CMB Anisotropies and Fundamental Physics

Lecture I

Alessandro Melchiorri
University of Rome «La Sapienza»

CMB Anisotropies and Fundamental Physics

Aims of these lectures:

- What is the physics behind the Cosmic Microwave Backround Anisotropies ?
- What we learned about cosmology from them ?

Lecture I

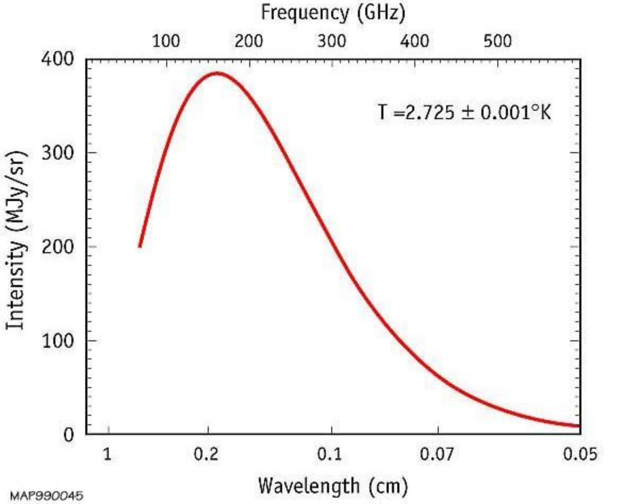
BASIC CMB & PARAMETERS

The Cosmic Microwave Background

Discovered By Penzias and Wilson in 1965.

It is an image of the universe at the time of recombination (near baryon-photons decoupling), when the universe was just a few thousand years old ($z \sim 1000$).

The CMB frequency spectrum is a perfect blackbody at T=2.73 K: this is an outstanding confirmation of the hot big bang model.



Earlier and «unknown» detections of the CMB

SÉANCE DU 17 JUIN 1957.

RADIOASTRONOMIE. — *Nouvelles observations du rayonnement du Ciel sur la longueur d'onde* 33 cm. Note de MM. JEAN-FRANÇOIS DENISSE, JAMES LEQUEUX et ÉMILE LE ROUX, présentée par M. André Danjon.

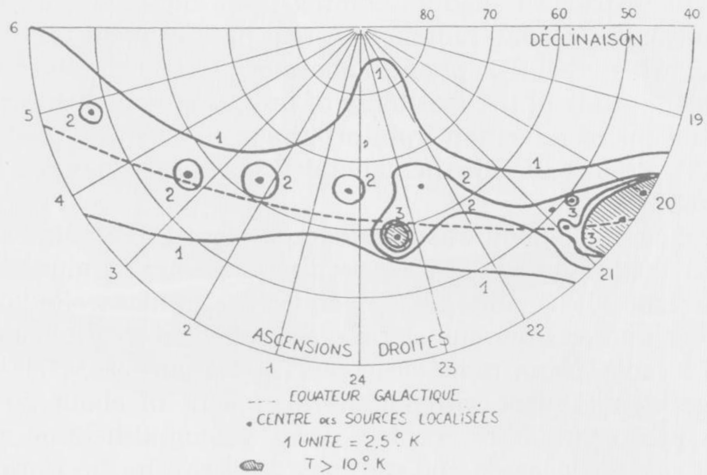


Fig. 1.

Rayonnement du Ciel. — En dehors des régions que l'on vient de décrire, la brillance du Ciel paraît uniforme. Son rayonnement est difficile à mesurer car on l'observe superposé aux emissions beaucoup plus intenses de l'environnement et au bruit propre du récepteur. Nos mesures ont toutefois permis de montrer que la température de brillance du Ciel est inférieure à 3° K et que ses variations d'un point à un autre sont inférieures à 0,5° K.

← Denisse, Lequeux, Le Roux scientific article (1957)

Reproduction of page 50 of (Leroux's thesis).

En résumé, on a trois équations donnant T_c :

$v_o = 0$ $\quad 137 = 138 - 0,485\ T_c \longrightarrow T_c = 2°K$

(25) $v_o = 5$ $\quad 50 = 51,3 - 0,485\ T_c \longrightarrow T_c = 2,7°K$

$v_o = -3$ $\quad 215 = 218 - 0,77\ T_c \longrightarrow T_c = 3,9°K.$

En fait, on devrait déduire, de plusieurs équations de ce genre, les coefficients 1/k, ρ, ρ', ρ'' et T_c. Mais la bonne cohérence des valeurs obtenues pour T_c montre que les valeurs prises pour ces coefficients sont correctes avec une bonne approximation. Si on diminuait le coefficient 1/k on obtiendrait des valeurs négatives pour T_c, quelles que soient les valeurs prises pour ρ' et ρ'' qui interviennent de façon différente dans les 3 équations précédentes, le coefficient ρ' intervenant notamment de façon opposée dans les deux dernières équations. De même, une augmentation de 1/k de quelques pour cent donnerait des valeurs de T_c incohérentes. Enfin, un coefficient de réflection du sol non nul donnerait $T_c < 0$.

Il est difficile de déterminer l'erreur sur cette valeur de T_c, basée sur la cohérence de différentes mesures. Nous pensons que l'erreur absolue doit être de l'ordre de 2° K, en prenant :

$$\boxed{T_c = 3°K}$$

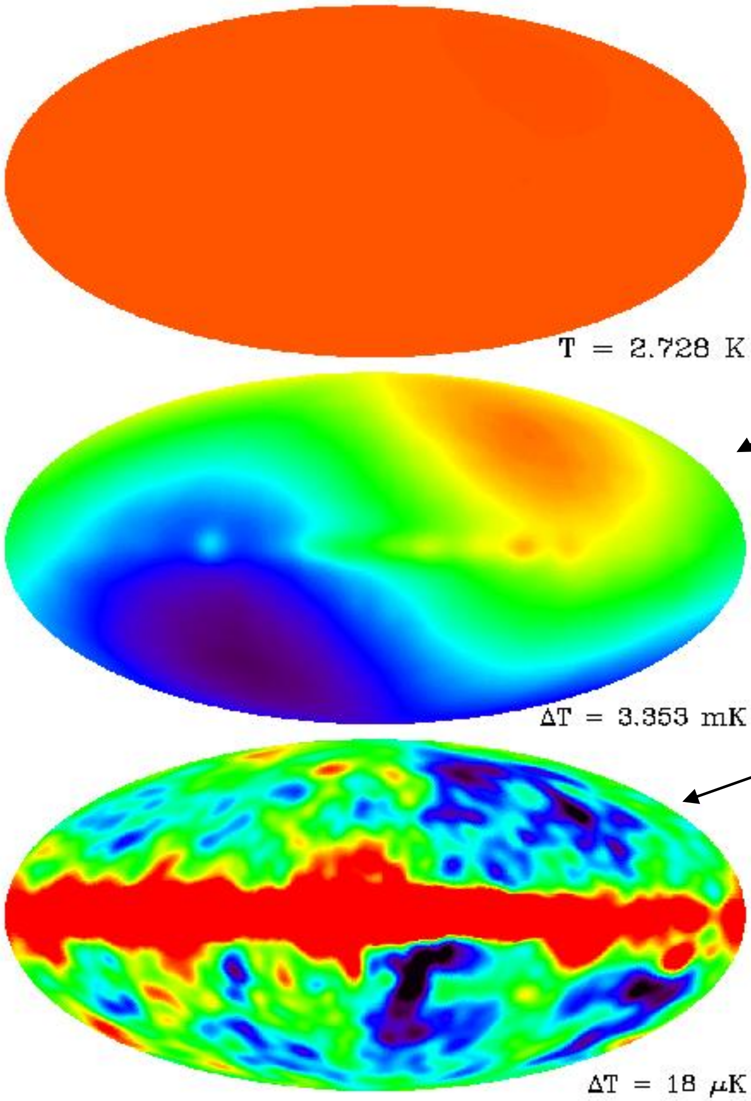
← Emile Le Roux Phd Thesis (1957)

Evidence of CMB removed from the article following the suggestion from the PhD supervisor.

Emile probably lost the nobel prize.

The Microwave Sky

COBE (circa 1995) @90GHz

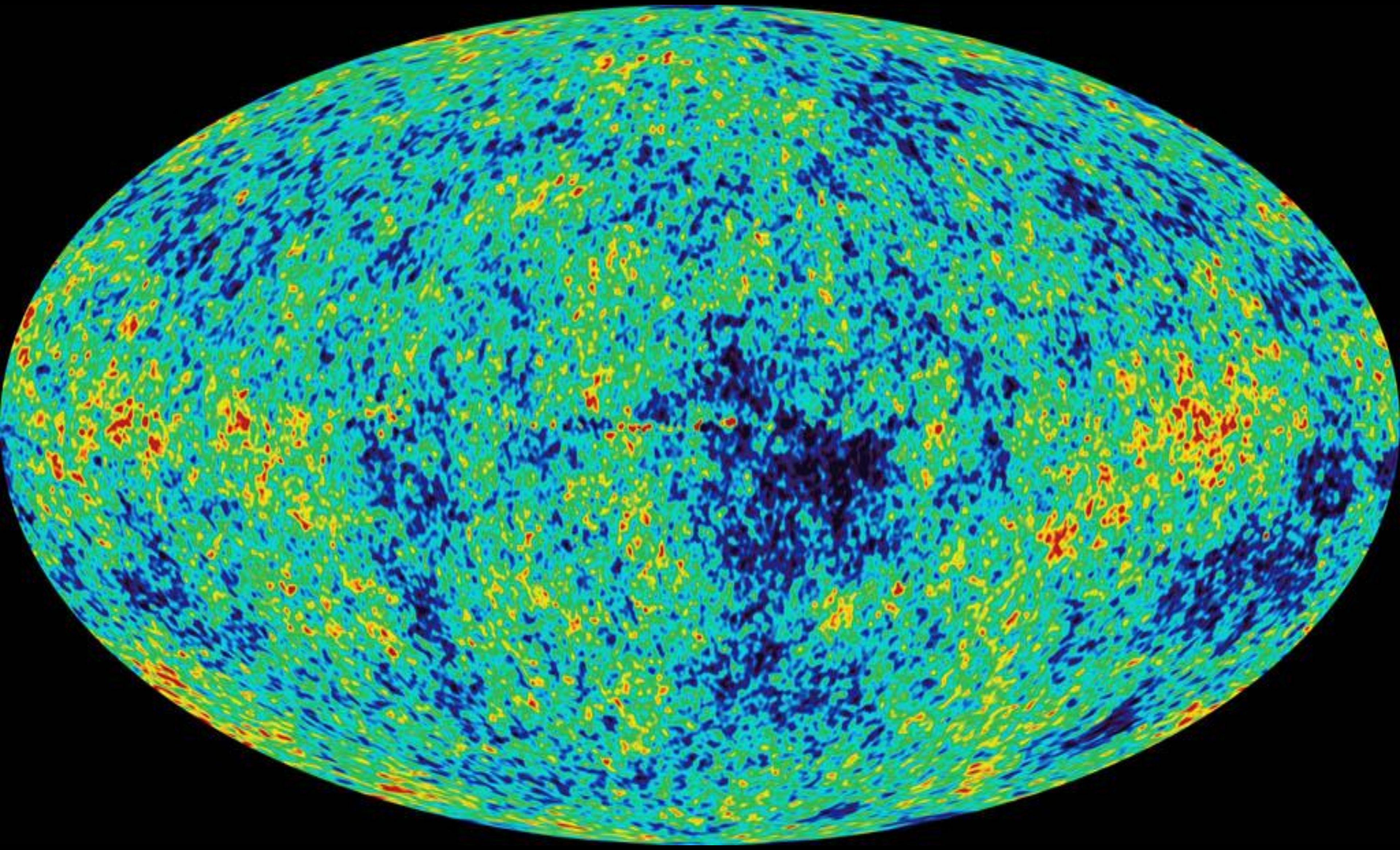


Uniform…

First Anisotropy we see is a Dipole anisotropy:
Implies solar-system barycenter has velocity v/c~0.00123 relative to 'rest-frame' of CMB.

If we remove the Dipole anisotropy and the Galactic emission, we see anisotropies at the level of $(\Delta T/T)_{rms}$~ 20 μK (smoothed on ~7° scale).
These anisotropies are the imprint left by primordial tiny density inhomogeneities (z~1000)..

Best Full Sky Map of the CMB so far: WMAP satellite (2002-2010)
(linear combination of 30,60 and 90 GHz channels)

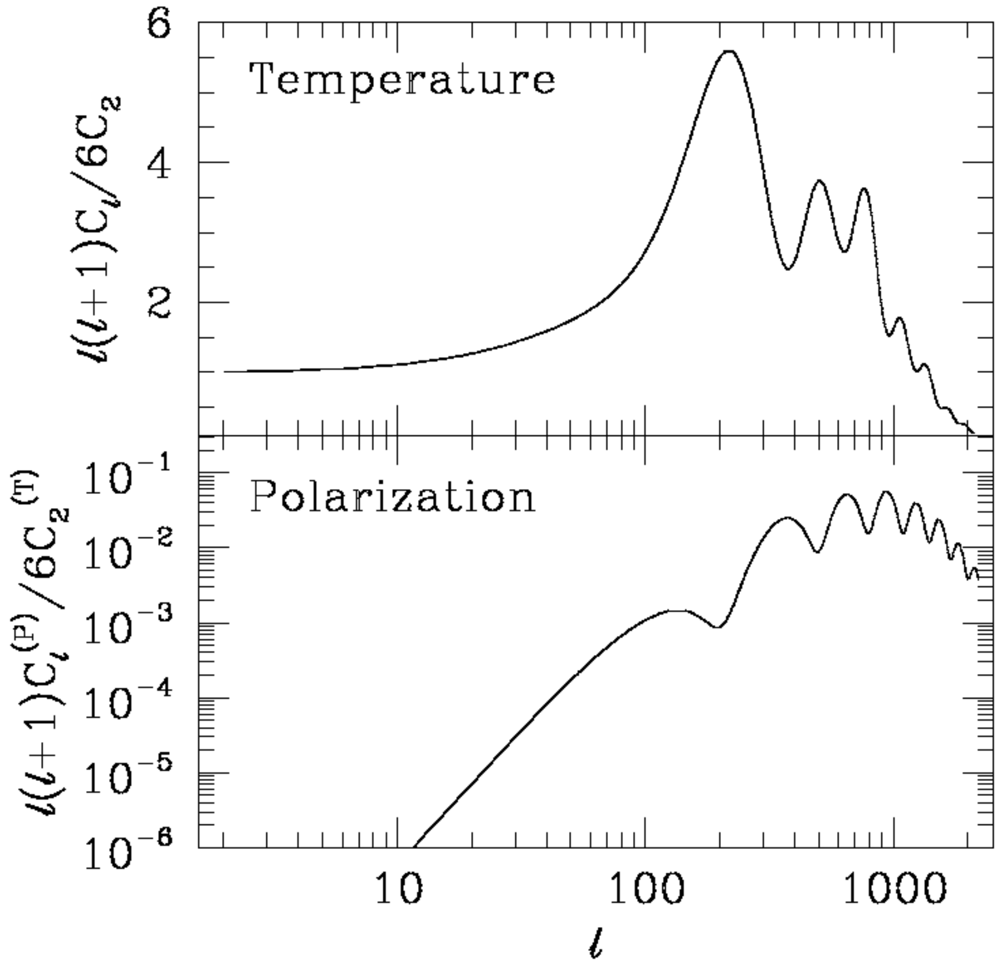


The CMB Angular Power Spectrum

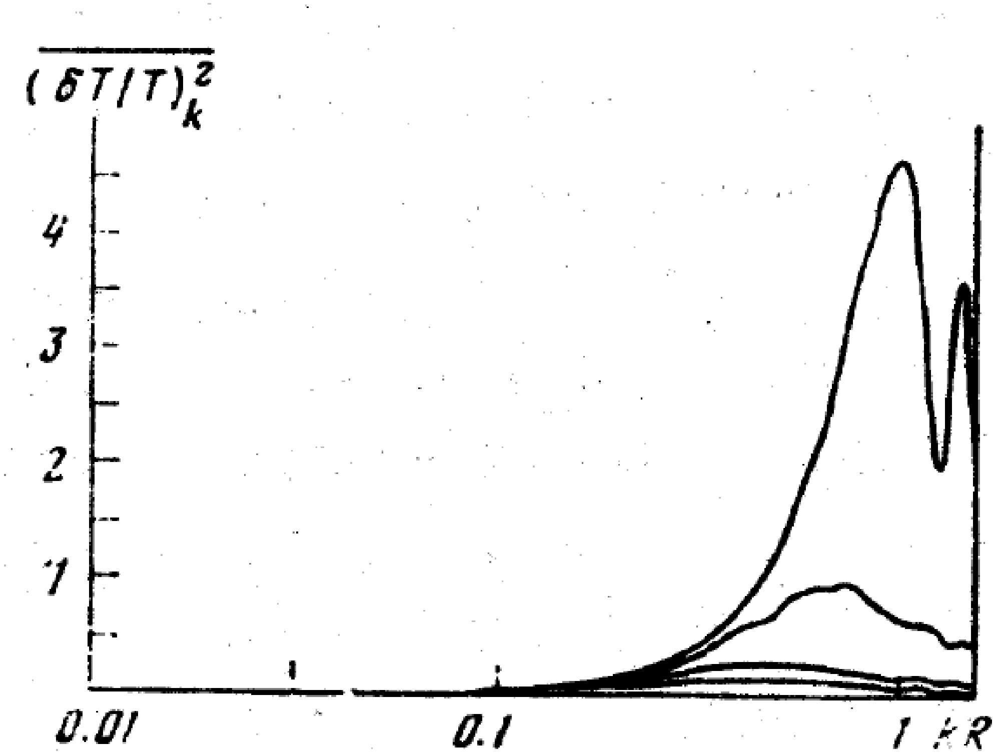
$$\left\langle \frac{\Delta T}{T}(\vec{\gamma}_1)\frac{\Delta T}{T}(\vec{\gamma}_2)\right\rangle = \frac{1}{2\pi}\sum_{\ell}(2\ell+1)C_\ell P_\ell(\vec{\gamma}_1\cdot\vec{\gamma}_2)$$

R.m.s. of $\Delta T / T$ has $l(l+1)C_l / 2\pi$ power per decade in l:

$$\left\langle(\Delta T/T)^2\right\rangle_{rms} = \sum_l \frac{(2l+1)}{4\pi}C_l \approx \int \frac{l(l+1)}{2\pi}C_l \, d\ln l$$



A Brief History of the CMB Anisotropies Angular Spectrum (Theoretical predictions)



Doroshkevich, A. G.; Zel'Dovich, Ya. B.; Syunyaev, R. A.

Soviet Astronomy, Vol. 22, p.523, 1978

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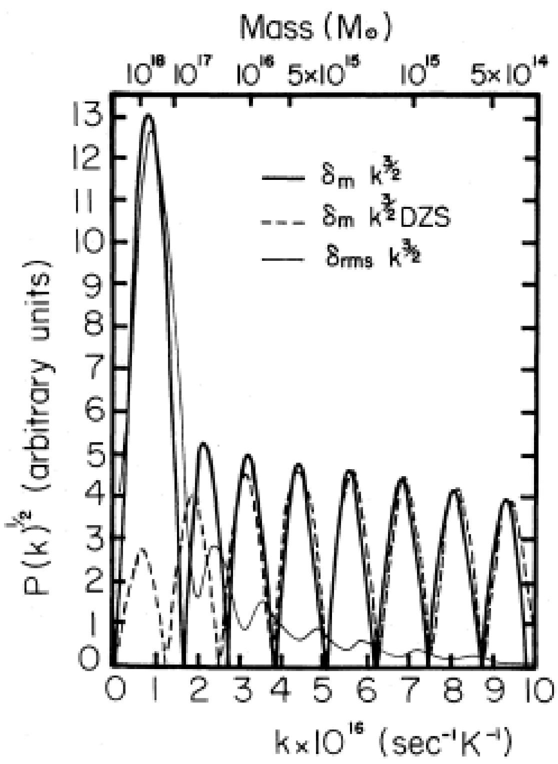

Wilson, M. L.; Silk, J., Astrophysical Journal, Part 1, vol. 243, Jan. 1, 1981, p. 14-25.
1981

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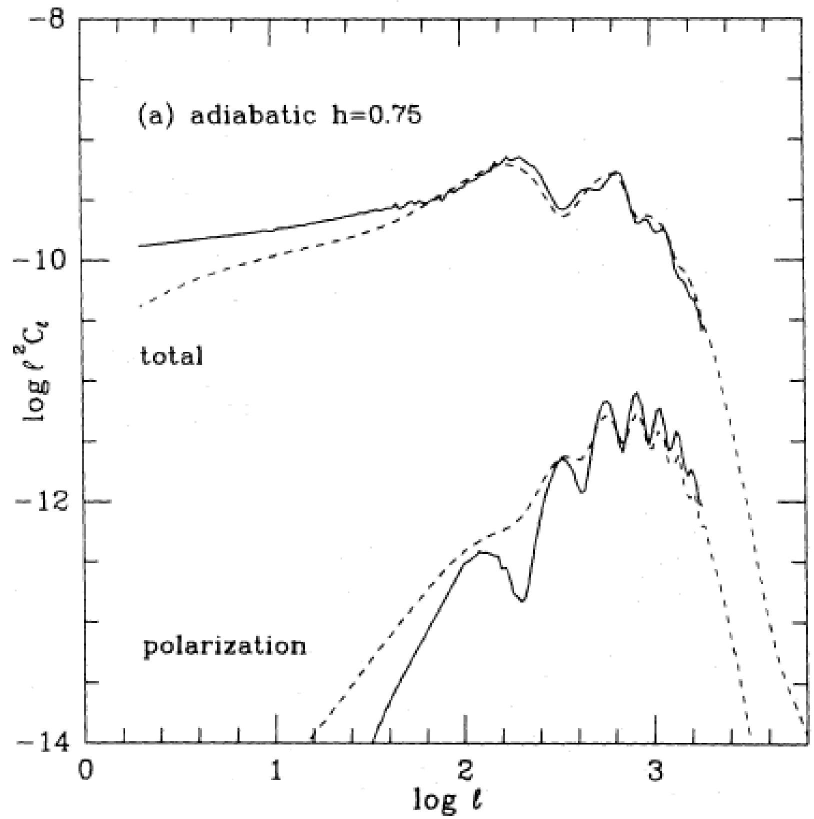

Bond, J. R.; Efstathiou, G.; Royal Astronomical Society, Monthly Notices (ISSN 0035-8711), vol. 226, June 1, 1987, p. 655-687, 1987

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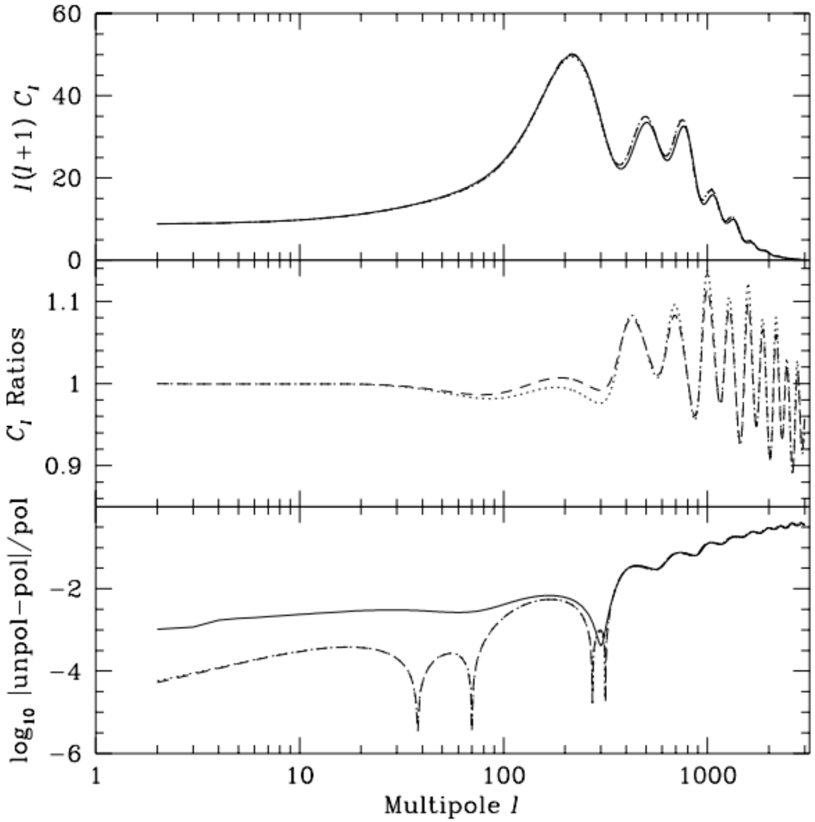
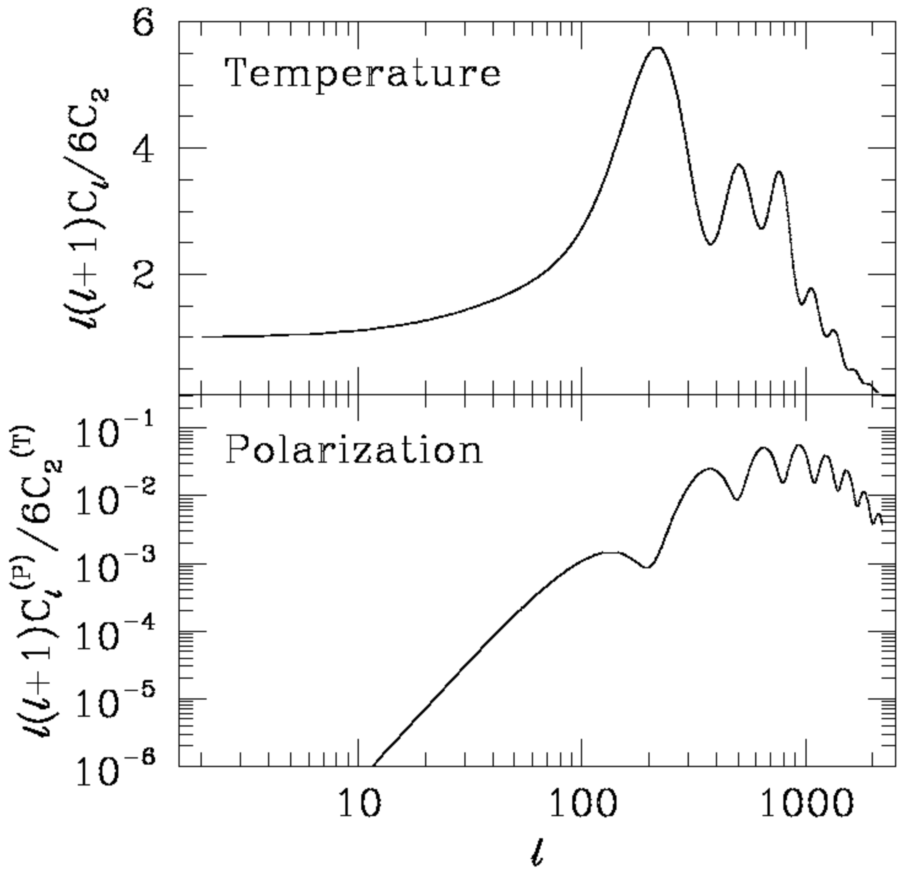


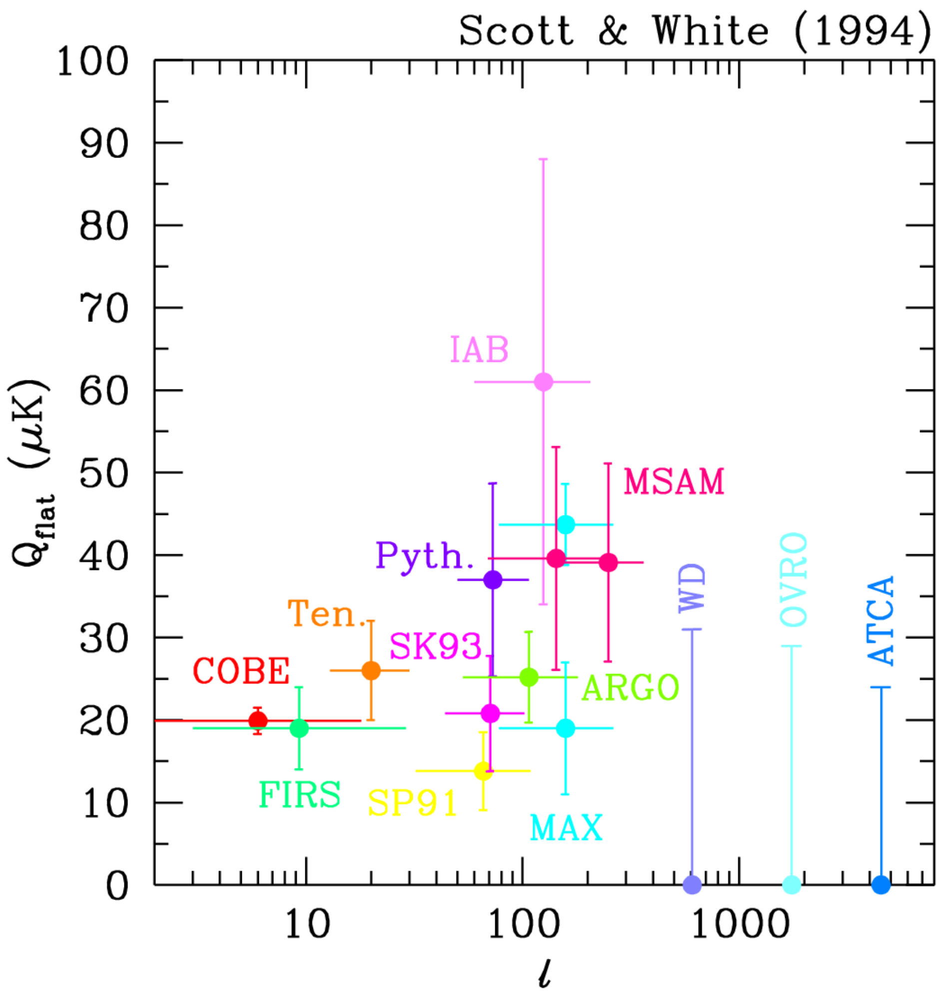
Chung-Pei Ma, Edmund Bertschinger, Astrophys.J. 455 (1995) 7-25

A Brief History of the CMB Anisotropies Angular Spectrum (Theoretical predictions)



Hu, Wayne; Scott, Douglas; Sugiyama, Naoshi; White, Martin.
Physical Review D, Volume 52, Issue 10, 15 November 1995, pp.5498-5515

A Brief History of the CMB Anisotropies Angular Spectrum (Experimental Data)



In 1995 Big Bang Model was nearly dead…

nature International weekly journal of science

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News and Views

Nature **377**, 99 (14 September 1995) |

Big Bang not yet dead but in decline

John Maddox

The latest measurements of the Hubble constant make the Big Bang account of the origin of the Universe more dependent on the coincidence of numbers than it has so far been. But it remains the only theory in the field.

▲ Top

Is there a crisis in cosmology, or is it that the latest measurement of the Hubble constant is yet another of those numeri-cal disagreements that plague the field from time to time? That is the question inevitably prompted by last week's article by N.

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- John Maddox

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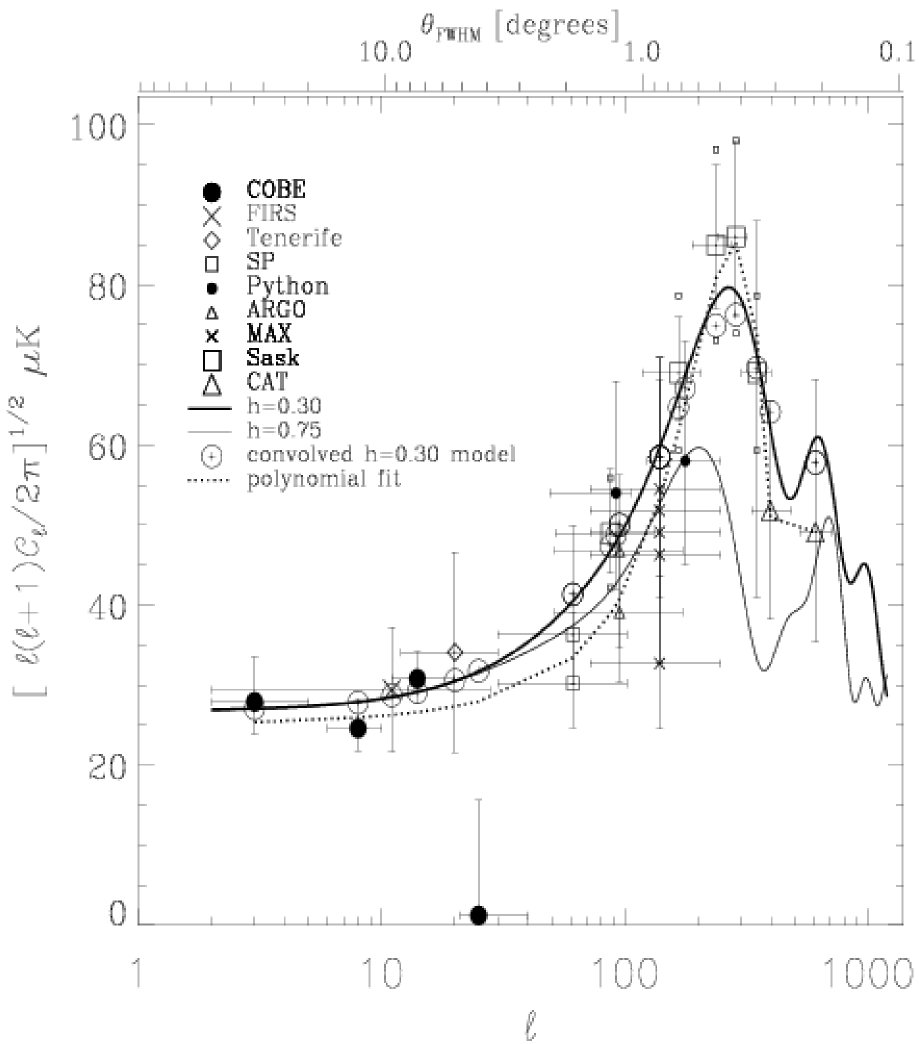
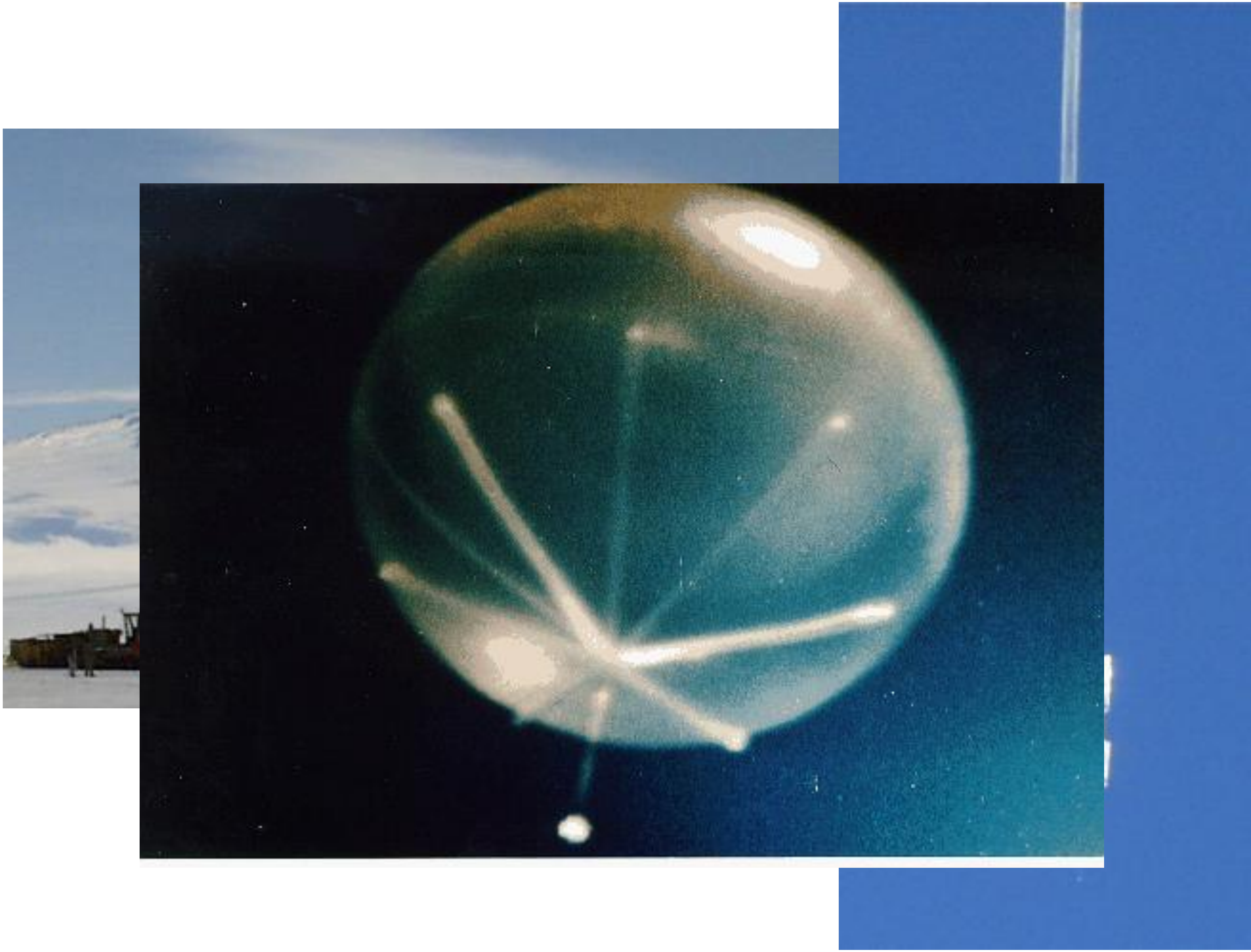
A Brief History of the CMB Anisotropies Angular Spectrum (Experimental Data)



Collection of CMB anisotropy data from C. Lineweaver et al., 1996

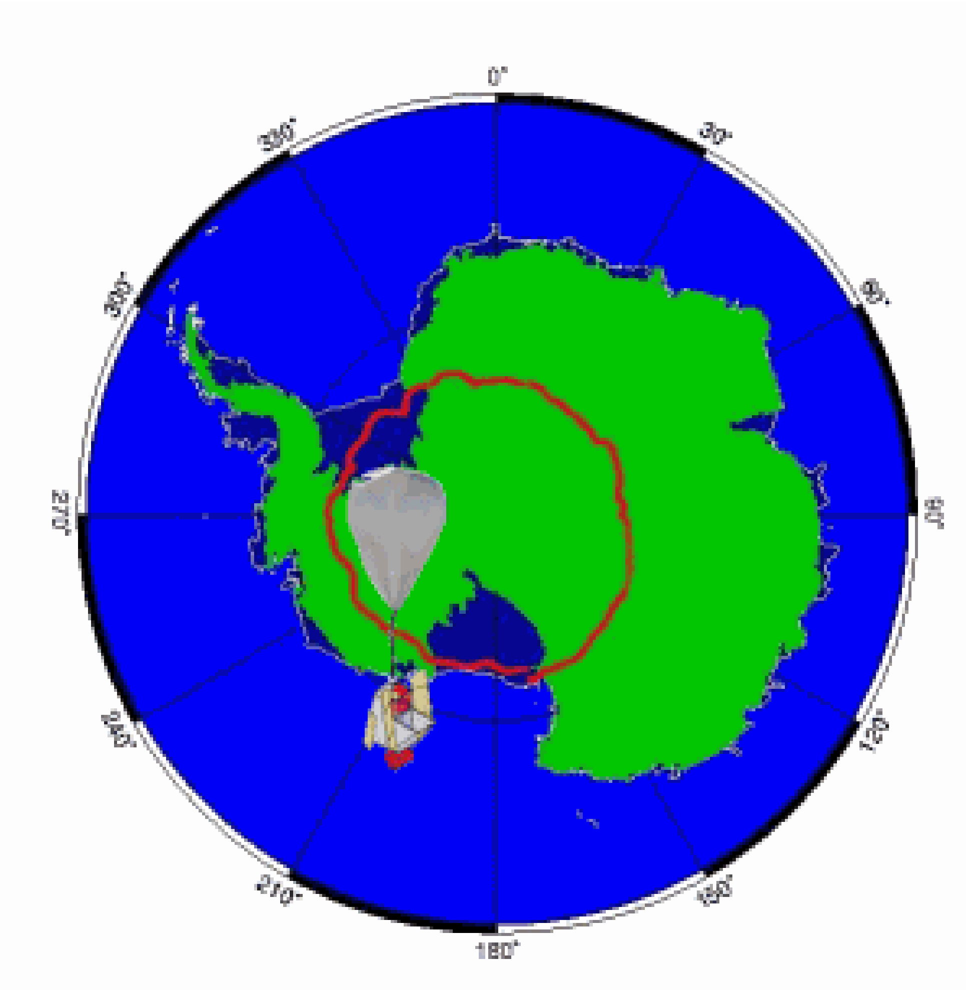
BOOMERanG Experiment

Test flight: P. Mauskopf et al, ApJ letters 536, L59, (2000), astro-ph/9911444
A. Melchiorri et al, ApJ letters 536, L63, (2000),astro-ph/9911445
LDB-B00: P. de Bernardis et al, Nature 404 (2000) 955-969, asro-ph/0004404
A. E. Lange et al, PRD D63 (2001) 042001, astro-ph/0005004
Maxima: A. Jaffe et al, Phys. Rev. Lett, (2001) 86, 3475-3479, astro-ph/0007333
LDB-B01: B. Netterfield et al, ApJ in press, astro-ph/0104460
P. de Bernardis et al, ApJ in press, astro-ph/0105296.

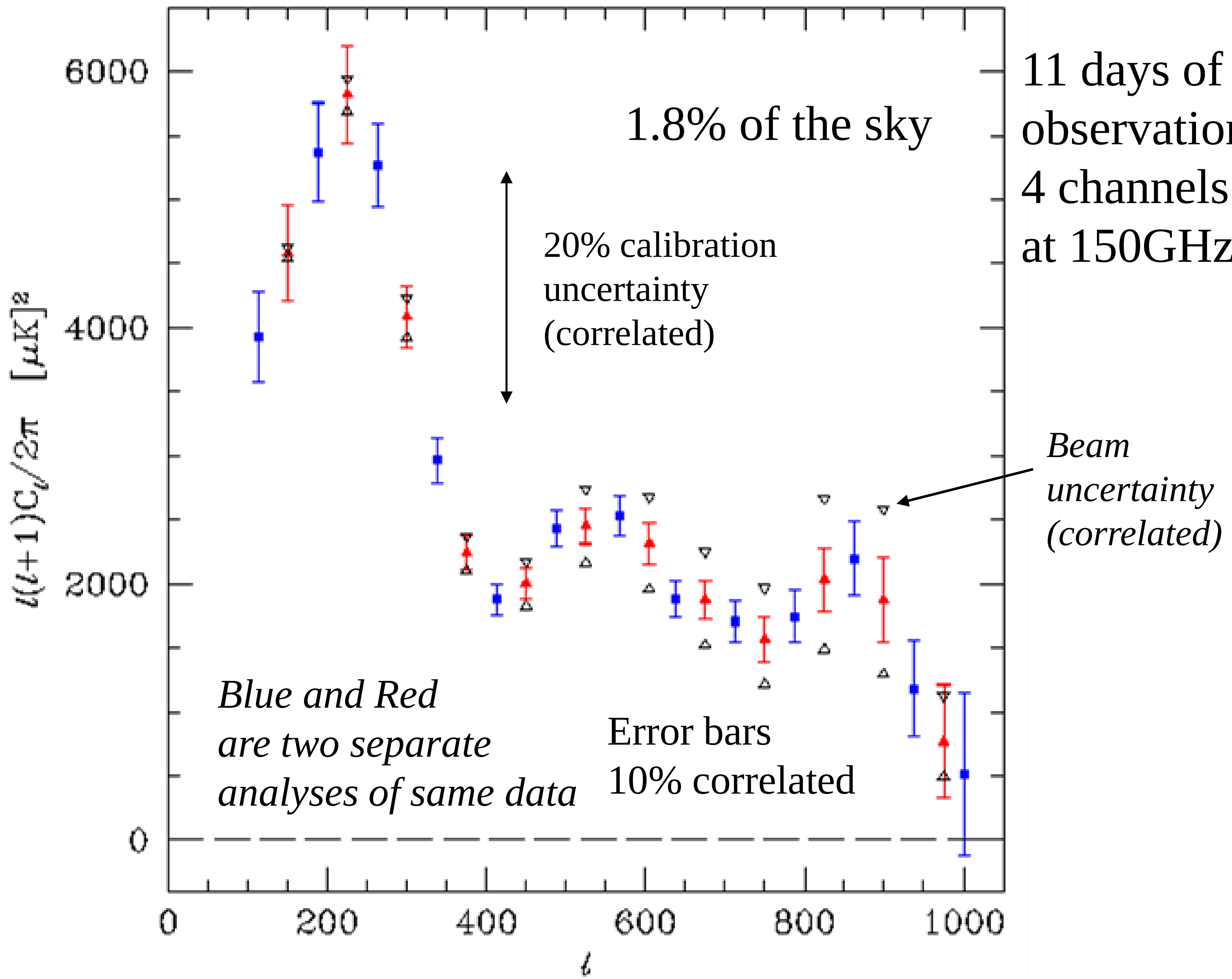


Boomerang's Track

(1 lap in 10.6 days...)

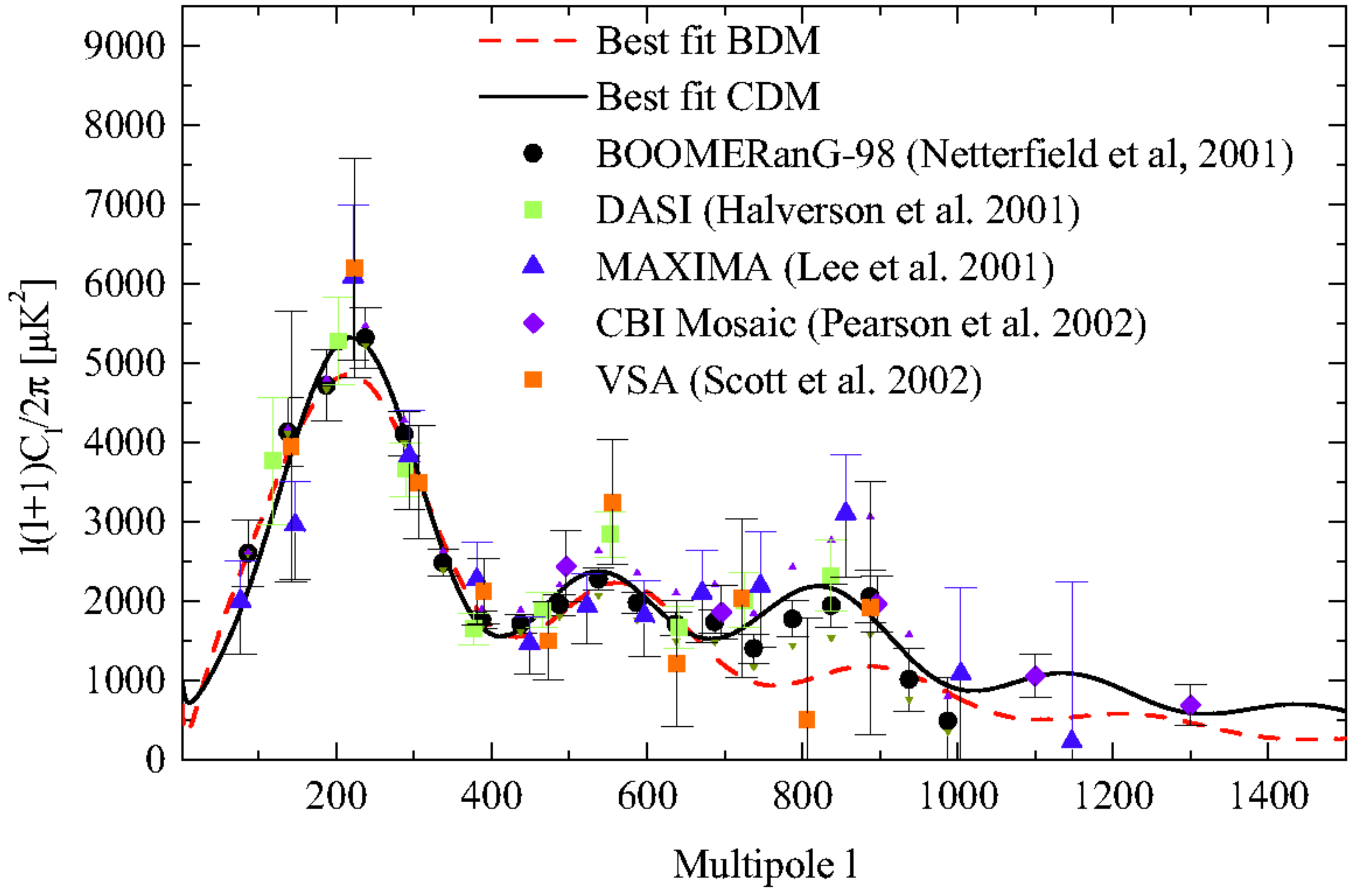


Boomerang Power Spectrum

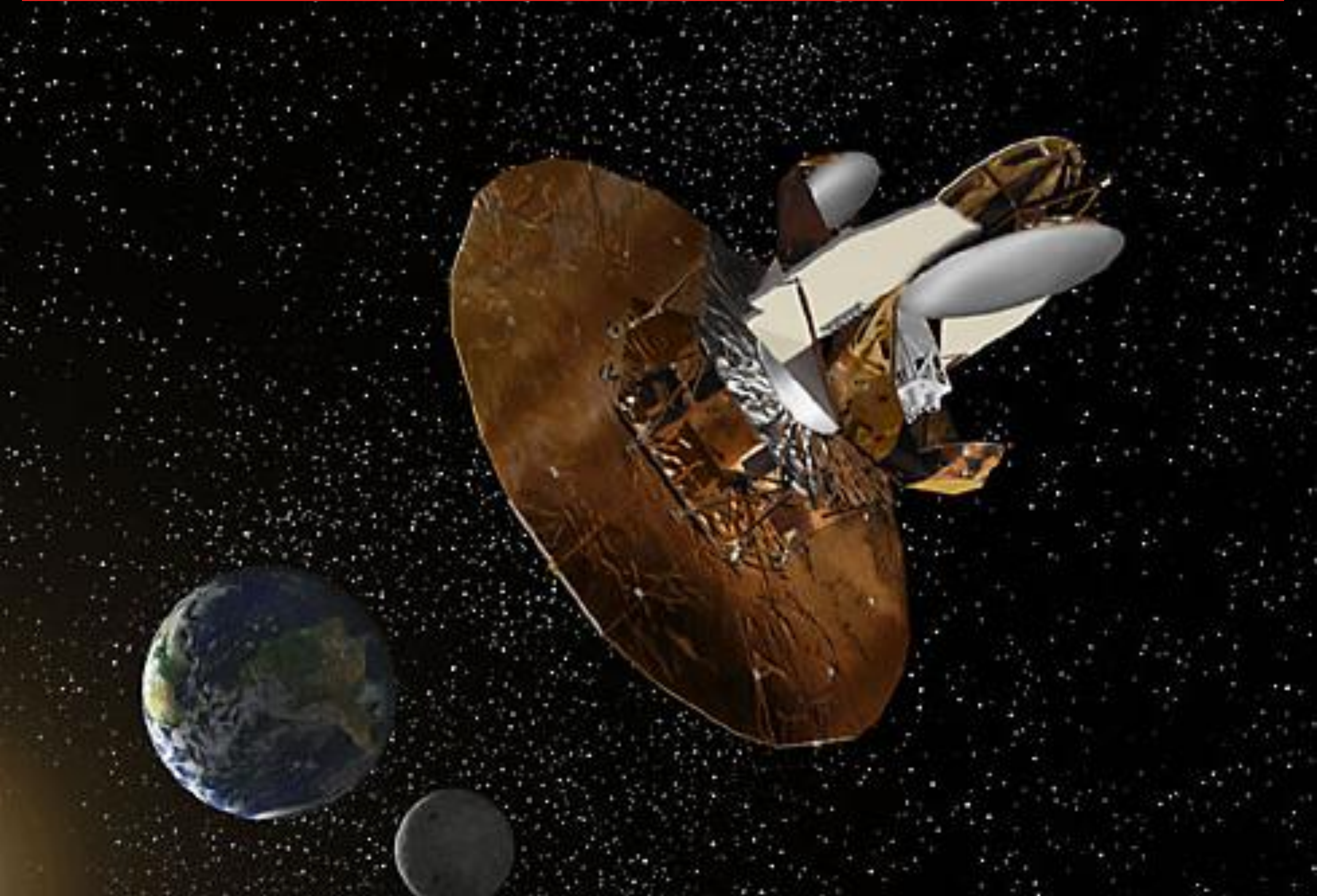


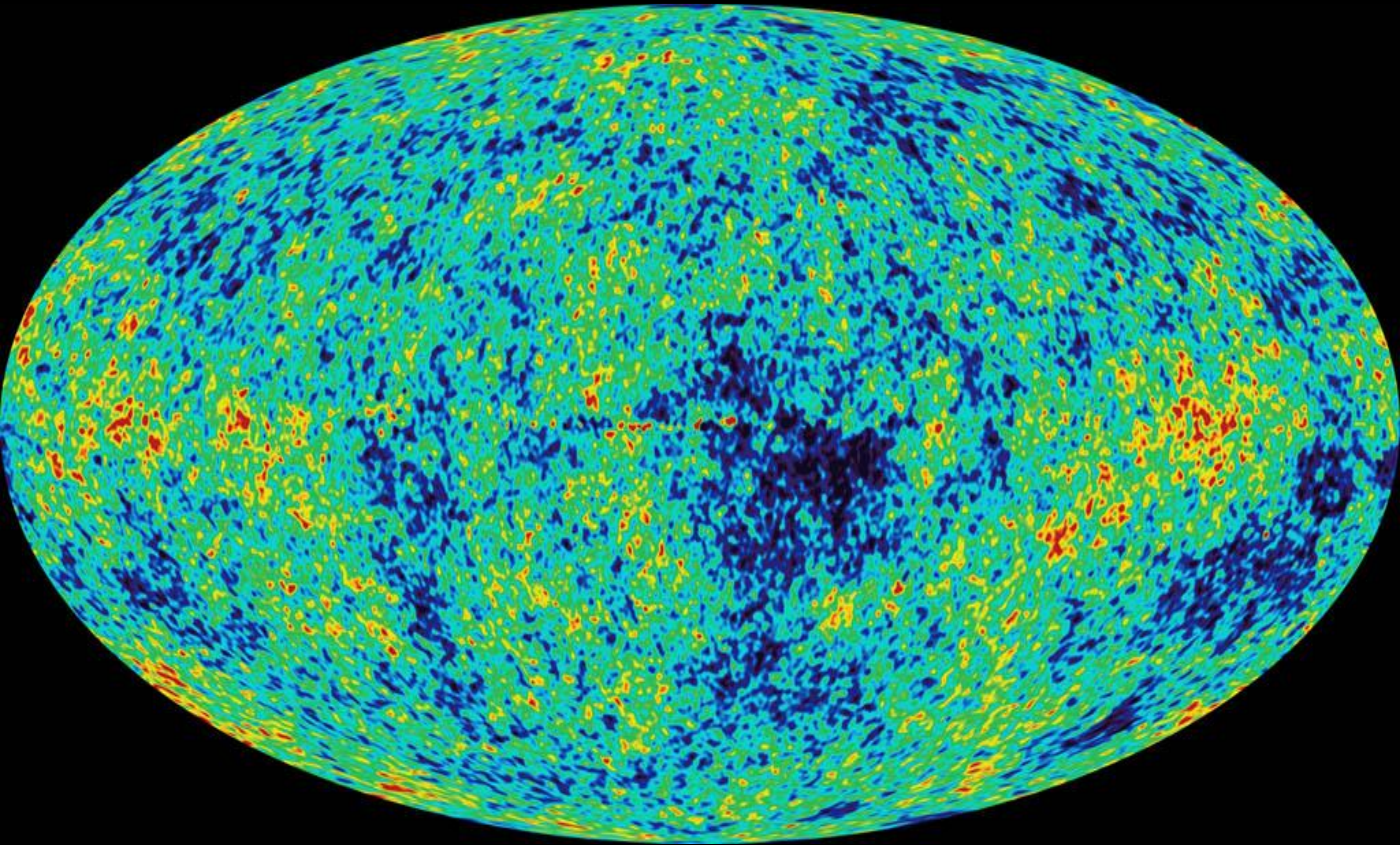
11 days of
observations
4 channels
at 150GHz

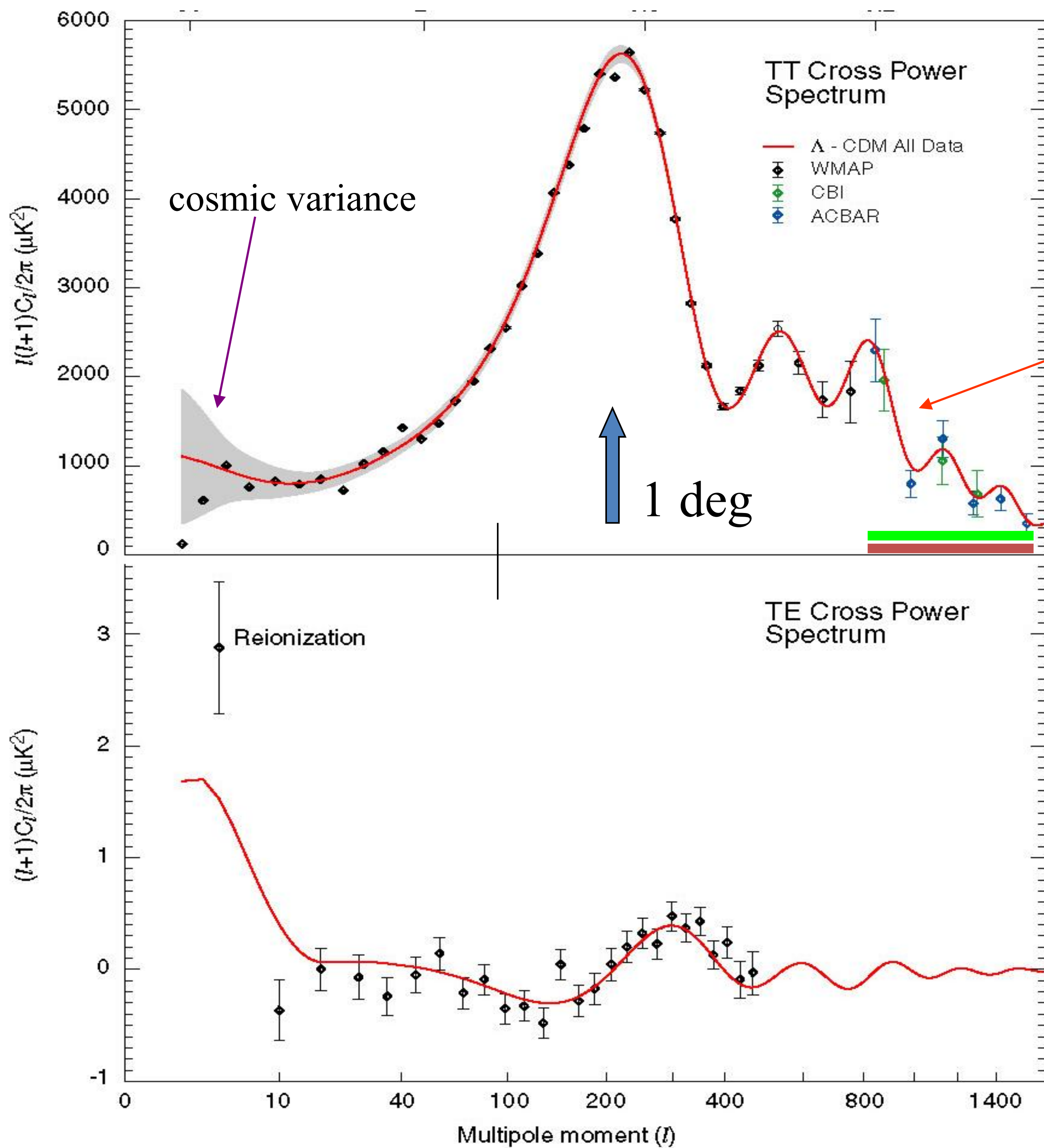
CMB anisotropies pre-WMAP (January 2003)



Wilkinson Microwave Anisotropy Probe







Temperature

85% of sky

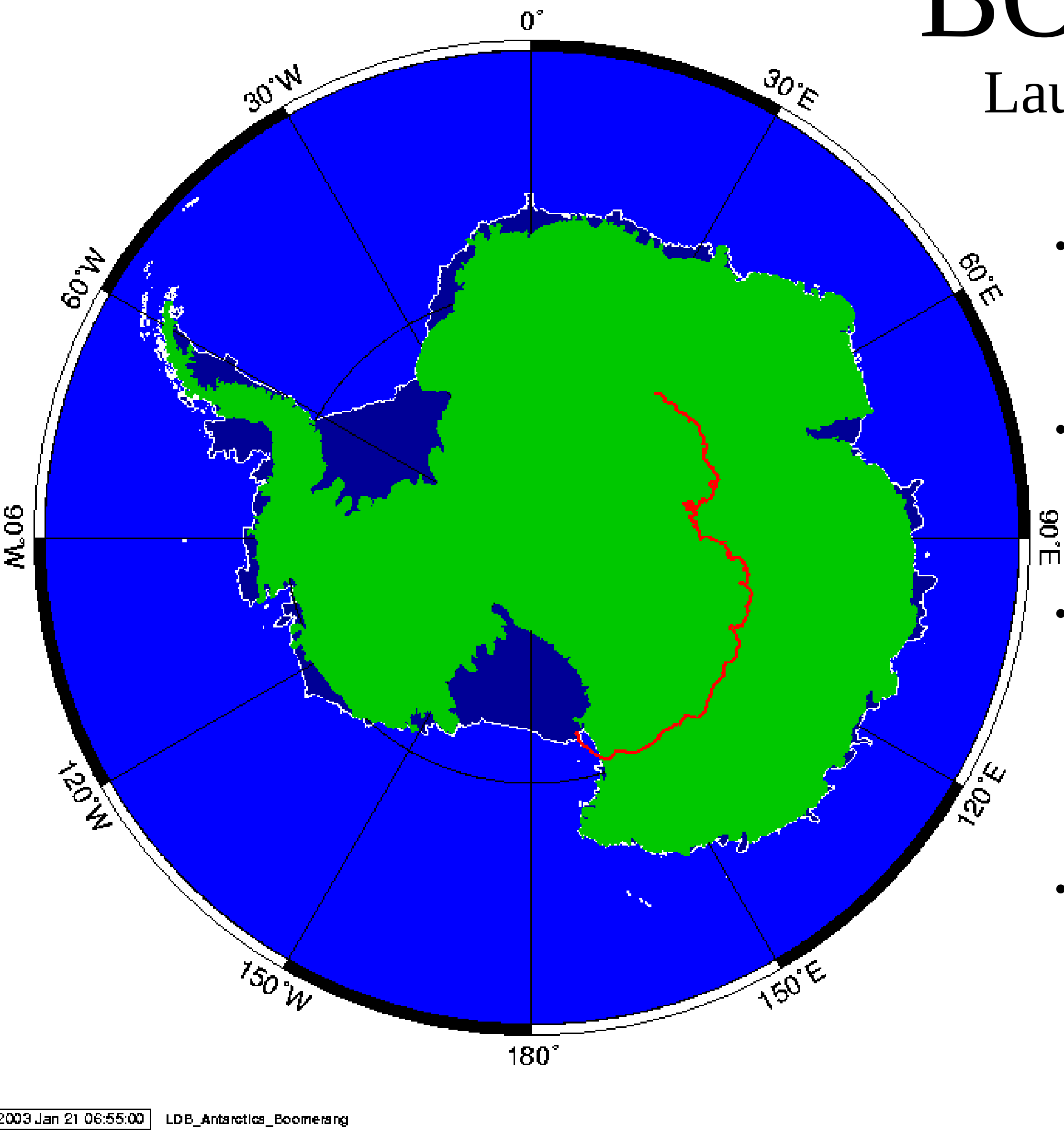
Best fit model

Spergel et al, 2003

Temperature-polarization

BOOM03 Flight

Launched: January 6, 2003



- Polarization sensitive receivers

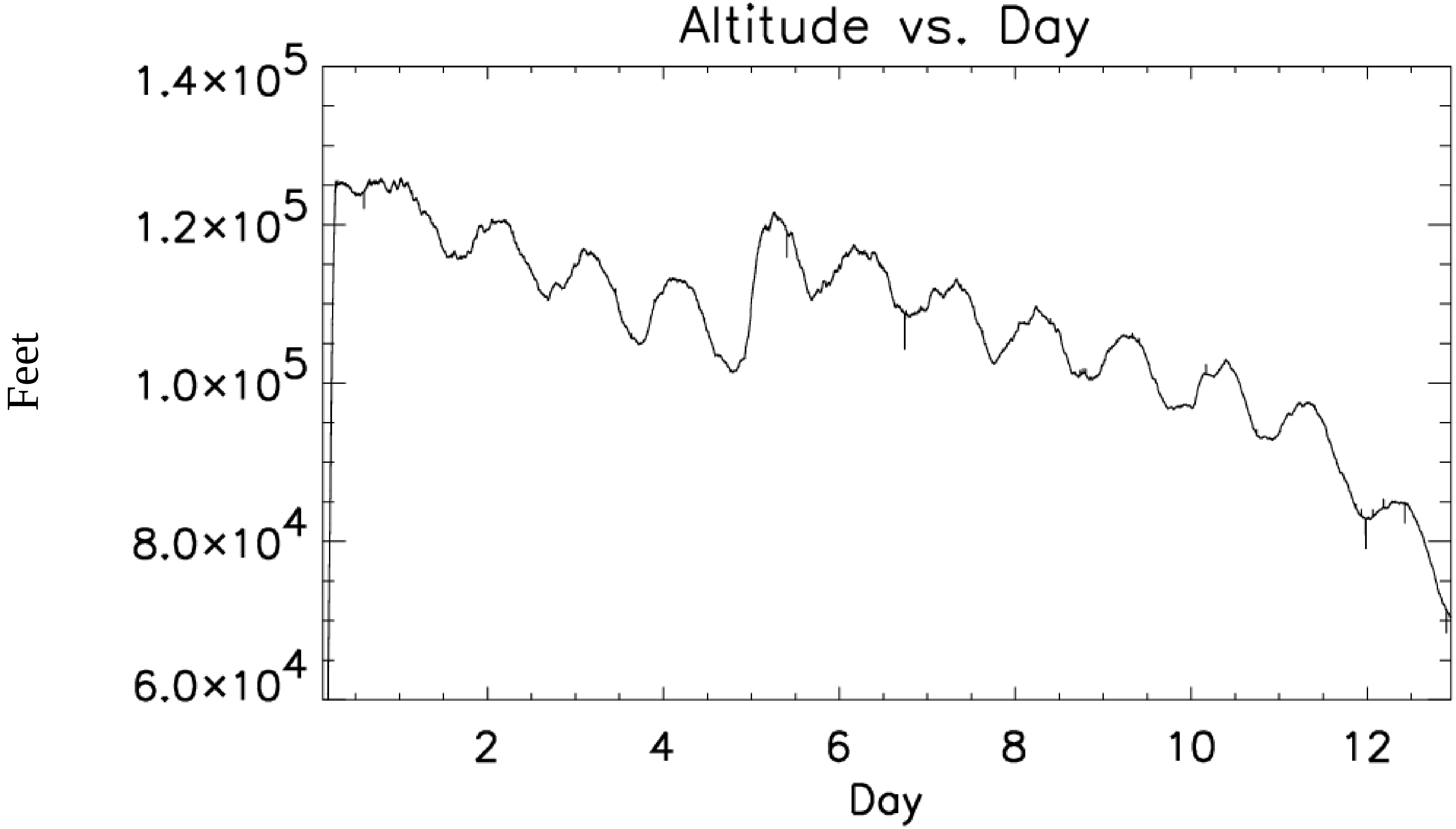
  145/245/345 GHz

- Flight January 2003

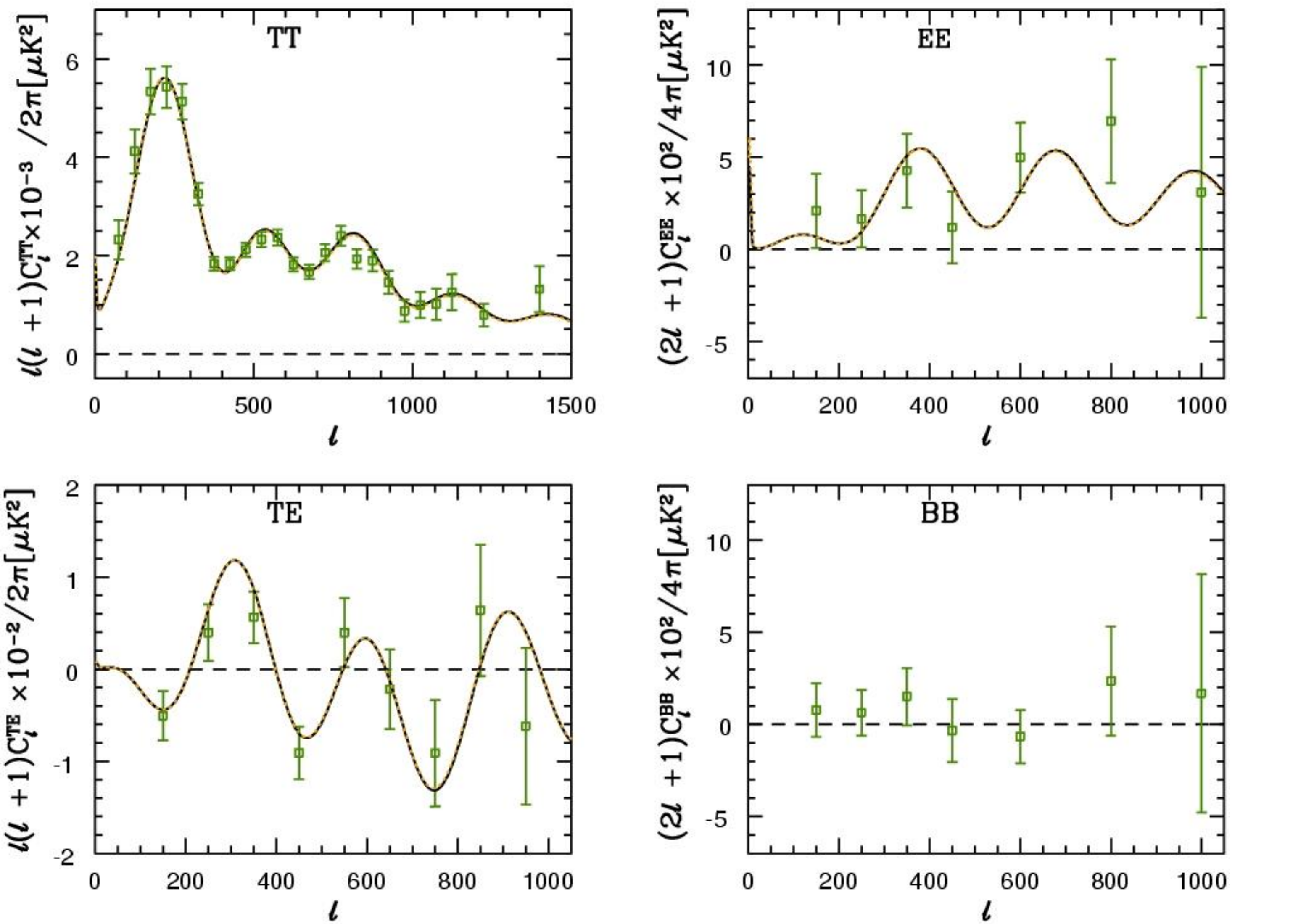
- 195 hours (11.7 days) of data

  f_{sky}= 1.8%

- First results published in July 2005
  - Masi et al. astro-ph/0507509
  - Jones et al. astro-ph/0507494
  - Piacentini et al. astro-ph/0507507
  - Montroy et al. astro-ph/0507514
  - MacTavish et al. astro-ph/0507503

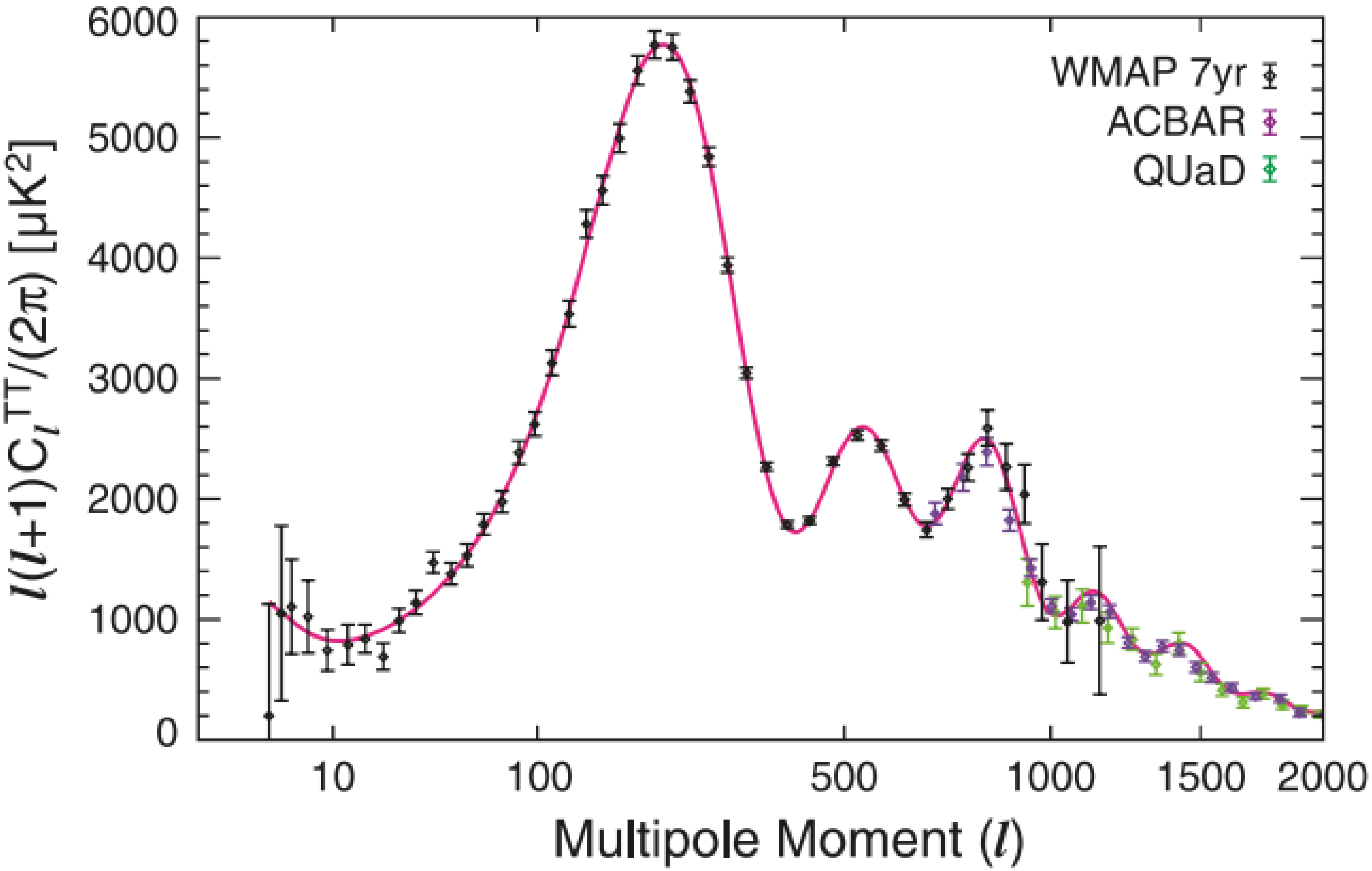


Altitude vs. Day
1.4×10^5
1.2×10^5
1.0×10^5
8.0×10^4
6.0×10^4
Feet
2
4
6
8
10
12
Day

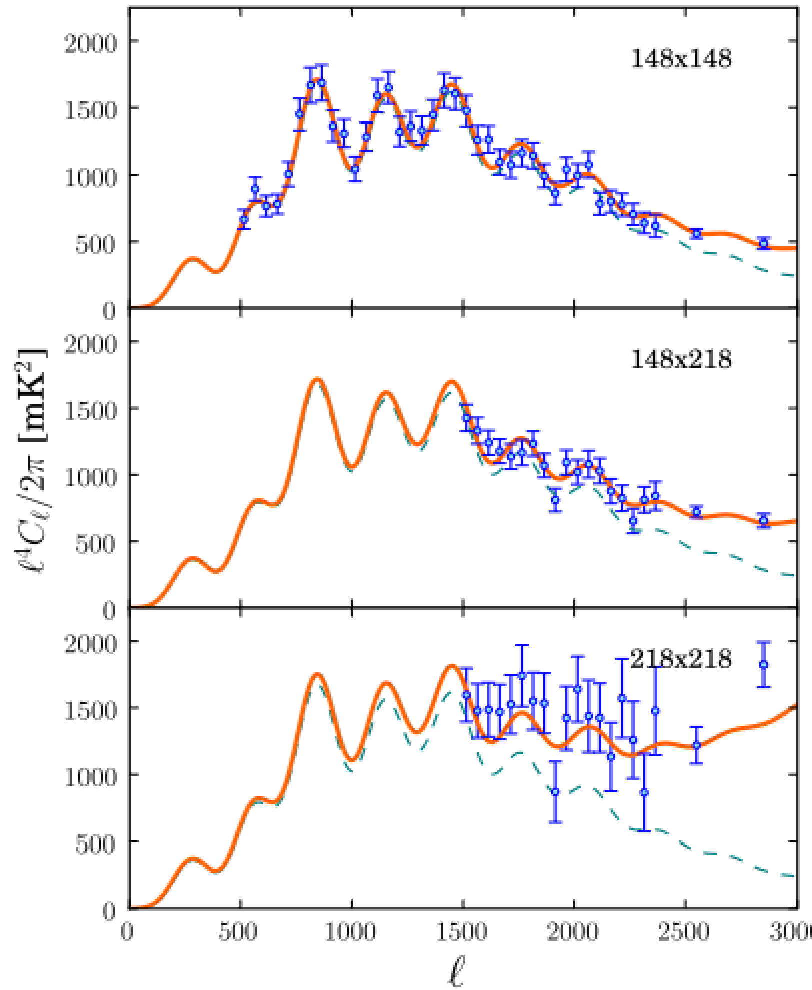


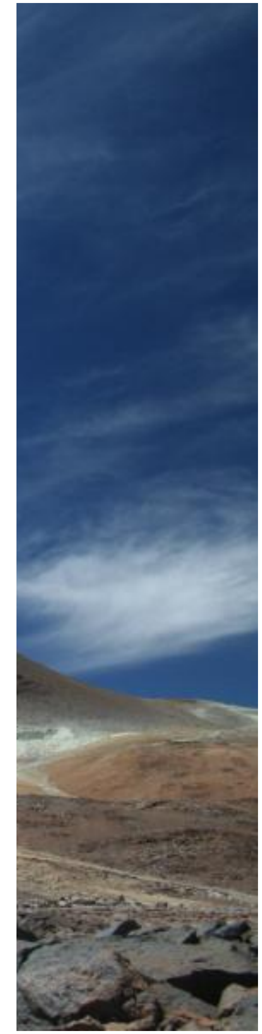
[MacTavish et al. 2005]

A Brief History of the CMB Anisotropies Angular Spectrum (Most Recent Experimental Data)



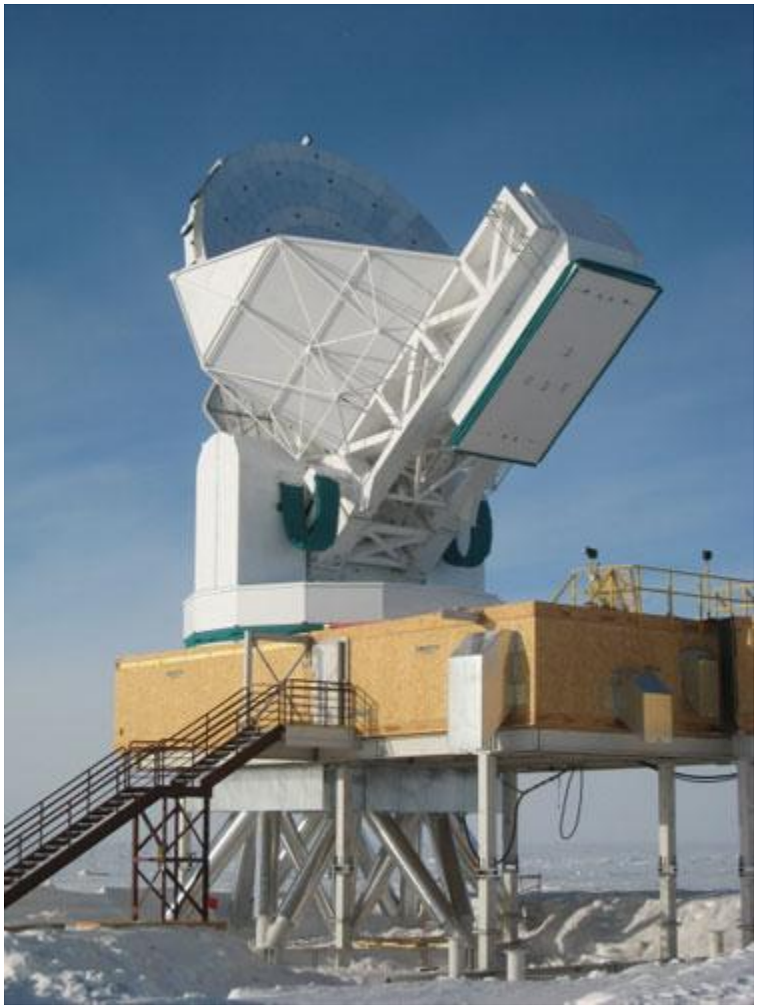
Atacama Cosmology Telescope

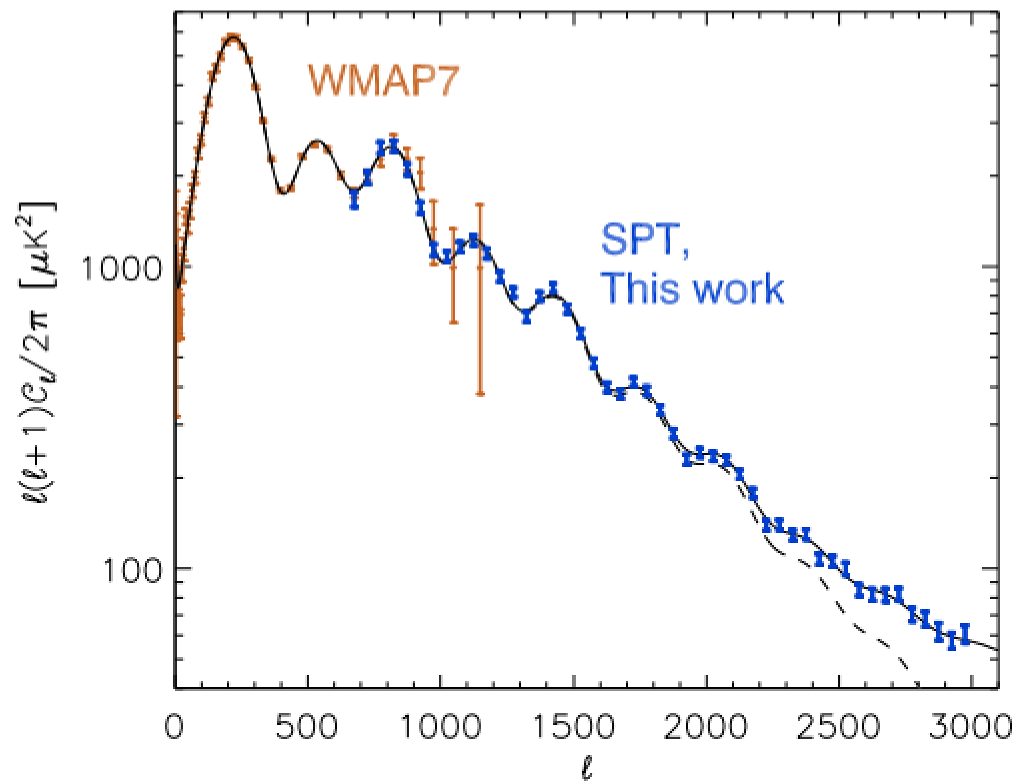




S. Das et al, 2011, [arXiv:1009.0847v1]()

South Pole Telescope





R. Keisler et al, 2011, arXiv:1105.3182


Next: Climbing to the Peak...

Interpreting the Temperature Angular Power Spectrum

Some suggested recent/old reviews:

Ted Bunn, **arXiv:astro-ph/9607088**

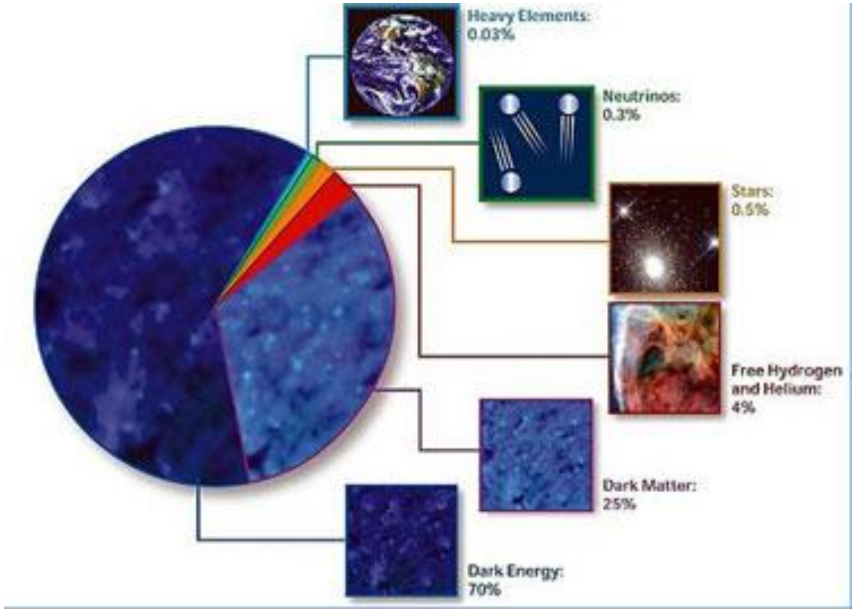
Arthur Kosowsky, **arXiv:astro-ph/9904102**

Hannu Kurki-Suonio, http://arxiv.org/abs/1012.5204

Challinor and Peiris, AIP Conf.Proc.1132:86-140, 2009, **arXiv:0903.5158**

CMB Anisotropy: BASICS

We work in a Friedmann Universe with 5 components: Baryons, Cold Dark Matter (w=0, always), Photons, Massless Neutrinos, Cosmological Constant.



- Linear Perturbation Theory. Newtonian Gauge. Scalar modes only.

$$ds^2 = a^2(\eta)\left\{-(1+2\psi)d\eta^2 + (1+2\phi)dx^i dx_i\right\}$$

CMB Anisotropy: BASICS

- Perturbation Variables:

  - $\delta_{\rm B} \equiv \delta\rho_{\rm B}/\rho_{\rm B}$, the baryon density perturbation.
  - $\delta_{\rm CDM} \equiv \delta\rho_{\rm CDM}/\rho_{\rm CDM}$, the perturbation in the CDM density.
  - $\mathbf{v}_{\rm B}$, the baryon peculiar velocity field.
  - $\mathbf{v}_{\rm CDM}$, the CDM peculiar velocity field.
  - Ψ, essentially the Newtonian gravitational potential.
  - Φ, the perturbation to the spatial curvature.[2]
  - f_γ, the photon phase-space distribution function.
  - f_ν, the neutrino phase-space distribution function.

Key point: we work in Fourier space :

$$\delta(\mathbf{x},t) = \sum_{\mathbf{k}} \delta_{\mathbf{k}}(t) \exp(i\mathbf{k}\cdot\mathbf{x})$$

CMB Anisotropy: BASICS

Their evolution is governed by a nasty set of coupled partial differential equations:

CDM:

$$\dot{\delta}_c = -\theta_c + 3\dot{\phi}\,, \quad \dot{\theta}_c = -\frac{\dot{a}}{a}\theta_c + k^2\psi$$

Baryons:

$$\begin{aligned}
\dot{\delta}_b &= -\theta_b + 3\dot{\phi}\,, \\
\dot{\theta}_b &= -\frac{\dot{a}}{a}\theta_b + c_s^2 k^2 \delta_b + \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b} a n_e \sigma_T (\theta_\gamma - \theta_b) + k^2\psi\,.
\end{aligned}$$

Photons:

$$\begin{aligned}
\dot{\delta}_\gamma &= -\frac{4}{3}\theta_\gamma + 4\dot{\phi}\,, \\
\dot{\theta}_\gamma &= k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) + k^2\psi + a n_e \sigma_T (\theta_b - \theta_\gamma)\,, \\
\dot{F}_{\gamma\,2} &= 2\dot{\sigma}_\gamma = \frac{8}{15}\theta_\gamma - \frac{3}{5}kF_{\gamma\,3} - \frac{9}{5}a n_e \sigma_T \sigma_\gamma + \frac{1}{10} a n_e \sigma_T \left(G_{\gamma\,0} + G_{\gamma\,2}\right)\,, \\
\dot{F}_{\gamma\,l} &= \frac{k}{2l+1}\left[lF_{\gamma\,(l-1)} - (l+1)F_{\gamma\,(l+1)}\right] - a n_e \sigma_T F_{\gamma\,l}\,, \quad l \geq 3 \\
\dot{G}_{\gamma\,l} &= \frac{k}{2l+1}\left[lG_{\gamma\,(l-1)} - (l+1)G_{\gamma\,(l+1)}\right] + a n_e \sigma_T \left[-G_{\gamma\,l} + \frac{1}{2}\left(F_{\gamma\,2} + G_{\gamma\,0} + G_{\gamma\,2}\right)\left(\delta_{l0} + \frac{\delta_{l2}}{5}\right)\right]
\end{aligned}$$

Neutrinos:

$$\begin{aligned}
\dot{\delta}_\nu &= -\frac{4}{3}\theta_\nu + 4\dot{\phi}\,, \\
\dot{\theta}_\nu &= k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right) + k^2\psi\,, \\
\dot{F}_{\nu\,l} &= \frac{k}{2l+1}\left[lF_{\nu\,(l-1)} - (l+1)F_{\nu\,(l+1)}\right]\,, \quad l \geq 2
\end{aligned}$$

Numerical Integration

- Early Codes (1995) integrate the full set of equations (about 2000 for each k mode, approx, 2 hours CPU time for obtaining one single spectrum).
  COSMICS first public Boltzmann code http://arxiv.org/abs/astro-ph/9506070.

- Major breakthrough with line of sight integration method with CMBFAST (Seljak&Zaldarriaga, 1996, http://arxiv.org/abs/astro-ph/9603033). (5 minutes of CPU time)

- Most supported and updated code at the moment CAMB (Challinor, Lasenby, Lewis), http://arxiv.org/abs/astro-ph/9911177 (Faster than CMBFAST).

- Both on-line versions of CAMB and CMBFAST available on LAMBDA website.

CMB Anisotropy: BASICS

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\end{aligned}$$

Neutrinos:

$$\begin{aligned}
\dot{\delta}_\nu &= -\frac{4}{3}\theta_\nu + 4\dot{\phi}\,, \\
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\dot{F}_{\nu\,l} &= \frac{k}{2l+1}\left[lF_{\nu\,(l-1)} - (l+1)F_{\nu\,(l+1)}\right]\,, \quad l \geq 2
\end{aligned}$$

First «Pilar» of the standard model of structure formation: Evolution of perturbations is passive and coherent.

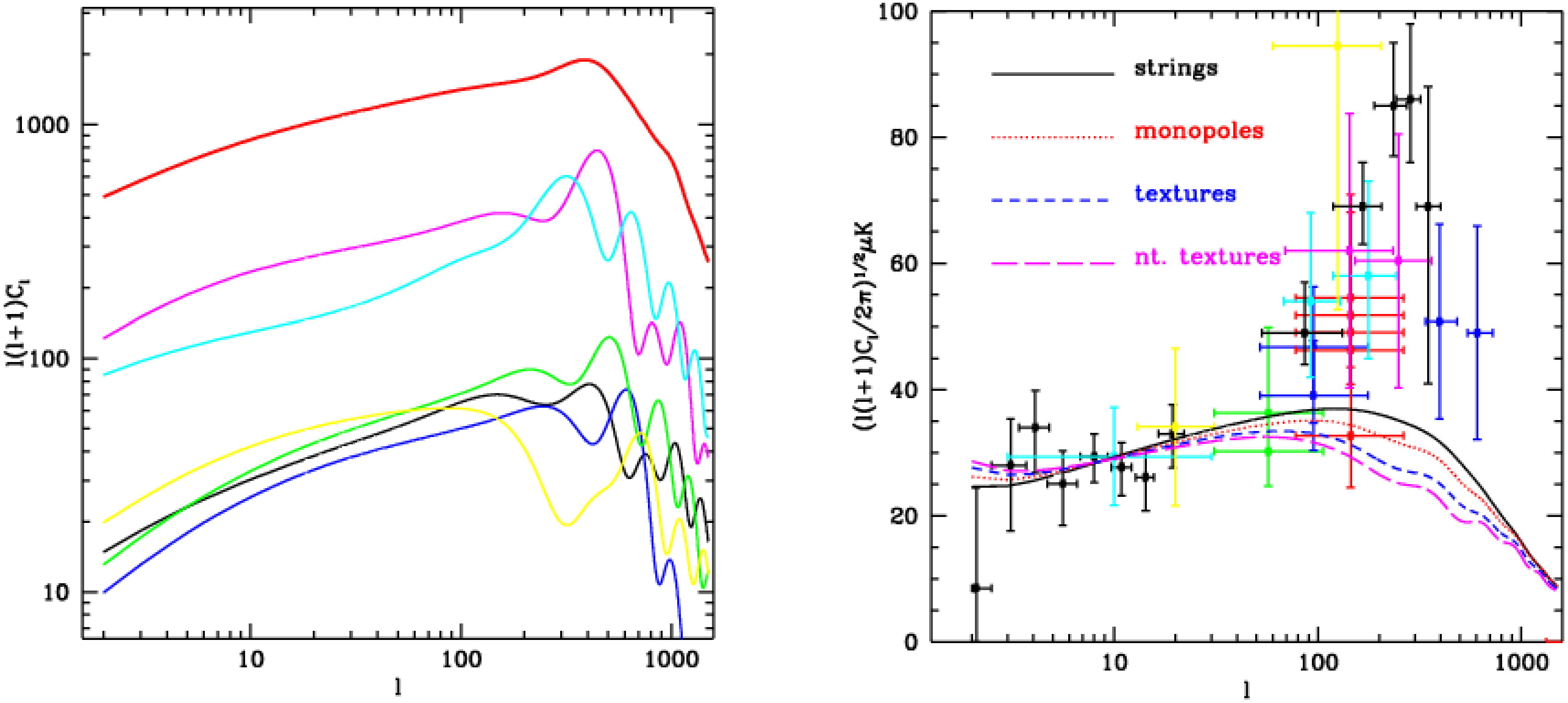
$$D_\eta \bar{f}(k,\eta) = \bar{0}$$

Linear differential operator

Perturbation Variables

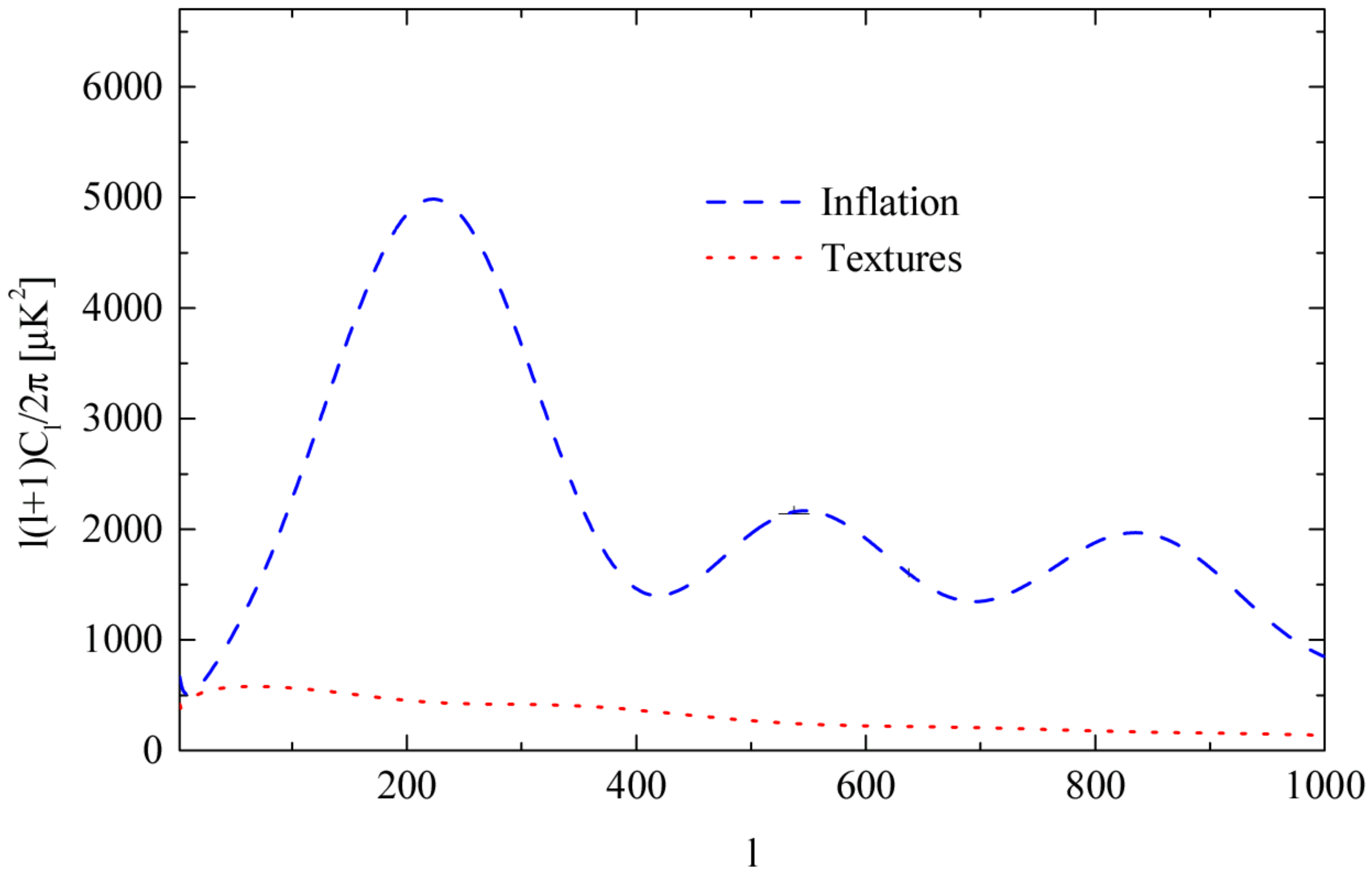
Active and decoherent models of structure formation exist (i.e. topological defects see Albrecht et al, http://arxiv.org/abs/astro-ph/9505030):

$$D_\eta \bar{f}(k,\eta) = \bar{F}(k,k',\eta,\eta')$$



Pen, Seljak, Turok, http://arxiv.org/abs/astro-ph/9704165
Expansion of the defect source term in eigenvalues. Final spectrum does'nt show any Feature or peak.

In active and decoherent models the structure of the peaks is generally not preserved.



Features in the CMB spectrum → supporting evidence for passive and coherent scheme.

Photon Geodesics

- Parameterise photon momentum with energy ε / a (seen by observer at rest in coordinates) and direction $\hat{e}$ ($e^2 = 1$) on orthonormal spatial triad:

$$p^{\mu} = a^{-2}\varepsilon\left[(1-\psi),(1+\phi)\ \hat{e}\right]$$

- Free photons move on geodesics of perturbed spacetime: $p^a \nabla_a p^b = 0$

$$d\varepsilon / d\eta = -\varepsilon\ d\psi / d\eta + \varepsilon\left(\dot{\phi} + \dot{\psi}\right)$$

$$d\hat{e} / d\eta = -\left(\overline{\nabla} - \hat{e}\hat{e}\cdot\overline{\nabla}\right)(\phi + \psi)$$

$$d\ \overline{x} / d\eta = (1 + \phi + \psi)\ \hat{e}$$

With no Perturbations: $d\varepsilon / d\eta = 0$

Deflection by Gravitational Lensing

Photons

Photons described by one-particle distribution function $f\left(\overline{x},\overline{p}\right)$

- Number of photons in proper phase space element $d^3x\ d^3p$

is $f\left(\overline{x},\overline{p}\right)d^3x\ d^3p$

- Frame-invariant and conserved along photon path in phase space (Liouville)

- In background model $f = f\left(\varepsilon\right)$

Photons

Thomson scattering around recombination ($k_B T_e << m_e c^2$) dominant scattering mechanism to affect CMB:

$$\frac{df}{d\eta} = \underbrace{-a n_e \sigma_T f}_{\text{Out-Scattering}} + \underbrace{\frac{3 a n_e \sigma_T}{16\pi} \int f(\varepsilon, \hat{m}) \left[1 + (\hat{e}\cdot\hat{m})^2\right] d\hat{m}}_{\text{In-Scattering}} \underbrace{- a n_e \sigma_T \left(\varepsilon df^B / d\varepsilon\right) \hat{e}\cdot\overline{\mathrm{v}}_{\mathrm{b}}}_{\text{Doppler term due to Electron bulk velocity}}$$

Derivative $df / d\eta$ along photon path in phase space (to first order):

$$\frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \hat{e}\cdot\overline{\nabla} f + \left(\dot{\phi} - \hat{e}\cdot\overline{\nabla}\psi\right)\varepsilon \frac{df^B}{d\varepsilon}$$

Temperature Anisotropies

Separate out background part and energy dependence of f as:

$$f(\eta, \varepsilon, \bar{x}, \bar{e}) = f^B(\varepsilon)\left[1 - \Theta(\eta, \bar{x}, \bar{e})\partial \ln f^B(\varepsilon)/\partial \ln \varepsilon\right]$$

Resulting Boltzmann equation for Θ is

$$\frac{\partial \Theta}{\partial \eta} + \hat{e} \cdot \overline{\overline{\nabla}} \Theta - \left(\dot{\phi} - \hat{e} \cdot \overline{\overline{\nabla}} \psi\right) = -a n_e \sigma_T \Theta + a n_e \sigma_T \hat{e} \cdot \bar{\mathrm{v}}_b + \frac{3 a n_e \sigma_T}{16\pi} \int \Theta(\hat{m})\left[1 + (\hat{e} \cdot \hat{m})^2\right] d\hat{m}$$

Temperature Anisotropies

$$\frac{\partial\Theta}{\partial\eta}+\hat{e}\cdot\overline{\nabla}\Theta-\left(\dot{\phi}-\hat{e}\cdot\overline{\nabla}\psi\right)=-an_e\sigma_T\Theta+an_e\sigma_T\hat{e}\cdot\overline{\mathrm{v}}_b+\frac{3an_e\sigma_T}{16\pi}\int\Theta(\hat{m})\left[1+(\hat{e}\cdot\hat{m})^2\right]d\hat{m}$$

since:

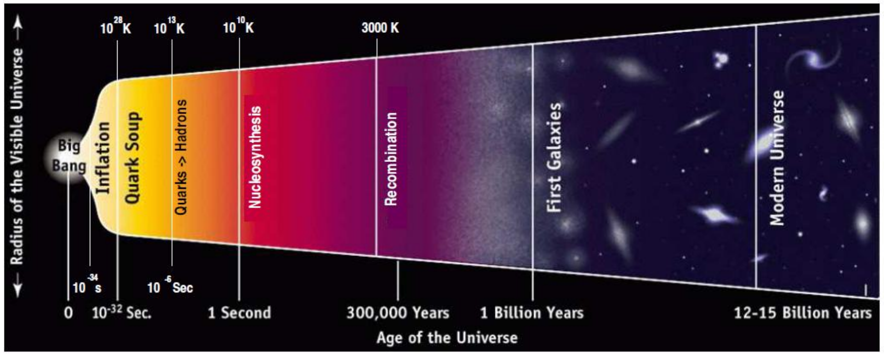
$$\frac{d\Theta}{d\eta}=\frac{\delta\Theta}{\delta\eta}+\hat{e}\cdot\overline{\nabla}\Theta$$

$$\frac{d\psi}{d\eta}=\frac{\delta\psi}{\delta\eta}+\hat{e}\cdot\overline{\nabla}\psi$$

We can rewrite it as ($\dot{\tau}=an_e\sigma_T$):

$$\frac{d\left[e^{-\tau}(\Theta+\psi)\right]}{d\eta}=-\dot{\tau}e^{-\tau}\left[\psi+\hat{e}\cdot\overline{\mathrm{v}}_b+\frac{3}{16\pi}\int\Theta(\hat{m})\left[1+(\hat{e}\cdot\hat{m})^2\right]d\hat{m}\right]+e^{-\tau}\left(\dot{\phi}+\dot{\psi}\right)$$

Thermal History and Recombination



- Dominant element hydrogen recombines rapidly around z 1000.

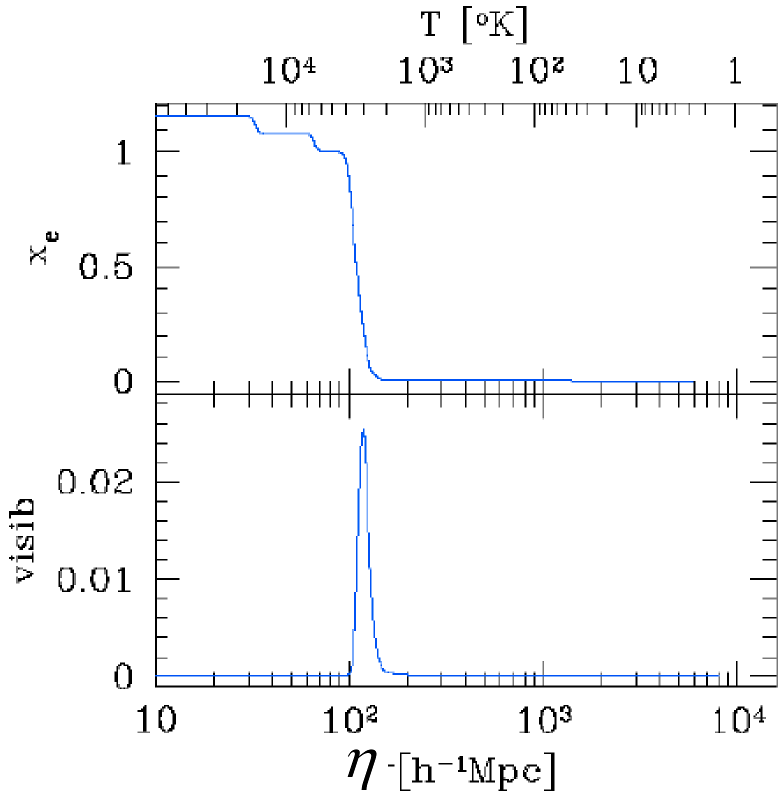
- Prior to recombination, Thomson scattering efficient and mean free path short cf. expansion time

- Little chance of scattering after recombination ! photons free stream keeping imprint of conditions on last scattering surface

• Optical depth back to (conformal) time η_0 for Thomson scattering:

$$\tau(\eta) = \int_{\eta}^{\eta_0} a n_e \sigma_T \, d\eta'$$

• The visibility function $-\dot{\tau} e^{-\tau}$ is the density probability of photon last scattering at time η



Temperature Anisotropies

Let us know expand Θ in Fourier and Legendre space:

$$\Theta(\hat{e},\bar{x},\eta)=\sum_{l\geq 0}\int\frac{d^3k}{(2\pi)^{3/2}}(-i)^l\,\Theta_l(\eta,k)P_l(\hat{k}\cdot\hat{e})e^{i\bar{k}\cdot\bar{x}}$$

We have that $\Theta_0=\delta_\gamma/4$ (density) $\Theta_1=-\mathrm{v}_\gamma$ (velocity) $\Theta_2=\frac{5}{3}\Pi_\gamma$ (stress)

Considering isotropic Thomson scattering, instantaneuos recombination and integrating the previous equation from last scattering to today we have:

$$\Theta(\hat{e},k,\eta_0)=\Theta_0(k,\eta_{rec})+[\psi(k,\eta_{rec})-\psi(k,\eta_0)]+\hat{e}\cdot\bar{\mathrm{v}}_b(k,\eta_{rec})+\int_{\eta_0}^{\eta_{rec}}(\dot{\phi}+\dot{\psi})d\eta$$

The temperature received along $\hat{e}$ is the isotropic temperature of the CMB at the last scattering event on the line of sight, Θ_0, corrected for the gravitational redshift due to the difference in the potential $[\psi(k,\eta_{rec})-\psi(k,\eta_0)]$ and the Doppler shift $\hat{e}\cdot\bar{\mathrm{v}}_b$ resulting from scattering off moving electrons .

Finally, there is the integrated Sachs-Wolfe contribution from evolution of the potentials along the line of sight.

Acoustic Oscillations

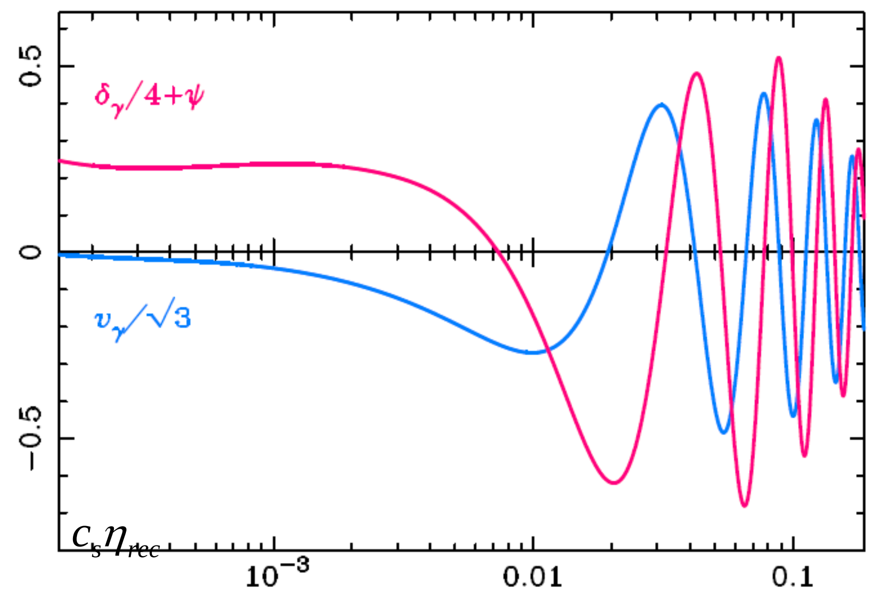
It is possible to show that the intrinsic temperature+gravity term has the solution (where $R \equiv 3\rho_B/4\rho_\gamma$ and $c_s = (3(1+R))^{-1/2}$ is the plasma sound speed) :

$$[\Theta_0 + \psi](k, \eta_{rec}) \approx \frac{1}{3}(1+3R)\psi \cos(kc_s\eta_{rec}) - R\psi$$

While the Doppler term (from the continuity equation) follows:

$$\hat{e} \cdot \overline{\mathrm{v}}_b(k, \eta_{rec}) \approx \frac{1}{3}\psi \sin(kc_s\eta_{rec})$$

There is a simple physical picture underlying this result.
The baryon-photon fluid wants to fall into the potential wells, but it is supported by radiation pressure. The balance between pressure and gravity sets up acoustic oscillations.



CMB Anisotropies and Baryons

The CMB spectrum is essentially the quadrature sum of the two contributions.

$$\Theta(k,\eta_0)^2 \approx \left[\frac{1}{3}(1+3R)\psi\cos(kc_s\eta_{rec}) - R\psi\right]^2 + \left[\frac{1}{3}\psi\sin(kc_s\eta_{rec})\right]^2 + ..$$

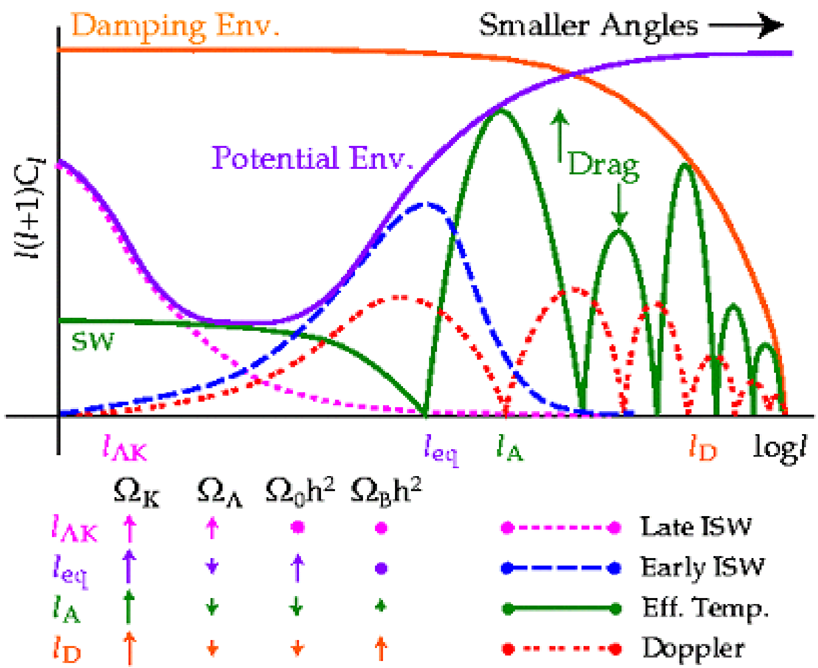
Note the following:

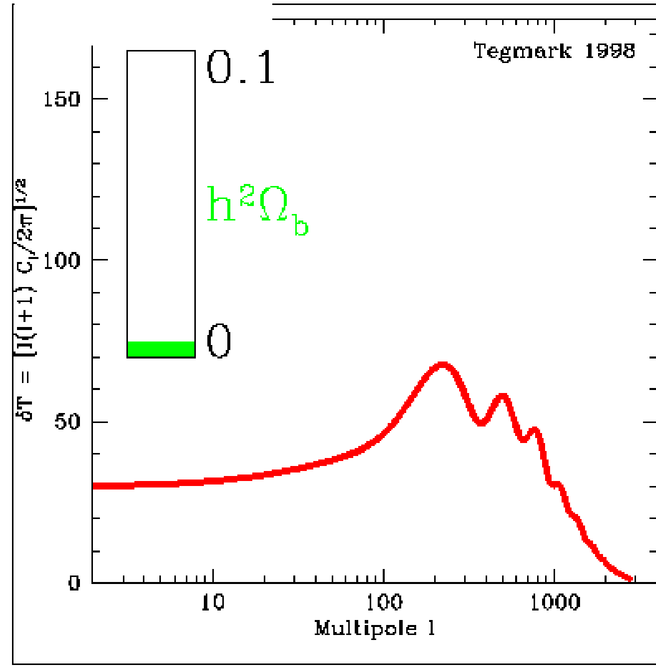
a) When R=0 (no baryons) the quadrature sum gives:

$$\Theta(k,\eta_0)^2 \approx \left[\frac{1}{3}\psi\cos(kc_s\eta_{rec})\right]^2 + \left[\frac{1}{3}\psi\sin(kc_s\eta_{rec})\right]^2 = 1$$

i.e. no oscillations !!!
Why does including the dynamical effect of the baryons change the solution?
The essential reason is that baryons contribute to the effective mass of the photon-baryon fluid, but not to the pressure.
The effect of the baryons, therefore, is to slow down the oscillations, and also to make the fluid fall deeper into the potential wells.

CMB Anisotropies and Baryons





The height of the peaks in the CMB anisotropy spectrum depends on the baryon density:

- The larger the baryon density, the larger R, and the greater the amplitude of the oscillations.
- Furthermore, because of the offset in the oscillations, we expect the odd-numbered peaks to be enhanced relative to the evennumbered ones.

Projection

A mode with wavelength λ will show up on an angular scale θ ~ λ/R, where R is the distance to the last-scattering surface, or in other words, a mode with wavenumber k shows up at multipoles l~k.

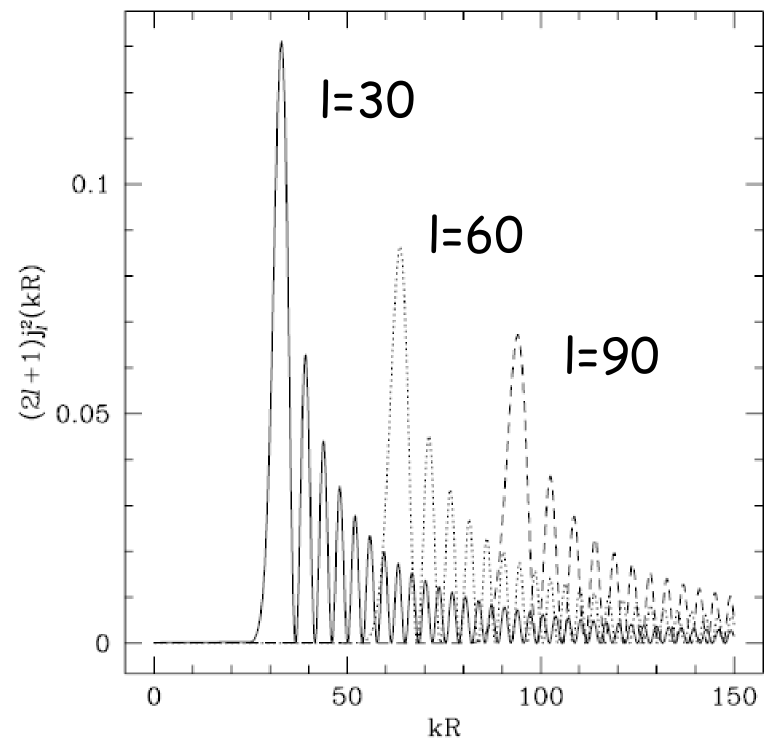
$$\frac{\Delta T}{T}(\hat{\mathbf{r}}) = \sum_{l,m} a_{lm} Y_{lm}(\hat{\mathbf{r}}), \qquad C_l = \langle |a_{lm}|^2 \rangle.$$

$$a_l^2 \equiv \sum_{m=-l}^{l} |a_{lm}|^2 = 4\pi(2l+1)\left|\Theta_{\mathbf{k}}^{(\mathrm{tot})}\right|^2 j_l^2(kR).$$

The spherical Bessel function $j_l(x)$ peaks at x ~ l, so a single Fourier mode k does indeed contribute most of its power around multipole l_k = kR, as expected. However, as the figure shows, jl does have significant power beyond the first peak, meaning that the power contributed by a Fourier mode "bleeds" to l-values different from lk.

Moreover for an open universe (K is the curvature) :

$$R_A = \frac{1}{\sqrt{|K|}} \sinh\left(\sqrt{|K|}R\right).$$



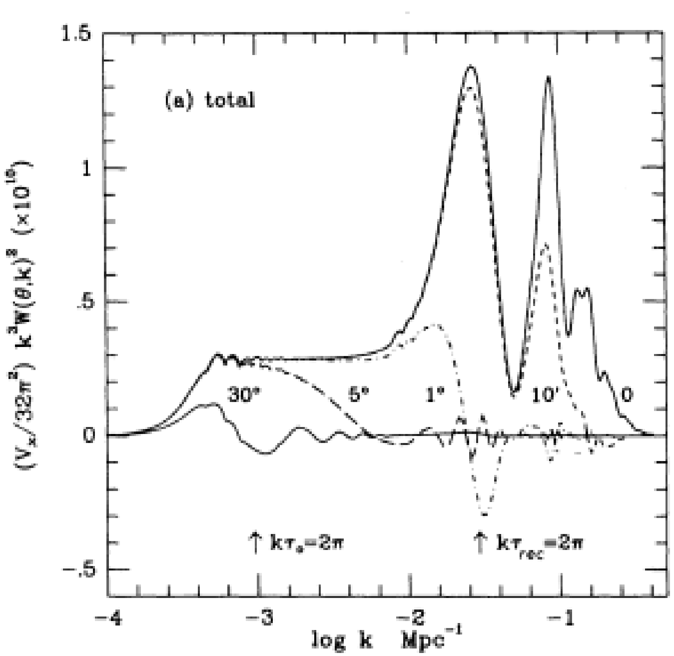
Projection

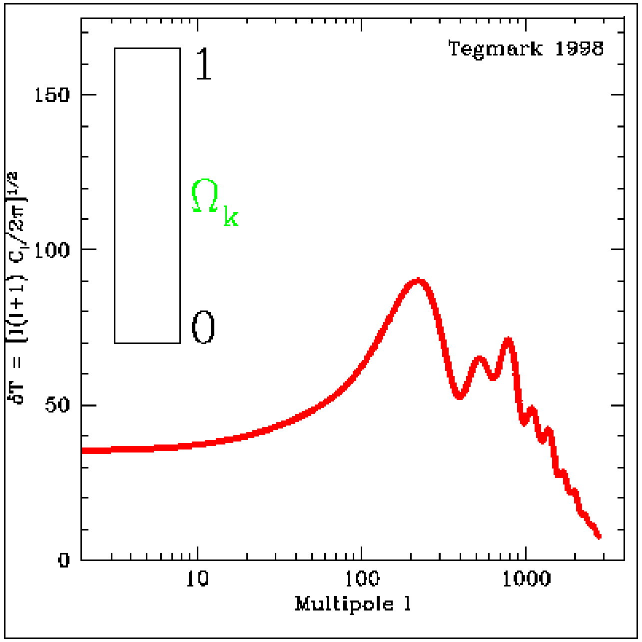
In Fourier space we have oscillations
With frequency (or physical scale):

$$\lambda = c_s \eta_{rec}$$

In Legendre space oscillations are smeared and have a frequency that Depends on the angular diameter distance at recombination.

$$\vartheta = \frac{c_s \eta_{rec}}{D_A(\eta_{rec})}$$



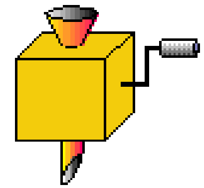


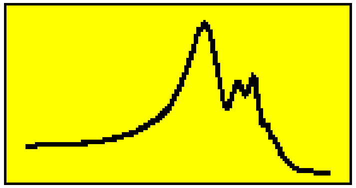
Conclusions to Lecture I

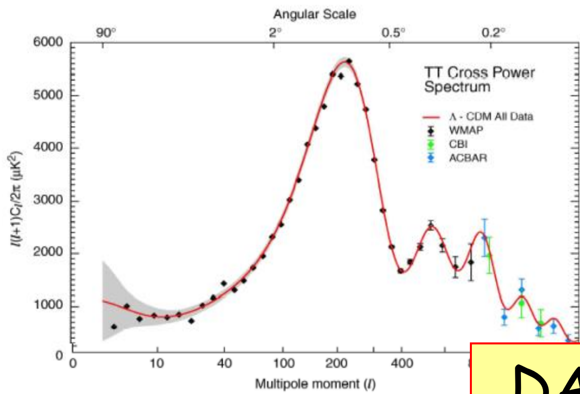
- Fantastic experimental progresses for CMB anisotropy in the past 15 years.
- CMB theory well developed (with 1% accuracy) since 1994.
- The theory of CMB anisotropies relies on passive and coherent perturbations.
- Recombination is crucial since it defines the visibility function.
- Acoustic oscillations are present because of gravitational collapse counterbalanced by photons pressure.
- Baryons are important for the formation of the peaks. CMB is very sensitive to the baryon abundance.
- Since we are measuring the CMB anisotropies in Legendre space, the angular diameter distance to recombination also affects the shape of the spectrum. Useful to measure the curvature of the universe or better constrain the (will see) dark energy equation of state.

How to get a bound on a cosmological parameter

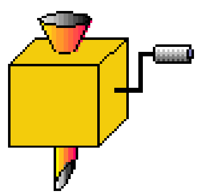
Fiducial cosmological model:
$(\Omega_b h^2 , \Omega_m h^2 , h , n_s , \tau , \Sigma m_\nu)$





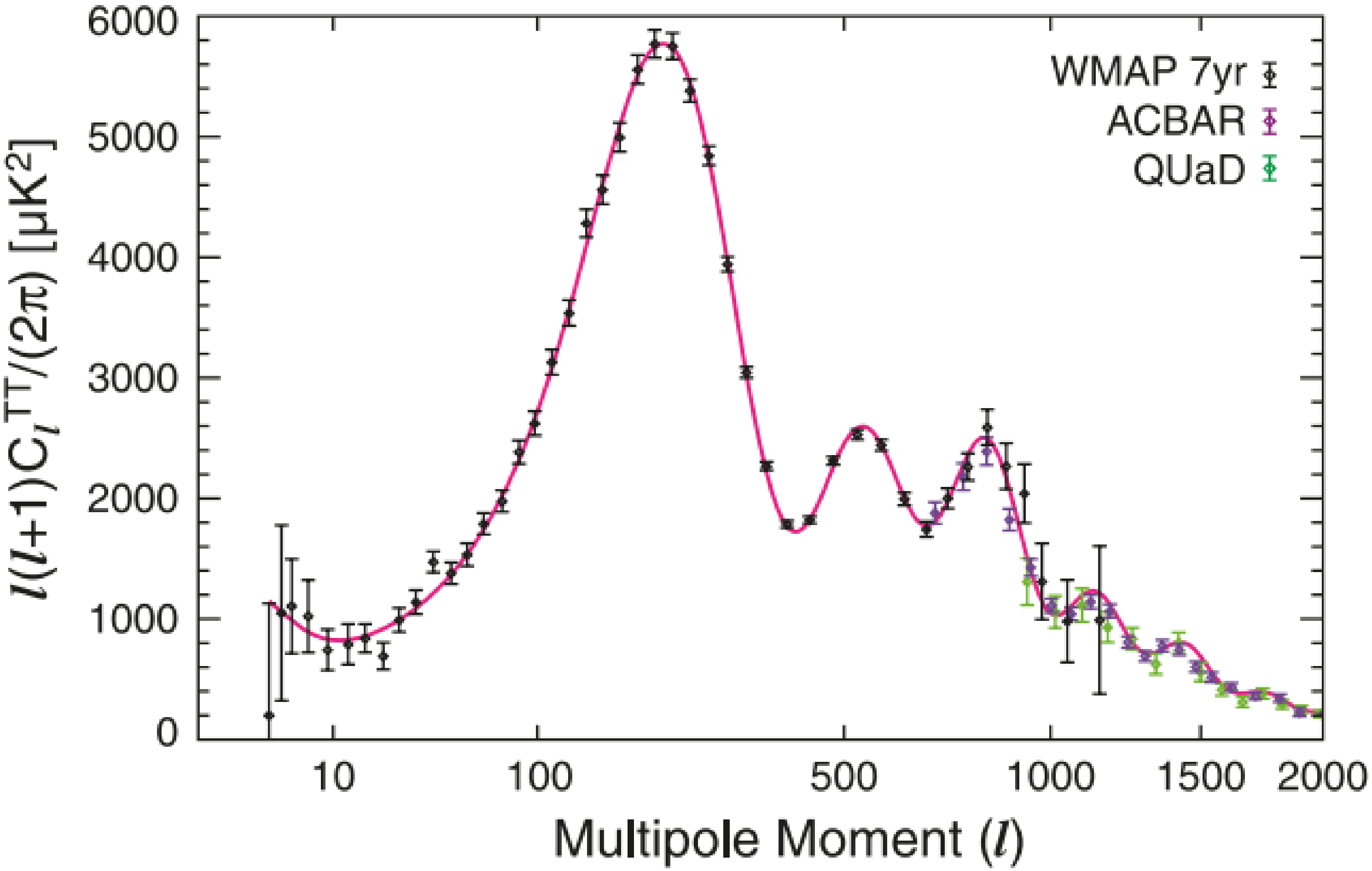


DATA

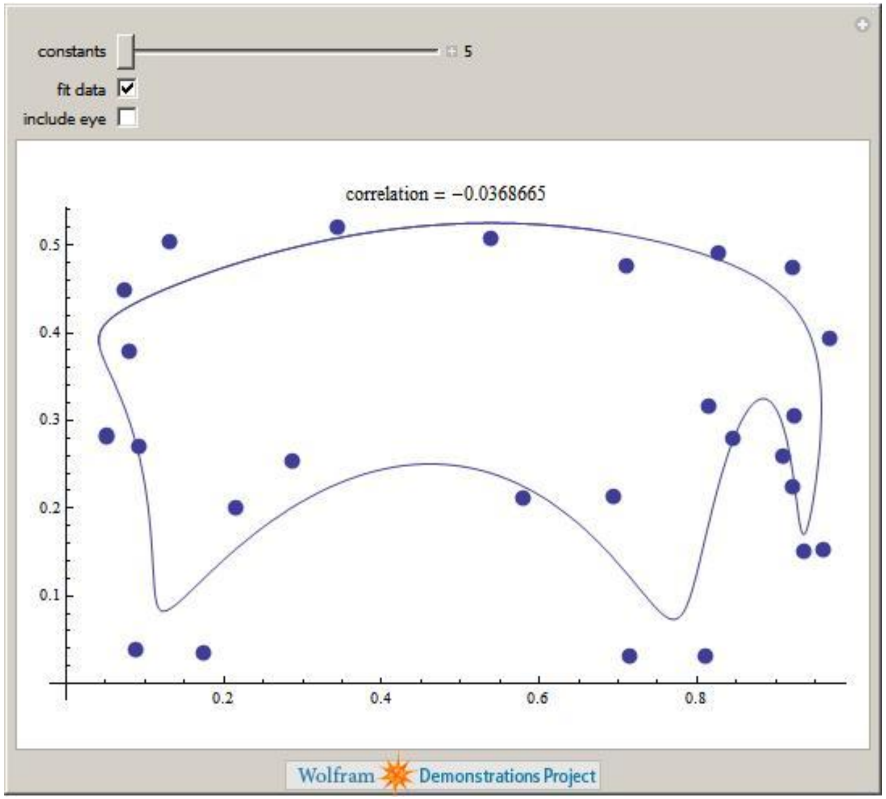


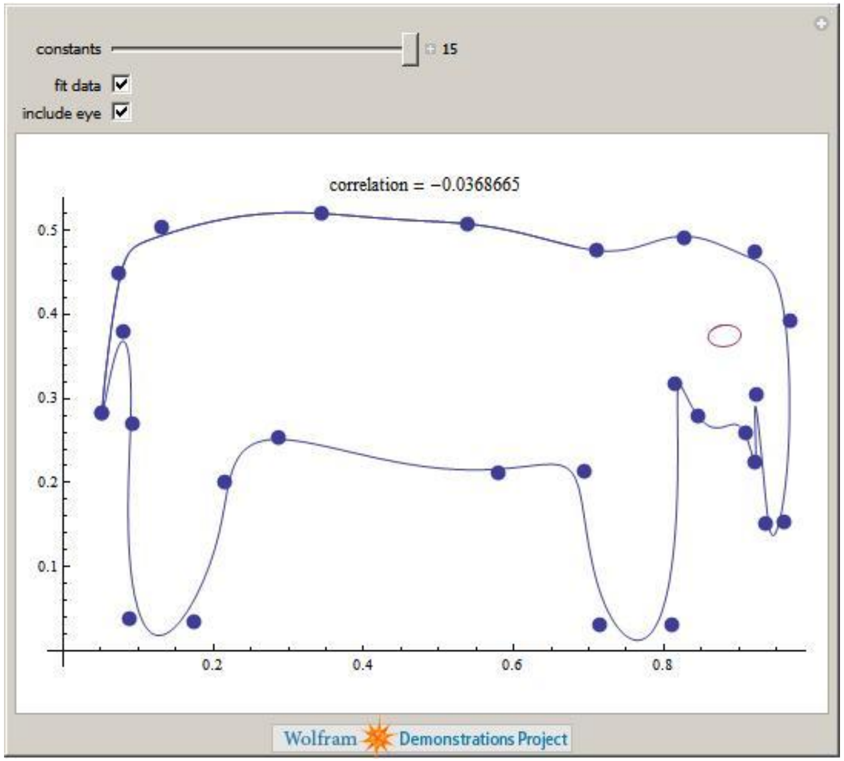
PARAMETER ESTIMATES

Too many parameters ?



Enrico Fermi:"I remember my friend Johnny von Neumann used to say, 'with four parameters I can fit an elephant and with five I can make him wiggle his trunk.'"



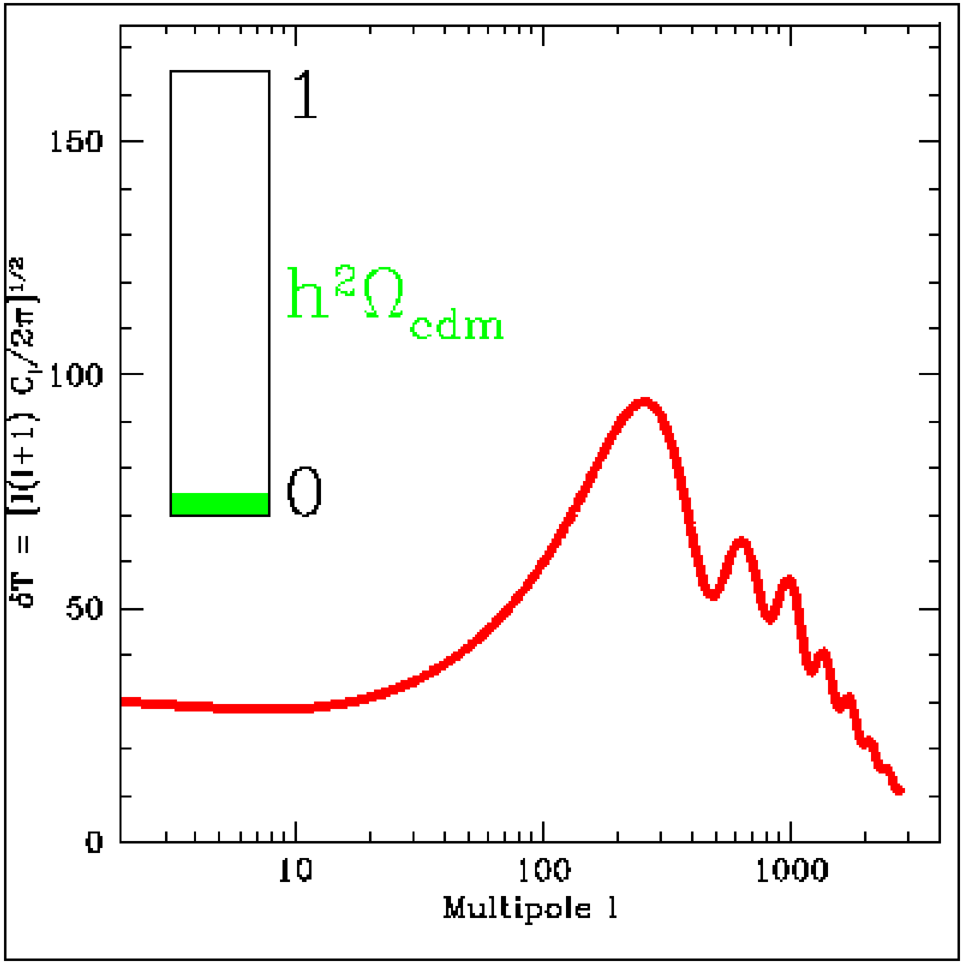


CMB Parameters

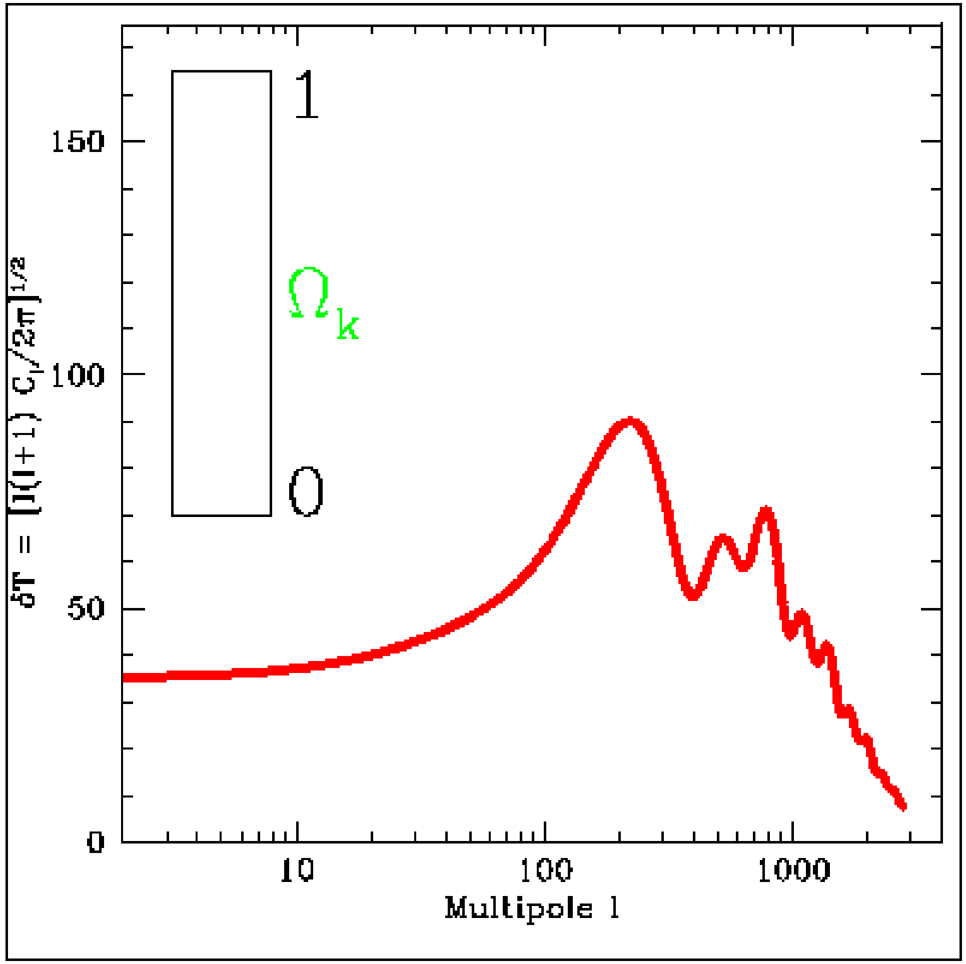
- Baryon Density $\Omega_b h^2$
- CDM Density $\Omega_{CDM} h^2$
- Distance to the LSS, «Shift Parameter» :

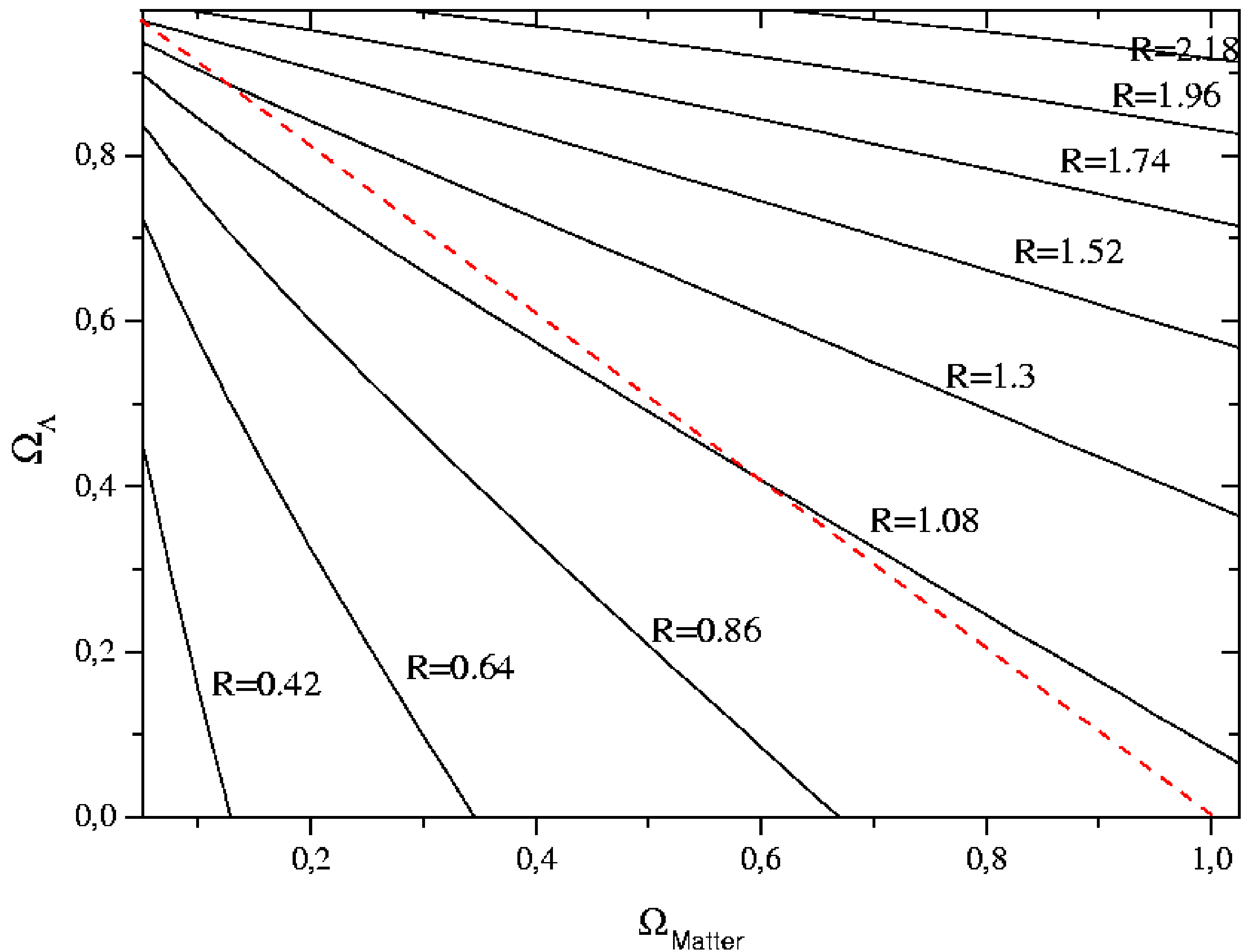
$$R = \sqrt{\frac{\Omega_M h^2}{|\Omega_k| h^2}} \chi(y)$$

$$\chi(y) = \begin{pmatrix} \sin y, k < 0 \\ y, k = 0 \\ \sinh y, k > 0 \end{pmatrix} \qquad y = \sqrt{|\Omega_K|} \int_0^{z_{dec}} \frac{dz}{\sqrt{\Omega_m (1+z)^3 + \Omega_K (1+z)^2 + \Omega_\Lambda}}$$



1
0
$h^2\Omega_{cdm}$
150
100
50
0
10
100
1000
Multipole l
$\delta T = [l(l+1)\ C_l/2\pi]^{1/2}$

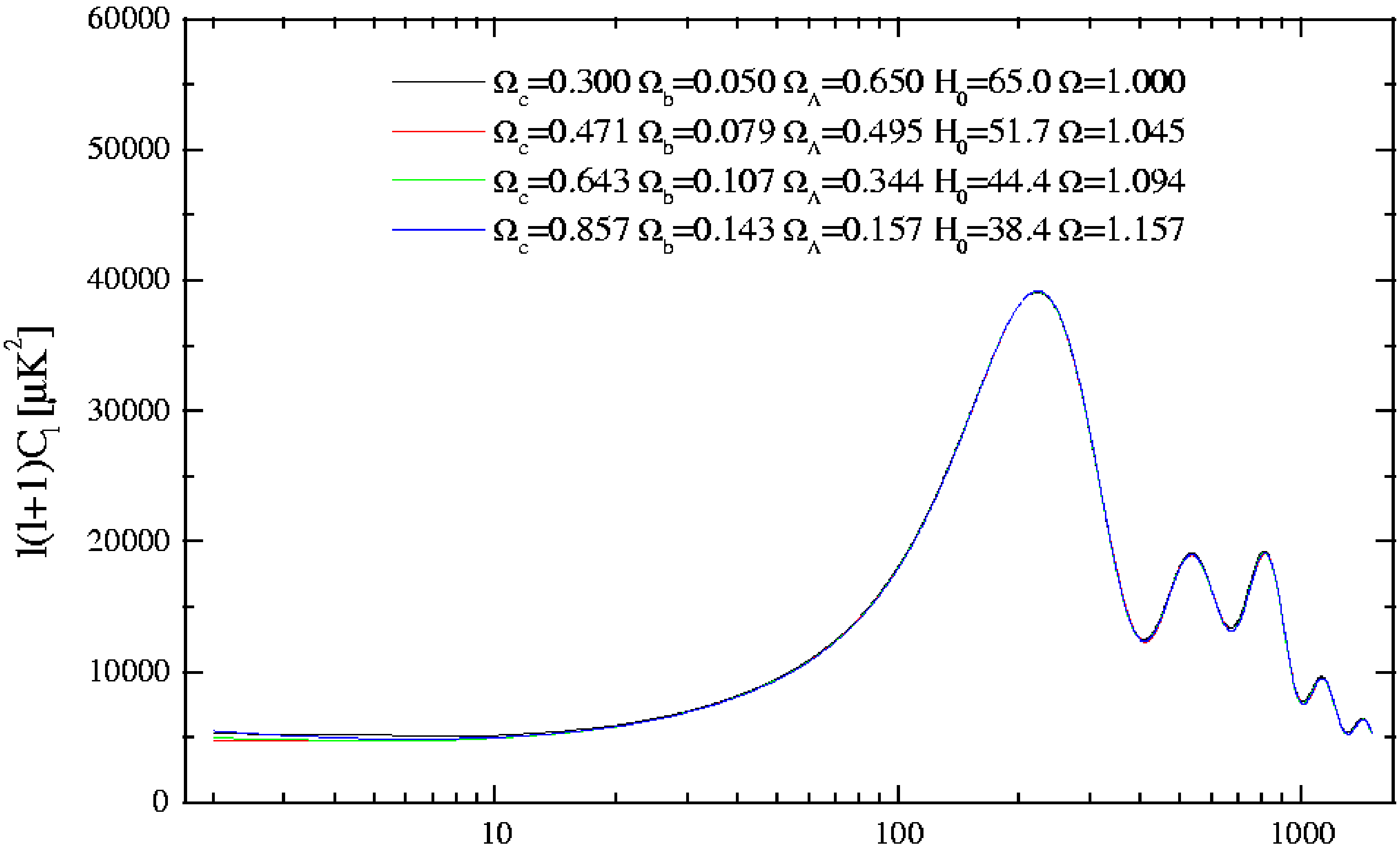

δT = [l(l+1) C_l/2π]^{1/2}
150
100
50
0
10
100
1000
Multipole l
1
Ω_k
0



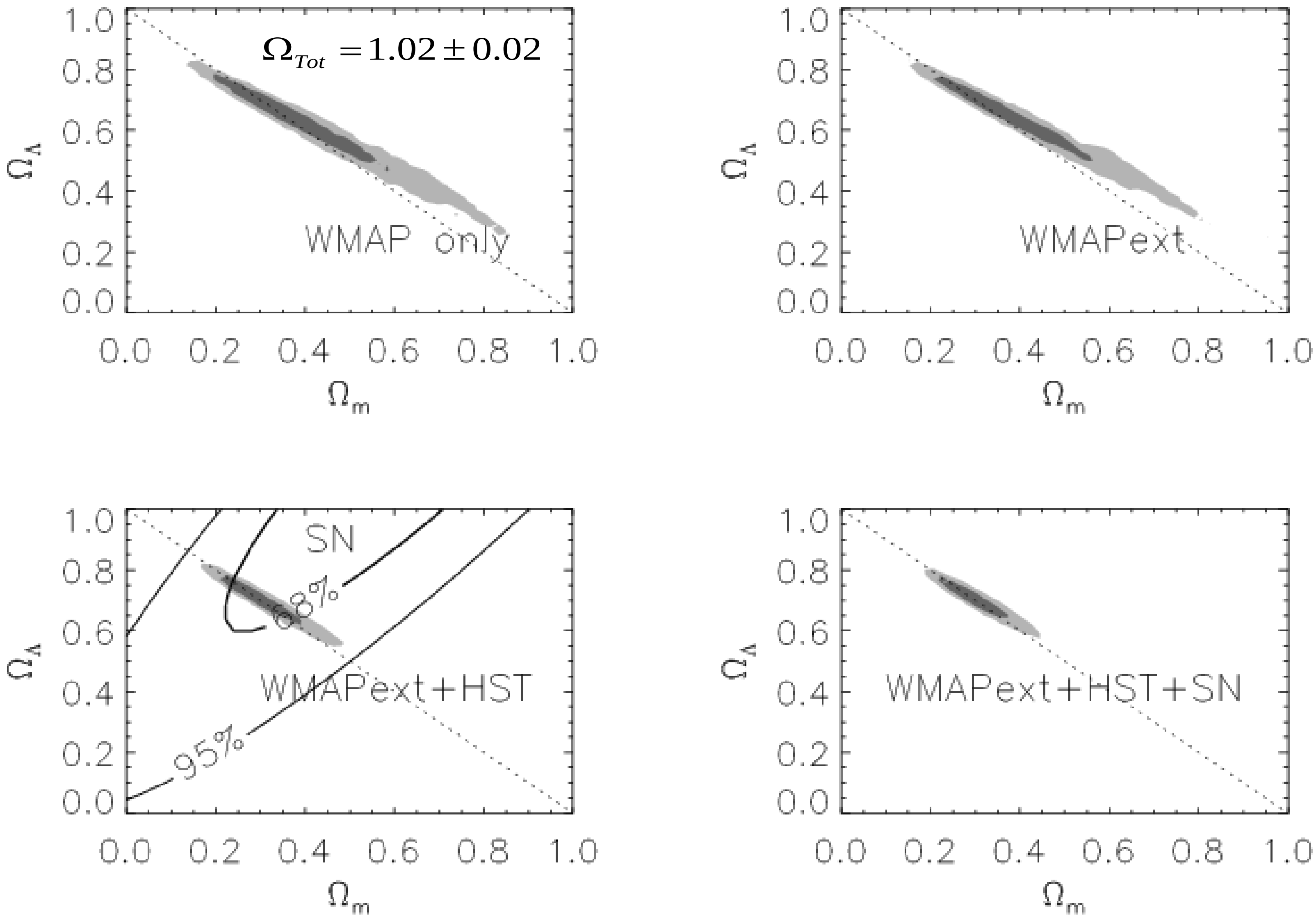
R=2.18
R=1.96
R=1.74
R=1.52
R=1.3
R=1.08
R=0.86
R=0.64
R=0.42
0,0
0,2
0,4
0,6
0,8
1,0
Ω_Λ
Ω_{Matter}

Geometrical degeneracy

See, e.g. Efstathiou and Bond 1998



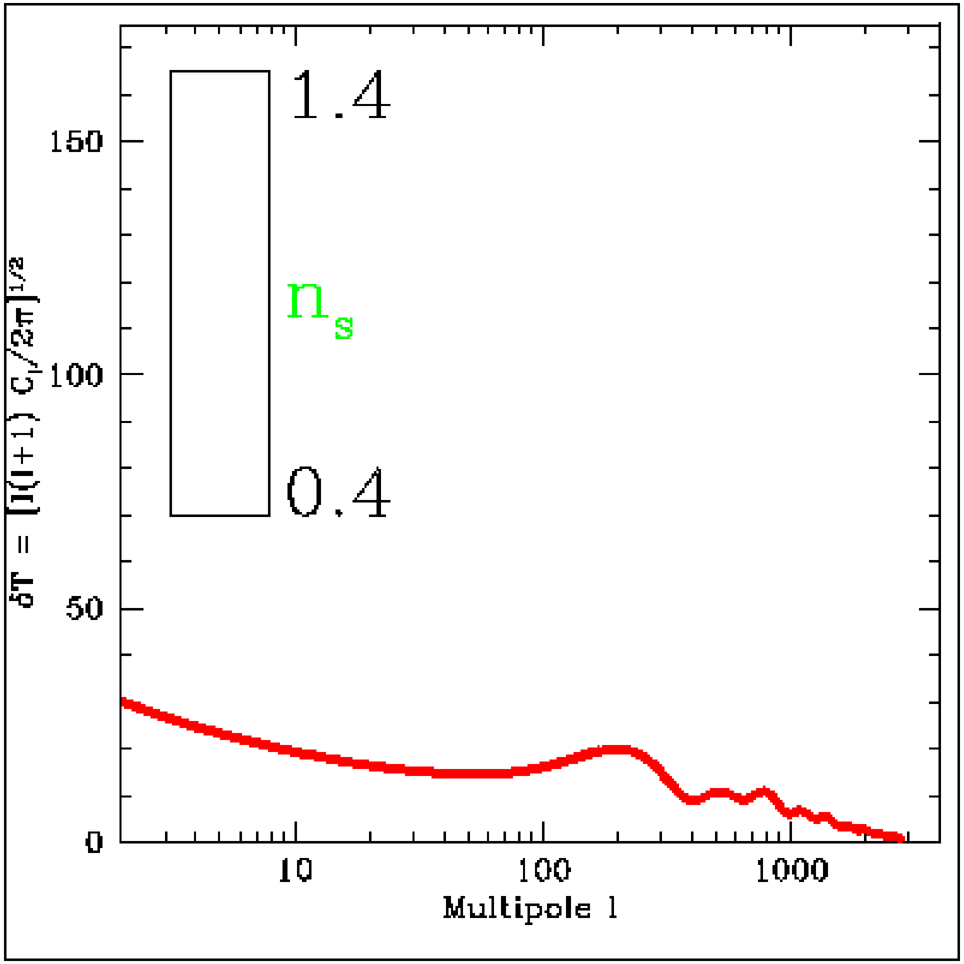
$$\Omega_b h^2, \Omega_m h^{2^1}, R = const$$



$\Omega_{Tot} = 1.02 \pm 0.02$
WMAP only
WMAPext
SN
68%
95%
WMAPext+HST
WMAPext+HST+SN
Ω_Λ
Ω_m
0.0
0.2
0.4
0.6
0.8
1.0

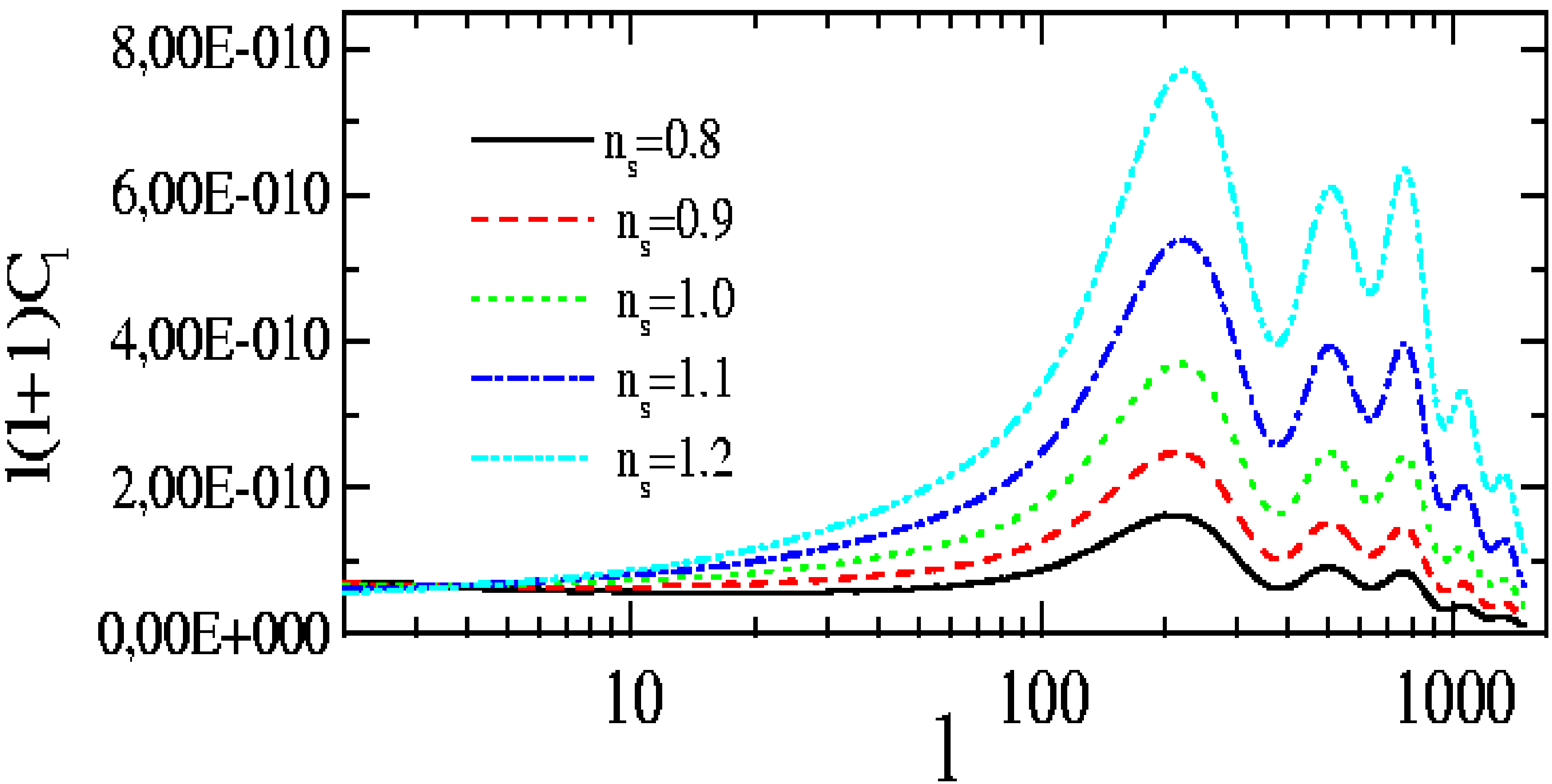
The «misleading» suggestion that WMAP data is preferring closed universes motivated theories for «finite universes»…

Inflationary parameters

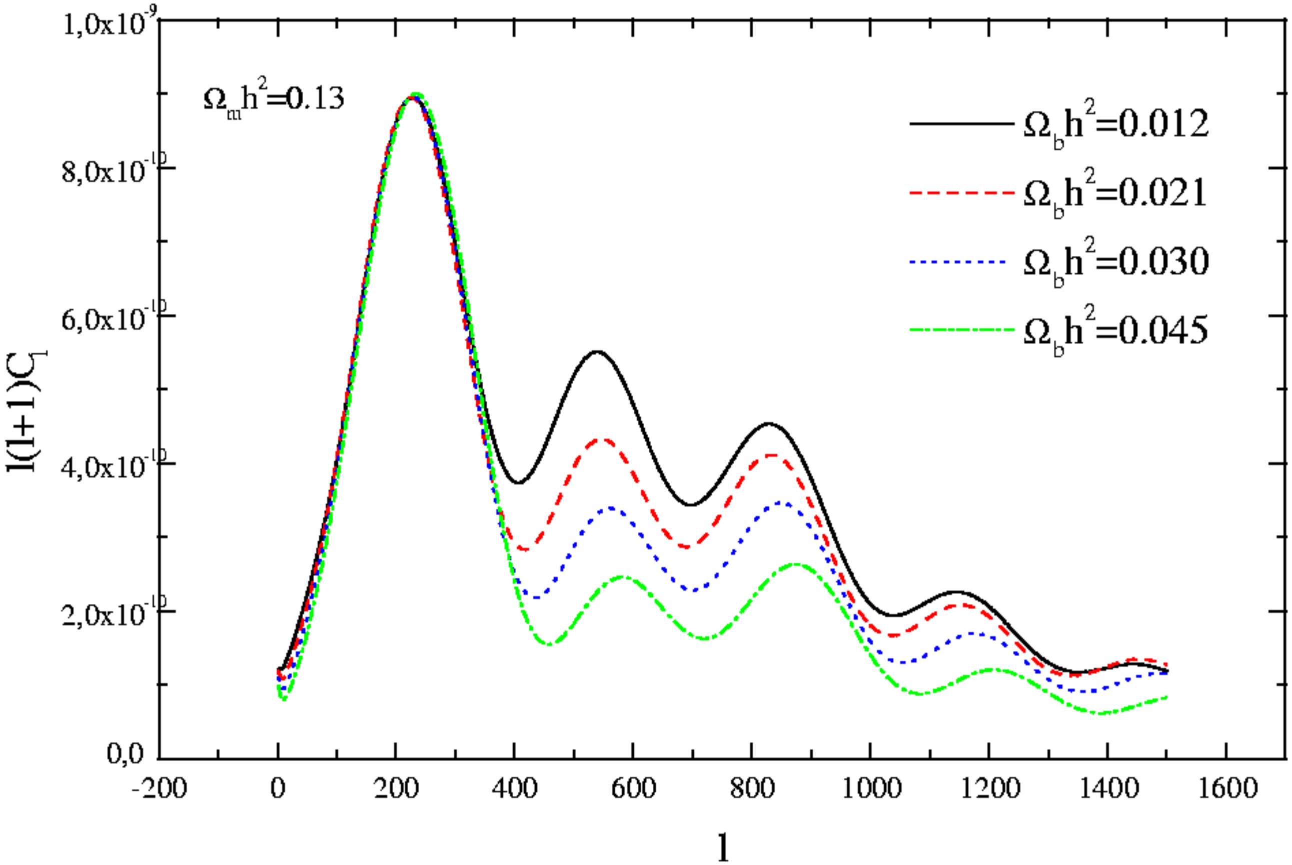


Inflationary parameters

$$n_S \downarrow, \Delta P_{12} \uparrow, \Delta P_{32} \downarrow$$

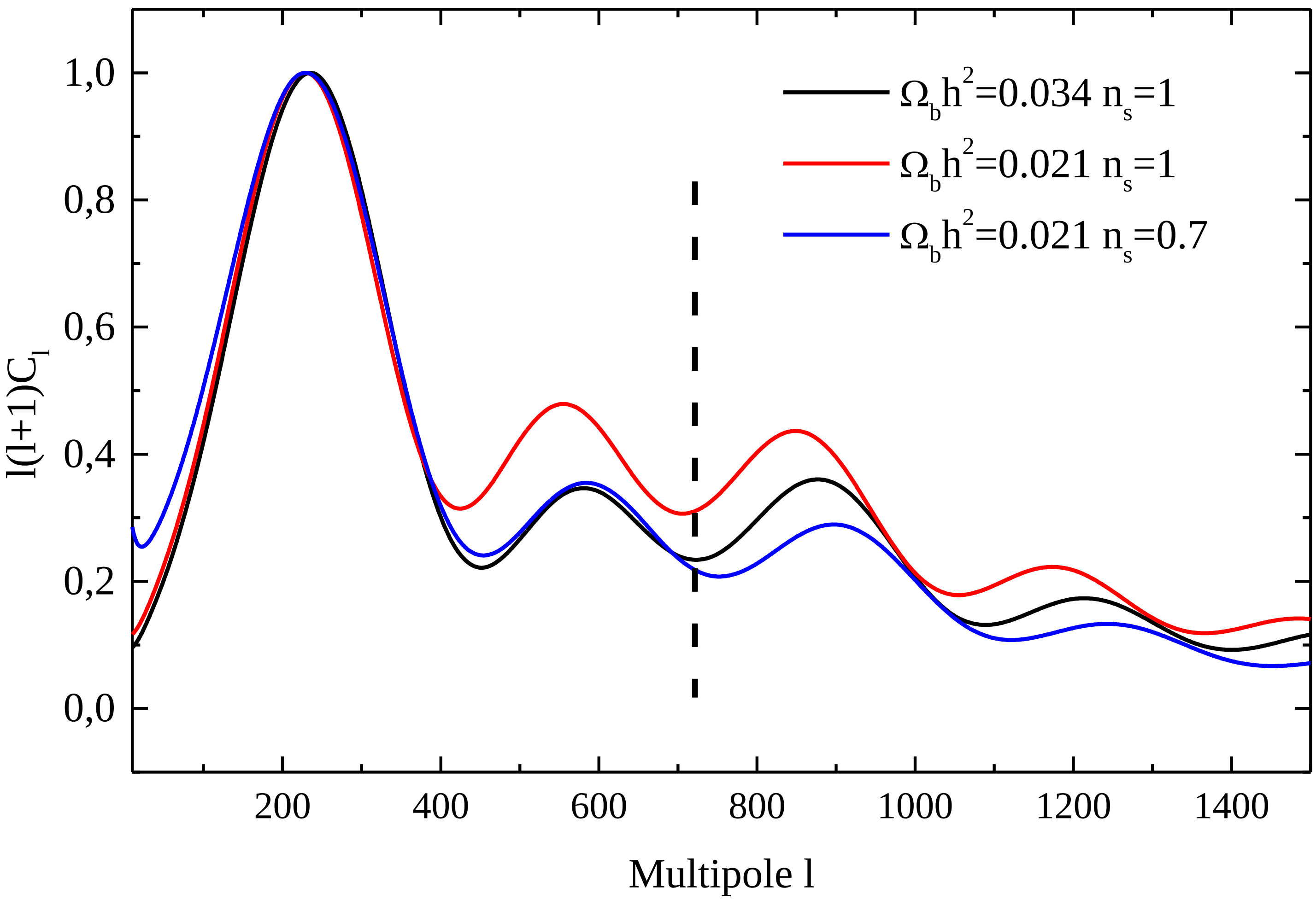


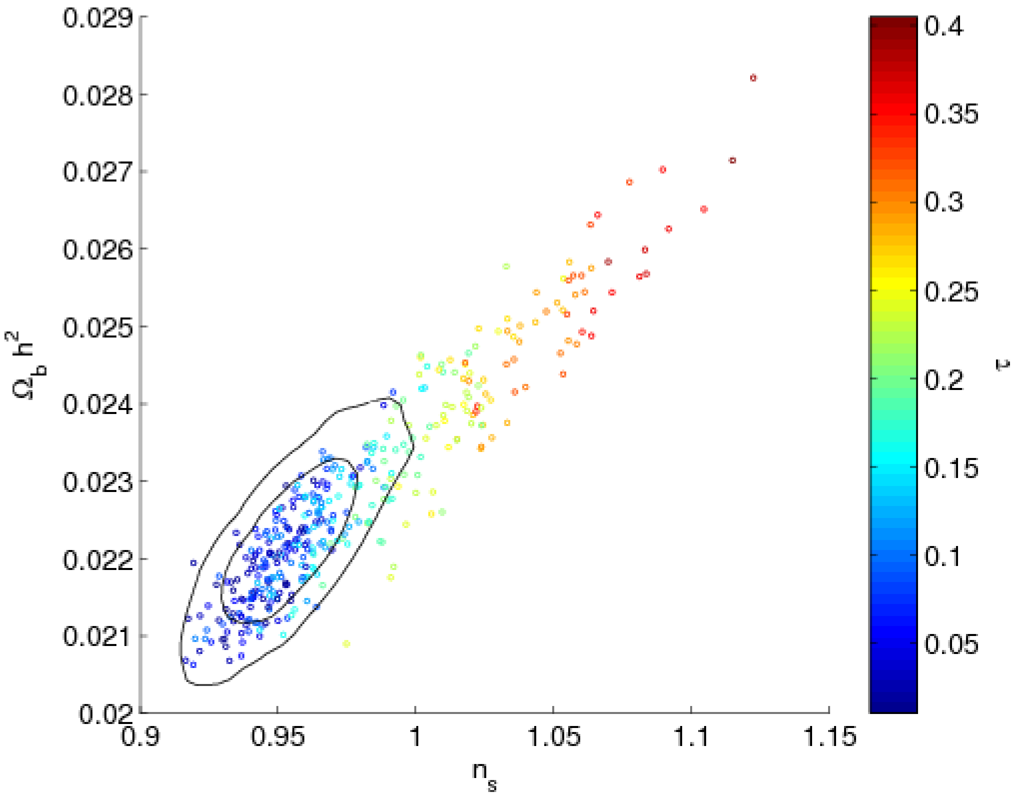
Baryonic Abundance



$$\Omega_B h^2 \uparrow, \Delta P_{12} \uparrow, \Delta P_{32} \uparrow$$

Up to the 2nd peak n and the baryon density are degenerate.





0.029
0.028
0.027
0.026
0.025
0.024
0.023
0.022
0.021
0.02
$\Omega_b h^2$
0.9
0.95
1
1.05
1.1
1.15
n_s
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
τ

The CMB Angular Power Spectrum

Decompose temperature anisotropies in spherical harmonics:

$$\frac{\Delta T}{T}(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

Gaussian fluctuations statistically isotropic imply:

$$\langle a_{lm} a^{*}_{l'm'} \rangle = C_l \ \delta_{ll'} \delta_{mm'}$$

Estimator

$$\hat{C}_l = \sum_m \frac{|a_{lm}|^2}{(2l+1)}$$

has mean C_l and cosmic variance

$$Var(\hat{C}_l) = \frac{2}{2l+1} C_l$$