#### Gravitational Waves Notes for Lectures at the Azores School on Observational Cosmology September 2011

**B F** Schutz

Albert Einstein Institute (AEI), Potsdam, Germany http://www.aei.mpg.de, Bernard.Schutz@aei.mpg.de

### Lecture 3 : A Stochastic Gravitational Wave Background

#### Introduction

The cosmic microwave background radiation (CMB) has yielded an enormous harvest of understanding of the Big Bang universe, and it has helped to firmly establish the standard ACDM model of cosmology, in which inflation plays a crucial role. The very early epoch of exponential expansion explains both the large-scale homogeneity and isotropy of the universe as well as the structure that has evolved on smaller scales.

The CMB is a thermal radiation, and it gives us our earliest electromagnetic view of the state of the young universe. It is thermal because photons couple relatively strongly to charges, so that the universe did not become optically thin until it was some  $3 \times 10^5$  y old. This is the moment that we study in the CMB.

Gravitational waves couple much more weakly to matter. Their coupling does not become O(1) ("gravitationally thick": total modification of the waveform, absorption, etc) unless they interact with gravitational fields where  $GM/rc^2$  1, i.e. with black holes. Although this can happen in rare, isolated locations, the only time it could happen to all the radiation in the universe is at the Planck time. And since the Planck time is before inflation, any thermal radiation that might have been produced at that time has been redshifted by inflation into oblivion. Any radiation that we might detect today will therefore have a non-thermal spectrum that contains detailed information about its origin, because the radiation has not been reprocessed or significantly distorted after it was produced.

That is not to say that the radiation comes from late epochs. The end of inflation, at  $10^{-40}$ (?) seconds, marks the beginning of the time when detectable radiation might have been generated. This is much earlier than the origin of the CMB, and the key thing about gravitational waves is that they will not have interacted with matter in any significant way since being generated. Whatever process generated them did so with low efficiency (otherwise they would have been thermal) and therefore the efficiency with which they subsequently scattered and were transformed by interactions was also low, and it got ever lower as the universe expanded. So gravitational waves, if we can detect them, bring us pristine information about the universe just as it was being born, 50 orders of magnitude younger than the universe we see in the CMB.

Our best chance to measure these primordial stochastic waves is by looking for their signature in the tensor modes of the CMB. The CMB is likely to show us what these waves looked like at the time of decoupling, at  $3 \times 10^5$  y, and on wavelength scales comparable to the then horizon size. If we want information about shorter wavelengths, which could reveal conditions in the universe at earlier times, we have to examine the spectrum at higher frequencies today. This is why all the gravitational wave projects are looking for or planning to look for stochastic waves: pulsar timing, LISA, and LIGO. We will look at these detection programs below. First we must try to understand how the waves arise in the first place.

There are three main ways that have been discussed for generating a stochastic background: inflation, post-inflation fundamental physics, and astrophysical systems. We shall treat them in separate discussions.

# Tensor modes from inflation, and their polarization signal

The tensor modes of gravitational waves arise from the same kind of parametric amplification process that leads to scalar density perturbations: a quantum fluctuation before inflation starts finds itself, after inflation has finished, with a much more localized wavefunction than a quantum fluctuation at the later time would have. So in the inflated universe the fluctuation is a superposition of a large number of higher-energy excitations. These excitations are real, and lead to the perturbations that are observed in the anisotropy of the CMB.

Gravitational waves act, as we saw, transversally to their propagation direction, and they act in an area-conserving way: if one direction stretches, the other direction compresses, and the resulting ellipse has the same area as the original circle. So gravitational waves do not produce density or temperature variations in the CMB. Instead, they can be detected in the polarization signal.

If the CMB were completely isotropic, and if (as we shall always assume) there are not significant magnetic fields, then we would expect CMB radiation to be unpolarized. But density perturbations lead to a polarization signal because the last scattering of a photon produces a scattered photon with a polarization perpendicular to the plane formed by the initial and final directions. if one imagines a point at the center of a quadrupolar temperature distribution, where there is one direction (and its opposite) leading to high-temperature regions and an orthogonal direction leading to low-temperature regions, then the radiation from the high-temperature regions will contribute more to the scattering at this point than the weaker radiation from the low-temperature regions. This leads to a slight bias in polarizations perpendicular to the line joining the high-temperature regions.

An acoustic (scalar) plane wave creates a simple pattern of polarization, which when converted to a Stokes vector has no curl. A superposition of random uncorrelated acoustic waves will preserve the curl-free property.

But gravitational waves also affect the polarization signal. Consider a gravitational wave traveling directly toward the observer through the primordial plasma. It is pushing the particles of the plasma apart in one direction (let's call that the x-direction on the sky at this location) and together in the perpendicular y-direction. This produces a bias in the polarization of the scattered photons even from an otherwise homogeneous plasma and produces a slight excess of polarization along the directions in which the scatterers are being pushed. But the fact that gravitational waves are polarized leads to a different kind of Stokes map. The polarization can rotate across the sky, leading to a rotation of the polarization map vectors, and this will be arbitrary. It will be a map with both divergence and curl. So detecting a curl in the Stokes map is the so-called B-mode signature of gravitational waves in the CMB polarization.

We should remember that the predictions of the inflation scenario

typically start with a pre-inflation spacetime in which the fluctuations that later get amplified are standard quantum fluctuations. But many authors have pointed out that this does not have to be the case, and physical conditions in the pre-inflationary universe, or different spectra for the pre-inflationary quantum fluctuations, can lead to (at times radically) different spectra of gravitational waves today.

### Post-inflation physics

After inflation has ended, the universe still has many transitions to go through before it becomes the modern universe. As the temperature falls, it will, according to current thinking, undergo a GUTs phase transition, where the strong interactions separates from the weak and electromagnetic interactions; an electroweak transition, where the weak interaction separates from the electromagnetic interaction, and a QCD phase transition, where protons and neutrons condense out from the quark soup. If any of these is first-order, so that there are density inhomogeneities, then these density regions can collide and produce gravitational waves.

The most important wavelengths for such gravitational waves will be comparable to the horizon size of the universe at the time of the event. These are then redshifted by the cosmological expansion into waves we can look for today. The higher the energy of the transition, the higher the frequency today. It happens that the EW transition should be within the LISA band (milliHertz) and the QCD transition could be in the band where pulsar timing experiments are most sensitive (periods of a few years).

Phase transitions could also lead to relics or defects, places where the pre-transition universe got trapped by topological effects and could

not accomplish the transition. The most commonly discussed relics are cosmic strings. Relic strings have nothing directly to do with "string theory", and were discussed extensively before string theory became popular. But string theory can also create strings, called cosmic superstrings, in which pre-inflationary "zero-point" string oscillations are inflated to macroscopic size. In principle both kinds of strings could produce detectable backgrounds of gravitational waves.

They do this by decaying: the string tension  $\mu$  is large, and waves that run along the string create quadrupolar changes in the external gravitational field, creating GWs. The strings then gradually shrink. In addition, strings can run into one another, and when they do there is a certain probability (model-dependent and hard to compute) that they will break and reconnect into smaller loops. These breaks can create slope discontinuities called kinks, which then run along the string at the speed of light, again disturbing the gravitational field and creating radiation. LIGO has searched for such events and put limits on the parameters of the relevant theories.

At one time cosmic strings were candidates for the seeds of structure formation, but the evidence now strongly supports cold dark matter instead. That has made them go out of fashion, but we should remember that they could still be present in a population that is too small to affect structure formation but nevertheless produces detectable gravitational radiation.

Besides these ideas that are grounded in relatively standard physics, the brane-world picture has opened up huge possibilities for generating gravitational waves from brane collisions and other scenarios. And they can lead to universes with no primordial gravitational radiation, like the ekpyrotic universe proposal of Steinhardt.

### Astrophysical backgrounds

Naturally the universe produces gravitational waves at all frequencies where binary systems can exist, and also from many catastrophic events. The nearest are the goals of detector projects, but the more distant systems can produce a background. It turns out that this background is likely to exceed a primordial background at low frequencies. So, although pulsar timing is well-placed in terms of observational frequency to look for a QCD transition, it might well be masked by radiation from all the massive binary black hole systems that have been formed in the universe.

Even at LISA's frequencies, compact binaries in distant galaxies form a background that is expected to be just below the LISA sensitivity – which is why LISA was designed with the laser power it has and not with much more. So if the primordial background is not above the LISA instrumental noise (corresponding to  $\Omega_{gw} \sim 10^{-10}$  it will not be detectable at those frequencies.

We expect the universe to become quieter above about 0.1 Hz. This is the band in which the NASA concept study for the Big Bang Observer placed its observing frequencies.

#### Current bounds

The most important bound is that no background should exceed  $\Omega_{gw} \sim 10^{-5}$ , at least if it was already present during the time of nucleosynthesis, since this would have disturbed the element abundances from their observed values. Astrophysically generated backgrounds are therefore exempt. Also there is room to move this bound around in frequency, but nevertheless it would have to be a rather

contrived and narrow spectrum that reached above this value and still complied with the overall bound.

Pulsar timing has set bounds around  $\Omega_{gw} \sim 10^{-7}$  on GW backgrounds of any kind in the one- to several-year period range. LIGO during its S5 run finally set a better bound than the nucleosynthesis bound ( $\Omega_{gw} < 7 \times 10^{-6}$ ) at frequencies around 100 Hz.

#### Detecting backgrounds

A stochastic background is by definition noise, so if it is stronger than the noise in a detector then it can be detected directly. This is how Penzias and Wilson detected the CMB in the first place. But ground-based GW detectors have noise levels that are uninterestingly high for this. LISA, as remarked above, and pulsar timing have the possibility to go to interesting values of  $\Omega_{gw}$  just by looking for noise larger than their instrumental noise. Just as with Penzias and Wilson, this method requires great confidence in the noise model of the detector.

But ground-based detectors can perform much better searches by doing a cross-correlation. Imagine two detectors right next to one another, with the same orientation. If their instrumental noise is uncorrelated, then they can look for the correlated noise created by a GW background by cross-correlation: this enhances the energy sensitivity by a factor  $\sqrt{BT}$ , where B is the bandwidth and T the observing time.

Co-located detectors might not, however, have uncorrelated instrumental noise. And in any case for other kinds of observing it is good to have detectors widely separated. In this case there is a penalty due to the fact that unless stochastic waves arrive at the same time at the two detectors, they will also be uncorrelated. This means that separated detectors respond only to a fraction of the sky perpendicular to the line joining them. The data analysis puts a filtering factor into the cross-correlation for this, called the overlap reduction function. Detector separations less than a reduced wavelength of the gravitational waves are close to optimal, but sensitivity falls for larger separations.

The most sensitive pair of detectors on the ground is the LIGO-Hanford and LIGO-Livingston combination. VIRGO is further away and therefore produces a smaller expected sensitivity. Advanced LIGO in this way hopes to reach close to  $\Omega_{gw} \sim 10^{-9}$  at 100 Hz.

LIGO is also doing stochastic searches in a directed way, by filtering for particular time-delays in the correlations. This could lead to upper bounds on astrophysical sources, such as a cluster that might contain a large number of pulsars radiating gravitational waves.

Although a detection of a primordial cosmic stochastic background by these instruments is not likely to happen soon, it is probably the most fundamental observation that GW detectors could make, and it will continue to be a primary goal of the experimental development of the field. The European design study for the lower-frequency and more-sensitive Einstein Telescope shows that, if there were two instruments, one in the US and one in Europe, one could get to  $10^{-13}$  – not enough to put a tight squeeze on inflation, but enough to detect something if the spectral index is favorable. So the search will continue for many decades.



# **Azores School on Cosmology**

# **Gravitational Waves**

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**B F Schutz** 

Albert Einstein Institute (AEI), Potsdam, Germany

and

School of Physics and Astronomy, Cardiff University

LISA



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• You can find the lecture notes available for download at

http://www.aei.mpg.de/~schutz/download/lectures/ AzoresCosmology

• There are 4 files: an outline, and the three lectures.





# How does polarization arise?

- Scattering: E-field of scattered photon must be perpendicular to both the incoming and outgoing photon directions, i.e. orthogonal to the plane formed by the two directions.
- Homogeneous plasma: no net polarization.
- Quadrupole inhomogeneities lead to polarization: flux along one direction larger than along the perpendicular direction.





# How does polarization arise?

- Density/temperature perturbations from acoustic waves:
  - Temperature anisotropies create flux anisotropies, polarization.
  - Wavelength of wave must match mean free path of photons during the decoupling process.
- Gravitational waves:
  - do not change local density, temperature tidal stretching ellipse preserves area, no compression.
  - Instead, plasma velocities are changed: along stretching direction, fluid elements are moving apart.
  - A photon that originates in one will be redshifted relative to rest frame of fluid element where it scatters, so it will have smaller flux. The stretching direction creates a direction that is like the low-temperature direction of a temperature map.





# E-mode polarization from density waves

### Scalar acoustic wave

- If it comes toward you, the temperature on the sky is uniform, no polarization.
- If it moves transversely to line of sight (i.e. wave is in the plane of sky), pattern at moment of decoupling:



When uncorrelated waves are superimposed, it remains curl-free.



![](_page_14_Figure_0.jpeg)

### **E-mode polarization from GWs**

- First consider a GW moving in the plane of the sky, transverse to observer's line of sight.
  - GW acts only transversely to its propagation direction, so will have an effect only from "+" polarization, where one axis of polarization ellipse is aimed toward observer and other is in the plane of the sky.

![](_page_14_Picture_4.jpeg)

In this way GWs contribute to the E-mode (curl-free) polarization signal.

![](_page_14_Picture_6.jpeg)

### **B-mode polarization from one GW**

![](_page_15_Figure_1.jpeg)

![](_page_16_Picture_0.jpeg)

### **Stochastic Predictions and Limits**

![](_page_16_Figure_2.jpeg)

# LISA band: possible backgrounds

![](_page_17_Figure_1.jpeg)