CMB Anisotropies and Fundamental Physics

Lecture II

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Lecture II

CMB & PARAMETERS

(Mostly Dark Energy)

Things we learned from lecture I

- Theory of CMB Temperature anisotropy was well established by 1995. No evidence of peaks in the angular spectrum at that time.

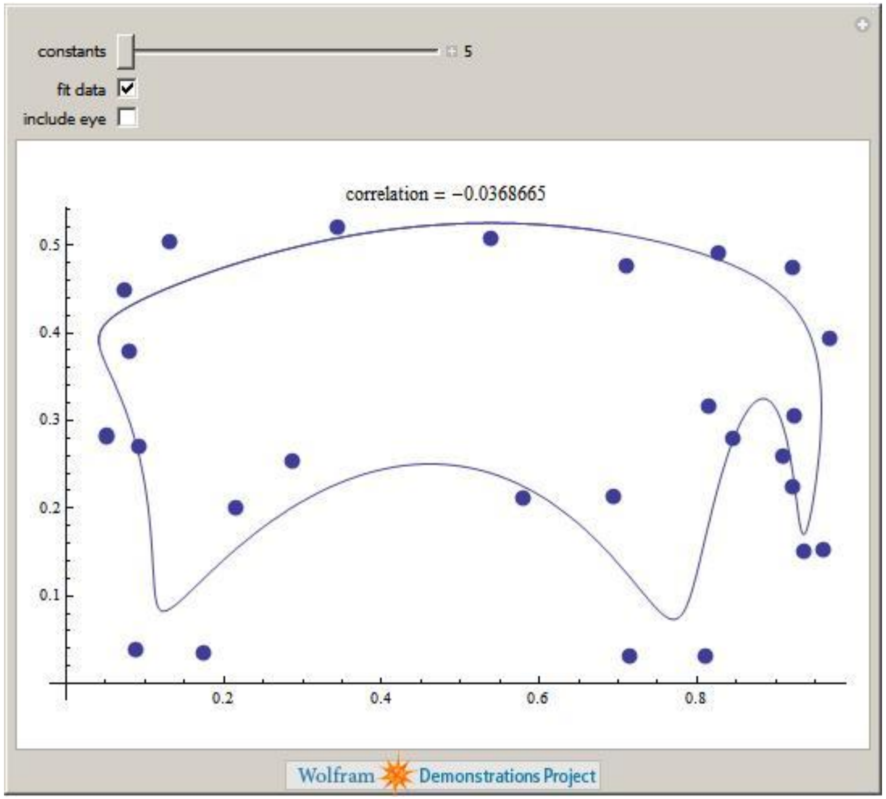
- This highly predictive theory motivated high accuracy measurements (Planck mission was approved and selected by ESA in 1996 !).

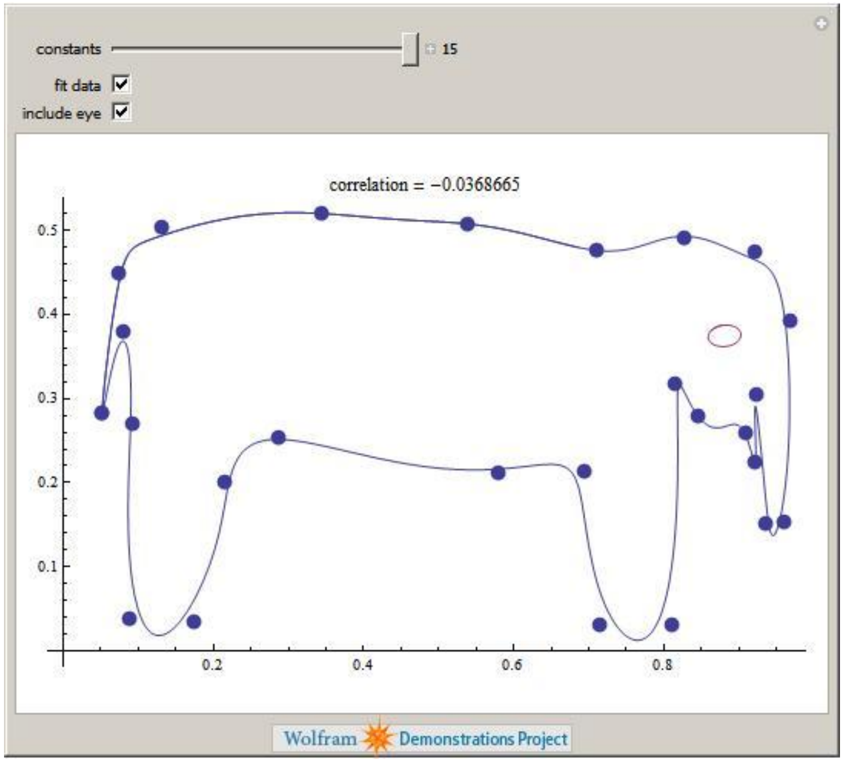
- Result: Success for the standard model based on passive and coherent perturbations. Textures and topological defects ruled out as the only mechanism of structure formation.

- Physics of the CMB. Acoustic oscillations related to the baryon density. Projection renders CMB anisotropy sensitive to the angular diameter distance at recombination.

How many parameters are needed to describe the CMB anisotropies ?

Enrico Fermi:"I remember my friend Johnny von Neumann used to say, 'with four parameters I can fit an elephant and with five I can make him wiggle his trunk.'"





The standard cosmological model

- Assumes General Relativity, Inflation, Adiabatic and Scalar Perturbations, flat universe.

- Friedmann-Robertson-Walker (or Friedmann-Lemaitre) metric. Hubble Constant (+1)

$$H_0 = 100h\ km/s/Mpc$$

- 3 Energy components: Baryons, Cold Dark Matter, Cosmological Constant (+3). Flat Universe (-1).

$$\omega_b = \Omega_b h^2 \qquad \omega_{CDM} = \Omega_{CDM} h^2$$

- Initial conditions for perturbations given by Inflation: Adiabatic, nearly scale invariant initial power spectrum, only scalar perturbations. Two free parameters (+2): Amplitude and Spectral index.

$$P(k) \approx A_S \left(\frac{k}{k_0}\right)^{n_S}$$

Pivot scale is usually fixed to:

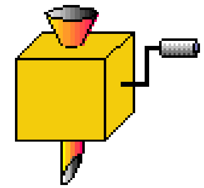
$$k_0 = 0.002\ \ hMpc^{-1}$$

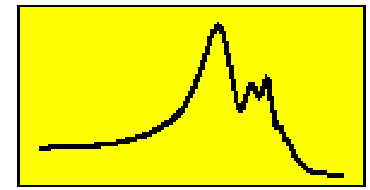
- Late universe reionization characterized with a single parameter(+1) : optical depth τ or reionization redshift z_r.

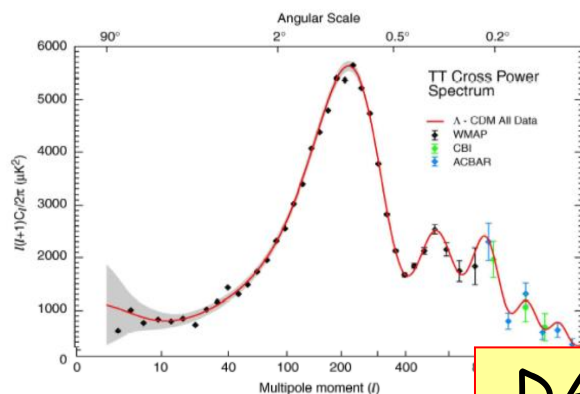
Total: 1+3-1+2+1= 6 parameters.

How to get a bound on a cosmological parameter

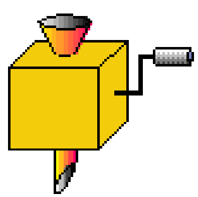
Fiducial cosmological model:
$(\Omega_b h^2 , \Omega_m h^2 , h , n_s , \tau , \Sigma m_\nu)$





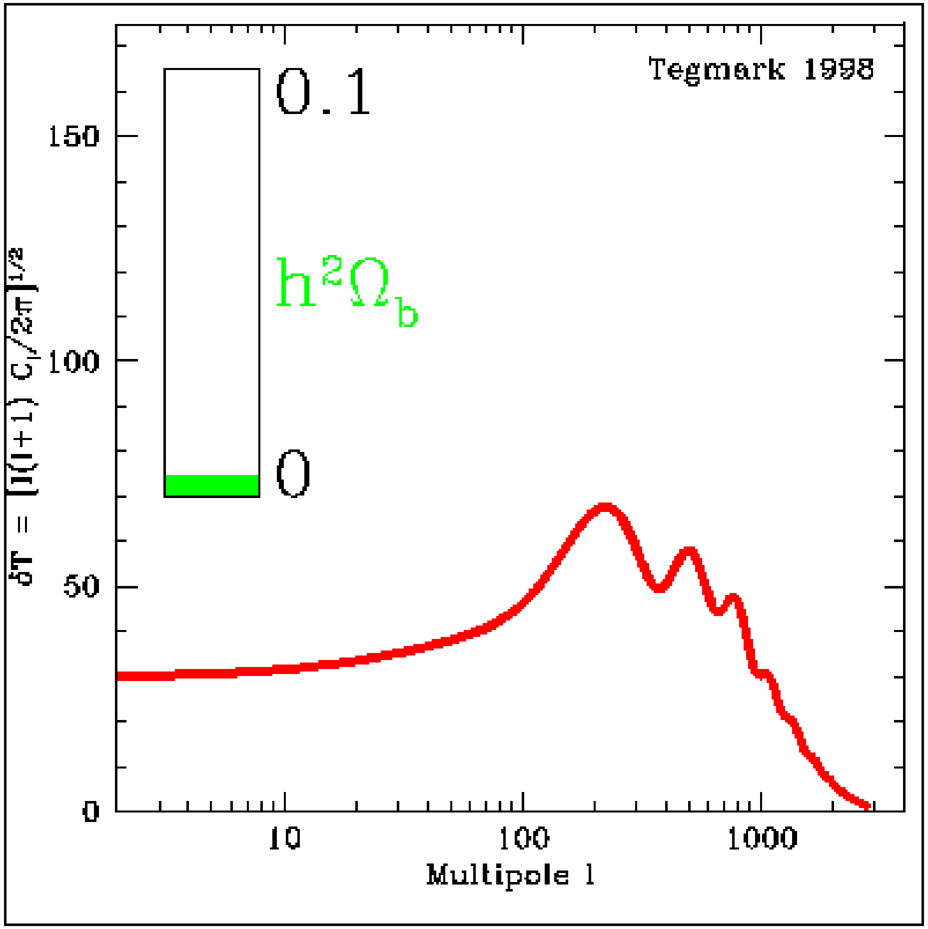


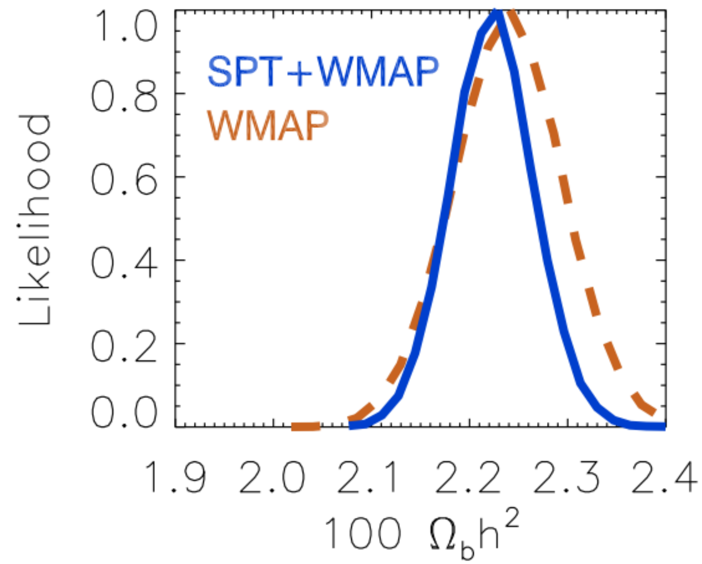
DATA



PARAMETER ESTIMATES

Constraints on the Baryon Abundance from Current CMB data



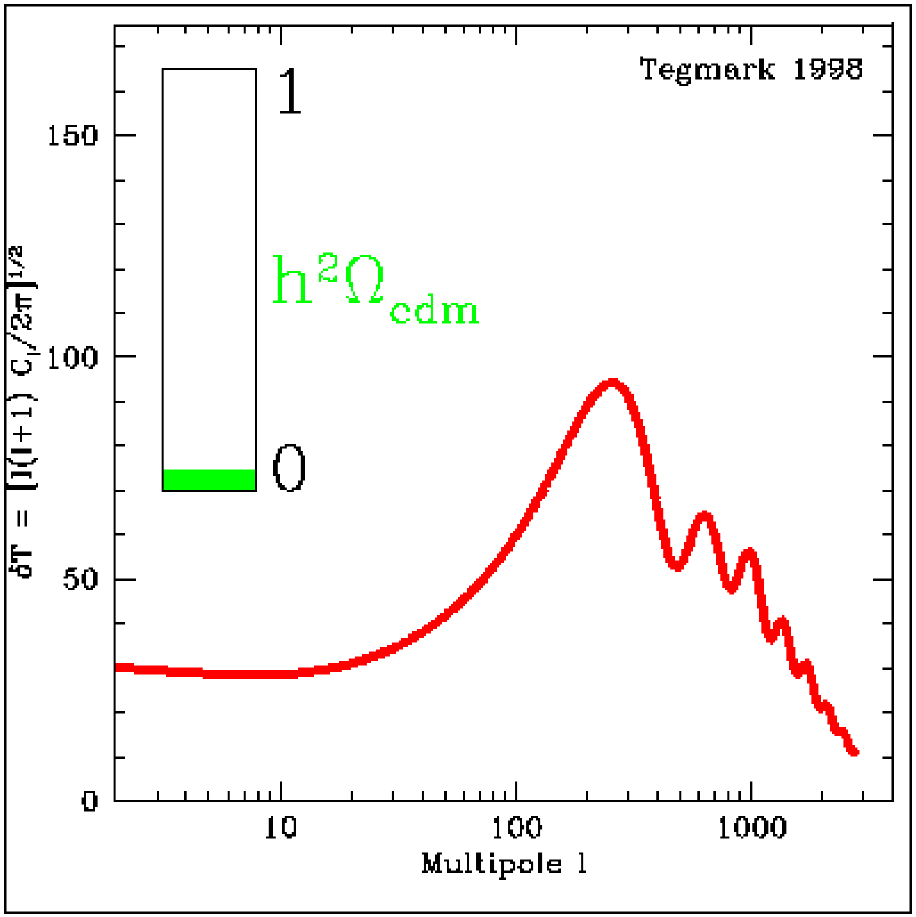


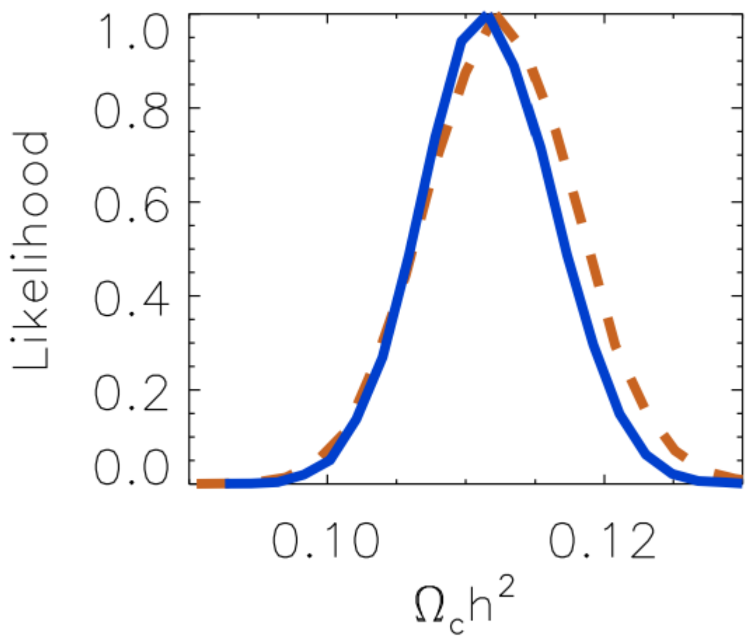
$$100\Omega_b h^2 \qquad 2.22 \pm 0.042$$

1.9% error !!

	CMB-based BBN prediction	Observed Value
$10^5 y_D^{FIT}$	$2.57^{+0.17}_{-0.13}$	1.6 - 4.0
$10^5 y_3$	$1.05 \pm 0.03 \pm 0.03$ (syst.)	$< 1.1 \pm 0.2$
Y_P	$0.24819^{+0.00029}_{-0.00040} \pm 0.0006$(syst.)	0.232 - 0.258
[Li]	2.64 ± 0.03	2.2 - 2.4

Constraints on the CDM Abundance from Current CMB data

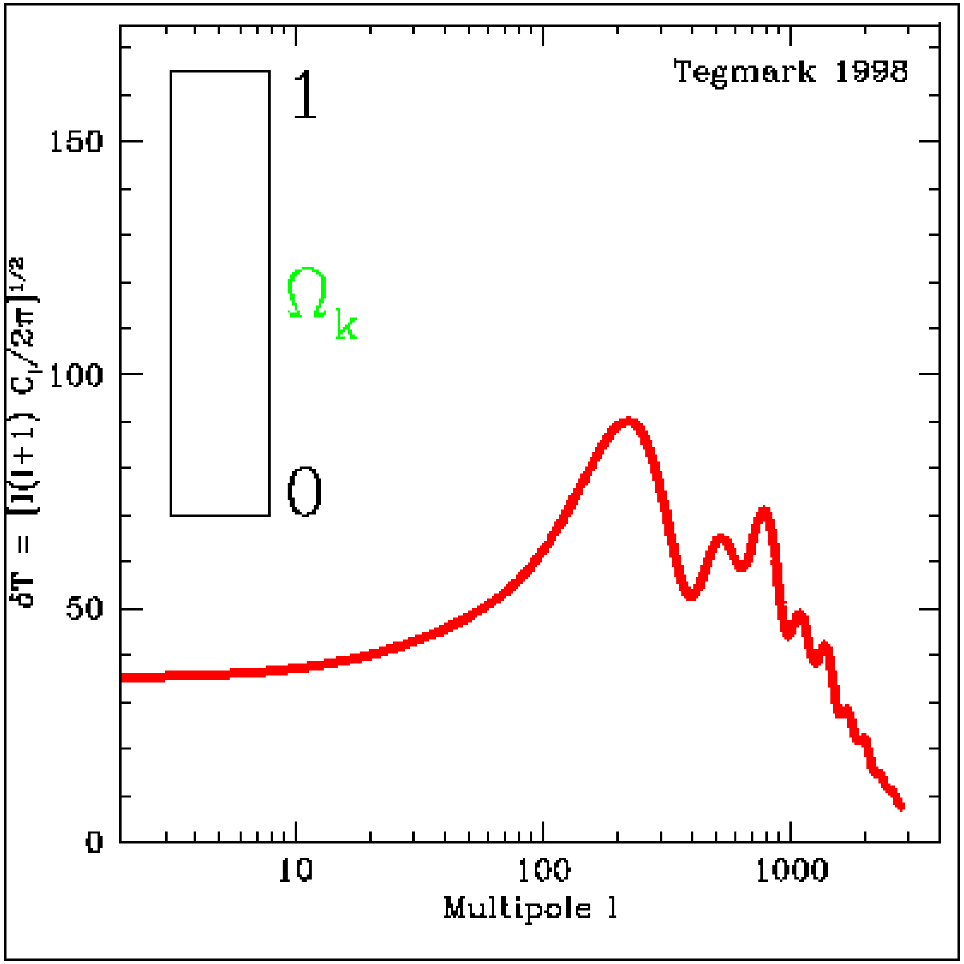




$\Omega_c h^2$ 0.112 ± 0.0048

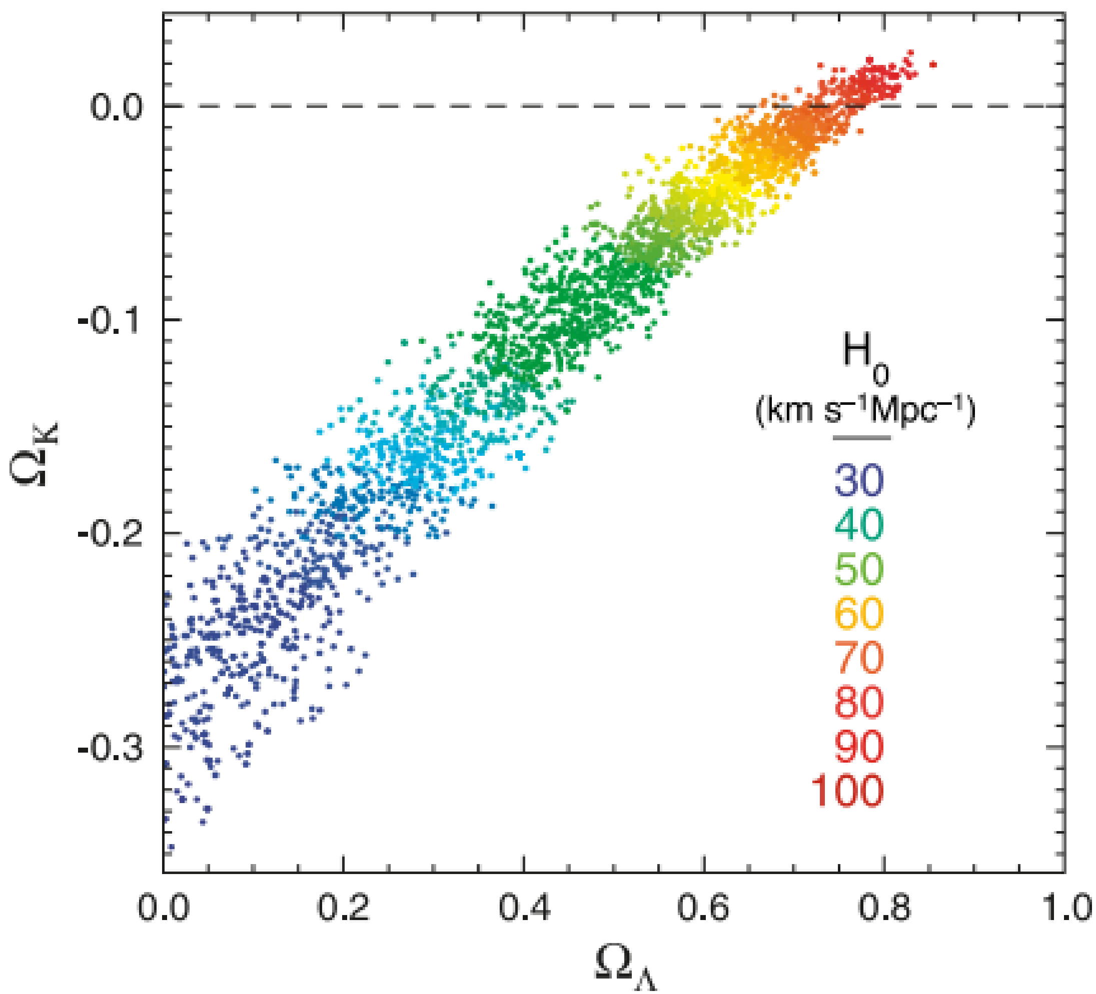
23 sigmas
Evidence for
CDM !!

Constraints on Curvature from Current CMB data



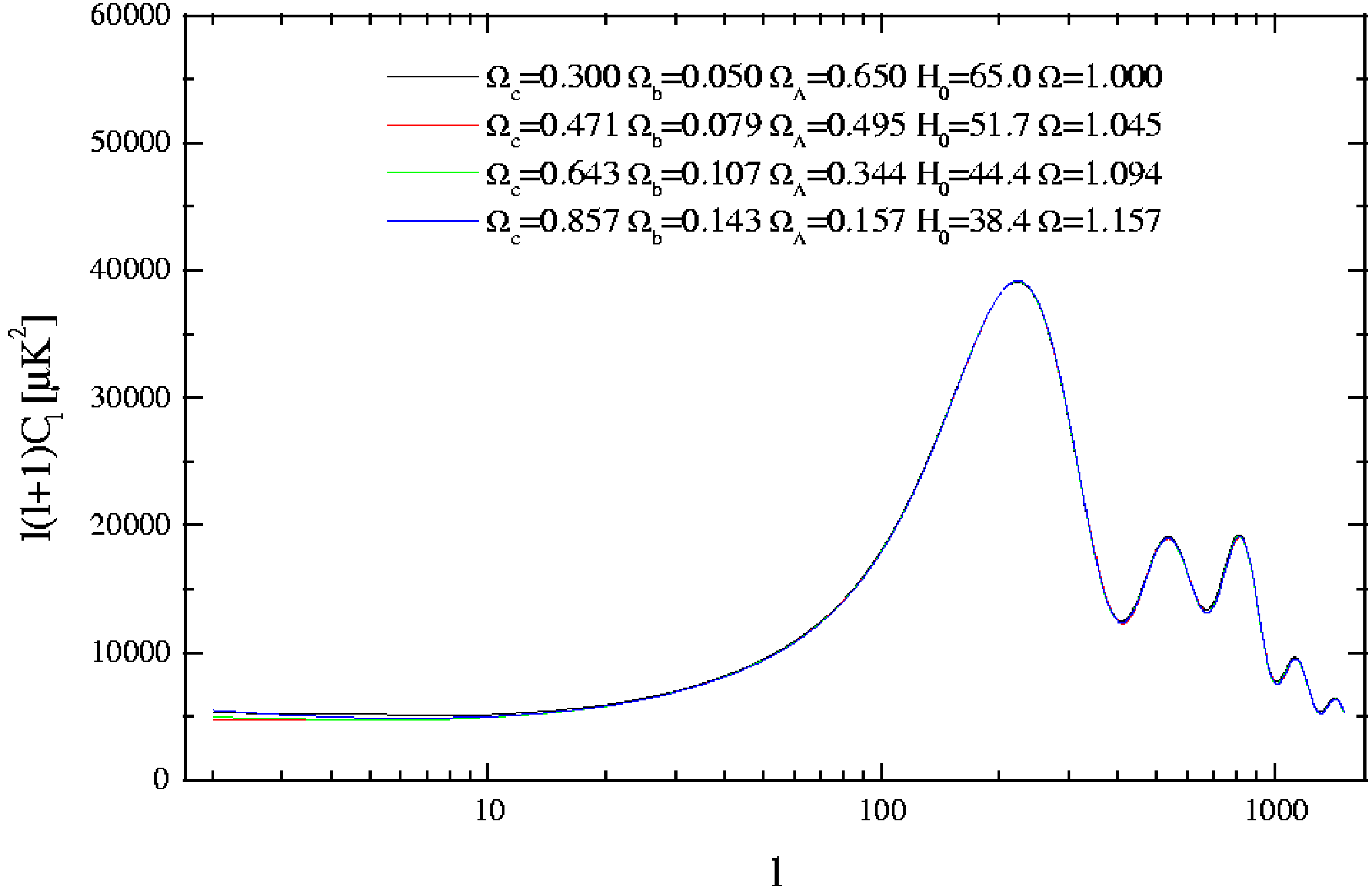
CMB does NOT constrain Curvature !!

Constraints on curvature from WMAP



Geometrical degeneracy

See, e.g. Efstathiou and Bond 1998
Melchiorri and Griffiths, 2001



CMB Parameters

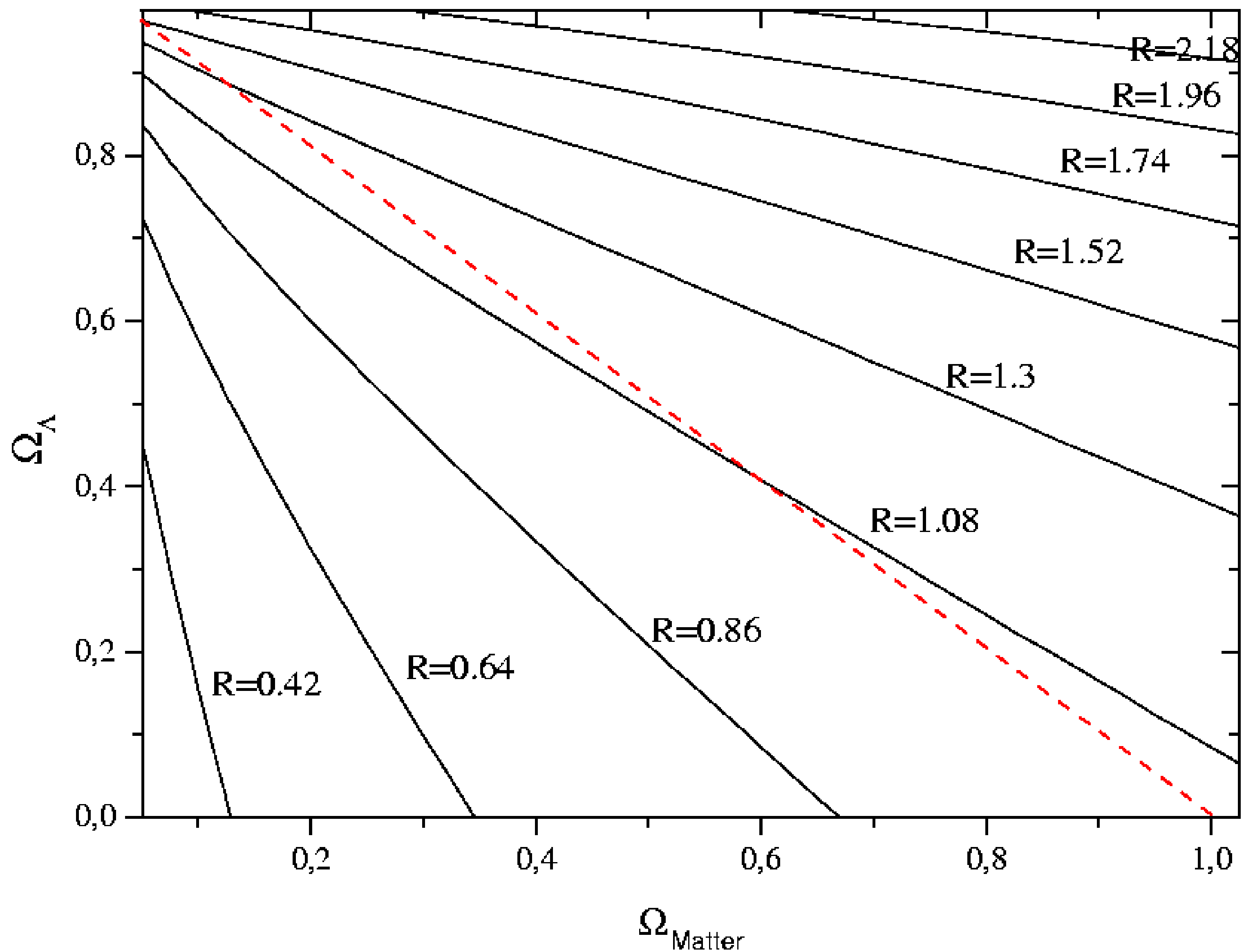
- Baryon Density $\Omega_b h^2$

- CDM Density $\Omega_{CDM} h^2$

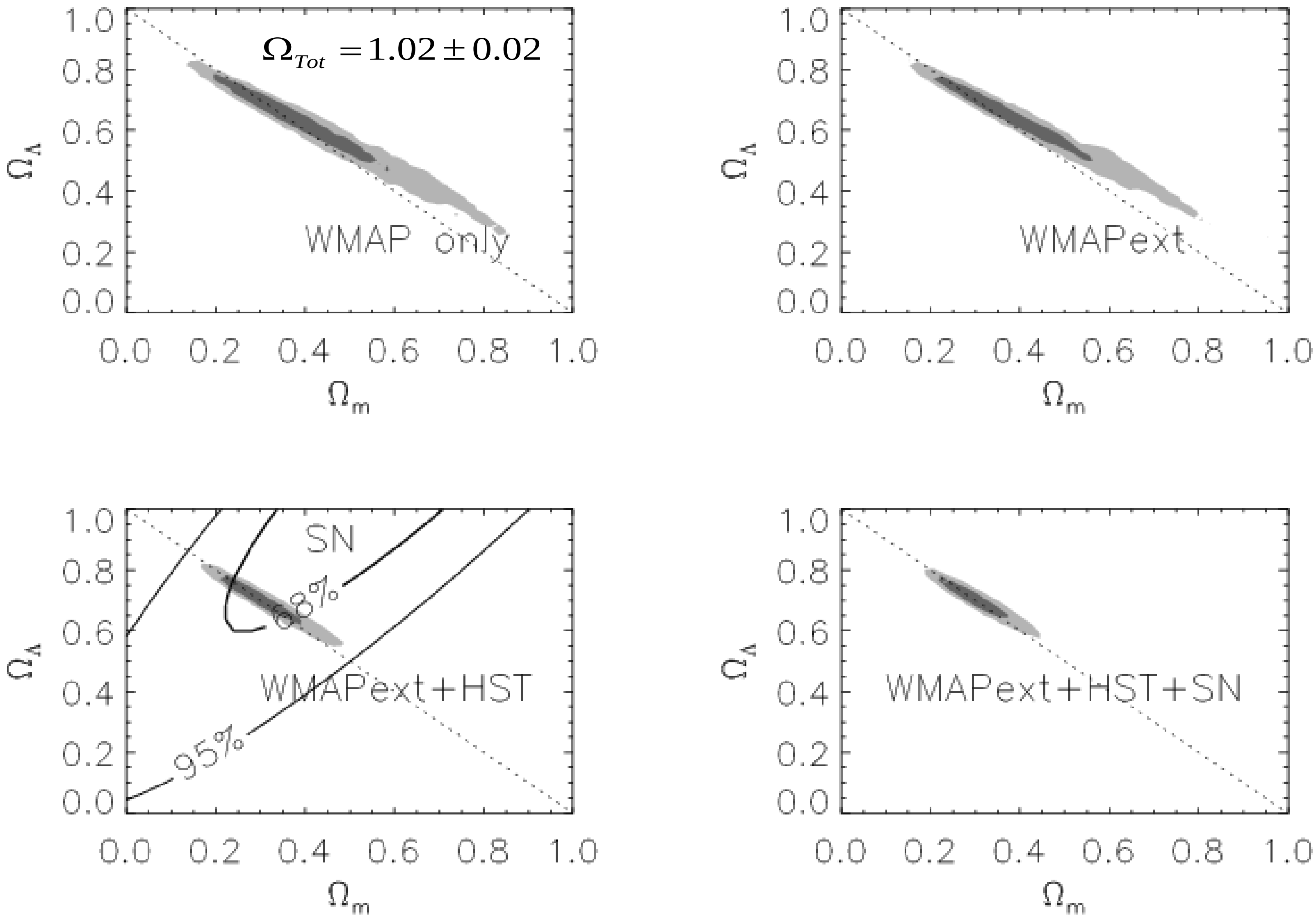
- Distance to the LSS, «Shift Parameter» :

$$R = \sqrt{\frac{\Omega_M h^2}{|\Omega_k| h^2}}\,\chi(y)$$

$$\chi(y) = \begin{pmatrix} \sin y, k < 0 \\ y, k = 0 \\ \sinh y, k > 0 \end{pmatrix} \qquad y = \sqrt{|\Omega_K|}\int_0^{z_{dec}} \frac{dz}{\sqrt{\Omega_m (1+z)^3 + \Omega_K (1+z)^2 + \Omega_\Lambda}}$$



R=2.18
R=1.96
R=1.74
R=1.52
R=1.3
R=1.08
R=0.86
R=0.64
R=0.42
0,0
0,2
0,4
0,6
0,8
1,0
Ω_Λ
Ω_{Matter}

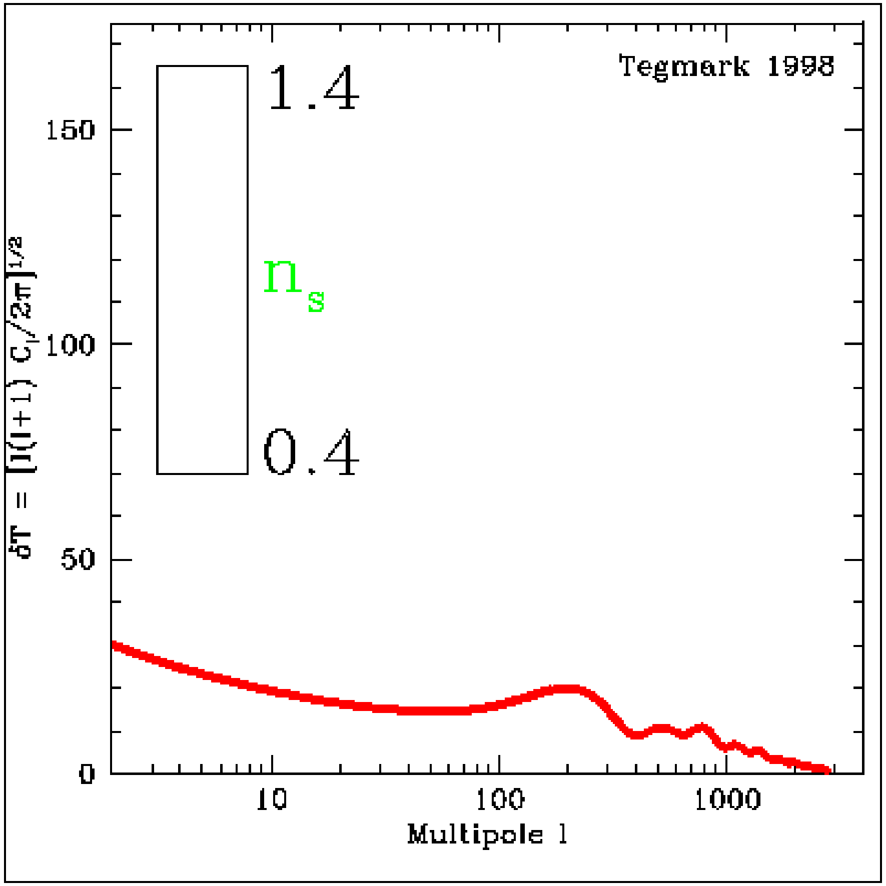


$\Omega_{Tot} = 1.02 \pm 0.02$
WMAP only
WMAPext
SN
68%
95%
WMAPext+HST
WMAPext+HST+SN
Ω_Λ
Ω_m

Important message: be careful in interpreting the CMB Constraints. Prior volume could bias your result.

The «misleading» suggestion that WMAP data is preferring closed universes motivated theories for «finite universes»…

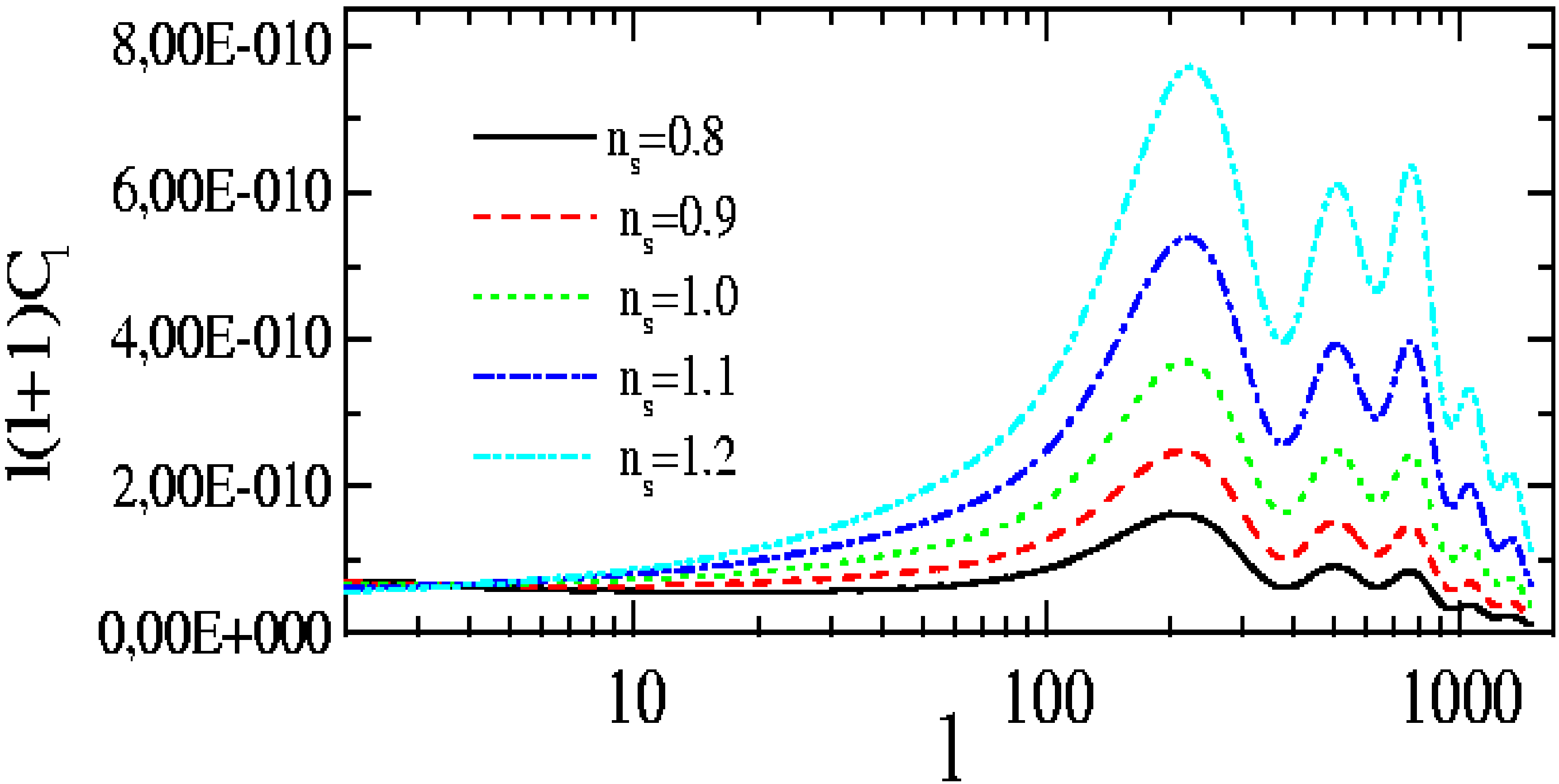
Inflationary parameters



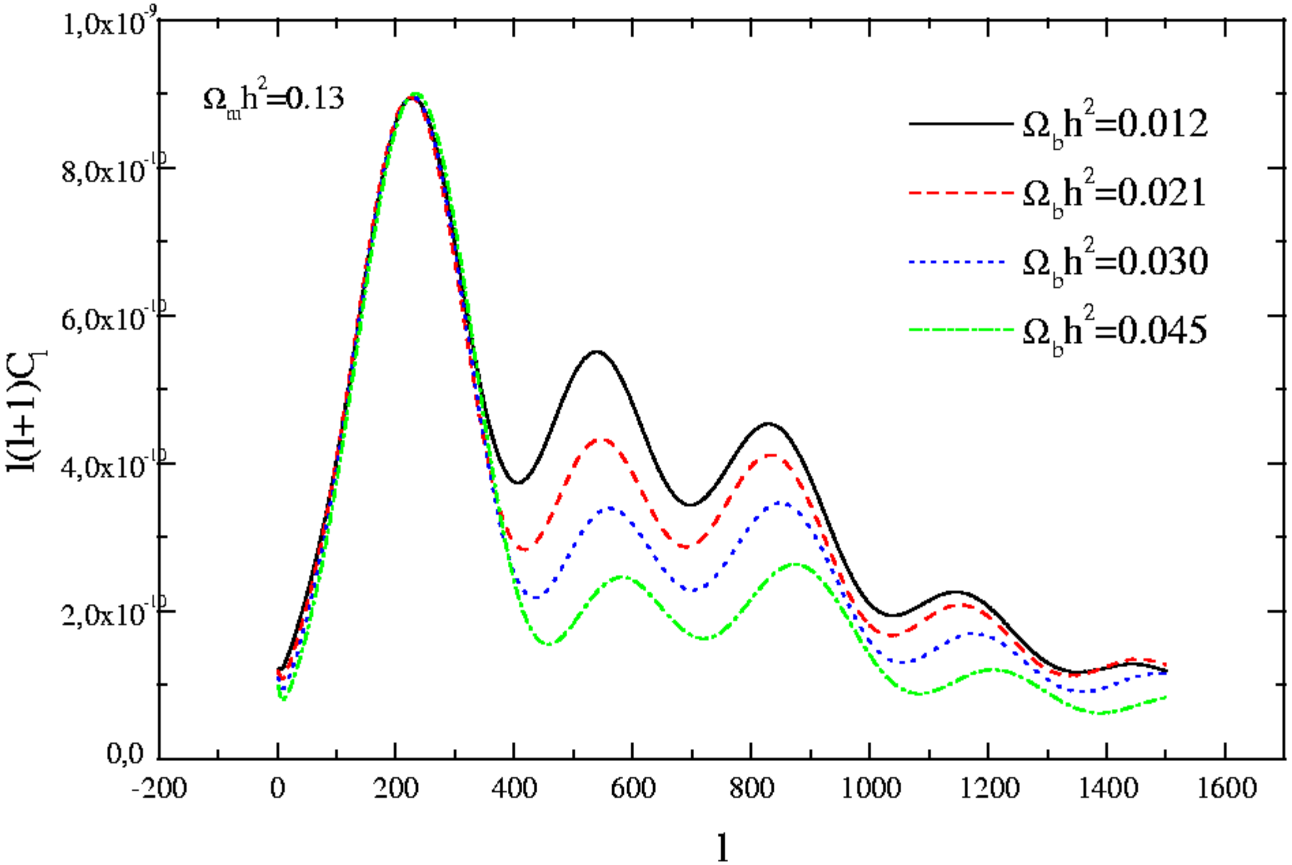
$$P(k) \approx A_S \left(\frac{k}{k_0} \right)^{n_S}$$

Inflationary parameters

$$n_S \downarrow, \Delta P_{12} \uparrow, \Delta P_{32} \downarrow$$

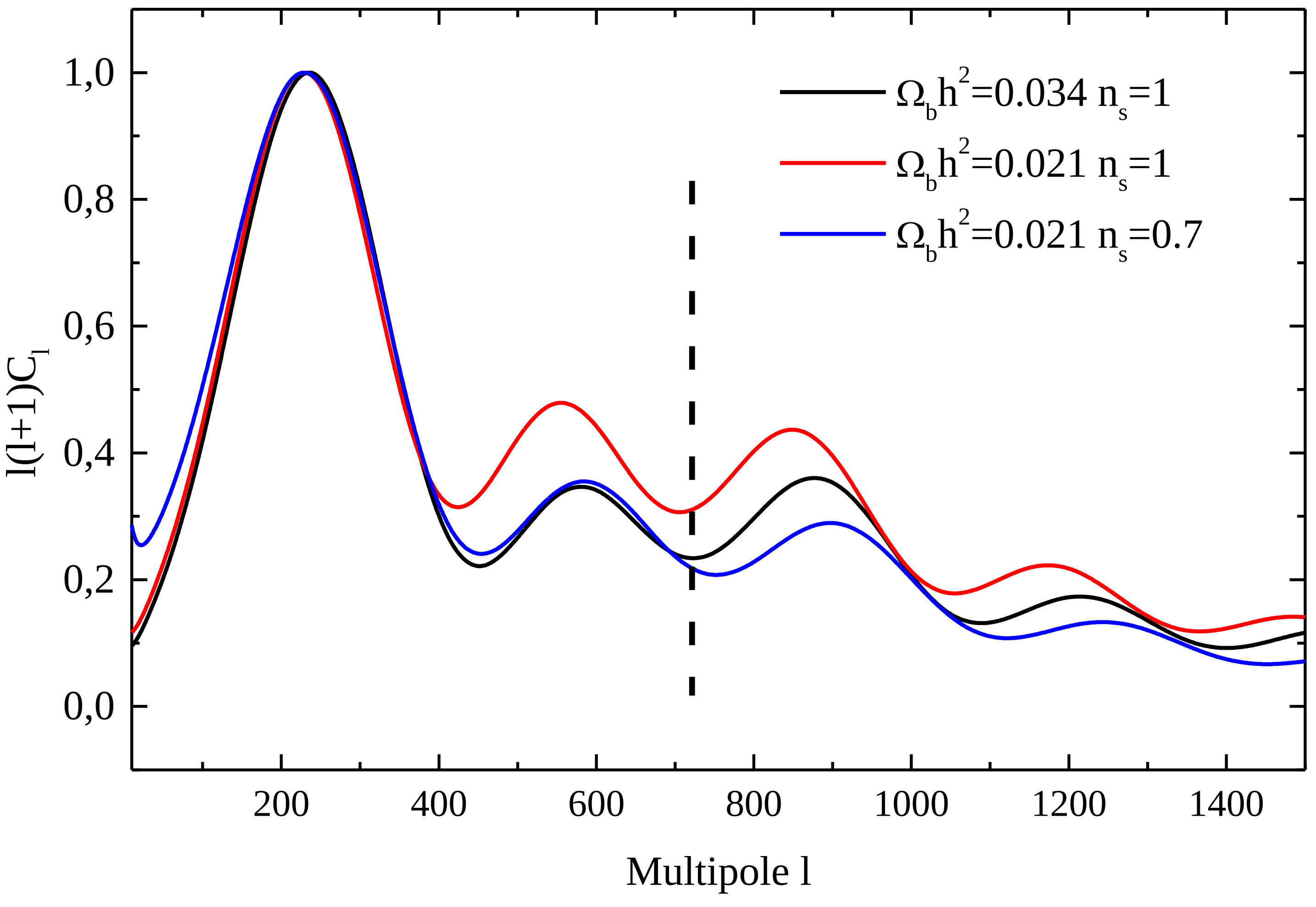


Baryonic Abundance

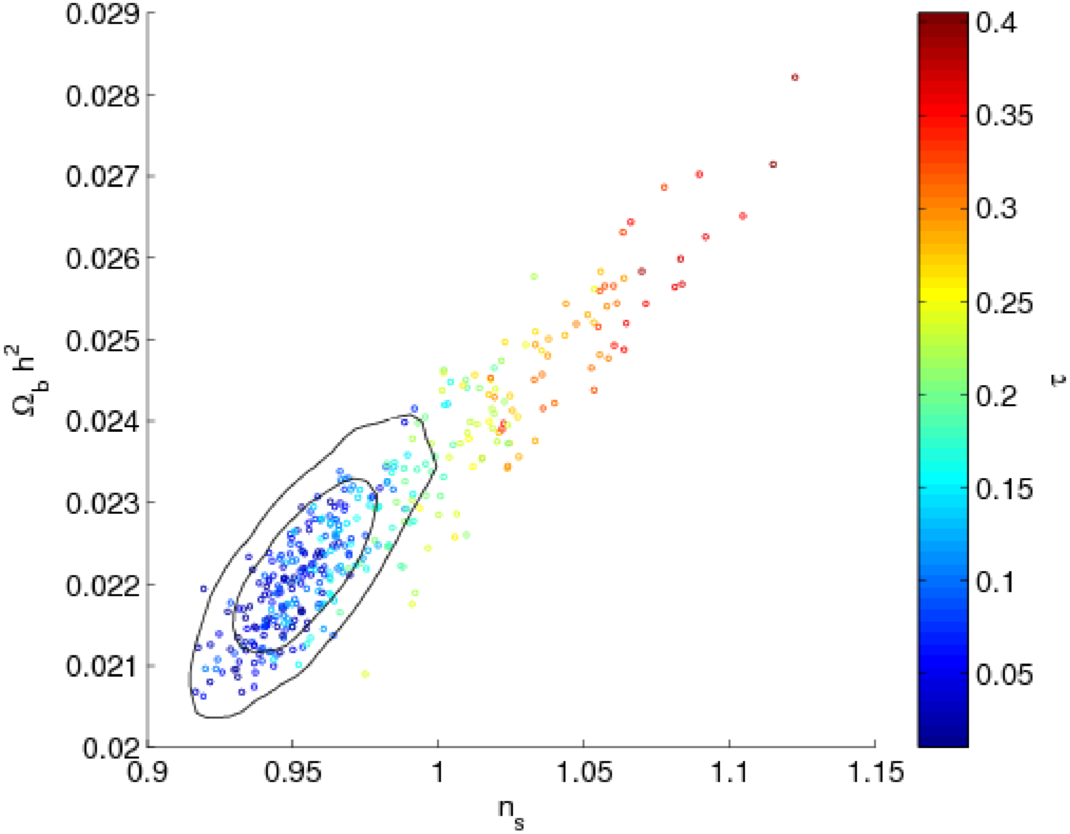


$$\Omega_B h^2 \uparrow, \Delta P_{12} \uparrow, \Delta P_{32} \uparrow$$

Up to the 2nd peak n and the baryon density are degenerate.



Degeneracy between spectral index and baryon density is seen in WMAP data.



During the slow-roll phase, the kinetic energy of the field is negligible and the potential is nearly constant:

$$\rho_\phi = V(\phi) + \frac{\dot{\phi}^2}{2} \approx V(\phi) \approx \text{const.}$$

This gives rise to a (quasi-) de Sitter phase: $H^2 = \frac{8\pi G}{3} V(\phi)$

The perturbations in the field are proportional to the value of the Hubble parameter *at the time of horizon crossing:*

$$(\Delta\phi)_k = \left(\frac{H_{hc}}{2\pi}\right)^2 = \frac{2G}{3\pi} V(\phi)^2$$

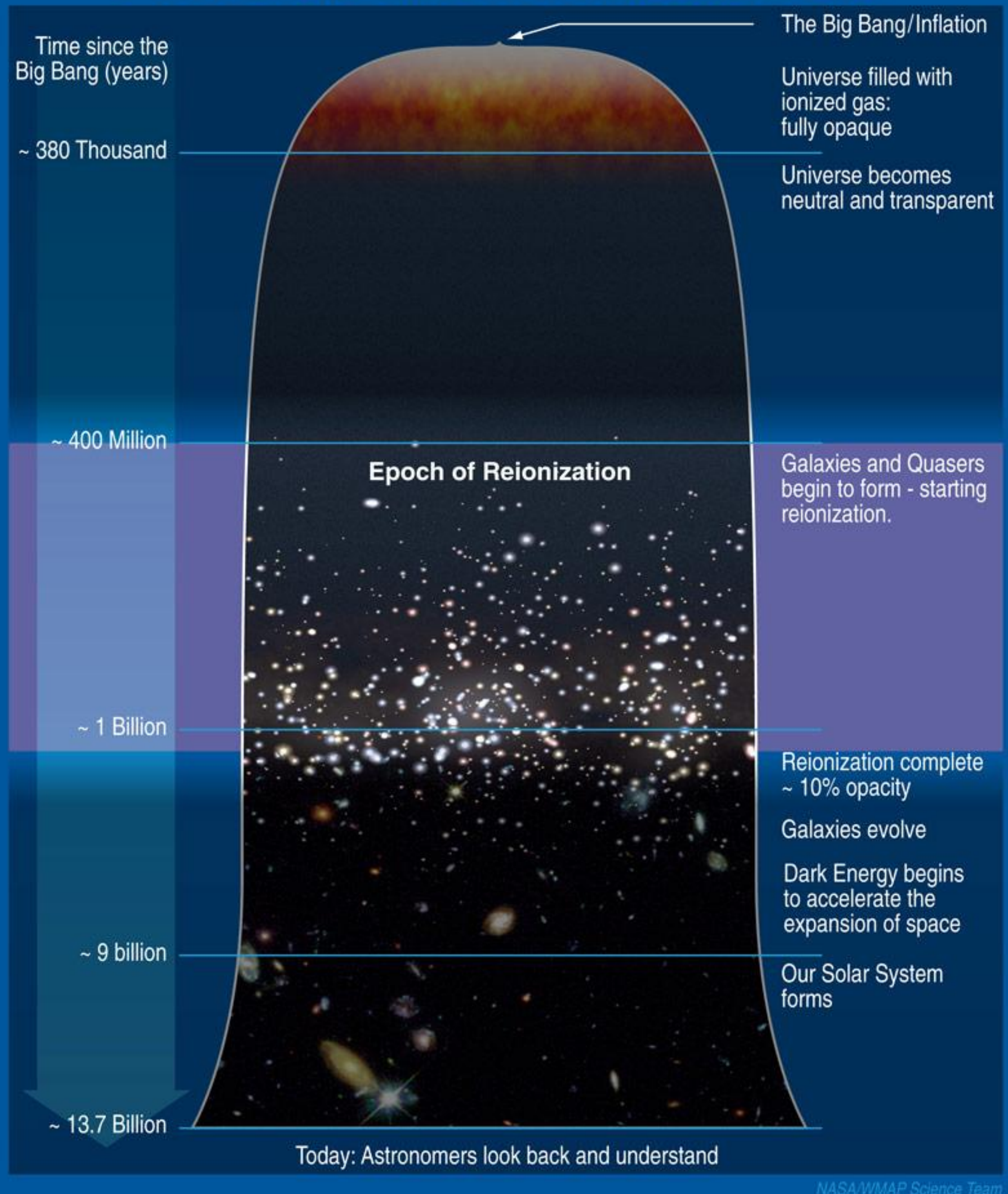
Since $V(\phi)$ is not actually constant, but slowly-varying, we expect a weak dependence of the amplitude of the perturbations on the wavenumber

⇨ If the perturbations were originated from the dynamics of a scalar field the spectrum should not be *exactly* scale invariant

The spectral index as a «test» for inflation

- Inflation predicts $n \approx 1$ but $n \neq 1$
- If $n \neq 1$ this would provide an indication for the dynamical evolution as perturbation are being produced

First Stars and Reionization Era

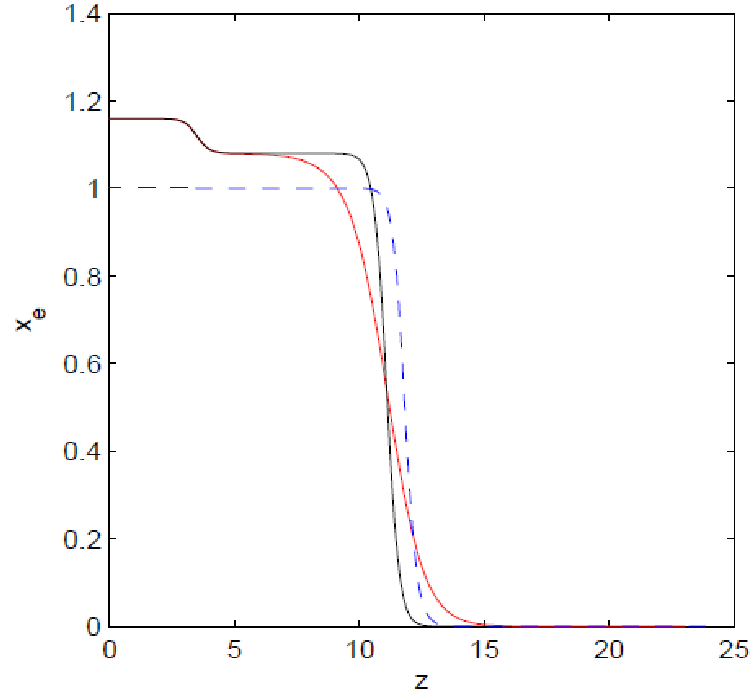


How reionization is implemented when CMB anisotropies are computed: simple «step» function !

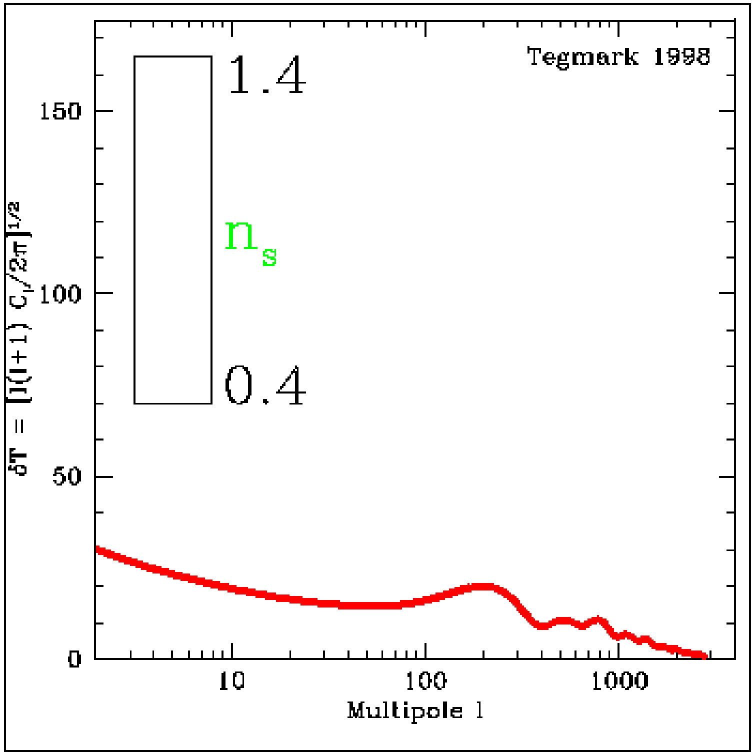
$$y(z_{re}) = (1 + z_{re})^{3/2}$$

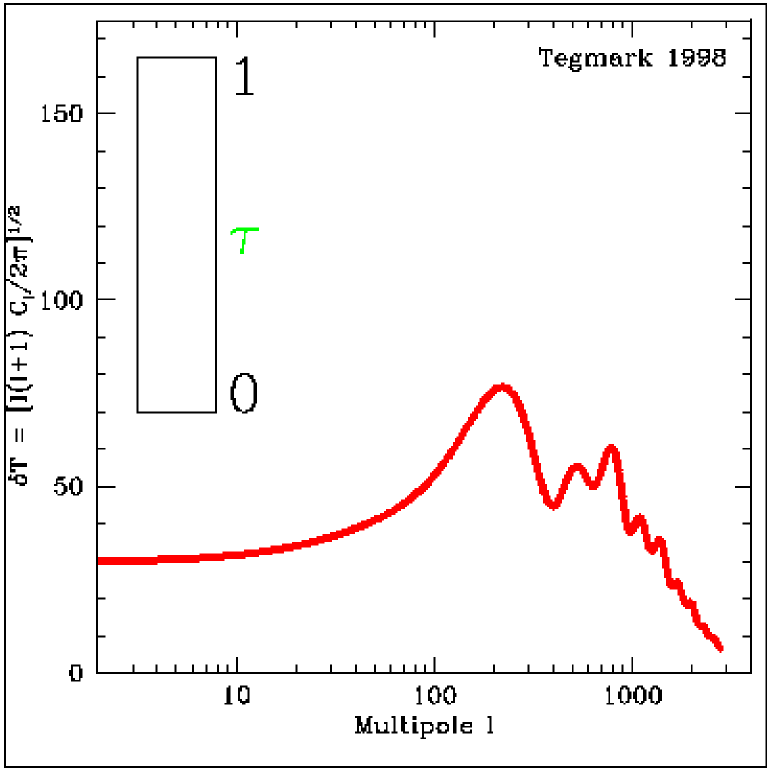
$$x_e(y) = \frac{f}{2}\left[1 + \tanh\left(\frac{y - y(z_{re})}{\Delta_y}\right)\right]$$

$$\tau = \int_0^{25} \frac{\sigma_T n_e x_e}{(1+z')H(z')} dz'$$

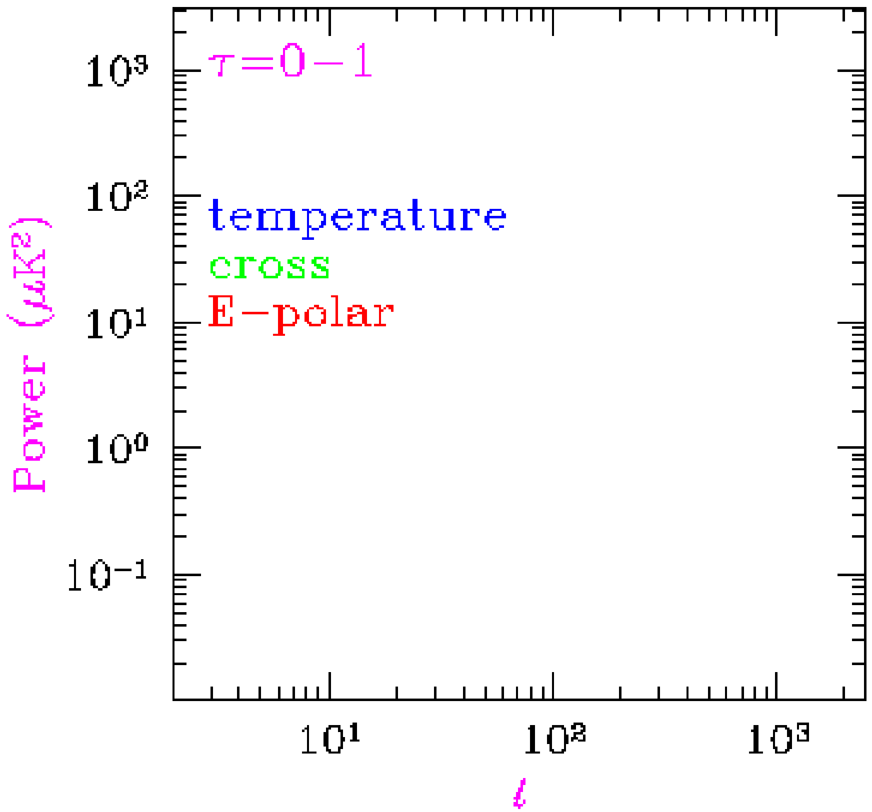


The spectral index is almost completely degenerate with the optical depth.





Measuring large scale CMB polarization can break this degeneracy.



WMAP7

WMAP 7-year Cosmological Interpretation 13

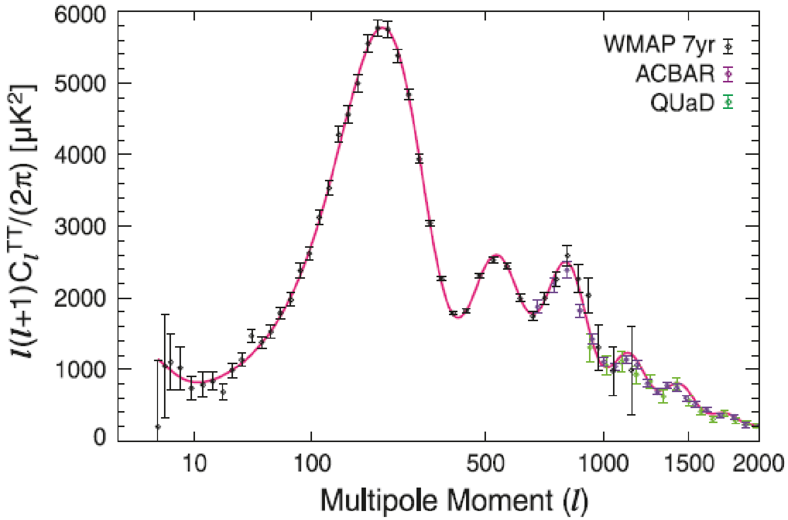


FIG. 7.— The *WMAP* 7-year temperature power spectrum (Larson et al. 2010), along with the temperature power spectra from the ACBAR (Reichardt et al. 2009) and QUaD (Brown et al. 2009) experiments. We show the ACBAR and QUaD data only at $l \geq 690$, where the errors in the *WMAP* power spectrum are dominated by noise. We do not use the power spectrum at $l > 2000$ because of a potential contribution from the SZ effect and point sources. The solid line shows the best-fitting 6-parameter flat ΛCDM model to the *WMAP* data alone (see the 3rd column of Table 1 for the maximum likelihood parameters).

4.1. *Primordial Spectral Index and Gravitational Waves*

The 7-year *WMAP* data combined with BAO and H_0 exclude the scale-invariant spectrum by more than 3σ, if we ignore tensor modes (gravitational waves).

For a power-law power spectrum of primordial curvature perturbations $\mathcal{R}_k$, i.e.,

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3\langle|\mathcal{R}_k|^2\rangle}{2\pi^2} = \Delta^2_{\mathcal{R}}(k_0)\left(\frac{k}{k_0}\right)^{n_s-1}, \qquad (29)$$

where $k_0 = 0.002\ \mathrm{Mpc}^{-1}$, we find

$$n_s = 0.963 \pm 0.012\ (68\%\ \mathrm{CL}).$$

Class	Parameter	*WMAP* 7-year Mean[b]	*WMAP*+BAO+H_0 Mean
Primary	$100\Omega_b h^2$	$2.258^{+0.057}_{-0.056}$	2.260 ± 0.053
	$\Omega_c h^2$	0.1109 ± 0.0056	0.1123 ± 0.0035
	Ω_Λ	0.734 ± 0.029	$0.728^{+0.015}_{-0.016}$
	n_s	0.963 ± 0.014	0.963 ± 0.012
	τ	0.088 ± 0.015	0.087 ± 0.014
	$\Delta^2_{\mathcal{R}}(k_0)$[c]	$(2.43 \pm 0.11) \times 10^{-9}$	$(2.441^{+0.088}_{-0.092}) \times 10^{-9}$

Constraints on the spectral index can be affected by Model of reionization.

Model	Parameter[a]	7-year *WMAP*[b]	*WMAP*+ACBAR+QUaD[c]	*WMAP*+BAO+H_0
Power-law	n_s	0.963 ± 0.014	$0.962^{+0.014}_{-0.013}$	0.963 ± 0.012

Komatsu, E., et.al (2010)

MH's Reionization

Following Mortonson & Hu we can parametrize the reionization history as a free function of the redshift by decomposing the free electron fraction as

$$x_e(z) = x_e^f(z) + \sum_\mu m_\mu S_\mu(z)$$

- The principal components $S_\mu(z)$ are the eigenfunctions of the Fisher matrix of an ideal, cosmic variance limited, experiment.

$$F_{ij} = \sum_{\ell=2}^{\ell_{\max}} \left(\ell + \frac{1}{2}\right) \frac{\partial \ln C_\ell^{EE}}{\partial x_e(z_i)} \frac{\partial \ln C_\ell^{EE}}{\partial x_e(z_j)} \qquad F_{ij} = (N_z+1)^{-2} \sum_{\mu=1}^{N_z} S_\mu(z_i) \sigma_\mu^{-2} S_\mu(z_j)$$

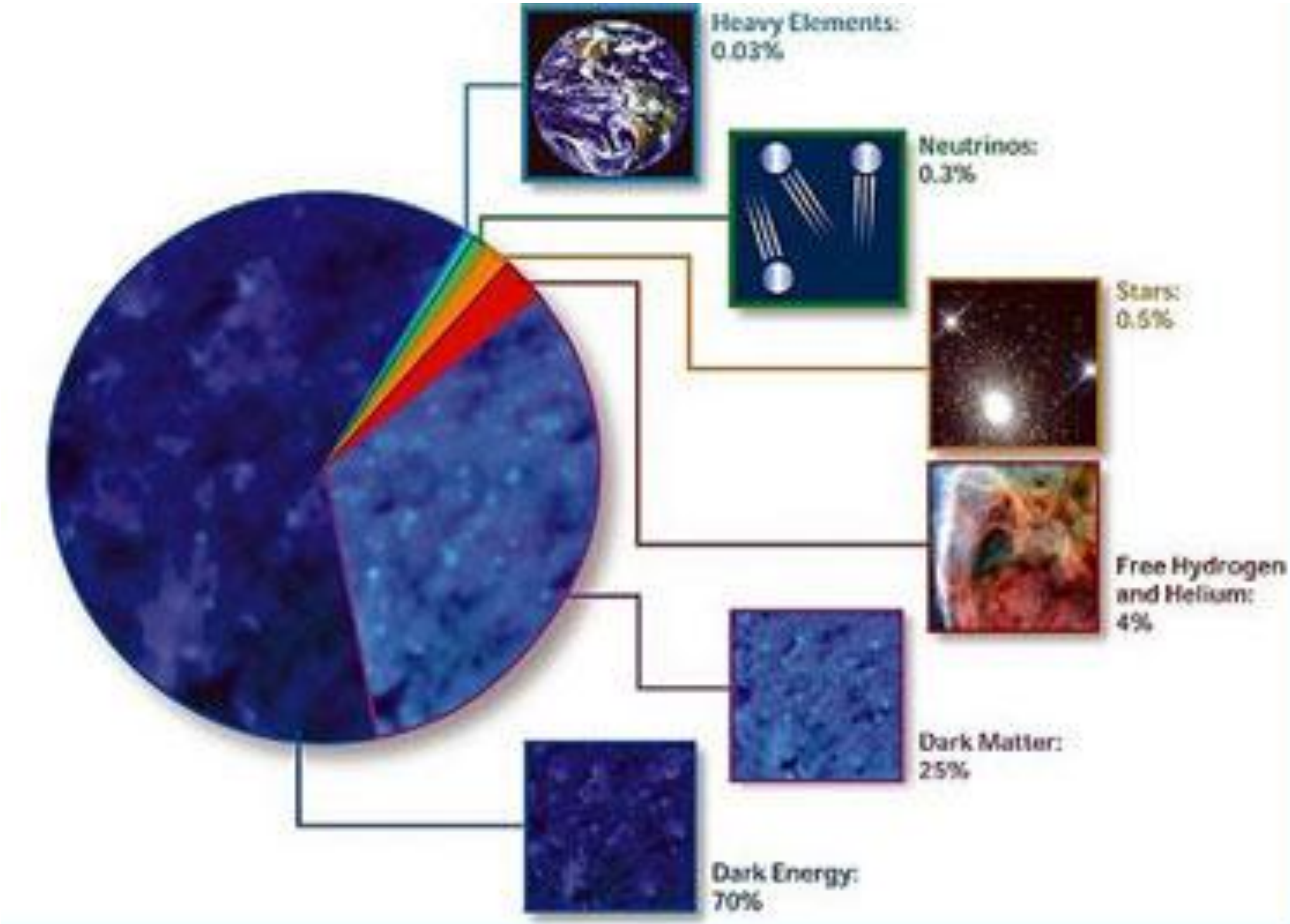
- m_μ are the amplitudes of the $S_\mu(z)$
- $x^{fid}(z)$ is the WMAP fiducial model at which the FM is computed

M. J. Mortonson and W. Hu ApJ 686, L53 (2008)

A more model independent parametrization of the reionization process based on principal components (Mortonson and Hu,2009) could change the results on n.

Dataset	Ionization	n (68% c.l.)	95% c.l.
WMAP7	sudden	0.965 ± 0.014	$n \leq 0.993$
CMB All	sudden	0.959 ± 0.013	$n \leq 0.984$
WMAP7	MH	0.993 ± 0.023	$n \leq 1.042$
CMB All	MH	0.975 ± 0.017	$n \leq 1.011$
CMB All+LRG-7	MH	0.966 ± 0.014	$n \leq 0.994$
CMB All+BAO	MH	0.965 ± 0.014	$n \leq 0.995$
CMB All+BAO	MH+$w(z)$	0.985 ± 0.018	$n \leq 1.025$
WMAP7	LWB	0.977 ± 0.018	$n \leq 1.01$
CMB All	LWB	0.963 ± 0.015	$n \leq 0.998$

S.Pandolfi et al, Phys.Rev.D81:123509,2010
S.Pandolfi et al, Phys.Rev.D82:123527,2010

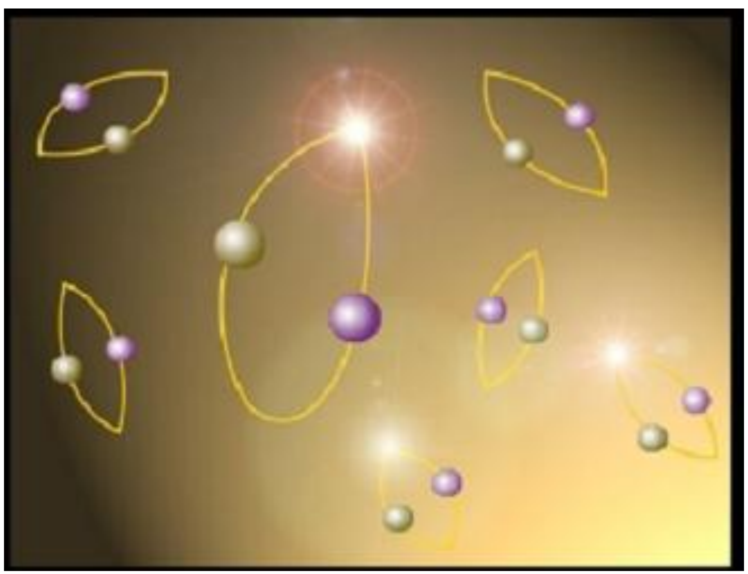


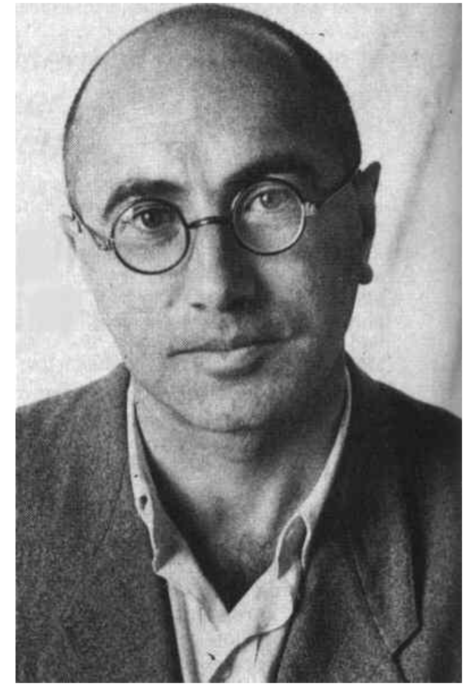
Heavy Elements:
0.03%
Neutrinos:
0.3%
Stars:
0.5%
Free Hydrogen
and Helium:
4%
Dark Matter:
25%
Dark Energy:
70%

The problem of the Cosmological Constant

Already in 1968 Zeldovich noticed that the vacuum energy in particle physics could be a source for a cosmological constant.

"The genie (Λ) has been let out of the bottle"





Zeldovich

"A new field of activity arises, namely the determination of Λ"

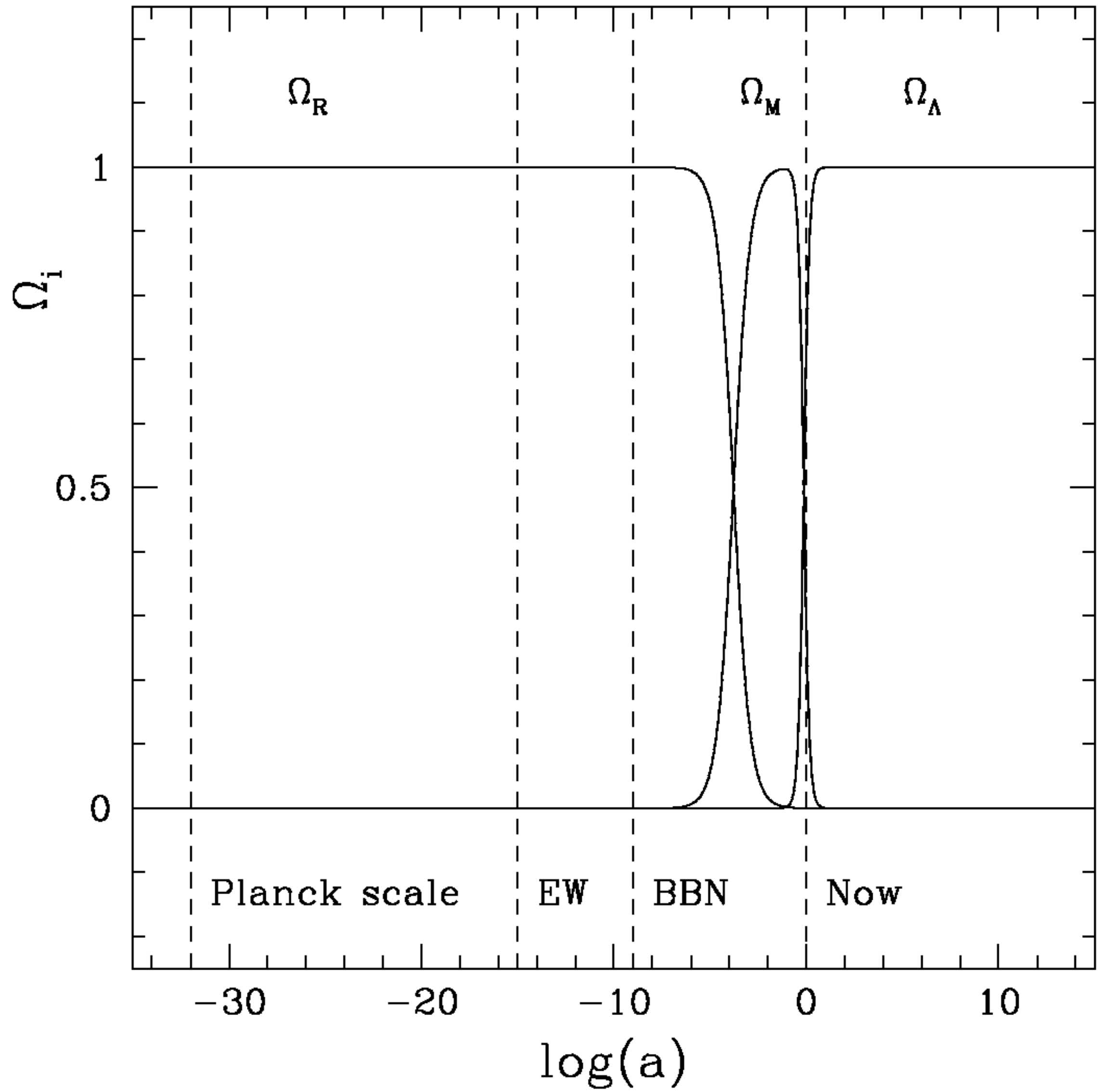
This anyway would lead to a great problem. The vacuum energy in particle physics is infinite. We may stop at Planck Scale but still we have a discrepancy of 120 orders of magnitude:

$$\rho_{vac} \approx \int_0^{M_P} \sqrt{k^2 + m^2}\, d^3k \approx M_p^4 \approx 10^{120} \rho_\Lambda$$

If we consider supersymmetry we go in the right direction but we are still 60 orders of magnitude away !

$$\rho_{vac} \approx \rho_{susy} \approx M_{susy}^4 \approx 10^{60} \rho_\Lambda$$

But there is a second problem, why the universe is accelerating Today ? ?



Why so small ? Why now ?

COSMOLOGICAL COSTANT vs "Something else"

$$\rho_\Lambda \equiv const$$
$$p_\Lambda = -\rho_\Lambda$$
$$\delta\rho_\Lambda = 0$$

Vs.

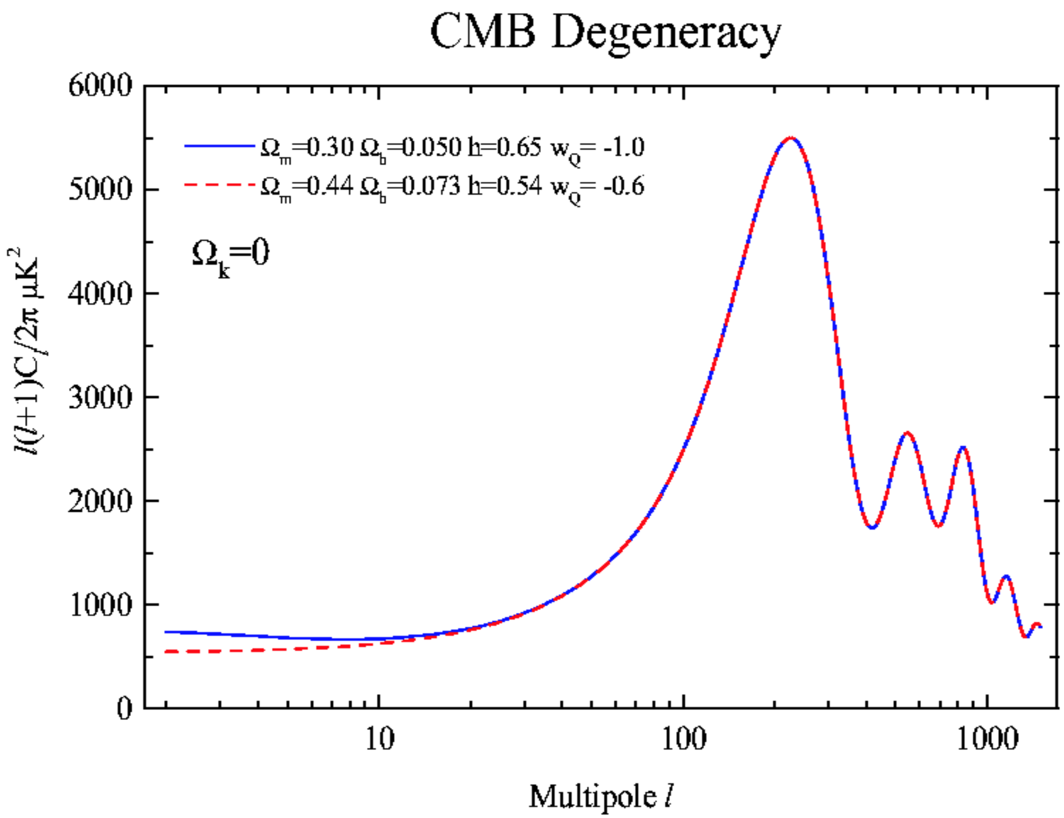
$$\rho_X(z) \equiv \rho_X(0)\exp\left(3\int_0^z dz'\frac{1+w(z')}{1+z'}\right)$$

$$p_X = w(z)\rho_X$$

$$\delta\rho_X \neq 0$$

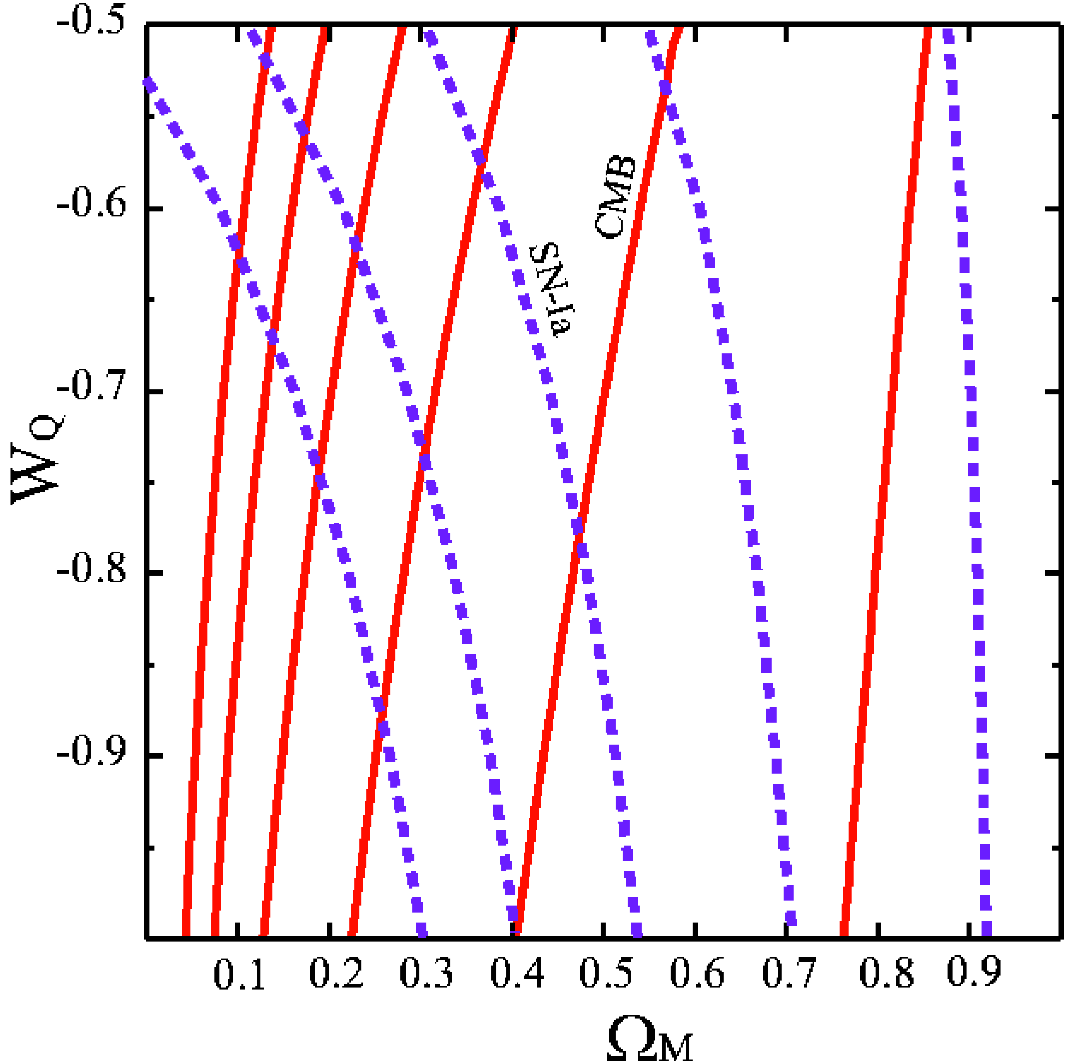
Dark Energy and the shift parameter

A variation in w changes
The angular diameter distance
At recombination and the
Shift parameter.
We have a degeneracy also
In the case of flat universes.

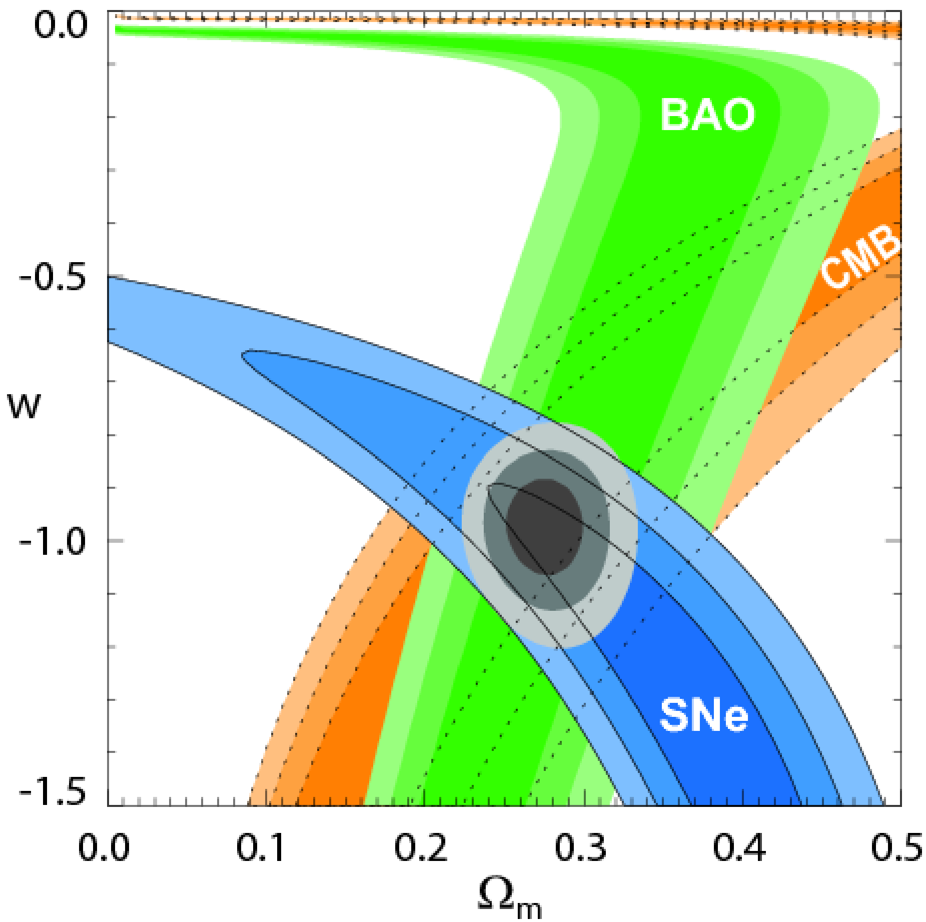


$$R = \frac{1}{\sqrt{\Omega_m}} \int_0^{z_{dec}} \frac{dz}{\sqrt{\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w_Q)}}}$$

Dark Energy and the shift parameter



Combining CMB and SN-Ia using the shift parameter



Kowalski et al, Astrophys.J.686:749-778,2008

CMB data is usually included in SN-Ia And BAO analysis by using the WMAP Constraint on the shift parameter.

WMAP constraints R

TABLE 9
WMAP DISTANCE PRIORS OBTAINED FROM THE *WMAP* 7-YEAR FIT TO MODELS WITH SPATIAL CURVATURE AND DARK ENERGY. THE CORRELATION COEFFICIENTS ARE: $r_{l_A,R} = 0.1956$, $r_{l_A,z_*} = 0.4595$, AND $r_{R,z_*} = 0.7357$.

	7-year ML[a]	7-year Mean[b]	Error, σ
l_A	302.09	302.69	0.76
R	1.725	1.726	0.018
z_*	1091.3	1091.36	0.91

[a] Maximum likelihood values (recommended).
[b] Mean of the likelihood.

↓

$$R = \frac{H_0}{\sqrt{\Omega_m}} \int_0^{z_{dec}} \frac{dz}{H(z)}$$

↓

Constraints on the w-Ω_m plane are combined as independent datasets.

Is the CMB constraint on the shift parameter model independent ?

Model	R
ΛCDM	1.707 ± 0.025
wCDM ($c^2_{DE} = 1$)	1.710 ± 0.029
wCDM ($c^2_{DE} = 0$)	1.711 ± 0.025
ΛCDM $m_\nu > 0$	1.769 ± 0.040
ΛCDM $N_{eff} \neq 3$	1.714 ± 0.025
ΛCDM $\Omega_k \neq 0$	1.714 ± 0.024
$w(z)$CDM CPL ($c^2_{DE} = 1$)	1.710 ± 0.026
ΛCDM + tensor	1.670 ± 0.036
ΛCDM + running	1.742 ± 0.032
ΛCDM + running + tensor	1.708 ± 0.039
ΛCDM + features	1.708 ± 0.028

Corasaniti, Melchiorri, Phys.Rev.D77:103507,2008
Elgaroy, Multamaki, http://arxiv.org/abs/astro-ph/0702343

Dark Energy and the shift parameter

A variation in w changes
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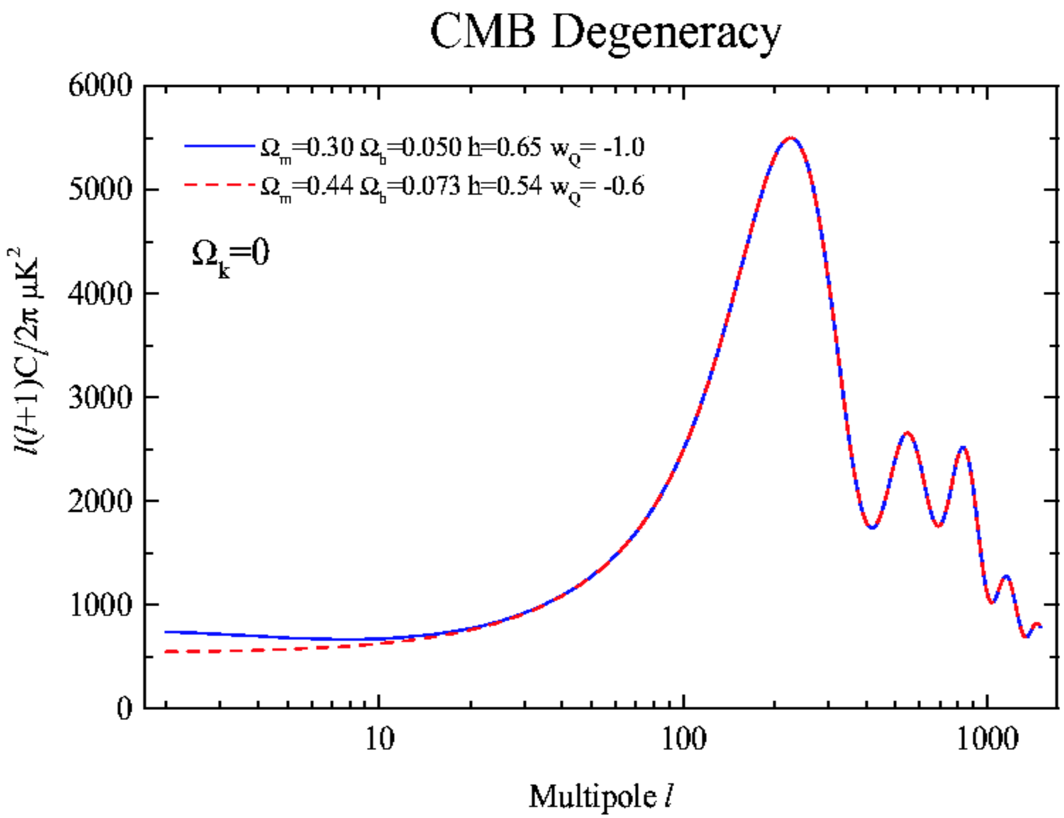


$$R = \frac{1}{\sqrt{\Omega_m}} \int_0^{z_{dec}} \frac{dz}{\sqrt{\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w_Q)}}}$$

Integrated Sachs-Wolfe effect

while most cmb anisotropies arise on the last scattering surface, some may be induced by passing through a time varying gravitational potential:

$$\frac{\delta T}{T} = -2\int d\tau\, \dot{\Phi}(\tau)$$

linear regime – integrated Sachs-Wolfe (ISW)

non-linear regime – Rees-Sciama effect

when does the linear potential change?

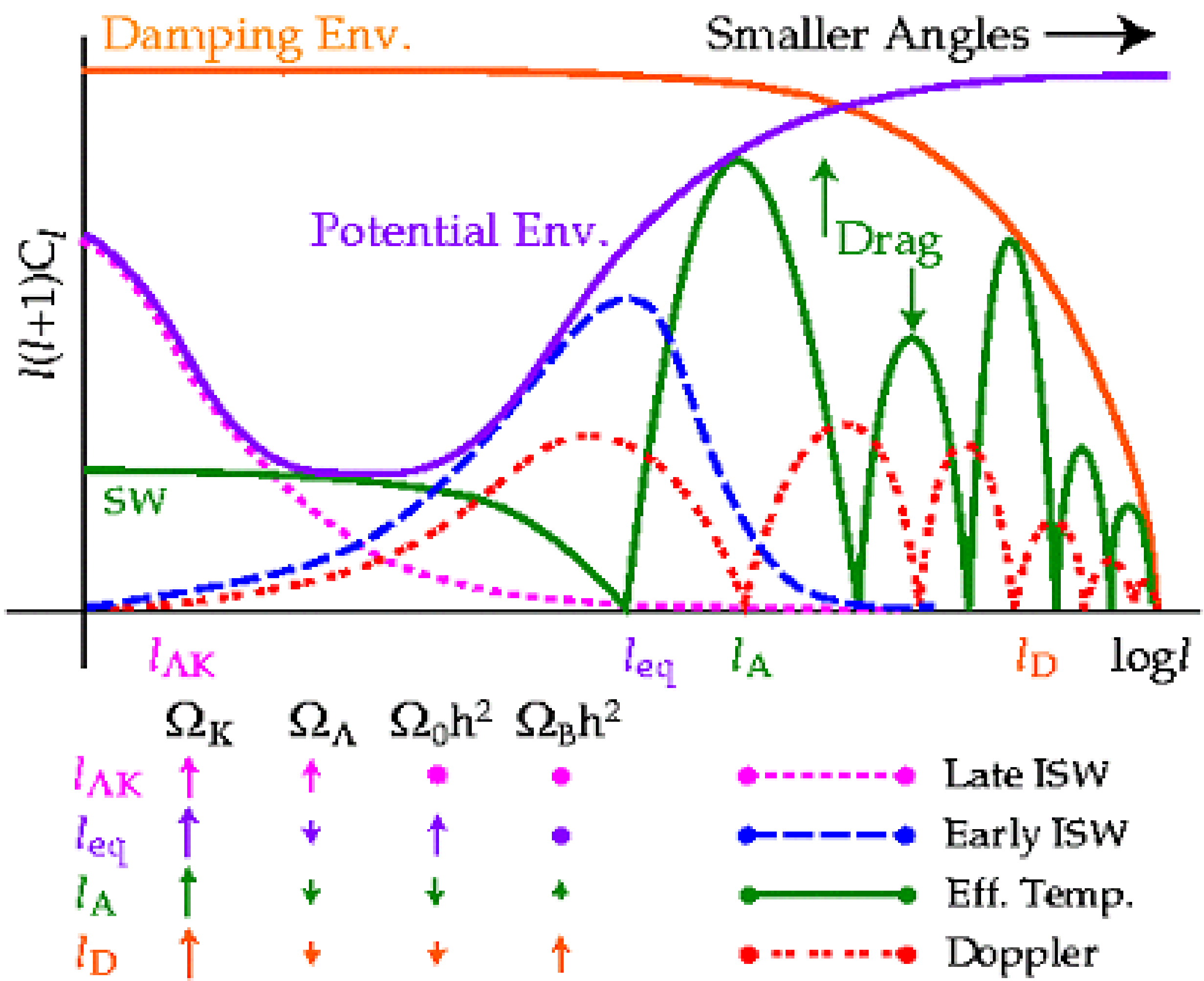
$$\nabla^2 \Phi = 4\pi G a^2 \overline{\rho}\, \delta$$

Poisson's equation

- constant during matter domination
- decays after curvature or dark energy come to dominate (z~1)

induces an additional, uncorrelated layer of large scale anisotropies

ISW



Damping Env.
Smaller Angles
Potential Env.
Drag
SW
$l(l+1)C_l$
$l_{\Lambda K}$
l_{eq}
l_A
l_D
log$l$
Ω_K
Ω_Λ
$\Omega_0 h^2$
$\Omega_B h^2$
$l_{\Lambda K}$
l_{eq}
l_A
l_D
Late ISW
Early ISW
Eff. Temp.
Doppler

Dark Energy Perturbations

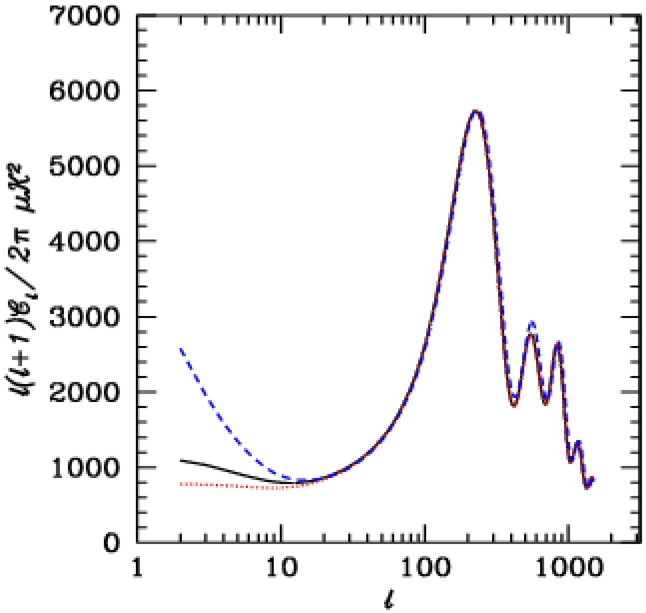
For a perfect fluid the speed of sound purely arises from adiabatic perturbations in the pressure, p, and energy density ρ and the adiabatic speed of sound is purely determined by the equation of state w:

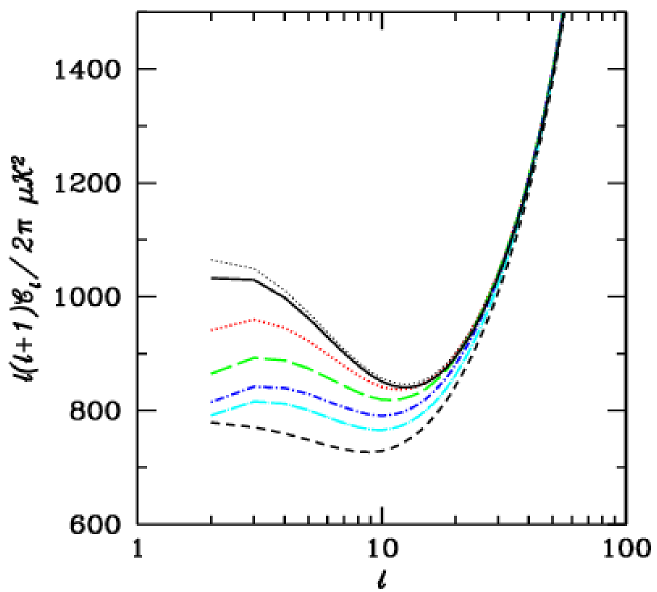
$$
\begin{aligned}
w_i &\equiv \frac{p_i}{\rho_i} \\
c_{ai}^2 &\equiv \frac{\dot{p}_i}{\dot{\rho}_i} = w_i - \frac{\dot{w}_i}{3\mathcal{H}(1+w_i)}
\end{aligned}
$$

In imperfect fluids, for example most scalar field or quintessence models, however, dissipative processes generate entropic perturbations in the fluid and this simple relation between background and the speed of sound breaks down.
This is usually parametrized by introducing the rest frame sound speed:

$$
\begin{aligned}
\dot{\delta} &= -(1+w)\left\{\left[k^2 + 9\mathcal{H}^2(\hat{c}_s^2 - c_a^2)\right]\frac{\theta}{k^2} + \frac{\dot{h}}{2}\right\} \\
&\quad -3\mathcal{H}(\hat{c}_s^2 - w)\delta \\
\frac{\dot{\theta}}{k^2} &= -\mathcal{H}(1-3\hat{c}_s^2)\frac{\theta}{k^2} + \frac{\hat{c}_s^2}{1+w}\delta\,.
\end{aligned}
$$

Dark Energy Perturbations

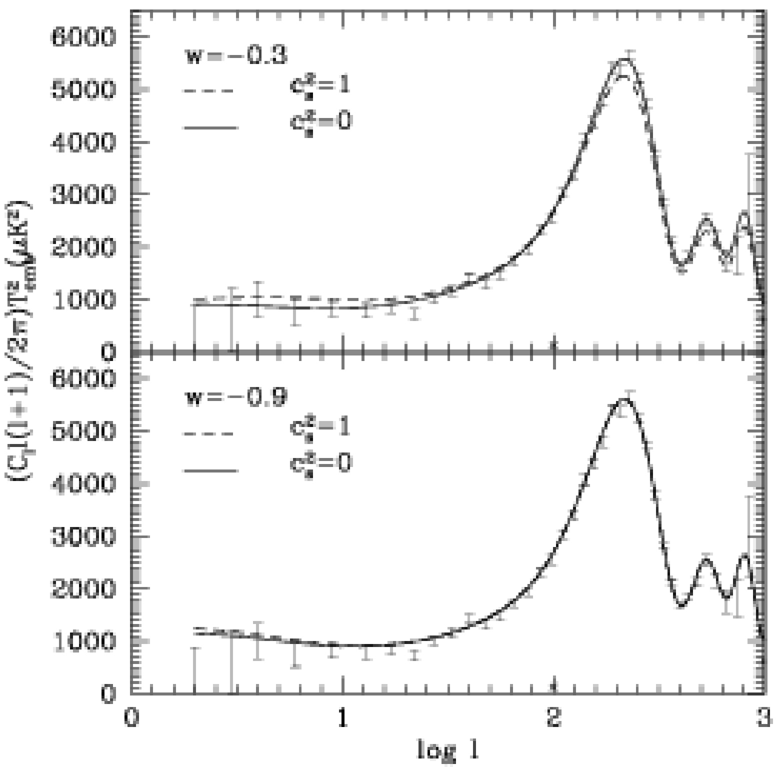


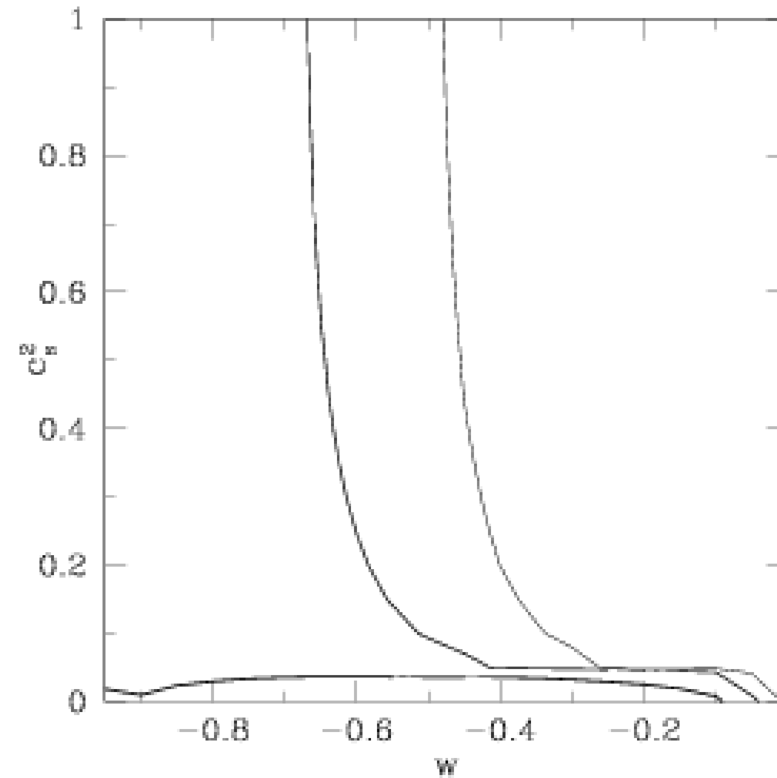


- Perturbations in the dark energy component must be included in order to avoid a too large ISW signal.
- Effects of variation in the sound speed are very small and substantially decrease for values of w closer to -1.

Weller, Lewis, Mon.Not.Roy.Astron.Soc.346:987-993,2003

Dark Energy Perturbations



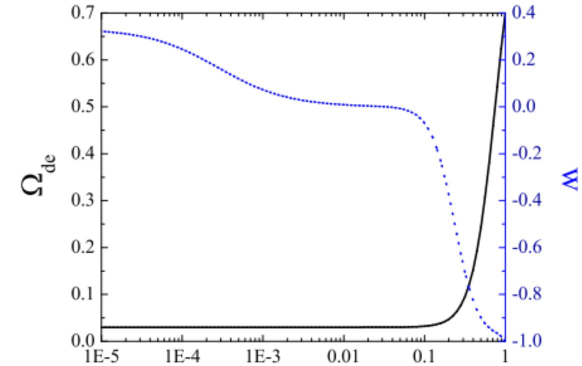


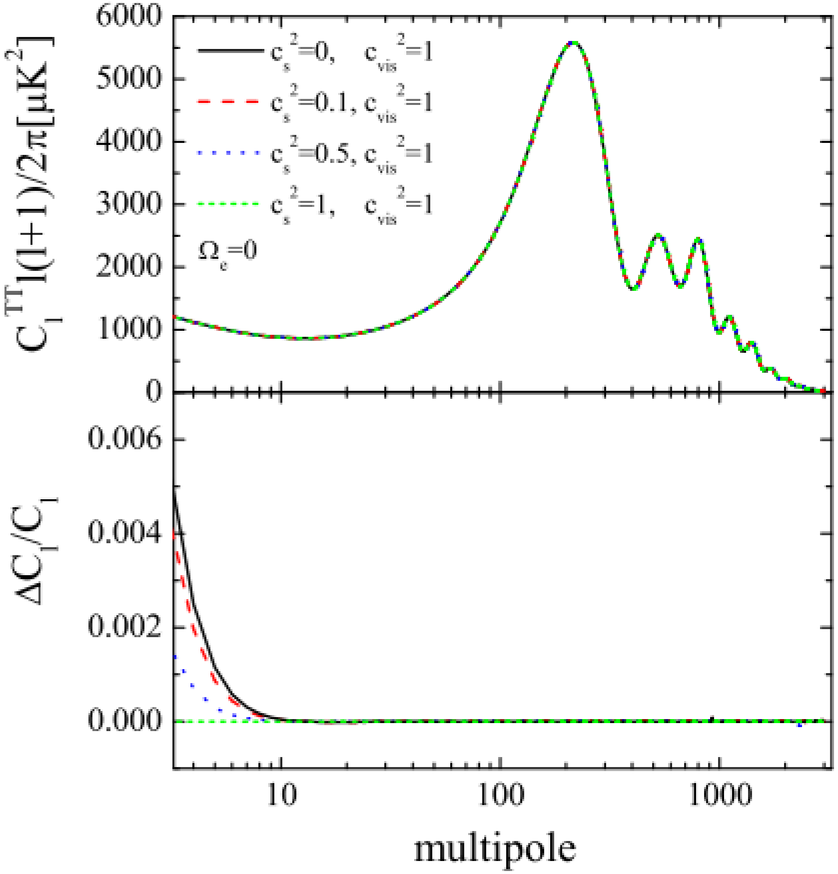
CMB alone is essentially unable to constrain the dark energy sound speed.

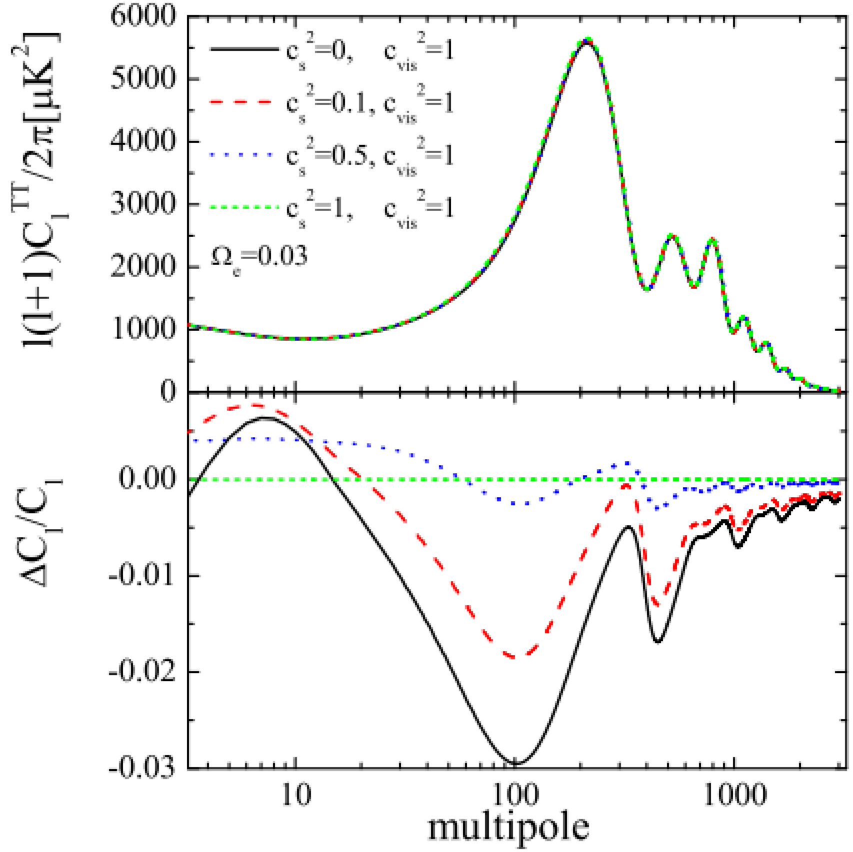
Bean, Dore’, Phys.Rev.D68:023515,2003

Early Dark Energy ?

Dark Energy can/must be more complicate than a constant with redshift w. In case of early dark energy perturbations have a larger impact on the CMB angular power spectrum.

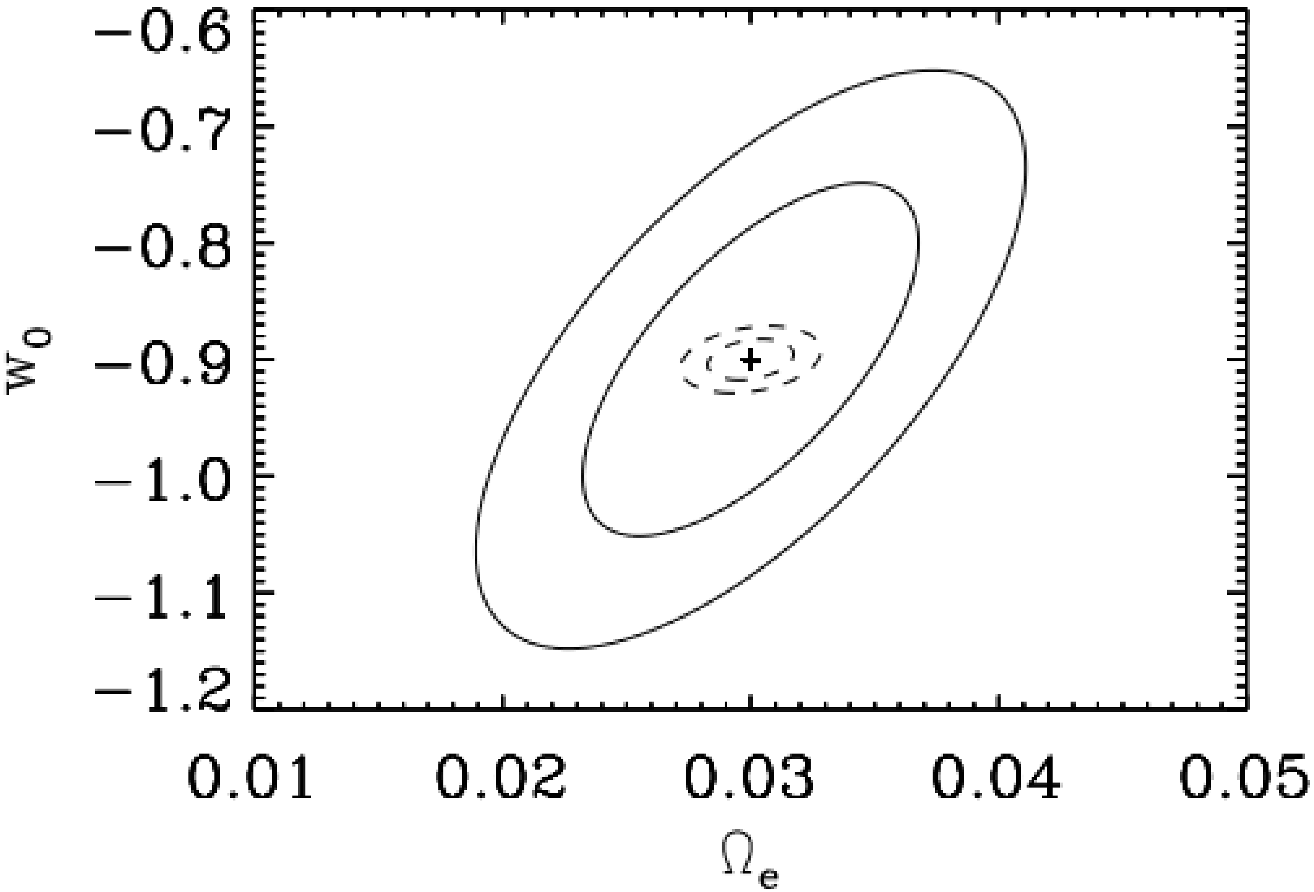






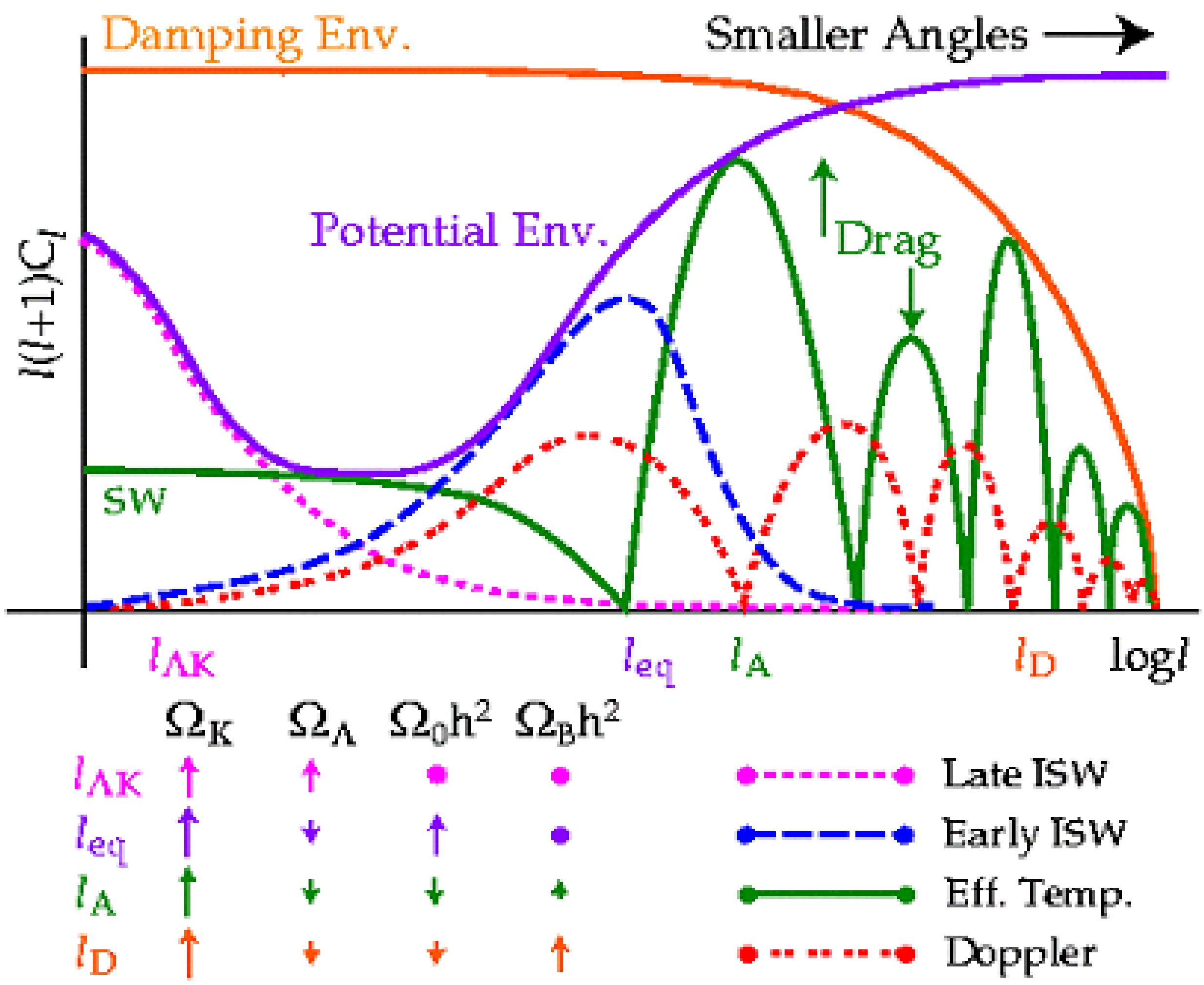
E. Calabrese et al., Phys.Rev.D83:023011,2011

Early Dark Energy can be well constrained with future CMB data.



E. Calabrese et al., Phys.Rev.D83:023011,2011

ISW



Damping Env.
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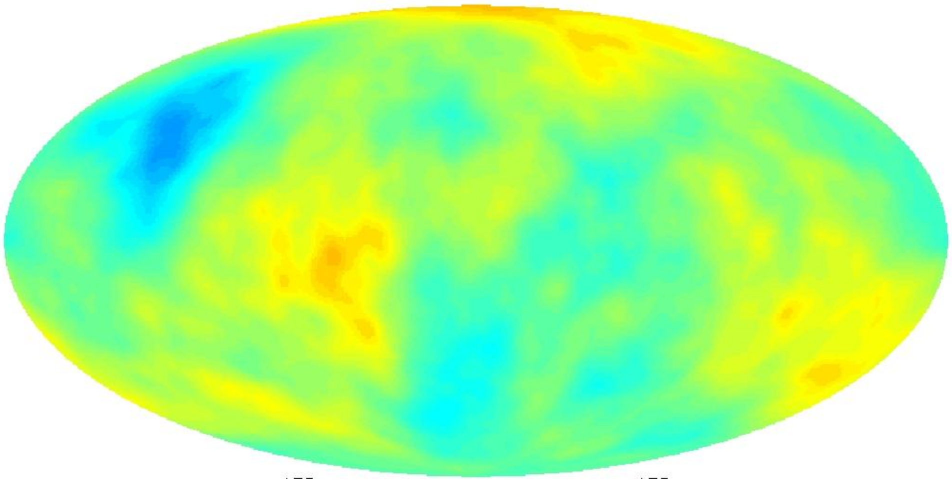
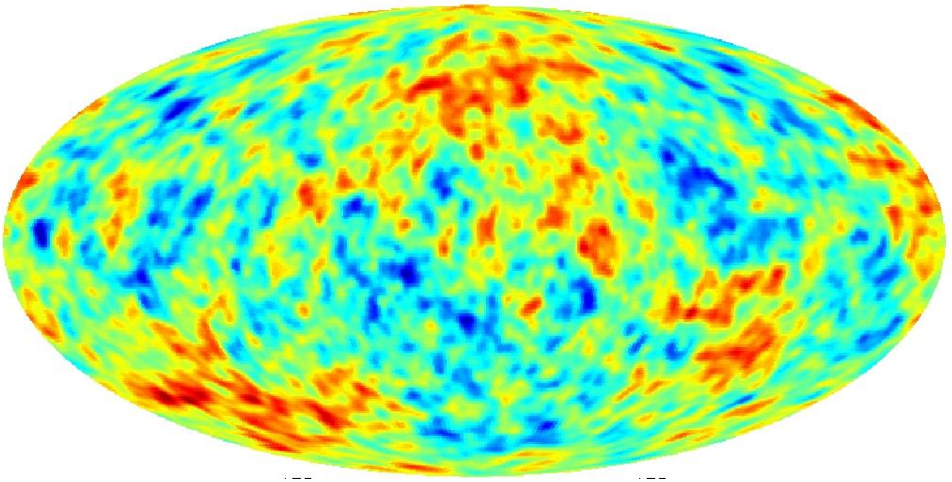
$\Omega_B h^2$
$l_{\Lambda K}$
l_{eq}
l_A
l_D
Late ISW
Early ISW
Eff. Temp.
Doppler

two independent maps

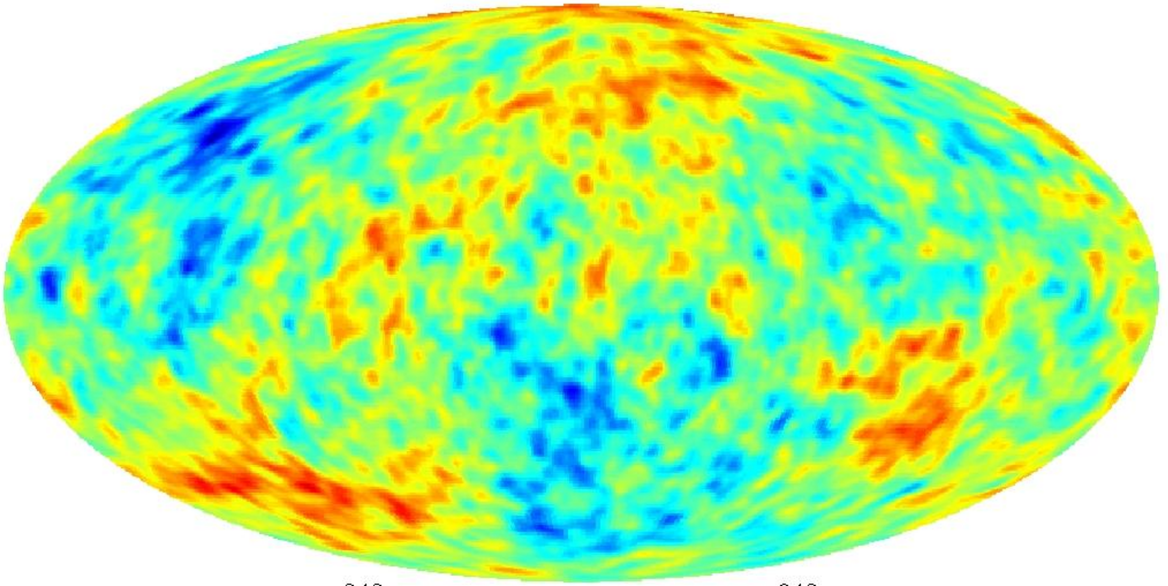


Integrated Sachs-Wolfe map
Mostly large angular features



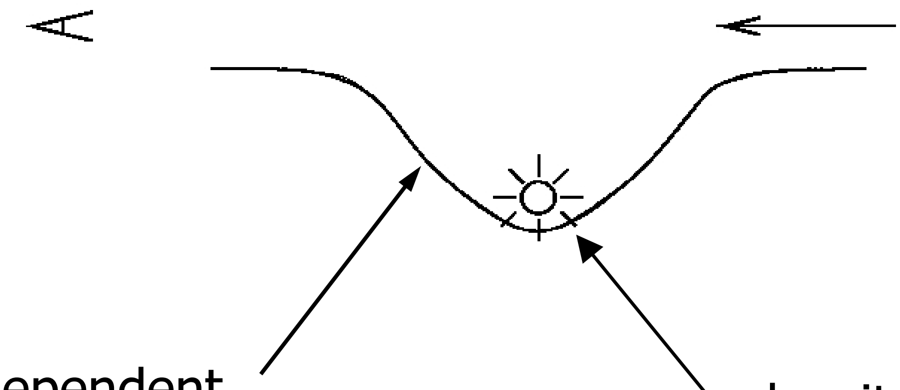
Early time map ($z > 4$)
Mostly from last scattering surface

Observed map is total of these, and has features of both (3 degree resolution)



compare with large scale structure

ISW fluctuations are correlated with the galaxy distribution!



since the decay happens slowly, we need to see galaxies at high redshifts (z~1)

- active galaxies (quasars, radio, or hard x-ray sources)
- possibility of accidental correlations means full sky needed

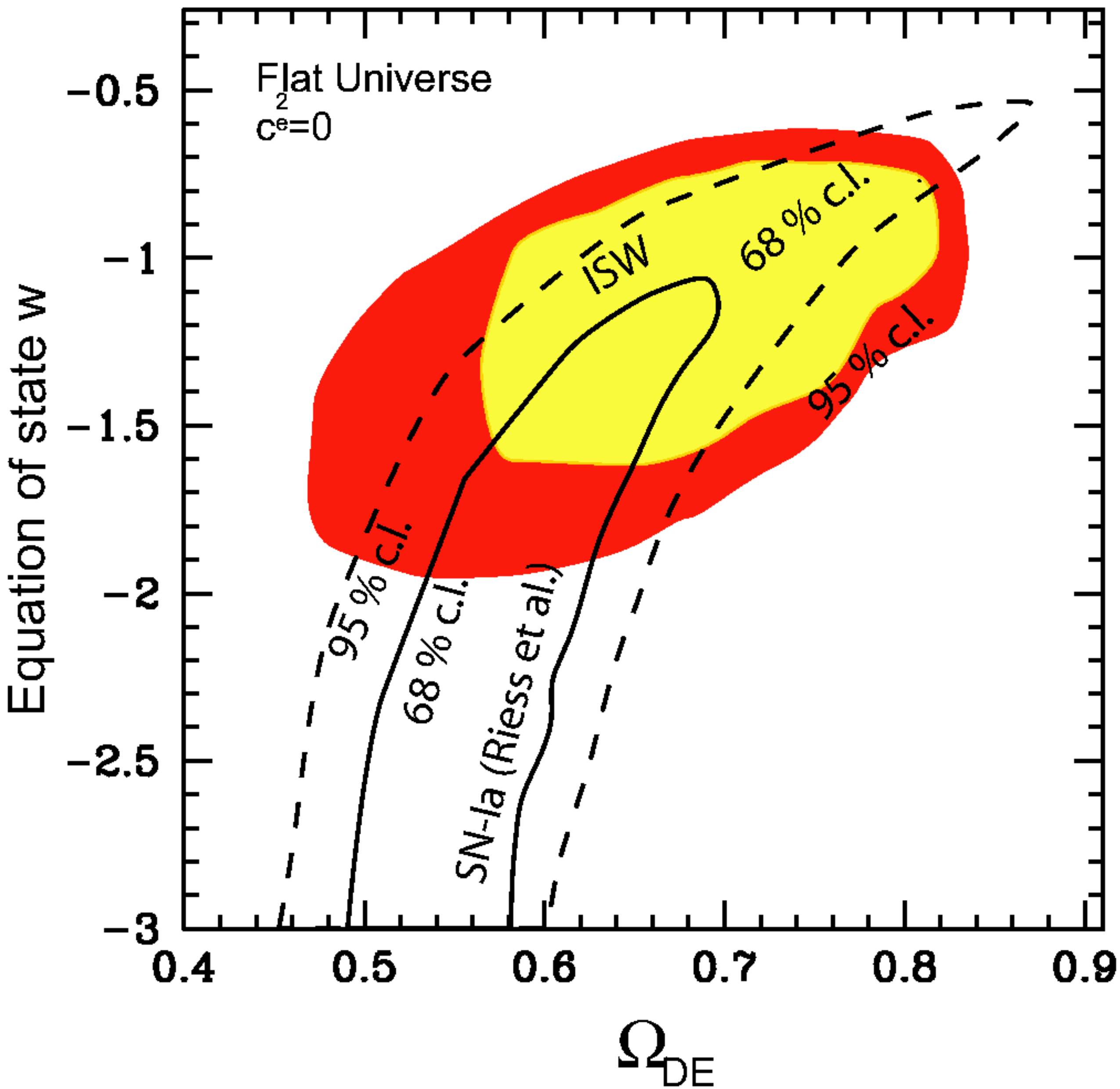
Current Observational Constraints

	5000 T-only Monte Carlos		5000 full Monte Carlos		JK - δ only		JK - δ and T	
catalogue	A	S/N	A	S/N	A	S/N	A	S/N
2MASS cut	1.22 ± 1.87	0.7σ	1.00 ± 1.96	0.5σ	0.66 ± 0.77	0.9σ	1.36 ± 1.10	1.2σ
SDSS	1.58 ± 0.70	2.2σ	1.48 ± 0.66	2.2σ	1.24 ± 0.42	3.0σ	1.59 ± 0.44	3.6σ
LRG	1.67 ± 0.76	2.2σ	1.73 ± 0.80	2.2σ	0.92 ± 0.50	1.8σ	1.22 ± 0.49	2.5σ
NVSS	1.12 ± 0.40	2.8σ	1.20 ± 0.37	3.3σ	0.68 ± 0.29	2.4σ	0.83 ± 0.27	3.1σ
HEAO	1.10 ± 0.41	2.7σ	1.22 ± 0.45	2.7σ	0.97 ± 0.26	3.7σ	1.00 ± 0.24	4.2σ
QSO	1.40 ± 0.53	2.6σ	1.33 ± 0.54	2.5σ	1.50 ± 0.58	2.6σ	1.33 ± 0.46	2.9σ
TOTAL	$\mathbf{1.02 \pm 0.23}$	$\mathbf{4.4}\sigma$	$\mathbf{1.24 \pm 0.27}$	$\mathbf{4.5}\sigma$	—	—	—	—

Late ISW is detected at about 4 standard deviations.

Giannantonio et al, '08
Ho et al, '08

Corasaniti, Giannantonio, AM, Phys.Rev. D71 (2005) 123521

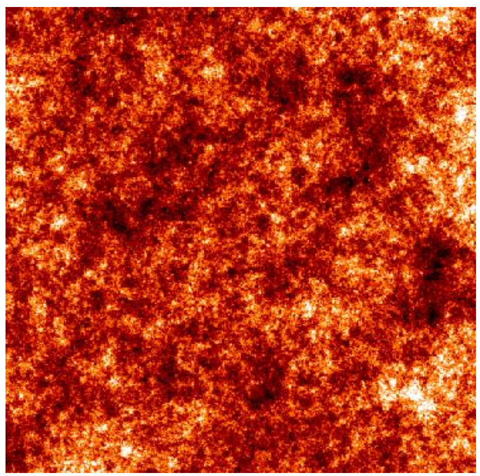


ISW is sensitive
to dark energy perturbations
but also to w, etc.
Prospects for determining
The dark energy sound
Speed are not really
Promising….

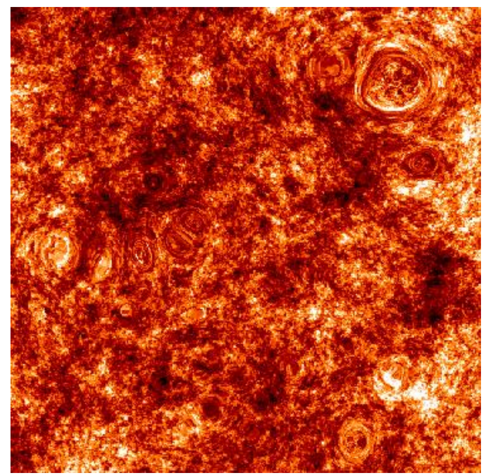
CMB Temperature Lensing

When the luminous source is the CMB, the lensing effect essentially re-maps the temperature field according to :

$$
\begin{aligned}
\tilde{\Theta}(\boldsymbol{x}) &= \Theta(\boldsymbol{x}') = \Theta(\boldsymbol{x}+\boldsymbol{\alpha}) = \Theta(\boldsymbol{x}+\nabla\psi) \\
&\approx \Theta(\boldsymbol{x}) + \nabla^a\psi(\boldsymbol{x})\nabla_a\Theta(\boldsymbol{x}) + \\
&\quad + \frac{1}{2}\nabla^a\psi(\boldsymbol{x})\nabla^b\psi(\boldsymbol{x})\nabla_a\nabla_b\Theta(\boldsymbol{x}) + \ldots
\end{aligned}
$$



unlensed



lensed

Taken from http://www.mpia-hd.mpg.de/
([Max Planck Institute for Astronomy at Heidelberg](http://www.mpia-hd.mpg.de/))

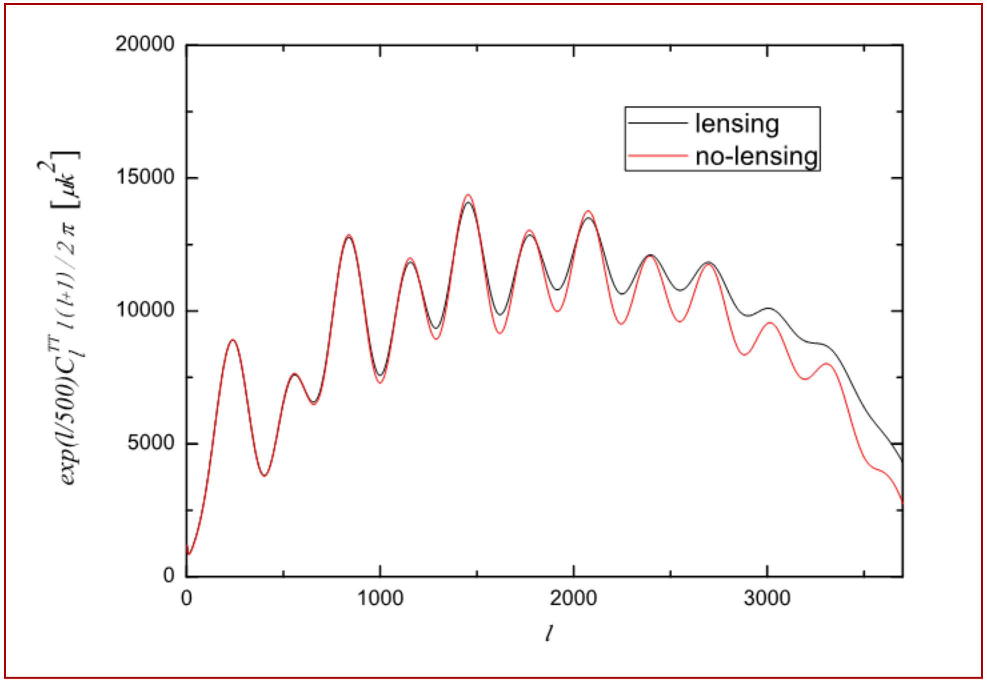
Lensing Effect on Temperature Power Spectrum

We obtain a convolution between the lensing potential power spectrum and the unlensed anisotropies power spectrum:

$$\tilde{C}_l^{\Theta} \approx C_l^{\Theta} + \int \frac{d^2 \boldsymbol{l}'}{(2\pi)^2} \left[\boldsymbol{l}' \cdot (\boldsymbol{l} - \boldsymbol{l}')\right]^2 C^{\psi}_{|\boldsymbol{l}-\boldsymbol{l}'|} C^{\Theta}_{l'} - C_l^{\Theta} \int \frac{d^2 \boldsymbol{l}'}{(2\pi)^2} (\boldsymbol{l} \cdot \boldsymbol{l}')^2 C^{\psi}_{l'}$$

Where the lensing potential power spectrum is given by :

$$C_l^{\psi} = 16\pi \int \frac{dk}{k} P_{\mathcal{R}}(k) \left[\int_0^{\chi_*} d\chi T_{\Psi}(k; \eta_0 - \chi) j_l(k_\chi) \left(\frac{\chi_* - \chi}{\chi_* \chi} \right) \right]^2$$

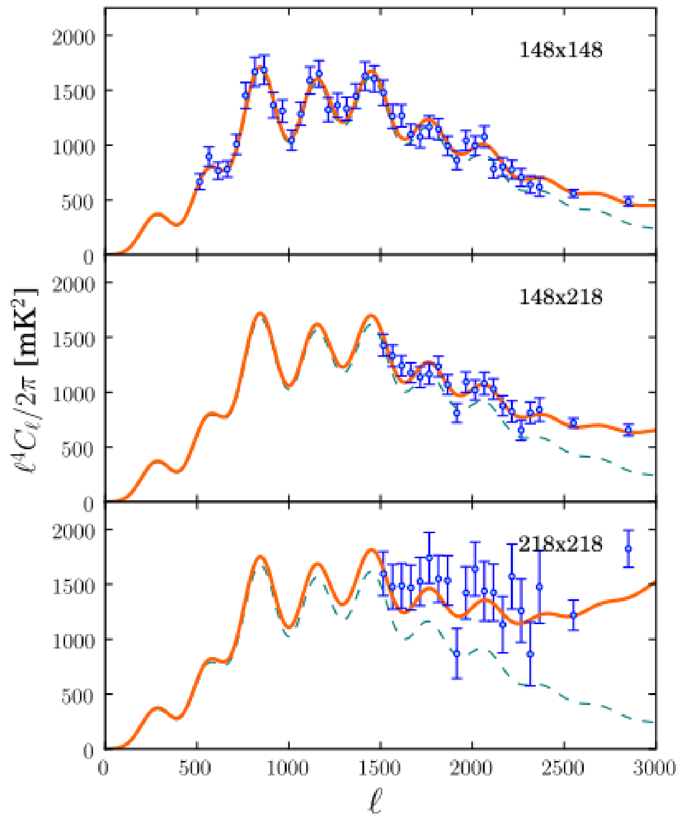


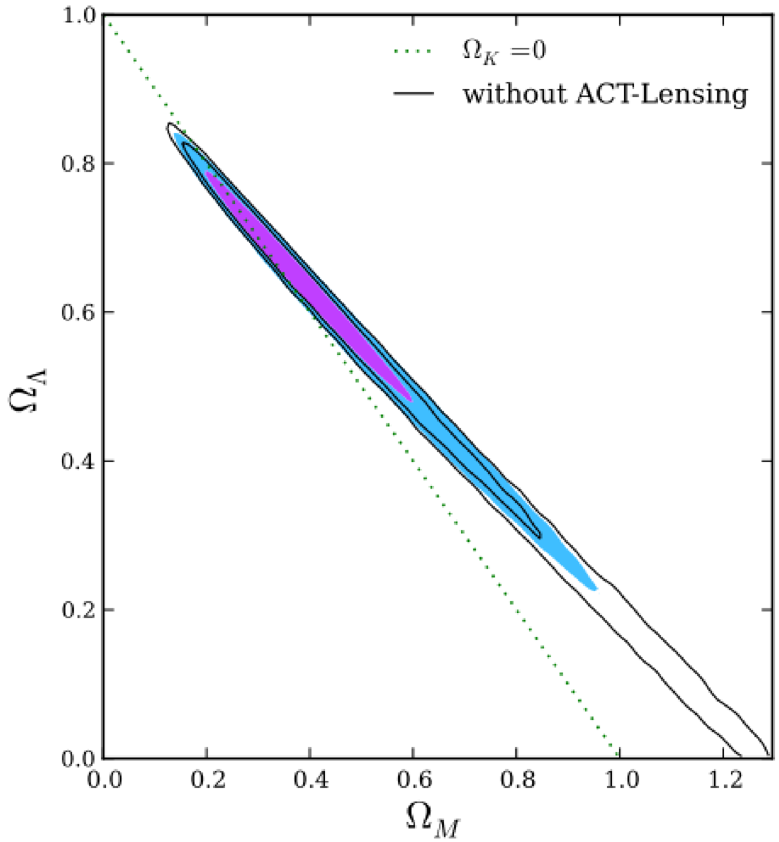
The net results are:

1- a 3% broadening of the CMB angular power spectrum acustic peaks

2- NON Gaussianity ! Trispectrum different from zero.

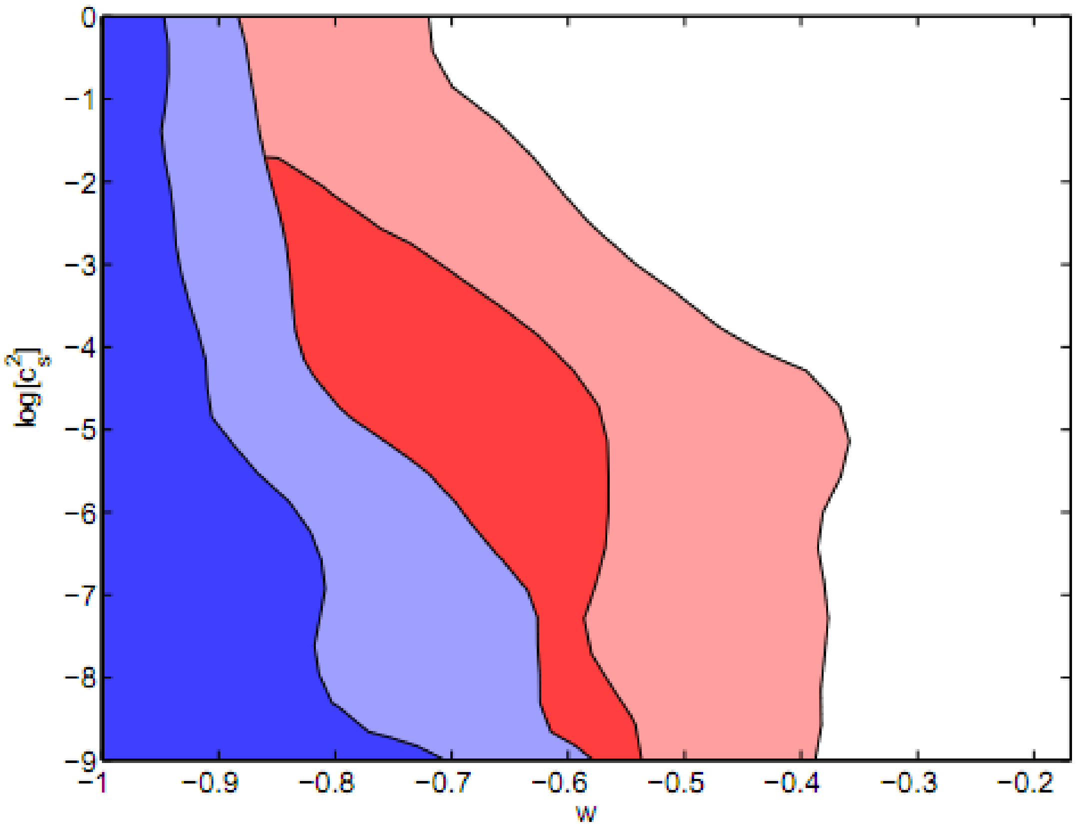
Recent CMB data from ACT are sensitive to lensing and break geometrical degeneracy.





Sherwin et al, Phys.Rev.Lett.107:021302,2011

Future data from Planck and CMBPol can also constrain w (but no bound on perturbation parameters)



Erminia Calabrese, Asantha Cooray, Matteo Martinelli, Alessandro Melchiorri, Luca Pagano, Anže Slosar, and George F. Smoot
Phys. Rev. D **80**, 103516 (2009)

Modified Gravity Models

The problems of the cosmological constant caused a burst of alternative models.
To produce an accelerated expansion without a cosmological constant, it is necessary to modify the equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

The need of a modification arise because standard energy components (matter and radiation) produce a decelerated expansion.
There are two ways to construct models alternative to the cosmological constant:

- to modify $T_{\mu\nu}$ introducing new energy components (eg. *Quintessence*).
- to modify the Einstein tensor $G_{\mu\nu}$ changing the Lagrangian of General Relativity.

Modified Gravity Models

There are several ways to modify gravity, changing the Einstein-Hilbert lagrangian or modifying space-time properties.

A few examples are:

- **DGP**
  "Leaking" of gravity in a 5 dimensional space-time.
- **Scalar-Tensor theories**
  Gravitation is not only given through the metric, but also through scalar fields.
- **$f(R)$ theories**
  The gravity lagrangian depends in a more general way on the Ricci scalar.

F(R) theories

The simplest way to modify the gravity lagrangian brings to $f(R)$ theories.
These models are constructed by simply introducing a general function of the Ricci scalar into the gravity Lagrangian

$$S_{GR} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + L_m\right] \rightarrow$$

$$\rightarrow S_{MG} = \int d^4x\sqrt{-g}\left[\frac{R + f(R)}{16\pi G} + L_m\right]$$

Important point: it is possible to construct f(R) models with w=-1 but with different behaviour for perturbations.
Clustering is needed if we want to discriminate between a cosmological constant and modified gravity !

Perturbations

In order to constrain $f(R)$ gravity we need to analyze the effect of these theories on the evolution of cosmic structure.
The structures we now see in the Universe evolve from small perturbations of the homogeneous and isotropic Friedmann metric.

$$ds^2 = [1 + 2\Psi(\vec{x}, t)]\, dt^2 - [1 + 2\Phi(\vec{x}, t)]\, d\vec{x}^2$$

Modified gravity changes the Newtonian and metric potentials Φ and Ψ.

Dark matter clustering, as well as the evolution of the metric potentials, is changed and can be scale-dependent. Moreover, typically there might be an effective anisotropic stress and the two potentials appearing in the metric element are not necessarily equal, as is in the ΛCDM model.

Parametrization

We use a parametrization that takes into account the modified relation between the metric potentials given by modified gravity

$$k^2\Psi = -\frac{a^2}{2M_P}\mu(a,k)\rho\Delta$$

$$\frac{\Phi}{\Psi} = \gamma(a,k)$$

where

$$\mu(a,k) = \frac{1+\beta_1\lambda_1^2 k^2 a^s}{1+\lambda_1^2 k^2 a^s}$$

$$\gamma(a,k) = \frac{1+\beta_2\lambda_2^2 k^2 a^s}{1+\lambda_2^2 k^2 a^s}$$

This parametrization is implemented in the MGCAMB code, which only consider theories that mimic the ΛCDM expansion history

G. Zhao et al., Phys. Rev. D **79** *(2009) 083513*

Parametrization

In the case of scalar-tensor theories the parameters are related as

$$\beta_1 = \frac{\lambda_2^2}{\lambda_1^2} = 2 - \beta_2 \frac{\lambda_2^2}{\lambda_1^2}$$

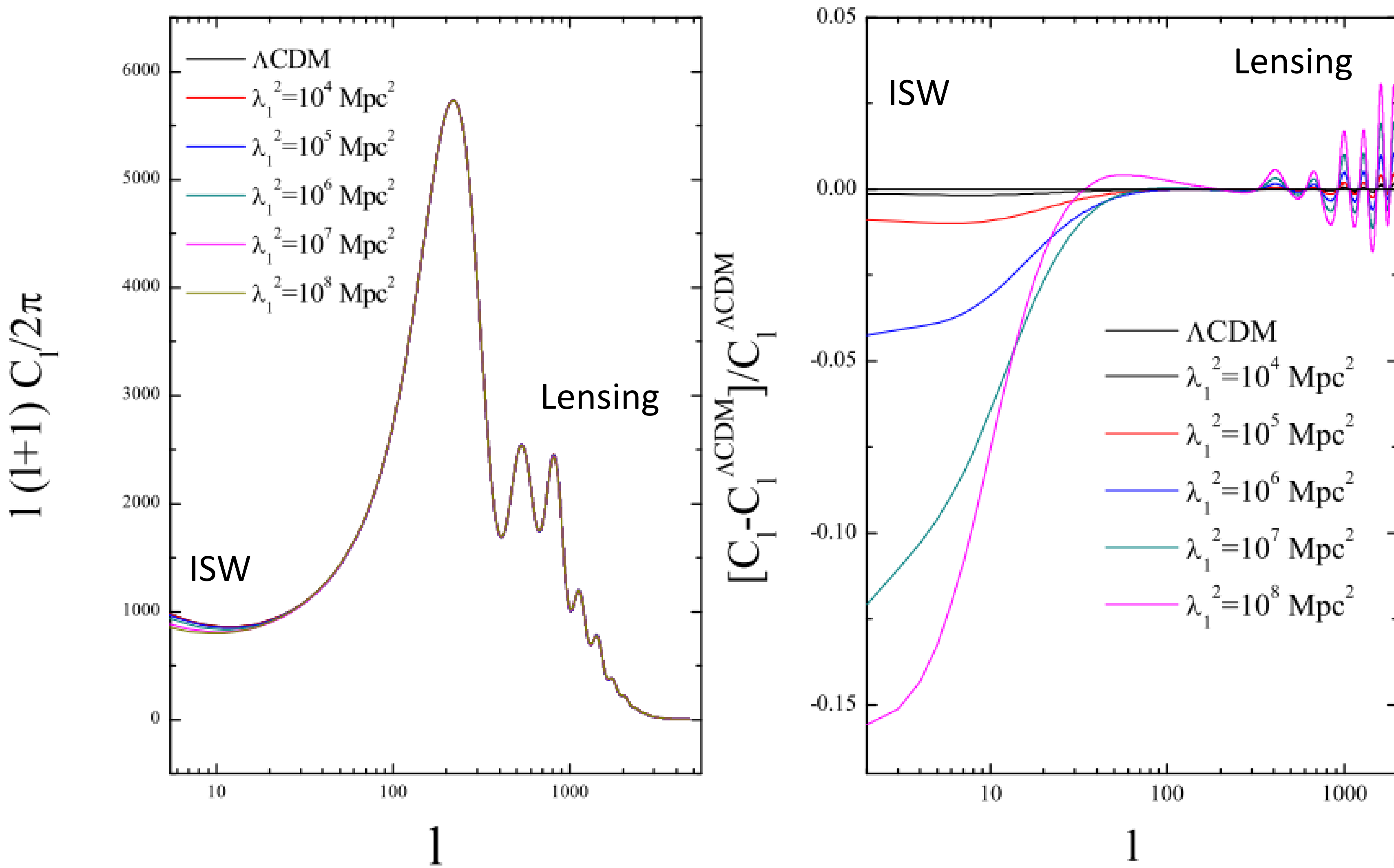
We focus on $f(R)$ theories, thus specifying the couplings β_i and the time evolution s of the length scale λ

$$\beta_1 = \frac{4}{3} \Rightarrow \beta_2 = \frac{2}{\beta_1} - 1 = \frac{1}{2}$$

$$s \approx 4$$

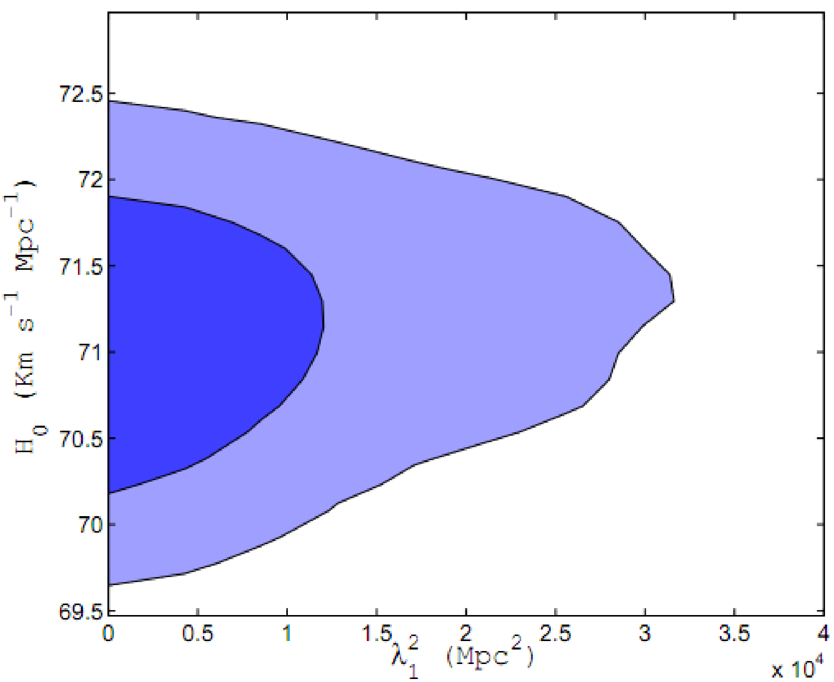
Thus the only free parameter for $f(R)$ theories is the length scale λ of the modified force.
When $\lambda_1^2 = 0$ we recover the cosmological constant model.

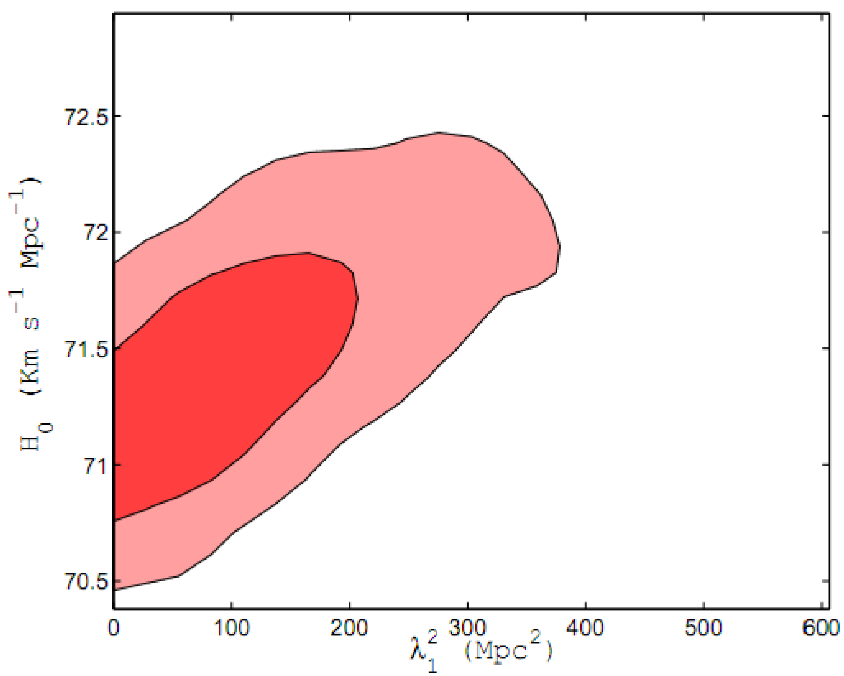
CMB anisotropies



Calabrese et al. Phys. Rev. D **80** *(2009) 103516*

Constraints from Planck and Planck+Euclid





M. Martinelli et al., Phys.Rev.D83:023012,2011

Conclusions to Lecture II

- CMB Anisotropy provides the strongest constraints on the baryon density and the greatest evidence for cold dark matter.
- CMB does NOT constrain Ω but the «shift» parameter . The CMB constraints on this parameter are model dependent. We should be careful when combining with other datasets (SN-Ia, BAO).
- CMB is sensitive to dark energy mainly because of the changes in the angular diameter distance at recombination by varying w. Geometrical degeneracy with other parameters does'nt let to put strong constraints on w from CMB only data.
- ISW and Lensing are also sensitive to dark energy.
- Modified gravity could be tested by its effects on CMB lensing.