Report on Nice Oscillation Code ESTA: Frequencies comparison

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Plan

- ⋆ Presentation and properties of NOC
- ⋆ Global and seismic properties of model Task2/Step1
- ⋆ NOC Internal accuracy tests – Task2/Step1 & Sun
- ⋆ Conclusions

Properties of the Nice Oscillation Code

- Variables from Unno et al.:

$$\frac{\xi_r}{r}, \frac{p'}{\rho g r}, \frac{\phi'}{g r}, \frac{1}{g}\frac{d\phi'}{dr}$$

- Difference scheme of second order

Richardson extrapolation (Shibahashi & Osaki 1981):

$$\nu_{Ri}(N) = \frac{1}{3}\left(4\nu(N) - \nu(N/2)\right)$$

- Internal check of accuracy
  - Comparison of ν and ν^{var}:
    check of computation for both model and oscillations

$$\left(\nu^{var}\right)^2 = \frac{\int \xi^* \mathcal{L}(\xi) \rho dv}{\int \xi^* \xi \rho dv}$$

  - Comparison of ν and ν_{Ri}
  - Effects of number N and distribution of mesh points?

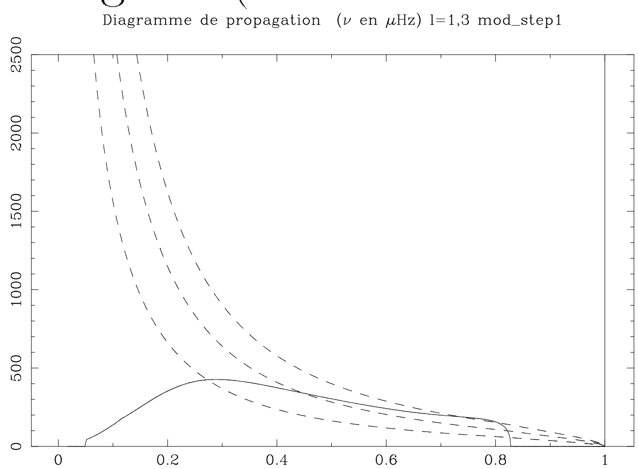
- Many comparisons in solar case

Properties of the model

- Global quantities:

$1.2M_\odot$, X_c=0.69 $r_{cc} \sim 0.05 R_\star$ $r_{ZC} \sim 0.83 R_\star$

- Propagation diagram (acoustic cut-off $\nu \sim 4373\mu$Hz)

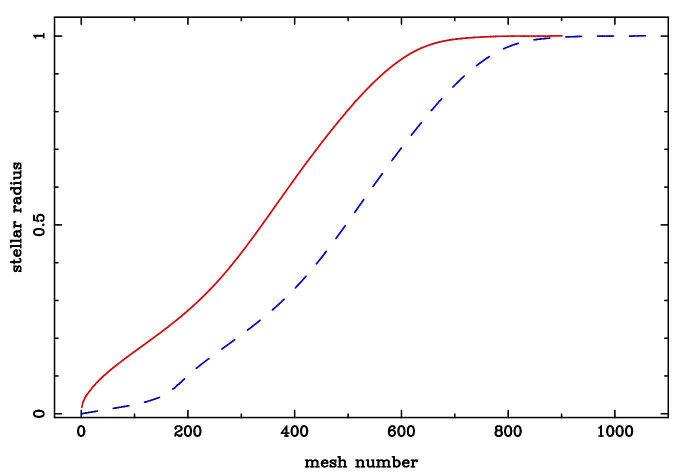


- Large spacing of p modes $\sim 120\mu$Hz.

ν from 100 to 4000μHz:

pressure modes n=1 to 30 for $\ell = 0, 1, 2\ 3$
+ gravity modes g2 g1 for $\ell = 1$
+ gravity modes g5 to g1 and f mode $\ell = 2$
+ gravity modes g7 to g1 and f mode for $\ell = 3$

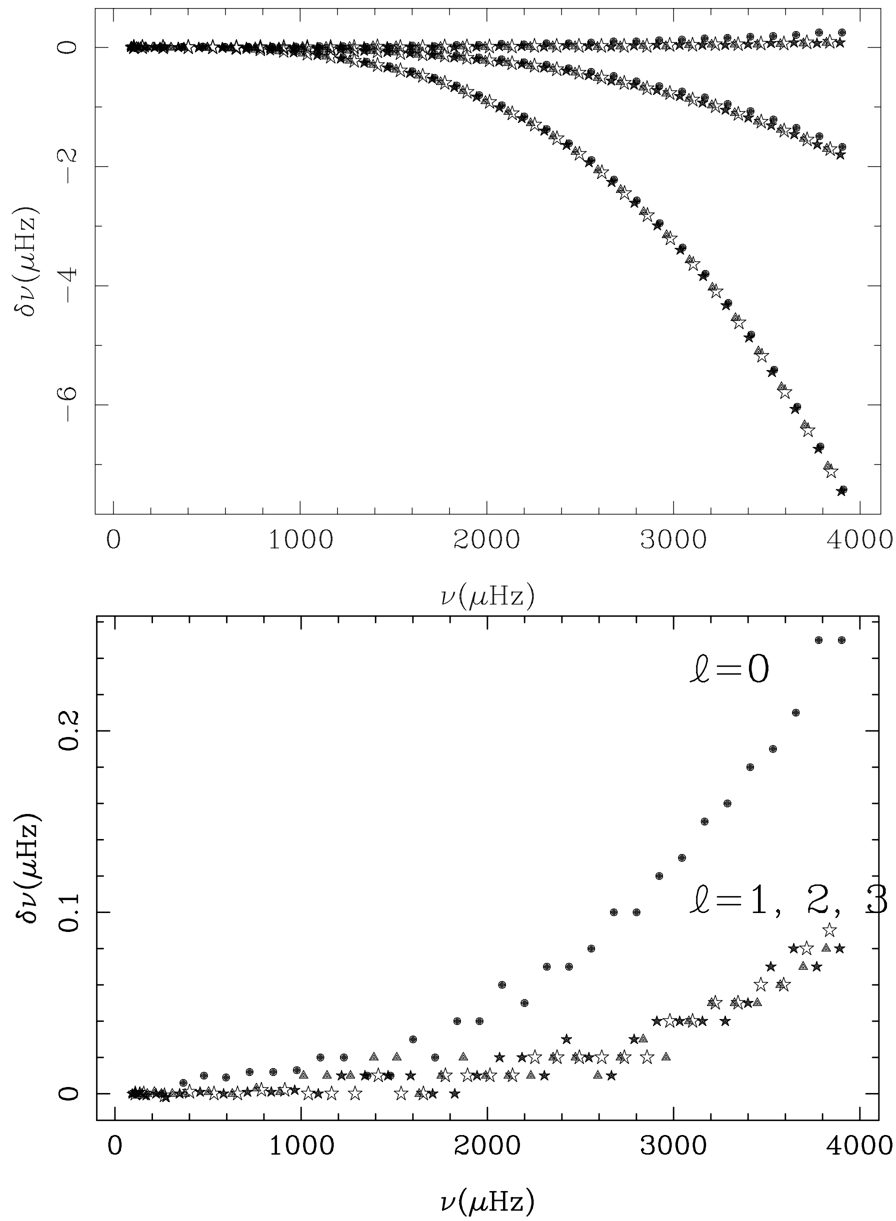
- Distribution of the N=902 mesh points



initial distrib. (red line) /adding points at center (blue line)

Eigenfrequency accuracy:

1. Effect of number of mesh points - Task2 - Step1



$\nu_{Ri}(N)$ - $\nu_{Ri}(2N)$

$\nu_{Ri}(N)$ - $\nu(2N)$

$\nu_{Ri}(N)$ - $\nu(N)$

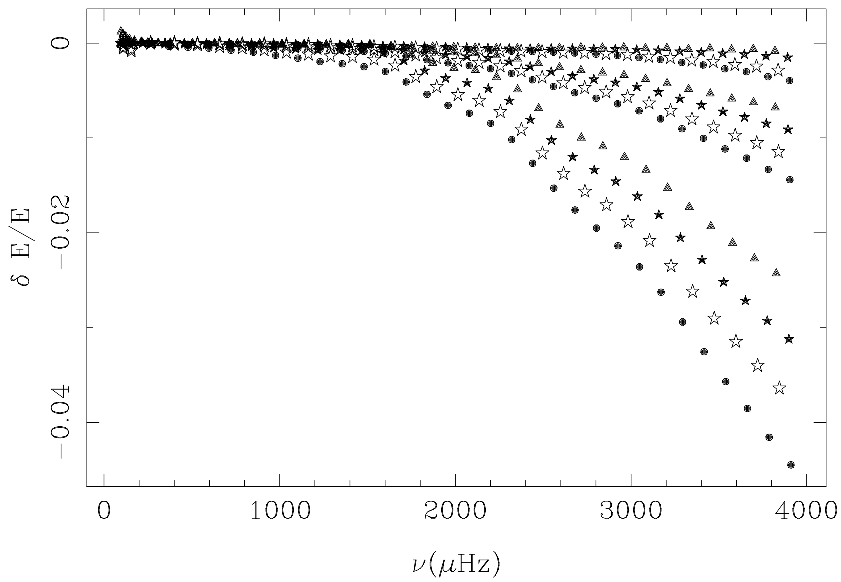
- large variation of the computed frequency with N
- $\nu_{Ri}(N)$ - $\nu_{Ri}(2N) \leq 0.2\mu$Hz small but still significant hence N $\sim$ 900 is too small
- different behavior of $\ell = 0$: mesh distribution?

Note that $\nu(2N)$ is obtained by interpolation of the model

Eigenfunction accuracy measured by mode energy

1. Effect of number of mesh points - Task2 - Step1

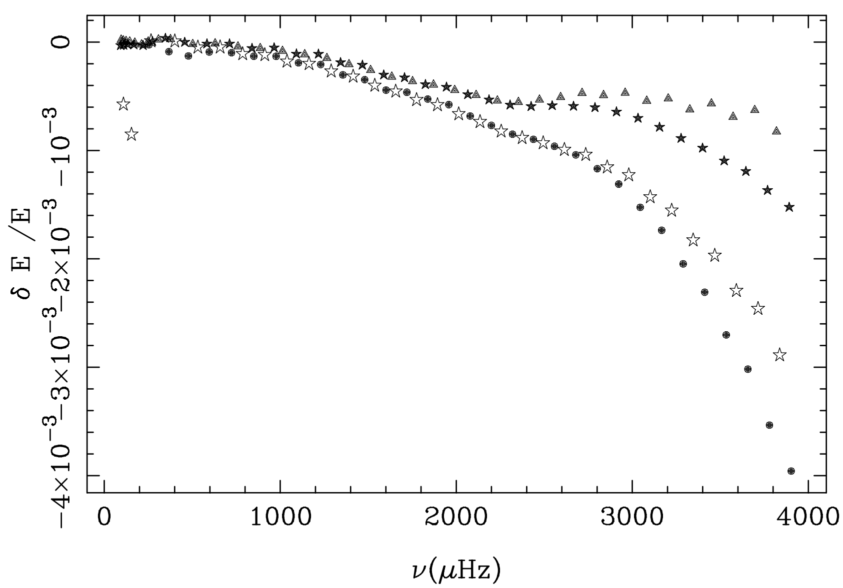
Comparison of $E(N)$ $E(2N)$ $E_{Ri}(N)$ $E_{Ri}(2N)$



$(E_{Ri}(N) - E_{Ri}(2N))/E$

$(E_{Ri}(N) - E(2N))/E$

$(E_{Ri}(N) - E(N))/E$



$(E_{Ri}(N) - E_{Ri}(2N))/E$

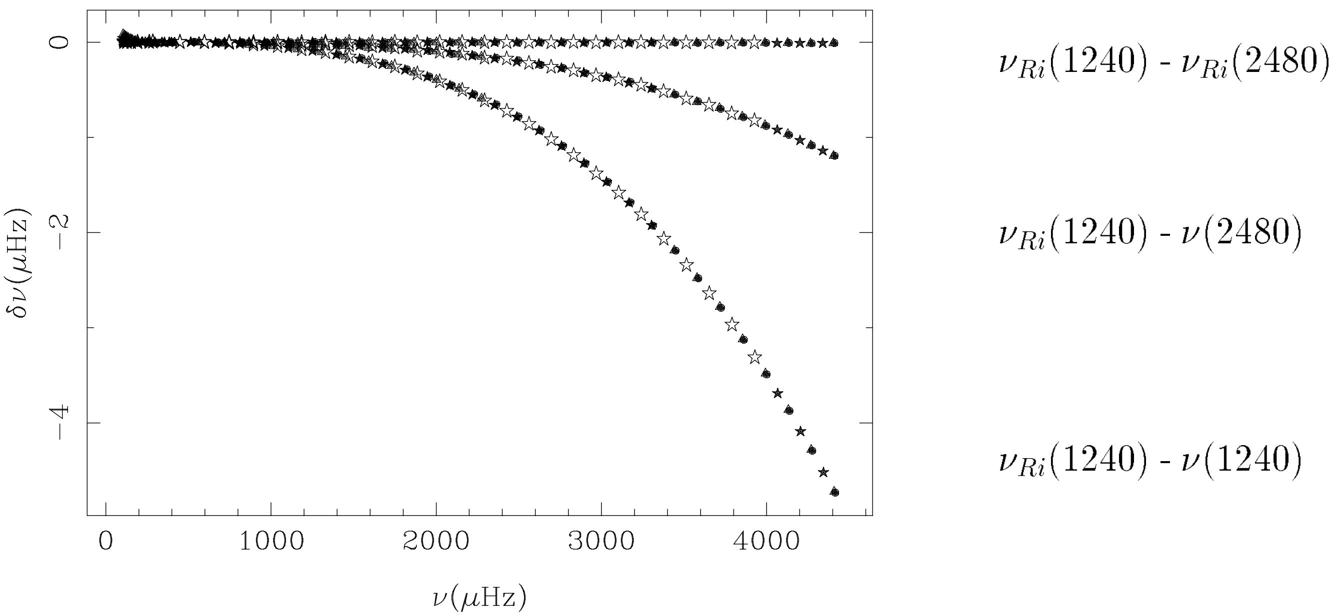
Same conclusions as for the frequency:

- large variation of the mode energy with N
- N $\sim$ 900 too small

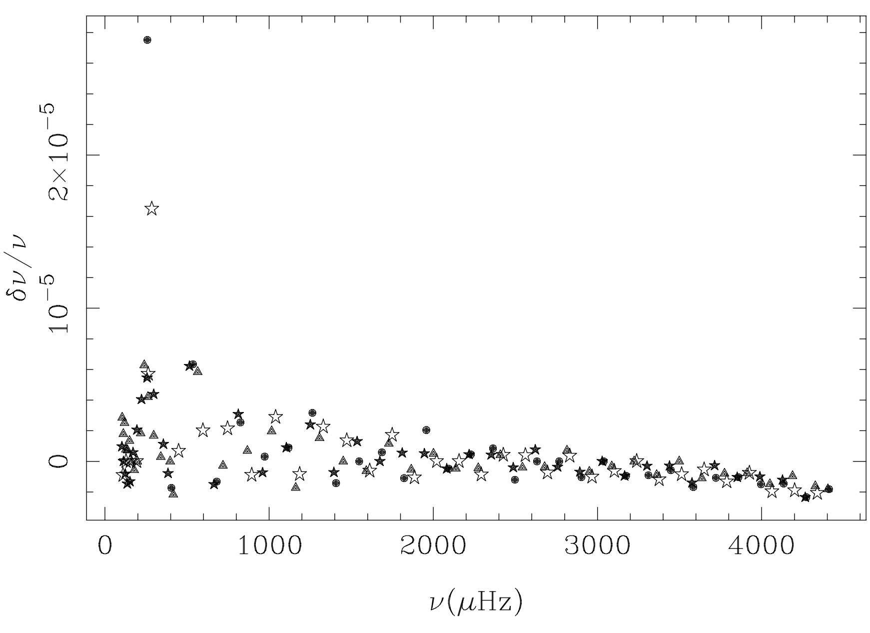
Symbols: $\ell = 0$ circle, 1 open star, 2 full star, 3 triangle

Eigenfrequency accuracy

2. Effect of number of mesh points - $\odot$ model S -N=2480



- $\nu_{Ri}(1240)$ - $\nu_{Ri}(2480)$ smaller than 0.01μHz



- Better accuracy that for case Task2-Step1:

N$\sim$2000 optimum (at least for second order code)

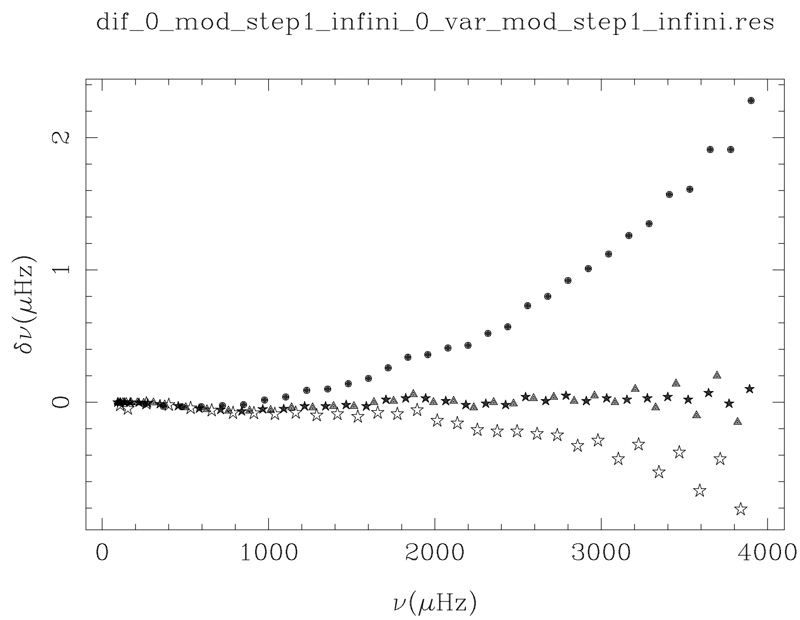
– no need of interpolation

– more mesh points towards the center of the model

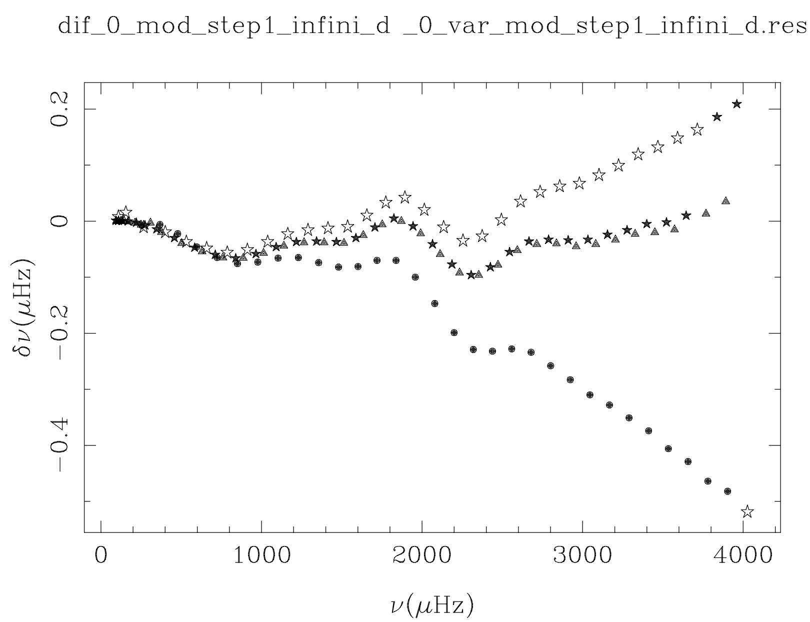
Internal consistency – Task2-Step1

Direct eigenfrequency/ its "variational" expression

$$\nu_{Ri}(N) - \nu_{Ri}^{var}(N)$$



$$\nu_{Ri}(2N) - \nu_{Ri}^{var}(2N)$$



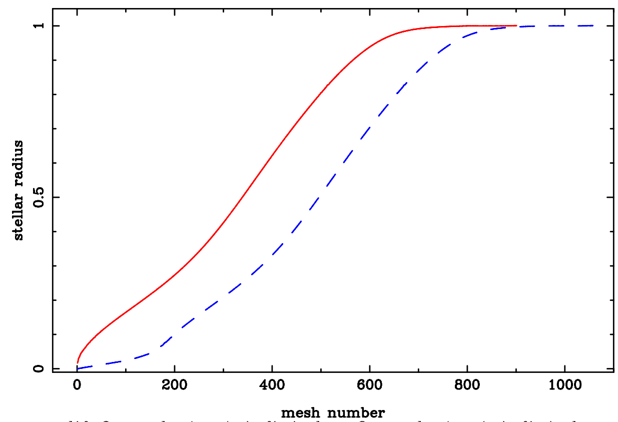
Better internal consistency with 2N

Much improved adding points in central part (see next page)

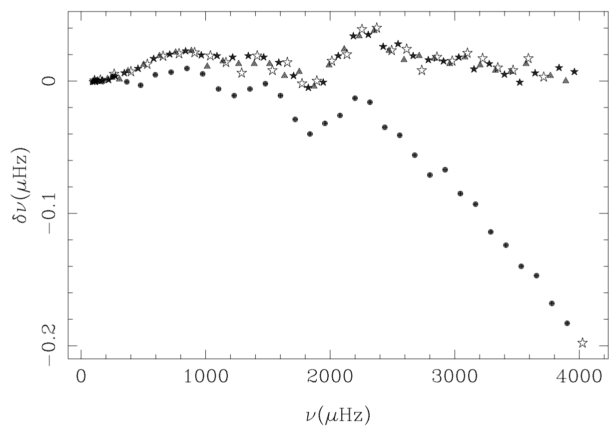
Symbols: ℓ =0 circle, 1 open star, 2 full star, 3 triangle

Sensitivity to mesh points distribution

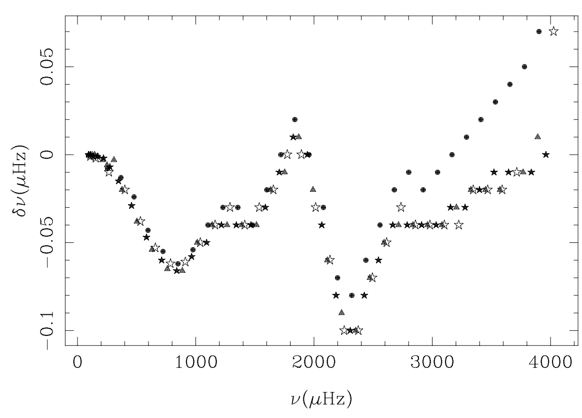
• Adding points in central part to insure enough mesh points over a wavelength new/initial distribution: blue/red line



dif_0_mod_step1_infini_d_a_0_mod_step1_infini_d.res



dif_a_0_mod_step1_infini_d_a_0_var_mod_step1_infini_d.res



Results:

• Main effect: change of frequency for $\ell = 0$ (middle figure)

• Better internal consistency: $\nu_{Ri}(2N) - \nu_{Ri}^{var}(2N)$

(lower figure compared to previous page)

Conclusions

We point out the effect on the numerical frequencies of the number N of mesh points and of their distribution.

For second order scheme code like NOC it is necessary to make a Richardson extrapolation

We emphasize the importance to estimate the internal consistency of the computation by comparing numerical frequency and its "variational expression"

An accurate frequency computation requires
– a large enough number of mesh points ($N \sim 2000$)
– a "good" distribution of mesh points, specially enough points close to the center and in central stellar interior.