Gravitational Waves
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Lecture 2. Gravitational Waves from Binary Systems: Probes of the Universe

Historical importance of orbiting systems.

Binary systems, or other orbiting systems, have not just been interesting astrophysical systems; they have historically played a key role in developing our understanding of physics.

- Galileo's observations of the moons of Jupiter put the final nail in the geocentric model of the solar system and suggested that there might be some natural law involved in producing orbits.
- Rømer in 1676 (even before Newton published the *Principia* in 1687) measured the speed of light for the first time by observing the retardation in the arrival time of light from Jupiter's moons when they were on the far side of Jupiter compared to when they were nearer the Earth. (His data give a value that is within 30% of today's accepted value for c.)
- Newton found the natural law governing orbits by studying the Moon's orbit around the Earth, and then applied it to the known

solar system to show that Kepler's laws of planetary motion were a natural consequence of $1/r^2$ gravity.

- Nineteenth-century astronomers discovered the small discrepancy in the precession of the orbit of Mercury that presaged general relativity. Einstein understood how important it was that he could derive this extra precession from his new theory: when he got the result, he reported later, he has palpitations of the heart for a full three days!

- Observations of the shrinking of the orbit of the Hulse-Taylor binary system (see below) have now tested the dynamical part of GR (i.e. tested gravitational wave theory) to better than 1%.

- Gravitational-wave observations of binaries that shrink, like the Hulse-Taylor system, will be able to measure their distances directly from the signal, and therefore provide a check on the usual astronomical distance ladder and a new way of measuring cosmological parameters (see below). They will also begin to test GR stringently in strong gravitational fields.

The utility of binaries for making fundamental measurements lies in their simplicity. Mostly they can be idealized as orbiting point particles, so very little modeling is necessary. Astrophysical models (of complex systems like stars, supernovae, galaxy formation, and so on) involve considerable physics, and often makes assumptions that are hard to test. This leads to uncertainties that prevent complex systems from being used to test physical laws. Binaries, however, are simple to observe and model, and so they are good testbeds.

We proceed now to compute the gravitational waves expected from a simple binary system.

Mass-quadrupole radiation.

First we consider the field of a source in linearized theory. We use a slow-motion approximation to compute the radiated field. The computation proceeds in close analogy to the derivation of the electric-dipole radiation from Maxwell's equations. Note that linearized theory is not very realistic: the orbits of a simple Newtonian system require an interaction between the body and the field, which is a second-order term and is thus not present in linearized theory. Nevertheless, remarkably, the approach we take will lead to a formula that is identical in Newtonian theory. The problem of radiation in linearized theory was first solved by Einstein in 1918, but it took until the 1950s and 1960s before the generalization to Newtonian systems was well understood.

- **Isolated source** The Einstein equation is

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}^{\alpha\beta} = -16\pi T^{\alpha\beta}.$$

  Its general solution is the following retarded integral for the field at a position x^i and a time t in terms of the source at a position y^i and the retarded time:

$$\bar{h}^{\alpha\beta}(x^i,t) = 4\int \frac{1}{R}T^{\alpha\beta}(t-R,y^i)d^3y,$$

  where we define

$$R^2 = (x^i - y^i)(x_i - y_i).$$

- **Expansion for the far field of a slow-motion source.** Let us suppose that the origin of coordinates is in or near the source, and the field point x^i is far away. Then we define $r^2 = x^i x_i$ and we have $r^2 \gg y^i y_i$. We can therefore expand the term R in the denominator in terms of y^i. The lowest order is r, and all higher-order terms are smaller than this by powers of r^{-1}. Therefore, they contribute terms to the field that fall off faster than $1/r$, and they are negligible in the far zone. So we can simply replace R by r in the denominator, and take it out of the integral.

The R inside the time-argument of the source term is not so simple. We handle that in the following way. Let us define $t' = t - r$ (the retarded time to the origin of coordinates) and expand

$$t - R = t - r + n^i y_i + O(1/r), \qquad \text{with } n^i = x^i/r, \quad n^i n_i = 1.$$

The terms of order $1/r$ are negligible for the same reason as above, but the first term in this expansion must be taken into account. It depends on the direction to the field point, given by the unit vector n^i. We use this by making a Taylor expansion in time on the time-argument of the source. The combined effect of these approximations is

$$\bar{h}^{\alpha\beta} = \frac{4}{r}\int \left[T^{\alpha\beta}(t', y^i) + T^{\alpha\beta}{}_{,0}(t', y^i) n^j y_j + \frac{1}{2} T^{\alpha\beta}{}_{,00}(t', y^i) n^j n^k y_j y_k + \ldots\right] d$$

We will need the Taylor expansion out to this order.

- **Moments of the source.** The integrals in the above expression contain moments of the components of the stress-energy. It is useful to give these names. Use M for moments of the density T^{00}, P for moments of the momentum T^{0i}, and S for moments of the stress T^{ij}. Here is our notation:

$$M(t') = \int T^{00}(t', y^i) d^3y, \quad M_j(t') = \int T^{00}(t', y^i) y_j d^3y,$$

$$M_{jk}(t') = \int T^{00}(t', y^i) y_j y_k d^3y;$$
$$P^\ell(t') = \int T^{0\ell}(t', y^i) d^3y, \quad P^\ell{}_j(t') = \int T^{0\ell}(t', y^i) y_j d^3y;$$
$$S^{\ell m}(t') = \int T^{\ell m}(t', y^i) d^3y.$$

These are the moments we will need.

Among these moments there are some identities that follow from the conservation law in linearized theory, $T^{\alpha\beta}{}_{,\beta} = 0$, which we use to replace time derivatives of components of T by divergences of other components and then integrate by parts. The identities we will need are

$$\dot{M} = 0, \quad \dot{M}^k = P^k, \quad \dot{M}^{jk} = P^{jk} + P^{kj};$$
$$\dot{P}^j = 0, \quad \dot{P}^{jk} = S^{jk}.$$

These can be applied recursively to show, for example, one further very useful relation:

$$\frac{d^2 M^{jk}}{dt^2} = 2S^{jk}.$$

- **Radiation zone expansions.** Using these relations and notation it is not hard to show that

$$\bar{h}^{00}(t, x^i) = \frac{4}{r} M + \frac{4}{r} P^j n_j + \frac{4}{r} S^{jk}(t') + \ldots;$$
$$\bar{h}^{0j}(t, x^i) = \frac{4}{r} P^j + \frac{4}{r} S^{jk}(t') n_k + \ldots;$$
$$\bar{h}^{jk}(t, x^i) = \frac{4}{r} S^{jk}(t') + \ldots.$$

In these expressions, one must remember that the moments are evaluated at the retarded time $t' = t - r$ (except for those moments that are constant in time), and they are multiplied by components of the unit vector to the field point $n_j = x/r$.

The next step is to apply the TT gauge to the mass quadrupole field. This has a close analogy to using the Lorentz gauge in electromagnetism.

- **Gauge transformations.** We are already in Lorentz gauge, and this can be checked by taking derivatives of the expressions for the field that we have derived. But we are manifestly not in TT gauge. Making a gauge transformation consists of choosing a vector field ξ^α and modifying the metric by
$$h_{\alpha\beta} \to h_{\alpha\beta} - \xi_{\alpha,\beta} - \xi_{\beta,\alpha}.$$
The corresponding expression for the potential $\bar{h}_{\alpha\beta}$ is
$$\bar{h}_{\alpha\beta} \to \bar{h}_{\alpha\beta} - \xi_{\alpha,\beta} - \xi_{\beta,\alpha} + \eta_{\alpha\beta}\xi^\mu{}_\mu.$$
For the different components this implies changes
$$\begin{aligned} \delta\bar{h}^{00} &= -\xi^0{}_{,0} + \xi^j{}_{,j}, \\ \delta\bar{h}^{0j} &= -\xi^0{}_{,j} + \xi^j{}_{,0}, \\ \delta\bar{h}^{jk} &= -\xi^{j,k} - \xi^{k,j} + \delta^{jk}\xi^\ell{}_\ell, \end{aligned}$$
where δ^{jk} is the Kronecker delta (unit matrix). In practice, when taking derivatives, the algebra is vastly simplified by the fact that we are keeping only the $1/r$ terms in the potentials. This means that spatial derivatives do not act on $1/r$ but only on t' in the arguments. Since $t' = t - r$, it follows that $\partial t'/\partial x^j = -n_j$, and $\partial h(t')/\partial x^j = -\dot{h}(t')n_j$.

- **The TT gauge transformation.** The following vector field puts the metric into TT gauge to the order we are working:
$$\begin{aligned} \xi^0 &= -\frac{1}{r}P^k{}_k - \frac{1}{r}P^{jk}n_jn_k, \\ \xi^i &= -\frac{4}{r}M^i - \frac{4}{r}P^{ij}n_j + \frac{1}{r}P^k{}_kn^i + \frac{1}{r}P^{jk}n_jn_kn^i. \end{aligned}$$

- **The wave amplitude in TT gauge.** The result of applying this gauge transformation to the original amplitudes is:

$$\bar{h}^{TT00} = \frac{4M}{r};$$

$$\bar{h}^{TT0i} = 0;$$

$$\bar{h}^{TTij} = \frac{4}{r}\left[\perp^{ik}\perp^{j\ell} S_{k\ell} + \frac{1}{2}\perp^{ij}\left(S_{k\ell}n^k n^\ell - S^k{}_k\right)\right],$$

where the notation $\perp^{jk}$ represents the projection operator perpendicular to the direction n^i to the field point,

$$\perp^{jk} = \delta^{jk} - n^j n^k.$$

It can be verified that this tensor is transverse to the direction n^i and is a projection, in the sense that it projects to itself:

$$\perp^{jk} n_k = 0, \quad \perp^{jk}\perp_k{}^\ell = \perp^{j\ell}.$$

The time component of the field is not totally eliminated in this gauge transformation: it must contain the Newtonian field of the source. (In fact we have succeeded in eliminating the momentum part of the field, which is also static. Our gauge transformation has incorporated a Lorentz transformation that has put us into the rest frame of the source.) But this is a constant term. Since waves are time-dependent, the time-dependent part of the field is now purely spatial, transverse (because everything is multiplied by $\perp$), and traceless (as can be verified by explicit calculation).

The expression for the spatial part of the field actually does not depend on the trace of S_{jk}, as can be seen by constructing the trace-free part of the tensor, defined as:

$$\not{S}^{jk} = S^{jk} - \frac{1}{3}\delta^{jk} S^\ell{}_\ell.$$

In fact, it is more conventional to use the mass moment here instead of the stress, so we also define

$$\not{M}^{jk} = M^{jk} - \frac{1}{3}\delta^{jk} M^{\ell}{}_{\ell}, \quad \not{S}^{jk} = \frac{1}{2}\frac{d^2 \not{M}^{jk}}{dt^2}.$$

In terms of $\not{M}$ the far field is:

$$\bar{h}^{TTij} = \frac{2}{r}\left(\perp^{ik}\perp^{j\ell}\ \ddot{\not{M}}_{k\ell} + \frac{1}{2}\perp^{ij}\ \ddot{\not{M}}_{k\ell} n^k n^\ell.\right)$$

This is the mass quadrupole field. In other books the notation is somewhat different than we have adopted here. In particular, our *quadrupole tensor* $\not{M}$ is what is called $\not{I}$ in Misner, Thorne, and Wheeler (1973) and Schutz (2009).

If we define the TT-part of the quadrupole tensor to be

$$M^{TT}_{ij} = \perp^{k}{}_{i} \perp^{l}{}_{j} M_{kl} - \frac{1}{2}\perp_{ij}\perp^{kl} M_{kl},$$

then we can rewrite the radiation field as

$$\bar{h}^{TTij} = \frac{2}{r}\ddot{M}^{TTij}.$$

- **Interpretation of the radiation.** It is useful to look at this expression and ask what actually generates the radiation. The source of the radiation is the second time-derivative of the second moment of the mass density T^{00}. The moments that are relevant are those in the plane perpendicular to the line of sight. So it is interesting that not only is the action of the wave transverse, but also the generation of radiation uses only the transverse distribution of mass. In fact we learn from this two equally important messages,

– the only motions that produce the radiation are the ones transverse to the line of sight; and
– the induced motions in a detector mirror the motions of the source projected onto the plane of the sky.

If most of the mass is static, then the time-derivatives allow us to concentrate only on the part that is changing.

Next we consider the energy carried away by the radiation, and then we consider relaxing the assumptions of linearized theory so we can treat the more realistic self-gravitating systems with Newtonian and post-Newtonian approximations.

- **Mass quadrupole radiation.** The radiation field we have computed can be put into our energy flux formula for the TT gauge, and this can be integrated over a sphere. It is not a difficult calculation, but it does require some simple angular integrals over the vector n^i, which depends on the angular direction on the sphere. These identities are

$$\int n^i n^j d\Omega = \frac{4\pi}{3}\delta^{ij}, \qquad \int n^i n^j n^k d\Omega = 0,$$

$$\int n^i n^j n^k n^\ell d\Omega = \frac{4\pi}{15}\left(\delta^{ij}\delta^{k\ell} + \delta^{ik}\delta^{j\ell} + \delta^{i\ell}\delta^{jk}\right).$$

Using these, one gets the following simple formula for the total luminosity of the source if only mass-quadrupole radiation is computed:

$$L_{gw}^{mass} = \frac{1}{5}\dddot{M\!\!\!/}^{jk}\dddot{M\!\!\!/}_{jk}.$$

Note that luminosity is dimensionless in our units because $\dddot{M\!\!\!/}$ is dimensionless: the three time derivatives just compensate the mass and two distances in the quadrupole moment. In conventional units the dimensions are c^5/G. This is a big number, of order 3×10^{59} erg/s. It is believed to be an upper bound on the luminosity of any physical system, and it is certainly far above any observed luminosity, in fact above the total luminosity of the universe.

If a binary system orbits in the very relativistic regime, then it can get relatively close to this bound. Numerical simulations of black hole inspiral and merger show that the luminosity of such a system reaches a peak that exceeds the luminosity of the rest of the entire universe.

- **Relaxation of restrictions of linearized theory.** The calculation so far has been within the assumptions of linearized theory. Real sources are likely to have significant self-gravity. This means, in particular, that there will be a significant component of the source energy in gravitational potential energy, and this must be taken into account.

  Fortunately, the formulas we have derived are robust. It turns out that the *leading order* radiation field from a Newtonian source has the same formula as in linearized theory.

Gravitational waves from a binary system

- **The quadrupole moment of a binary system.** The motion of two stars in a binary is a classic source calculation. We shall calculate here only for two equal-mass stars in a circular orbit, governed by Newtonian dynamics. If the stars have mass m and an orbital radius R, orbiting in the $x-y$ plane with angular velocity ω, then it is easy to show that their quadrupole moment components are

$$M_{xx} = 2mR^2\cos^2(\omega t),\ M_{yy} = 2mR^2\sin^2(\omega t),$$

$$M_{xy} = 2mR^2\cos(\omega t)\sin(\omega t).$$

By using trigonometric identities, we convert these to functions of a frequency 2ω and discard the parts that do not depend on time:

$$M_{xx} = mR^2\cos(2\omega t), \quad M_{yy} = -mR^2\cos(2\omega t),$$

$$M_{xy} = mR^2\sin(2\omega t).$$

This shows that the radiation will come out at twice the orbital frequency, essentially because in half an orbital period the mass distribution has returned to its original configuration.

The trace of the quadrupole tensor is already zero.

- **The radiated field in different directions.** The general expression for the radiation field is $h^{TTij} = (2/r)\ddot{M}^{TTij}$.

  1. *Radiation perpendicular to the orbital plane.* This is the z-direction, and the tensor M is already transverse to it. So the radiation components can be read off of M. We see that $h_+ = -(8m\omega^2R^2/r)\cos(2\omega t)$ and $h_\times = (8m\omega^2R^2/r)\sin(2\omega t)$.

Both polarisations are present but are out of phase, so this represents purely circularly polarised radiation.

2. *Radiation along the x-axis.* The xx and xy components of M will be projected out, and when M is made trace-free again its components become $M^{TTyy} = -(mR^2/2)\cos(2\omega t)$ and $M^{TTzz} = (mR^2/2)\cos(2\omega t)$. This is pure $+$-polarised radiation with amplitude $4m\omega^2 R^2/r$. This is half the amplitude of each of the polarisation components in the z-direction, so the radiation is much weaker here. The energy flux will be only 1/8 of the flux up the rotation axis. By symmetry this conclusion holds for any direction in the orbital plane.

At directions between the ones we have calculated there will be a mixture of polarisations, which leads to a general elliptically polarised wave. By measuring the polarisation received, a detector (or network of detectors) can measure the angle of inclination of the orbital plane of the binary to the line of sight. This is often one of the hardest things to measure with optical observations of binaries, so gravitational wave observations are complementary to other observations of binaries.

- **The energy radiated by the orbital motion.** If we put our quadrupole moment into the luminosity formula we get

$$L_{gw} = \frac{16}{5} m^2 R^4 \omega^6.$$

The various factors are not independent, however, because the angular velocity is determined by the masses and radius of the orbit. When observing such a system, we can't usually measure R directly, but we can infer ω from the observations and often make a guess at m. So we eliminate R using the Newtonian orbit equation:

$$R^3 = \frac{m}{4\omega^2}.$$

If in addition we use the gravitational wave frequency $\omega_{gw} = 2\omega$, we get

$$L_{gw} = \frac{1}{20}(m\omega_{gw})^{10/3}.$$

- **Back-reaction on the orbit.** This energy must come from the orbital energy, $E = -m\omega^2 R^2$. The result is that we can predict the rate of change of ω_{gw}:

$$\frac{dE}{dt} = -L, \qquad \Rightarrow \qquad \dot{\omega}_{gw} = \frac{1}{10} m^{5/3} \omega_{gw}^{11/3}.$$

This is the key formula for interpreting the observations of the Hulse-Taylor binary pulsar system, PSR B1913+16. Its confirmation at the level of 1% by long-term radio timing of the pulsar won Hulse and Taylor the Nobel Prize for Physics in 1993.

- **Binaries as standard candles or *standard sirens*.** Remarkably, if we can measure the chirp rate $\dot{\omega}_{gw}$, we can infer from it the distance to the binary system (B F Schutz, *Nature*, **323**, 310, 1986). It is very unusual in astronomy to be able to observe a system and infer its distance; systems for which this is possible are called "standard candles", since basically one must know their intrinsic luminosity and compare that with their apparent brightness in order to measure the distance.

In the binary system we have studied, we can understand how this works if we note that the amplitude of the radiated field depends, as above, on m, R, and r. Since we can solve for R in terms of the other variables, we can take the radiated field to depend on m, ω_{gw}, and r. Since we measure ω_{gw}, if in addition we can measure the chirp rate then we can infer m. Then it follows that when we also measure the amplitude of the waves we can determine r, the distance to the binary. Essentially, the chirp rate is a measure of the intrinsic luminosity of the system (its frequency is changing because of the energy it loses), while the observed amplitude is a measure of the apparent luminosity.

Because of the analogy with sound, gravitational wave astrophysicists have begun calling chirping binaries "standard sirens".

- **More general binary systems.**
  - Our assumption of *equal masses* may seem restrictive, but it actually is not. With unequal masses one replaces m by something called the *chirp mass* $\mathcal{M}$, formed from the reduced mass μ and the total mass M in the following way:

    $$\mathcal{M} = \mu^{3/5} M^{2/5}.$$

    This combination appears in both the formula for $\dot{\omega}_{gw}$ and for h, so that it is possible to infer distances from these observations even when the masses of the two stars are unequal. This is a remarkable coincidence; it allows the method to work even though a counting argument would suggest that we do not have enough observables to determine all the unknowns about the system.
  - Another assumption we made, for simplicity, is that the *orbit is circular*. Coalescing binary neutron stars and black holes have probably evolved into circular orbits by the time they coalesce, but stellar binaries observed by LISA may not always be circular. The Hulse-Taylor binary pulsar is not

in a circular orbit. The eccentricity of an orbit brings the stars closer together than they get in a circular orbit of the same semi-major axis. Because the gravitational wave energy emission is such a strong power of the velocity (see the factor ω^6 in the formula above), radiation is stronger in eccentric orbits, and they shrink faster.

– We have only worked in Newtonian theory for the orbits. *Post-Newtonian orbit corrections* will be very important in observations. This might at first seem puzzling, since ground-based detectors will have low signal-to-noise ratios for these observations. But the key fact is that the corrections to the orbital radiation have a *cumulative* effect on the waveform, steadily changing its phase from what might be expected from Newtonian orbits. If the phase of an orbit changes by as much as π in the whole evolution (half an orbit) then the template being used to search for the signal becomes useless. Since observations will follow the orbital evolution of such systems for thousands of orbits, very precise templates are required. By measuring the post-Newtonian effects on an orbit, one can measure the individual masses of the stars, their spins, and possibly even their equations of state.

– Even more extreme are orbits of small black holes falling into massive black holes, such as are seen in the centers of galaxies. Here one needs to solve the full relativistic orbit equation with corrections to the geodesic motion that are first-order in the mass of the infalling object. This is a problem that has not yet been completely solved, although progress on it is very rapid indeed. It is key for LISA or any other space-based detector. (When one says that one corrects the geodesic equation even for a freely falling black hole, that does not mean we are abandoning the equivalence principle. We are finding corrections to the geodesic equation of the background black hole; the infalling black hole follows a geodesic of the time-dependent geometry which it helps to create.) These systems are called Extreme Mass-Ratio Inspiral Systems (EMRIs).

– The most difficult phase of the binary orbit is the merger of the two objects. This must be calculated entirely numerically. For many years the field of numerical relativity painstakingly addressed the many problems and instabilities associated with numerical integrations of the Einstein equations, and with the presence of causality boundaries (horizons) in the numerical domain. About 5 years ago the last piece of the puzzle was put into place, and since then many groups around the world routinely produce accurate simulations of the mergers of two black holes, and accurate predictions of the radiation emerging from the event and the subsequent ringdown oscillations of the product black hole.

  The frontiers of this research are: pushing to more unequal mass ratios (5:1 is difficult); exploring the entire parameter space of spins and masses; attaching waveform predictions onto those from the post-Newtonian studies of the same systems before they enter the merger phase, so that detectors have one unified waveform prediction to look for; performing merger simulations for neutron stars with all the associated new physics, like magnetic fields and neutrino transport.

Expected science from Advanced Detectors

- Last year the LSC and VIRGO collaborations published their best-estimate predictions of the rates of detections of binary systems to be expected when the Advanced LIGO and Advanced VIRGO detectors reach their expected sensitivity and operational duty cycle [*Class. Quant. Grav.* **27**, 173001 (2010), arXiv:1003.2480]. All binaries that are in the detectable frequency band (above 40 Hz) are orbiting so fast that they have only at most a few seconds before gravitational wave energy losses bring them together into a coalescence. While there are big astrophysical uncertainties in the populations, the "most-likely" numbers are encouraging: the network could detect 40 coalescences of neutron stars with neutron stars each year, and 20 coalescences of black holes with black holes. The rates for neutron stars with black holes are more uncertain but in the same range. The neutron-star events probably will involve stars with masses around 1.4 $M_{\odot}$. The black holes in these events are probably between 8 and 20 $M_{\odot}$, but this range could go much higher.

  The neutron star coalescence population is estimated from observed binary neutron stars in our Galaxy. Pulsar astronomers now know about six such systems that are tightly enough bound to coalesce within a Hubble time. Extrapolating the numbers out to the detection range of Advanced Detectors gives the rate of about 40 detectable events per year (at a signal-to-noise ratio of 8 or more in at least one detector). The black hole population is not so well related to observations, and the paper quoted here simply relies on population synthesis models. Such systems evolve through binary evolution, in a way analogous to the way the neutron star binaries evolve, but also are formed in globular clusters by three-body interactions occasionally resulting in bound black-hole binaries. The black hole rate is comparable to the neutron star rate because, although the population densities are much smaller because we believe that black holes are rarer, the systems are more massive and can therefore be detected in a much larger volume. There are recent suggestions that the black hole rate could be much higher, but these have not yet been thoroughly tested by the community.

- Observation of 40 NS-NS events per year would bring a great deal of astronomical and astrophysical information. A typical event would determine the chirp mass of the system very accurately and the individual masses to perhaps 20%. So we would soon build up a large sample of accurately known neutron star masses. Some information about spins might come from the data, but this remains to be seen. Very interestingly, NS-NS mergers are the leading candidate for the systems that create short hard gamma-ray bursts. Because gamma-ray bursts are beamed, one would not expect to associate a burst with each detection. But perhaps one or two of the 40 per year would be associated with a burst. And the other events might still be associated with some kind of X-ray, optical, or radio activity, which would lead to the identification of the host galaxy. One can hope, therefore, that a good fraction of the 40 NS-NS events each year will be identified to a particular galaxy. With that information, it will be possible to measure the redshift of these events, and couple that with the distance information provided by the GW signal itself. This would allow an accurate measurement (to a few percent) of the local Hubble constant, i.e. the expansion of the Universe within about 300 Mpc, which is about $z = 0.05$.

Observation of 20 BH-BH systems per year would provide us with our first census of black hole masses and spins, and finally provide observational data to constrain models of black hole formation. Although we will not have counterparts in the electromagnetic spectrum, it will still be possible to find a value of the Hubble constant from statistical methods, as I showed as long ago as 1986. Recent simulations using realistic galaxy catalogues show that we are likely to achieve values of the Hubble constant accurate to a few percent after 10-20 NS-NS events even without identifications (del Pozzo, arXiv:1108.1317), and similarly on a larger distance scale with BH-BH mergers. This is comparable to the best accuracy from other methods, but it has the great advantage of having very different systematic errors.

Expected science from space-based detectors like LISA

Binary systems are even more important for LISA than for ground-based detectors. While the details of the sensitivity of the mission depends on the outcome of the ongoing LISA re-design, the capabilities of the new detector will differ from those of LISA more in terms of numbers of signals that can be detected rather than in the nature of the sources.

Any space-based detector will register the waves from every binary systems of compact objects in the Galaxy that is tight enough to radiate in its frequency band. This amounts to tens or even hundreds of thousands of systems, and most of them will be so closely overlapping that they will blend into a confusion noise background. But thousands might still be resolvable.

A space detector will also detect coalescences of massive black holes, essentially throughout the universe and therefore back in time to very high redshifts, say up to $z = 10$ or 15. The mass range from 10^4 to $10^7\,M_\odot$ will provide an important window into the distribution of black holes today, their formation history, and their relationship to galaxy formation. By comparing the detected population with synthetic population models based on theories of early black hole formation and growth, a space-based detector will be able to discriminate between models and provide our earliest evidence of how the black holes that now inhabit galaxies like our own first formed. (Sesana, et al, Phys.Rev.D83:044036, 2011)

The massive black hole mergers also can be used to measure cosmological parameters, in the same way in principle as we described for NS-NS and BH-BH coalescences measured by LIGO an VIRGO.

LISA would be able to measure the Hubble constant to fractions of a percent, and the dark energy parameter w to a few percent, in only one or two years of observing. A longer mission could lead to a determination of the time-variation of w. (Petiteau, et al, Astrophys.J.732:82,2011)

However, the effectiveness of a space-based detector will depend on the configuration that is launched. On the ground, the distance to a source can be determined only once the other properties of the signal, such as its polarization and direction, are determined, and that requires a network. LISA's design also has such a network, because it has a triangular array that returns three different gravitational wave signals. But if the descoped mission is launched with only two active arms, and only one gravitational wave measurement, then it will not be able to determine polarization so easily and its ability to determine cosmological parameters will be degraded.

A detector in space could detect a stochastic cosmological background of gravitational waves provided the noise was larger than the noise in the detector. LISA would be able to reach an energy density in gravitational waves of $\Omega_{gw} \sim 10^{-10}$ in the milliHertz frequency range. This would be very interesting, because this frequency band is where radiation from a 1 TeV electroweak phase transition would be concentrated, if such a transition produced significant radiation. But it does not reach the standard inflation value of around 10^{-15}.

Partly because of this there has been discussion of much more sensitive detectors in space. NASA's concept study, called the Big Bang Observer (BBO), would reach 10^{-15} by putting four LISA-like constellations in orbit around the Sun, each with much more powerful lasers. This is not yet technically feasible, but the study shows just how hard it will be to construct such a detector. Similarly, a

Japanese proposal called DECIGO could, in its most sensitive form, do the same thing.

But such missions could have another cosmological payoff. Besides detecting an inflation-generated background, such missions would inevitably detect every NS-NS binary in the universe that happens to pass through its frequency band (many of these systems are already orbiting tightly enough to coalesce within a few years or less). With the sensitivity of these systems it would be possible to use these binaries to measure the expansion rate of the universe at the time they existed, perhaps at redshift 3 or more! This is because it would be possible to measure the extra differential redshift placed on the signal between its beginning and its termination a year or more later (N Seto, S Kawamura, T Nakamura *Phys. Rev. Lett.*, **87**,221103, 2001). This would be another way of constraining dark energy in the early universe!

Azores School on Cosmology

Gravitational Waves

1-5 September 2011

B F Schutz

Albert Einstein Institute (AEI), Potsdam, Germany

and

School of Physics and Astronomy, Cardiff University

Download Lecture Notes

- You can find the lecture notes available for download at

  http://www.aei.mpg.de/~schutz/download/lectures/AzoresCosmol

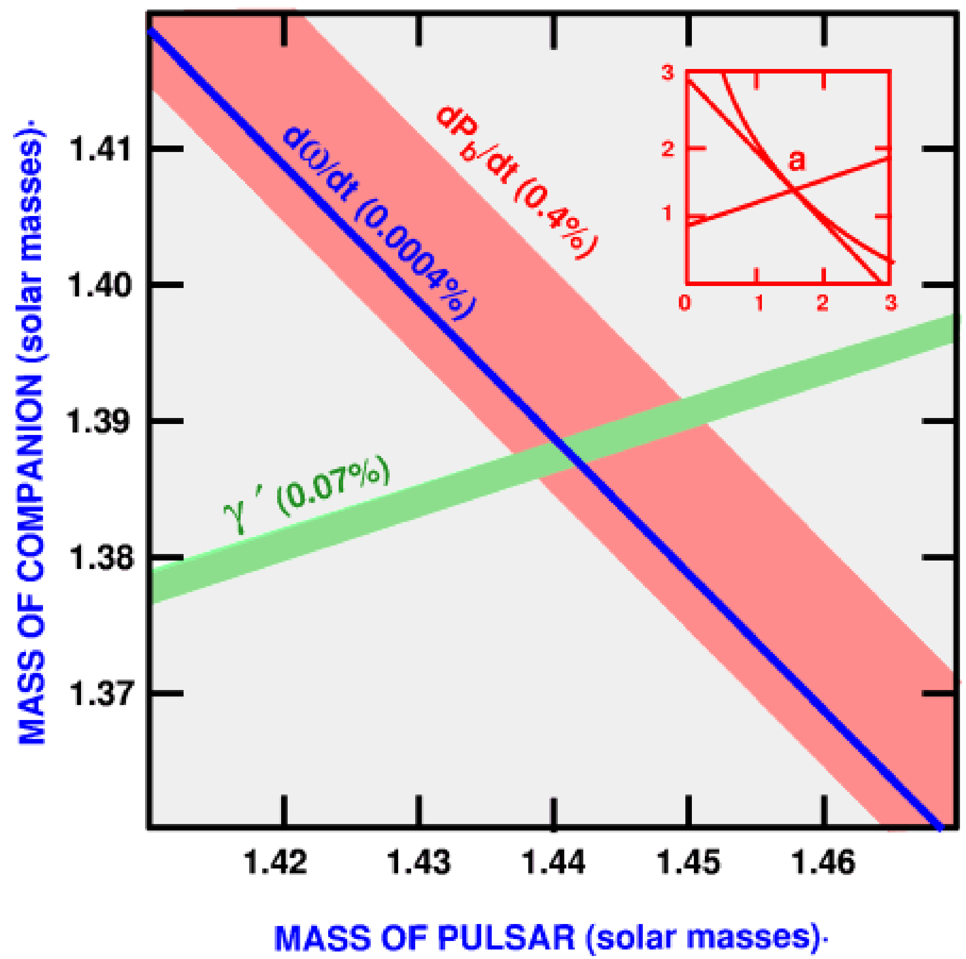
- There are 4 files: an outline, and the three lectures.

Testing Gravitational Wave Theory

- In 1993 Hulse and Taylor were awarded the Nobel Prize for the discovery and scientific exploitation of PSR1913+16.
- Now there are about 6 similar systems, and the spectacular “double pulsar” PSR J0737-3039 overtook 1913 in precision before it became a single pulsar system again.
- Since the GW frequency is just slightly lower than the LISA waveband, LISA and LIGO should have confidence that the GWs they detect will not be too different from the GR prediction.

Confirmation of GR

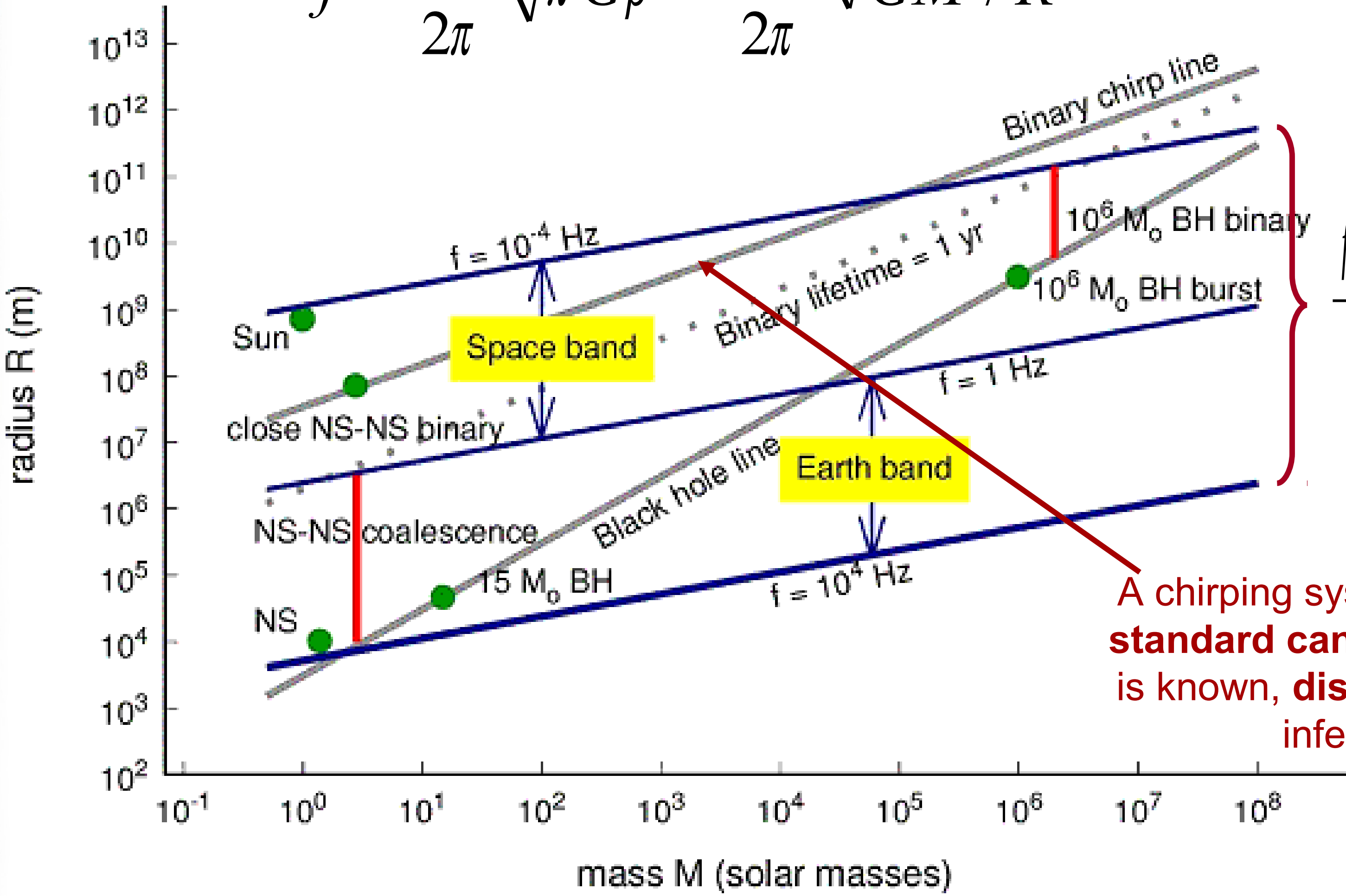
This is a parameter-free confirmation. Perihelion shift, Shapiro delay, redshift by companion star determine *all* orbital parameters. GW energy loss is therefore a *strong test.*



(Will LRR)

GW physics across the spectrum

$$f = \frac{1}{2\pi}\sqrt{\pi G\rho} \sim \frac{1}{2\pi}\sqrt{GM/R^3}$$

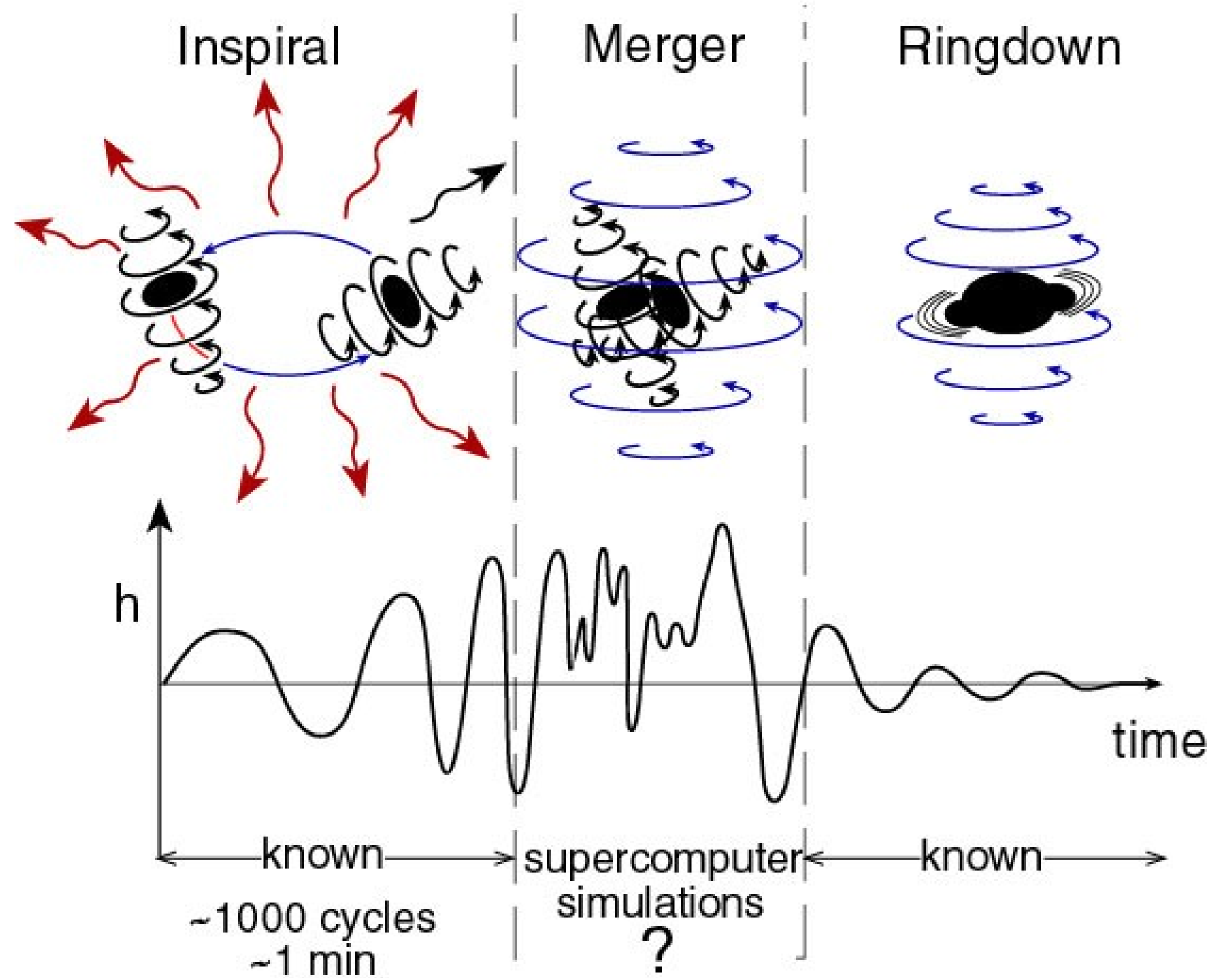


$$\frac{f_{high}}{f_{low}} = 10^8$$

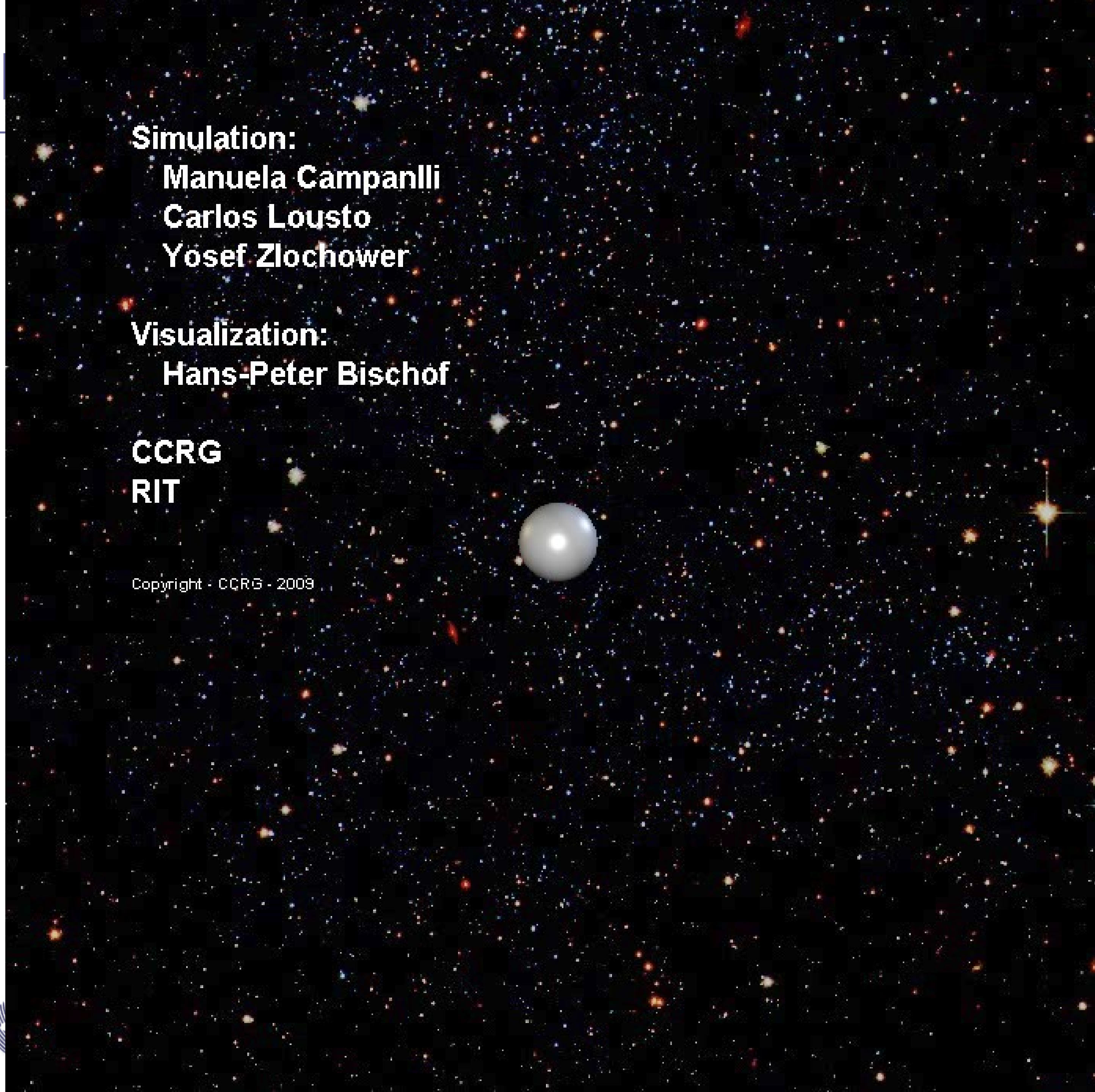
A chirping system is a GW **standard candle**: if position is known, **distance** can be inferred.

Three Phases of BH Merger with Comparable Masses

(Courtesy Kip Thorne)



- Inspiral phase gives primary SNR for detecting signals, measuring masses.
- Merger and ringdown contain extra spin information, plus final state.
- Hybrid pN-Numrel waveforms used now in searches.



g
rge

at

to
ss

BH

Listen to Binary Mergers

- Inspiral (no frequency conversion if M ~ 10 solar).