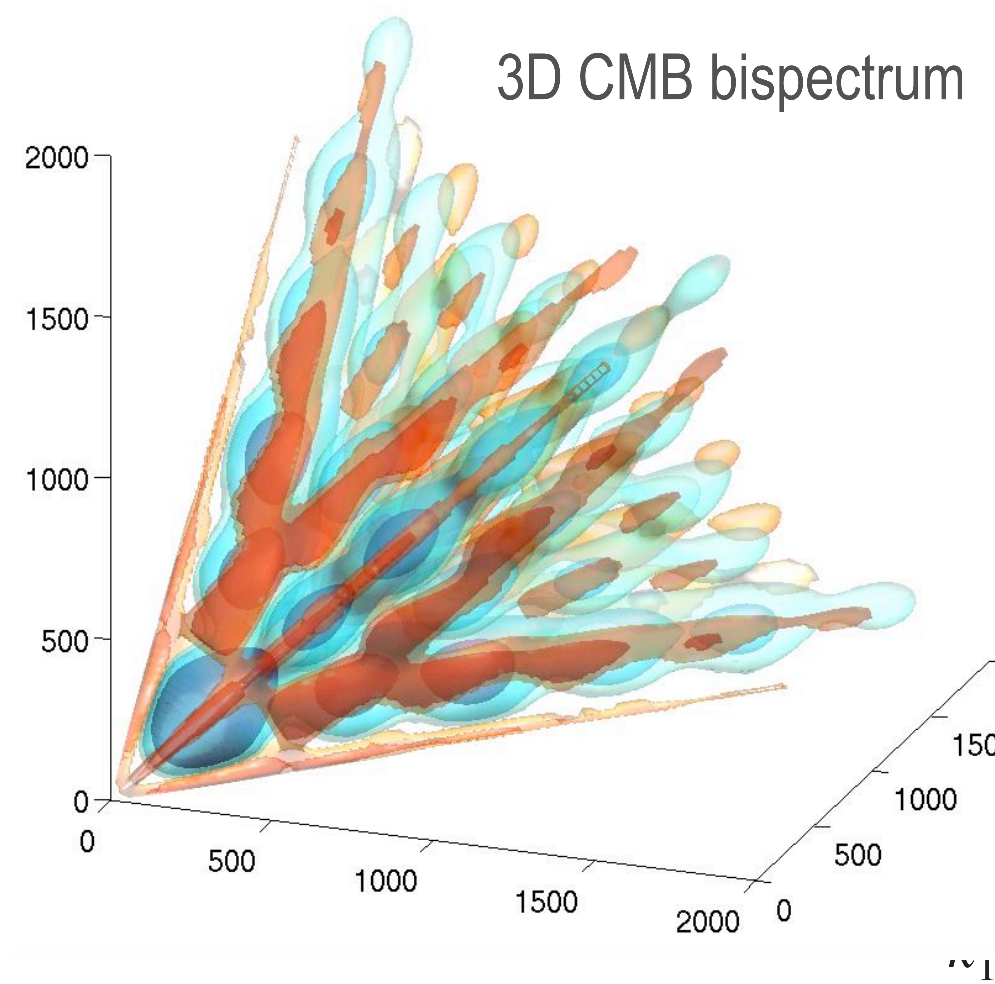


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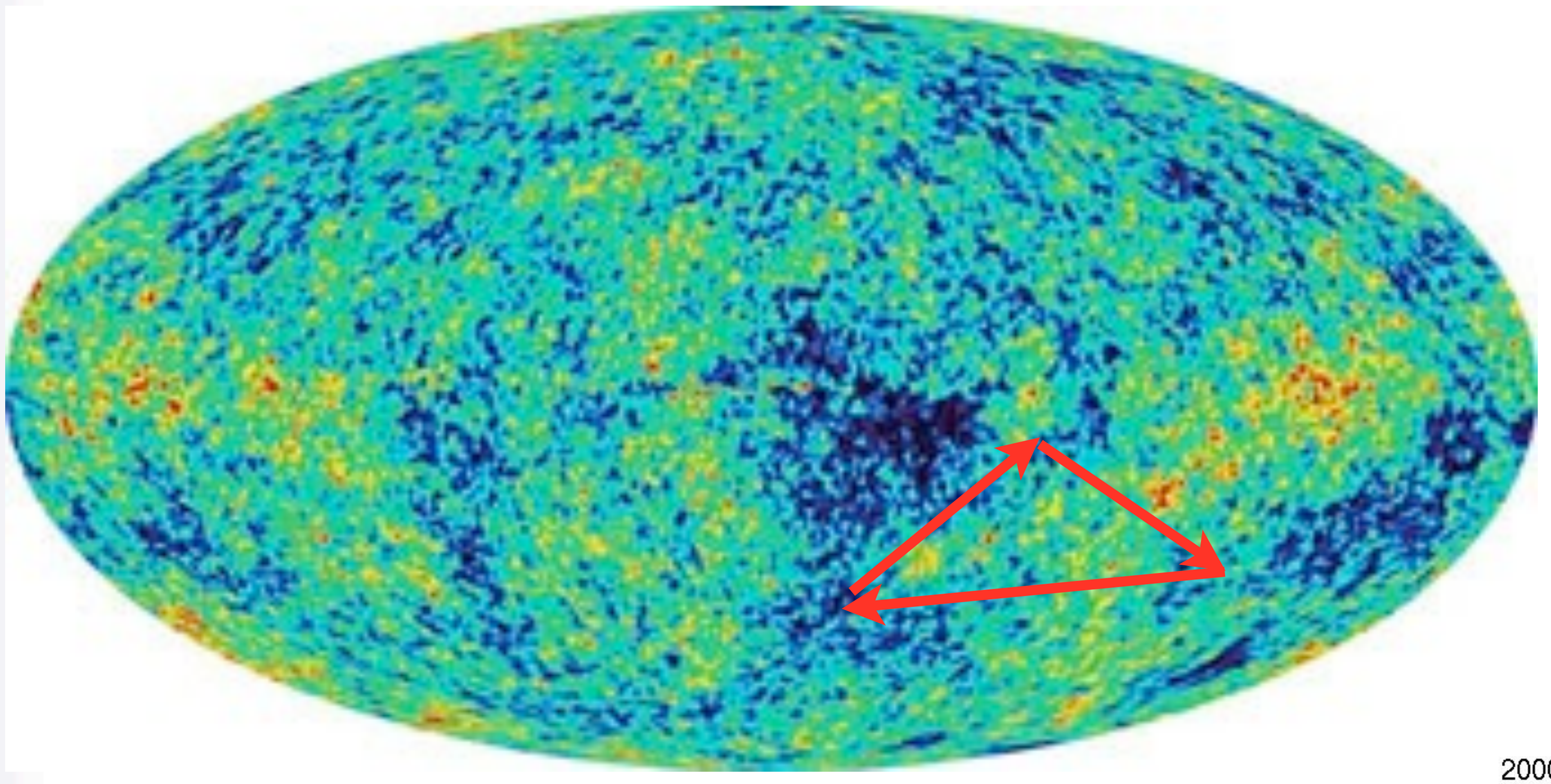

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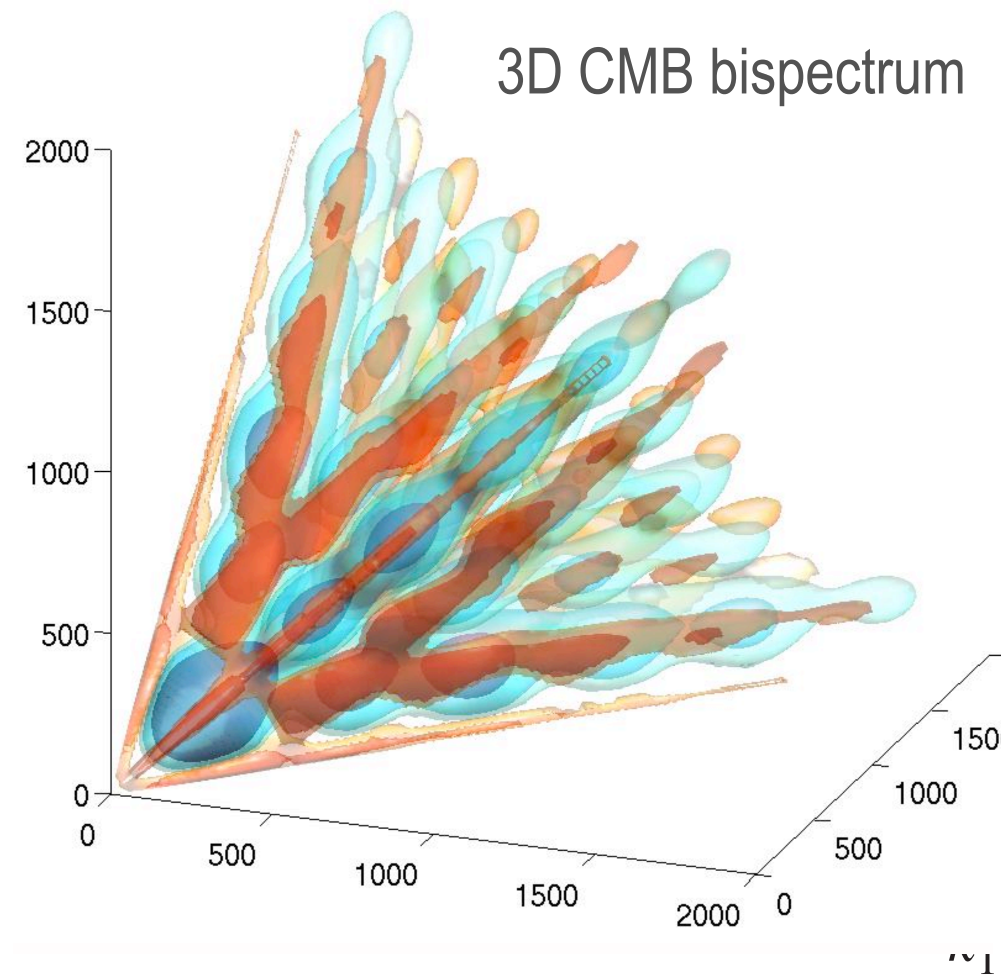


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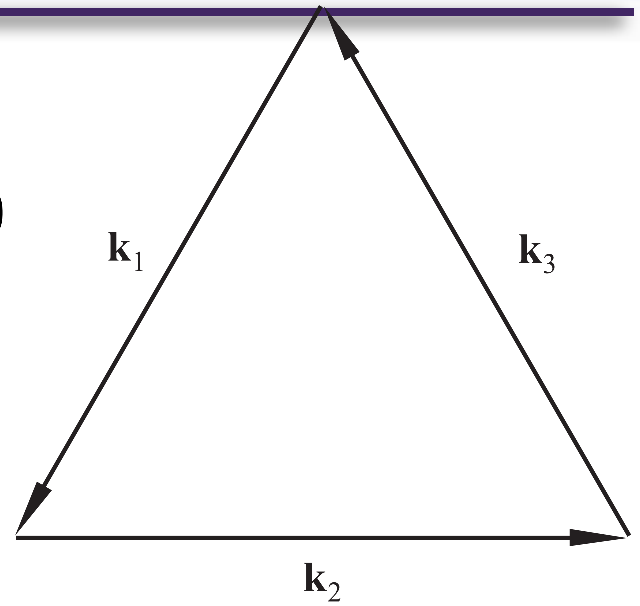
# A Non-Gaussian Universe?

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$$\hat{\Phi}_{\rm lin} = \Phi_{\rm lin}\,\hat{a}^\dagger + \Phi^*_{\rm lin}\,\hat{a} \quad \Rightarrow \quad \text{Gaussian with } \langle \hat{\Phi}\hat{\Phi}\hat{\Phi}\rangle = 0$$

## *Non-Gaussianity (local model with parameter $f_{NL}$)*

$$\hat{\Phi} = \hat{\Phi}_{\rm lin} + \hat{\Phi}_{\rm NL} \text{ where } \hat{\Phi}_{\rm NL} = f_{\rm NL}\,\hat{\Phi}^2_{\rm lin}$$

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Perturbative higher order correlators $f_{NL} \sim O(1)$

$$\text{Leading order term } \langle \Phi^{(1)}\Phi^{(1)}\Phi^{(2)}\rangle \;\approx\; \langle \Phi^{(1)}\Phi^{(1)}\,(\Phi^{(1)} * \Phi^{(1)})\rangle$$

Second-order gravitational perturbations also ~ $O(1)$

## *Non-perturbative models - e.g. cosmic strings*

Poor suppression of higher orders with signals $f_{NL} \sim P(k)^{-1/2} \sim 10^5 >> 1$

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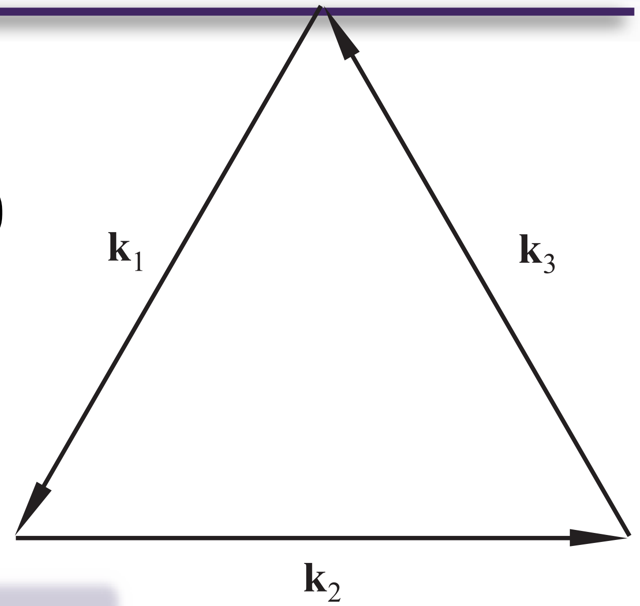


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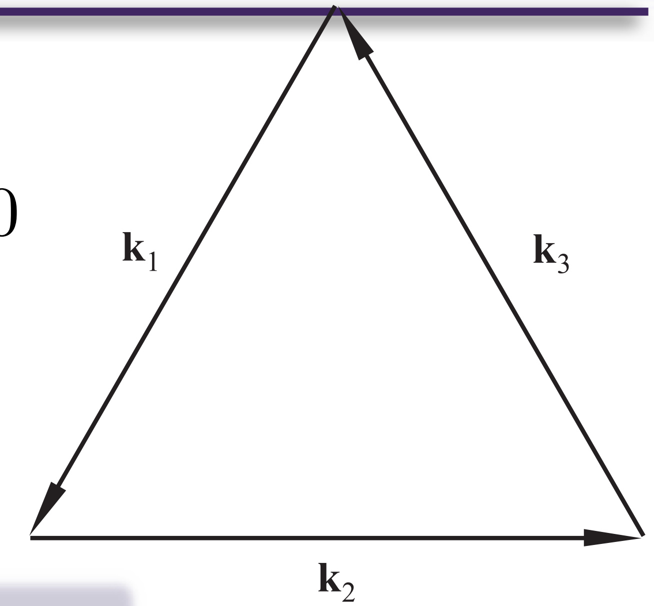


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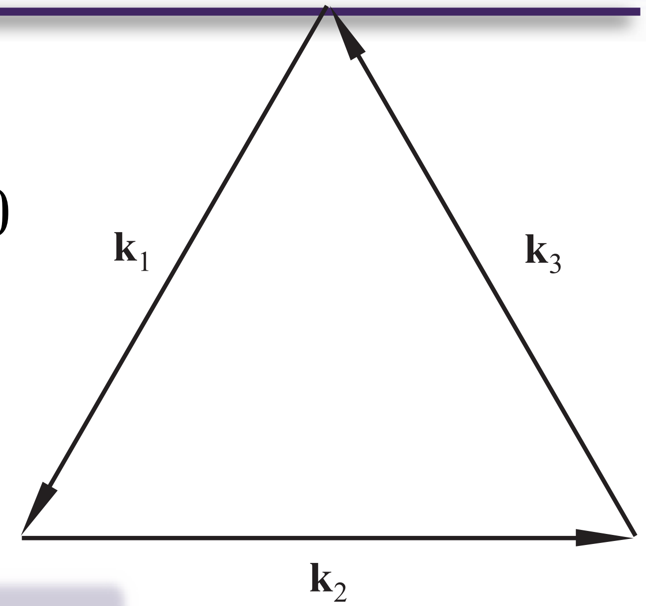


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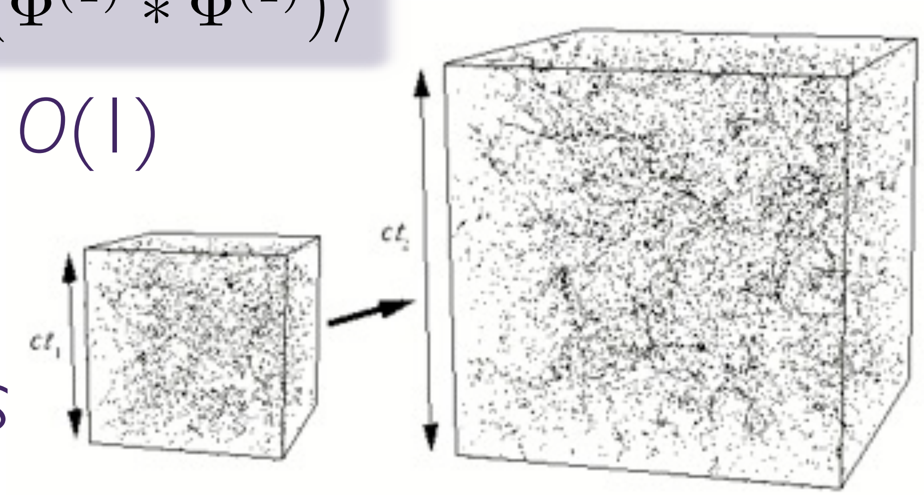

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# Exercise I: Local non-Gaussianity

Assume that higher order corrections to the linear pert. solution are

$$\zeta(\mathbf{x}) = \zeta^{(1)}(\mathbf{x}) + \zeta^{(2)}(\mathbf{x}) + ... = \zeta^{(1)} + f_{\rm NL}\left((\zeta^{(1)})^2 - \langle(\zeta^{(1)})^2\rangle\right) + ...$$

where the linear solution obeys $\langle\zeta^{(1)}(\mathbf{k})\,\zeta^{(1)}(\mathbf{k}')\rangle = (2\pi)^3 P(k)\,\delta(\mathbf{k}+\mathbf{k}')$

Using the convolution theorem, show that second-order solution can be expressed in the form:

$$\zeta^{(2)}(\mathbf{k}) = \int \frac{d^3\mathbf{k}'}{(2\pi)^3}\int \frac{d^3\mathbf{k}''}{(2\pi)^3}\left[\zeta^{(1)}(\mathbf{k}')\,\zeta^{(1)}(\mathbf{k}'') - \langle\zeta^{(1)}(\mathbf{k}')\,\zeta^{(1)}(\mathbf{k}'')\rangle\right]\,\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')$$

Hence, find the bispectrum for the local model defined by

$$\langle\zeta(\mathbf{k}_1)\,\zeta(\mathbf{k}_2)\,\zeta(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,B(k_1,\,k_2,\,k_3)$$

with

$$B(k_1,k_2,k_3) = 2f_{\rm NL}\left[P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)\right]\,.$$

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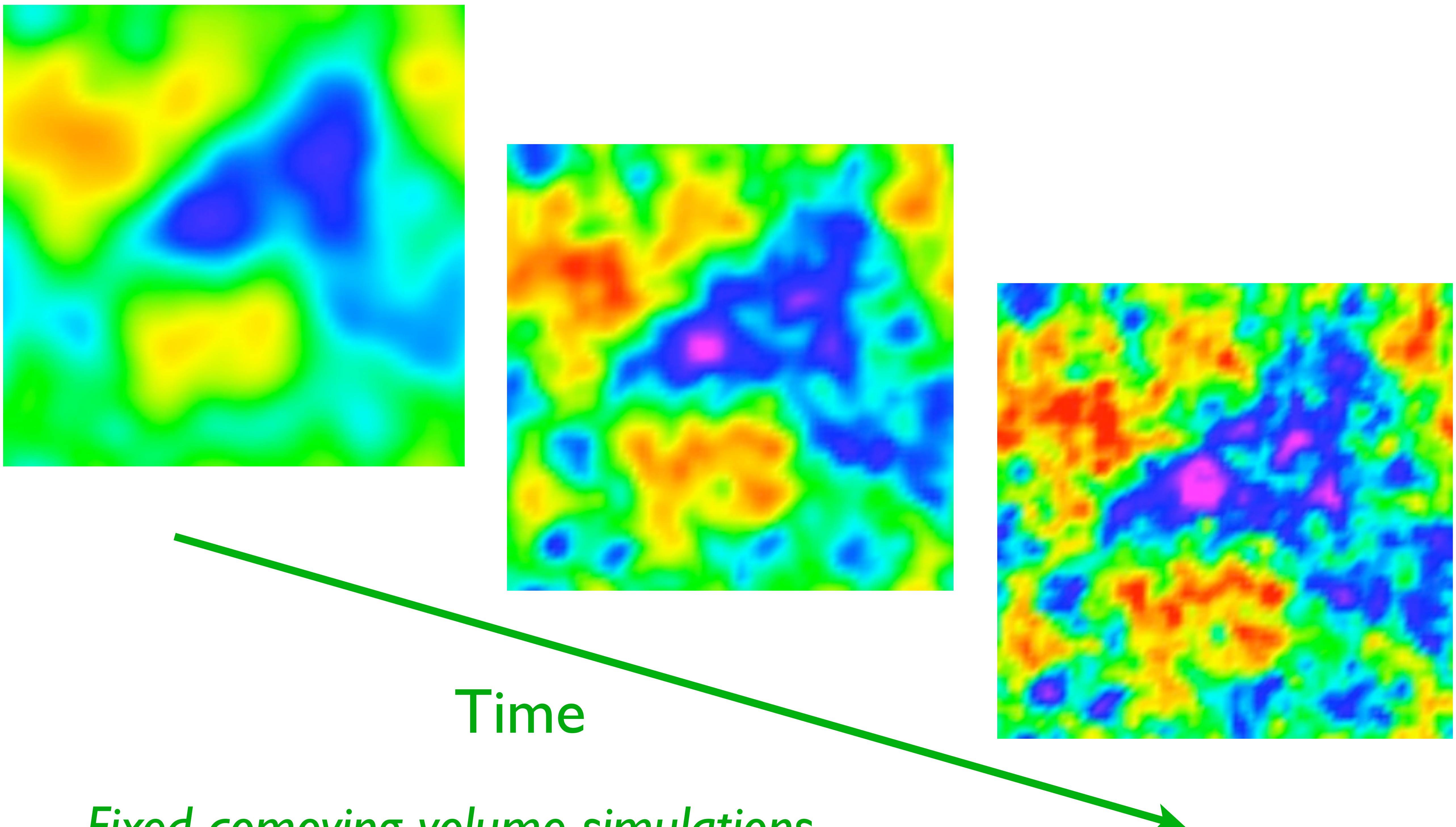
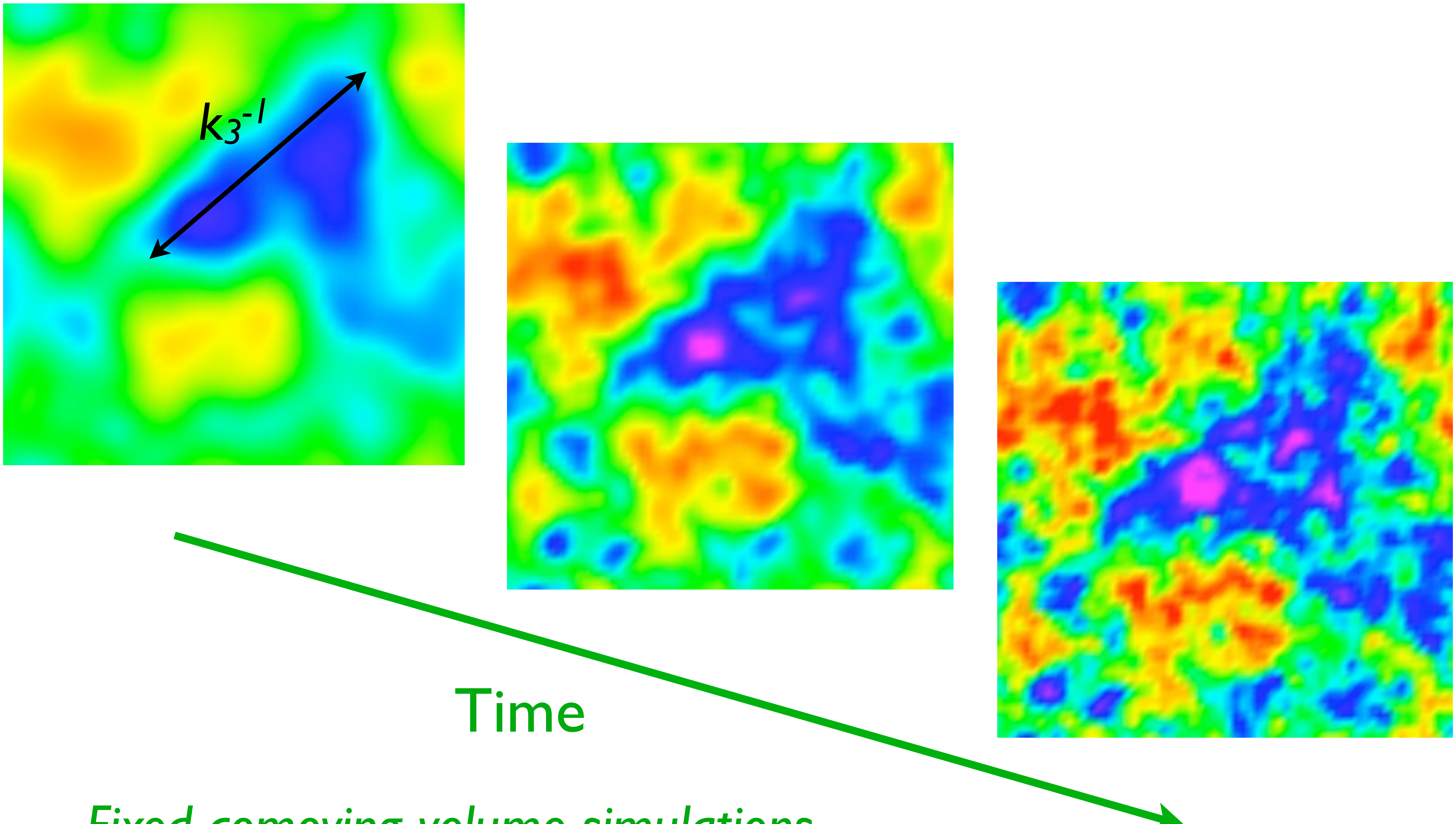
# Origin of local non-Gaussianity




*Fixed comoving volume simulations*

# Origin of local non-Gaussianity

*Long wavelength $k_3$ freezes out*

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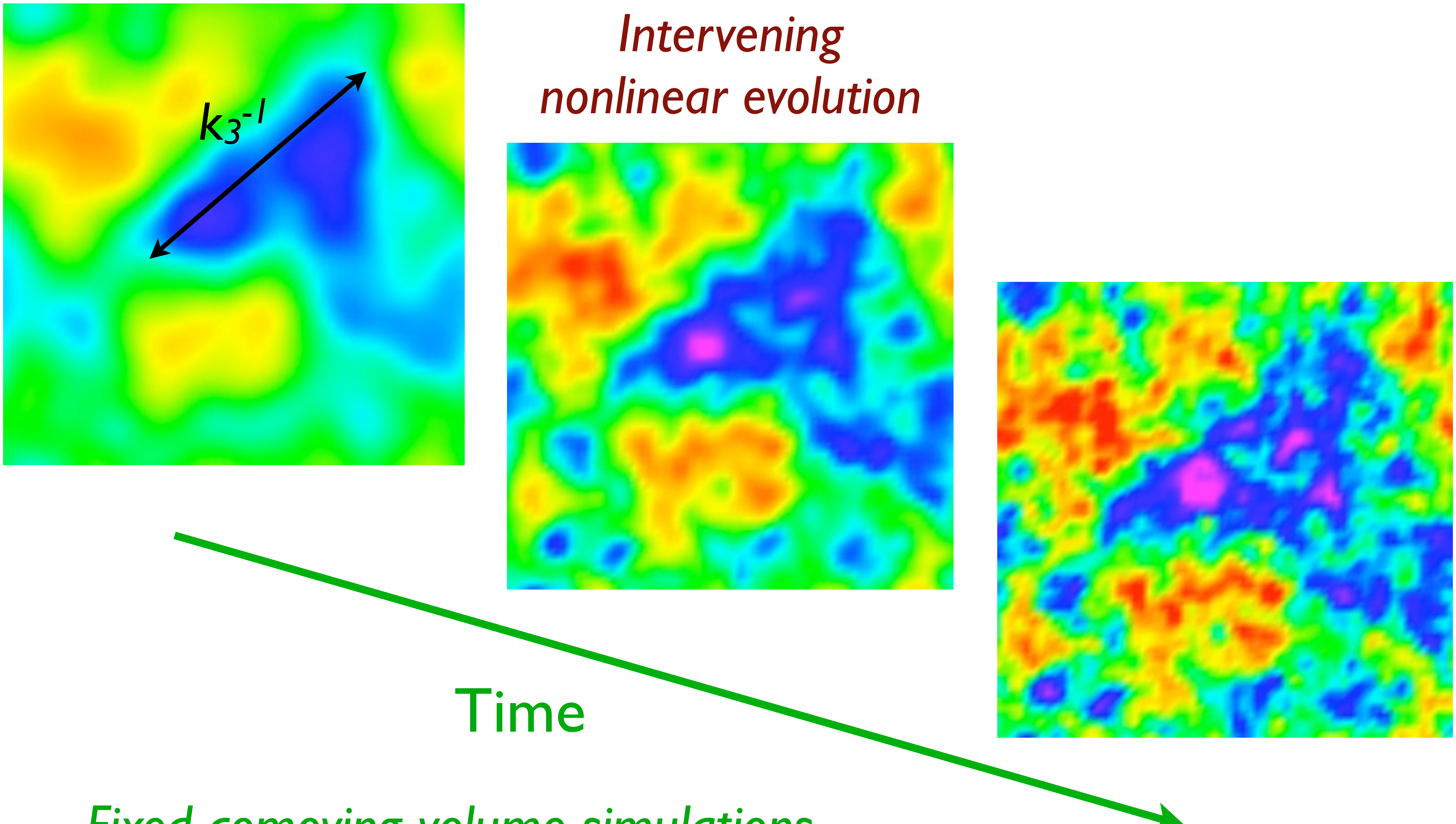
*Intervening nonlinear evolution*

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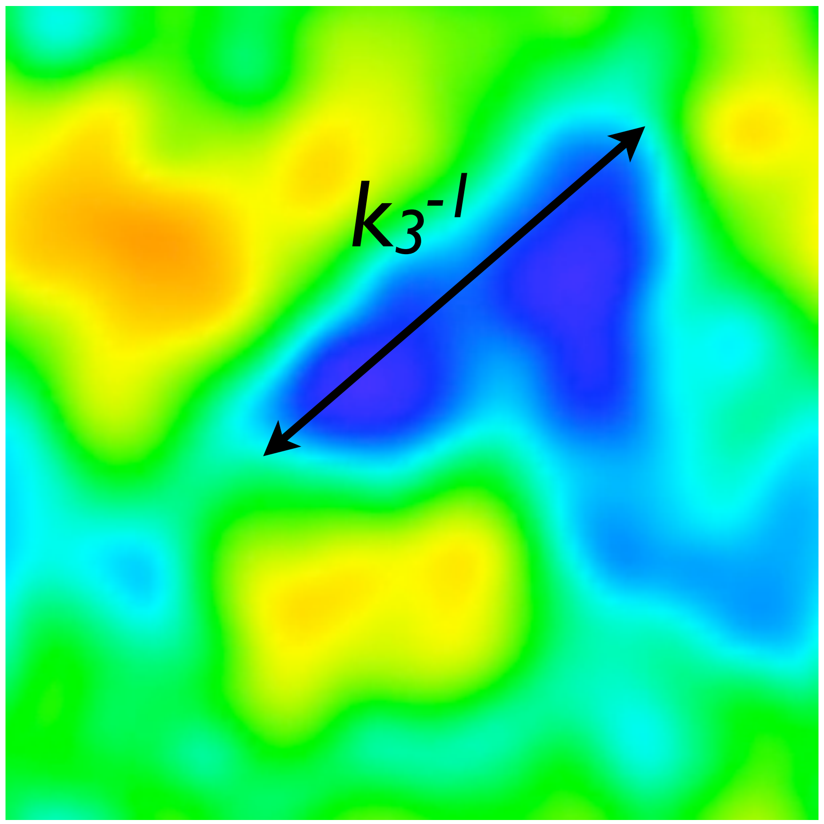


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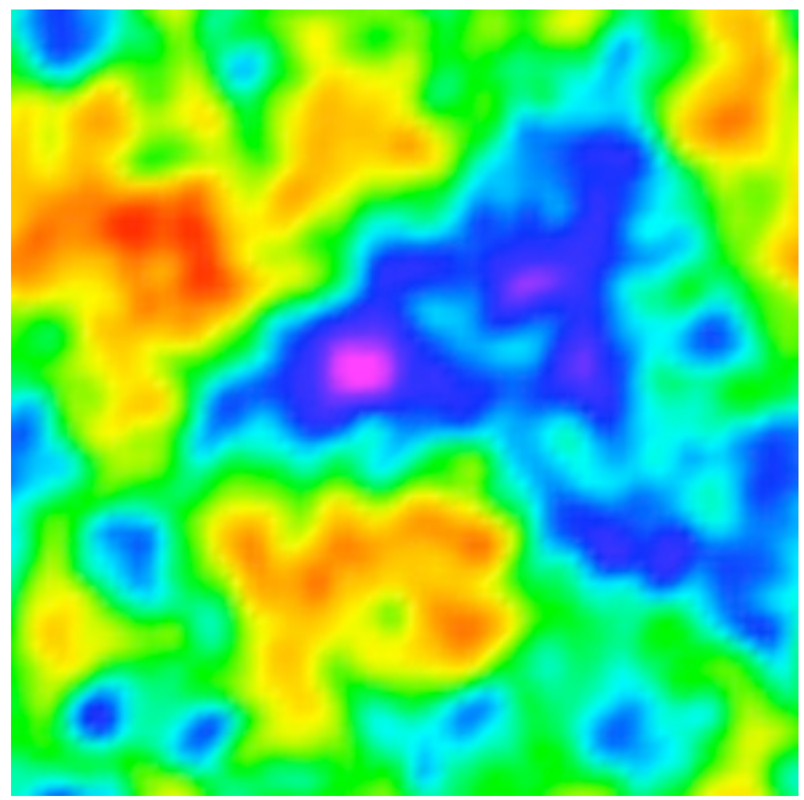
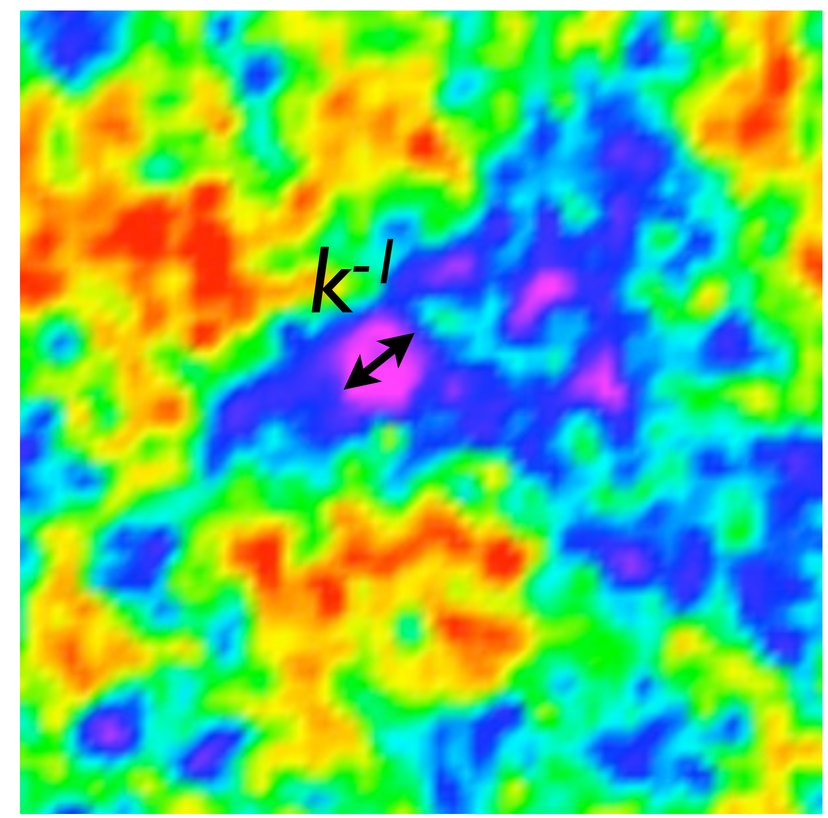


*Short wavelengths $k$ influenced by $k_3$*




Time

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# Local Non-Gaussianity

*Single field inflation non-Gaussianity (Maldacena, 2003)*

*Simple route to NG, Rigopoulos, et al, 2004*

Nonlinearity for $\zeta_i = \frac{1}{a\sqrt{2\epsilon}} Q_i$ only in prefactor of source $S_i[Q_{\rm lin}]$

$$\mathcal{S}_i = \frac{-\kappa}{2a\sqrt{\tilde\epsilon}} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} \dot{\mathcal{W}}(k)\mathrm{i}k_i \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} Q_{\rm lin}(k)\alpha(\mathbf{k}) + \mathrm{c.c.}$$

Expand to leading order ...

$$\partial_i \epsilon = -2\epsilon(2\epsilon - \eta) \longrightarrow \zeta_i^{(2)} = \underline{\overbrace{(4\epsilon - 2\eta)}^{1 - n_s \text{ (spectral index)}}\, \zeta^{(1)}}\; S_i^{(1)}$$

Squeezed limit with $k_3 << k_1, k_2 = k$

2nd order solution

Long mode $k_3$ background

Short mode $k$ source

Schematically 2nd order solution is yields 3-pt correlator or bispectrum:

$$\zeta^{(2)}(k) \sim (4\epsilon - 2\eta)\, P(k)^{1/2}\, P(k_3)^{1/2}$$

$$\langle \zeta^{(1)}(k)\zeta^{(1)}(k_3)\zeta^{(2)}(k)\rangle \sim (4\epsilon - 2\eta)\, P(k)P(k_3) \sim (4\epsilon - 2\eta)\, \frac{1}{k^3\, k_3^3} \left.\frac{H^2}{\epsilon}\right|_{k_3} \left.\frac{H_k^2}{\epsilon_k}\right|_k$$

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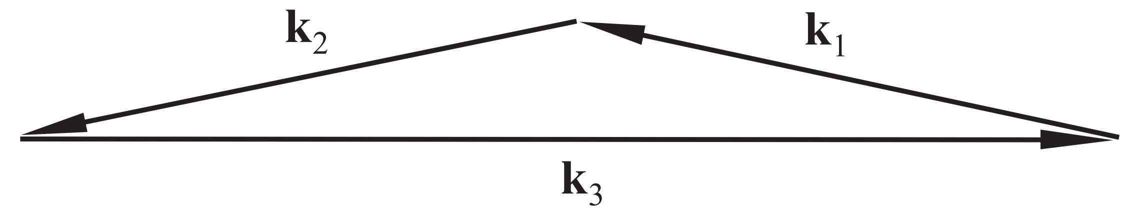
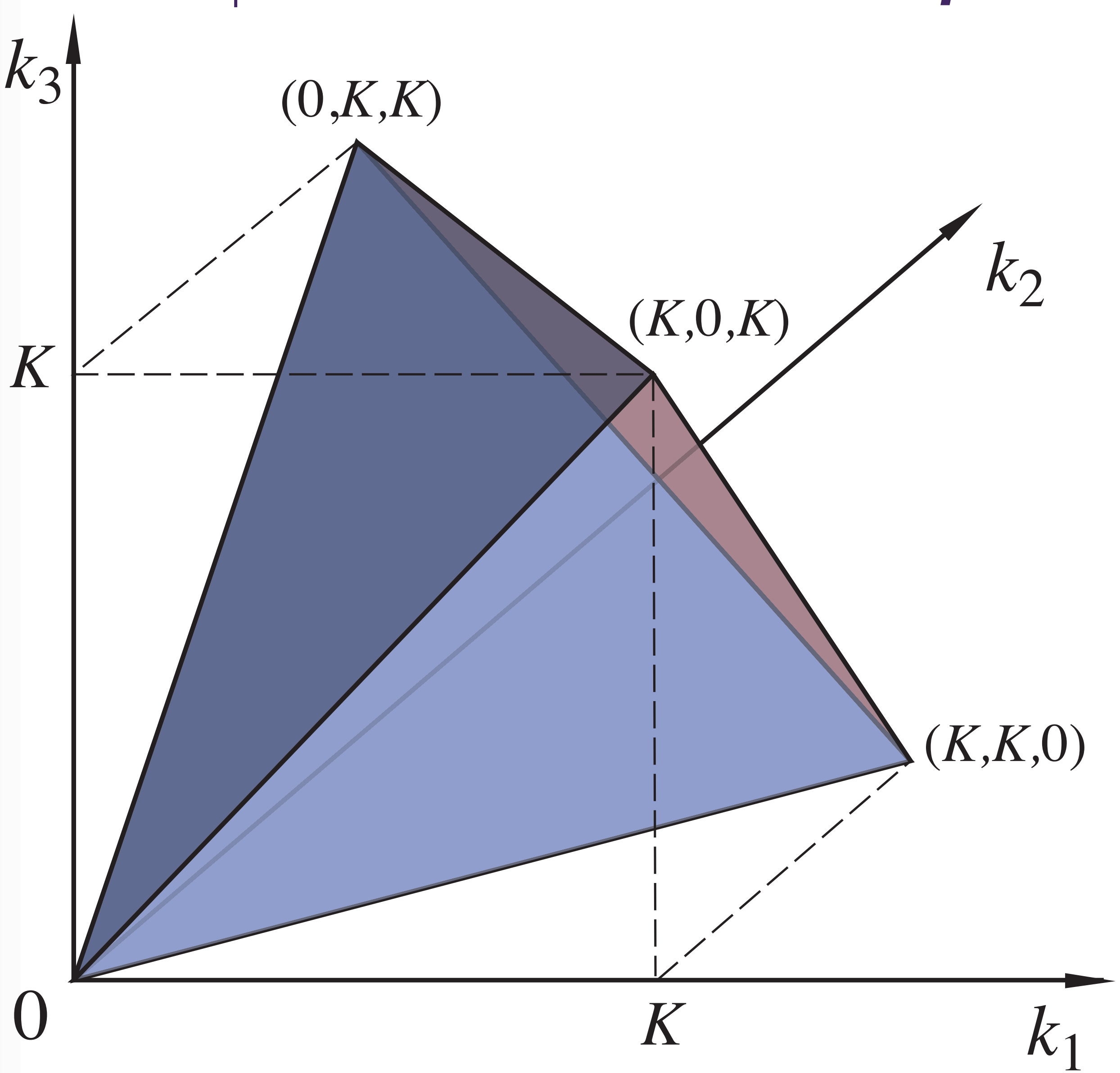
# Primordial Bispectrum

Defined by $$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3 B_\Phi(k_1,k_2,k_3)\,\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

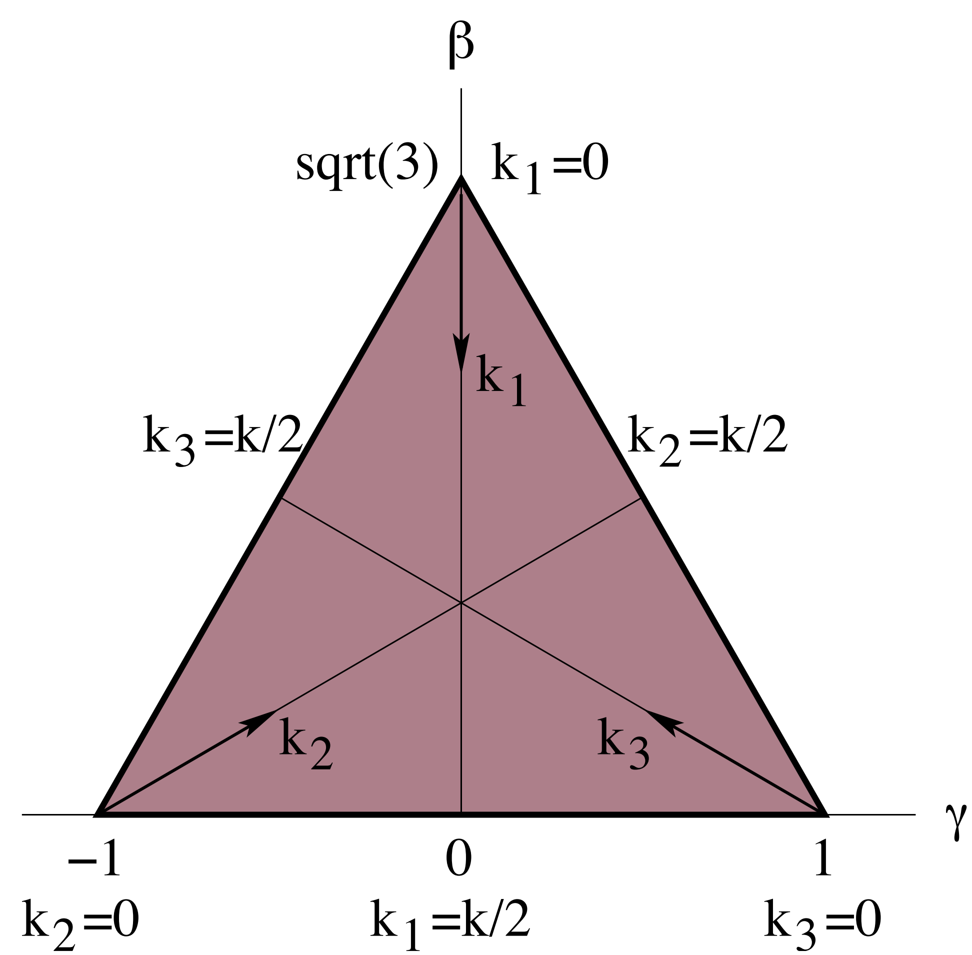
Note the triangle condition




Imposes ***3D tetrahedral bispectrum domain*** for the wavenumbers $k_1, k_2, k_3$




Slices $\tilde{k} \equiv \frac{1}{2}(k_1+k_2+k_3) = \text{const.}$



# Local shape function

There is an overall $k^{-6}$ scaling in the bispectrum, so define

$$S(k_1, k_2, k_3) = \frac{1}{N}(k_1 k_2 k_3)^2 \, B(k_1, k_2, k_3)$$

which is the (scale-invariant) *shape function*

For the local $B(k_1,k_2,k_3) = P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)$

$$S(k_1, k_2, k_3) \approx \tfrac{1}{3}\left(\frac{k_1^3 + k_2^3 + k_3^3}{k_1 k_2 k_3}\right)$$

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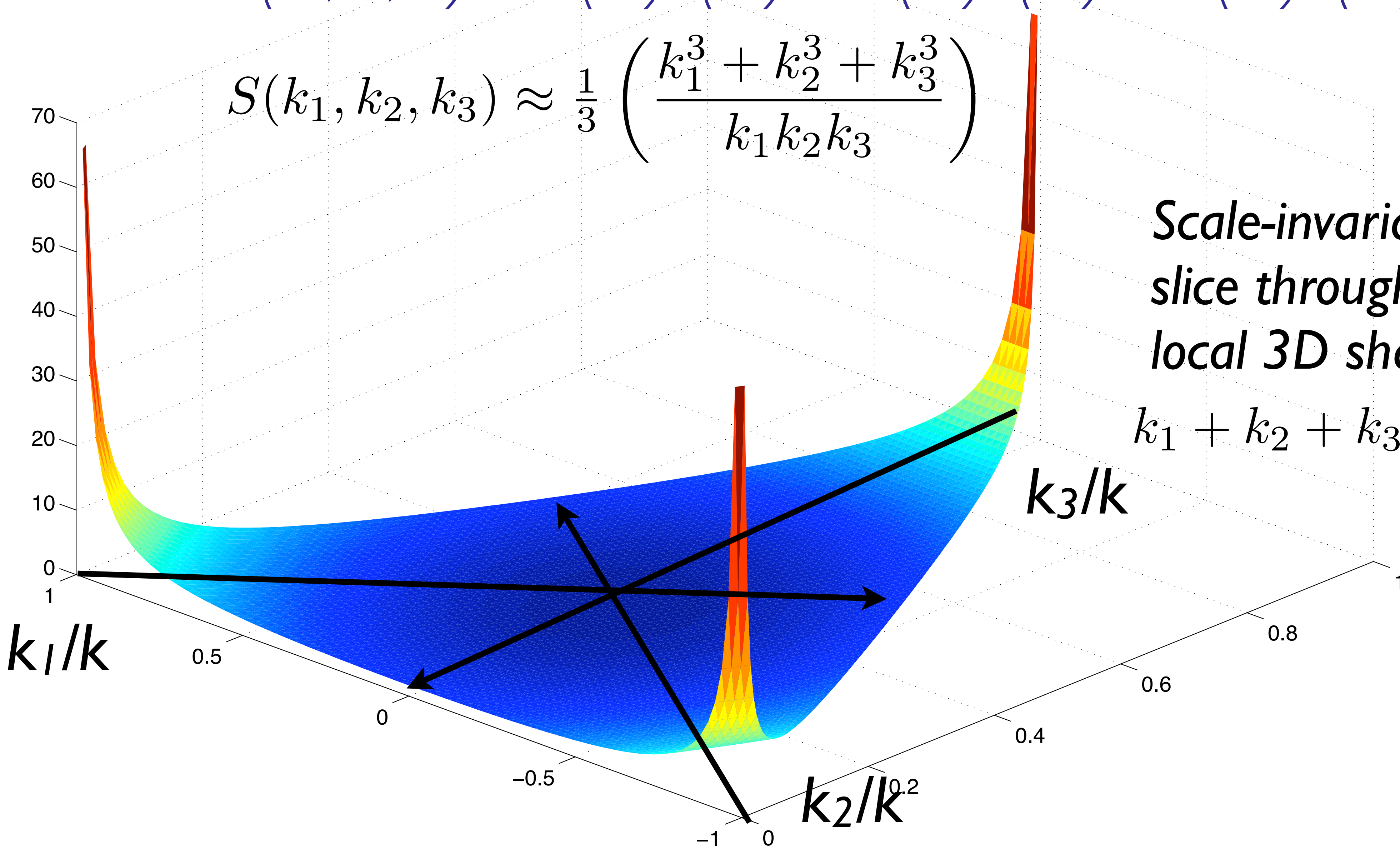



*Scale-invariant slice through the local 3D shape*

$k_1 + k_2 + k_3 = \text{const.}$

# No-Go Theorem

*Simple* inflation models *cannot* generate observable non-Gaussianity:

- single scalar field
- canonical kinetic terms
- always slow roll
- ground state initial vacuum
- standard Einstein gravity

Non-Gaussianity is arguably the most stringent test of the standard picture

But simple inflation model-building faces rigorous challenges in fundamental theory (e.g. *eta problem and super-Planckian field values*). Many new ideas/ solutions violate these conditions!

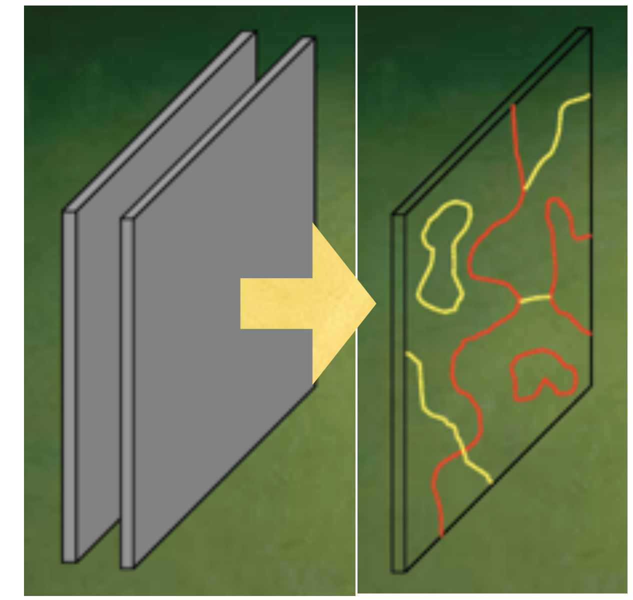
# Inflation in string theory

Motivated by deficiencies of inflation

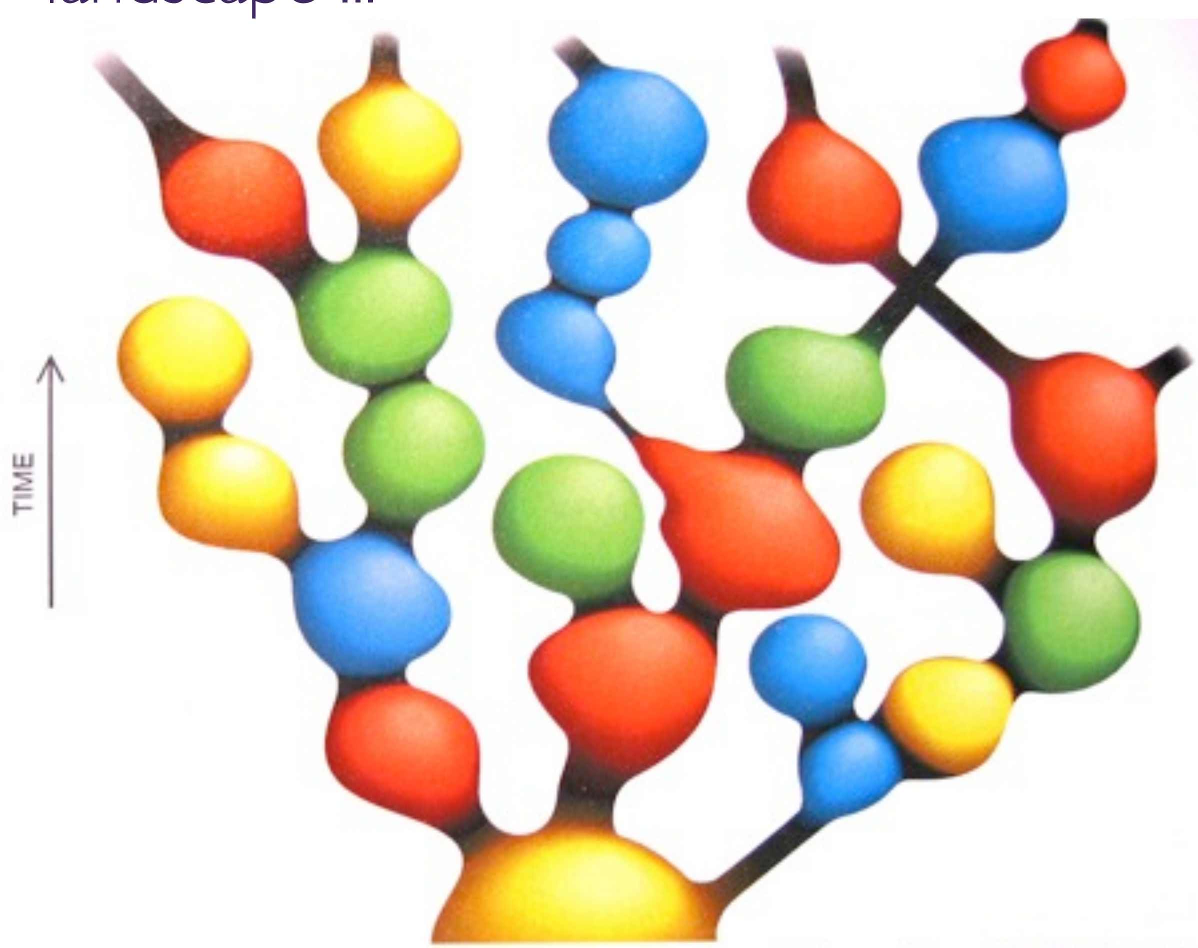
- UV completeness sensitivity
- Super-Planckian fields
- Eta problem - quantum corrections
- Challenges to effective field theory

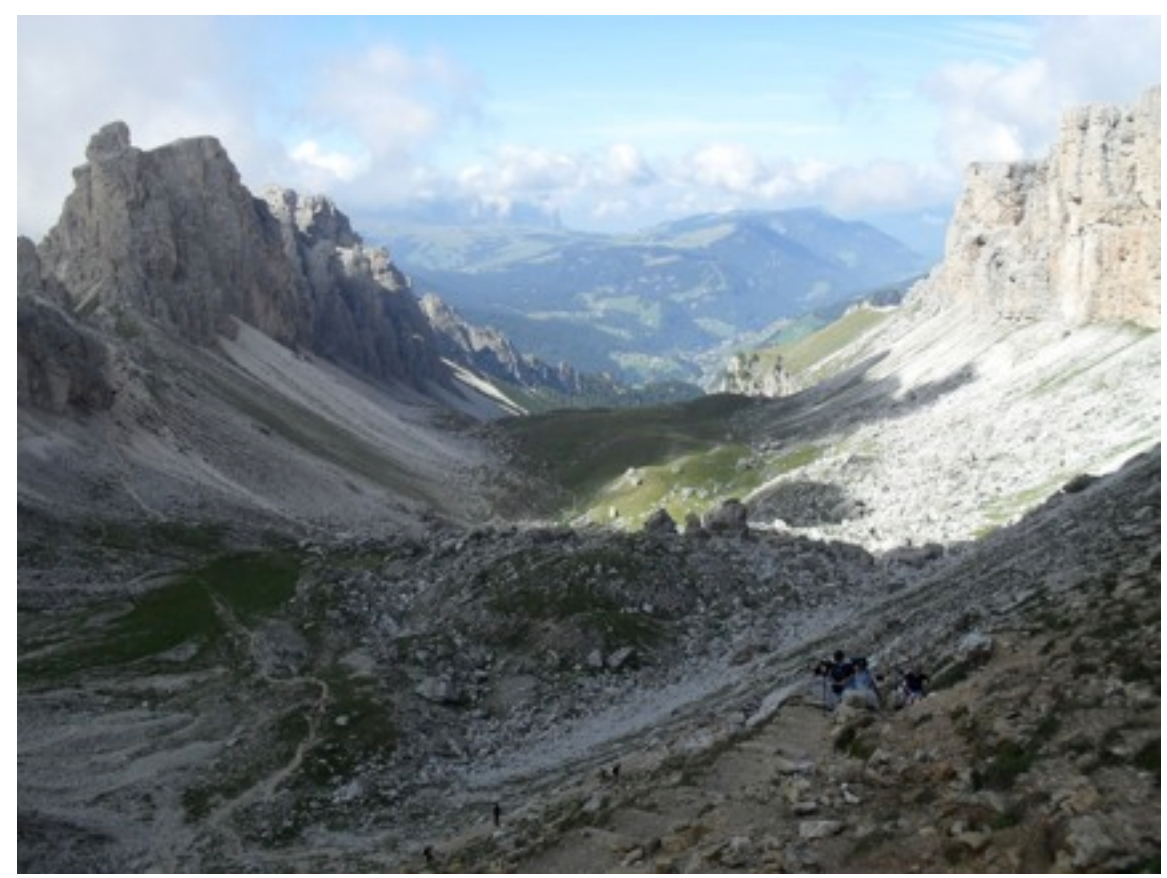
Rich structure

- warped branes, axions, eternal inflation
- landscape ...



Dvali & Tye, 2000
Sarangi & Tye, 2002
KKLMMT, 2003

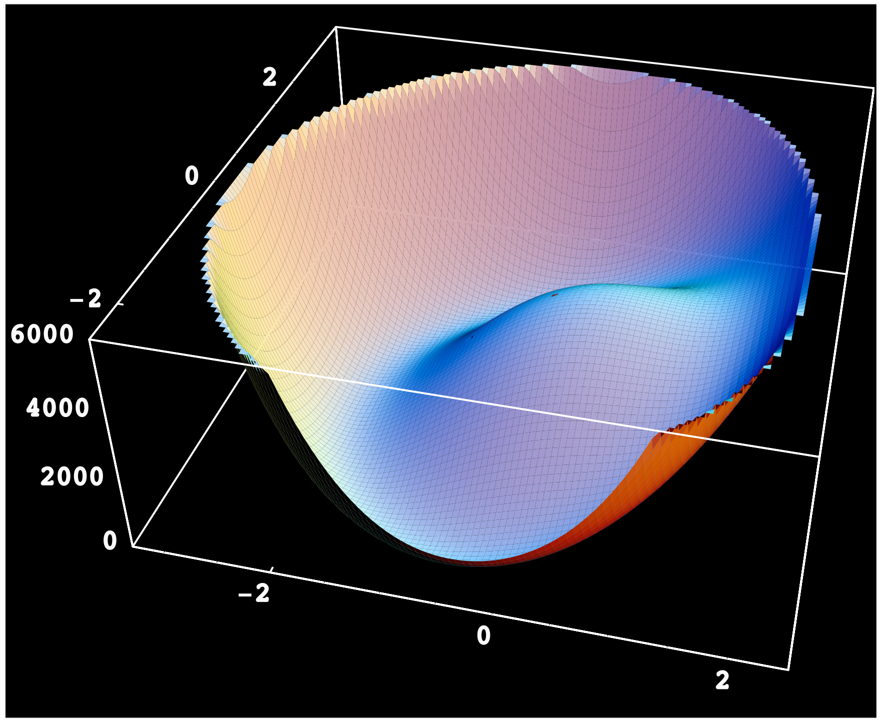


# Multiple field inflation

## Non-Gaussianity from interacting potentials

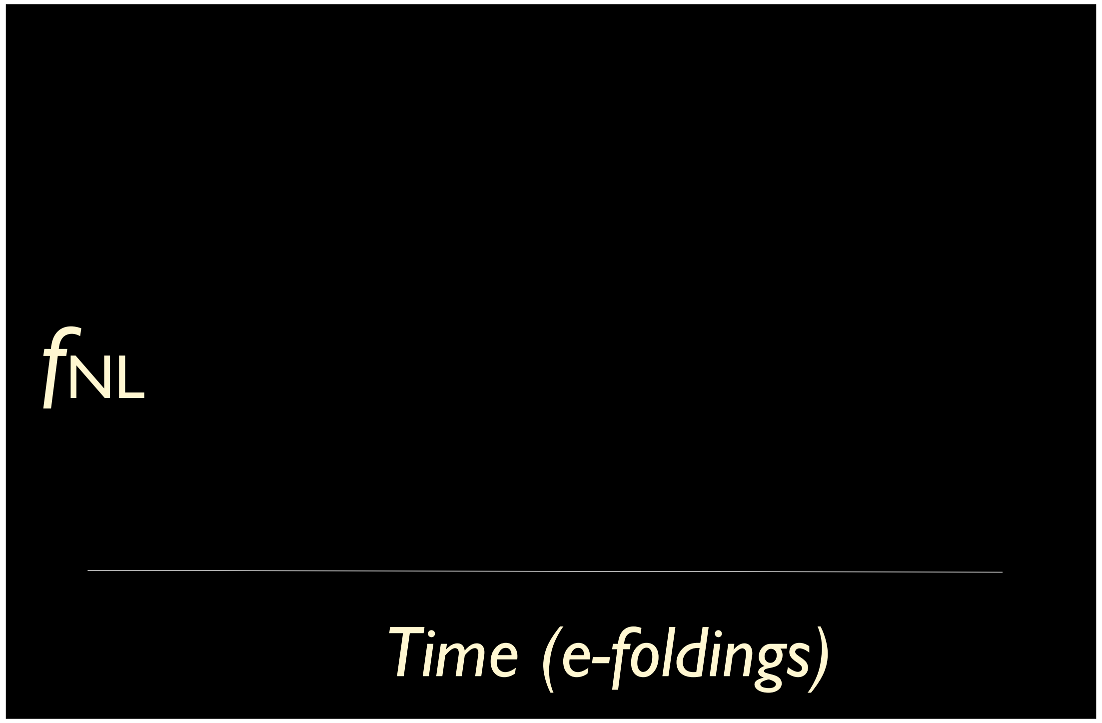
$$V(\phi_1, \phi_2) = \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2 - m^2)^2 + \nu(\phi_1 + m)^3$$




## Significant final $f_{NL}$ ingredients:

- *corner turning*
- *nontrivial potential*
- *or breakout (hybrid models)*

*Rigopoulos, EPS, van Tent 05, 06*
*see also Vernizzi & Wands 06,*
*and Bernadeau & Uzan 02*



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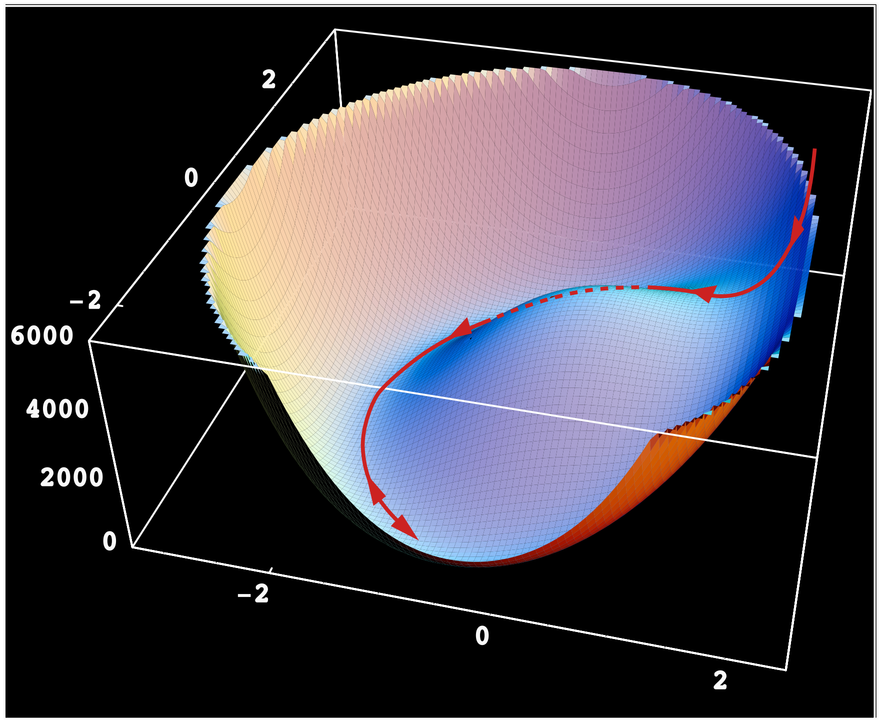


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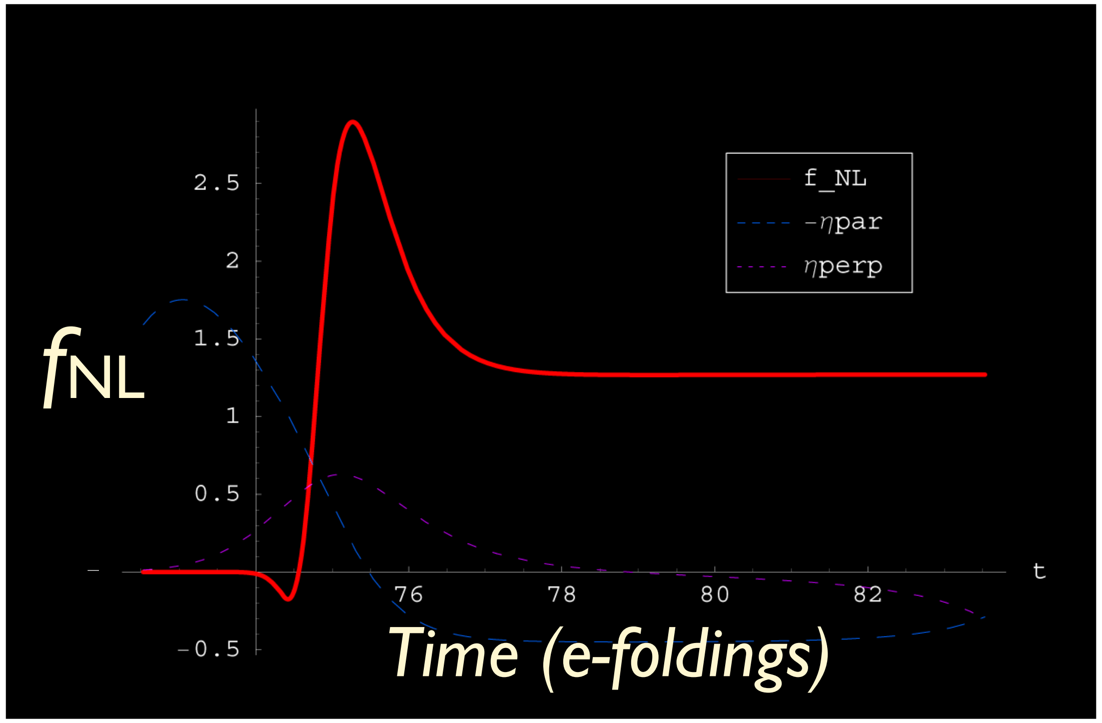

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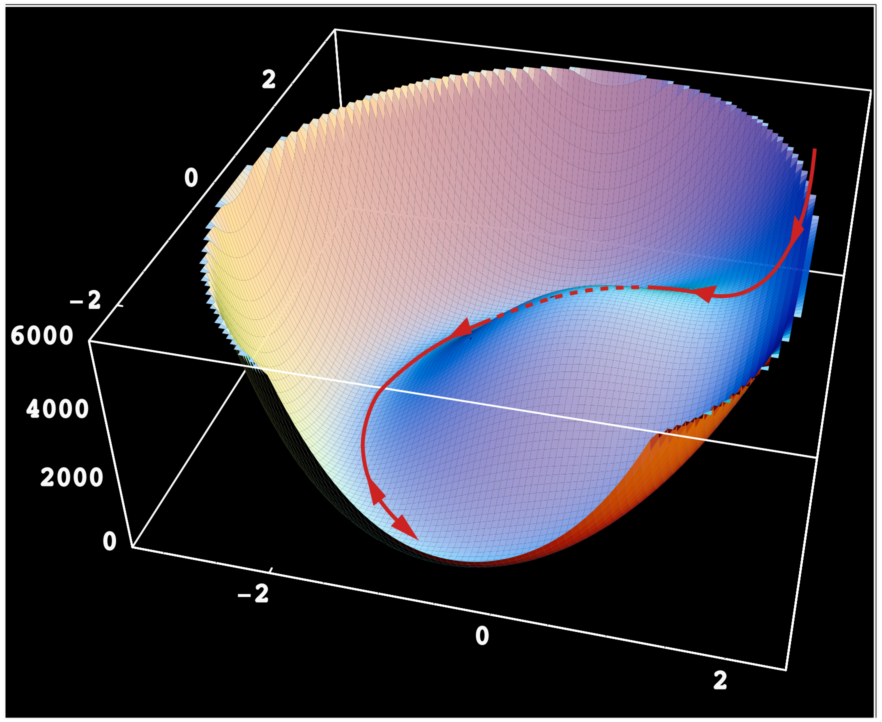


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## *Large NG from multifield inflation*

<u>Curvatons</u> - *non-participating, later dominant*

*e.g. Linde & Mukhanov 96; Lyth & Wands 01; Moroi &Takahashi 01*

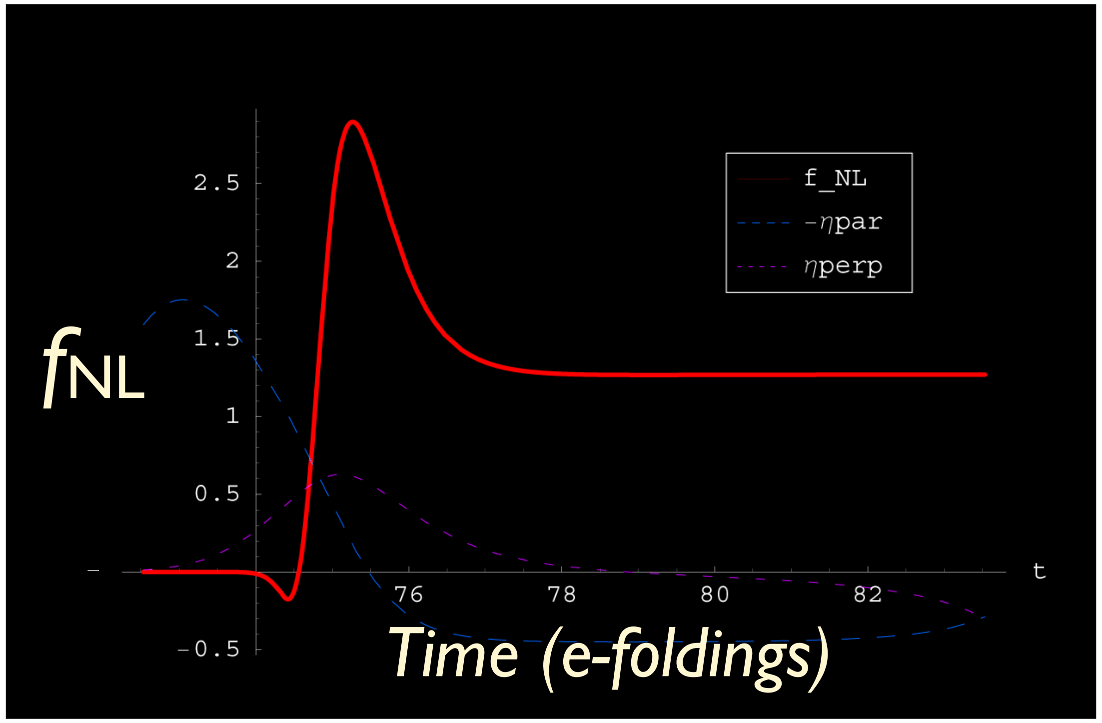
Simple curvatons local, but corrections:

*e.g. Fassielo et al, 11; Lerner et al, 11; Byrnes et al, 11.*

<u>Corner-turning multifield models</u>

*Slow-roll: Byrnes, Choi, Hall, 08a, 08b; see also Seery & Lidsey, 04*

*General: RSvT, 07; Peterson & Tegmark, 10; Meyer & Sivanandam*




<u>Hybrid or waterfall inflation models</u> *Barnaby & Cline, 06; Naruko & Sasaki, 08; Mulryne et al 11.*

# Non-Gaussian Sources

## Multiple field inflation

- *Complex corner-turning dynamics during multifield inflation*
  *e.g. Rigopoulos et al 2005; Vernizzi & Wands, 2006; Byrnes, 2010*
- *End of inflation, reheating and preheating*
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## Higher derivative kinetic terms

- *K-inflation, DBI inflation - modified sound speed*
  *e.g. Silverstein & Tong 2003; Alishaha et al 2004; Chen et al 2006*

**Excited initial states** - *trans-Planckian effects*

*e.g. Chen, et al, 2006; Holman & Tolley, 2008; Meerburg et al 2008*

**Feature and periodic models** *(non slow roll)*

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**Secondary anisotropies** - *e.g. ISW, cosmic strings*

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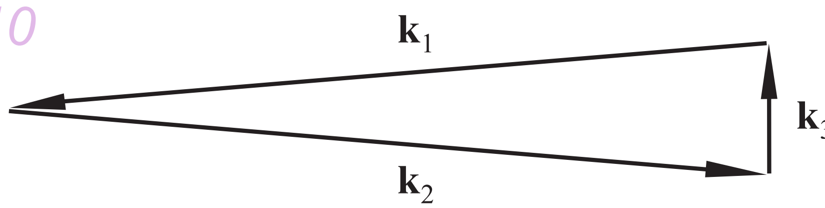


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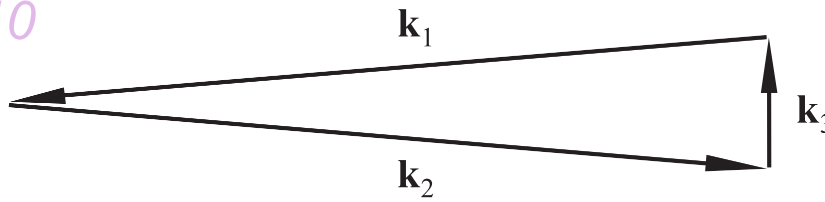


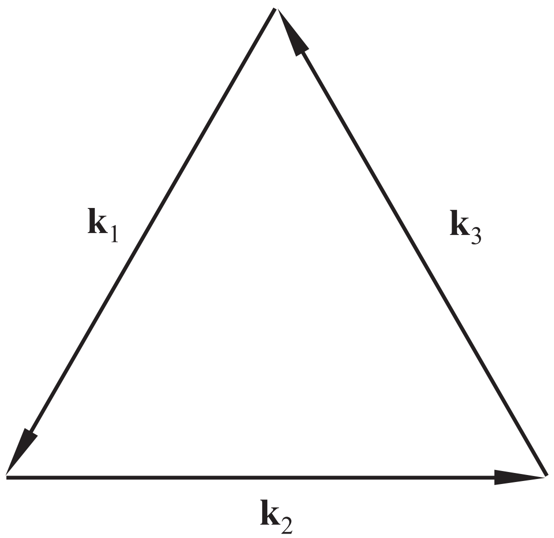


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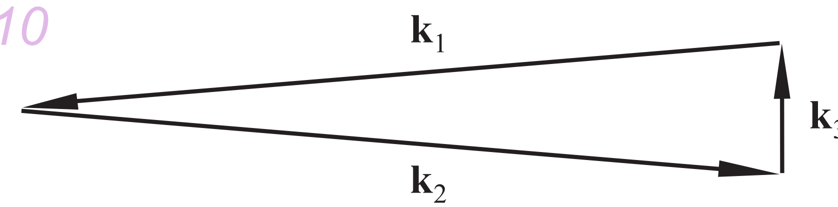


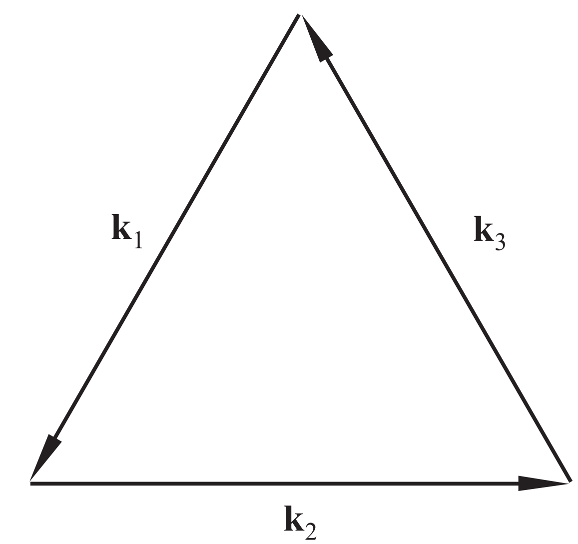


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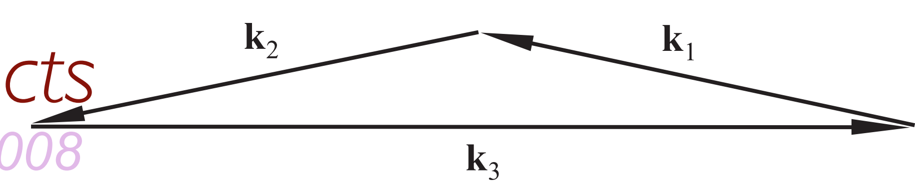


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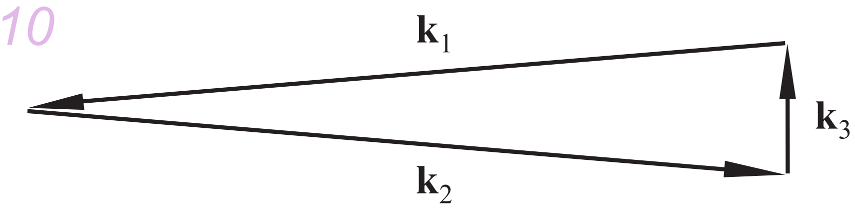


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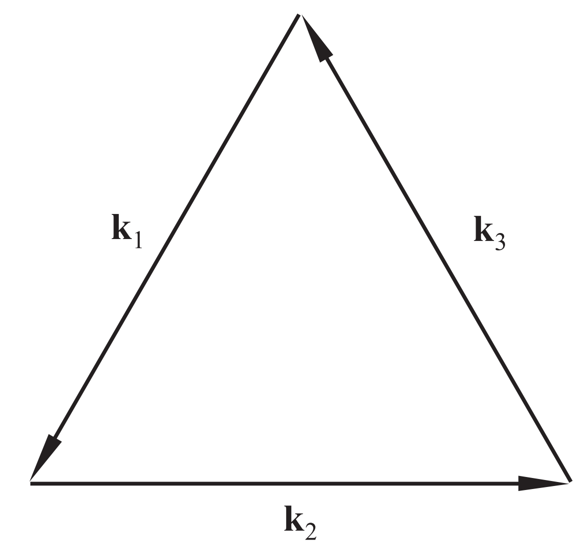
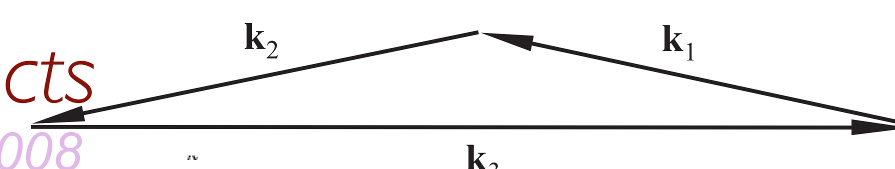



## ***Higher derivative kinetic terms***

- *K-inflation, DBI inflation - modified sound speed*
  *e.g. Silverstein & Tong 2003; Alishaha et al 2004; Chen et al 2006*

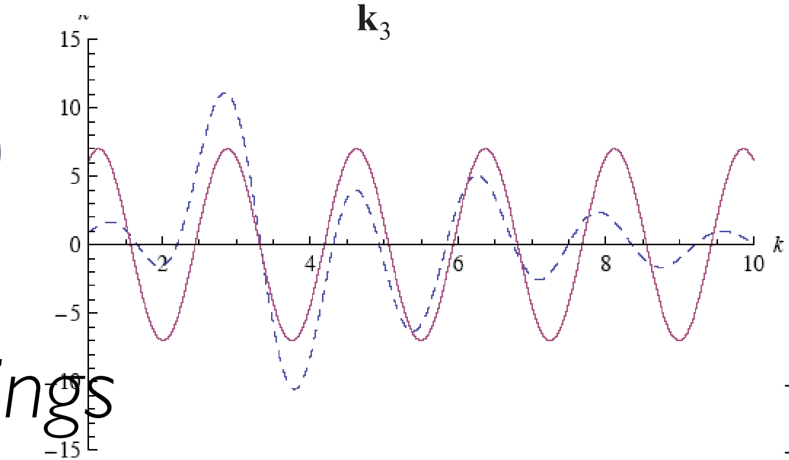



## ***Excited initial states*** *- trans-Planckian effects*

*e.g. Chen, et al, 2006; Holman & Tolley, 2008; Meerburg et al 2008*

## ***Feature and periodic models*** *(non slow roll)*

*e.g. Chen, Easther & Lim, 2005; Meerburg, 2010*



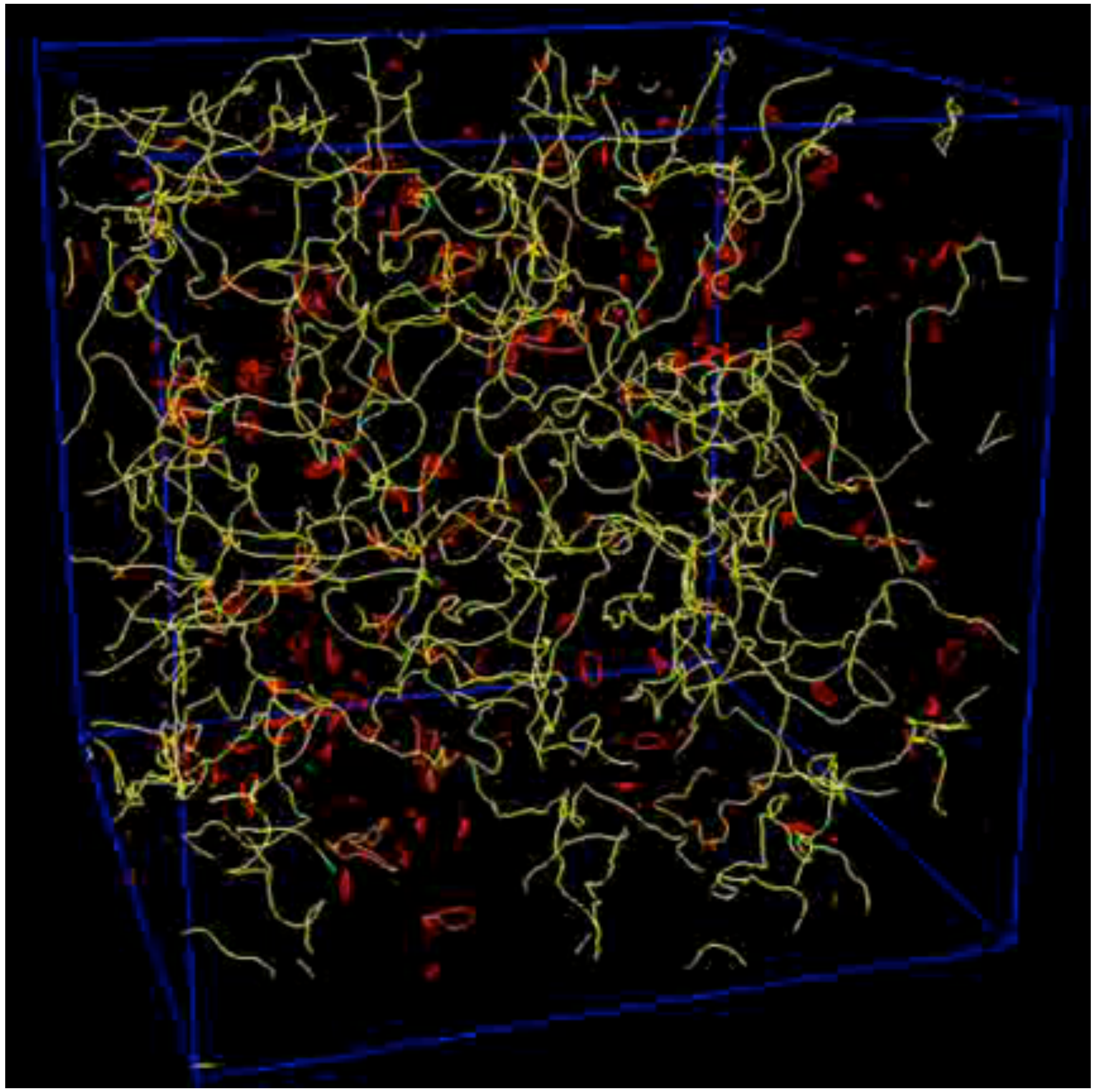
## ***Secondary anisotropies*** - e.g. ISW, cosmic strings

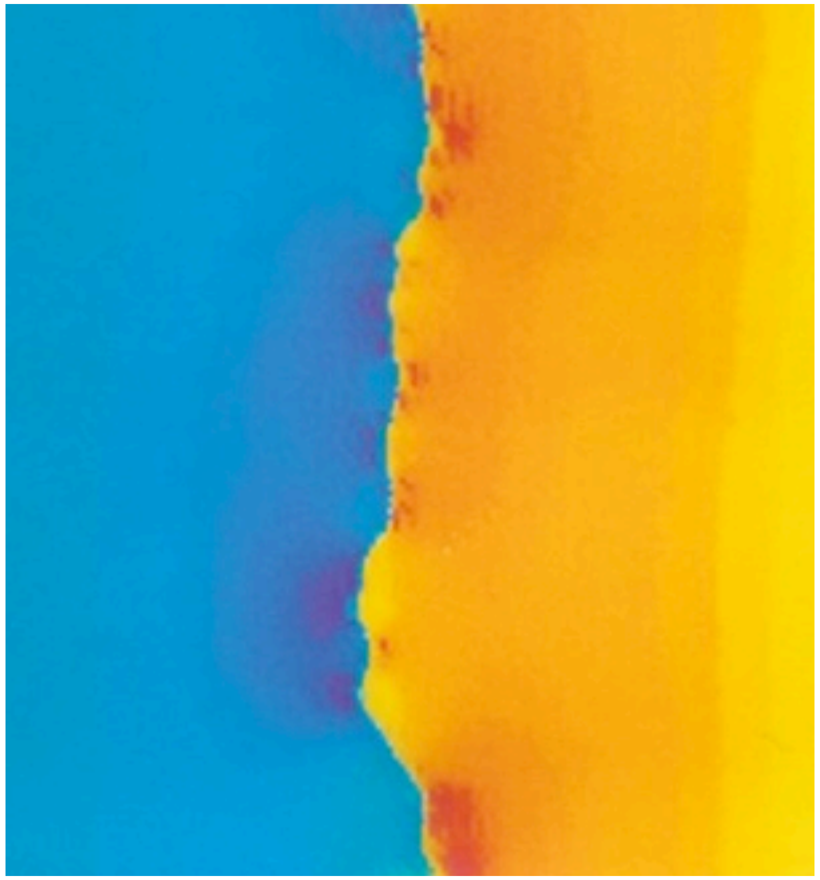
# Cosmic (super)strings

Brane inflation collisions (F- and D-strings) or SUSY GUT models

Scale-invariant evolution

Non-Gaussian CMB signature





STRING MOTION

Trispectrum key, not bispectrum

Analytic approximations for string bispectrum *Hindmarsh & Ringeval, 09*

String polyspectra calculated for Planck - *T competitive w. P(k)* *Regan & EPS, 09*

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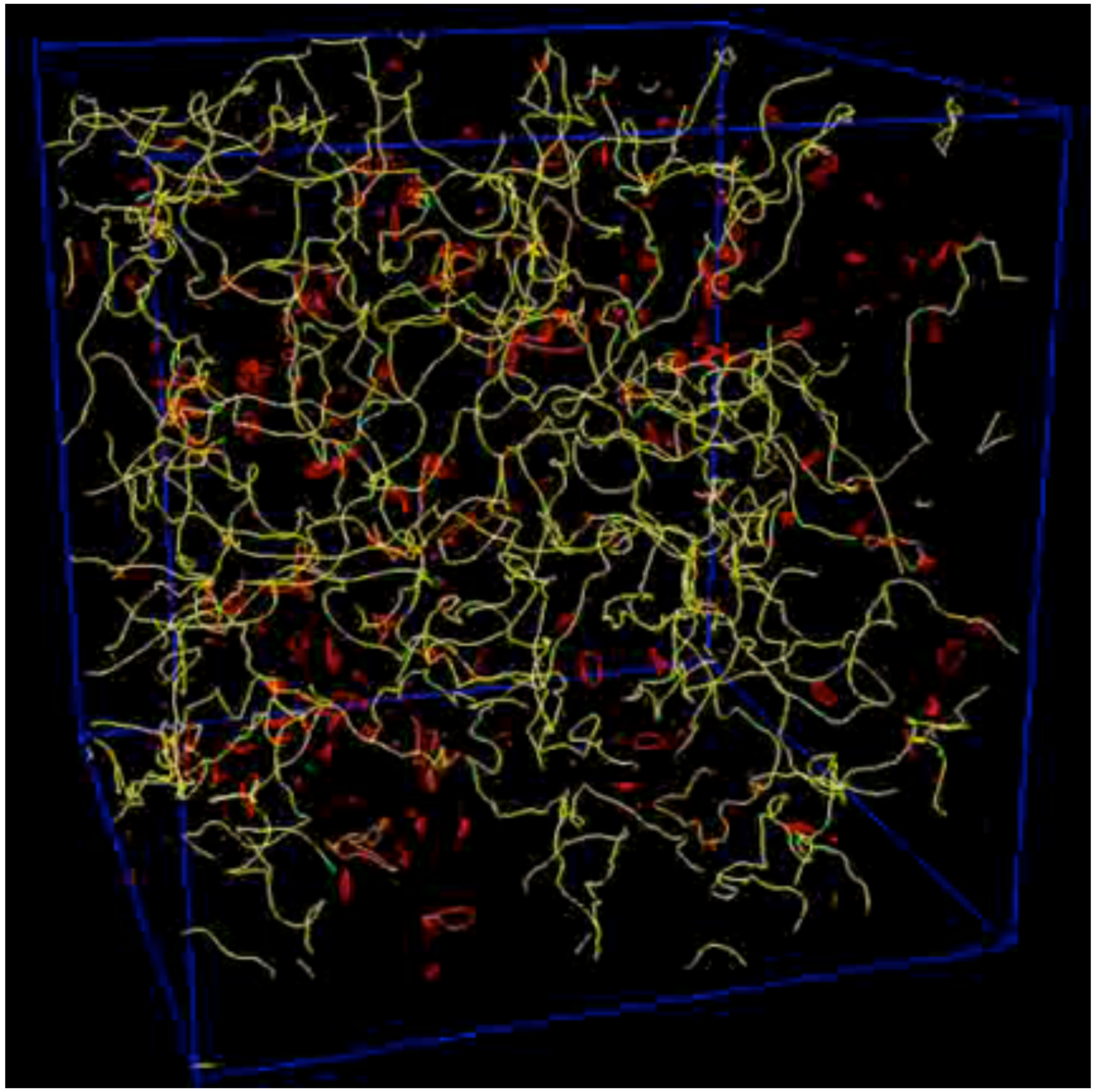

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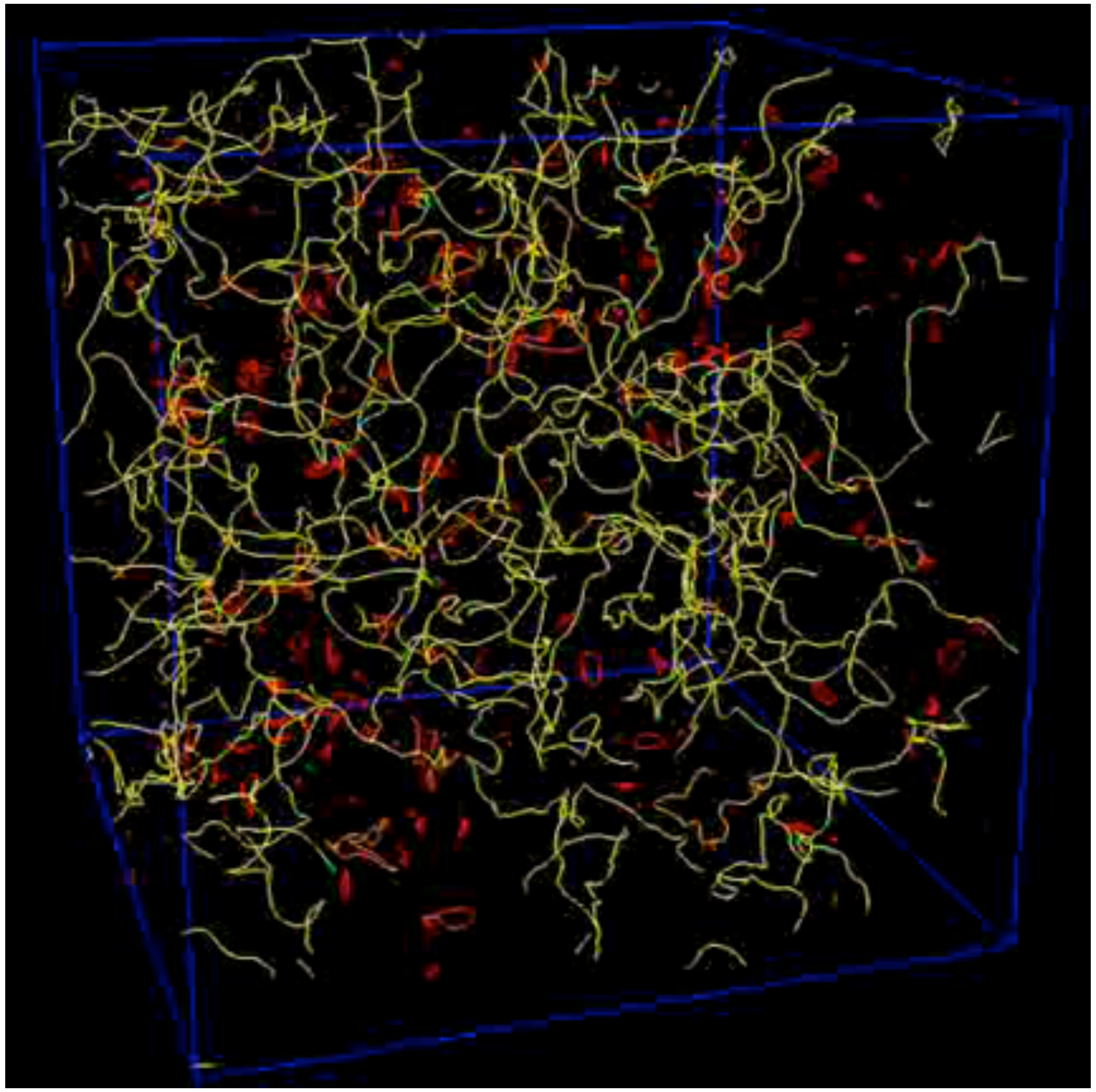

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# Primordial & CMB Bispectra

A primordial bispectrum $B(k_1,k_2,k_3) = S(k_1,k_2,k_3)/(k_1k_2k_3)^2$ induces

$$b_{l_1l_2l_3} = \left(\frac{2}{\pi}\right)^3 \int x^2 dx \int dk_1 dk_2 dk_3 \, S(k_1,k_2,k_3) \times \Delta_{l_1}(k_1)\,\Delta_{l_2}(k_2)\,\Delta_{l_3}(k_3)\, j_{l_1}(k_1x)\, j_{l_2}(k_2x)\, j_{l_3}(k_3x)$$

For the CMB, the bispectrum and trispectrum* are defined by

$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle = \left(\int d^2\hat{n} Y_{l_1m_1}(\hat{\mathbf{n}})Y_{l_2m_2}(\hat{\mathbf{n}})Y_{l_3m_3}(\hat{\mathbf{n}})\right) b_{l_1l_2l_3}$$

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* For simplicity we give formulae only for diagonal-free trispectra.

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*Primordial shape*

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Primordial shape

Transfer functions

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*Primordial shape*

*Transfer functions*

*Oscillatory*

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# Exercise II: CMB Local non-Gaussianity

The reduced CMB bispectrum arises from the primordial bispectrum

$$b_{l_1 l_2 l_3} = \left(\frac{2}{\pi}\right)^3 \int dx dk_1 dk_2 dk_3 (x k_1 k_2 k_3)^2 B(k_1, k_2, k_3) \Delta_{l_1}(k_1) \Delta_{l_2}(k_2) \Delta_{l_3}(k_3) j_{l_1}(k_1 x) j_{l_2}(k_2 x) j_{l_3}(k_3 x)$$

In the large-angle approximation (*l* << 200), we can approximate the transfer functions by $\Delta_l(k) = \frac{1}{3} j_l(\Delta\tau k)$ with $\Delta\tau = \tau_0 - \tau_{\mathrm{dec}}$.

In the local model with $P(k) \propto k^{-3}$, show that the 4D bispectrum integral above becomes separable yielding the large-angle analytic result

$$b_{l_1 l_2 l_3} \propto \frac{1}{27\pi^2} \left( \frac{1}{l_1(l_1+1) l_2(l_2+1)} + \frac{1}{l_2(l_2+1) l_3(l_3+1)} + \frac{1}{l_3(l_3+1) l_1(l_1+1)} \right) .$$

*Hint:* Note the results

$$\int dk\, k^2 j_l(ak) j_l(bk) = \frac{\pi}{2a^2} \delta(a-b) \qquad \int dk\, k^{-1} j_l(ak) j_l(bk) = (2l(l+1))^{-1}$$

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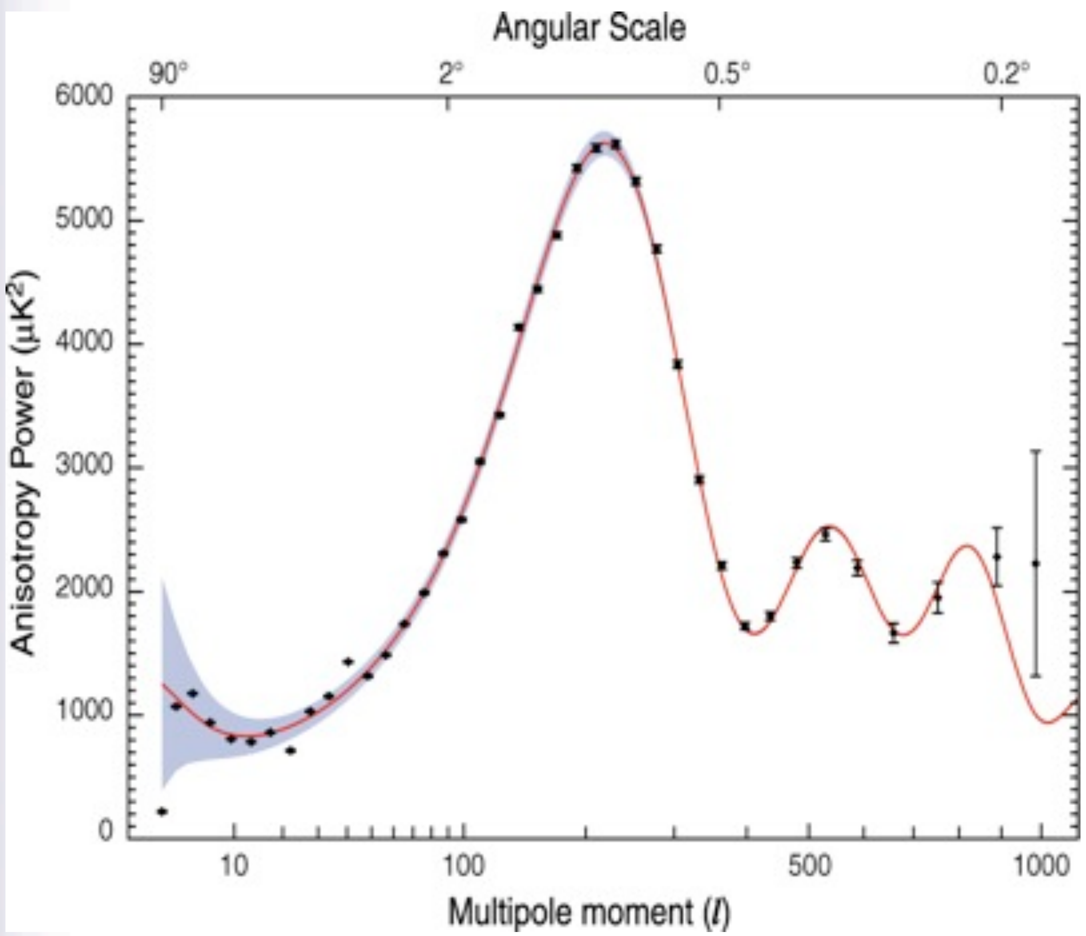
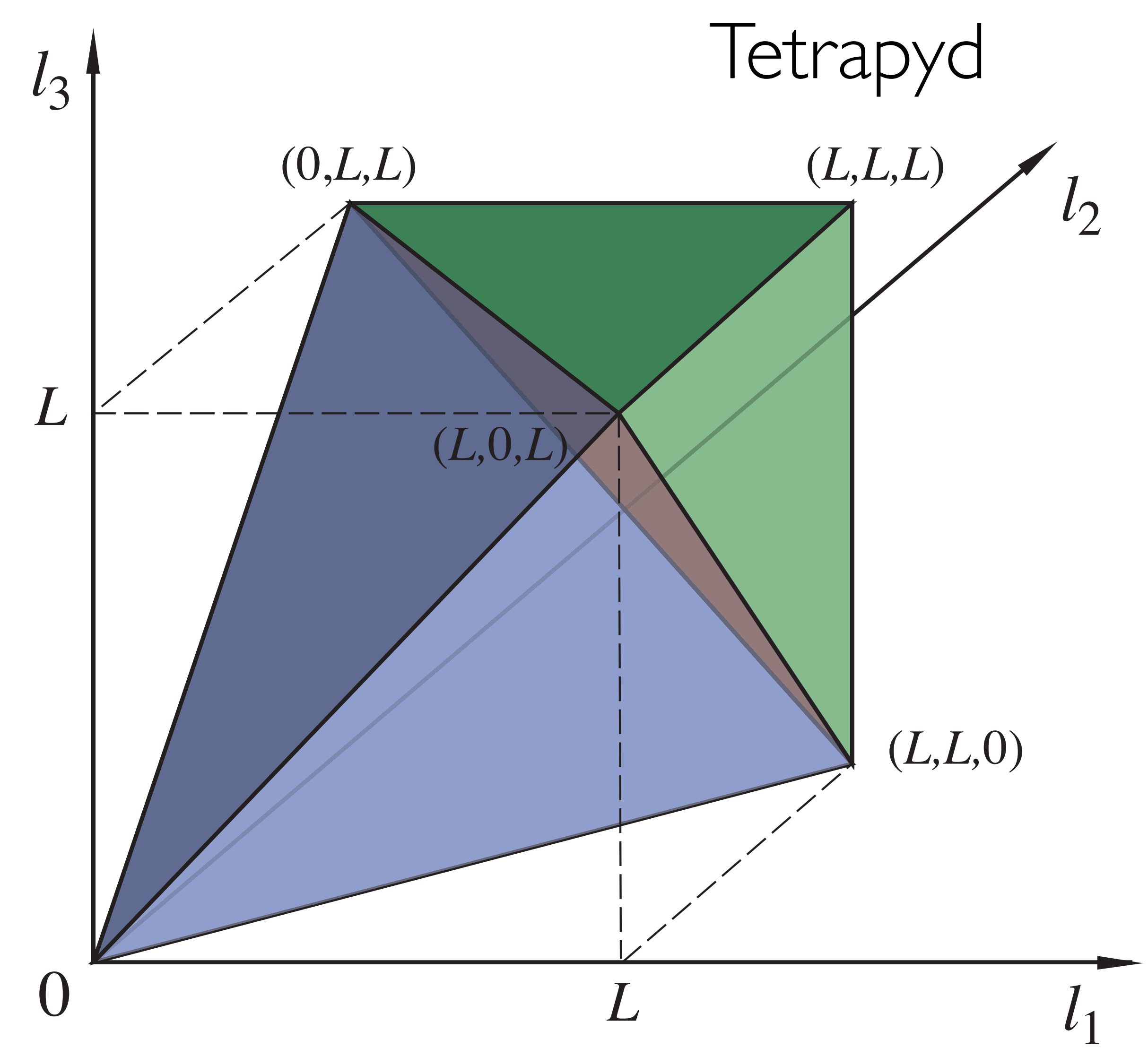
# CMB polyspectra




*Power spectrum (2pt correlator)*




*Bispectrum (3pt correlator)*

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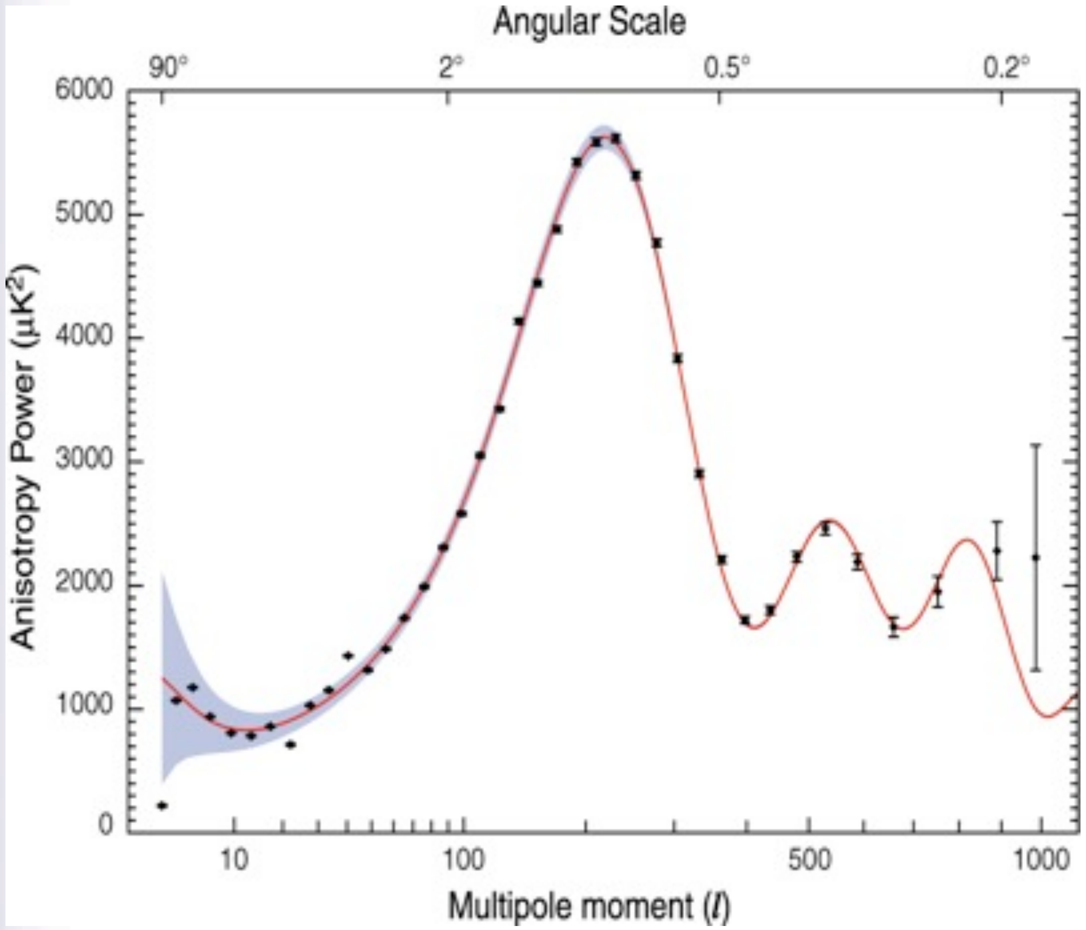


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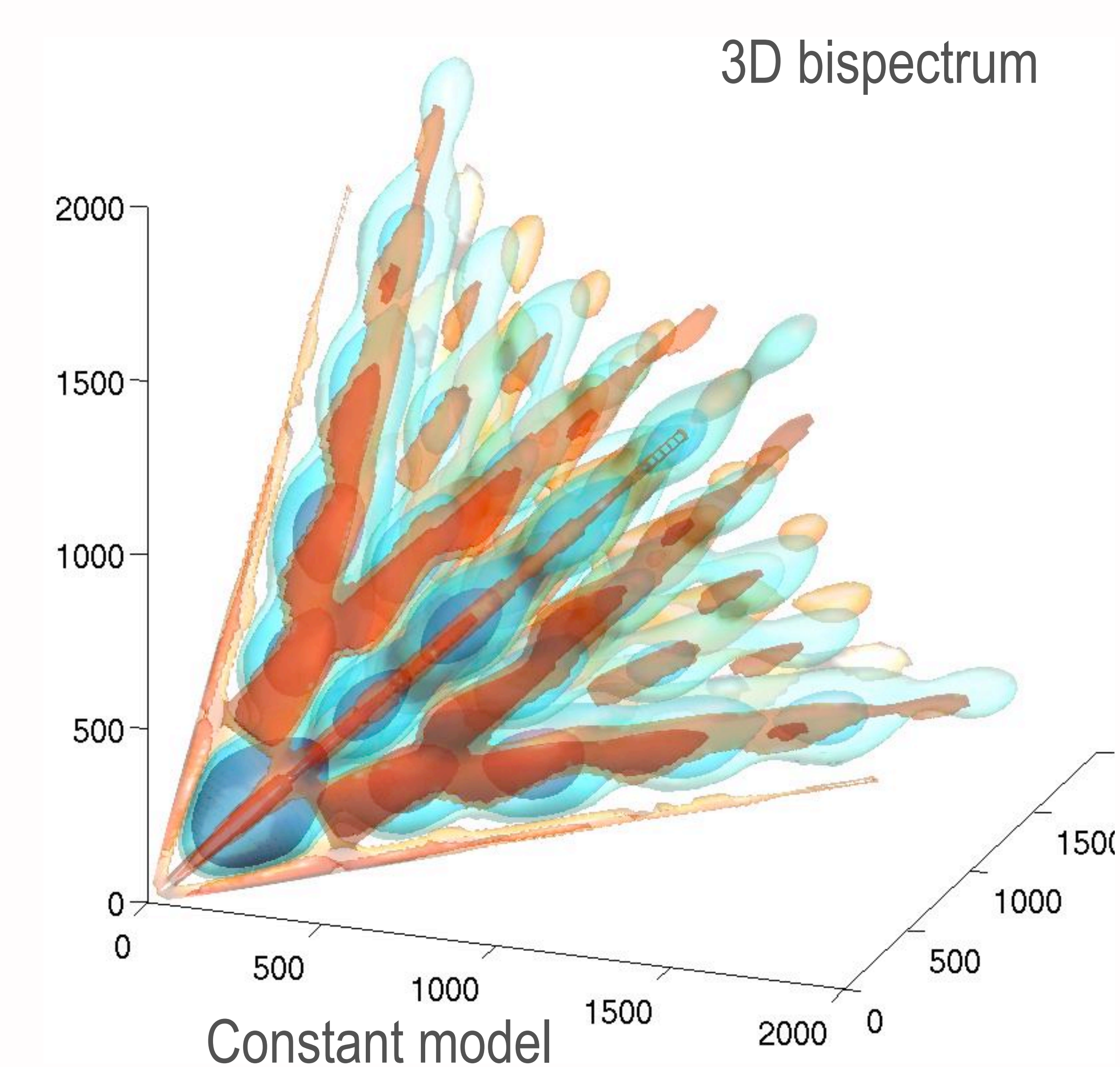


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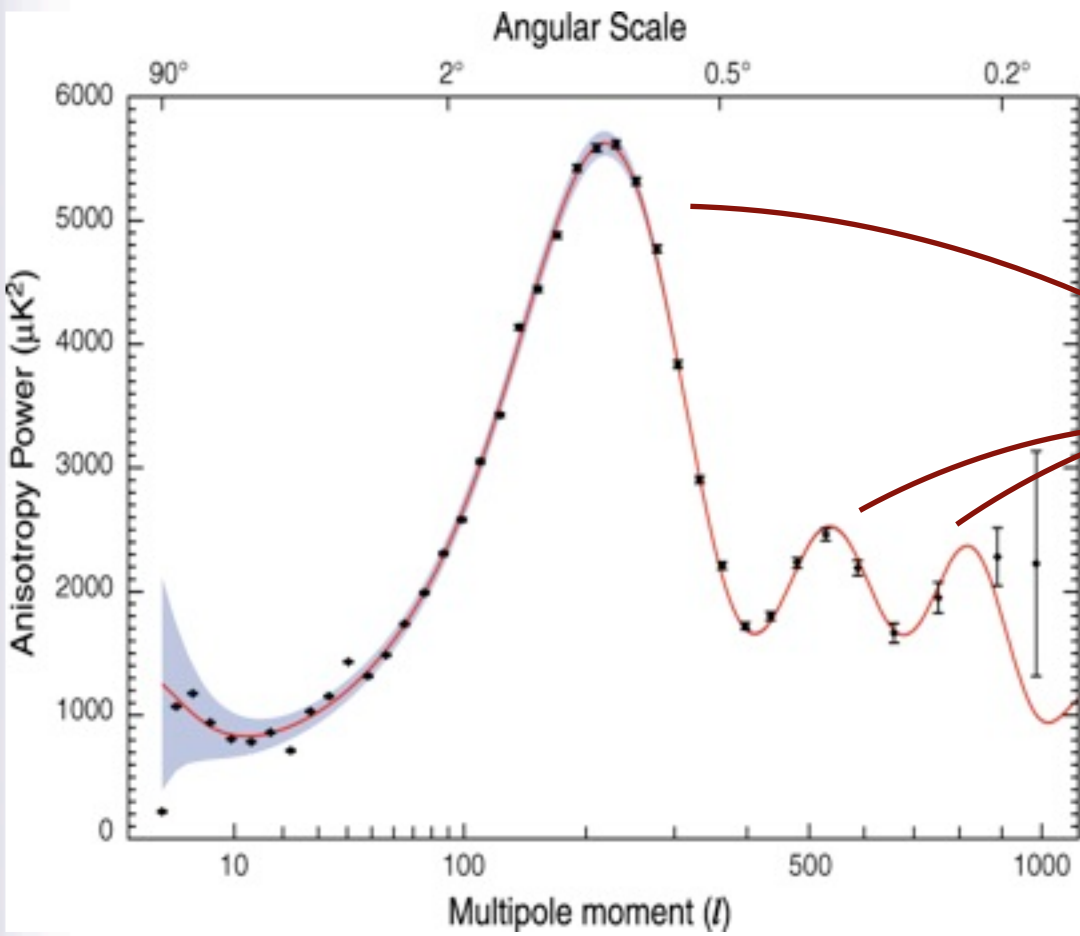


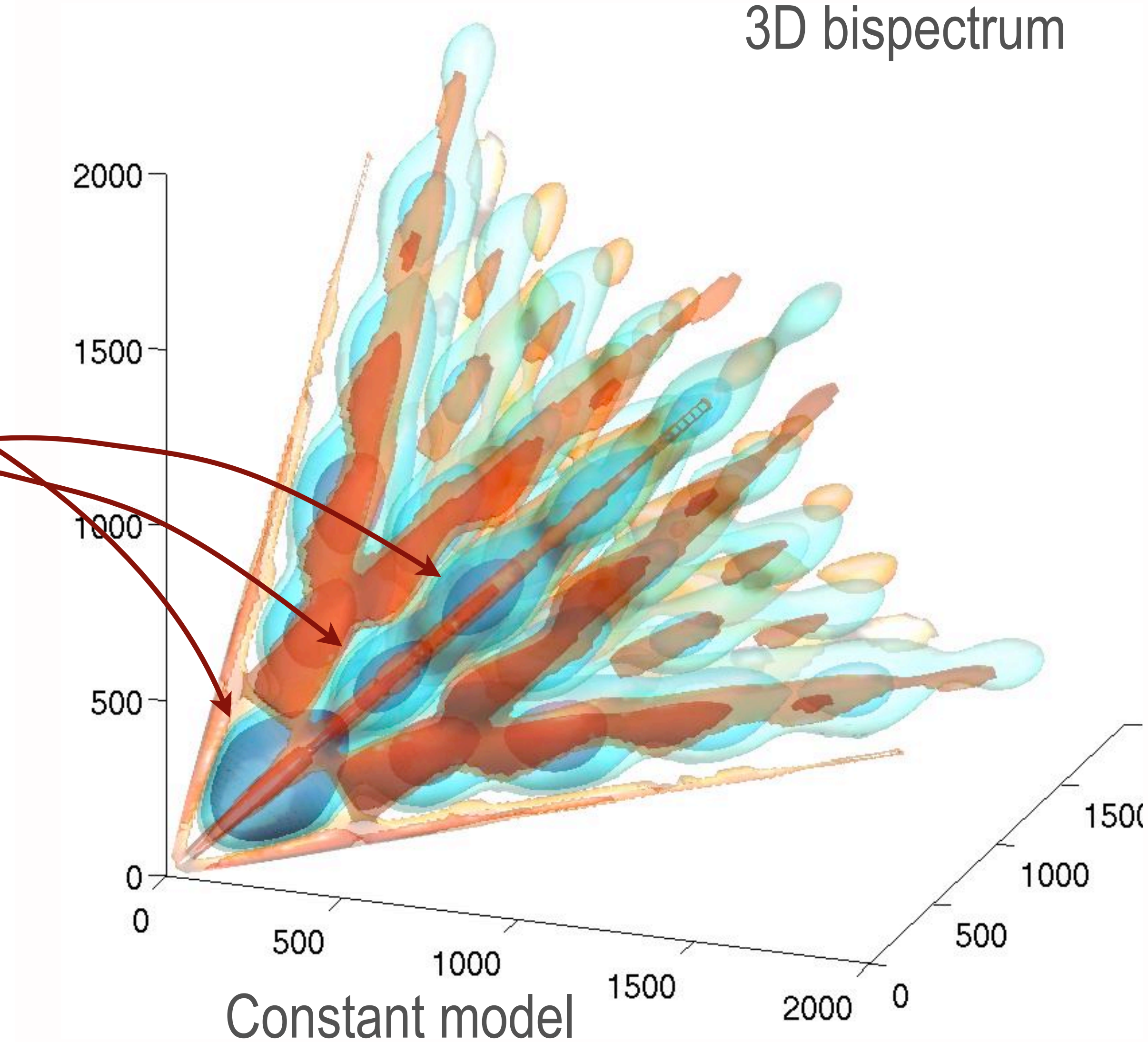


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*Bispectrum (3pt correlator)*

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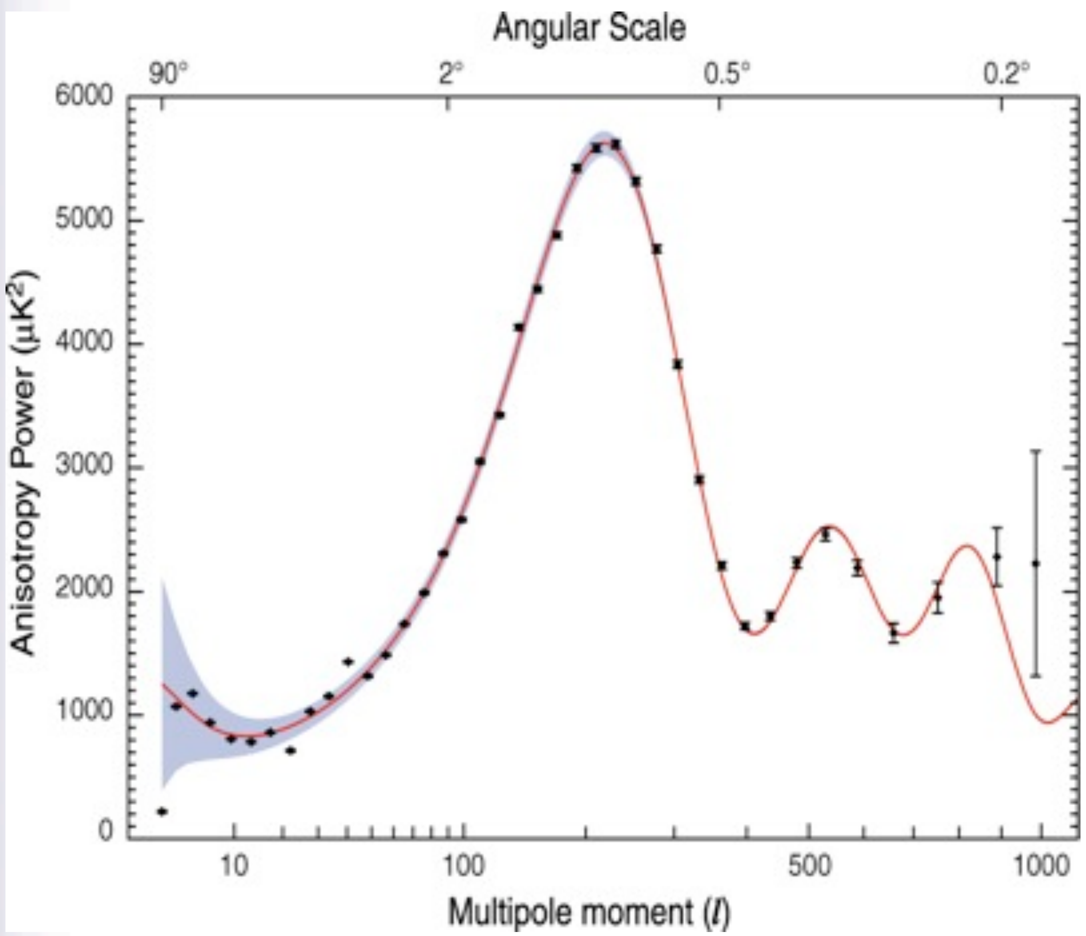
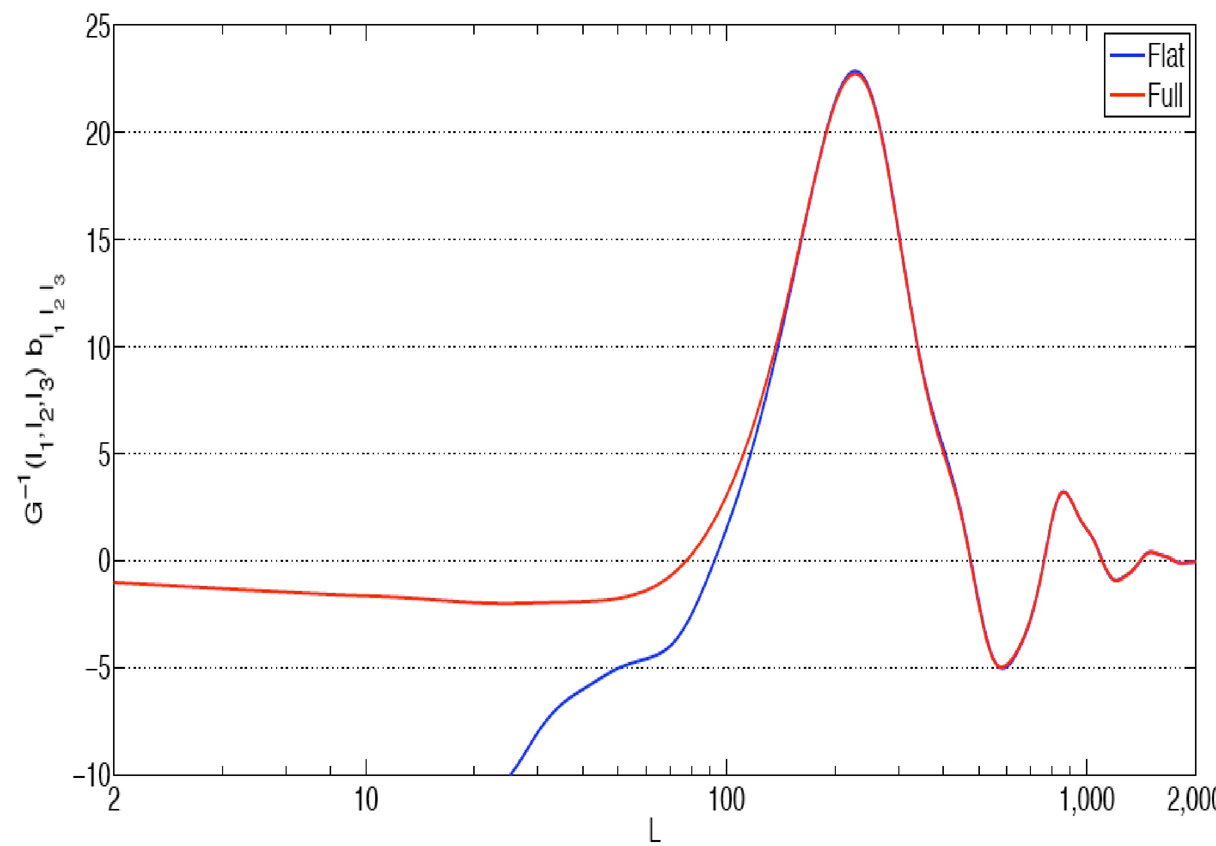
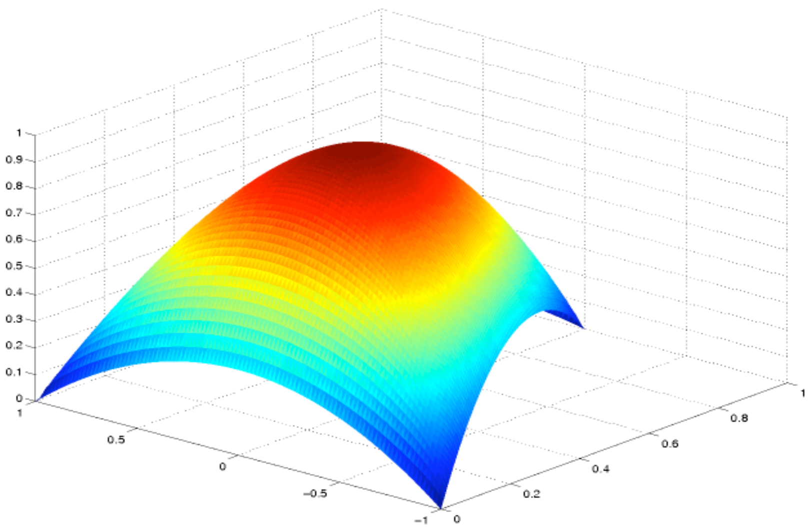


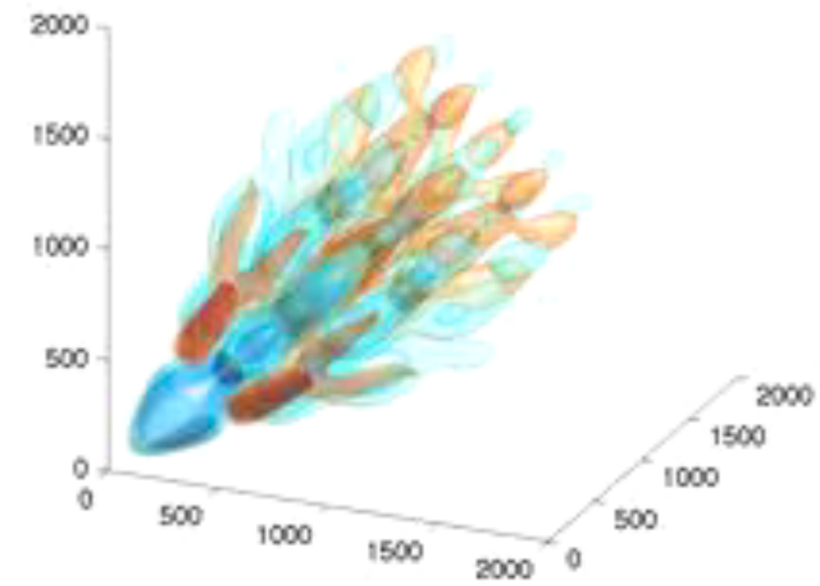



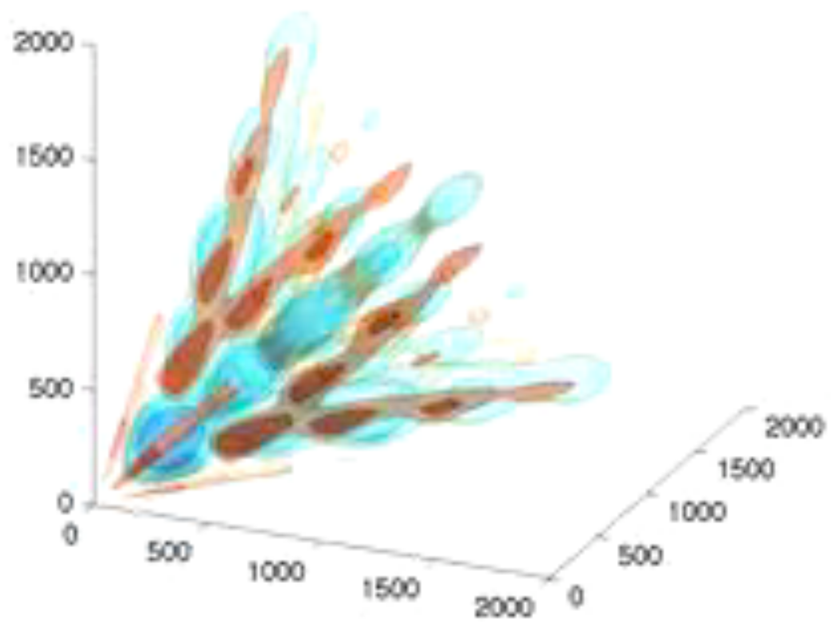

*Power spectrum (2pt correlator)*

*Equal-l $B_{lll}$ Bispectrum*

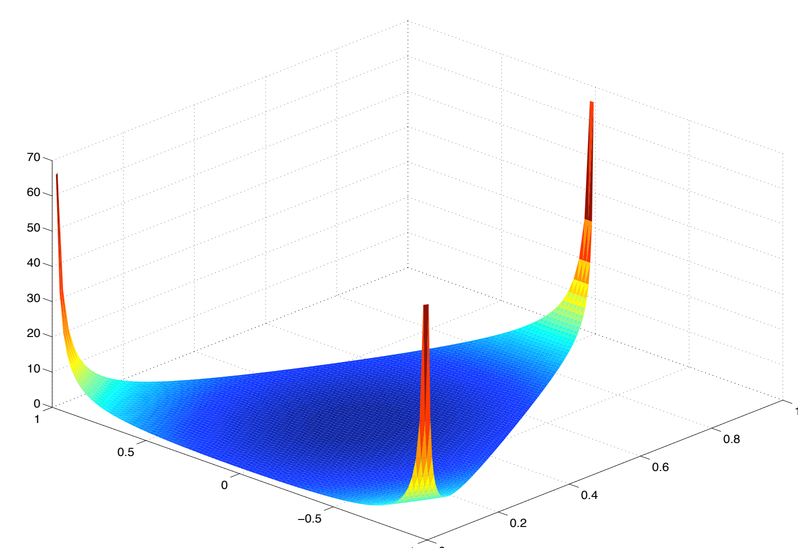
# DBI Inflation »»»»




# «« Multifield Inflation




*Robust CMB $b_{lll}$ calculation – see arXiv:1008.1730*

# Bispectrum estimator

**Purpose:** Test a model with predicted theoretical bispectrum

$$b^{\rm th}_{l_1 l_2 l_3} = \sum_{m_i} \mathcal{G}^{l_1\, l_2\, l_3}_{m_1 m_2 m_3} \langle a^{\rm th}_{l_1 m_1} a^{\rm th}_{l_2 m_2} a^{\rm th}_{l_3 m_3} \rangle$$

**Estimator** gives a least squares fit to the data

$$\begin{aligned} \mathcal{E} &= \frac{1}{N^2} \sum_{l_i, m_i} \langle a^{\rm th}_{l_1 m_1} a^{\rm th}_{l_2 m_2} a^{\rm th}_{l_3 m_3} \rangle (C^{-1} a)_{l_1 m_1} (C^{-1} a)_{l_2 m_2} (C^{-1} a)_{l_3 m_3} \\ &= \frac{1}{N^2} \sum_{l_i m_i} \frac{\mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3}\, b^{\rm th}_{l_1 l_2 l_3}\, a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}} \end{aligned}$$

*Babich, 2005*

with covariance matrix $C_{lm,l'm'} = \langle a_{lm} a_{l'm'} \rangle$

with inverse weighting $(C^{-1} a)_{lm} = C^{-1}_{lm,l'm'} a_{l'm'} \approx \dfrac{a_{lm}}{C_l}$ (ideal case)

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Model

Babich, 2005

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*Babich, 2005*

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Model

Signal

Noise

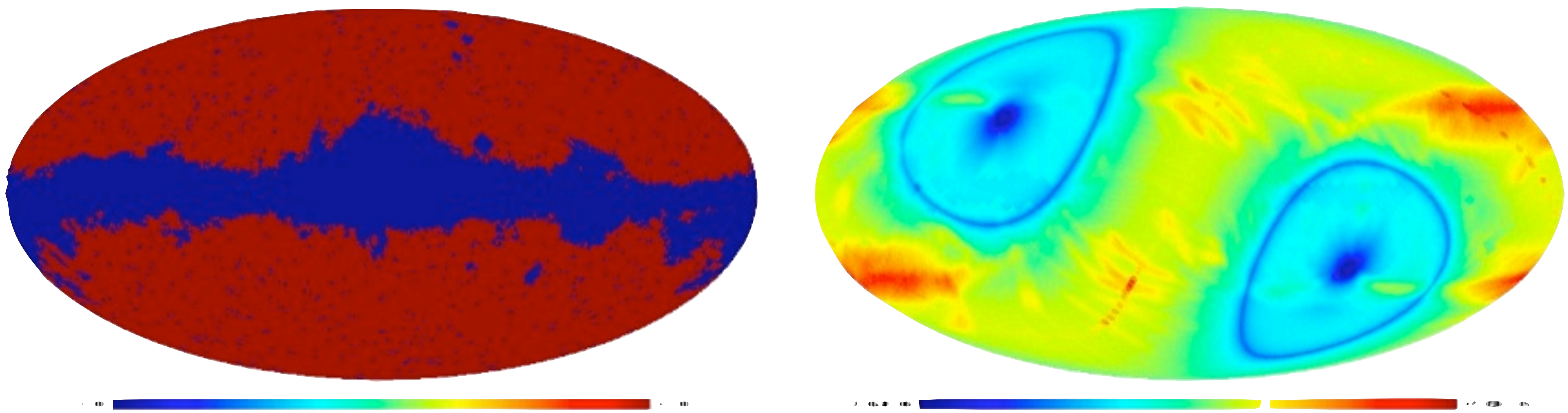
*Babich, 2005*

with covariance matrix $\quad C_{lm,l'm'} = \langle a_{lm} a_{l'm'} \rangle$

with inverse weighting $\quad (C^{-1}a)_{lm} = C^{-1}_{lm,l'm'} a_{l'm'} \approx \frac{a_{lm}}{C_l}$ $\qquad$ (ideal case)

# Realistic CMB estimator

Need to incorporate effects of masks, noise, and beams



WMAP KQ75 Mask

WMAP inhomogeneous noise

Linear term to subtract out spurious contributions *Creminelli et al, 2006*

$$\mathcal{E} = \frac{1}{\tilde{N}^2} \sum_{l_i m_i} \frac{\mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3}\; \tilde{b}^{\rm th}_{l_1 l_2 l_3}}{\tilde{C}_{l_1} \tilde{C}_{l_2} \tilde{C}_{l_3}} \left( a_{l_1 m_1} a_{l_2 m_2} - 6\, C^{\rm sim}_{l_1 m_1, l_2 m_2} \right) a_{l_3 m_3}$$

where $C_l^{\rm sim}$ is an average over many realistic Gaussian realizations

Include effects of noise, beams and masking through

$$\tilde{C}_l = b_l^2 C_l + N_l \qquad \text{and} \qquad \tilde{b}_{l_1 l_2 l_3} = b_{l_1} b_{l_2} b_{l_3}\, b_{l_1 l_2 l_3}$$

$$b^{\rm mask}_{l_1 l_2 l_3} = f_{\rm sky} b_{l_1 l_2 l_3} \qquad \text{and} \qquad C_l^{\rm mask} = f_{\rm sky} C_l$$

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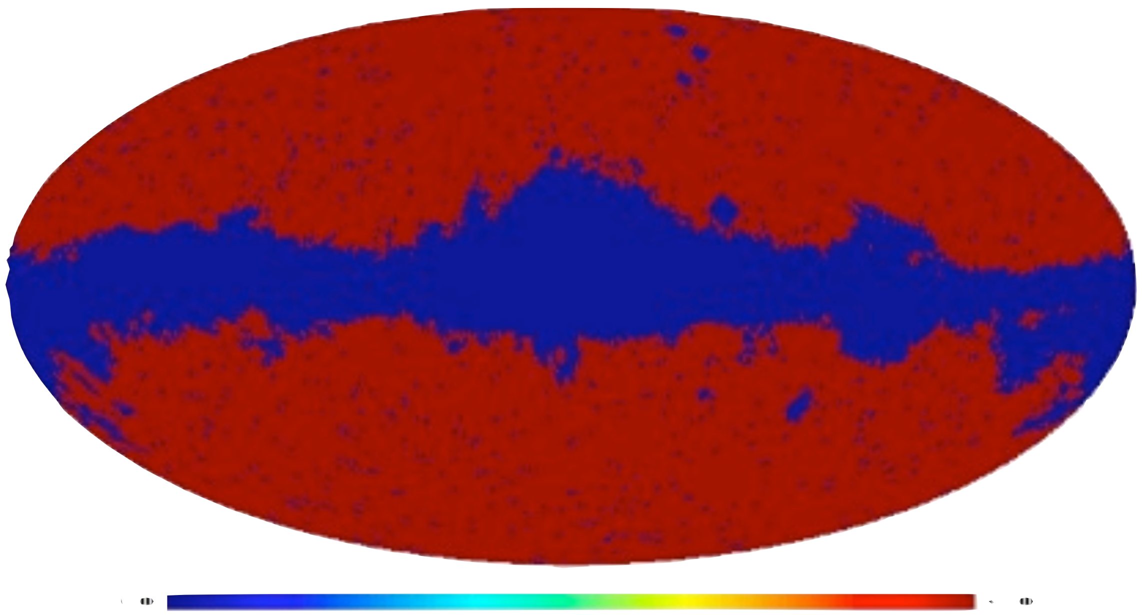

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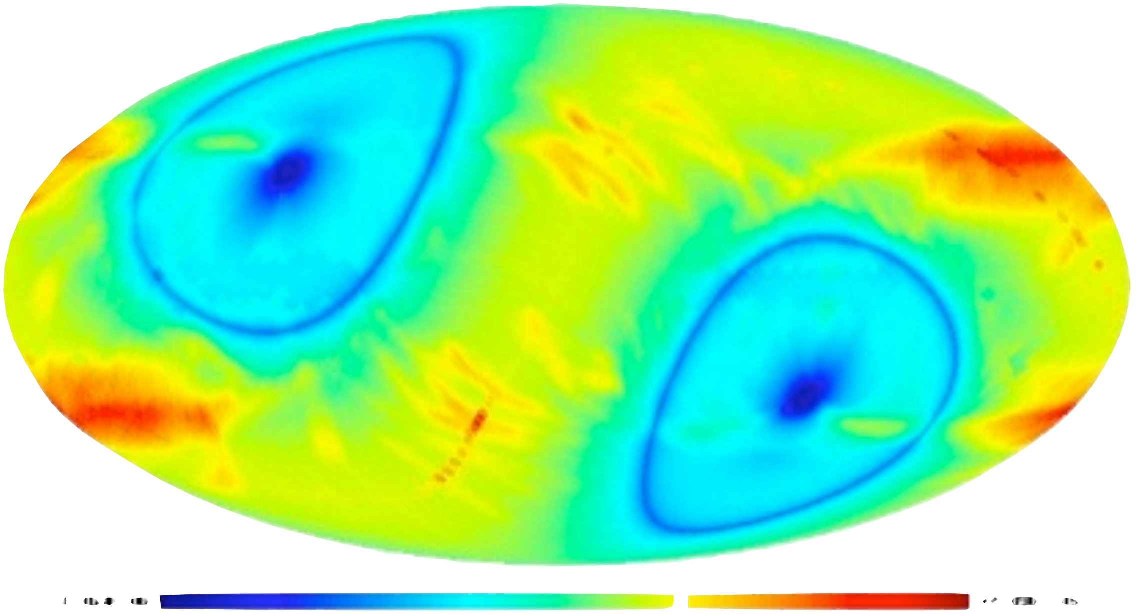

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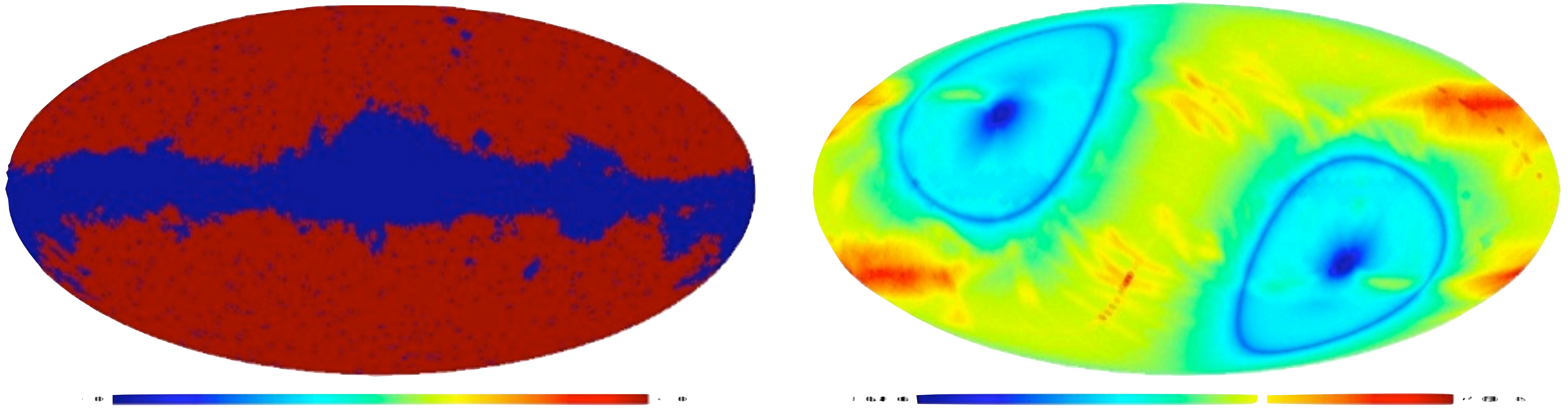

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# Bispectrum constraints

Computational cost of direct CMB estimation is Operations $\sim 10^3 \times L^5$

Separable primordial models: $B(k_1,k_2,k_3) = X(k_1)Y(k_2)Z(k_3) + \text{perms.}$

these require only Operations $\sim 10^3 \times L^2$ ($L=l_{max}$)

*TRACTABILITY = SEPARABILITY!*

*Example:* Local model where $B(k_1,k_2,k_3) = P(k_1)P(k_2) + \text{perms.}$

Recall the 4D bispectrum integral

$$b_{l_1 l_2 l_3} = \left(\frac{2}{\pi}\right)^3 \int x^2 dx \int dk_1 dk_2 dk_3\, S(k_1,k_2,k_3) \times \Delta_{l_1}(k_1)\,\Delta_{l_2}(k_2)\,\Delta_{l_3}(k_3)\, j_{l_1}(k_1 x)\, j_{l_2}(k_2 x)\, j_{l_3}(k_3 x)$$

which reduces to

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e.g. local model $-10 < f_{\rm NL} < 74$ (Komatsu et al, 2010)

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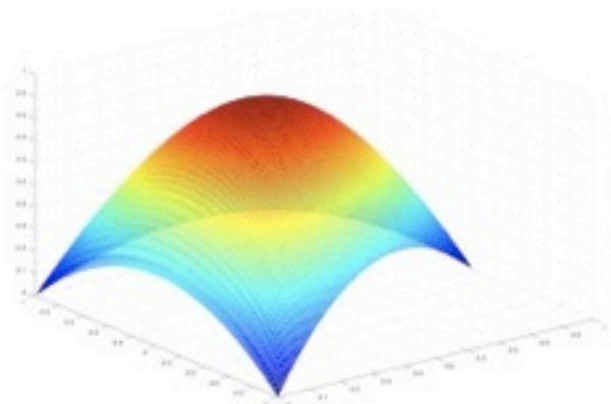
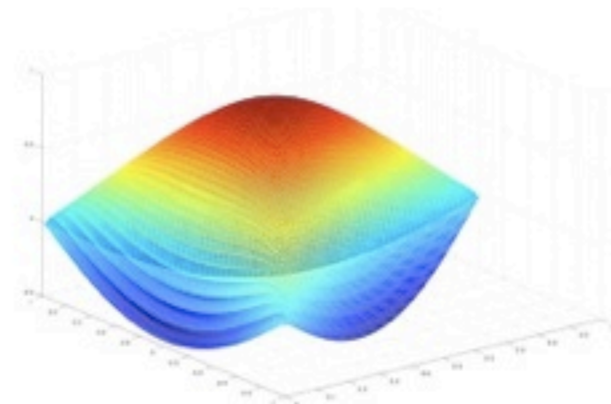
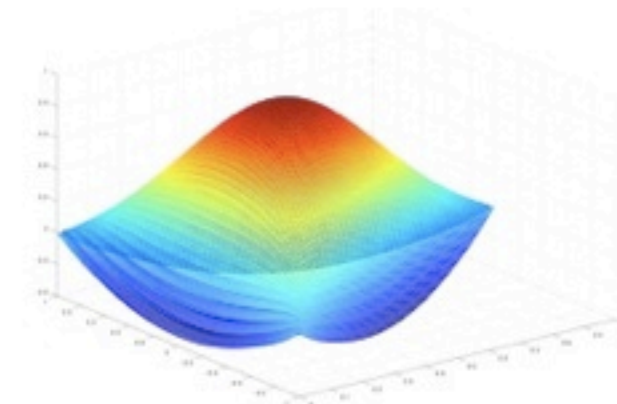
# Three models down, ∞ to go ....



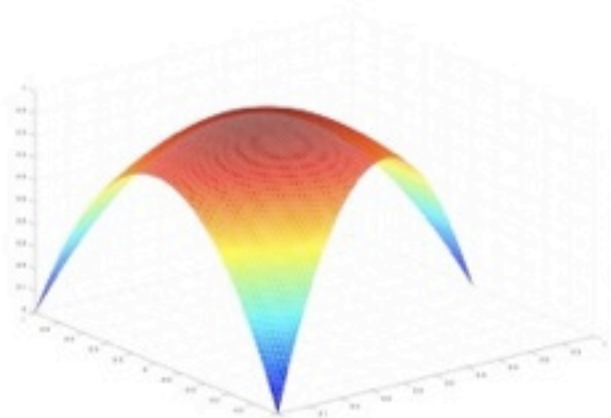
DBI



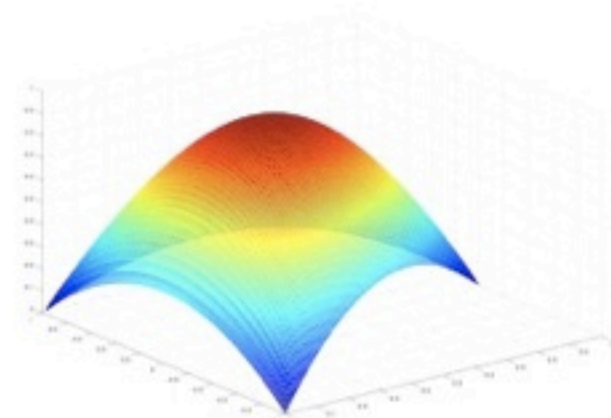
DBI with angular momentum



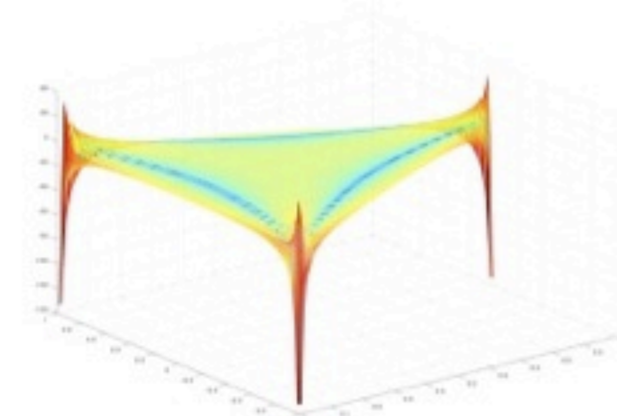
Ghost



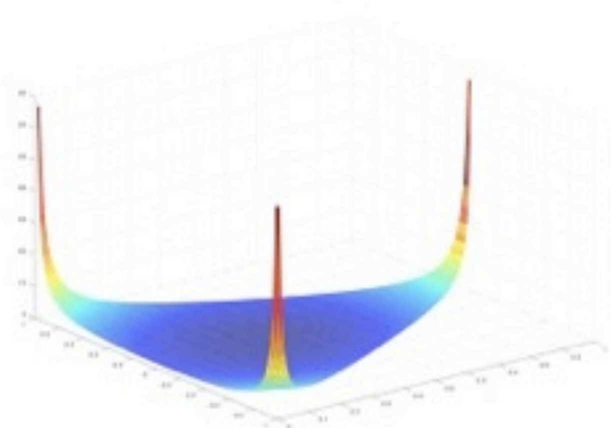
Single Field 1



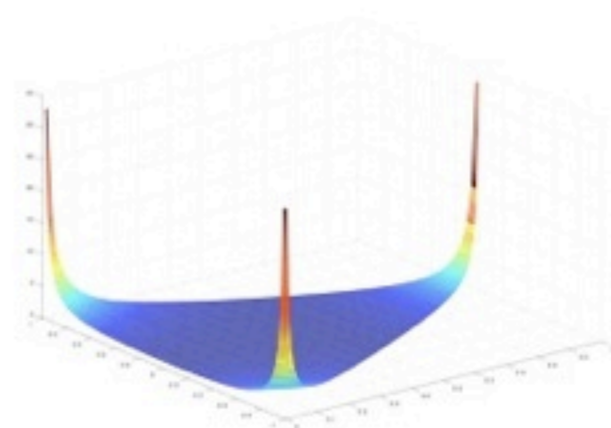
Single Field 2



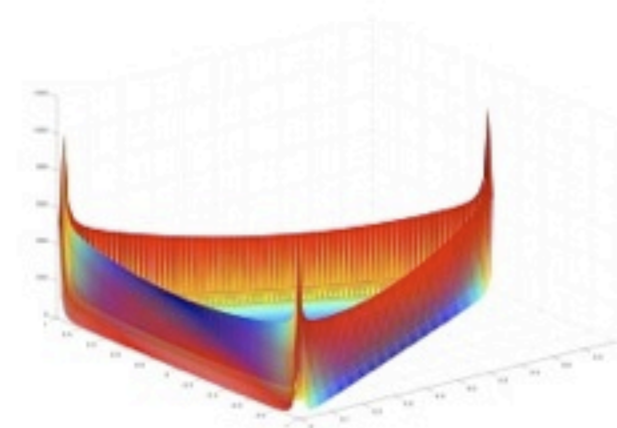
Warm



Non-Local



Exited Initial States 1

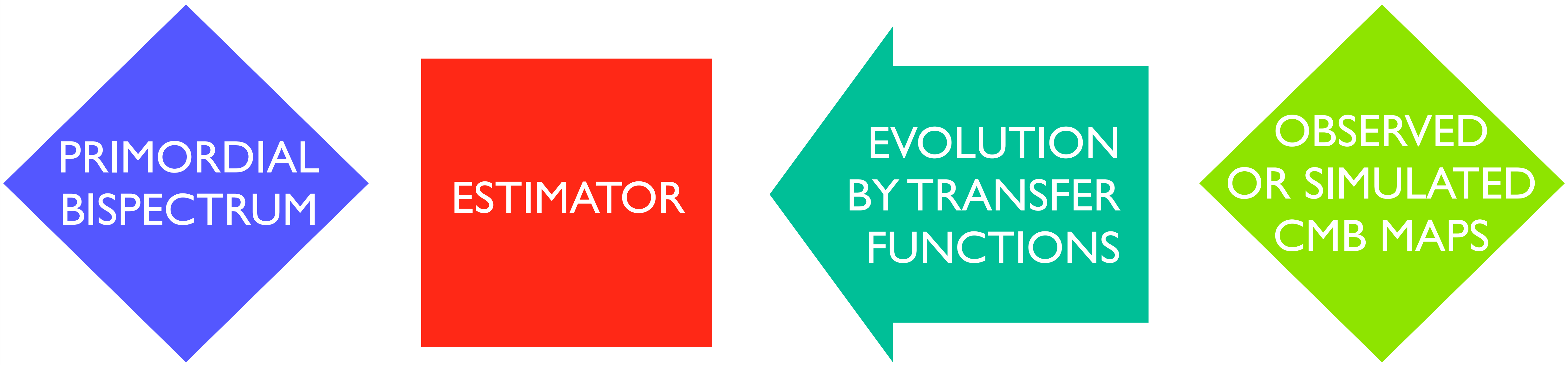


Exited Initial States 2

# General Modal Estimator

astro-ph/0612713, arXiv:1012.6039

Current estimators (e.g. KSW) extract primordial signals from the CMB




1. **Late-time estimators** filter for polyspectra in the CMB map

   ***Efficient:*** Transfer functions evaluated only once for primordial theory

   ***Flexible:*** Can seek late-time contributions (e.g. strings) / contaminants

2. **Modal expansion** allows efficient separable study of ***all bispectra***

$$b_{l_1 l_2 l_3} = \sum_p \sum_r \sum_s \alpha_{prs}\, q_p(l_1)\, q_r(l_2)\, q_s(l_3)$$

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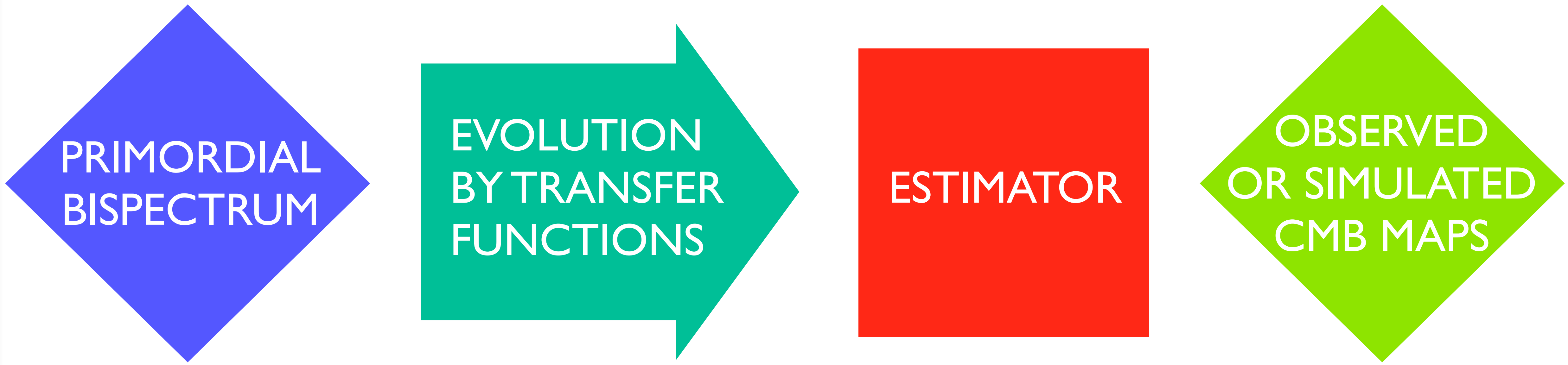


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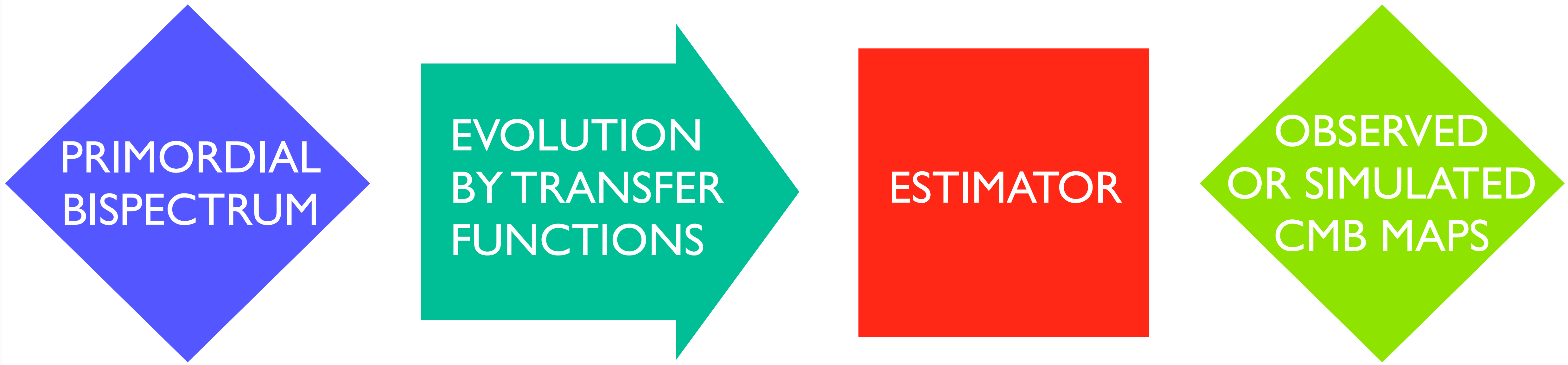


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# Tetrapyd - bispectrum domain

Allowed multipoles $l_1, l_2, l_3$ for the CMB bispectrum live in the domain

| | |
|---|---|
| Resolution: | $l_1, l_2, l_3 \leq l_{\max}\,, \quad l_1, l_2, l_3 \in \mathbb{N}\,,$ |
| Triangle condition: | $l_1 \leq l_2 + l_3$ for $l_1 \geq l_2,\ l_3,\quad$ + cyclic perms. |
| Parity condition: | $l_1 + l_2 + l_3 = 2n\,, \quad n \in \mathbb{N}\,.$ |

## Inner product:

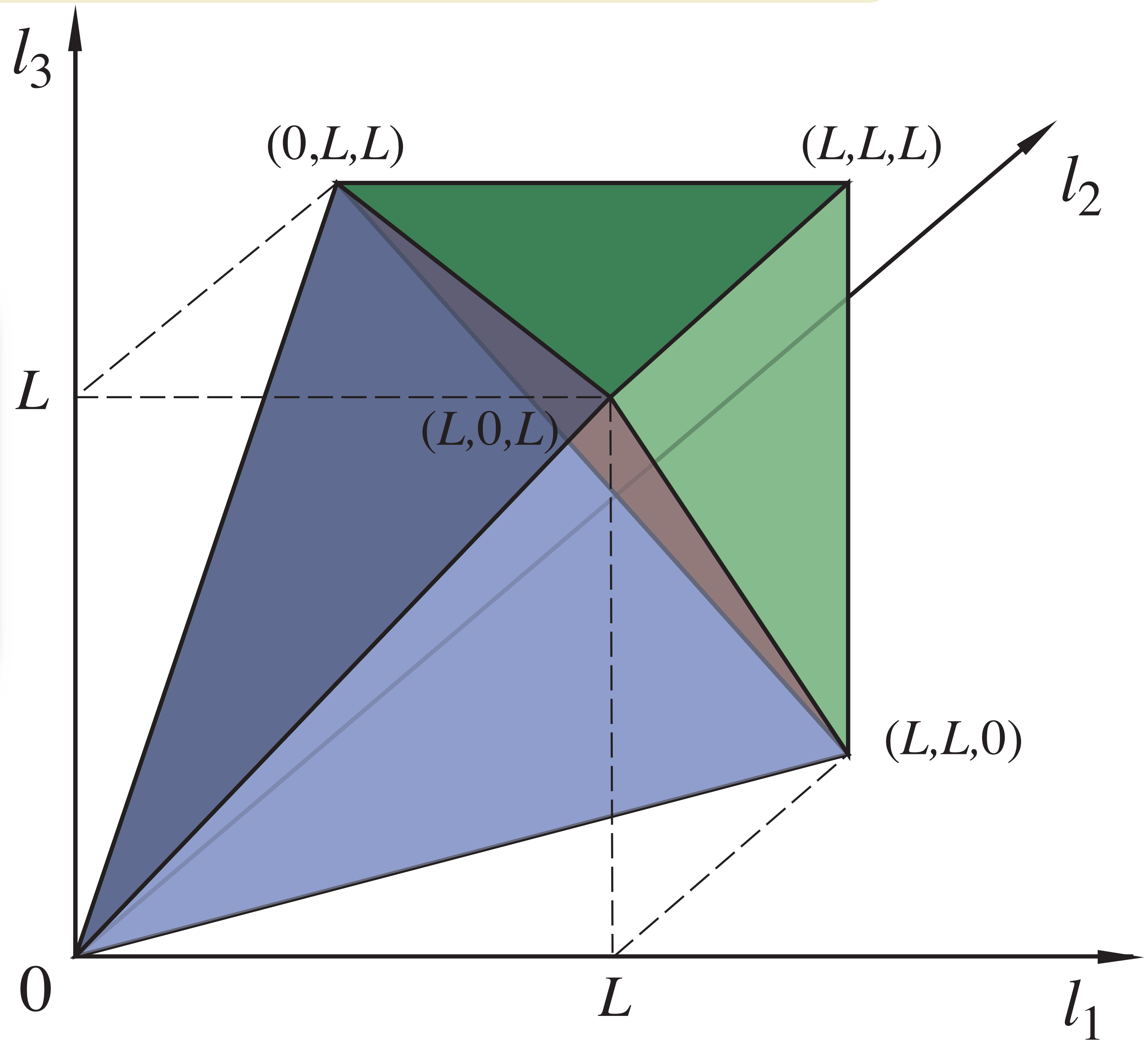
*Defined by estimator sum*

$$\langle b,\, b' \rangle \;\equiv\; \sum_{l_1,l_2,l_3 \in \mathcal{V}_{\mathcal{T}}} w_{l_1 l_2 l_3}\, b_{l_1 l_2 l_3}\, b'_{l_1 l_2 l_3}$$

*with weight* $\quad w_{l_1 l_2 l_3} = h^2_{l_1 l_2 l_3}$

*an (uninteresting) geometric factor*

$$h_{l_1 l_2 l_3} = \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}$$



# Separable Q modes

Need complete set of separable eigenmodes spanning tetrapyd

Define **separable basis functions $\mathbf{Q}_n$**

$$\begin{aligned}\overline{\mathcal{Q}}_n(l_1,l_2,l_3) &= \tfrac{1}{6}[\bar{q}_p(l_1)\,\bar{q}_r(l_2)\,\bar{q}_s(l_3) + \bar{q}_r(l_1)\,\bar{q}_p(l_2)\,\bar{q}_s(l_3) + \text{cyclic perms in } prs] \\ &\equiv \bar{q}_{\{p}q_r q_{s\}} \quad \text{with} \quad n \leftrightarrow \{prs\}\,,\end{aligned}$$

where $q_p(l)$ can be Legendre polynomials, trig. functions, wavelets etc.

Note: $\langle \overline{\mathcal{Q}}_n,\, \overline{\mathcal{Q}}_p\rangle \equiv \gamma_{np} \neq \delta_{np}$ are not in general orthogonal

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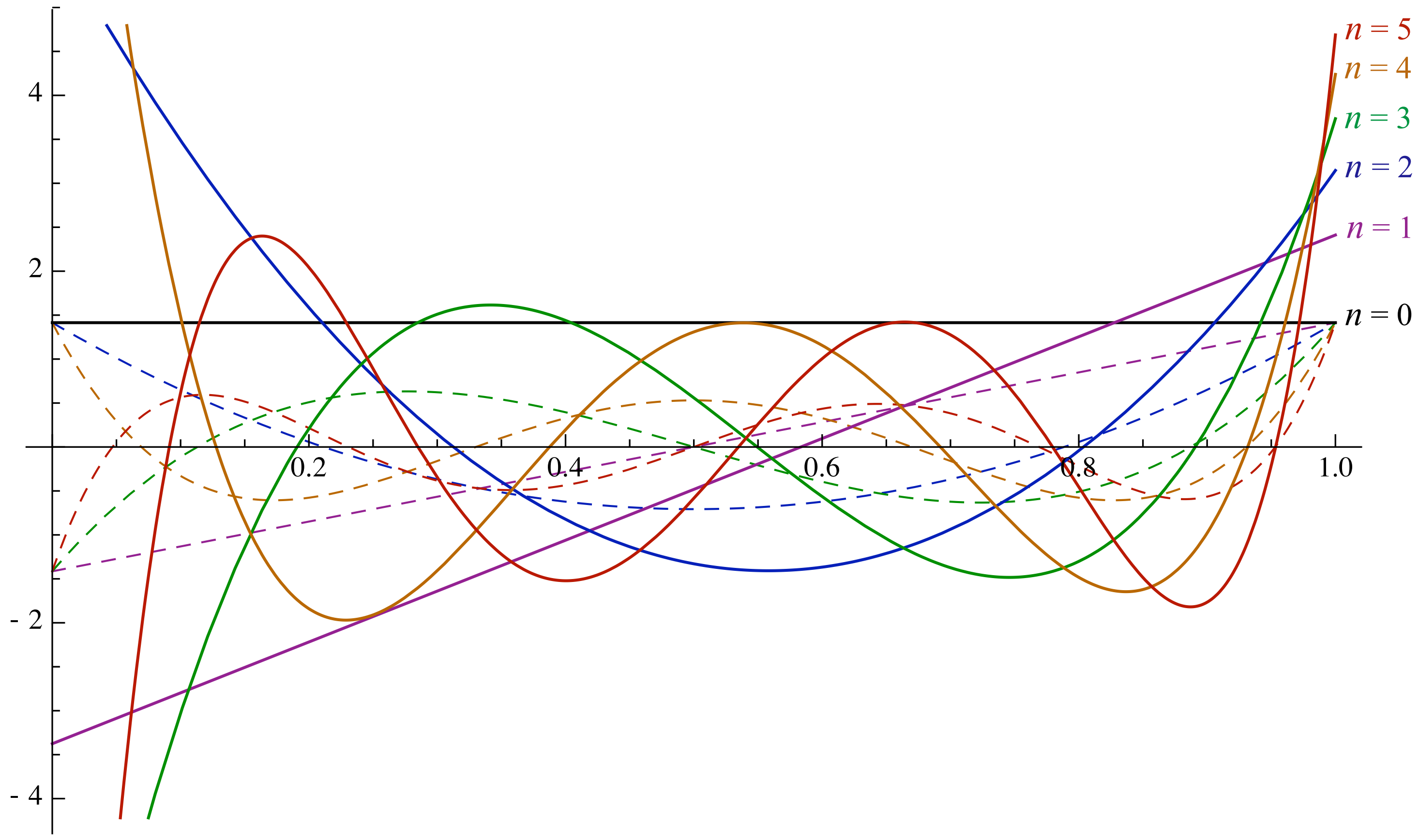


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# Orthonormal R basis

Finally, we orthonormalise the *Q* to create the basis *R*

$$\langle \bar{\mathcal{R}}_n, \bar{\mathcal{R}}_p \rangle = \delta_{np} \quad \text{with} \quad \mathcal{R}_n = \sum_{p=0}^{n} \lambda_{mp} \mathcal{Q}_p$$

We need to calculate $\lambda_{nm}$

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We need to calculate $\lambda_{nm}$

*This can be achieved step-by-step with Gram-Schmidt*

*Or (better) by Cholesky decomposition,*

*since* $\lambda_{nm}$ *is lower triangular*

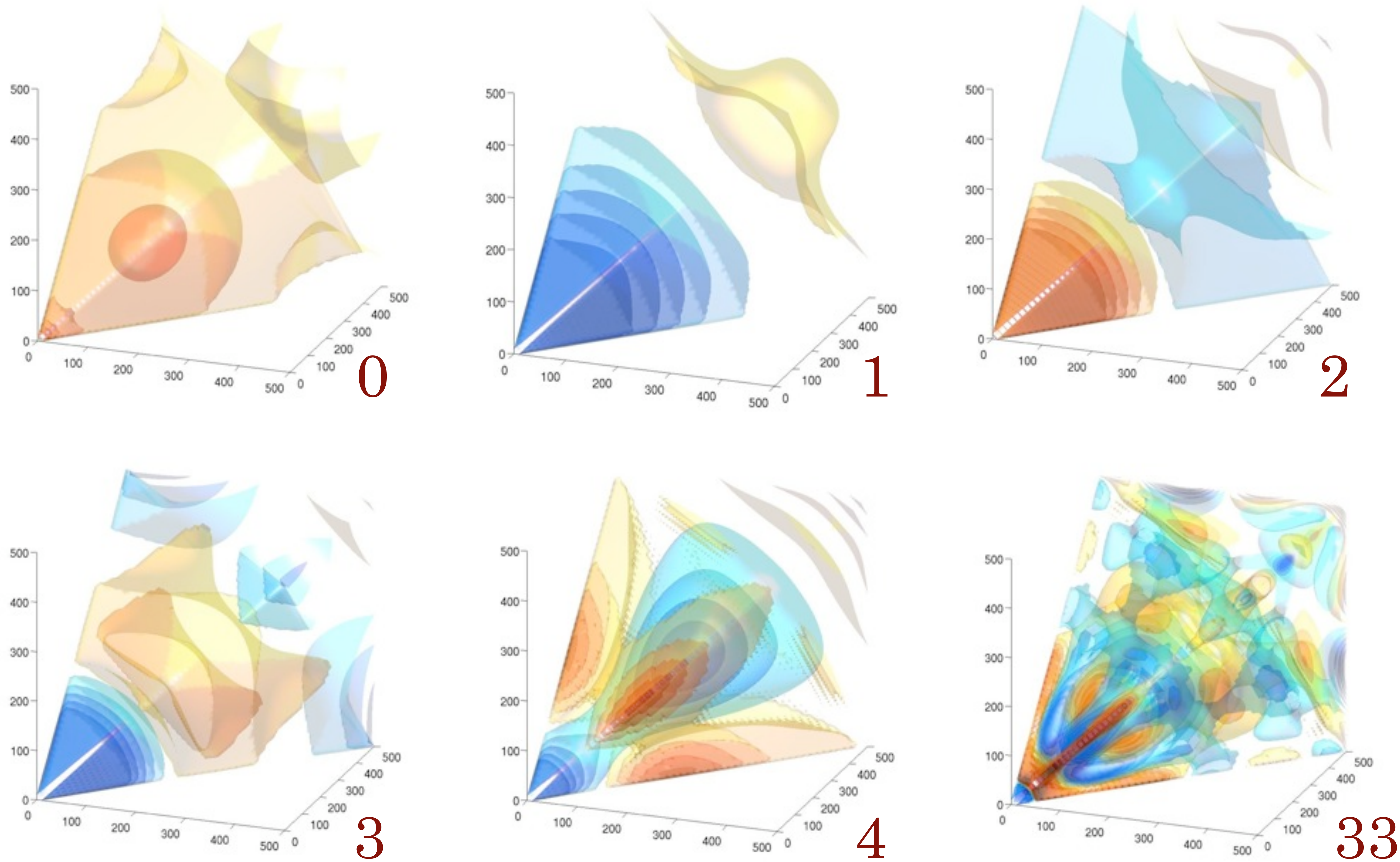
*Here* $\langle R_n R_m \rangle = \lambda_{nr} \lambda_{ms} \langle Q_r Q_s \rangle$

$$\langle Q_r Q_s \rangle = \gamma_{rs}$$

*so* $I = \lambda \gamma \lambda^T$ *and* $\gamma = \lambda^{-1} {\lambda^{-1}}^T$

*(See lecture 2.)*

# Orthogonal bispectrum modes

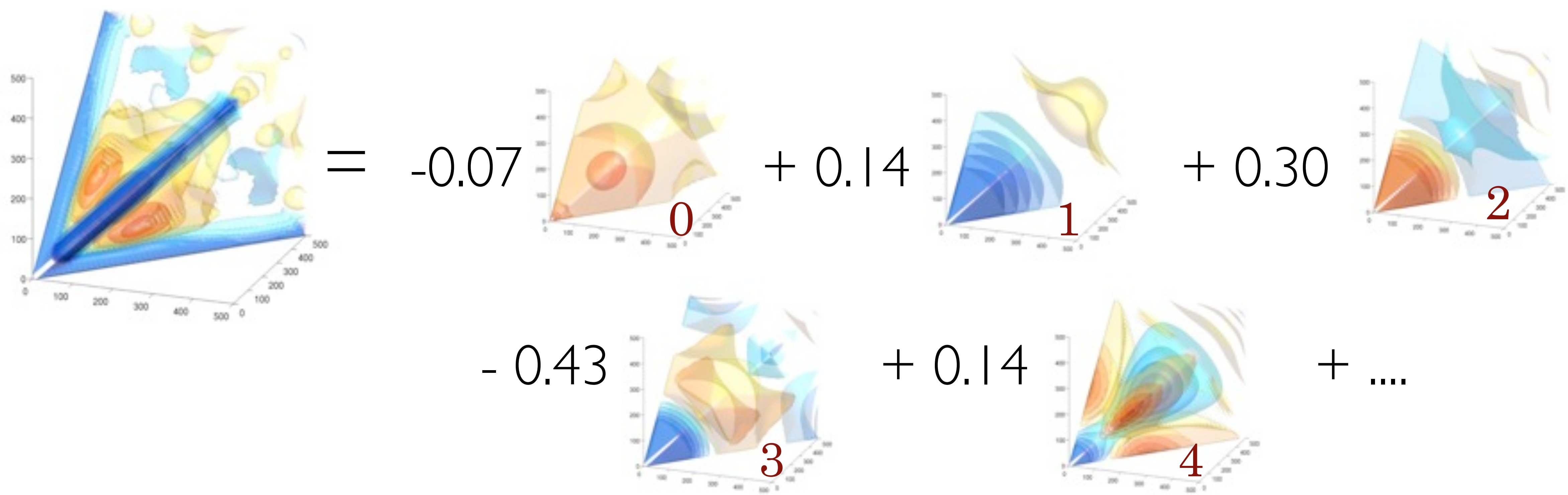


# Arbitrary bispectrum expansion

Expand the bispectrum signal strength as

$$\frac{v_{l_1} v_{l_2} v_{l_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}} b_{l_1 l_2 l_3} = \sum_n \bar{\alpha}_n^{\mathcal{R}} \overline{\mathcal{R}}_n$$

E.g. Local $f_{NL}$ Model expansion:

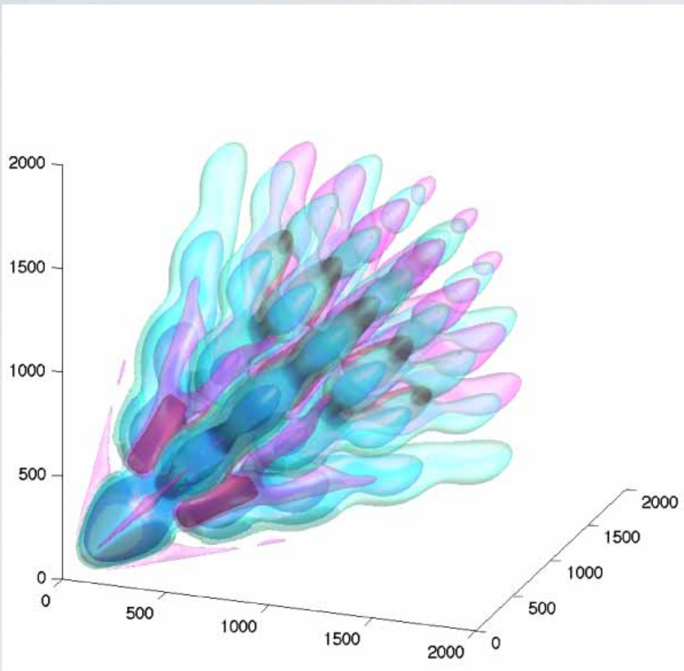



Almost all theoretical bispectra require $n_{max} < 30$ at WMAP resolution!

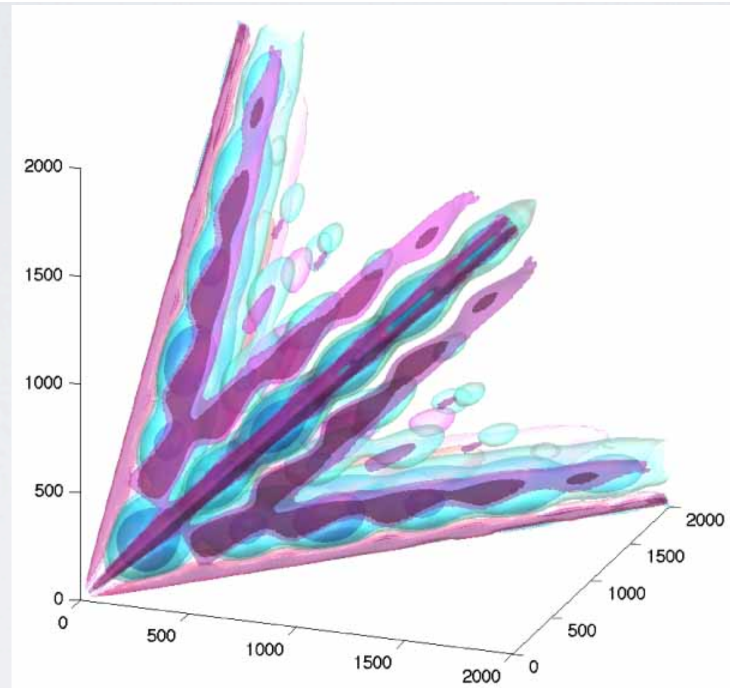
# *Applicable to inflation, defects, secondaries*

*Fergusson & EPS, arXiv:1008.1730*

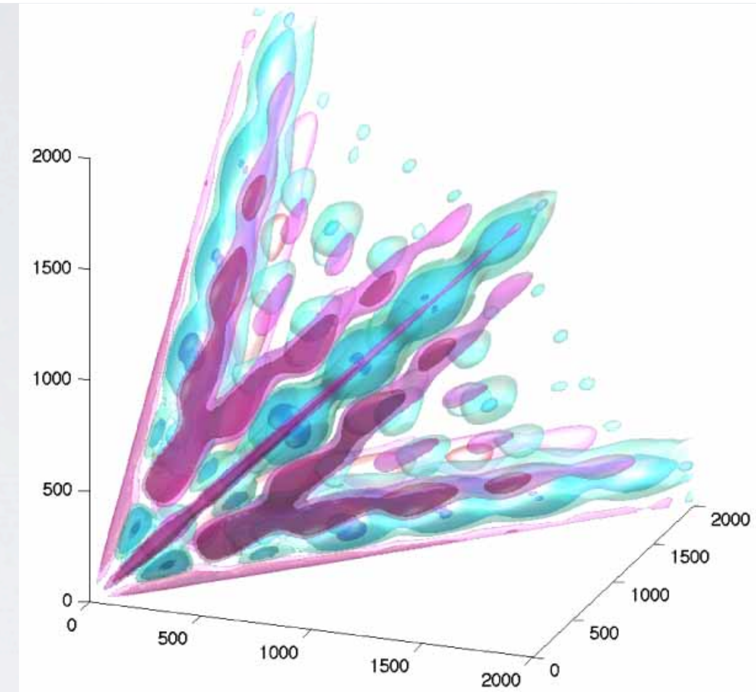
Planck resolution CMB bispectra (multipoles $l_1, l_2, l_3$)



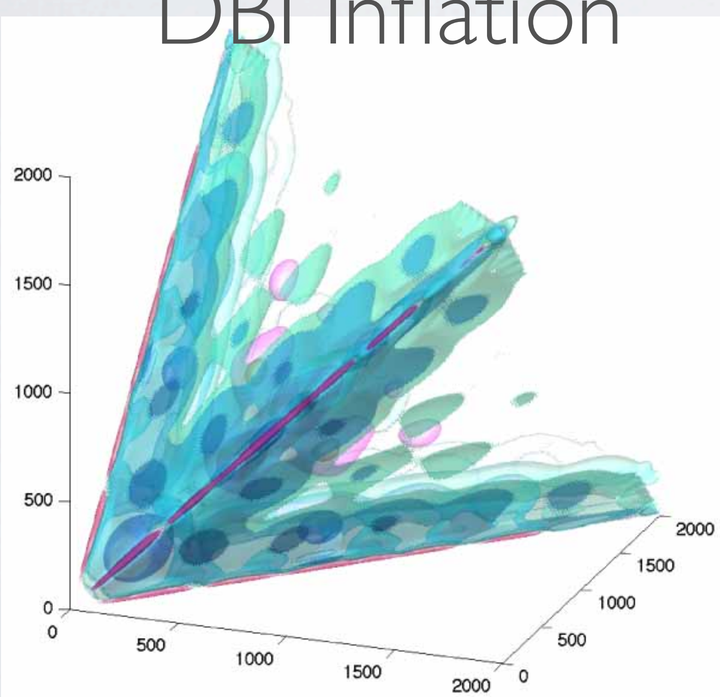

DBI Inflation



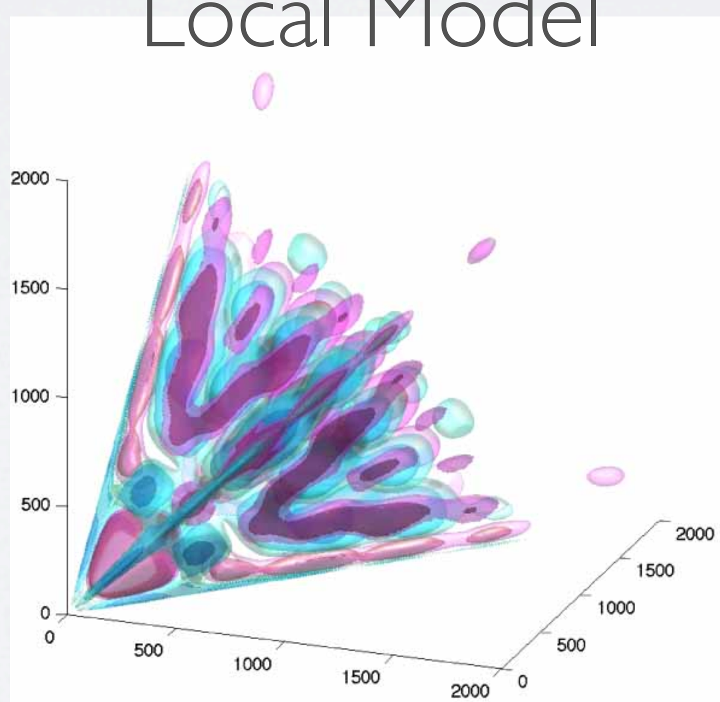

Local Model



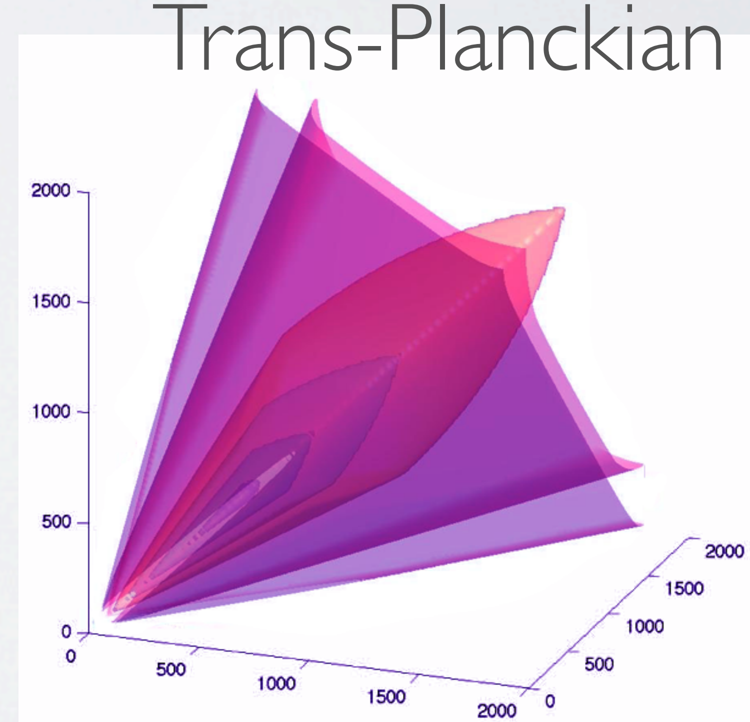

Trans-Planckian




Warm inflation

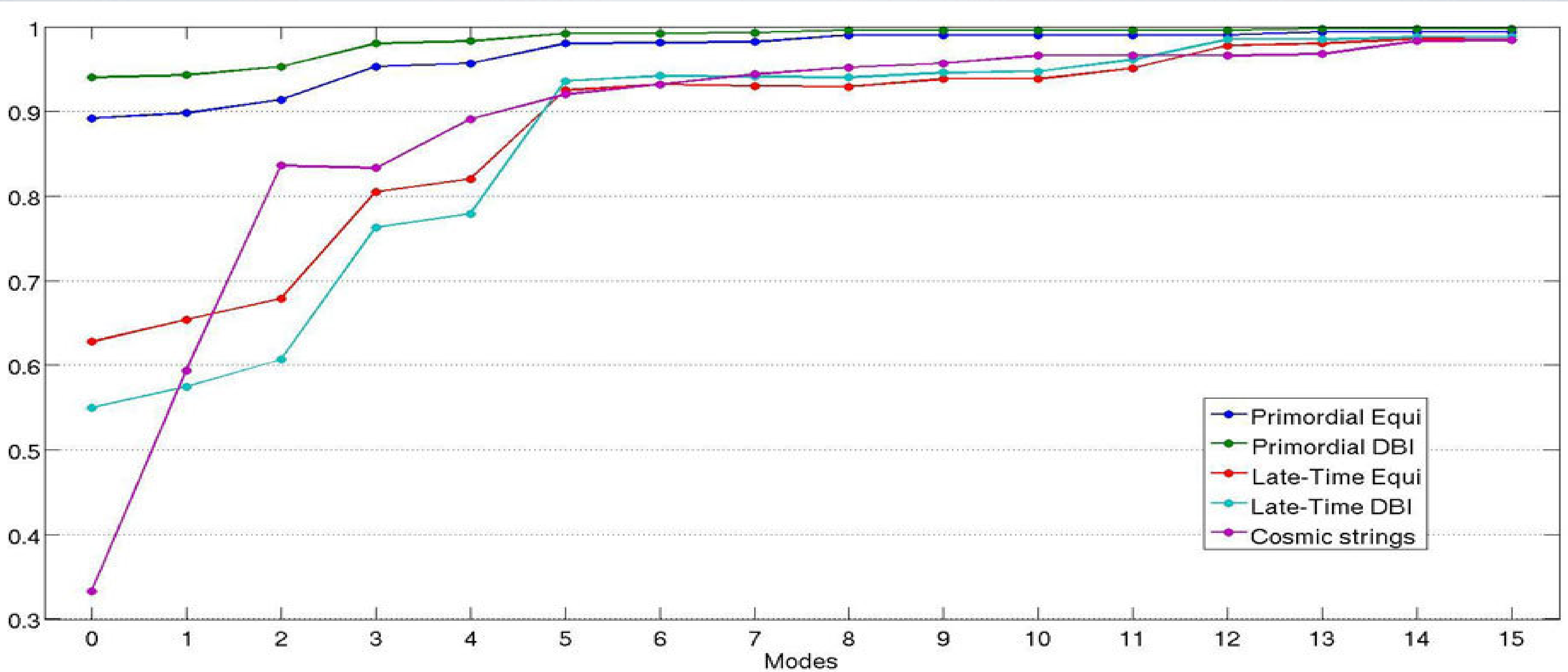



Feature Model




Cosmic strings

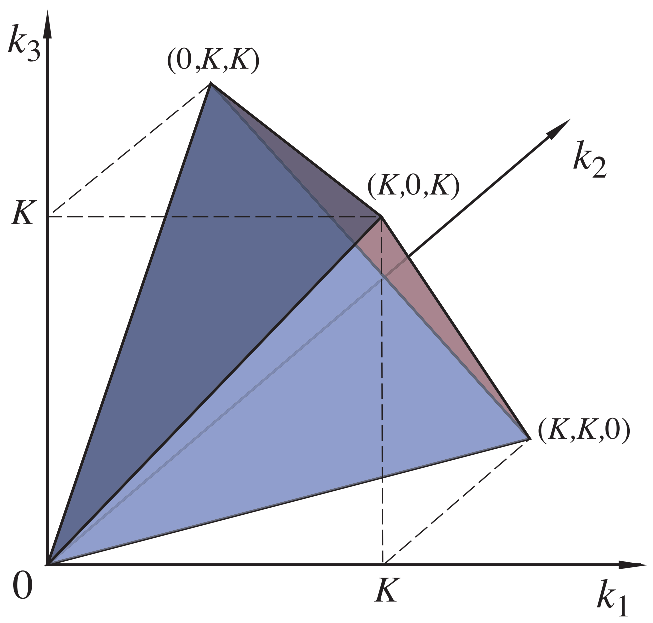
# Rapid modal convergence




Correlation of the separable approximation to the original bispectra, both primordial and CMB

# Primordial to CMB basis

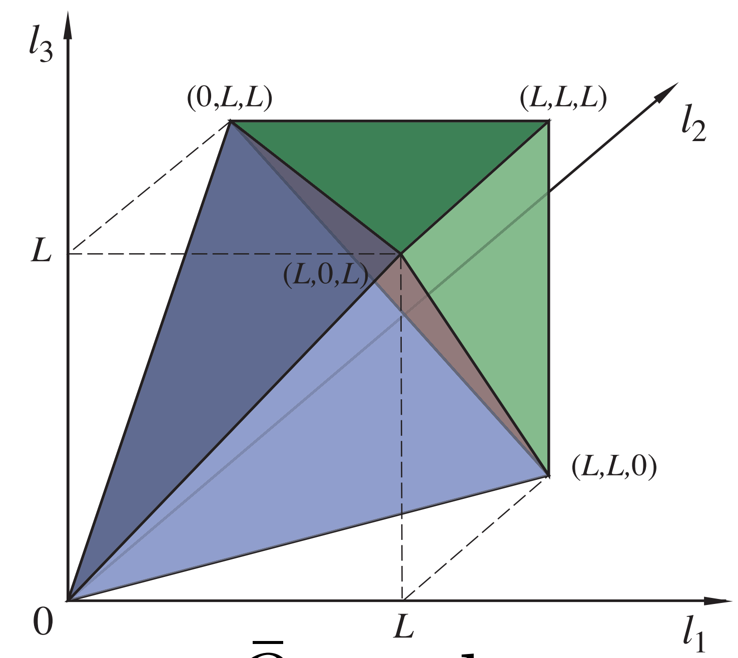
***Primordial bispectrum*** *(k-space)*




$Q$-modes

*Transfer functions*
$\Delta_l(k)$

***CMB bispectrum*** *(l-space)*




$\bar{Q}$-modes

Use transfer functions <u>*once*</u> to project forward primordial modes $Q \longrightarrow \tilde{Q}$, so we calculate

$$\Gamma_{nm} = \left\langle \bar{Q}^n \frac{vvv\tilde{Q}^m}{\sqrt{CCC}} \right\rangle$$

Then we can transform between the primordial and CMB expansions

$$\bar{\alpha}^Q = \bar{\gamma}^{-1}\Gamma\alpha^Q$$

# Modal estimator

Substitute bispectrum mode expansion

$$\frac{v_{l_1} v_{l_2} v_{l_3}}{\sqrt{C_{l_1} C_{l_2} C_{l_3}}}\, b_{l_1 l_2 l_3} = \sum_n \bar{\alpha}_n^{\mathcal{Q}} \bar{\mathcal{Q}}_n(l_1, l_2, l_3)\,,$$

into the estimator $\mathcal{E} = \frac{1}{N^2} \sum_{l_i m_i} \frac{\mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3}\, b^{\mathrm{th}}_{l_1 l_2 l_3}\, a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}{C_{l_1} C_{l_2} C_{l_3}}$

$$\mathcal{E} = \sum_n \bar{\alpha}_n^{\mathcal{Q}} \bar{\beta}_n^{\mathcal{Q}}$$

where $\bar{\beta}_n^{\mathcal{Q}} = \int d^2\hat{\mathbf{n}}\, \bar{M}_{\{p}(\hat{\mathbf{n}}) \bar{M}_r(\hat{\mathbf{n}}) \bar{M}_{s\}}(\hat{\mathbf{n}}) \quad \text{with} \quad \bar{M}_p(\hat{\mathbf{n}}) = \sum_{lm} \bar{q}_p(l) \frac{a_{lm}}{v_l \sqrt{C_l}} Y_{lm}(\hat{\mathbf{n}})$

***Bonus:*** Full CMB bispectrum reconstruction since

$$\langle \bar{\beta}_n^{\mathcal{R}} \rangle = \bar{\alpha}_n^{\mathcal{R}}$$

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Substitute bispectrum mode expansion

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*Theory*

$$\boxed{\mathcal{E} = \sum_n \bar{\alpha}_n^{\mathcal{Q}} \bar{\beta}_n^{\mathcal{Q}}}$$

where $\bar{\beta}_n^{\mathcal{Q}} = \int d^2\hat{\mathbf{n}} \, \bar{M}_{\{p}(\hat{\mathbf{n}}) \bar{M}_r(\hat{\mathbf{n}}) \bar{M}_{s\}}(\hat{\mathbf{n}}) \quad \text{with} \quad \bar{M}_p(\hat{\mathbf{n}}) = \sum_{lm} \bar{q}_p(l) \frac{a_{lm}}{v_l \sqrt{C_l}} Y_{lm}(\hat{\mathbf{n}})$

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Theory

Observed

$$\boxed{\mathcal{E} = \sum_n \bar{\alpha}_n^{\mathcal{Q}} \bar{\beta}_n^{\mathcal{Q}}}$$

where $\bar{\beta}_n^{\mathcal{Q}} = \int d^2\hat{\mathbf{n}} \, \bar{M}_{\{p}(\hat{\mathbf{n}}) \bar{M}_r(\hat{\mathbf{n}}) \bar{M}_{s\}}(\hat{\mathbf{n}}) \quad \text{with} \quad \bar{M}_p(\hat{\mathbf{n}}) = \sum_{lm} \bar{q}_p(l) \frac{a_{lm}}{v_l \sqrt{C_l}} Y_{lm}(\hat{\mathbf{n}})$

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Substitute bispectrum mode expansion

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Theory

Observed

$$\boxed{\mathcal{E} = \sum_n \bar{\alpha}_n^{\mathcal{Q}} \bar{\beta}_n^{\mathcal{Q}}}$$

Mode filtered maps

where $\bar{\beta}_n^{\mathcal{Q}} = \int d^2\hat{\mathbf{n}}\, \bar{M}_{\{p}(\hat{\mathbf{n}}) \bar{M}_r(\hat{\mathbf{n}}) \bar{M}_{s\}}(\hat{\mathbf{n}}) \quad \text{with} \quad \bar{M}_p(\hat{\mathbf{n}}) = \sum_{lm} \bar{q}_p(l) \frac{a_{lm}}{v_l \sqrt{C_l}} Y_{lm}(\hat{\mathbf{n}})$

**Bonus:** Full CMB bispectrum reconstruction since

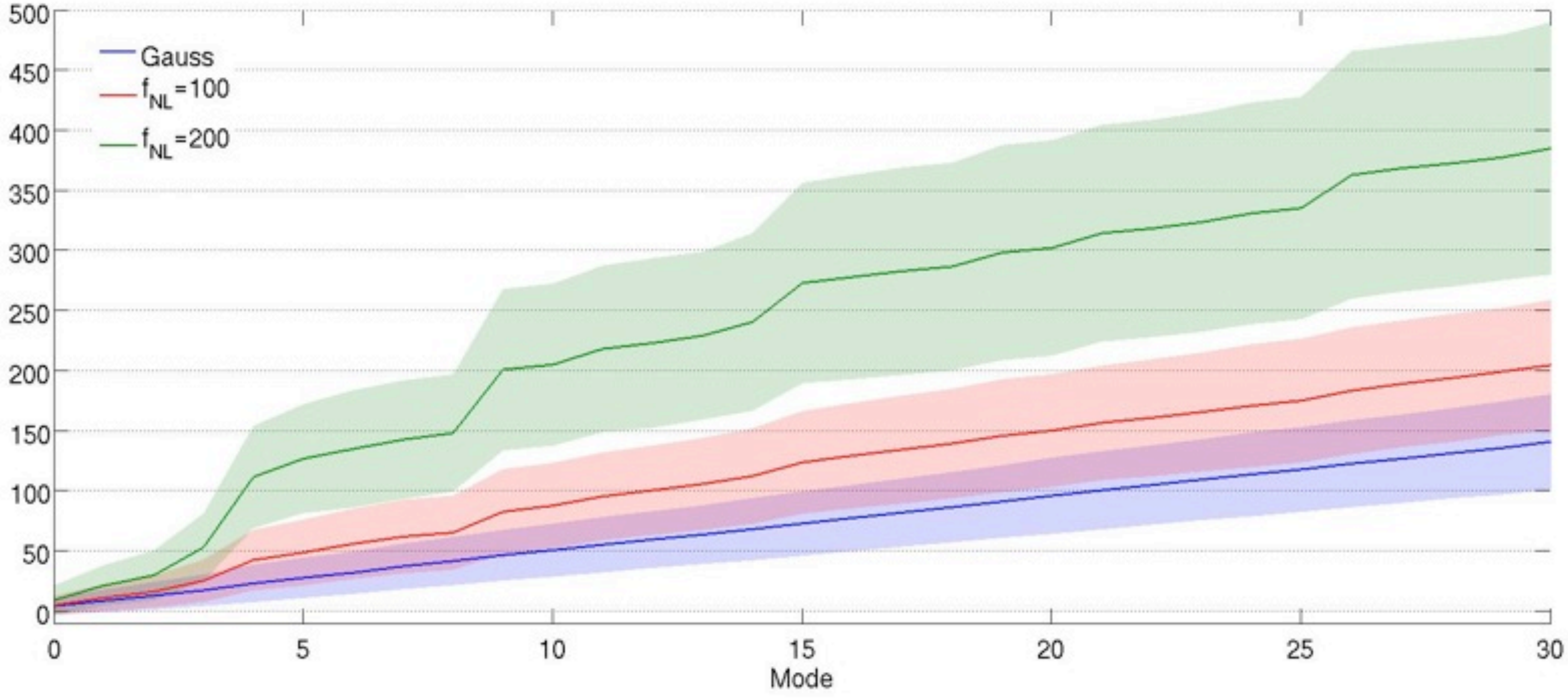
$$\langle \bar{\beta}_n^{\mathcal{R}} \rangle = \bar{\alpha}_n^{\mathcal{R}}$$

# Total bispectrum measure

Total integrated bispectrum (with Parseval's theorem)

$$\bar{F}_{\rm NL}^2 \equiv \frac{1}{N_{\rm loc}^2} \sum_{l_i} \frac{h_{l_1 l_2 l_3}^2 b_{l_1 l_2 l_3}^2}{C_{l_1} C_{l_2} C_{l_3}} = \frac{1}{N_{\rm loc}^2} \sum_n \bar{\alpha}_n^{\mathcal{R}\,2}$$

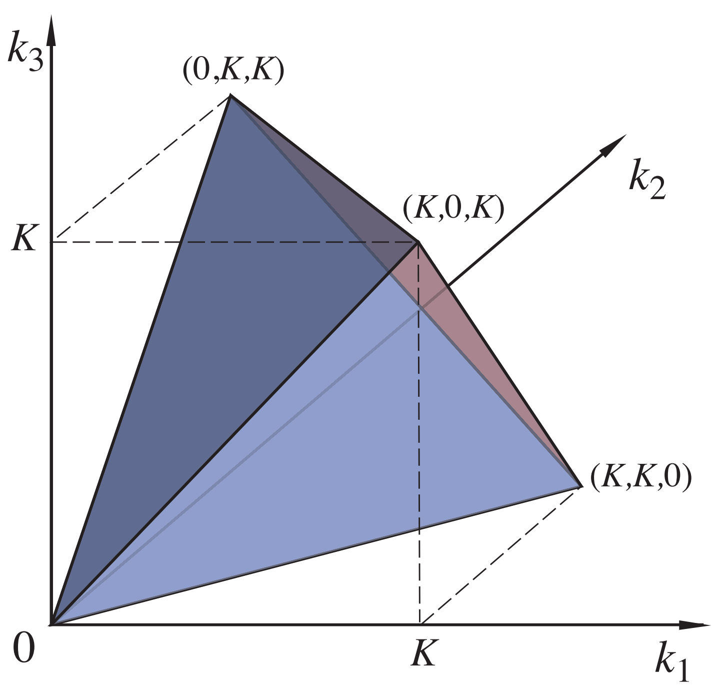
Remove the Gaussian mean $$\bar{F}_{\rm NL}{}^2 = \frac{1}{N_{\rm loc}^2} \left[ \sum_n \bar{\beta}_n^{\mathcal{R}\,2} - n_{\rm max} \sigma^2 \right]$$



# *Modal Polyspectra Estimation*

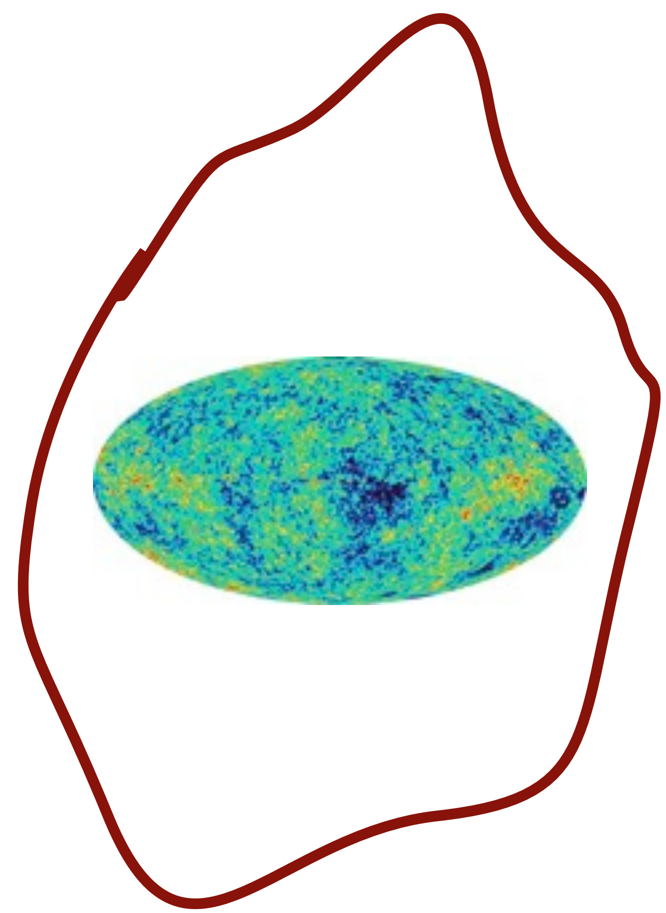
<u>THEORY</u>

Space of (primordial) isotropic polyspectra ($k$-space)




<u>OBSERVATION</u>

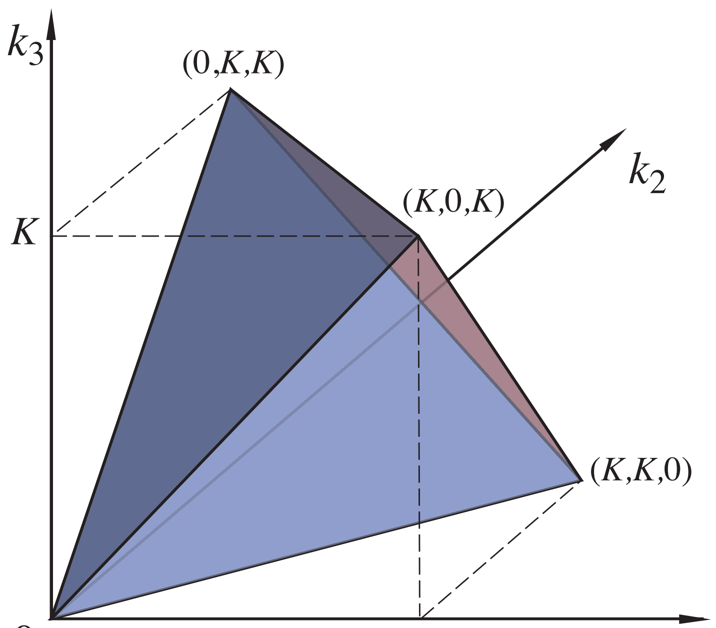
Space *V of possible* CMB polyspectra

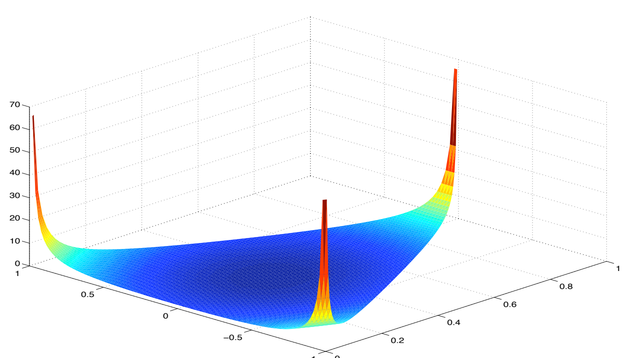


# Modal Polyspectra Estimation

<u>THEORY</u>

Space of (primordial) isotropic polyspectra ($k$-space)

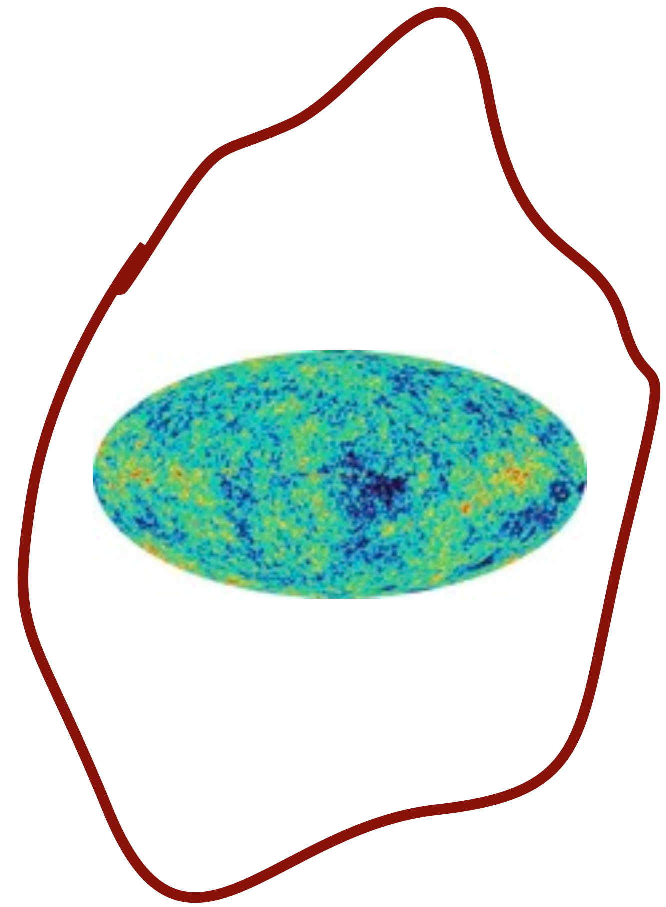


*Expand <u>any</u> model with primordial modes* $\alpha_n$

<u>OBSERVATION</u>

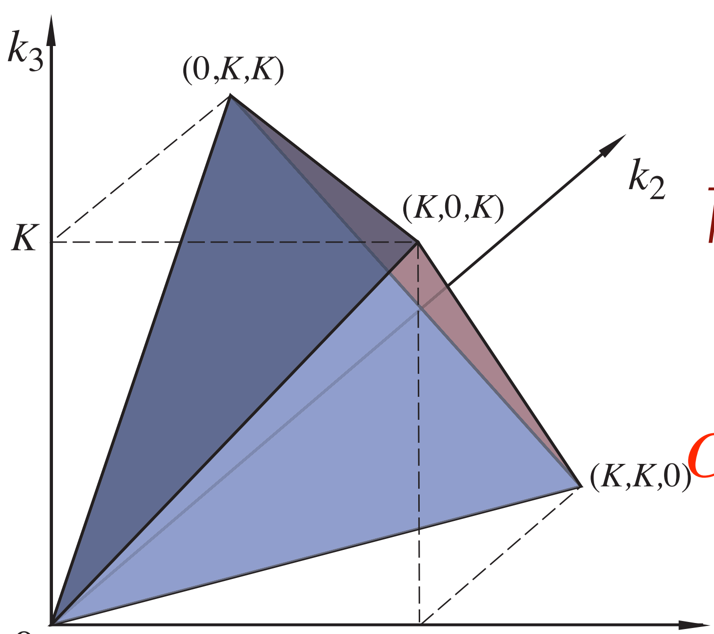
Space *V of possible* CMB polyspectra

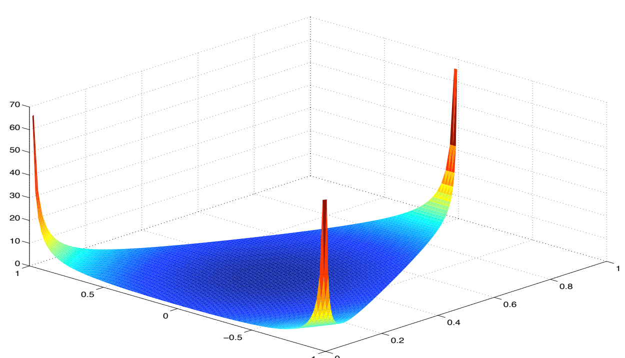


# Modal Polyspectra Estimation

THEORY

Space of (primordial) isotropic polyspectra ($k$-space)



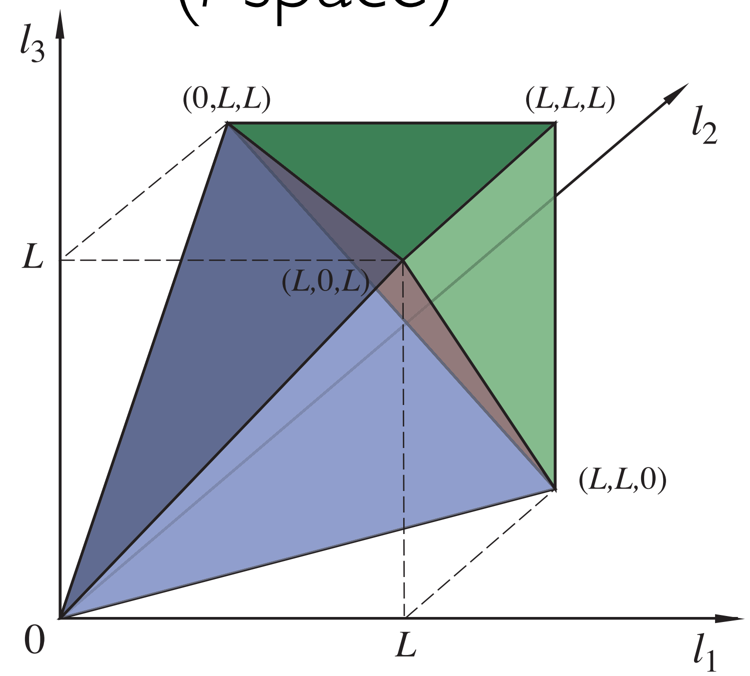



*Expand any model with primordial modes* $\alpha_n$

*Mode transfer functions*

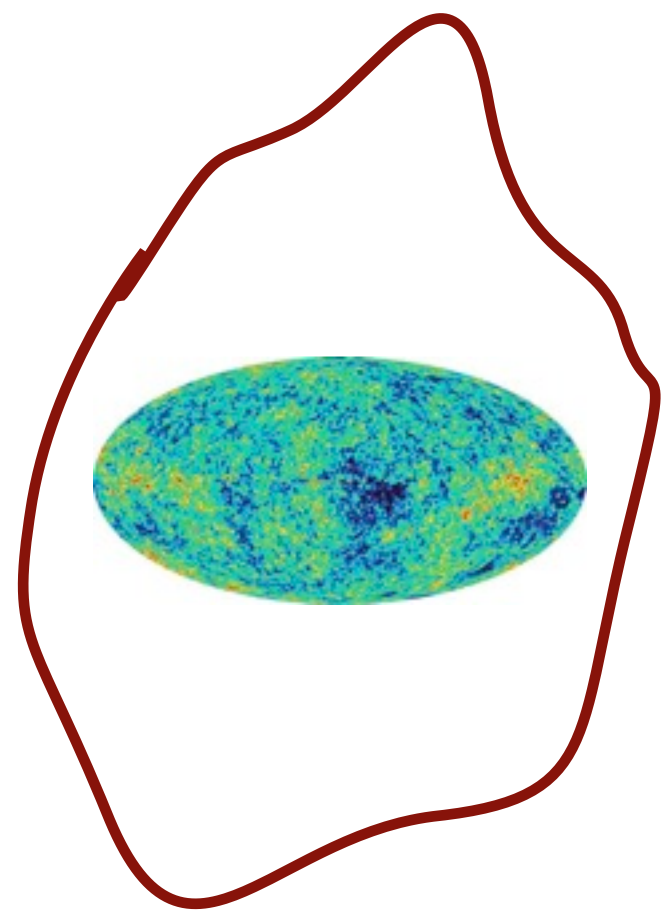
$\alpha_n \rightarrow \bar{\alpha}_n$

Projected space $V_P$ of CMB polyspectra ($l$-space)




OBSERVATION

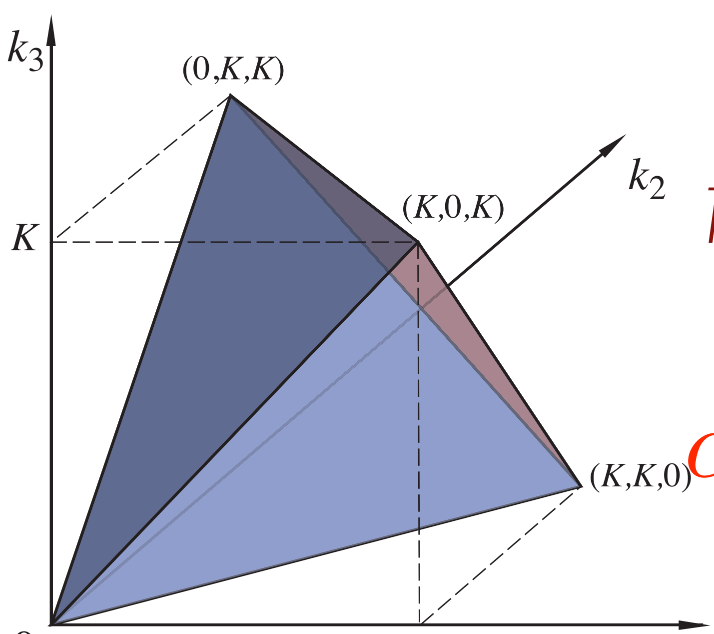
Space $V$ *of possible* CMB polyspectra

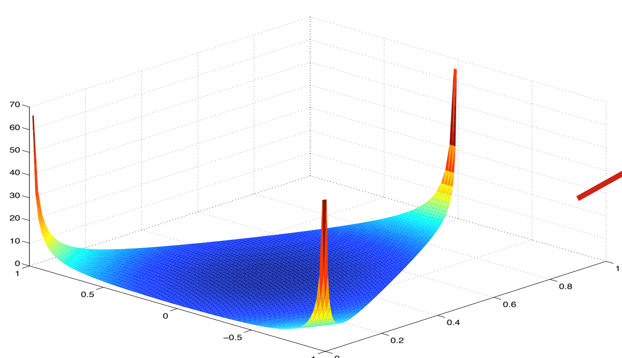


# Modal Polyspectra Estimation

THEORY

Space of (primordial) isotropic polyspectra ($k$-space)



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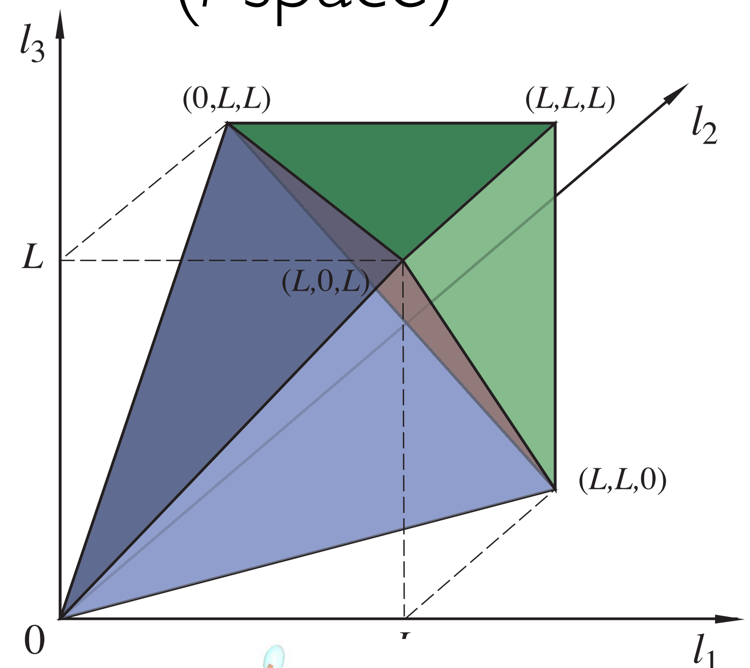
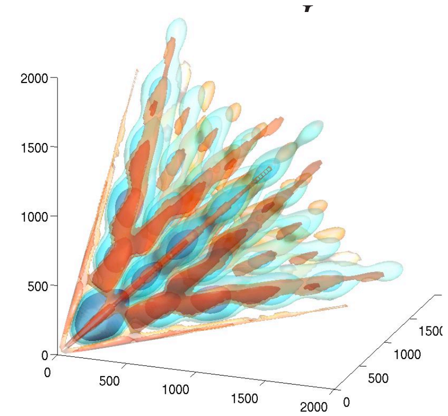
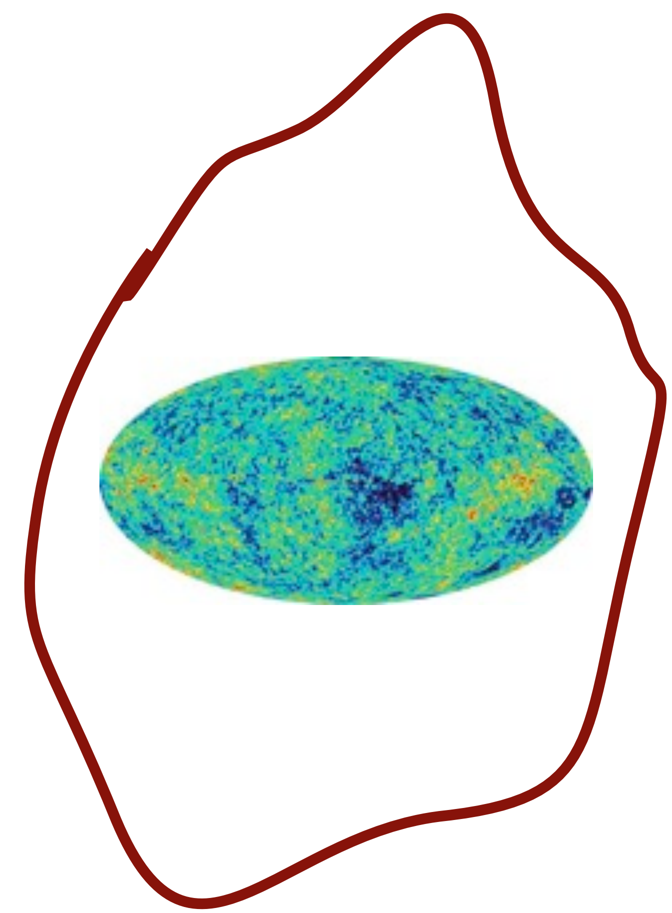
Projected space $V_P$ of CMB polyspectra ($l$-space)




OBSERVATION

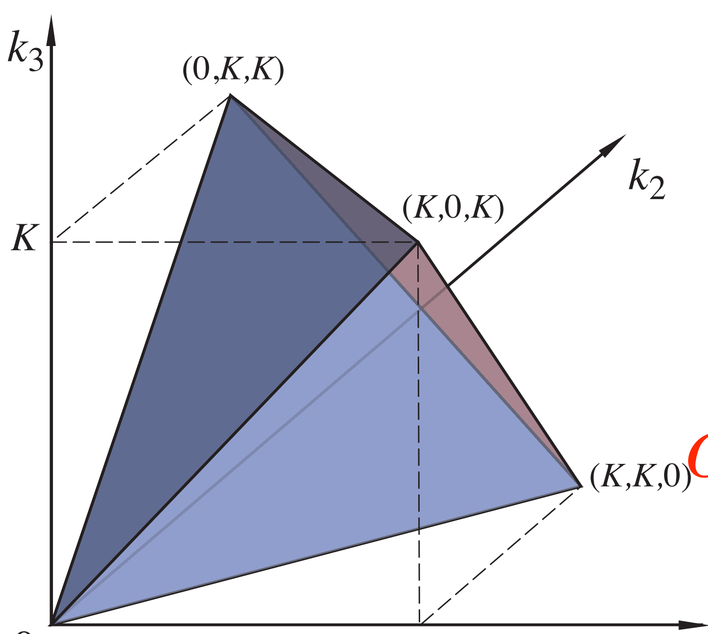
Space $V$ *of possible* CMB polyspectra

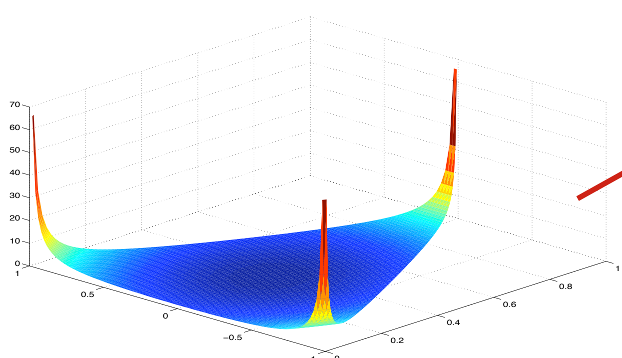


# Modal Polyspectra Estimation

<u>THEORY</u>

Space of (primordial) isotropic polyspectra (*k*-space)



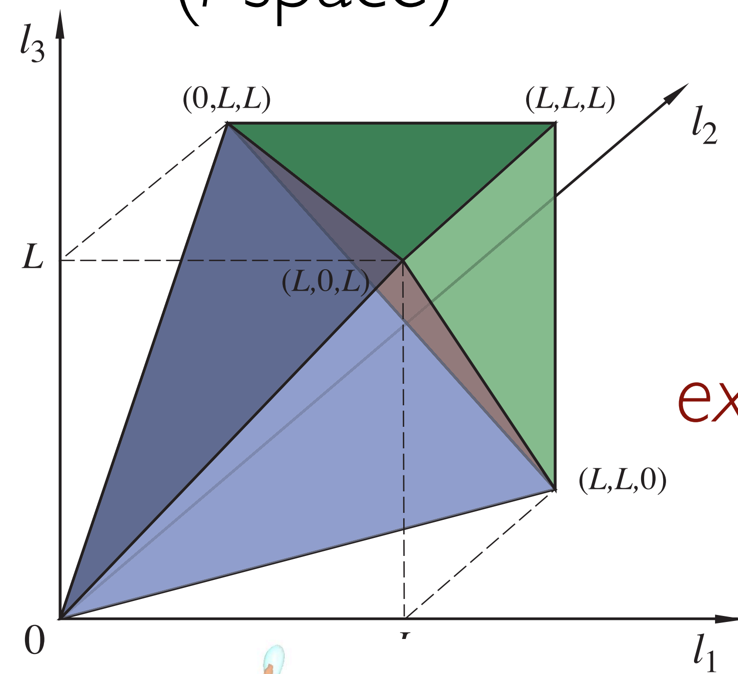



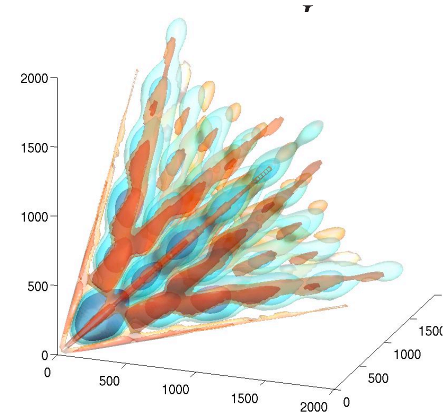
*Expand <u>any</u> model with primordial modes* $\alpha_n$

*Mode transfer functions*

$\alpha_n \rightarrow \bar{\alpha}_n$

Projected space $V_P$ of CMB polyspectra (*l*-space)



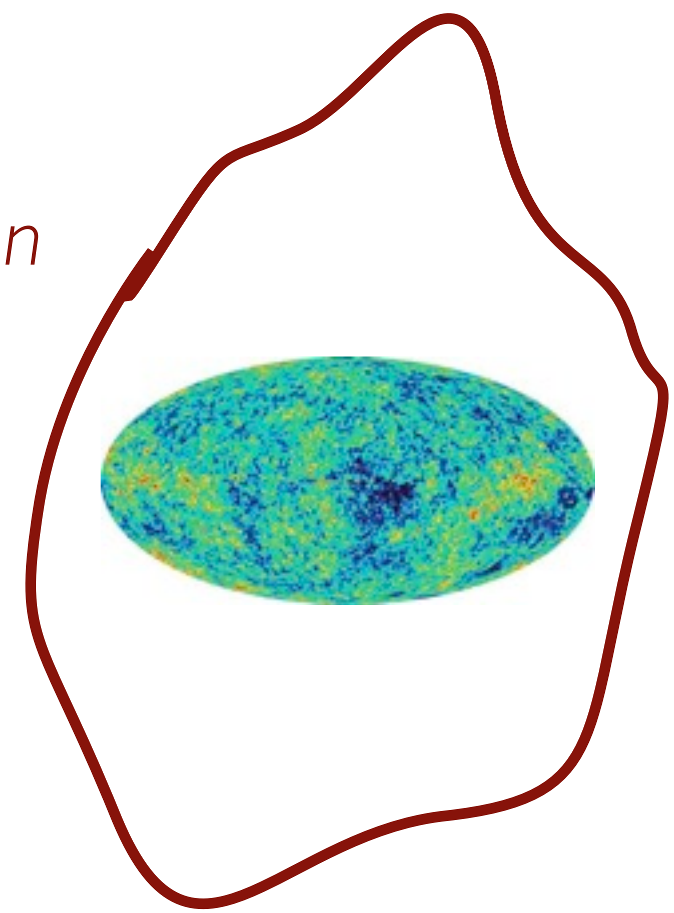




*Map extraction*

$\bar{\beta}_n$

<u>OBSERVATION</u>

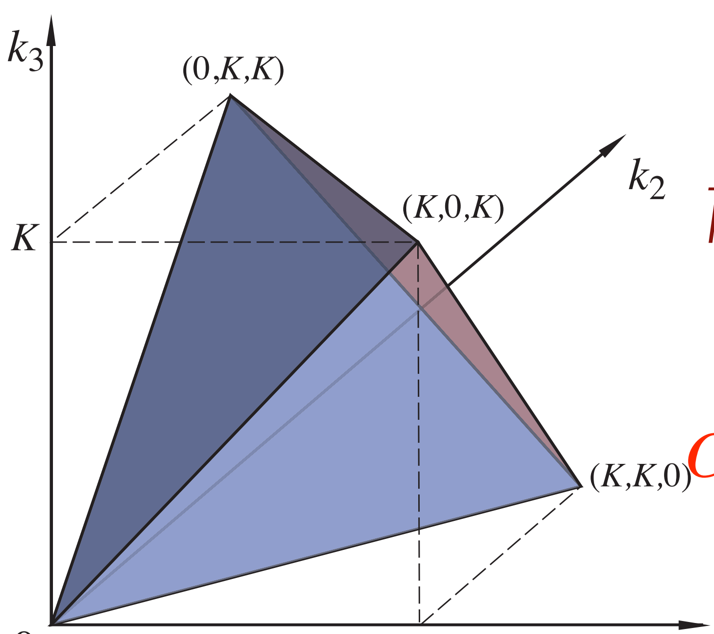
Space *V of possible* CMB polyspectra

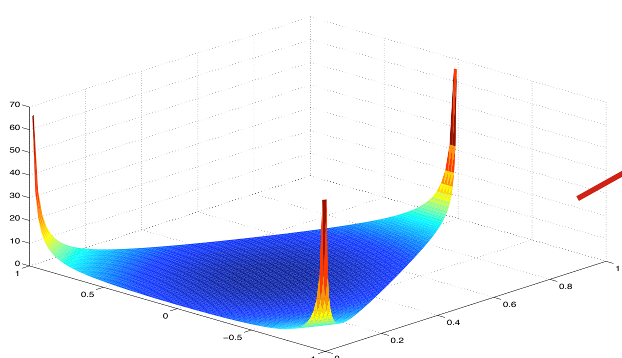


*Filter with sufficient separable eigenmodes*

# Modal Polyspectra Estimation

<u>THEORY</u>
Space of (primordial) isotropic polyspectra ($k$-space)



*Expand <u>any</u> model with primordial modes* $\alpha_n$

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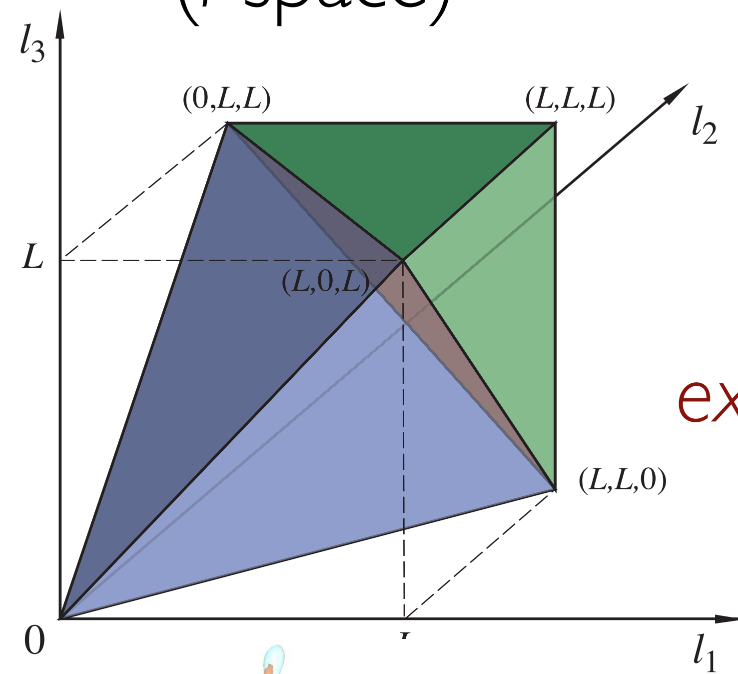
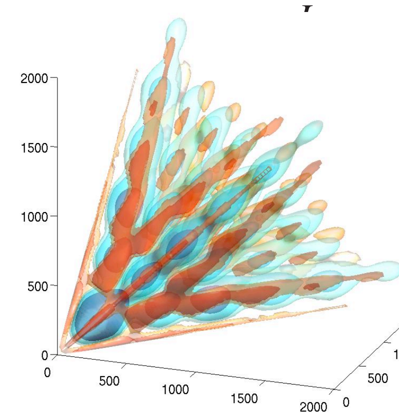
Projected space $V_P$ of CMB polyspectra ($l$-space)



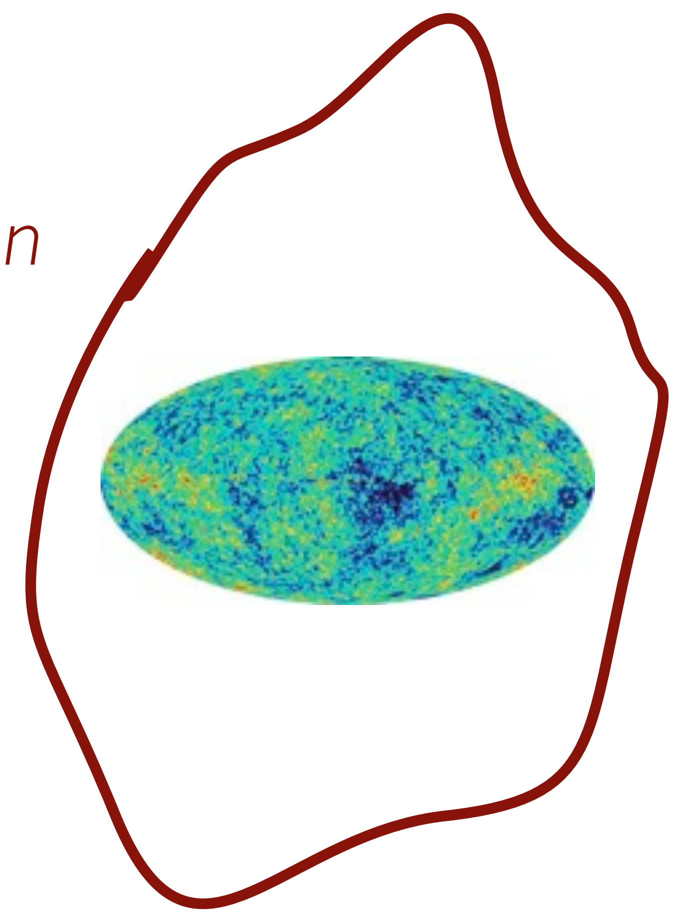




**Modal estimator** $$\mathcal{E} = \frac{\sum_n \bar{\alpha}_n^R \bar{\beta}_n^R}{\sum_n \left(\bar{\alpha}_n^R\right)^2}$$

<u>OBSERVATION</u>
Space $V$ *of possible* CMB polyspectra

*Map extraction*

$\bar{\beta}_n$

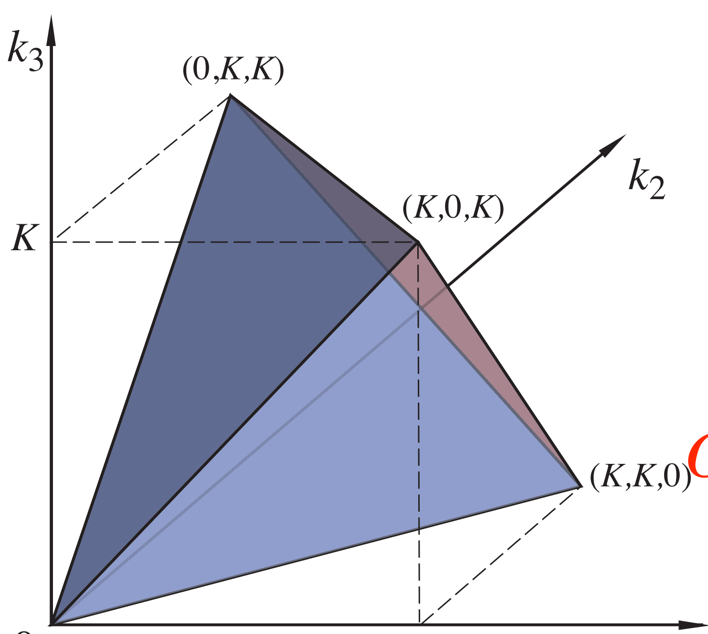


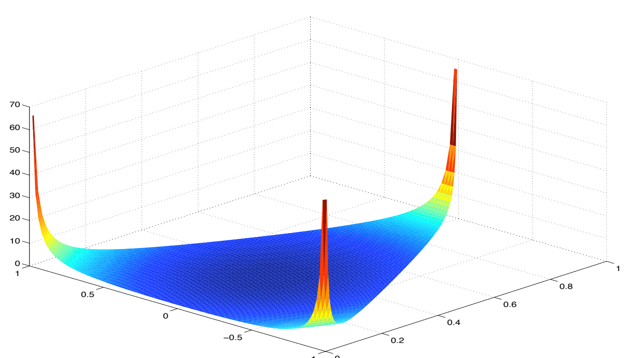
*Filter with sufficient separable eigenmodes*

# Modal Polyspectra Estimation

THEORY

Space of (primordial) isotropic polyspectra ($k$-space)



*Expand* <u>*any*</u> *model with primordial modes* $\alpha_n$

*Mode transfer functions*

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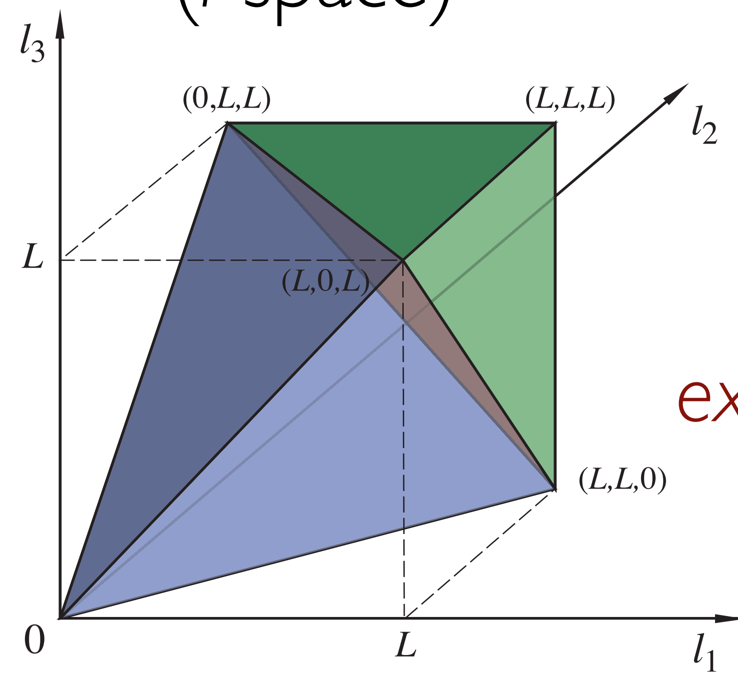
Projected space $V_P$ of CMB polyspectra ($l$-space)

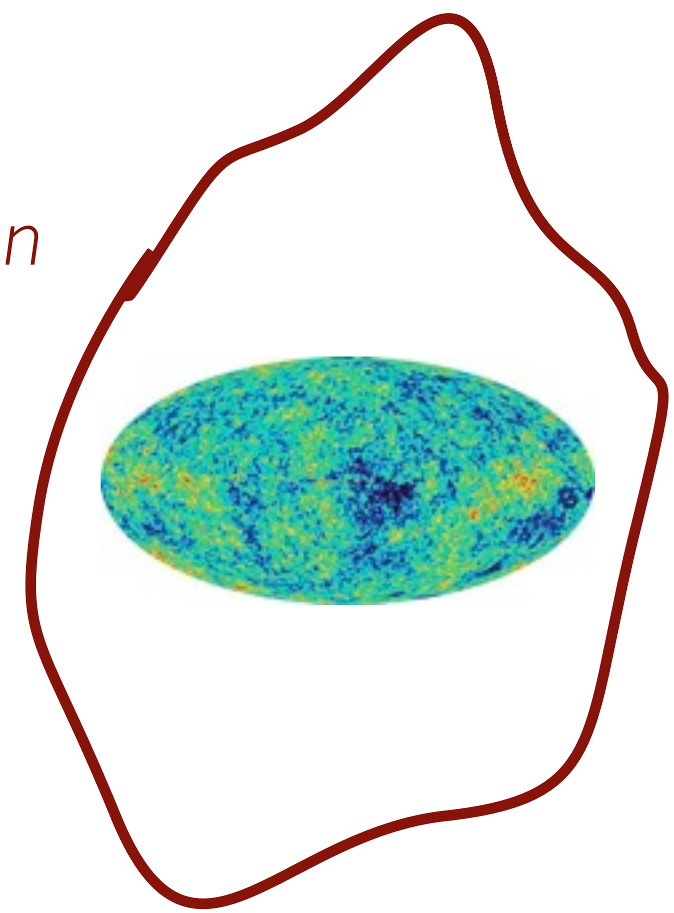



*Map extraction*

$\bar{\beta}_n$

OBSERVATION

Space $V$ *of possible* CMB polyspectra



*Filter with sufficient separable eigenmodes*

***Optimal modal estimator***

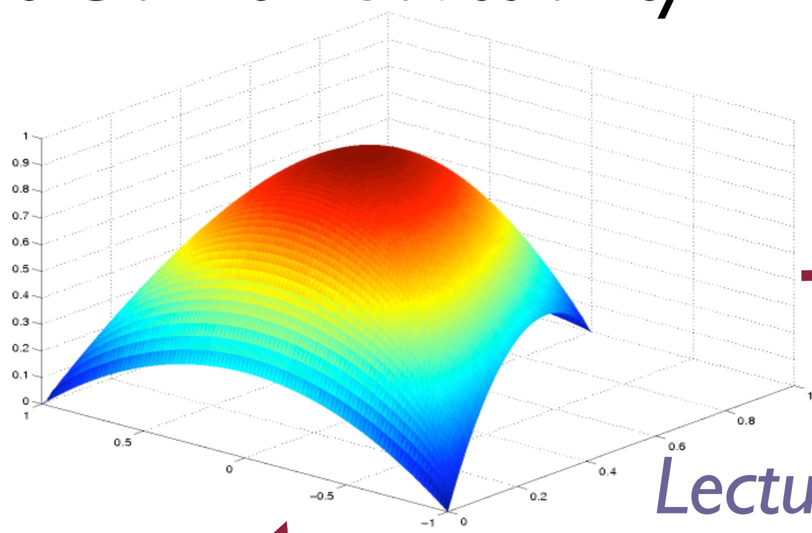
$$\mathcal{E} = \frac{\alpha^T \zeta^{-1} \beta}{\alpha^T \zeta^{-1} \alpha}$$

***with weight*** $\zeta = \mathcal{R}\mathcal{C}\mathcal{R}^T = \langle \beta \beta^T \rangle$

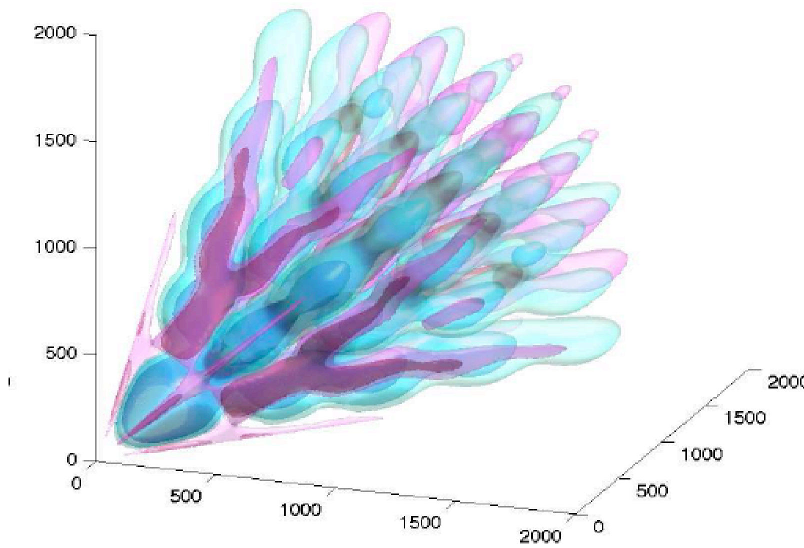
*see Ferguson & EPS arXiv:1105.2791*

# Motivation

Primordial non-Gaussianity



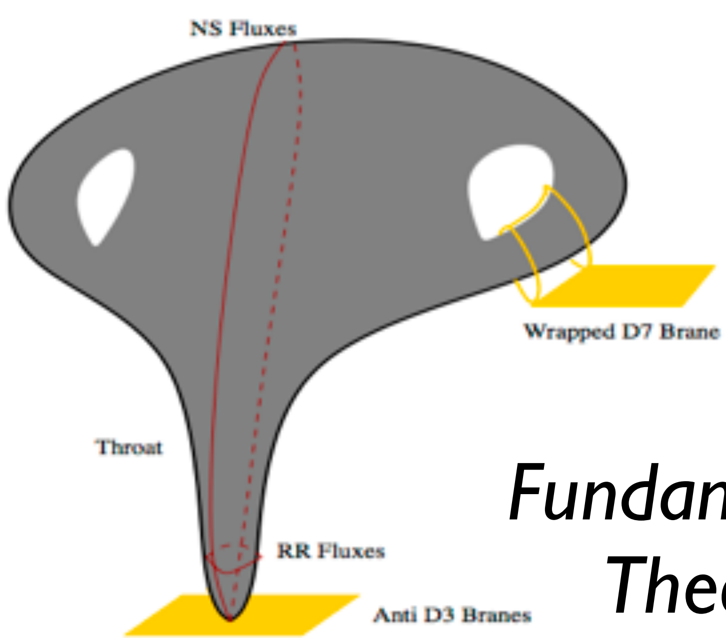
CMB (or LSS) fingerprint



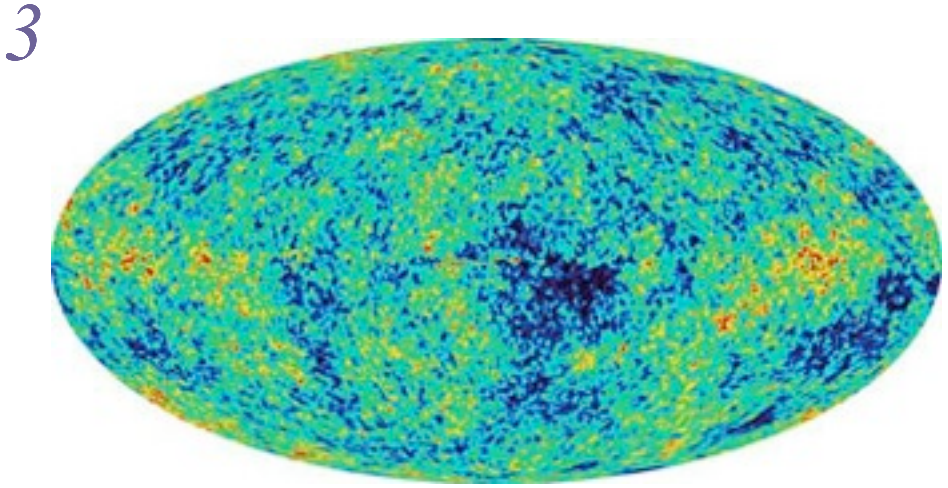
Lecture 1

Lecture 2

Lecture 3




Fundamental Theory



Observational Data